SCAD Office
Version 23

VERIFICATION EXAMPLES
Volume 1

Electronic edition

Kyiv • 2024

TABLE OF CONTENTS

Introduction

This document contains verification examples, which are used to assess the reliability of the results obtained in **SCAD**. In the verification examples the numerical results obtained in **SCAD** are compared with known theoretical solutions (exact and approximate) in the fields of statics, dynamics and stability of structures, as well as with experimental data and numerical results obtained with the help of other independent software.

All verification examples are provided with exhaustive initial data with design models, necessary explanations and descriptions of finite element models, as well as the references to publications which are the sources of the adopted target solutions (theoretical and experimental). There are analytical formulas for the calculation of the results based on the theoretical solutions for most verification examples. Results of the calculation in **SCAD** are given in tabular and graphical form.

The differences between the results obtained in **SCAD** and the target results (theoretical and experimental) are given as relative deviations in %, primarily for the extreme values (maximum and minimum) of the target solution, as well as for values that have a significant contribution to the stress-strain state of the structure, which, for example, can be estimated as the ratio of the considered value to the maximum extreme value according to a certain strength theory. The calculation of deviations was not performed in the areas of close proximity to zero solutions and to solutions with singularities, as well as in the areas where there is a distortion of solutions by the accepted boundary conditions.

SCAD. VERIFICATION MATRIX

	Code	Name of the test	Combination of loads and actions	Type of check of the results	Finite elements	Checked parameters	Deviation %
Linear Statics							
1.	SSLL09	Plane Truss Subjected to a Concentrated Force	Concentrated static load	Based on the analytical solution	10	Displacements	0.00
						Forces	0.00
2.	SSLL11	Plane Hinged Bar System Subjected to a Concentrated Force	Concentrated static load	Based on the analytical solution	10	Displacements	0.00
3.	SSLL12	Plane Truss Subjected to Force, Thermal and Kinematic Actions	Concentrated static load, initial displacement, thermal action	Based on the analytical solution	1	Displacements	0.02
						Forces	0.00
4.	T1	Plane Hinged Bar System with Elements of Different Material Subjected to Temperature Variation	Thermal action	Based on the analytical solution	1	Forces	0.00
5.	T2	Plane Hinged Bar System with Elements of the Same Material Subjected to Temperature Variation	Thermal action	Based on the analytical solution	1	Stresses	0.00
6.	CS01	Spatial Hinged Bar System Subjected to a Concentrated Force	Concentrated static load	Based on the analytical solution	4	Forces	0.00
7.	4.1	Cantilever Beam Subjected to a Concentrated Load	Concentrated static load	Based on the analytical solution	5	Displacements	0.00
						Forces	0.00
8.	CS06	Cantilever Beam Subjected to a Concentrated Shear Force	Concentrated static load	Based on the analytical solution	10	Displacements	0.00
					30	Displacements	0.07
9.	4.9	Vertical Cantilever Bar of Square Cross-Section with Longitudinal and Transverse Concentrated Loads at Its Free End	Concentrated static load	Based on the analytical solution	5	Displacements	0.00
						Stresses	0.00
					50	Displacements	0.12
						Stresses	1.67
					37	Displacements	0.06
						Stresses	1.29
10.	4.3	Simply Supported Beam Subjected to a Concentrated Force and Uniformly Distributed Pressure	Concentrated and distributed static loads	Based on the analytical solution	2	Displacements	0.00
						Forces	0.00
11.	4.5	Three-Step Simply Supported Beam Subjected to Concentrated Forces	Concentrated static load	Based on the analytical solution	5	Displacements	0.00
12.	4.4	Doubly Clamped Beam Subjected to a Uniformly Distributed Load	Distributed static load	Based on the analytical solution	2	Displacements	0.00
						Forces	0.00
13.	SSLL01	Doubly Clamped Beam Subjected to a Uniformly Distributed Load, Concentrated	Concentrated and distributed static loads	Based on the analytical solution	10	Displacements	0.05

	Code	Name of the test	Combination of loads and actions	Type of check of the results	Finite elements	Checked parameters	Deviation %
		Longitudinal and Shear Forces and a Bending Moment				Forces	0.00
14.	SSLL03	Two-Span Simply Supported Beam with an Intermediate Compliant Support Subjected to Concentrated Shear Forces Applied in the Middle of the Spans	Concentrated static load	Based on the analytical solution	10, 51	Displacements	0.00
						Forces	0.00
15.	SSLL15	Beam on the Elastic Horizontal Subgrade Subjected to Concentrated Vertical Forces	Concentrated static load	Based on the analytical solution	3	Displacements	0.06
						Forces	0.00
					3, 51	Displacements	1.63
						Forces	0.28
16.	SSLL16	Simply Supported Beam on the Elastic Horizontal Subgrade Subjected to a Vertical Uniformly Distributed Load, Concentrated Vertical Force and Bending Moment	Concentrated and distributed static loads	Based on the analytical solution	3	Displacements	0.00
						Forces	0.00
					3, 51	Displacements	0.00
						Forces	0.08
17.	CS09	Doubly Clamped Beam Subjected to the Transverse Displacement of One of its Ends	Initial displacement	Based on the analytical solution	2	Forces	0.00
18.	B1	Plane System of Two Coaxial Bars Subjected to Temperature Variation	Thermal action	Based on the analytical solution	2	Stresses	0.00
19.	4.8	Stress Strain State of a Simply Supported Beam Subjected to Longitudinal-Transverse Bending	Concentrated static load	Based on the analytical solution	2, 51	Displacements	0.11
						Forces	0.03
					2, 51	Displacements	0.07
						Forces	0.01
20.	SSLL10	System of Cross Bars Subjected to a Distributed Load and a Concentrated Force in Their Plane	Concentrated and distributed static loads	Based on the analytical solution	10	Displacements	0.12
						Forces	0.12
21.	SSLL05	Cantilever Frame Subjected to a Concentrated Force	Concentrated static load	Based on the analytical solution	10	Displacements	0.02
						Forces	0.00
22.	SSLL14	Single-Span Simply Supported Plane Frame with a Dual-Pitched Girder Subjected to a Vertical Uniformly Distributed Load,	Concentrated and distributed static loads	Based on the analytical solution	2	Displacements	0.10

	Code	Name of the test	Combination of loads and actions	Type of check of the results	Finite elements	Checked parameters	Deviation %
		Concentrated Vertical and Horizontal Forces and a Bending Moment				Forces	0.00
23.	SSLL04	Spatial Bar System with Elastic Constraints Subjected to a Concentrated Force	Concentrated static load	Based on the analytical solution	10, 51	Displacements	0.01
						Forces	0.01
24.	4.7	Ring Subjected to a Distributed Load Acting in Its Plane	Distributed static load	Based on the analytical solution	10	Displacements	0.00
						Forces	0.86
25.	SSLL08	Simply Supported Semicircular Arch of Constant Cross-Section Subjected to a Concentrated Force Acting in Its Plane	Concentrated static load	Based on the analytical solution	10	Displacements	0.05
26.	4.6	Strain State of a Split Circular Ring Subjected to Two Mutually Perpendicular Forces P_x and P_y, Acting in the Plane of the Ring	Concentrated static load	Based on the analytical solution	5	Displacements	0.00
27.	4.38	Cantilever Curved Beam with a Transverse Concentrated Force at Its Free End	Concentrated static load	Based on the analytical solution	5	Displacements	0.03
					50	Displacements	0.03
					37	Displacements	0.03
28.	SSLL06	Cantilever Circular Bar of Constant Cross-Section with Concentrated Forces and a Moment Acting in Its Plane at Its Free End	Concentrated static load	Based on the analytical solution	10	Displacements	0.07
29.	SSLL07	Cantilever Circular Bar of Constant Cross-Section with a Concentrated Force out of Its Plane at Its Free End	Concentrated static load	Based on the analytical solution	10	Displacements	0.07
						Forces	0.18
30.	SSLL13	Single-Span Beam with a Prestressed Tie Subjected to a Uniformly Distributed Load	Distributed static load, prestressing	Based on the analytical solution	1, 2	Displacements	0.00
						Forces	0.00/
31.	Influen ce Line	Two-Span Single-Storey Frame Subjected to a Constant Transverse Unit Force Moving Along the Girder Spans with a Small Speed. Plotting of Influence Lines of Internal Forces in the Frame Sections	Concentrated static load	Based on the analytical solution	2	Forces	0.69
32.	KSLS0 1	Bending of a Rectangular Deep Beam Rigidly Suspended along the Sides Subjected to a Uniformly Distributed Load Applied to Its Upper Side	Static load distributed along the line	Based on the analytical solution	21	Displacements	4.56

	Code	Name of the test	Combination of loads and actions	Type of check of the results	Finite elements	Checked parameters	Deviation %
33.	4.29	Pure Bending of a Square Plate in the Plane Stress State Clamped on One Side and Simply Supported in the Center of the Opposite Side	Concentrated static load	Based on the analytical solution	30, 2	Stresses	1.69
					30, 100	Stresses	1.69
34.	4.22	Compression and Bending of a Symmetric Wedge by Concentrated Forces Applied to Its Vertex (Michell's Problem)	Concentrated static load	Based on the analytical solution	50, 100	Stresses	0.16
35.	4.23	Bending of a Symmetric Wedge by a Concentrated Moment Applied to Its Vertex (Inglis Problem)	Concentrated static load	Based on the analytical solution	50, 100	Stresses	2.21
36.	4.24	Bending of a Symmetric Wedge by a Uniformly Distributed Load Applied to the Surface of One of the Faces of the Wedge (Levi Problem)	Static load distributed along the line	Based on the analytical solution	50	Stresses	0.89
37.	4.25	Triangular Dam Subjected to Its Self-Weight and Hydrostatic Pressure	Distributed surface static load and static load distributed along the line	Based on the analytical solution	30, 25	Stresses	1.70
38.	4.26	Plane Subjected to a Concentrated Moment and a Concentrated Force	Concentrated static load	Based on the analytical solution	30, 100	Stresses	6.60
39.	4.21	Bending of a Curved Beam of a Narrow Rectangular Cross-Section by a Force Applied to Its Free End (Golovin's Problem)	Concentrated static load	Based on the analytical solution	50, 100	Stresses	1.67
40.	4.27	Unilateral Tension of a Plate with a Small Circular Hole (Kirsch Problem)	Static load distributed along the line	Based on the analytical solution	30, 25	Stresses	5.15
					30, 25	Stresses	1.17
41.	4.14	Stress-Strain State of a Simply Supported Circular Plate Subjected to a Uniformly Distributed Transverse Load	Distributed surface static load	Based on the analytical solution	50, 45	Displacements	0.46
						Forces	2.70
42.	4.15	Stress-Strain State of a Clamped Circular Plate Subjected to a Uniformly Distributed Transverse Load	Distributed surface static load	Based on the analytical solution	50, 45	Displacements	0.59
						Forces	6.48
43.	4.16	Stress-Strain State of a Simply Supported Annular Plate Subjected	Distributed surface static load	Based on the analytical solution	50	Displacements	0.78

	Code	Name of the test	Combination of loads and actions	Type of check of the results	Finite elements	Checked parameters	Deviation %
		to a Uniformly Distributed Transverse Load				Forces	1.72
44.	SSLS01	Rectangular Narrow Cantilever Plate Subjected to a Uniformly Distributed Transverse Load	Distributed surface static load	Based on the analytical solution	11	Displacements	0.00
45.	SSLS27	Torsion of a Rectangular Narrow Cantilever Plate by a Pair of Concentrated Forces	Concentrated static load	Based on the analytical solution	11	Displacements	0.20
46.	4.17	Square Plate Simply Supported along the Perimeter Subjected to a Uniformly Distributed Load	Distributed surface static load	Based on the analytical solution	20	Displacements	0.09
						Forces	0.09
47.	SSLS24	Rectangular Plate Simply Supported along the Perimeter Subjected to a Uniformly Distributed Transverse Load	Distributed surface static load	Based on the analytical solution	20	Displacements	0.38
						Forces	1.57
					20	Displacements	0.18
						Forces	0.60
					20	Displacements	0.00
						Forces	0.64
48.	SSLS26	Rectangular Plate Simply Supported at Three Vertices Subjected to a Concentrated Force and Concentrated Moments out of Its Plane	Concentrated static load	Based on the analytical solution	20	Displacements	0.00
49.	4.19	Stress-Strain State of a Clamped Hexagonal Plate Subjected to a Uniformly Distributed Load	Distributed surface static load	Based on the analytical solution	44, 42	Displacements	0.77
						Forces	0.69
50.	4.20	Clamped Rectangular Plate of Constant Thickness Subjected to Thermal Loading	Temperature gradient across the thickness	Based on the analytical solution	41	Displacements	–
						Forces	0.00
						Stresses	0.00
51.	RMP	Simply Supported Thick Square Plate Subjected to a Uniformly Distributed Transverse Load	Distributed surface static load	Based on the analytical solution	150	Displacements	0.07
					150	Displacements	0.00
					150	Displacements	0.00
52.	4.34	Two-Ribbed Beam Subjected to Uniformly Distributed Loads Applied in the Plane of the Ribs	Static load distributed along the line	Based on the analytical solution	27	Stresses	0.92
53.	4.35	Curved Hollow Section Beam of a Bridge Superstructure Subjected to a Concentrated Force	Concentrated static load	Based on the experiment	150	Displacements	9.9
						Stresses	10,9
54.	4.31	Cylindrical Shell with Simply Supported Edges	Distributed surface static	Based on the analytical	44	Displacements	0.19

	Code	Name of the test	Combination of loads and actions	Type of check of the results	Finite elements	Checked parameters	Deviation %
		Subjected to Uniform Internal Pressure	load	solution		Forces	0.83
55.	4.32	Cylindrical Vertical Tank with a Wall of Constant Thickness with a Flat Bottom Subjected to Internal Fluid Pressure	Distributed surface static load	Based on the analytical solution	44	Displacements	1.47
						Forces	5.73
56.	4.33	Cylindrical Shell with Free Edges at a Temperature Gradient across the Thickness (in the Radial Direction)	Temperature gradient across the thickness	Based on the analytical solution	44	Displacements	6.67
						Stresses	1.04
57.	4.36a	Thick Square Slab Simply Supported along the Sides Subjected to a Transverse Load Distributed over the Upper Face According to the Cosine Law	Distributed surface static load	Based on the analytical solution	36	Displacements	0.3
						Stresses	1.65
58.	4.37	Thick Circular Slab Clamped along the Side Surface Subjected to a Load Uniformly Distributed over the Upper Face	Distributed surface static load	Based on the analytical solution	35, 37	Displacements	7.59
						Stresses	9.12
59.	SSLV01	Cylindrical Body Free from Restraints Subjected to a Longitudinal Load Uniformly Distributed over the Edges	Distributed surface static load	Based on the analytical solution	61	Displacements	0.00
60.	Flate_plate_Circular_column.spr	Square Panel of a Flat Slab Rigidly Connected to a Column of a Circular Cross-Section Subjected to a Uniformly Distributed Transverse Load	Distributed surface static load	Based on the analytical solution	15, 20, 100	Stresses	2.3
61.	Flate_plate_Square_column.spr	Square Panel of a Flat Slab Rigidly Connected to a Column of a Square Cross-Section Subjected to a Uniformly Distributed Transverse Load	Distributed surface static load	Based on the analytical solution	20, 100	Stresses	9.45
62.	Lave.spr	Elastic Half-Space Subjected to a Transverse Load Uniformly Distributed over a Rectangular Surface. Love's Problem.	Distributed surface static load	Based on the analytical solution	37	Displacements	3.2
						Stresses	7.75
Linear Dynamics							
1.	5.11	Plane Truss Subjected to Instantaneous Pulses Concentrated in Non-Supporting Nodes of the Bottom Chord	Concentrated dynamic load	Based on the analytical solution	1	Natural frequencies	5.10
						Displacements	2.46
2.	5.1	Natural Oscillations of a Spatial Pipeline Clamped at the Edges (Hougaard's Problem)	Modal analysis	Based on the analytical solution	5	Natural frequencies	8.26

	Code	Name of the test	Combination of loads and actions	Type of check of the results	Finite elements	Checked parameters	Deviation %
3.	5.12_Sudd_L	Simply Supported Weightless Beam with Two Concentrated Masses and Transverse Sudden Constant Load Applied to One of Them	Concentrated dynamic load	Based on the analytical solution	2	Natural frequencies	0.00
						Displacements	0.09
						Forces	0.91
4.	5.12_Harm_L	Simply Supported Weightless Beam with Two Concentrated Masses and Transverse Harmonic Exciting Force Applied to One of Them	Concentrated dynamic load	Based on the analytical solution	2	Natural frequencies	0.00
						Displacements	6.16
5.	Test 5.12 Harm L Damp	Simply Supported Weightless Beam with Two Concentrated Masses and Transverse Harmonic Exciting Force Applied to One of Them Taking into Account the Energy Dissipation due to Internal Friction	Concentrated dynamic load	Based on the analytical solution	2	Natural frequencies	0.00
						Displacements	6.39
6.	Test 5.13	Simply Supported Beam with a Distributed Mass Subjected to a Transverse Harmonic Exciting Force Applied in the Middle of the Span	Concentrated dynamic load	Based on the analytical solution	3	Natural frequencies	0.73
						Displacements	3.68
7.	Test DIN B ML	Simply Supported Beam with a Distributed Mass Subjected to a Constant Shear Force Moving along the Span of the Beam at a Constant Speed	Concentrated dynamic load	Based on the analytical solution	3	Natural frequencies	0.73
						Displacements	0.18
8.	Test DIN B IL	Simply Supported Beam with a Distributed Mass Subjected to a Uniformly Distributed Instantaneous Pulse (Impact of a Beam with Immovable Supports)	Distributed dynamic load	Based on the analytical solution	3	Natural frequencies	0.73
						Displacements	0.36
						Forces	14.06
9.	Test DIN B SL	Simply Supported Beam with a Distributed Mass Subjected to a Kinematic Excitation of Supports (Seismic Action)	Dynamic displacement	Based on the analytical solution	3	Natural frequencies	0.05
						Displacements	0.80
						Forces	0.73
10.	5.14	Cantilever Weightless Column with a Concentrated Mass at the Free End Subjected to a Horizontal Kinematic	Dynamic displacement	Based on the analytical solution	5	Natural frequencies	0.00

	Code	Name of the test	Combination of loads and actions	Type of check of the results	Finite elements	Checked parameters	Deviation %
		Displacement of a Support (Seismogram Based Analysis)				Displacements	0.39
11.	5.7	Natural Oscillations of a Simply Supported Circular Plate	Modal analysis	Based on the analytical solution	20, 15	Natural frequencies	1.57
12.	5.6	Natural Oscillations of a Clamped Circular Plate	Modal analysis	Based on the analytical solution	20, 15	Natural frequencies	1.89
13.	5.5	Natural Oscillations of a Square Cantilever Plate	Modal analysis	Based on the analytical solution	20	Natural frequencies	1.62
14.	5.2	Natural Oscillations of a Simply Supported Square Plate	Modal analysis	Based on the analytical solution	20	Natural frequencies	0.82
15.	5.3	Natural Oscillations of a Simply Supported Rectangular Plate	Modal analysis	Based on the analytical solution	20	Natural frequencies	0.50
16.	5.4	Natural Oscillations of a Clamped Square Plate	Modal analysis	Based on the analytical solution	20	Natural frequencies	0.81
17.	Test 5.8 S	Natural Oscillations of a Simply Supported Circular Cylindrical Shell	Modal analysis	Based on the analytical solution	50	Natural frequencies	0.86
18.	Test 5.8 C	Natural Oscillations of a Clamped Circular Cylindrical Shell	Modal analysis	Based on the analytical solution	50	Natural frequencies	2.38
19.	Test 5.9	Natural Oscillations of a Cantilever Open Cylindrical Shell	Modal analysis	Based on the experiment	50	Natural frequencies	5.02
20.	DI_F.SPR	Plane Frame Subjected to a Uniformly Distributed Instantaneous Pulse	Pulse	Based on the analytical solution	2	Natural frequencies	0.00
						Displacements	0.00
						Forces	1.25
21.	LinSpectral	Seismic Response of a Beam according to the Linear Spectral Theory	Dynamic displacement	Based on the analytical solution	3	Natural frequencies	0,06
						Displacements	1.75
						Stresses	1.85
22.		Non-uniform Damping. Return to the Static Equilibrium Position					
23.		Non-uniform Damping					
		Linear Stability					
1.	CB01	Stability of a Simply Supported Beam Subjected to a Concentrated Longitudinal Force	Concentrated static load	Based on the analytical solution	10	Critical force	0.00
2.	CB02	Stability of a Clamped Beam Subjected to a Concentrated Longitudinal Force	Concentrated static load	Based on the analytical solution	10	Critical force	0.00
3.	Leg of varying section	Stability of a Cantilever Column with a Step Change in Cross-Section Subjected to Longitudinal Compressive Forces Applied to the Intermediate and End Sections	Concentrated static load	Based on the analytical solution	2	Critical force	0.00
						Unsupported length of columns	0.00
					150	Critical force	2.93
						Unsupported length of columns	—
4.	Frame 5a1	Stability of the System of Three Equally Loaded	Concentrated static load	Based on the analytical	2, 100	Critical force	0.01

	Code	Name of the test	Combination of loads and actions	Type of check of the results	Finite elements	Checked parameters	Deviation %
		Columns of Different Rigidity Hingedly Interconnected by Girders		solution		Unsupported length of columns	0.00
5.	Frame 5a2	Stability of the System of Three Differently Loaded Columns of the Same Rigidity Hingedly Interconnected by Girders	Concentrated static load	Based on the analytical solution	2, 100	Critical force	0.00
						Unsupported length of columns	0.00
6.	Frame 5б	Stability of the System of Three Differently Loaded Columns of Different Rigidity Interconnected by Girders Infinitely Rigid in Bending	Concentrated static load	Based on the analytical solution	2, 100	Critical force	0.00
						Unsupported length of columns	0.00
7.	Frame leg hard	Stability of the Frame of Two Simply Supported Equally Loaded Rigid Columns Rigidly Interconnected by a Girder	Concentrated static load	Based on the analytical solution	2, 100	Critical force	0.00
8.	6.1	Stability of a Three-Span Two-Storey Frame Subjected to Concentrated Longitudinal Forces Applied to the Columns in the Joints with Girders	Concentrated static load	Based on the analytical solution	2, 100	Critical force	0.00
9.	Arch hinged	Stability of a Circular Two-Hinged Arch of a Constant Cross-Section Subjected to Hydrostatic Pressure	Distributed static load	Based on the analytical solution	2	Critical load	0.23
10.	6.2	Stability of In-Plane Bending of a Cantilever Strip of a Rectangular Cross-Section by a Shear force Applied at the Free End	Concentrated static load	Based on the analytical solution	150	Critical force	4.50
11.	Stabilit y Bar 1	Stability of a Cantilever Beam of a Square Cross-Section Subjected to a Concentrated Longitudinal Compressive Force Centrally Applied at the Free End (Central Compression)	Concentrated static load	Based on the analytical solution	5	Critical force	0.01
					150	Critical force	0.48
					37	Critical force	0.58
12.	Stabilit y Bar 2	Stability of a Cantilever Beam of a Square Cross-Section Subjected to a Concentrated Transverse Bending Force Centrally Applied at the Free End	Concentrated static load	Based on the analytical solution	5	Critical force	0.76
					150	Critical force	2.28
					37	Critical force	0.31
13.	Stabilit y Bar 3	Stability of a Cantilever Beam of a Square Cross-Section Subjected to a	Concentrated static load	Based on the analytical solution	5, 100	Critical force	0.64

	Code	Name of the test	Combination of loads and actions	Type of check of the results	Finite elements	Checked parameters	Deviation %
		Concentrated Transverse Bending Force Applied to the Upper Edges of the Free End			150	Critical force	1.80
					37	Critical force	4.26
14.	Stability Bar 4	Stability of a Beam of a Square Cross-Section Simply Supported in and out of the Bending Plane Subjected to Concentrated Bending Moments Applied at the Ends and Equal in Value (Pure Bending)	Concentrated static load	Based on the analytical solution	5	Critical force	0.59
					150, 5	Critical force	1.28
15.	Stability Bar 5	Stability of a Beam of a Square Cross-Section Simply Supported in the Bending Plane and Clamped out of the Bending Plane Subjected to Concentrated Bending Moments Applied at the Ends and Equal in Value (Pure Bending)	Concentrated static load	Based on the analytical solution	5	Critical force	0.64
					150, 5	Critical force	5.36
16.	Stability Bar 6	Stability of a Beam of a Square Cross-Section Simply Supported in and out of the Bending Plane Subjected to Concentrated Longitudinal Bending Forces Applied to the Upper Edges of the Ends and Equal in Value (Longitudinal Bending)	Concentrated static load	Based on the analytical solution	5, 100	Critical force	0.01
					150, 5	Critical force	0.48
17.	Stability Bar 7	Stability of a Beam of a Square Cross-Section Simply Supported in the Bending Plane and Clamped out of the Bending Plane Subjected to Concentrated Longitudinal Bending Forces Applied to the Upper Edges of the Ends and Equal in Value (Longitudinal Bending)	Concentrated static load	Based on the analytical solution	5, 100	Critical force	0.02
					150, 5	Critical force	8.18
18.	Stability Bar 8	Stability of a Cantilever Beam of a Square Cross-Section Subjected to a Load Uniformly Distributed along Its Longitudinal Axis	Distributed static load	Based on the analytical solution	5	Critical load	0.45
					150	Critical load	3.30
					37	Critical load	5.73
19.	Stability Bar 9	Stability of a Cantilever Beam of a Square Cross-Section Subjected to a Load Uniformly Distributed along the Longitudinal Axis of Its Upper Face	Distributed static load	Based on the analytical solution	5, 100	Critical load	0.20
					150	Critical load	1.71
					37	Critical load	3.67

	Code	Name of the test	Combination of loads and actions	Type of check of the results	Finite elements	Checked parameters	Deviation %
20.	Stability Bar 10	Stability of a Beam of a Square Cross-Section Simply Supported in and out of the Bending Plane Subjected to a Concentrated Transverse Bending Force Applied in the Middle of the Span at the Level of the Longitudinal Axis (Transverse Bending)	Concentrated static load	Based on the analytical solution	5	Critical force	0.52
					150, 5	Critical force	2.91
21.	Stability Bar 11	Stability of a Beam of a Square Cross-Section Simply Supported in and out of the Bending Plane Subjected to a Concentrated Transverse Bending Force Applied in the Middle of the Span at the Level of the Longitudinal Axis of the Upper Face (Transverse Bending)	Concentrated static load	Based on the analytical solution	5, 100	Critical force	1.21
					150, 5	Critical force	4.03
22.	Stability Bar 12	Stability of a Beam of a Square Cross-Section Simply Supported in the Bending Plane and Clamped out of the Bending Plane Subjected to a Concentrated Transverse Bending Force Applied in the Middle of the Span at the Level of the Longitudinal Axis (Transverse Bending)	Concentrated static load	Based on the analytical solution	5	Critical force	3.19
					150, 5	Critical force	9.42
23.	Stability Bar 13	Stability of a Beam of a Square Cross-Section Simply Supported in and out of the Bending Plane Subjected to a Transverse Load Uniformly Distributed along Its Longitudinal Axis	Distributed static load	Based on the analytical solution	5	Critical load	0.53
					150, 5	Critical load	2.44
24.	Stability Bar 14	Stability of a Beam of a Square Cross-Section Simply Supported in the Bending Plane and Clamped out of the Bending Plane Subjected to a Transverse Load Uniformly Distributed along Its Longitudinal Axis	Distributed static load	Based on the analytical solution	5	Critical load	2.27
					150, 5	Critical load	7.25
25.	Stability Flanged Beam 1	Stability of an I-beam Simply Supported in and out of the Bending Plane Subjected to Concentrated Bending Moments Applied at the Ends and Equal in Value (Pure Bending)	Concentrated static load	Based on the analytical solution	5	Critical force	1.19
					150	Critical force	3.52

	Code	Name of the test	Combination of loads and actions	Type of check of the results	Finite elements	Checked parameters	Deviation %
26.	Stabilit y Flanged Beam 2	Stability of an I-beam Simply Supported in and out of the Bending Plane Subjected to a Concentrated Transverse Bending Force Applied in the Middle of the Span at the Level of the Longitudinal Axis (Transverse Bending)	Concentrated static load	Based on the analytical solution	5	Critical force	1.38
					150	Critical force	1.65
27.	Stabilit y Flanged Beam 3	Stability of an I-beam Simply Supported in and out of the Bending Plane Subjected to a Transverse Load Uniformly Distributed along Its Longitudinal Axis	Distributed static load	Based on the analytical solution	5	Critical load	1.21
					150	Critical load	0.98
28.	Stabilit y Flanged Beam 4	Stability of an I-beam Simply Supported in and out of the Bending Plane Subjected to a Load Uniformly Distributed along the Longitudinal Axis of Its Upper Flange	Distributed static load	Based on the analytical solution	5	Critical load	1.54
					150	Critical load	1.87
29.	6.6	Stability of a Simply Supported Rectangular Plate Uniformly Compressed in One Direction	Static load distributed along the line	Based on the analytical solution	44	Critical stresses	1.95
					50		0.00
30.	6.7	Stability of a Simply Supported Square Plate Uniformly Compressed in One Direction	Static load distributed along the line	Based on the analytical solution	44	Critical stresses	1.26
					50		0.00
31.	6.8	Stability of a Simply Supported Square Plate Uniformly Compressed in One Direction under Kinematic Action	Initial displacement	Based on the analytical solution	44	Critical stresses	1.27
					50		0.00
32.	6.9	Stability of a Rectangular Simply Supported Plate under Pure Shear	Static load distributed along the line	Based on the analytical solution	44	Critical stresses	3.26
					50		0.05
33.	6.10 a model 1	Stability of a Rectangular Simply Supported Plate with Longitudinal Stiffeners Uniformly Compressed in the Longitudinal Direction (Model 1)	Distributed along the line and concentrated static loads	Based on the analytical solution	50, 5	Critical stresses	0.10
34.	6.10 a model 2	Stability of a Rectangular Simply Supported Plate with Longitudinal Stiffeners Uniformly Compressed in the Longitudinal Direction (Model 2)	Static loads distributed along the line	Based on the analytical solution	50	Critical stresses	2.55

	Code	Name of the test	Combination of loads and actions	Type of check of the results	Finite elements	Checked parameters	Deviation %
35.	6.10 b	Stability of a Rectangular Simply Supported Orthotropic Plate Uniformly Compressed in One Direction	Static loads distributed along the line	Based on the analytical solution	50	Critical stresses	0.03
36.	6.11 S	Stability of a Cylindrical Thin-Walled Shell with Simply Supported Edges Subjected to Uniform External Pressure	Distributed surface static load	Based on the analytical solution	50	Critical pressure	1.94
			Nonlinear Statics				
1.	Contact 1	Three-Span Beam with One Clamped End and Three Rigid One-Sided Supports Subjected to Concentrated Forces above Them	Concentrated static load	Based on the analytical solution	2, 352	Displacements	0.00
						Forces	1.67
2.	Contact 2	Rigid Body Restrained by Five Springs of the Same Rigidity Working Only in Tension Subjected to a Concentrated Force	Concentrated static load	Based on the analytical solution	100, 352	Forces	0.07
3.	Tunnel lining	Circular Tunnel Lining Subjected to the Given Active Vertical and Horizontal Earth Pressure and Passive Lateral Earth Pressure in the Contact Area	Concentrated static load	Based on the analytical solution	5, 352	Forces	0.01
4.	Contact 3	Contact with Detachment for a Layer and Subgrade with a Concentrated Shear force Applied to the Layer	Concentrated static load	Based on the analytical solution	30, 352	Size of the contact area	3.77
5.	NL CANA T	Flexible Thread with Supports in One Level Subjected to a Uniformly Distributed Transverse Load	Distributed static load	Based on the analytical solution	302	Displacements	0.02
						Forces	0.34
6.	Ring	Flexible Ring Subjected to Two Mutually Balanced Radially Compressive Forces	Concentrated static load for a non-inflectional elastic curve	Based on the analytical solution	310	Displacements	3.29
						Forces	0.05
7.	NEL	Flexible Long Rectangular Plate Simply Supported along the Longitudinal Edges Subjected to a Uniformly Distributed Transverse Load	Distributed surface static load	Based on the analytical solution	341	Displacements	0.06
						Stresses	4.26
8.	7.6	Flexible Square Plate Simply Supported along the Perimeter Subjected to a Uniformly Distributed Transverse Load	Distributed surface static load	Based on the analytical solution	344	Displacements	1.87
						Stresses	1.80
9.	7.7	Simply Supported Flexible Circular Plate Subjected to a Uniformly	Distributed surface static load	Based on the analytical solution	342, 344	Displacements	1.14

	Code	Name of the test	Combination of loads and actions	Type of check of the results	Finite elements	Checked parameters	Deviation %
		Distributed Transverse Load				**Stresses**	**1.67**
10.	**Mast**	**Double-Guyed Mast Subjected to Static Loads and Prestressing Forces**	**Concentrated and distributed static loads**	**Based on the analytical solution**	**5, 308**	**Forces**	**0.23**
11.	**Plate-membrane 4**	**Square Membrane with a Compliant Contour**	**Distributed surface static load**	**Experimental data**	**341**	**Displacements**	**5.84**
		Pathological Tests					
1.	**Patch test Constant stress Shell**	**Rectangular Plate under the Constant Stresses on the Midsurface**	**Initial displacement**	**Based on the analytical solution**	**42**	**Stresses**	**0.00**
					44	**Stresses**	**0.00**
					45	**Stresses**	**0.00**
					50	**Stresses**	**0.00**
2.	**Patch test Constant curvature Shell**	**Rectangular Plate with Constant Curvature**	**Initial displacement**	**Based on the analytical solution**	**42**	**Stresses**	**0.00**
					44	**Stresses**	**0.00**
					45	**Stresses**	**0.00**
					50	**Stresses**	**0.00**
3.	**Patch test Constant stress Solid**	**Cube under the Constant Stresses throughout the Volume**	**Initial displacement**	**Based on the analytical solution**	**32**	**Stresses**	**0.00**
					34	**Stresses**	**0.00**
					36	**Stresses**	**0.00**
					37	**Stresses**	**0.00**
4.	**Straight cantilever beam**	**Rectilinear Cantilever Beam with Concentrated Longitudinal and Shear Forces and a Torque at Its Free End**	**Concentrated static load**	**Based on the analytical solution**	**42 regular mesh**	**Displacements**	**96.85**
					42 trapezoidal mesh	**Displacements**	**98.52**
					42 parallelogram mesh	**Displacements**	**97.78**
					142 regular mesh	**Displacements**	**96.85**
					142 trapezoidal mesh	**Displacements**	**98.52**
					142 parallelogram mesh	**Displacements**	**97.78**
					44 regular mesh	**Displacements**	**90.65**
					44 trapezoidal mesh	**Displacements**	**97.31**
					44 parallelogram mesh	**Displacements**	**96.57**
					144 regular mesh	**Displacements**	**90.37**
					144 trapezoidal mesh	**Displacements**	**97.22**

	Code	Name of the test	Combination of loads and actions	Type of check of the results	Finite elements	Checked parameters	Deviation %
					144 parallelogram mesh	Displacements	96.11
					45 regular mesh	Displacements	3.29
					45 trapezoidal mesh	Displacements	3.69
					45 parallelogram mesh	Displacements	2.97
					145 regular mesh	Displacements	4.08
					145 trapezoidal mesh	Displacements	4.25
					145 parallelogram mesh	Displacements	4.13
					50 regular mesh	Displacements	2.51
					50 trapezoidal mesh	Displacements	2.79
					50 parallelogram mesh	Displacements	2.78
					150 regular mesh	Displacements	3.37
					150 trapezoidal mesh	Displacements	3.53
					150 parallelogram mesh	Displacements	3.43
					36 regular mesh	Displacements	97.48
					36 trapezoidal mesh	Displacements	98.96
					36 parallelogram mesh	Displacements	98.59
					37 regular mesh	Displacements	15.05
					37 trapezoidal mesh	Displacements	24.81
					37 parallelogram mesh	Displacements	15.09
5.	Curved cantilever beam	Curvilinear Cantilever Beam with Concentrated Shear Forces at Its Free End	Concentrated static load	Based on the analytical solution	42	Displacements	97.50
					142	Displacements	97.50
					44	Displacements	92.76
					144	Displacements	92.59
					45	Displacements	2.74
					145	Displacements	2.57
					50	Displacements	1.41
					150	Displacements	1.95
					36	Displacements	92.77
					37	Displacements	6.01

	Code	Name of the test	Combination of loads and actions	Type of check of the results	Finite elements	Checked parameters	Deviation %
6.	Twisted cantilever beam	Twisted Cantilever Beam with Concentrated Shear Forces at Its Free End	Concentrated static load	Based on the analytical solution	42	Displacements	16.21
					142	Displacements	14.63
					44	Displacements	65.00
					144	Displacements	65.23
					45	Displacements	0.70
					145	Displacements	24.74
					50	Displacements	27.24
					150	Displacements	24.55
					36	Displacements	79.38
					37	Displacements	0.63
7.	Bending of square flat plate Simply supported	Simply Supported Flat Square Plate Subjected to a Transverse Load Uniformly Distributed over the Entire Area and a Concentrated Shear Force Applied in the Center	Distributed surface static load	Based on the analytical solution	42, FE mesh 8x8	Displacements	0.39
					44, FE mesh 8x8	Displacements	0.32
					45, FE mesh 8x8	Displacements	0.00
					50, FE mesh 8x8	Displacements	0.00
					36, FE mesh 128x128	Displacements	24.64
					37, FE mesh 128x128	Displacements	0.39
			Distributed surface static load	Based on the analytical solution	42, FE mesh 8x8	Displacements	0.95
					44, FE mesh 8x8	Displacements	0.54
					45, FE mesh 8x8	Displacements	0.01
					50, FE mesh 8x8	Displacements	0.02
					36, FE mesh 128x128	Displacements	25.08
					37, FE mesh 128x128	Displacements	0.38
8.	Bending of square flat plate Clamped supported	Flat Square Plate Clamped along the Outer Edges and Subjected to a Transverse Load Uniformly Distributed over the Entire Area and a Concentrated Shear Force Applied in the Center	Distributed surface static load	Based on the analytical solution	42, FE mesh 8x8	Displacements	0.71
					44, FE mesh 8x8	Displacements	0.63
					45, FE mesh 8x8	Displacements	0.00
					50, FE mesh 8x8	Displacements	0.00
					36, FE mesh 128x128	Displacements	27.91
					37, FE mesh 128x128	Displacements	0.08
			Concentrated static load	Based on the analytical solution	42, FE mesh 8x8	Displacements	1.71
					44, FE mesh 8x8	Displacements	1.05
					45, FE mesh 8x8	Displacements	0.04
					50, FE mesh 8x8	Displacements	0.07
					36, FE mesh 128x128	Displacements	27.66

	Code	Name of the test	Combination of loads and actions	Type of check of the results	Finite elements	Checked parameters	Deviation %
					37, FE mesh 128x128	Displacements	0.18
9.	Bending of rectangular flat plate Simply supported	Simply Supported Flat Rectangular Plate Subjected to a Transverse Load Uniformly Distributed over the Entire Area and a Concentrated Shear Force Applied in the Center	Distributed surface static load	Based on the analytical solution	42, FE mesh 8x8	Displacements	0.10
					44, FE mesh 8x8	Displacements	0.45
					45, FE mesh 8x8	Displacements	0.00
					50, FE mesh 8x8	Displacements	0.00
					36, FE mesh 128x128	Displacements	28.81
					37, FE mesh 128x128	Displacements	0.02
			Concentrated static load	Based on the analytical solution	42, FE mesh 8x8	Displacements	12.54
					44, FE mesh 8x8	Displacements	7.68
					45, FE mesh 8x8	Displacements	0.65
					50, FE mesh 8x8	Displacements	0.68
					36, FE mesh 128x128	Displacements	43.08
					37, FE mesh 128x128	Displacements	0.09
10.	Bending of rectangular flat plate Clamped supported	Flat Rectangular Plate Clamped along the Outer Edges and Subjected to a Transverse Load Uniformly Distributed over the Entire Area and a Concentrated Shear Force Applied in the Center	Distributed surface static load	Based on the analytical solution	42, FE mesh 8x8	Displacements	1.34
					44, FE mesh 8x8	Displacements	0.04
					45, FE mesh 8x8	Displacements	0.04
					50, FE mesh 8x8	Displacements	0.04
					36, FE mesh 128x128	Displacements	30.29
					37, FE mesh 128x128	Displacements	0.00
			Concentrated static load	Based on the analytical solution	42, FE mesh 8x8	Displacements	20.79
					44, FE mesh 8x8	Displacements	12.04
					45, FE mesh 8x8	Displacements	2.02
					50, FE mesh 8x8	Displacements	1.85
					36, FE mesh 128x128	Displacements	45.90
					37, FE mesh 128x128	Displacements	0.21
11.	Scordelis-Lo roof	Open Cylindrical Shell Rectangular in Plan and Simply Supported along the Curvilinear Edges Subjected to a Transverse Load Uniformly Distributed over the Entire Area	Distributed surface static load	Based on the analytical solution	42, FE mesh 8x8	Displacements	15.04
					44, FE mesh 8x8	Displacements	4.70
					45, FE mesh 8x8	Displacements	0.94
					50, FE mesh 8x8	Displacements	0.87

	Code	Name of the test	Combination of loads and actions	Type of check of the results	Finite elements	Checked parameters	Deviation %
					36, FE mesh 128x128	Displacements	4.86
					37, FE mesh 128x128	Displacements	0.58
12.	Quadrant of a spherical shell	Free Hemispherical Shell with a Circular Pole Hole Subjected to Two Orthogonal Pairs of Mutually Balanced Radial Tensile and Compressive Forces at the Equator	Concentrated static load	Based on the analytical solution	42, FE mesh 32x32	Displacements	1.38
					44, FE mesh 32x32	Displacements	0.85
					45, FE mesh 32x32	Displacements	1.38
					50, FE mesh 32x32	Displacements	0.85
					36, FE mesh 128x128	Displacements	62.77
					37, FE mesh 128x128	Displacements	0.32
13.	Nearly incompressible thick cylinder	Nearly Incompressible Thick-Walled Cylinder under Plane Deformation Subjected to Uniformly Distributed Internal Pressure	Distributed surface static load	Based on the analytical solution	42, Poisson’s ratio 0.49	Displacements	1.05
					42, Poisson’s ratio 0.499	Displacements	1.15
					42, Poisson’s ratio 0.4999	Displacements	1.17
					44, Poisson’s ratio 0.49	Displacements	1.94
					44, Poisson’s ratio 0.499	Displacements	2.04
					44, Poisson’s ratio 0.4999	Displacements	2.05
					45, Poisson’s ratio 0.49	Displacements	3.08
					45, Poisson’s ratio 0.499	Displacements	3.20
					45, Poisson’s ratio 0.4999	Displacements	3.22
					50, Poisson’s ratio 0.49	Displacements	3.04
					50, Poisson’s ratio 0.499	Displacements	3.20
					50, Poisson’s ratio 0.4999	Displacements	3.18
					36, Poisson’s ratio 0.49	Displacements	15.48

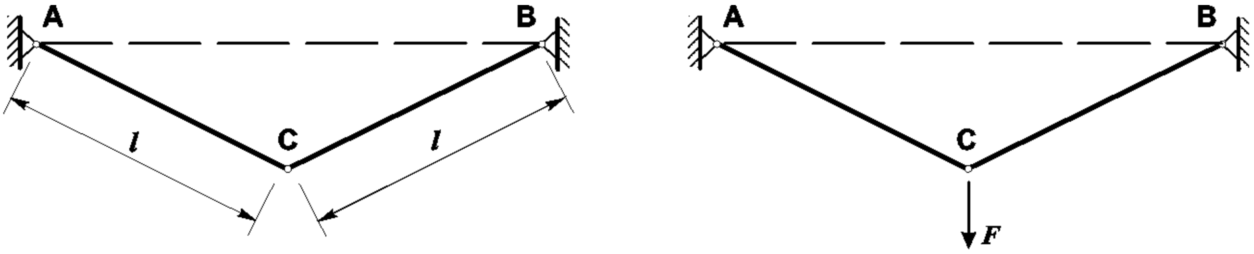
	Code	Name of the test	Combination of loads and actions	Type of check of the results	Finite elements	Checked parameters	Deviation %
					36, Poisson's ratio 0.499	Displacements	64.07
					36, Poisson's ratio 0.4999	Displacements	94.65
					37, Poisson's ratio 0.49	Displacements	0.54
					37, Poisson's ratio 0.499	Displacements	1.19
					37, Poisson's ratio 0.4999	Displacements	11.36
Energy Analysis							
1.	Energy 94A	Frame Subjected to Various Vertical Forces	Nodal load	Based on the analytical solution	2	Estimation of the role of subsystems in the case of buckling	—
2.	Energy 94B	Frame Subjected to Vertical Forces	Nodal load	Based on the analytical solution	2	Estimation of the role of subsystems in the case of buckling	—
3.	Energy	Symmetric Frame Subjected to Vertical Forces — Detection of "Weak" Elements	Nodal load	Based on the analytical solution	2	Estimation of the role of subsystems in the case of buckling	—
Erection							
1.	Test-01	Static Analysis of Stress-Strain State of a Building Taking into Account Genetic Nonlinearity	Distributed loads	Comparison with the ANSYS solution	44,5	displacements	2.5
						forces	6.6
2.	Truss	Determination of Stress-Strain State Taking into Account Genetic Nonlinearity	Nodal loads	Based on the analytical solution	1	displacements	0.21
						forces	0.91
3.	Rearrange_Frame	Replacement of a Column of a Two-Span Single-Storey Frame Subjected to a Constant Load	Distributed loads	Based on the analytical solution	5	forces	0.0
4.	Wiring_Girder.MPR	Sequential Erection of a Steel Reinforced Concrete Single-Span Beam	Distributed loads	Based on the analytical solution	5,44,100	displacements	1.29
Response Spectra							
1.	DIN_B_RS	Determination of the Response Spectrum of Response Accelerations of a Linear Oscillator	Accelerogram	Comparison with the Abaqus calculation	5	Frequency at which the maximum acceleration occurs	0.0
						Maximum acceleration	0.95

	Code	Name of the test	Combination of loads and actions	Type of check of the results	Finite elements	Checked parameters	Deviation %
Amplitude-Frequency Characteristics							
1.	**АЧХ**	**Plotting the Amplitude-Frequency Characteristic of a Single-Mass Elastic System under Harmonic Excitation**	**Concentrated dynamic load**	**Based on the analytical solution**	**51**	**Frequency at which the maximum displacement occurs**	**0.0**
						Maximum displacement	**0.65**
Steel Structural Members							
1.	**4.1 Section Resistance_Example_4.1**	**Strength and Stiffness Analysis of a Welded I-beam**	**Nodal loads**	**Based on the analytical solution**	**5**	**Utilization factors of restrictions**	**0.0**
2.	**4.2 Section Resis-tance_Example_4.2**	Strength and Stiffness Analysis of a Rolled I-beam	**Nodal loads**	**Based on the analytical solution**	**5**	**Utilization factors of restrictions**	**0.0**
3.	**4.3 Section Resistance_Example_4.3**	**Strength and Stiffness Analysis of a Rolled I-beam**	**Nodal loads**	**Based on the analytical solution**	**5**	**Utilization factors of restrictions**	**0.0**
4.	**4.4 Section Resis-tance_Example_4.4**	**Strength and Stiffness Analysis of a Rolled I-beam**	**Nodal loads**	**Based on the analytical solution**	**5**	**Utilization factors of restrictions**	**0.0**
5.	**4.5 Section Resis-tance_Example_4.5**	**Strength and Stiffness Analysis of a Welded I-beam**	**Nodal loads**	**Based on the analytical solution**	**5**	**Utilization factors of restrictions**	**0.0**
6.	**4.6 Section Resistance_Example_4.6**	**Analysis of an Axially Compressed Welded I-beam Column**	**Nodal loads**	**Based on the analytical solution**	**5**	**Utilization factors of restrictions**	**0.0**
7.	**3.1 Beam_Example_3.1**	**Strength and Stiffness Analysis of Stringers for a Normal Stub Girder System**	**Nodal loads**	**Based on the analytical solution**	**5**	**Utilization factors of restrictions**	**0.0**
8.	**3.2 Beam_Example_3.2**	**Strength and Stiffness Analysis of Stringers for a Complex Stub Girder System**	**Nodal loads**	**Based on the analytical solution**	**5**	**Utilization factors of restrictions**	**0.0**
9.	**3.3 Beam_Example_3.3**	**Strength and Stiffness Analysis of Secondary Beams for a Complex Stub Girder System**	**Nodal loads**	**Based on the analytical solution**	**5**	**Utilization factors of restrictions**	**0.0**
10.	**3.4 Beam_Example_3.4**	**Strength and Stiffness Analysis of Main Beams of Complex Stub Girder Systems**	**Nodal loads**	**Based on the analytical solution**	**5**	**Utilization factors of restrictions**	**0.0**

	Code	Name of the test	Combination of loads and actions	Type of check of the results	Finite elements	Checked parameters	Deviation %
11.	5.1 Column _Example_5.1	Analysis of an Axially Compressed Welded I-beam Column	Nodal loads	Based on the analytical solution	5	Utilization factors of restrictions	0.0
12.	5.3 Column _Example_5.3	Analysis of an Axially Compressed Electric Welded Circular Hollow Section Column		Based on the analytical solution	5	Utilization factors of restrictions	0.0
13.	7.1 Truss_Element_Example_7.1	Analysis of a Top Truss Chord from Unequal Angles		Based on the analytical solution	5	Utilization factors of restrictions	0.0
			Reinforced Concrete Structural Members				
			Calculations according to SNiP 2.03.01-84*				
1.	SCAD 3 SNiP	Strength Analysis of a Rectangular Beam	Nodal loads	Based on the analytical solution	2	Utilization factors of restrictions	4.2
2.	SCAD 7 SNiP	Strength Analysis of a T-section	Distributed loads	Based on the analytical solution	2	Utilization factors of restrictions	3.0
3.	SCAD 12 SNiP	Strength Analysis of a Wall Panel	Distributed loads	Based on the analytical solution	5	Utilization factors of restrictions	4.1
			Calculations according to SNiP 52-01-2003				
1.	SCAD 6 SP	Strength Analysis of a Rectangular Beam	Distributed loads	Based on the analytical solution	2	Utilization factors of restrictions	1.9
2.	SCAD 12.1.SP и SCAD 12.2.SP	Calculation of a Rib of a TT-shaped Floor Slab for Load-bearing Capacity under Lateral Forces	Distributed loads	Based on the analytical solution	2	Utilization factors of restrictions	1.4
3.	SCAD 13 SP	Calculation of a Simply Supported Rectangular Beam under Lateral Forces	Distributed loads	Based on the analytical solution	2	Utilization factors of restrictions	1.7
4.	SCAD 34 SP	Calculation of a Column of a Multi-storey Frame for Load-bearing Capacity under a Lateral Force	Nodal loads	Based on the analytical solution	5	Utilization factors of restrictions	0.4
5.	SCAD 41 SP-2003 и SCAD 41 SP-2012	Example of Punching Near the Edge of the Slab	Nodal loads	Based on the analytical solution	5, 41, 51	Utilization factors of restrictions	0.1
6.	SCAD 43 SP	Analysis of a Reinforced Concrete Foundation Slab for Normal Crack Opening	Concentrated moment	Based on the analytical solution	2	Utilization factors of restrictions	4.9

Linear Statics

Plane Truss Subjected to a Concentrated Force



Objective: Determination of the stress-strain state of a plane truss subjected to a concentrated force.

Initial data file: **SSLL09_v11.3.SPR**

Problem formulation: The plane truss consists of two inclined downward bars of the same length and rigidity of the cross-section arranged symmetrically with respect to the vertical axis, connected by hinges in the common node (point C) and simply supported at the opposite nodes (points A and B). A vertical concentrated force F is applied in the common node of the truss bars. Determine the vertical displacement of the common node of the truss bars Z and longitudinal forces in the truss bars N.

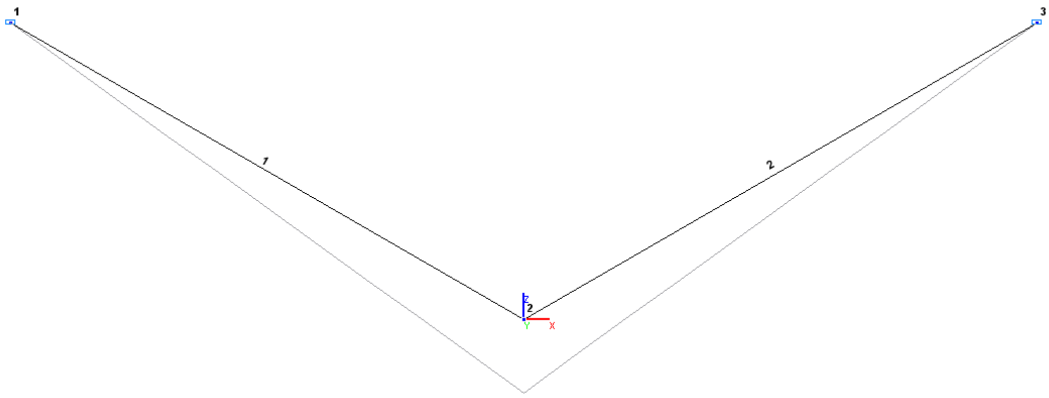
References: S. Timoshenko, Resistance des materiaux, t.1, Bruxelles, Edition Polytechnique Beranger, 1963, p. 10.

Initial data:

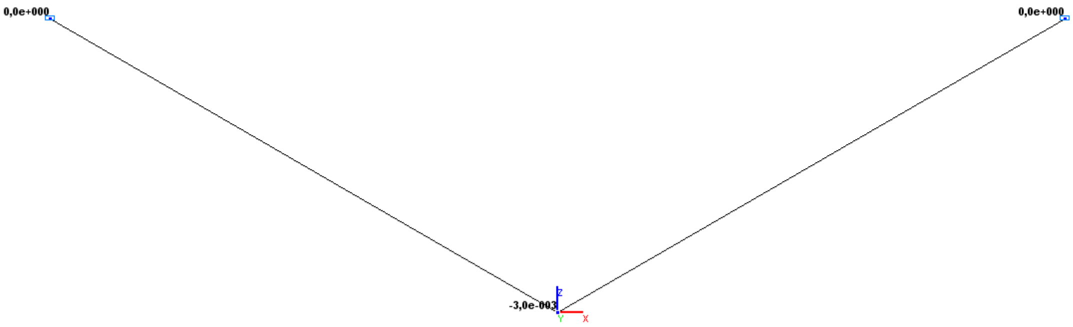
$E = 2.1\cdot10^{11}$ Pa-	- elastic modulus of truss bars;
l = 4.5 m	- length of truss bars;
θ = 30º	- inclination angle of the bars to the horizon;
$A = 3.0\cdot10^{-4}$ m^2	- cross-sectional area of the bars;
$F = 2.1\cdot10^4$ N	- value of the vertical concentrated force.

Finite element model: Design model – plane hinged bar system, 2 bar elements of type 10. Boundary conditions are provided by imposing constraints in the directions of the degrees of freedom X, Z for pinned support nodes. Number of nodes in the design model – 3.

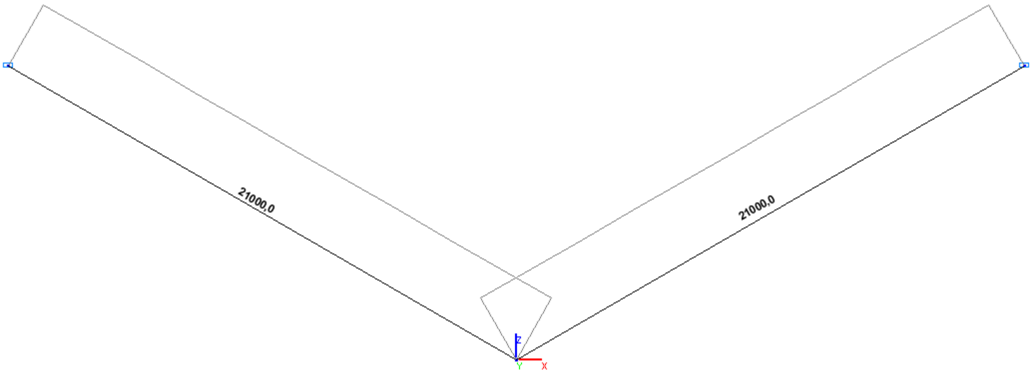
Results in SCAD



Design and deformed models



Values of vertical displacements Z (m)



Values of longitudinal forces N (N)

Comparison of solutions:

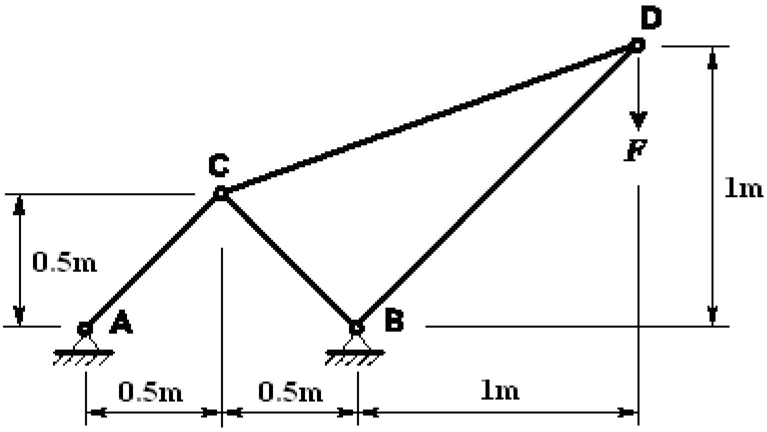
Parameter	Theory	SCAD	Deviations, %
Vertical displacement Z (point C), m	$-3.0000 \cdot 10^{-3}$	$-3.0000 \cdot 10^{-3}$	0.00
Longitudinal force *N* (bar AC), N	21000.0	21000.0	0.00
Longitudinal force *N* (bar BC), N	21000.0	21000.0	0.00

Notes: In the analytical solution, the vertical displacement of the common node of the truss bars Z and longitudinal forces in the truss bars *N* are determined according to the following formulas:

$$Z = \frac{F \cdot L}{2 \cdot E \cdot A \cdot sin^2(\theta)};$$

$$N = \frac{F}{2 \cdot sin(\theta)}.$$

Plane Hinged Bar System Subjected to a Concentrated Force



Objective: Determination of the strain state of a plane hinged bar system subjected to a concentrated force.

Initial data file: **SSLL11_v11.3.SPR**

Problem formulation: The plane hinged bar system consists of four inclined bars. The bars in the first pair have the same lengths and rigidities of the cross-section, go upward to the common node (point C) and are simply supported in the opposite nodes (points A and B). The bars in the second pair have the same rigidities of the cross-section, go upward to the common node (point D) and are attached to one of the bars of the first pair at the opposite nodes (points C and B). A vertical concentrated force F is applied in the common node of the second pair of bars. Determine horizontal X and vertical Z displacements of the common nodes of the first (point C) and second (point D) pairs of bars of the system.

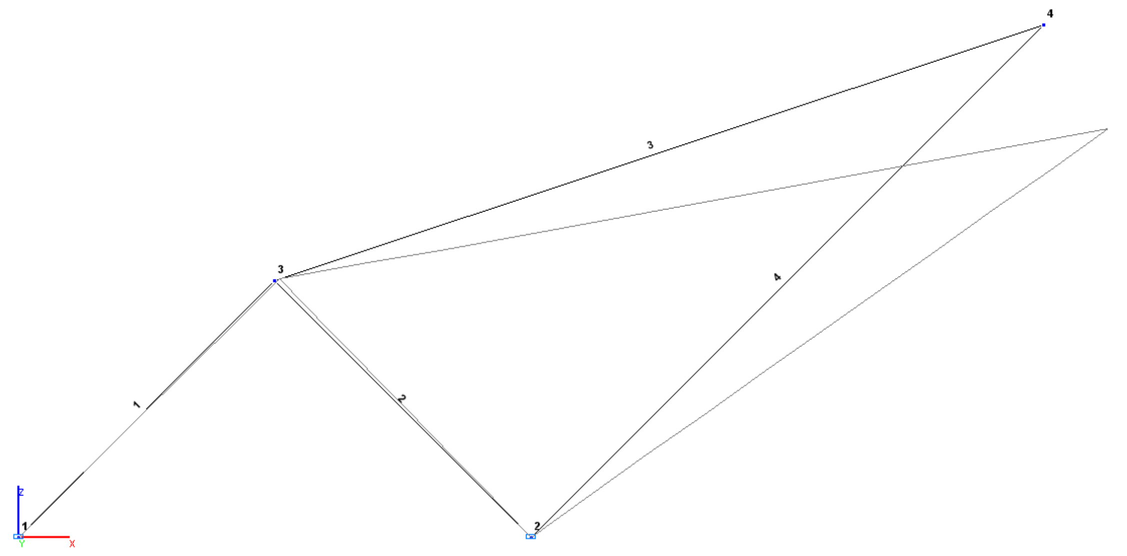
References: S. S. Rao, The finite element method in engineering, 4 ed, Elsevier science end technology books, 2004, p. 313.

Initial data:

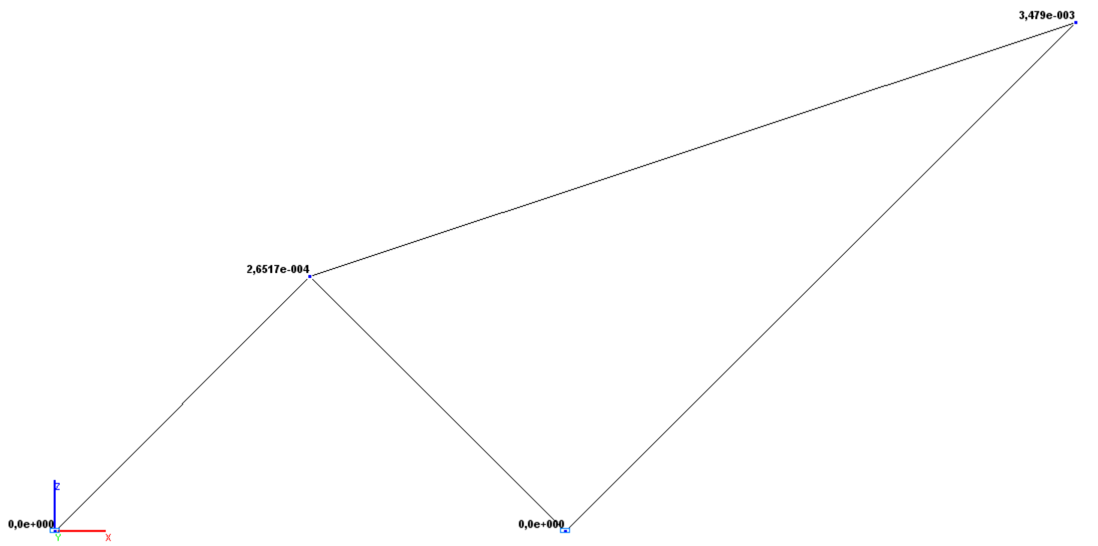
$E = 2.0 \cdot 10^{10}$ Pa-	- elastic modulus of the bars of the system;
$X_A = 0.0$ m	- coordinates of the node A;
$Y_A = 0.0$ m	
$X_B = 1.0$ m	- coordinates of the node B;
$Y_B = 0.0$ m	
$X_C = 0.5$ m	- coordinates of the node C;
$Y_C = 0.5$ m	
$X_D = 2.0$ m	- coordinates of the node D;
$Y_D = 1.0$ m	
$A_{AC} = 2.0 \cdot 10^{-4}$ m^2	- cross-sectional area of the bar AC;
$A_{BC} = 2.0 \cdot 10^{-4}$ m^2	- cross-sectional area of the bar BC;
$A_{CD} = 1.0 \cdot 10^{-4}$ m^2	- cross-sectional area of the bar CD;
$A_{BD} = 1.0 \cdot 10^{-4}$ m^2	- cross-sectional area of the bar BD;
$F = 1.0 \cdot 10^3$ N	- value of the vertical concentrated force.

***Finite element model*:** Design model – plane hinged bar system, 4 bar elements of type 10. Boundary conditions are provided by imposing constraints in the directions of the degrees of freedom X, Z for pinned support nodes (points A and B). Number of nodes in the design model – 4.

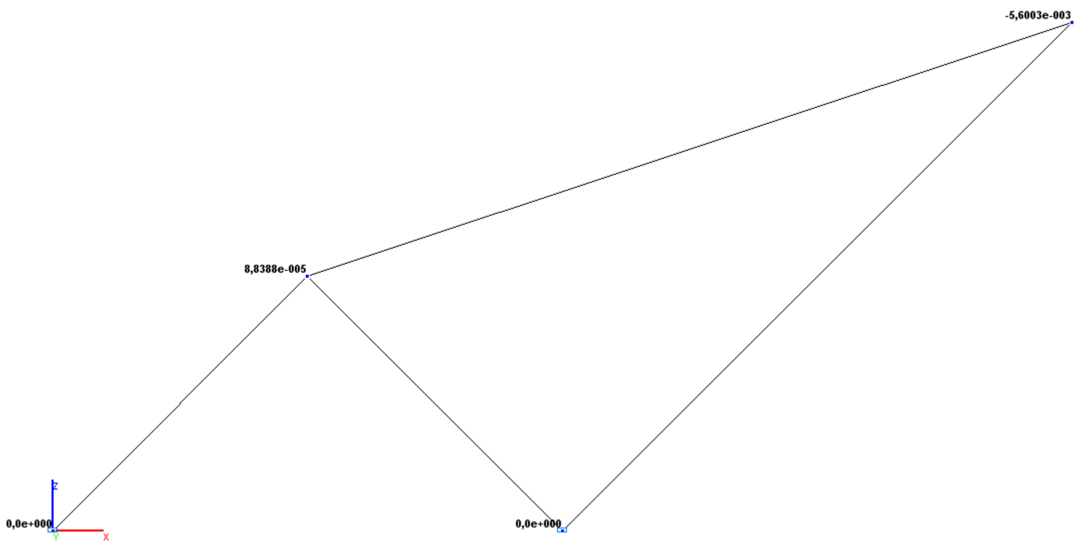
Results in SCAD



Design and deformed models



Values of horizontal displacements X (m)

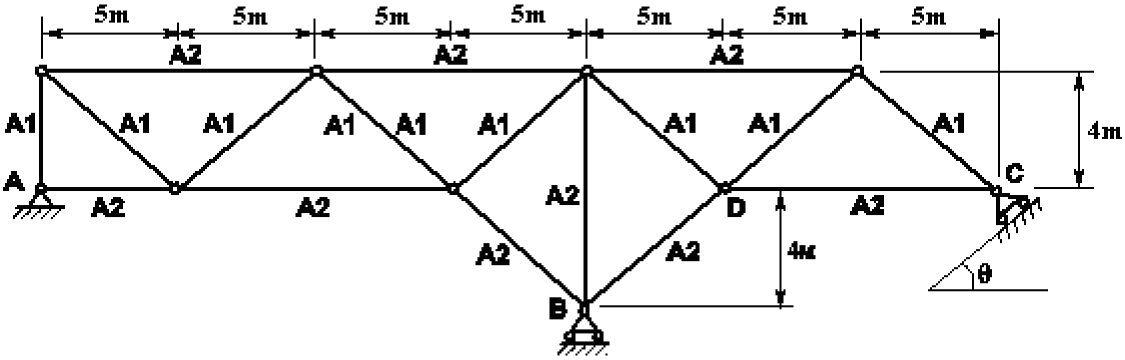


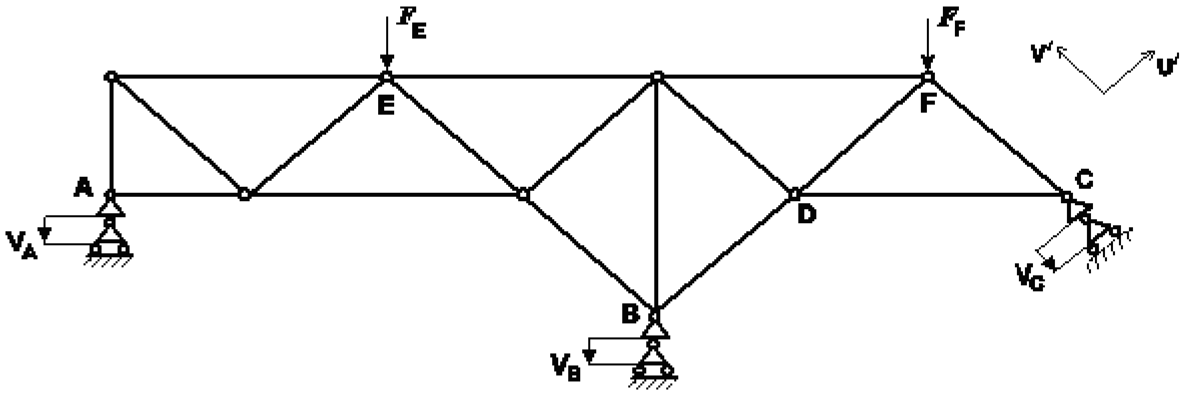
Values of vertical displacements Z (m)

Comparison of solutions:

Parameter	Theory	SCAD	Deviations, %
Horizontal displacement X (point C), m	$2.6517 \cdot 10^{-4}$	$2.6517 \cdot 10^{-4}$	0.00
Vertical displacement Z (point C), m	$0.8839 \cdot 10^{-4}$	$0.8839 \cdot 10^{-4}$	0.00
Horizontal displacement X (point D), m	$34.7903 \cdot 10^{-4}$	$34.7903 \cdot 10^{-4}$	0.00
Vertical displacement Z (point D), m	$-56.0035 \cdot 10^{-4}$	$-56.0035 \cdot 10^{-4}$	0.00

Plane Truss Subjected to Force, Thermal and Kinematic Actions





Objective: Determination of the stress-strain state of a truss subjected to force, thermal and kinematic actions.

Initial data file: **SSLL12_v11.3.spr**

Problem formulation: The two-span truss is loaded by two concentrated forces F_E and F_F in the nodes of the top chord, uniformly heated across all cross-sections of its elements by the value of ΔT and subjected to the displacement of its supports by the values of v_A, v_B and v_C. Determine the longitudinal force *N* in the support diagonal BD and vertical displacement *v* (Z) in the point D of its joint with the bottom chord and the lattice members.

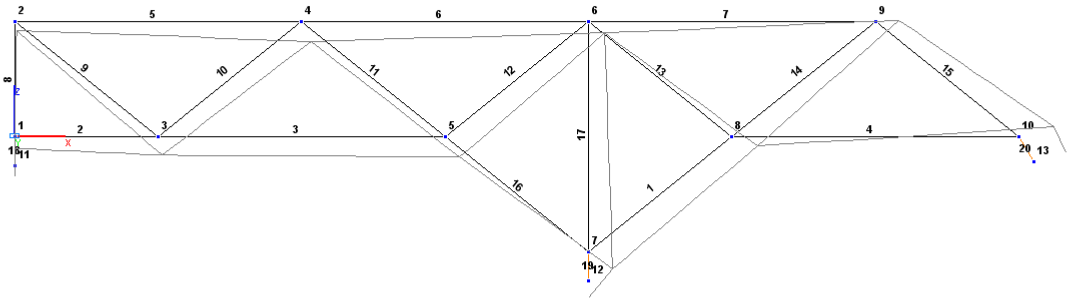
References: M. Laredo, Resistence des materiaux, Paris, Dunod, 1970, p.579.

Initial data:
Lattice members A1:
$EF = 2.961 \cdot 10^8$ N – axial stiffness;
Elements of the top and bottom chords, support diagonals and support vertical A2:
$EF = 5.922 \cdot 10^8$ N – axial stiffness;
Elements modeling the constraints in the support nodes in the directions v_A, v_B and v_C (null elements):
$EF = 10^{10}$ N – axial stiffness;
Boundary conditions:
$\theta = 30°$ – angle of the support area in the node C;
Properties of the material:
$\alpha = 10^{-5}$ 1/°C – linear expansion coefficient;
Loads and actions:
$F_E = 1.5 \cdot 10^5$ N
$F_F = 1.0 \cdot 10^5$ N
$\Delta T = 150$ °C
$v_A = 0.020$ m
$v_B = 0.030$ m
$v_C = 0.015$ m.

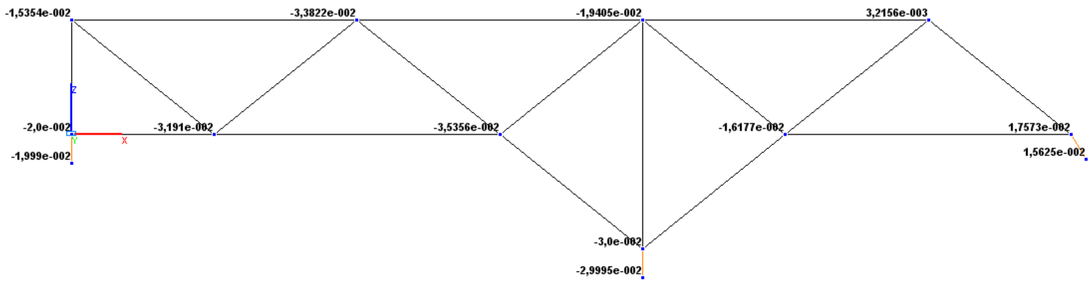
Finite element model: Design model – plane hinged bar system. Lattice members: A1 – 8 elements of type 1, elements of the top and bottom chords, support diagonals and support vertical A2 – 9 elements of type 1;

elements modeling the constraints in the support nodes in the directions v_A, v_B and v_C – 3 elements of type 154. Boundary conditions in the direction u_A are provided by imposing the respective rigid constraint. Number of nodes in the design model – 13.

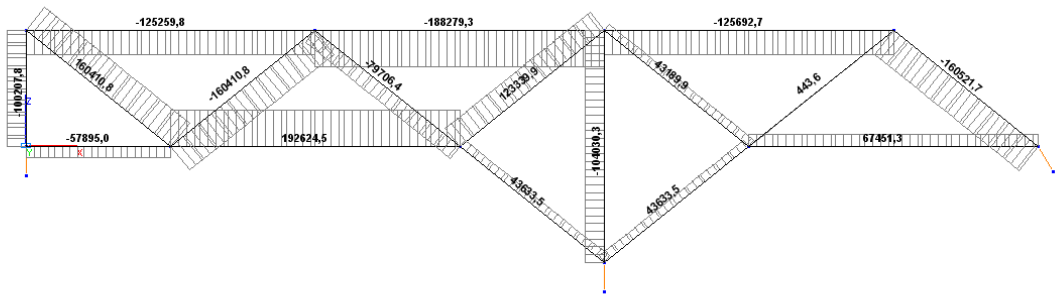
Results in SCAD



Design and deformed models



Values of vertical displacements v (Z) (m)

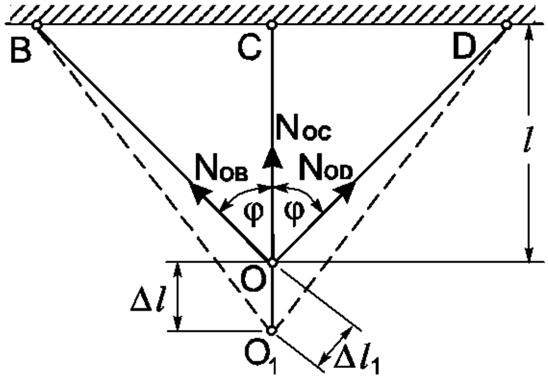


Values of longitudinal forces N (N)

Comparison of solutions:

Parameter	Theory	SCAD	Deviations, %
Vertical displacement v_D (Z), m	$-1.6180 \cdot 10^{-2}$	$-1.6177 \cdot 10^{-2}$	0.02
Longitudinal force N_{BD}, N	43633.0	43633.5	0.00

Plane Hinged Bar System with Elements of Different Material Subjected to Temperature Variation



***Objective*:** Determination of the stress state of a plane hinged bar system with elements of different material subjected to temperature variation.

Initial data file: **T1_v11.3.spr**

Problem formulation:
Three bars of the plane system are connected by hinges in the common node (O) and are simply supported in the opposite nodes (B, C, D). Support nodes are arranged on one horizontal straight line symmetrically with respect to the vertical axis (OC), the common node lies on the vertical axis. The vertical bar (OC) is made of steel, the inclined bars (OB, OD) are made of copper. The system is subjected to the temperature variation Δt relative to the assembly temperature. Determine longitudinal forces *N* in each bar.

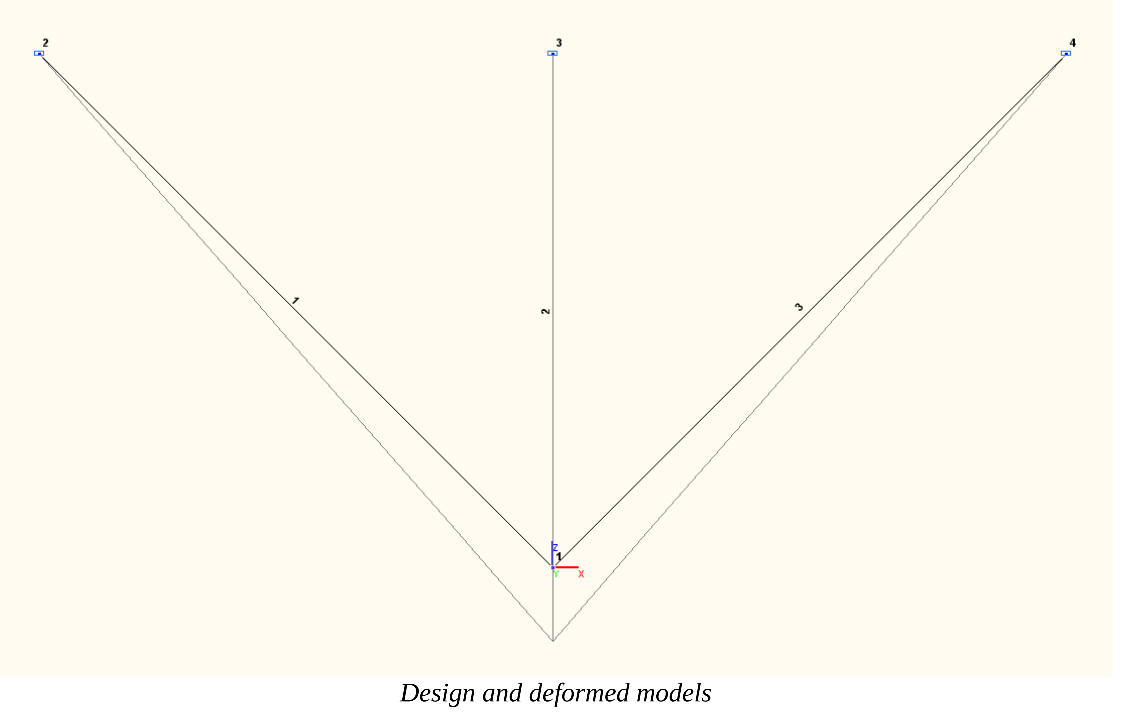
References: S.P. Timoshenko, Strength of Materials, Volume 1: Elementary Theory and Problems, Moscow, Nauka, 1965, p. 34.

Initial data:

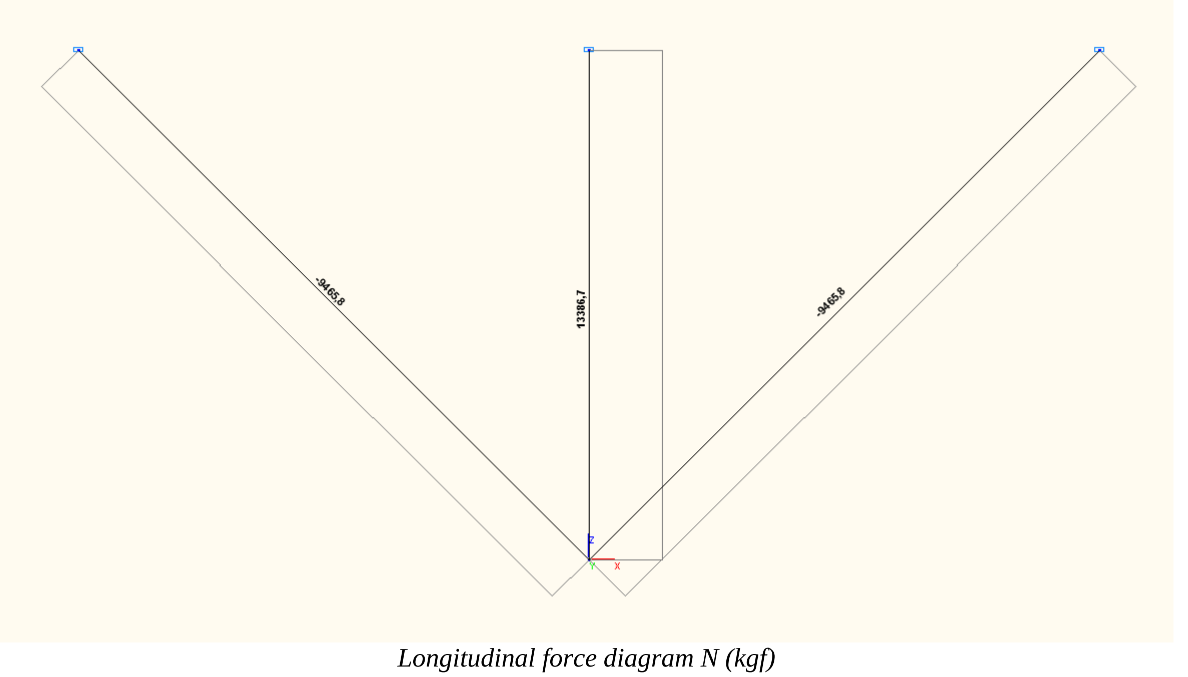
$E_s = 2.0 \cdot 10^6$ kgf/cm^2	– elastic modulus of steel;
$E_c = 1.0 \cdot 10^6$ kgf/cm^2	– elastic modulus of copper;
$\alpha_s = 1.25 \cdot 10^{-5}$ 1/ ºC	– linear thermal expansion coefficient of steel;
$\alpha_c = 1.65 \cdot 10^{-5}$ 1/ ºC	– linear thermal expansion coefficient of copper;
l = 100.0 cm	– length of the vertical bar;
φ = 45 º	– angle between inclined and vertical bars;
$A_s = 5.0 \cdot 5.0$ cm^2	– cross-sectional area of a vertical steel bar;
$A_c = 5.0 \cdot 5.0$ cm^2	– cross-sectional area of an inclined copper bar;
Δt = 50 ºC	– temperature variation of the system.

Finite element model: Design model – plane hinged bar system, 3 elements of type 1. Boundary conditions are provided by imposing constraints in the support nodes in the directions of the degrees of freedom X, Z. The effect of the temperature variation of the system Δt relative to the assembly temperature is specified as uniform along the longitudinal axes of all bar elements. Number of nodes in the design model – 4.

Results in SCAD



Design and deformed models



Longitudinal force diagram N (kgf)

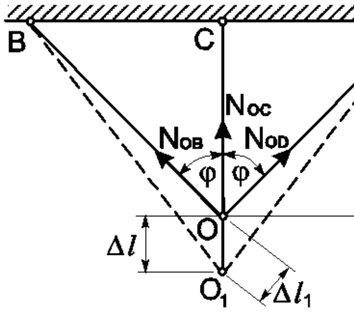
Comparison of solutions:

Parameter	Theory	SCAD	Deviations, %
Longitudinal force N (bar OC), kgf	13386.7	13386.7	0.00
Longitudinal force N (bars OB and OD), kgf	-9465.8	-9465.8	0.00

Notes: In the analytical solution, the longitudinal forces N in the bars of the system are determined according to the following formulas:

$$N_{OC} = \frac{\Delta t \cdot \left(\frac{\alpha_c}{cos^2(\varphi)} - \alpha_s \right) \cdot E_s \cdot A_s}{1 + \frac{1}{2 \cdot cos^3(\varphi)} \cdot \frac{E_s \cdot A_s}{E_c \cdot A_c}}; \qquad N_{OB} = N_{OD} = -\frac{\Delta t \cdot \left(\frac{\alpha_c}{cos^2(\varphi)} - \alpha_s \right) \cdot E_s \cdot A_s}{2 \cdot cos(\varphi) + \frac{1}{cos^2(\varphi)} \cdot \frac{E_s \cdot A_s}{E_c \cdot A_c}}.$$

Plane Hinged Bar System with Elements of the Same Material Subjected to Temperature Variation



Objective: Determination of the stress state of a plane hinged bar system with elements of the same material subjected to temperature variation.

Initial data file: **T2_v11.3.spr**

Problem formulation: Three bars of the plane system are connected by hinges in the common node (O) and are simply supported in the opposite nodes (B, C, D). Support nodes are arranged on one horizontal straight line symmetrically with respect to the vertical axis (OC), the common node lies on the vertical axis. Vertical (OC) and inclined (OB, OD) bars are made of steel. The system is subjected to the temperature variation Δt relative to the assembly temperature. Determine normal stresses σ in the cross-sections of the bars of the system.

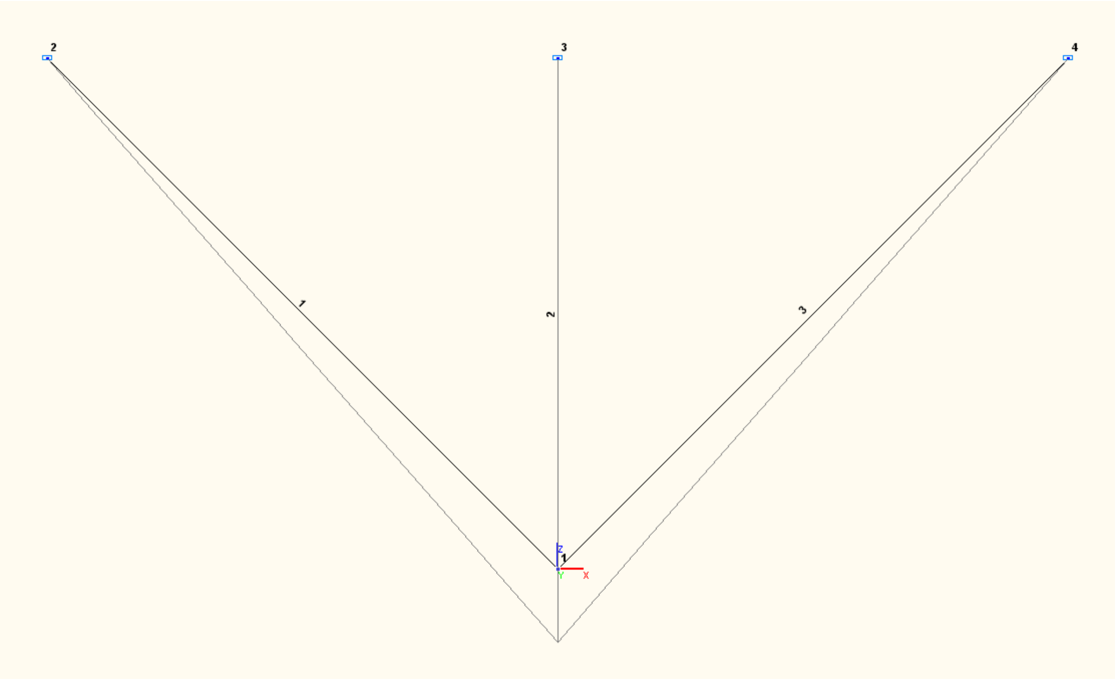
References: S.P. Timoshenko, Strength of Materials, Volume 1: Elementary Theory and Problems, Moscow, Nauka, 1965, p. 35.

Initial data:

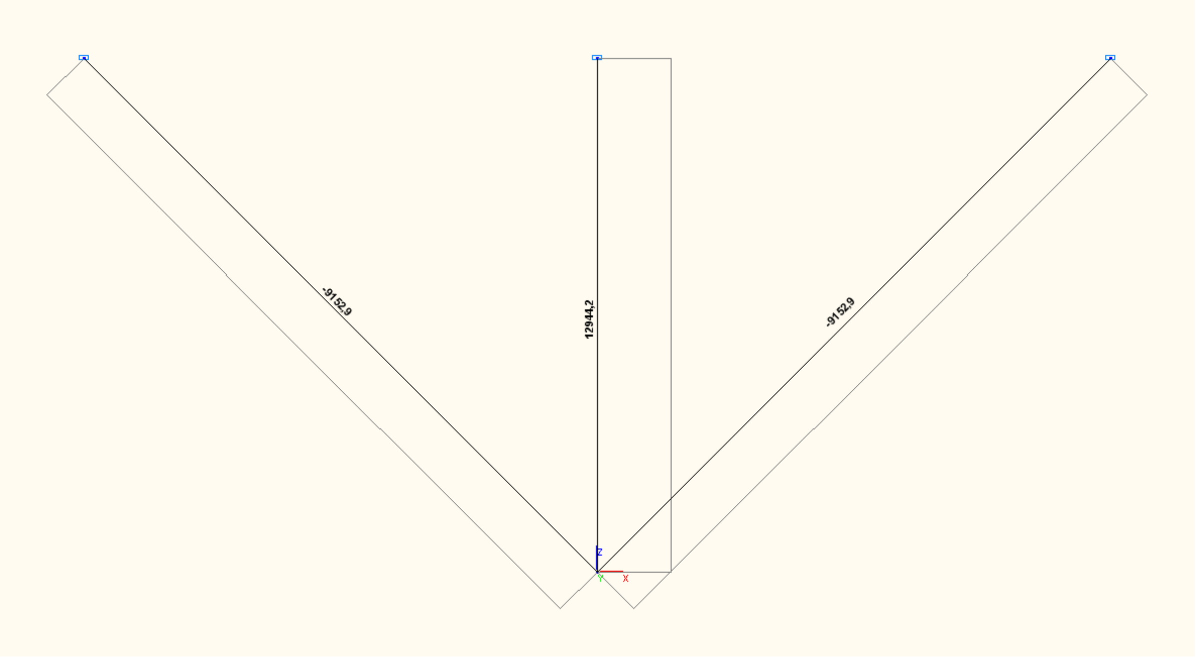
$E_s = 2.0 \cdot 10^6$ kgf/cm^2	- elastic modulus of steel;
$\alpha_s = 1.25 \cdot 10^{-5}$ 1/ ºC	- linear thermal expansion coefficient of steel;
l = 100.0 cm	- length of the vertical bar;
φ = 45 º	- angle between inclined and vertical bars;
A = 5.0·5.0 cm^2	- cross-sectional area of vertical and inclined bars;
Δt = 50 ºC	- temperature variation of the system.

Finite element model: Design model – plane hinged bar system, 3 elements of type 1. Boundary conditions are provided by imposing constraints in the support nodes in the directions of the degrees of freedom X, Z. The effect of the temperature variation of the system Δt relative to the assembly temperature is specified as uniform along the longitudinal axes of all bar elements. Number of nodes in the design model – 4.

Results in SCAD



Design and deformed models



Longitudinal force diagram N (kgf)

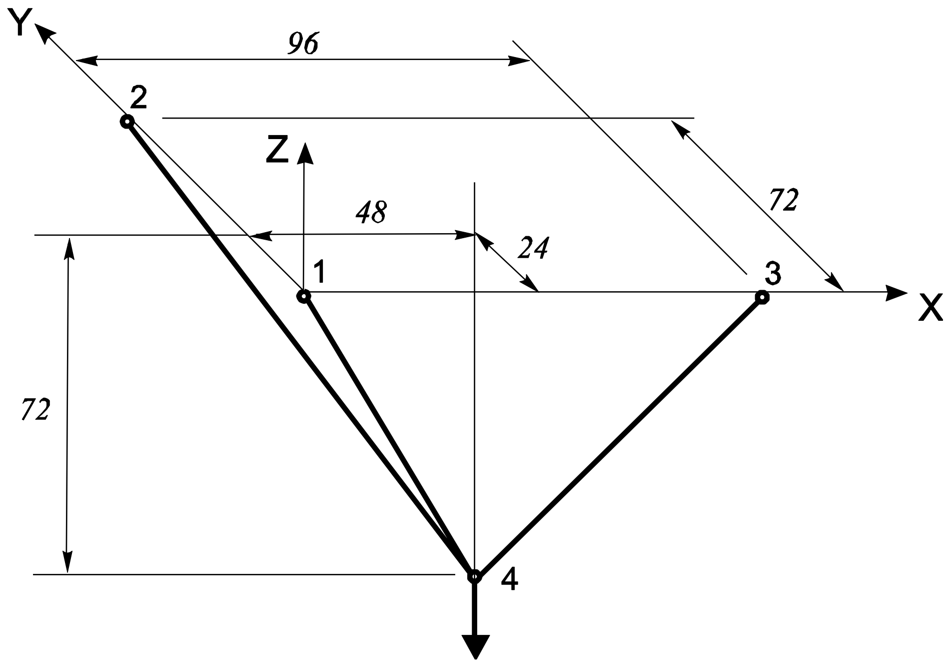
Comparison of solutions:

Parameter	Theory	SCAD	Deviations, %
Normal stresses σ (bar OC), kgf/cm^2	517.768	12944.2 / (5.0 * 5.0) = = 517.768	0.00
Normal stresses σ (bars OB and OD), kgf/cm^2	-366.116	-9152.9 / (5.0 * 5.0) = = -366.116	0.00

Notes: In the analytical solution the normal stresses σ in the cross-sections of bars of the system are determined according to the following formulas:

$$\sigma_{OC} = \frac{2 \cdot \Delta t \cdot \alpha_S \cdot E_S \cdot cos(\varphi) \cdot sin^2(\varphi)}{2 \cdot cos^3(\varphi) + 1}; \qquad \sigma_{OB} = \sigma_{OD} = \frac{\Delta t \cdot \alpha_S \cdot E_S \cdot sin^2(\varphi)}{2 \cdot cos^3(\varphi) + 1}.$$

Test CS01 Spatial Hinged Bar System Subjected to a Concentrated Force



Objective: Determination of the stress state in the elements of a spatial hinged-bar system subjected to a concentrated force.

Initial data file: **CS01_v11.3.SPR**

Problem formulation: Three bars of the spatial system are connected by hinges in a common node (4) and are simply supported in the opposite nodes (1, 2, 3). Support nodes are arranged in one horizontal plane, the common node lies outside this plane and is loaded with a vertical concentrated force *P*. Determine longitudinal forces *N* in each bar.

References: F. P. Beer, E. R. Johnston Jr., D. F. Mazurek, P. J. Cornwell, E. R. Eisenberg, Vector Mechanics for Engineers, Statics and Dynamics, New York, McGraw-Hill Co., 1962, p. 47.

Initial data:

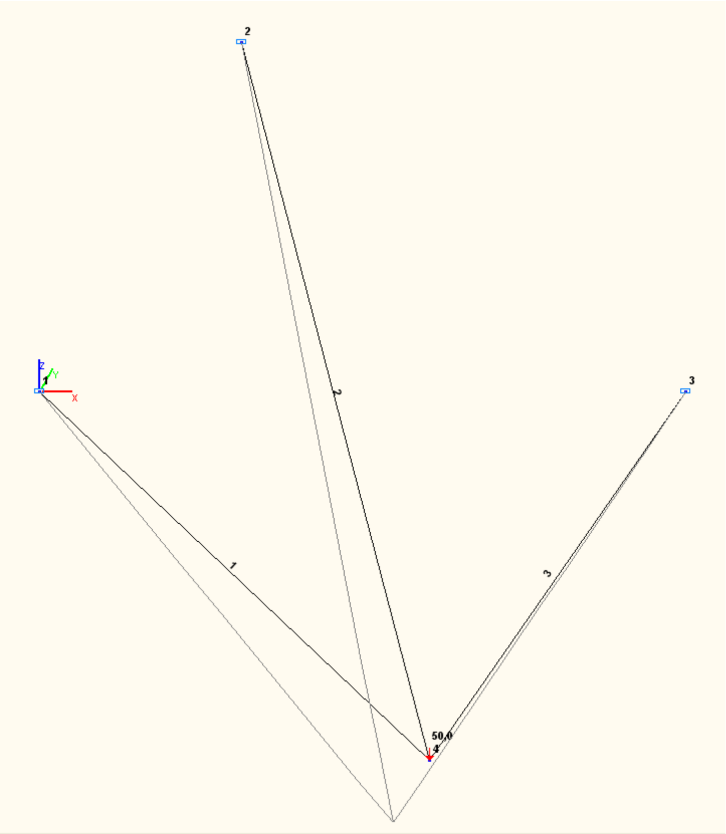
$E = 3.0 \cdot 10^7$ Pa – elastic modulus,
$A = 1.0$ m^2 – cross-sectional area of the bars;
P = 50 N – value of the concentrated force.

Finite element model: Design model - spatial hinged bar system, 3 bar elements of type 4. Boundary conditions in the support nodes are provided by imposing constraints in the directions of the degrees of freedom: X, Y, Z. Number of nodes in the design model – 4.

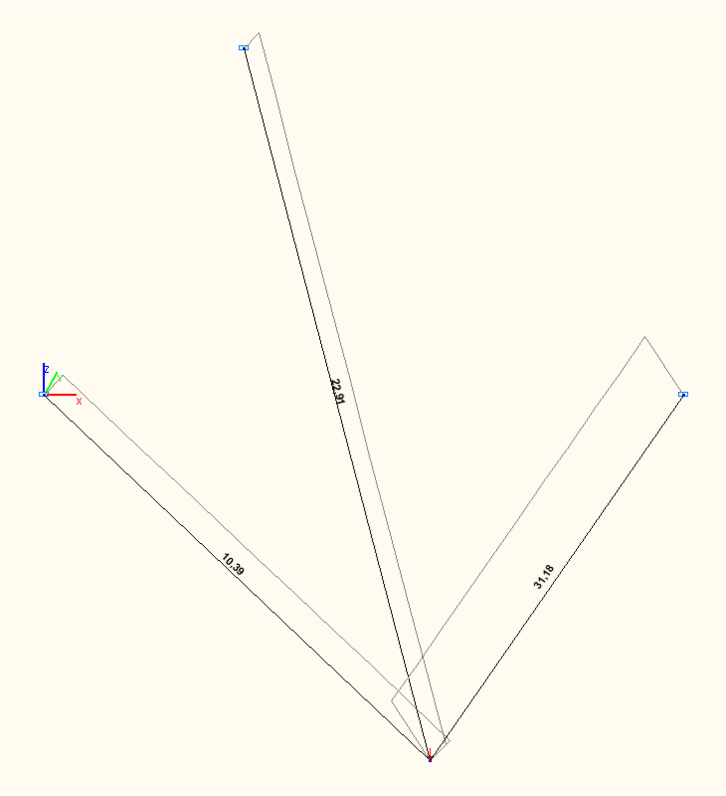
Coordinates of nodes:

Node	X (m)	Y (m)	Z (m)
1	0.0	0.0	0.0
2	0.0	72.0	0.0
3	96.0	0.0	0.0
4	48.0	24.0	-72.0

Results in SCAD



Design and deformed models



Longitudinal force diagram N (N)

Comparison of solutions:

Values of longitudinal forces *N* (N)

Bar (nodes)	**Theory**	**SCAD**	**Deviations, %**
1 (1-4)	10.39	10.39	0.00
2 (2-4)	22.91	22.91	0.00
3 (3-4)	31.18	31.18	0.00

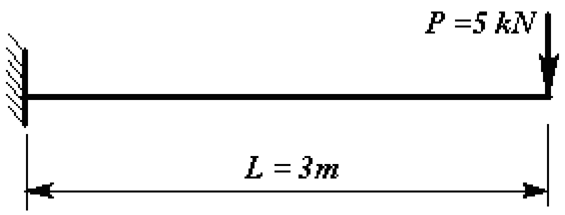
Notes: In the analytical solution, the longitudinal forces *N* in the elements of the spatial hinged-bar system subjected to a concentrated load are determined according to the following formulas:

$$N_1 = -\frac{P \cdot (x_3 \cdot y_2 - x_3 \cdot y_4 - x_4 \cdot y_2) \cdot \sqrt{x_4^2 + y_4^2 + z_4^2}}{x_3 \cdot y_2 \cdot z_4};$$

$$N_2 = -\frac{P \cdot y_4 \cdot \sqrt{x_4^2 + (y_2 - y_4)^2 + z_4^2}}{y_2 \cdot z_4};$$

$$N_3 = -\frac{P \cdot x_4 \cdot \sqrt{(x_3 - x_4)^2 + y_4^2 + z_4^2}}{x_3 \cdot z_4}.$$

Cantilever Beam Subjected to a Concentrated Load



Objective: Analysis for bending in the force plane under a concentrated force without taking into account the transverse shear deformations. The values of the maximum transverse displacement, rotation angle and bending moment are checked.

Initial data file: **Example 4.1.SPR**

Problem formulation: The cantilever beam is loaded by a concentrated force P applied to its free end. Determine the maximum values of the transverse displacement w, rotation angle θ and bending moment M.

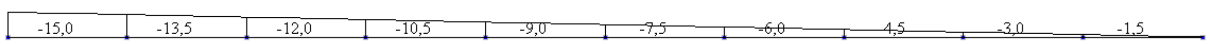
References: G.S. Pisarenko, A.P. Yakovlev, V.V. Matveev, Handbook on Strength of Materials. — Kiev: Naukova Dumka, 1988, p. 263.

Initial data:

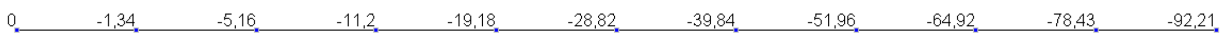
$E = 2.0\cdot10^{11}$ Pa	- elastic modulus,
$\nu = 0.3$	- Poisson's ratio,
$L = 3$ m	- beam length;
$I = 2.44\cdot10^{-6}$ m^4	- cross-sectional moment of inertia;
$P = 5$ kN	- value of the concentrated force.

Finite element model: Design model – general type system, 10 bar elements of type 5, 11 nodes.

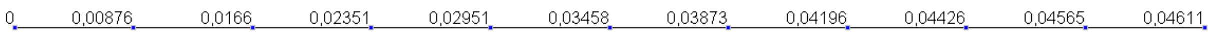
Results in SCAD:



Bending moment diagram M (kN·m)



Values of transverse displacements w(mm)



Values of rotation angles θ (rad)

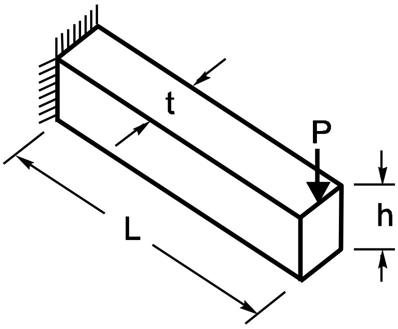
Comparison of solutions:

Parameter	Theory	SCAD	Deviations, %
Transverse displacement w, mm	-92.21	-92.21	0.00
Rotation angle Θ, rad	0.04611	0.04611	0.00
Bending moment M, kN·m	-15.0	-15.0	0.00

Notes: In the analytical solution, the maximum values of the transverse displacement *w*, rotation angle θ and bending moment *M* are determined according to the following formulas:

$$w = -\frac{P \cdot L^3}{3 \cdot E \cdot I}; \qquad \theta = \frac{P \cdot L^2}{2 \cdot E \cdot I}; \qquad M = -P \cdot L .$$

Cantilever Beam Subjected to a Concentrated Shear Force



Objective: Determination of the strain state of a cantilever beam subjected a concentrated shear force.

Initial data files:

CS06_c_v11.3.SPR	bar model
CS06_п_v11.3.SPR	plane-stress model

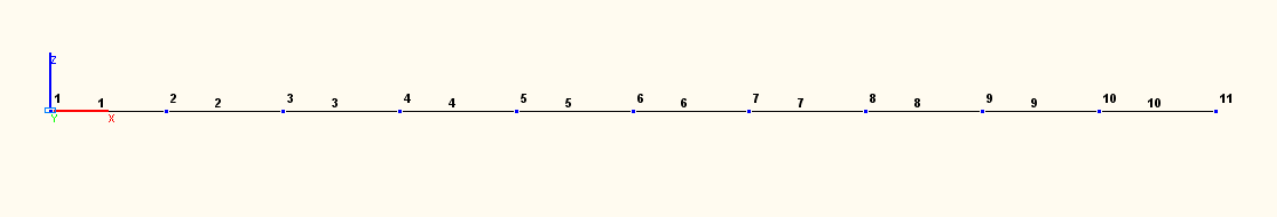
Problem formulation: The cantilever beam of a rectangular cross-section is subjected to a concentrated shear force *P* applied at its free end. Determine the displacement z of the free end of the beam taking into account the effect of the transverse shear.

Initial data:

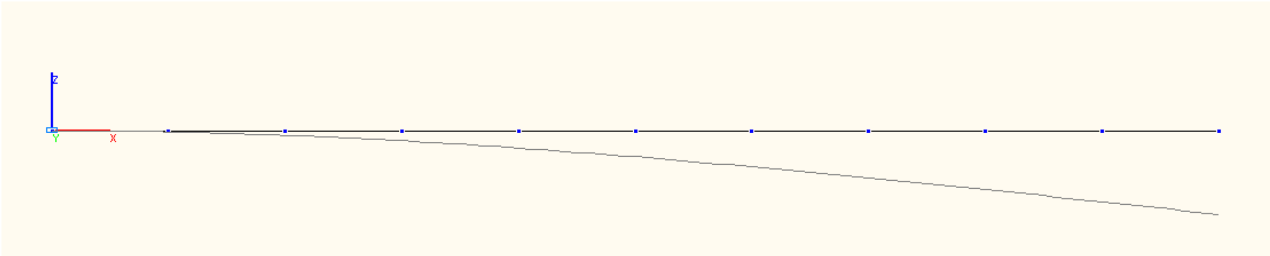
$E = 3.0 \cdot 10^7$ Pa	- elastic modulus,
ν = 0.0	- Poisson's ratio,
L = 10.0 m	- beam length;
t = 0.1 m	- width of the beam cross-section;
h = 1.0 m	- height of the beam cross-section;
k = 1.2	- shear coefficient;
P = 1.0 N	- value of the concentrated force

Finite element model: Two design models are considered:
Bar model (B), design model – plane frame, 10 elements of type 10. The spacing of the finite element mesh along the longitudinal axis is 1.0 m. Boundary conditions at the clamped end are provided by imposing constraints in the directions of the degrees of freedom: X, Z, UY. Number of nodes in the design model – 11.
Plane-stress model (P), 10 eight-node elements of type 30. The spacing of the finite element mesh along the longitudinal axis is 1.0 m. Boundary conditions at the clamped end are provided by imposing constraints in the directions of the degrees of freedom: X, Z. Number of nodes in the design model – 53.

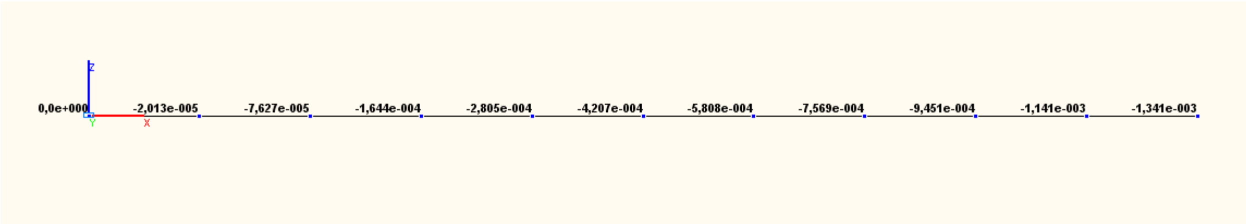
Results in SCAD



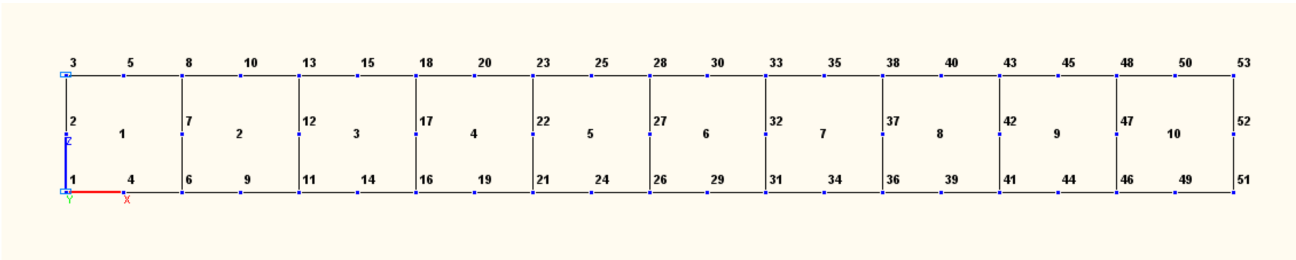
Design model. Bar model



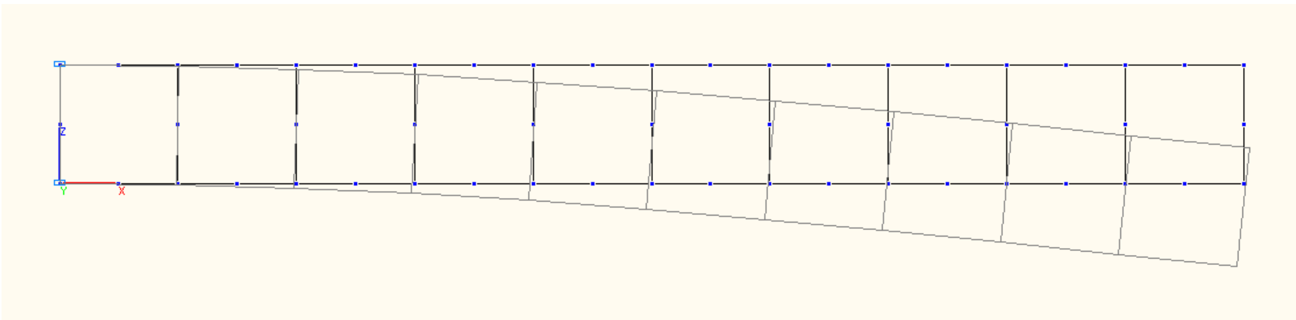
Deformed model. Bar model



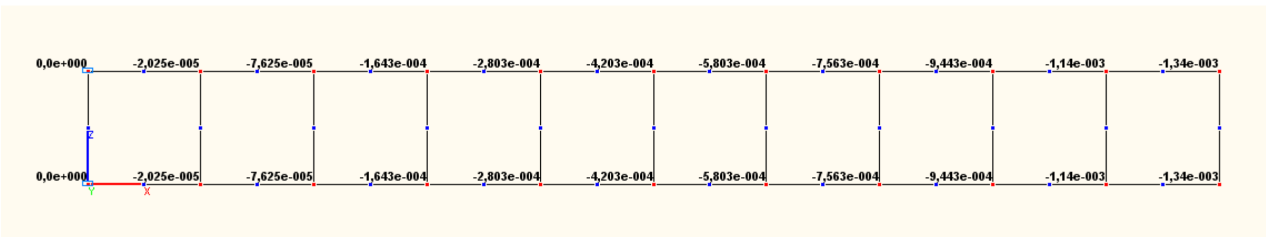
Displacements z (m). Bar model



Design model. Plane-stress model



Deformed model. Plane-stress model



Displacements z (m). Plane-stress model

Comparison of solutions:

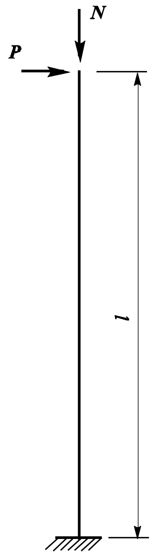
Model	Displacements z, m	Deviations, %
Bar (B)	$-1.341 \cdot 10^{-3}$	0.00

Model	Displacements z, m	Deviations, %
Plane-stress (P)	$-1.340 \cdot 10^{-3}$	0.07
Theory	$-1.341 \cdot 10^{-3}$	–

Notes: In the analytical solution, the displacement z of the free end of the beam taking into account the effect of the transverse shear is determined according to the following formula:

$$z = \frac{4 \cdot P \cdot L^3}{E \cdot t \cdot h^3} \cdot \left(1 + \frac{k \cdot (1 + \nu) \cdot h^2}{2 \cdot L^2} \right).$$

Vertical Cantilever Bar of Square Cross-Section with Longitudinal and Transverse Concentrated Loads at Its Free End



Objective: Check of the consistency of the results for models of different dimensions.

Initial data files:

File name	Description
Задача 4.9_c.SPR	Bar model
Задача 4.9_п.SPR	Shell element model
Задача 4.9_o.SPR	Solid element model

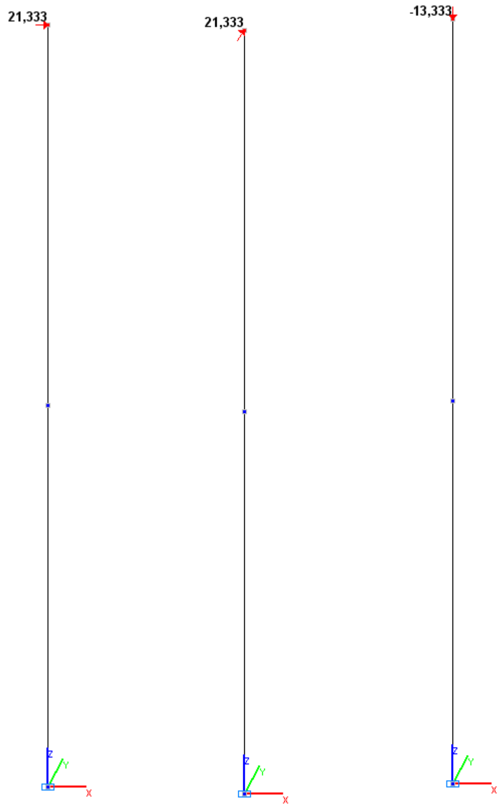
Problem formulation: Determine the displacements of the free end x, y, z and maximum stresses in the clamped section σ_z.

Initial data:

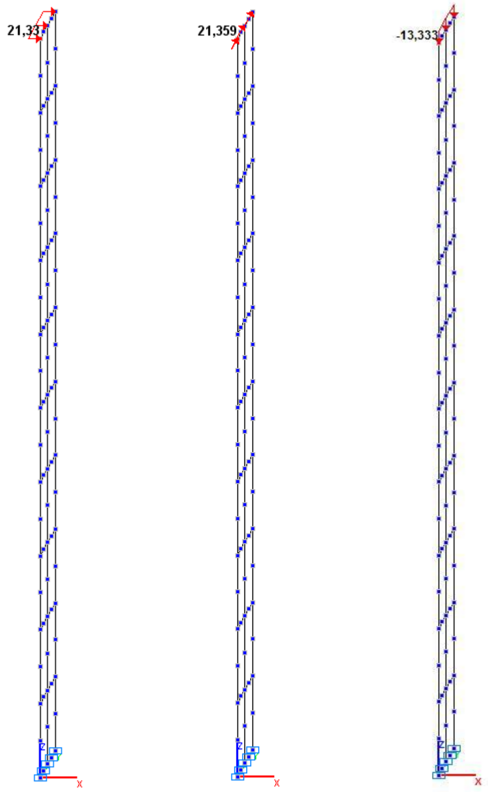
$E = 3.0 \cdot 10^7$ kPa - elastic modulus;
$\mu = 0.2$ - Poisson's ratio;
b = h = 0.5 m - cross-sectional dimensions of the cantilever bar;
l = 10 m - height of the cantilever bar;
P_x = 10 kN - value of the concentrated force acting along the X axis of the global coordinate system (loading 1);
P_y = 10 kN - value of the concentrated force acting along the Y axis of the global coordinate system (loading 2);
N = 10000 kN - value of the concentrated force acting along the Z axis of the global coordinate system (loading 3).

Finite element model: Design model – general type system. Three design models are considered:
Bar model (B), 2 elements of type 5, 3 nodes;
Shell element model (P), 20 elements of type 50, 85 nodes;
Solid element model (S), 10 elements of type 37, 128 nodes.

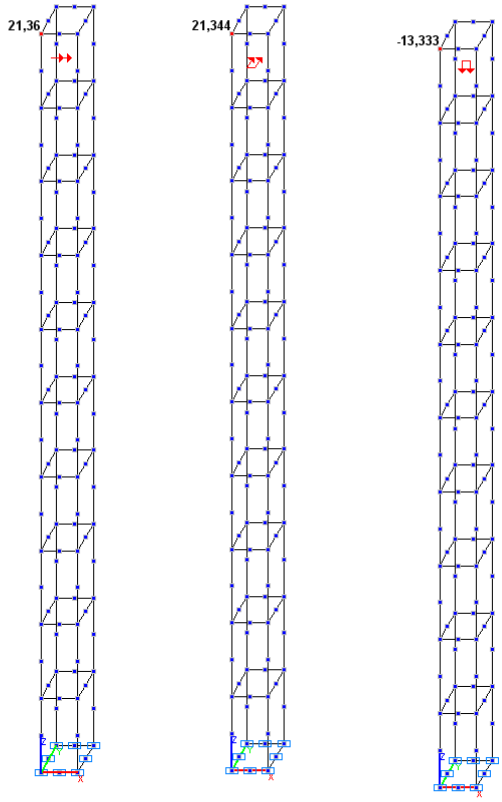
Results in SCAD



Values of the displacements x, y , z in the bar model (mm)



Values of the displacements x, y , z in the shell element model (mm)



Values of the displacements x, y , z in the solid element model (mm)

Comparison of solutions:

Model	**Loading 1**			
	Displacements x (mm)	**Deviations, %**	**Stresses σ_z (kPa)**	**Deviations, %**
Bar (B)	21.333	0.00	4800	0.00
Shell element (P)	21.330	0.01	4819	0.40
Solid element (S)	21.336	0.01	4738	1.29
Theory	21.333	—	4800	—

Model	**Loading 2**			
	Displacements y (mm)	**Deviations, %**	**Stresses σ_z (kPa)**	**Deviations, %**
Bar (B)	21.333	0.00	4800	0.00
Shell element (P)	21.359	0.12	4720	1.67
Solid element (S)	21.345	0.06	4743	1.19
Theory	21.333	—	4800	—

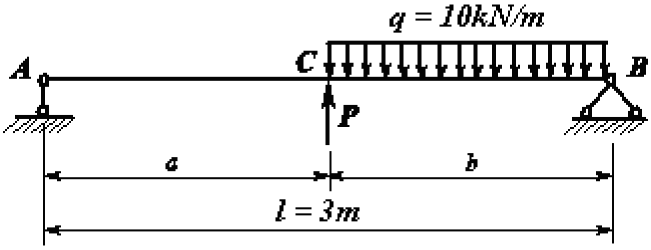
Model	**Loading 3**			
	Displacements z (mm)	**Deviations, %**	**Stresses σ_z (kPa)**	**Deviations, %**
Bar (B)	-13.333	0.00	-40000	0.00
Shell element (P)	-13.333	0.00	-40000	0.00
Solid element (S)	-13.333	0.00	-40000	0.00
Theory	-13.333	—	-40000	—

Notes: In the analytical solution for non-deformed models, the displacements of the free end x, y, z and the maximum stresses in the clamped section σ_z are determined according to the following formulas:

$$x = \frac{4 \cdot Px \cdot l^3}{E \cdot b \cdot h^3}; \qquad y = \frac{4 \cdot Py \cdot l^3}{E \cdot h \cdot b^3}; \qquad z = \frac{N \cdot l}{E \cdot b \cdot h};$$

$$\sigma_z(Px) = \frac{6 \cdot Px \cdot l}{b \cdot h^2}; \qquad \sigma_z(Py) = \frac{6 \cdot Py \cdot l}{h \cdot b^2}; \qquad \sigma_z(N) = \frac{N}{b \cdot h}.$$

Simply Supported Beam Subjected to a Concentrated Force and Uniformly Distributed Pressure



Objective: Combined loading (lateral pressure, concentrated force) in one plane without taking into account the transverse shear deformations. Displacements and forces are checked.

***Initial data file*: 4.3.SPR**

Problem formulation: The simply supported beam is subjected to a concentrated force *P* and uniformly distributed pressure *q*. Displacements *w*, rotation angles *θ*, shear forces *Q* and bending moments *M* are determined.

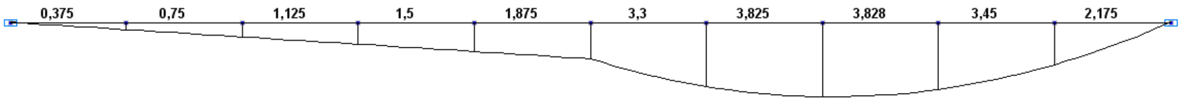
References: G.S. Pisarenko, A.P. Yakovlev, V.V. Matveev, Handbook on Strength of Materials. — Kiev: Naukova Dumka, 1988.

Initial data:

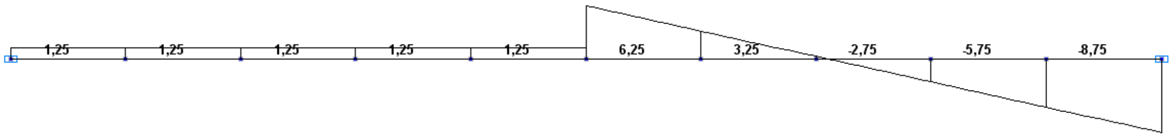
$E = 2.0 \cdot 10^{11}$ Pa	- elastic modulus;
$\mu = 0.3$	- Poisson's ratio;
l = 3 m	- beam length;
$F = 14.2 \cdot 10^{-4}$ m^2	- cross-sectional area;
$I = 2.44 \cdot 10^{-6}$ m^4	- moment of inertia;
P = −5 kN	- value of the concentrated force;
q = 10 kN/m	- value of pressure;
a = b = 1.5 m	- geometric size.

Finite element model:
Design model – plane frame, 10 bar elements, 11 nodes.

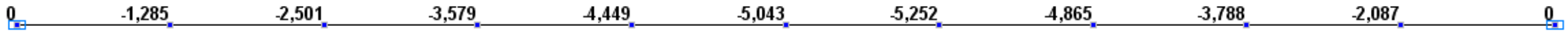
Results in SCAD



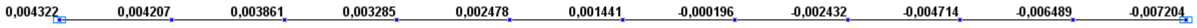
*Bending moment diagram M (kN*m)*



Shear force diagram Q (kN)



Values of transverse displacements w (mm)



Values of rotation angles θ (rad)

Comparison of solutions:

Parameter	Theory	SCAD	Deviations, %
Deflection in the point C, mm	-5.043	-5.043	0.00
Rotation angle in the point B, rad	$-7.204\cdot10^{-3}$	$-7.204\cdot10^{-3}$	0.00
Bending moment in the point C, kN·m	1.875	1.875	0.00
Shear force in the point A, kN	1.25	1.25	0.00
Shear force in the point B, kN	-8.75	-8.75	0.00

Notes: In the analytical solution, the deflection in the point C can be calculated according to the following formula (“Handbook on Strength of Materials” p. 295, 297):

$$w_C = \frac{P\cdot a^2\cdot b^2}{3\cdot E\cdot I\cdot(a+b)} + \frac{q\cdot a\cdot b^3\cdot(4\cdot a+b)}{24\cdot E\cdot I\cdot(a+b)}.$$

The rotation angle in the point B can be calculated according to the following formula (“Handbook on Strength of Materials” p. 295, 297):

$$\theta_B = \frac{P\cdot b\cdot(2\cdot a^2+a\cdot b)}{6\cdot E\cdot I\cdot(a+b)} - \frac{q\cdot b^2\cdot(4\cdot a^2+4\cdot a\cdot b+b^2)}{24\cdot E\cdot I\cdot(a+b)}.$$

The bending moment in the point C can be calculated according to the following formula:

$$M_C = \frac{P\cdot a\cdot b}{a+b} + \frac{q\cdot a\cdot b^2}{2\cdot(a+b)}.$$

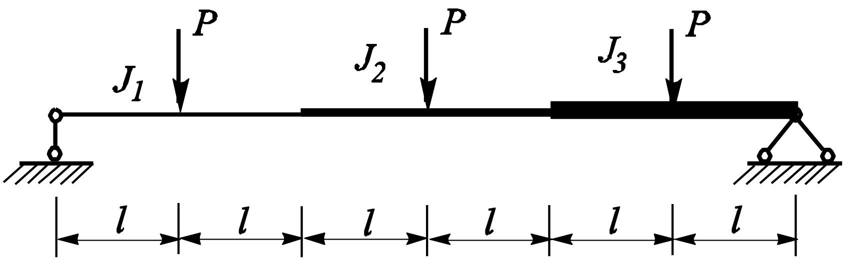
The shear force in the point A can be calculated according to the following formula:

$$Q_A = \frac{P\cdot b}{a+b} + \frac{q\cdot b^2}{2\cdot(a+b)}.$$

The shear force in the point B can be calculated according to the following formula:

$$Q_B = -\frac{P\cdot a}{a+b} - \frac{q\cdot(2\cdot a+b)\cdot b}{2\cdot(a+b)}.$$

Three-Step Simply Supported Beam Subjected to Concentrated Forces



Objective: Strain state of a three-step simply supported beam subjected to concentrated forces without taking into account the transverse shear deformations. Transverse displacements and rotation angles are checked.
Initial data file: **4.5.SPR**

Problem formulation: The three-step simply supported beam is subjected to three concentrated forces *P*. Determine the rotation angles of support sections and transverse displacements in the force application points.

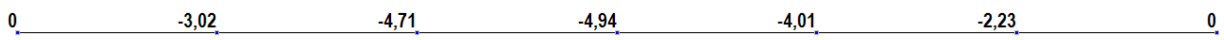
References: G.S. Pisarenko, A.P. Yakovlev, V.V. Matveev, Handbook on Strength of Materials. — Kiev: Naukova Dumka, 1988.

Initial data:

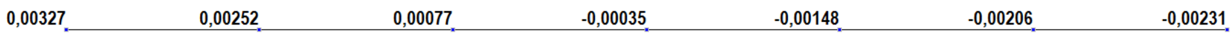
$E = 2.0\cdot10^{11}$ Pa-	- elastic modulus,
$l = 1$ m	- half length of the beam span of each section;
$F = 1\cdot10^{-2}$ m^2	- cross-sectional area;
$I_1 = 5\cdot10^{-6}$ m^4	- moment of inertia;
P = 1 kN	- load value.
$I_1 : I_2 : I_3 = 1 : 2 : 3$	
$F_1 : F_2 : F_3 = 1 : 2 : 3$	

Finite element model: Design model – general type system, 6 bar elements of type 5, 7 nodes.

Results in SCAD



Values of transverse displacements w (mm)



Values of rotation angles θ (rad)

Comparison of solutions:

Parameter	Theory	SCAD	Deviations, %

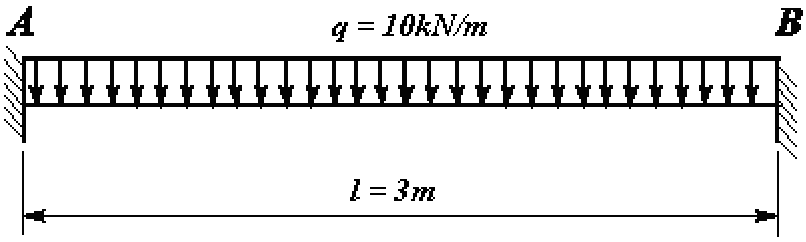
Transverse displacements, mm *w* (*l*) *w* (3*l*) *w* (5*l*)	 -3.02 -4.94 -2.23	 -3.02 -4.94 -2.23	 0.00 0.00 0.00
Rotation angles, rad θ (0) θ (6*l*)	 0.00327 -0.00231	 0.00327 -0.00231	 0.00 0.00

Notes: In the analytical solution, the rotation angles of support sections and deflections in the force application points are determined according to the following formulas:

$$w(l) = -\frac{653 \cdot P \cdot l^3}{216 \cdot E \cdot I_1}; \qquad w(3 \cdot l) = -\frac{89 \cdot P \cdot l^3}{18 \cdot E \cdot I_1}; \qquad w(5 \cdot l) = -\frac{481 \cdot P \cdot l^3}{216 \cdot E \cdot I_1};$$

$$\theta(0) = \frac{707 \cdot P \cdot l^2}{216 \cdot E \cdot I_1}; \qquad \theta(6 \cdot l) = -\frac{499 \cdot P \cdot l^2}{216 \cdot E \cdot I_1}.$$

Doubly Clamped Beam Subjected to a Uniformly Distributed Load



Objective: Loading of a doubly clamped beam in one plane without taking into account the transverse shear deformations. The values of the maximum transverse displacement and the bending moments are checked.

Initial data file: **4.4.SPR**

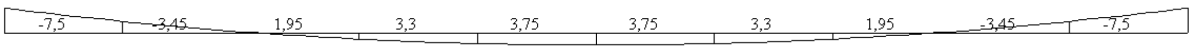
Problem formulation: The doubly-clamped beam is subjected to a uniformly distributed load *q*. Determine the maximum transverse displacement *w* and bending moments *M*.

References: G.S. Pisarenko, A.P. Yakovlev, V.V. Matveev, Handbook on Strength of Materials. — Kiev: Naukova Dumka, 1988.

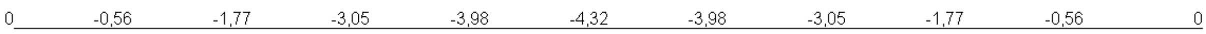
Initial data:
$E = 2.0 \cdot 10^{11}$ Pa - elastic modulus,
$\mu = 0.3$ - Poisson's ratio,
l = 3 m - beam length;
$F = 14.2 \cdot 10^{-4}$ m^2 - cross-sectional area;
$I = 2.44 \cdot 10^{-6}$ m^4 - moment of inertia;
q = 10 kN/m - load value.

Finite element model: Design model – plane frame, 10 bar elements of type 2, 11 nodes.

Results in SCAD



*Bending moment diagram M (kN*m)*



Values of transverse displacements w (mm).

Comparison of solutions:

Parameter	Theory	SCAD	Deviations, %
Transverse displacement in the middle of the beam span, mm	-4.32	-4.32	0.00
Bending moment in the middle of the beam span, kN·m	3.75	3.75	0.00
Bending moment at the beam support, kN·m	-7.5	-7.5	0.00

Notes: In the analytical solution, the deflection at the center of the beam can be calculated according to the following formula ("Handbook on Strength of Materials" p. 352):

$$w = -\frac{q \cdot l^4}{384 \cdot E \cdot I};$$

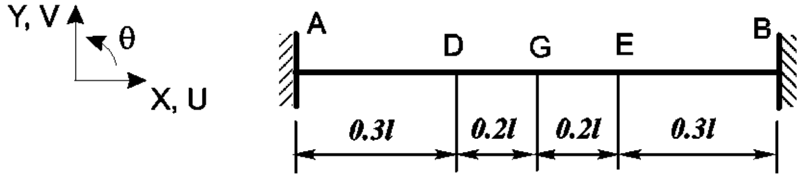
Bending moments at the clamping are calculated according to the following formula:

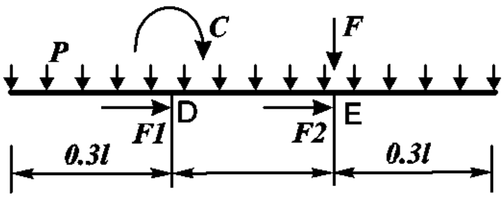
$$M = -\frac{q \cdot l^2}{12};$$

Bending moment in the middle of the beam:

$$M = \frac{q \cdot l^2}{24}.$$

Doubly Clamped Beam Subjected to a Uniformly Distributed Load, Concentrated Longitudinal and Shear Forces and a Bending Moment





Objective: Determination of the stress-strain state of a doubly clamped beam subjected to a uniformly distributed load, concentrated longitudinal and shear forces and a bending moment.

Initial data file: **SSLL01_v11.3.SPR**

Problem formulation: The doubly clamped beam is subjected to a load *P* uniformly distributed over the entire length of the span *l*, unidirectional concentrated longitudinal forces *F1* and *F2*, applied at the distance of 0.3*l* from the left and right end respectively, concentrated shear force F, applied at the distance of 0.3*l* from the right end, and a concentrated bending moment C, applied at the distance of 0.3*l* from the left end. Determine the vertical displacement Z, longitudinal force *N* and bending moment *M* in the middle of the beam span (point G), and the horizontal reaction at the left end of the beam *H* (point A).

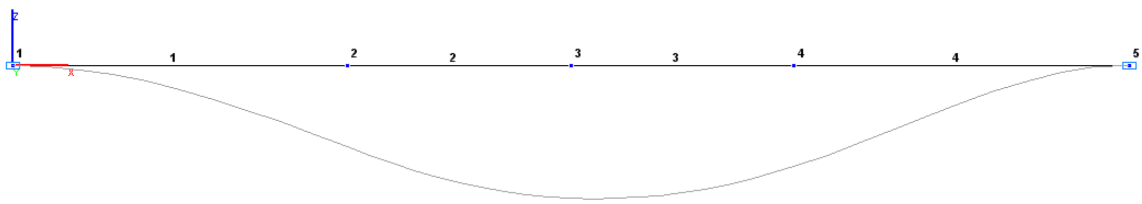
References: S. Timoshenko, Resistance des materiaux, t.1, Paris, Eyrolles, 1976, p. 26. M. Courtand et P. Lebelle, Formulaire du beton arme, t.2, Paris, Eyrolles, 1976, p. 219.

Initial data:

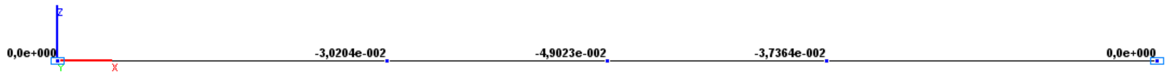
$E = 2.0\cdot10^{11}$ Pa	- elastic modulus,
$\mu = 0.2$	- Poisson's ratio,
$l = 1.0$ m	- beam length;
$J = 1.7\cdot10^{-8}$ m^4	- cross-sectional moment of inertia cross-sectional moment of inertia;
P = 24000 N/m	- value of the uniformly distributed load;
F1 = 30000 N	- value of the concentrated longitudinal force;
F2 = 10000 N	- value of the concentrated longitudinal force;
F = 20000 N	- value of the concentrated shear force;
C = 24000 N·m	- value of the concentrated bending moment.

Finite element model: Design model – general type system, 4 bar elements of type 10. Boundary conditions at the clamped ends are provided by imposing constraints in the directions of the degrees of freedom: X, Y, Z, UX, UY, UZ. Number of nodes in the design model – 5.

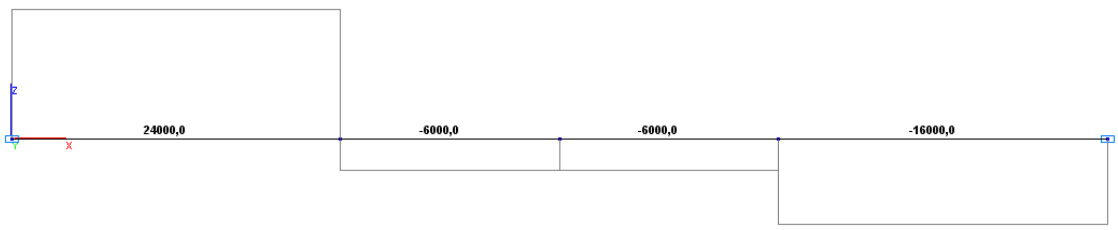
Results in SCAD



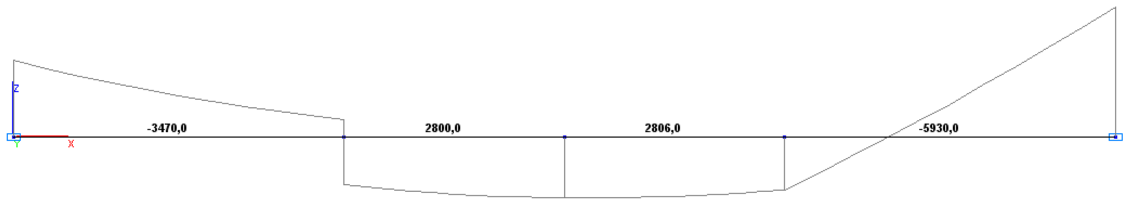
Design and deformed models



Values of vertical displacements Z (m)



Longitudinal force diagram N (N)

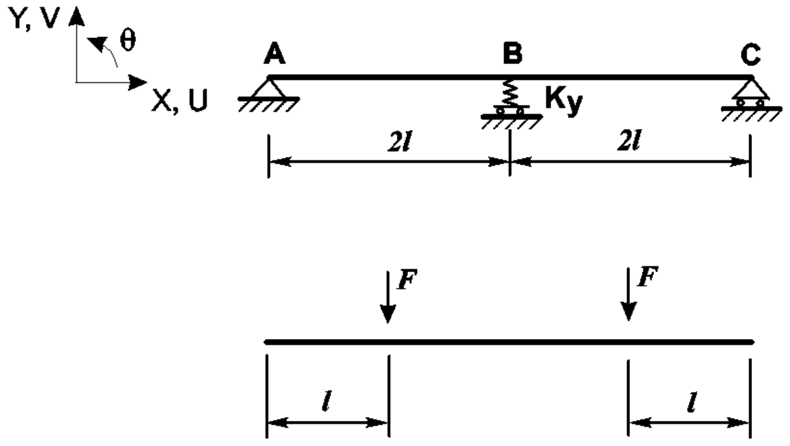


*Bending moment diagram M (kN*m)*

Comparison of solutions:

Parameter	Theory	SCAD	Deviations, %
Vertical displacement Z (point G), m	$-4.9023\cdot10^{-2}$	$-4.9000\cdot10^{-2}$	0.05
Longitudinal force *N* (point G), N	-6000.0	-6000.0	0.00
Bending moment *M* (point G), N·m	2800.0	2800.0	0.00
Horizontal reaction *H* (point A), N	24000.0	24000.0	0.00

Two-Span Simply Supported Beam with an Intermediate Compliant Support Subjected to Concentrated Shear Forces Applied in the Middle of the Spans



Objective: Determination of the stress-strain state of a two-span simply supported beam with an intermediate compliant support subjected to concentrated shear forces applied in the middle of the spans.

Initial data file: **SSLL03_v11.3.SPR**

Problem formulation: The two-span simply supported beam with an intermediate compliant support is subjected to concentrated shear forces *F*, applied in the middle of the spans (at the distance *l* from the end supports). Determine the vertical displacement Z and the vertical reaction *N* of the intermediate compliant support, and the bending moment *M* in the beam above the intermediate compliant support (point B).

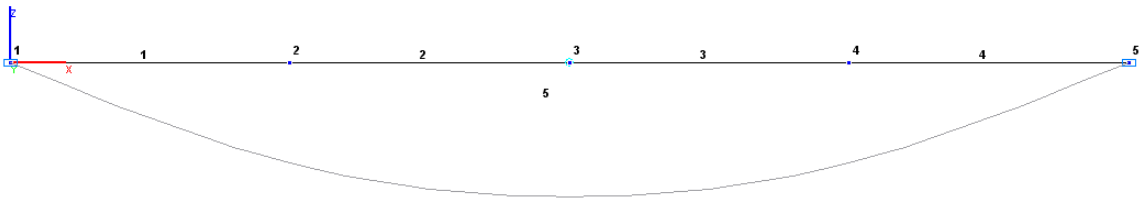
References: C. Massonnet, Application des ordinateurs au calcul des structures, Paris, Eyrolles, 1968, p. 233.

Initial data:

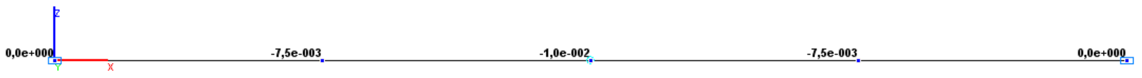
$E = 2.1 \cdot 10^{11}$ Pa	- elastic modulus,
$2 \cdot l = 6.0$ m	- length of the beam span;
$A = 0.4762 \cdot 10^{-3}$ m^2	- cross-sectional area;
$I = 6{,}3 \cdot 10^{-4}$ m^4	- cross-sectional moment of inertia;
$k = 2.1 \cdot 10^{11}$ N/m	- stiffness of the intermediate compliant support;
$F = 4.2 \cdot 10^4$ N	- value of the concentrated shear forces.

Finite element model: Design model – plane frame, 4 bar elements of type 2. Boundary conditions are provided by imposing constraints in the directions of the degrees of freedom: X, Z – for the left support; Z – for the right support, and by imposing a constraint of finite rigidity in the direction of the degree of freedom Z – for the intermediate support (member type 51). Number of nodes in the design model – 5.

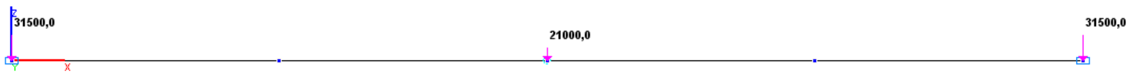
Results in SCAD



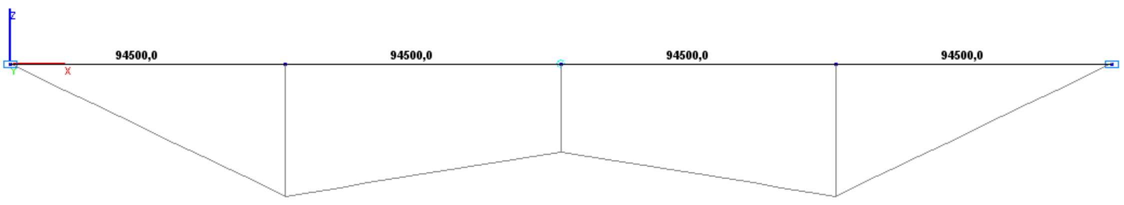
Design and deformed models



Values of vertical displacements Z (m)



Values of vertical support reactions N (N)



*Bending moment diagram M (kN*m)*

Comparison of solutions:

Parameter	Theory	SCAD	Deviations, %
Vertical displacement Z (point B), m	$-1.0000\cdot10^{-2}$	$-1.0000\cdot10^{-2}$	0.00
Vertical reaction *H* (point B), N	21000.0	21000.0	0.00
Bending moment *M* (point B), N·m	63000.0	63000.0	0.00

Beam on the Elastic Horizontal Subgrade Subjected to Concentrated Vertical Forces

Objective: Determination of the stress-strain state of a beam on the elastic horizontal subgrade subjected to concentrated vertical forces.

Initial data files:

File name	Description
SSLL15_var_1_v11.3.SPR	Design model – bar elements on the elastic subgrade
SSLL15_var_2_v11.3.SPR	Design model – bar elements on elastic supports in the form of elements of constraints of finite rigidity of type 51

Problem formulation: The beam on the elastic horizontal subgrade with the stiffness k constant along the length is subjected to three concentrated vertical forces of the same value F, applied at the edges (points A and B) and in the middle of the span (point C). Determine the vertical displacements Z in the middle of the beam span (point C) and at its edges (points A and B), rotation angles UY of the beam edges, as well as the bending moment *M* in the middle of the beam span.

References: M. Courtand et P. Lebelle, Formulaire du beton arme, t.2, Paris, Eyrolles,1976, p. 382.

Initial data:

$E = 2.1 \cdot 10^{11}$ Pa	- elastic modulus;
$l = 0.5 \cdot \pi \cdot (10.0)^{0.5} = 4.967294133$ m	- beam length;
$b = 1.0$ m	- beam width;
$I_y = 1.0 \cdot 10^{-4}$ m^4	- cross-sectional moment of inertia of the beam;
$k_z = 8.4 \cdot 10^5$ N/m^3	- subsoil parameter;
$F = 1.0 \cdot 10^4$ N	- value of the concentrated vertical force.

Finite element model: Two variants of the design model are considered.

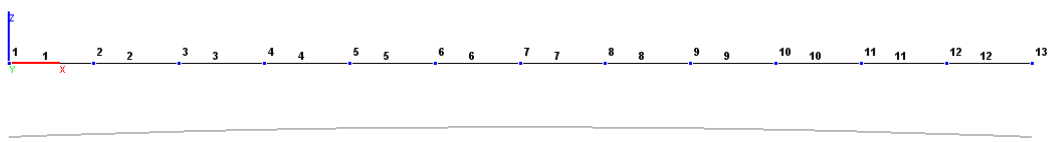
Variant 1:

Design model – grade beam / plate, 12 bar elements of type 3 on the elastic subgrade directed along the Z1 axis of the local coordinate system. Number of nodes in the design model – 13.

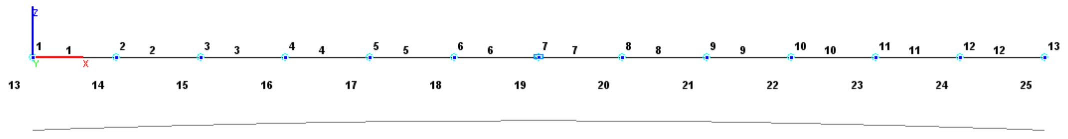
Variant 2:

Design model – grade beam / plate, 12 bar elements of type 3 on the elastic supports in the form of 13 elements of constraints of finite rigidity of type 51 directed along the Z axis of the global coordinate system. Stiffness of intermediate elastic supports: $k_z \cdot b \cdot l/12 = 347711$ N/m, stiffness of end elastic supports: $0.5 \cdot k_z \cdot b \cdot l/12 = 173855$ N/m. In order to prevent the dimensional instability of the system, a constraint in the direction of the degree of freedom UX is imposed along the beam symmetry axis and the minimum torsional stiffness of the beam is introduced $GI_x = 1.0$ N·m^2. Number of nodes in the design model – 13.

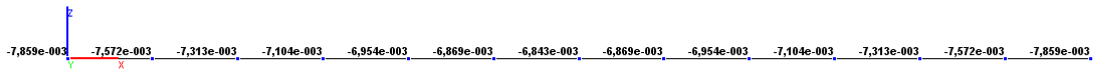
Results in SCAD



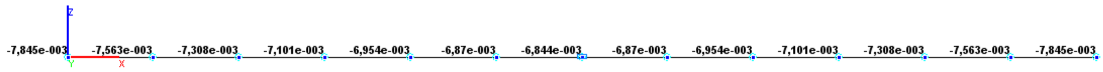
Design and deformed models. Variant 1



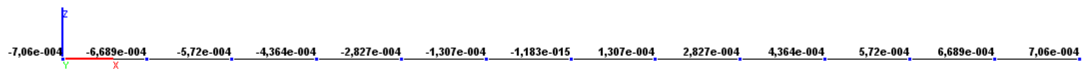
Design and deformed models. Variant 2



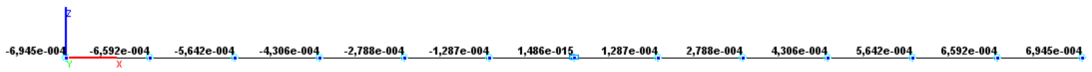
Values of vertical displacements Z (m) for the design model according to variant 1



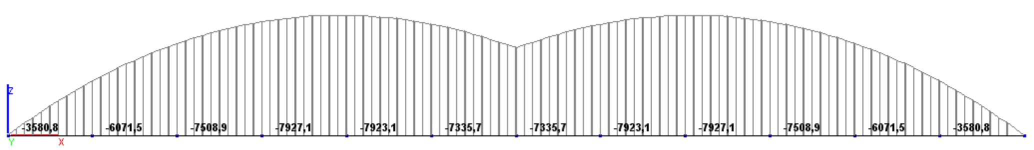
Values of vertical displacements Z (m) for the design model according to variant 2



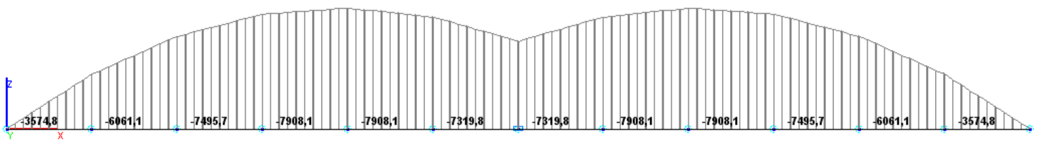
Values of rotation angles UY (rad) for the design model according to variant 1



Values of rotation angles UY (rad) for the design model according to variant 2



Values of bending moments M (N·m) for the design model according to variant 1

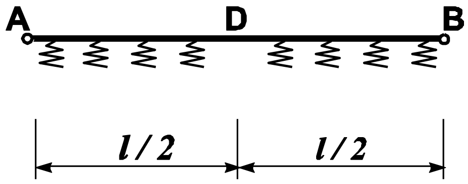


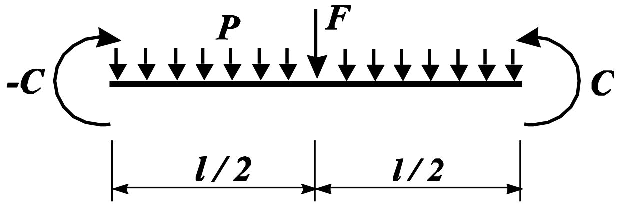
Values of bending moments M (N·m) for the design model according to variant 2

Comparison of solutions:

Parameter	**Theory**	**SCAD DM according to variant 1**	**Deviations, %**	**SCAD DM according to variant 2**	**Deviations, %**
Vertical displacement Z_C, m	$-6.844 \cdot 10^{-3}$	$-6.843 \cdot 10^{-3}$	0.01	$-6.844 \cdot 10^{-3}$	0.00
Vertical displacement Z_A, m	$-7.854 \cdot 10^{-3}$	$-7.859 \cdot 10^{-3}$	0.06	$-7.845 \cdot 10^{-3}$	0.11
Rotation angle UY_A, rad	$-7.060 \cdot 10^{-4}$	$-7.060 \cdot 10^{-4}$	0.00	$-6.945 \cdot 10^{-4}$	1.63
Bending moment M_C, N·m	-5759.0	-5758.8	0.00	-5742.6	0.28

Simply Supported Beam on the Elastic Horizontal Subgrade Subjected to a Vertical Uniformly Distributed Load, Concentrated Vertical Force and Bending Moment





Objective: Determination of the stress-strain state of a simply supported beam on the elastic horizontal subgrade subjected to a vertical uniformly distributed load, concentrated force and bending moment.

Initial data files:

File name	Description
SSLL16_var_1_v11.3.SPR	Design model – bar elements on the elastic subgrade
SSLL16_var_2_v11.3.SPR	Design model – bar elements on elastic supports in the form of elements of constraints of finite rigidity of type 51

Problem formulation: The simply supported beam on the elastic horizontal subgrade with the stiffness k constant along the length is subjected to a vertical uniformly distributed load P, concentrated vertical force F, applied in the middle of the span (point D) and concentrated bending moments –C and C, applied at the edges (points A and B). Determine the vertical displacement Z in the middle of the beam span (point D), rotation angles UY of the beam edges (points A and B), as well as the bending moment *M* in the middle of the beam span and the shear force *Q* at the edge of the beam.

References: M. Courtand et P. Lebelle, Formulaire du beton arme, t.2, Paris, Eyrolles,1976, p. 385.

Initial data:

$E = 2.1 \cdot 10^{11}$ Pa - elastic modulus;
$l = 0.5 \cdot \pi \cdot (10.0)^{0.5} = 4.967294133$ m - beam length;
$b = 1.0$ m - beam width;
$I_y = 1.0 \cdot 10^{-4}$ m^4 - cross-sectional moment of inertia of the beam;
$k_z = 8.4 \cdot 10^5$ N/m^3 - subsoil parameter;
$P = 5.0 \cdot 10^3$ N/m - value of the vertical uniformly distributed load;
$F = 1.0 \cdot 10^4$ N - value of the concentrated vertical force;
$C = 1.5 \cdot 10^4$ N·m - value of the concentrated bending moment.

Finite element model: Two variants of the design model are considered.

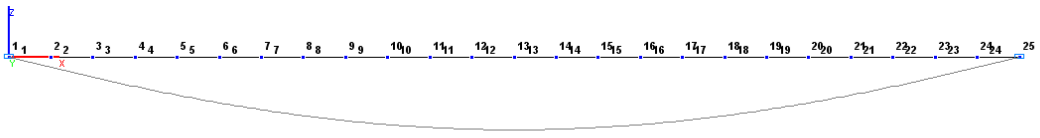
Variant 1:

Design model – grade beam / plate, 24 bar elements of type 3 on the elastic subgrade directed along the Z1 axis of the local coordinate system. Boundary conditions are provided by imposing constraints in the direction of the degree of freedom Z for roller support nodes. Number of nodes in the design model – 25.

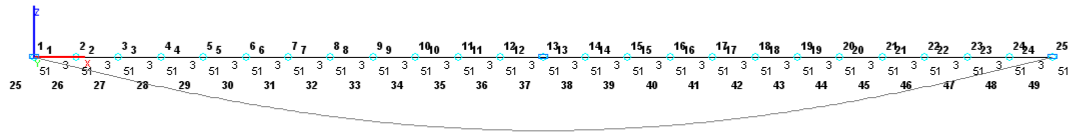
Variant 2:

Design model – grade beam / plate, 24 bar elements of type 3 on the elastic supports in the form of 25 elements of constraints of finite rigidity of type 51 directed along the Z axis of the global coordinate system. Stiffness of intermediate elastic supports: $k_z \cdot b \cdot l/24 = 173855$ N/m, stiffness of end elastic supports: $0.5 \cdot k_z \cdot b \cdot l/12 = 86928$ N/m. Boundary conditions are provided by imposing constraints in the direction of the degree of freedom Z for roller support nodes. In order to prevent the dimensional instability of the system, a constraint in the direction of the degree of freedom UX is imposed along the beam symmetry axis and the minimum torsional stiffness of the beam is introduced $GI_x = 1.0$ $N \cdot m^2$. Number of nodes in the design model – 25.

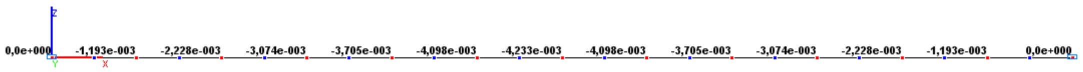
Results in SCAD



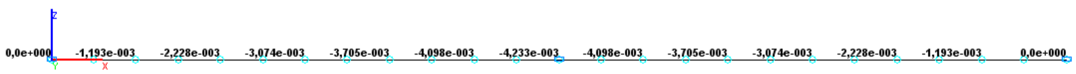
Design and deformed models. Variant 1



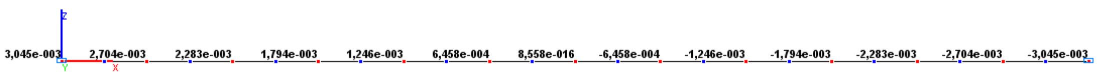
Design and deformed models. Variant 2



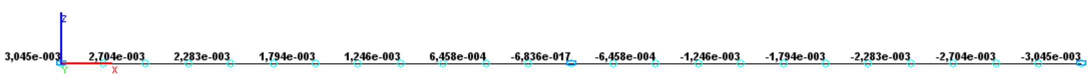
Values of vertical displacements Z (m) for the design model according to variant 1



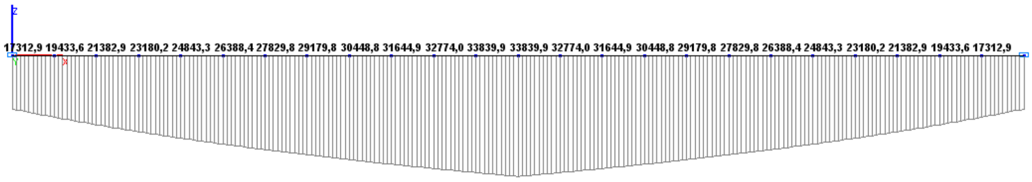
Values of vertical displacements Z (m) for the design model according to variant 2



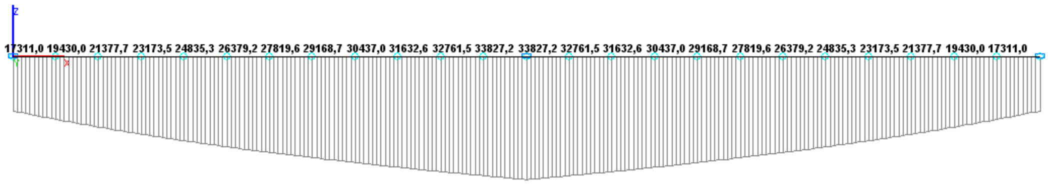
Values of rotation angles UY (rad) for the design model according to variant 1



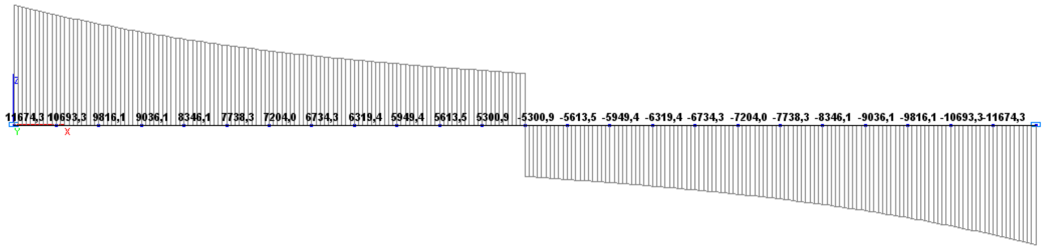
Values of rotation angles UY (rad) for the design model according to variant 2



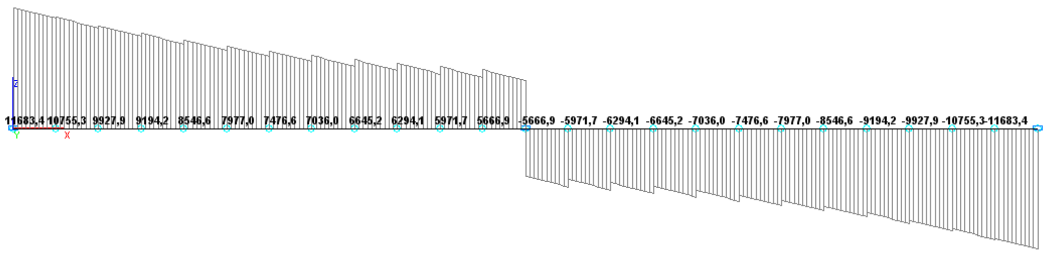
Values of bending moments M (N·m) for the design model according to variant 1



Values of bending moments M (N·m) for the design model according to variant 2



Values of shear forces Q (N) for the design model according to variant 1

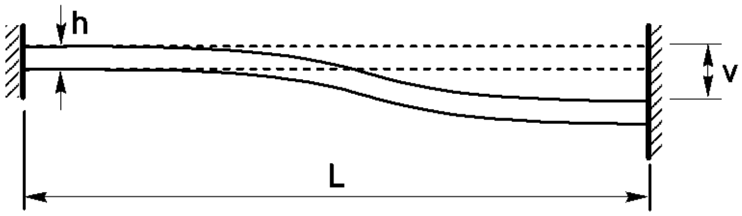


Values of shear forces Q (N) for the design model according to variant 2

Comparison of solutions:

Parameter	**Theory**	**SCAD DM according to variant 1**	**Deviations, %**	**SCAD DM according to variant 2**	**Deviations, %**
Vertical displacement Z_D, m	$-4.233 \cdot 10^{-3}$	$-4.233 \cdot 10^{-3}$	0.00	$-4.233 \cdot 10^{-3}$	0.00
Rotation angle UY_A, rad	$3.045 \cdot 10^{-3}$	$3.045 \cdot 10^{-3}$	0.00	$3.045 \cdot 10^{-3}$	0.00
Bending moment M_D, N·m	33840.0	33839.9	0.00	33827.2	0.04
Shear force Q_A, N	11674.0	11674.3	0.00	11683.4	0.08

Doubly Clamped Beam Subjected to the Transverse Displacement of One of its Ends



Objective: Determination of the stress state of a doubly clamped beam subjected to the transverse displacement of one of its ends.

Initial data file: **CS09_v11.3.SPR**

Problem formulation: The doubly clamped beam of a rectangular cross-section is subjected to a transverse displacement v of one of its ends. Determine the shear force *Q* and the bending moment *M* at the displaced end.

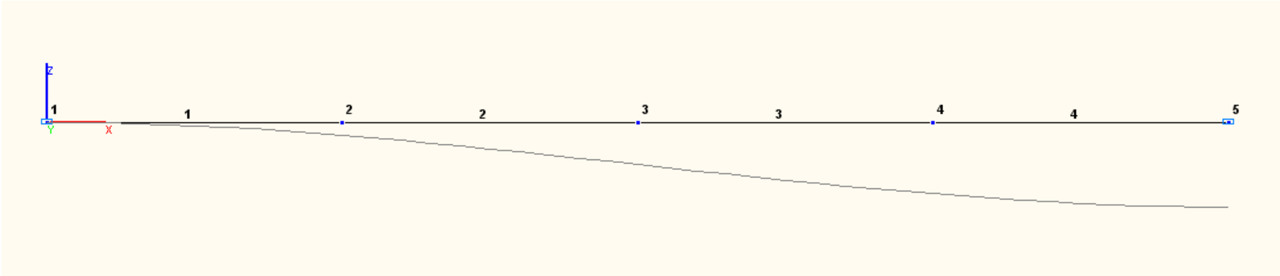
References: J. M. Gere and W. Weaver, Jr., Analysis of Framed Structures, New York, D. Van Nostrand Co., 1965.

Initial data:

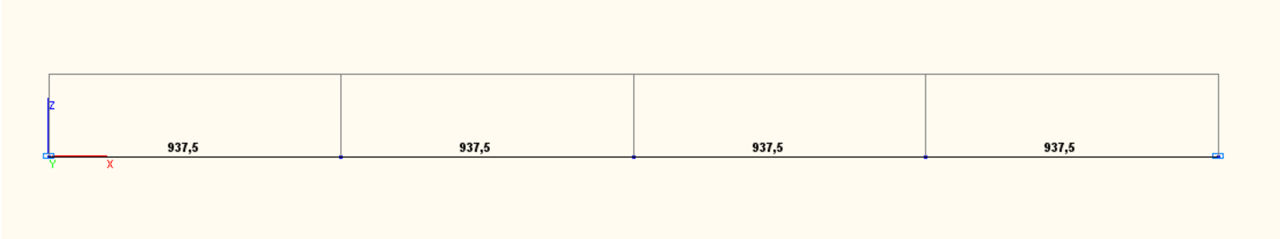
$E = 3.0 \cdot 10^7$ Pa	- elastic modulus,
L = 80.0 m	- beam length;
b = 2.0 m	- width of the beam cross-section;
h = 2.0 m	- height of the beam cross-section;
v = 1.0 m	- value of the transverse displacement.

Finite element model: Design model – plane frame, 4 elements of type 2. The spacing of the finite element mesh along the longitudinal axis (along the X axis of the global coordinate system) is 20.0 m. Boundary conditions at the clamped ends are provided by imposing constraints in the directions of the degrees of freedom: X, Z, UY. The action of the given transverse displacement is specified by the displacement of the respective constraint along the Z axis of the global coordinate system. Number of nodes in the design model – 5.

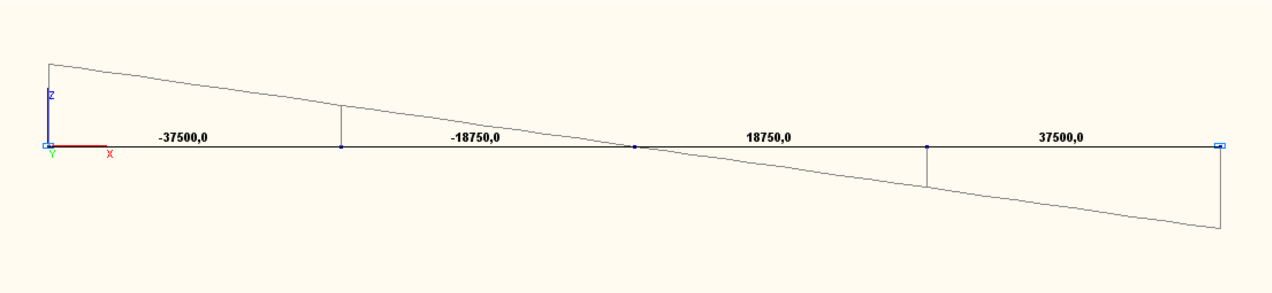
Results in SCAD



Design and deformed models



Shear force diagram Q (N)



Bending moment diagram M (N·m)

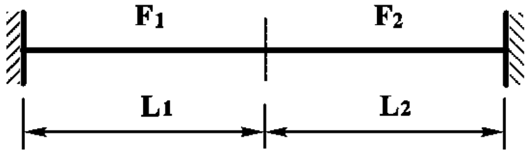
Comparison of solutions:

Parameter	Theory	SCAD	Deviations, %
Shear force Q at the displaced end , N	937.5	937.5	0.00
Bending moment M at the displaced end, N·m	37500.0	37500.0	0.00

Notes: In the analytical solution, the shear force Q and the bending moment M at the displaced end are determined according to the following formulas:

$$Q = \frac{12 \cdot E \cdot I}{L^3}; \qquad M = \frac{6 \cdot E \cdot I}{L^2}, \text{ where: } \qquad I = \frac{b \cdot h^3}{12}.$$

Plane System of Two Coaxial Bars Subjected to Temperature Variation



Objective: Determination of the stress state of a plane system of two coaxial bars subjected to temperature variation.

***Initial data file*: B1_v11.3.SPR**

Problem formulation: The system consists of two coaxial horizontal bars of square cross-section, rigidly connected in the common node and clamped at the opposite nodes. The system is subjected to the temperature variation Δt relative to the assembly temperature. Determine normal stresses σ in the cross-sections of the bars of the system.

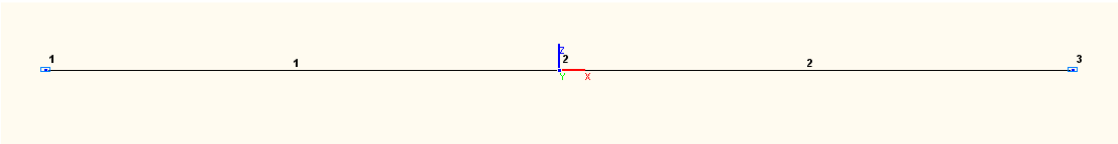
References: S.P. Timoshenko, Strength of Materials, Volume 1: Elementary Theory and Problems, Moscow, Nauka, 1965, p.35.

Initial data:

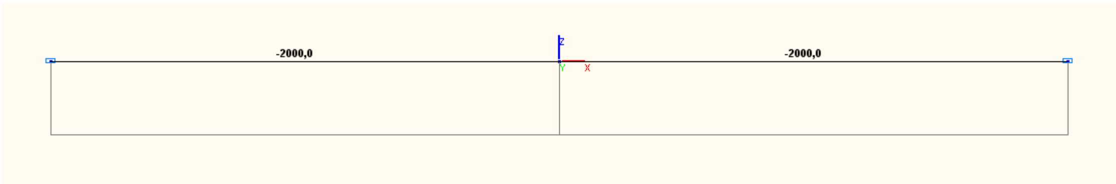
$E_s = 2.0 \cdot 10^6$ kgf/cm^2	- elastic modulus of steel;
$\alpha_s = 1.25 \cdot 10^{-5}$ 1/ ºC	- linear thermal expansion coefficient of steel;
$L_1 = 100.0$ cm	- length of the left bar;
$F_1 = 1.0 \cdot 1.0$ cm^2	- cross-sectional area of the left bar;
$L_2 = 100.0$ cm	- length of the right bar;
$F_2 = 1.0 \cdot 2.0$ cm^2	- cross-sectional area of the right bar;
$\Delta t = 60$ ºC	- temperature variation of the system.

Finite element model: Design model – plane frame, 2 elements of type 2. Boundary conditions are provided by imposing constraints in the end nodes of the system in the directions of the degrees of freedom X, Z, UY. The effect of the temperature variation of the system Δt relative to the assembly temperature is specified as uniform along the longitudinal axes of all bar elements. Number of nodes in the design model – 3.

Results in SCAD



Design model



Longitudinal force diagram N (kgf)

Comparison of solutions:

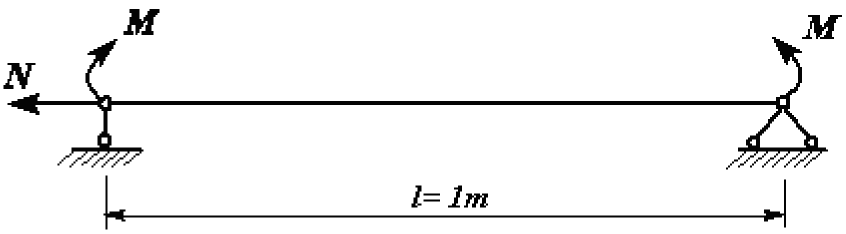
Parameter	Theory	SCAD	Deviations, %

Normal stresses σ (left bar), kgf/cm^2	-2000.000	-2000.0 / (1.0 * 1.0) = = -2000.000	0.00
Normal stresses σ (right bar), kgf/cm^2	-1000.000	-2000.0 / (1.0 * 2.0) = = -1000.000	0.00

Notes: In the analytical solution, the normal stresses σ in the cross-sections of the bars of the system are determined according to the following formulas:

$$\sigma_l = \frac{\Delta t \cdot \alpha_s \cdot E_s \cdot (L_1 + L_2) \cdot F_2}{L_1 \cdot F_2 + L_2 \cdot F_1}\ ; \qquad \sigma_r = \frac{\Delta t \cdot \alpha_s \cdot E_s \cdot (L_1 + L_2) \cdot F_1}{L_1 \cdot F_2 + L_2 \cdot F_1}\ .$$

Stress-Strain State of a Simply Supported Beam Subjected to Longitudinal-Transverse Bending



Objective: Longitudinal-transverse bending in one plane.

Initial data files:

File name	Description
4.8_s_c.SPR	Longitudinal-transverse bending under a longitudinal compressive force
4.8_s_t.SPR	Longitudinal-transverse bending under a longitudinal tensile force

Problem formulation: A simply supported beam under pure bending is additionally loaded by a longitudinal force. Determine the maximum transverse displacements *w*(x) and bending moments *M*(x) under a longitudinal compressive and tensile force.

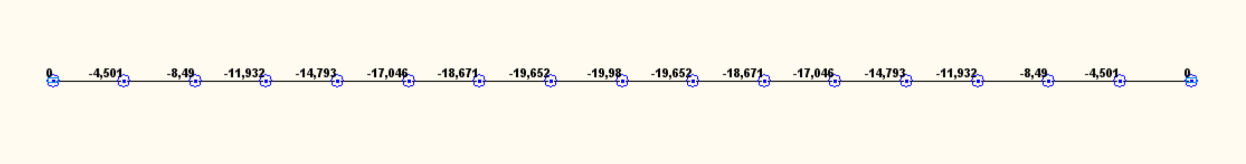
References: Strength Analysis in Mechanical Engineering / S. D. Ponomarev, V. L. Biderman, K. K. Likharev, et al., In three volumes. Volume 1. M.: Mashgiz, 1956.

Initial data:

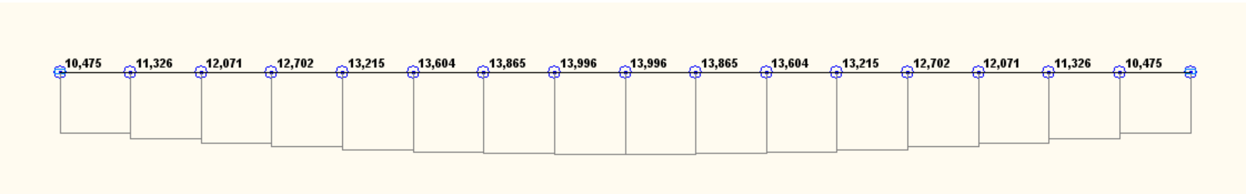
$E = 1.0 \cdot 10^{10}$ Pa - elastic modulus;
$\mu = 0.3$ - Poisson's ratio;
$F = 1 \cdot 10^{-2}$ m^2 - cross-sectional area;
$I = 8.333 \cdot 10^{-6}$ m^4 - cross-sectional moment of inertia;
M = 10 kN·m - value of the bending moment;
N = ±200 kN - value of the concentrated force;
l = 1.0 m - beam length.

Finite element model: The calculation is performed in the geometrically linear formulation for an energetically equivalent model in the form of a bar on the elastic subgrade resisting the rotations of its sections with a linear stiffness parameter $k_\varphi = N$. Design model – plane frame, 16 bar elements of type 2, 17 elements of concentrated rotational (clock) springs with stiffness C_{UY} = -12.5 kN·m/rad (-6.25 kN·m/rad) for a bar under compression and bending and C_{UY} = 12.5 kN m/rad (6.25 kN·m/rad) for a bar under tension and bending of type 51, 17 nodes.

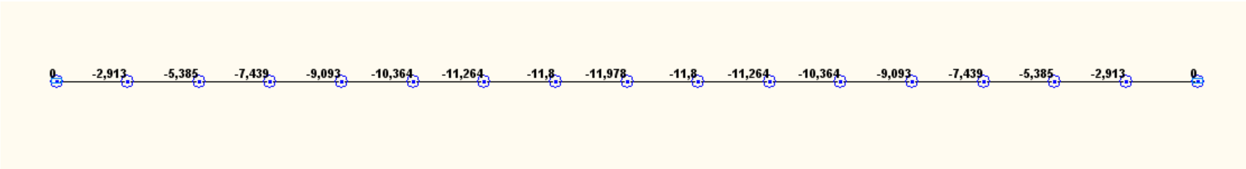
Results in SCAD



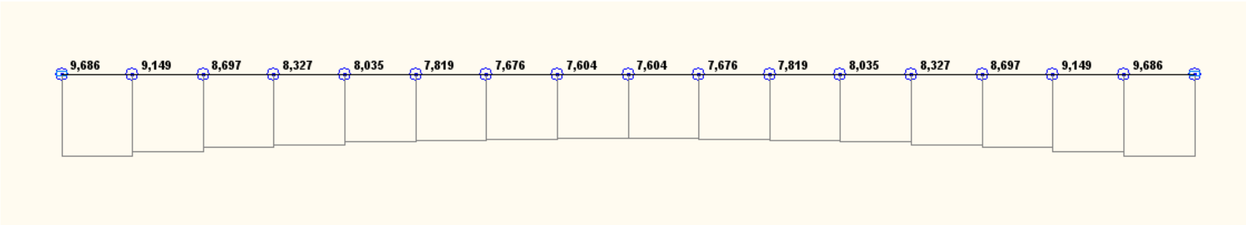
Values of transverse displacements w under a longitudinal compressive force (mm)



Bending moment diagram M under a longitudinal compressive force (kN·m)



Values of transverse displacements w under a longitudinal tensile force (mm)



Bending moment diagram M under a longitudinal tensile force (kN·m)

Comparison of solutions:

Parameter	Longitudinal compressive force			Longitudinal tensile force		
	Theory	**SCAD**	**Deviations, %**	**Theory**	**SCAD**	**Deviations, %**
Transverse displacements $w(0.5\cdot l)$, mm	-19.959	-19.980	0.11	-11.986	-11.978	0.07
Bending moment $M(0.5\cdot l)$, kN·m	13.992	13.996	0.03	7.603	7.604	0.01

Notes: In the analytical solution, the equation of the elastic line *w*(x) and the equation of the bending moment *M*(x) under a longitudinal compressive force are determined according to the following formulas:

$$w(x) = \frac{M}{N}\cdot\left[\frac{cos(k\cdot l)-1}{sin(k\cdot l)}\cdot sin(k\cdot x) - cos(k\cdot x) + 1\right];$$

$$M(x) = M\cdot\left[\frac{1-cos(k\cdot l)}{sin(k\cdot l)}\cdot sin(k\cdot x) + cos(k\cdot x)\right],$$

where:

$$k = \sqrt{\frac{N}{E\cdot I}}\;.$$

In the analytical solution, the equation of the elastic line *w*(x) and the equation of the bending moment *M*(x) under a longitudinal tensile force are determined according to the following formulas:

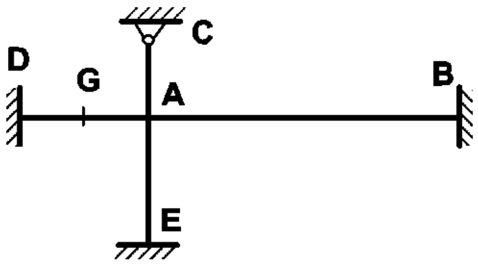
$$w(x) = \frac{M}{N}\cdot\left[\frac{1-ch(k\cdot l)}{sh(k\cdot l)}\cdot sh(k\cdot x) + ch(k\cdot x) - 1\right];$$

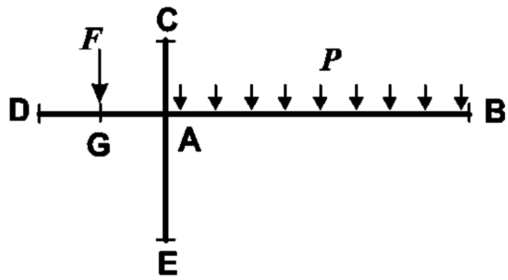
$$M(x) = M\cdot\left[\frac{ch(k\cdot l)-1}{sh(k\cdot l)}\cdot sh(k\cdot x) + ch(k\cdot x)\right],$$

where:

$$k = \sqrt{\frac{N}{E\cdot I}}\;.$$

System of Cross Bars Subjected to a Distributed Load and a Concentrated Force in Their Plane





Objective: Determination of the stress-strain state of a system of cross bars subjected to a distributed load and a concentrated force in their plane.

***Initial data file*: SSLL10_v11.3.SPR**

Problem formulation: The system consists of two cross bars of square cross-section, horizontal (BD) and vertical (CE), rigidly connected in the common node (point A). The horizontal bar is clamped in the left and right nodes (points D and B). The vertical bar is clamped in the lower node (point E) and simply supported in the upper one (point C). A vertical concentrated force *F* is applied in the middle of the left span of the horizontal bar (point G), and a vertical uniformly distributed load *p* is applied to the right span of the horizontal bar (AB). Determine the rotation angle UY in the common node of cross bars (point A) and bending moments *M* in the bars on both sides of the node.

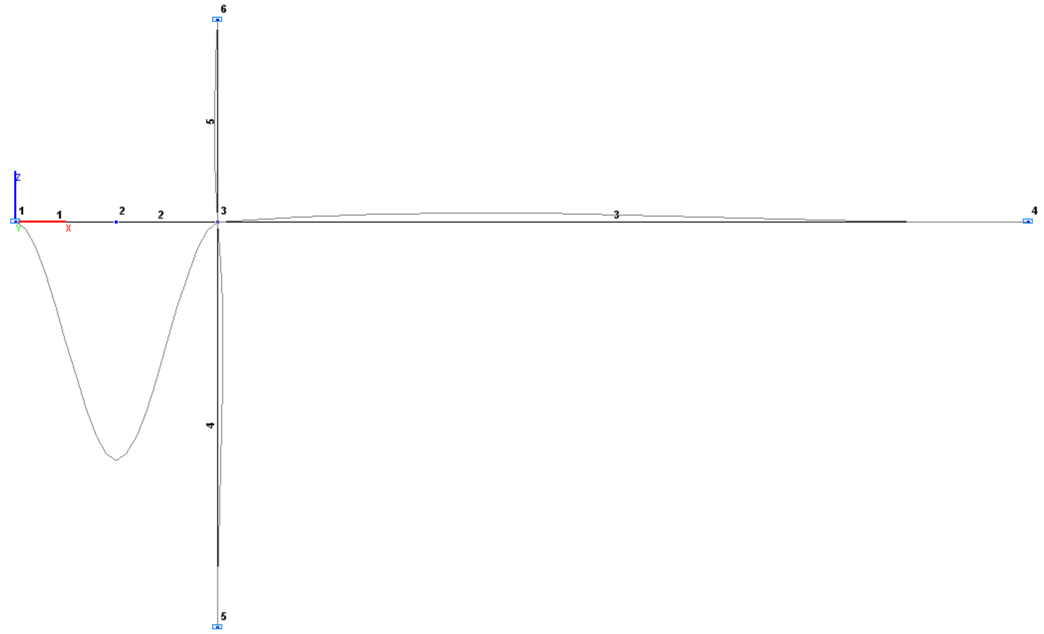
References: S. Timoshenko et D.H. Young, Theorie des constructions, Paris, Librairie Polytechnique Beranger, 1949, p. 412-416.

Initial data:

$E = 2.0 \cdot 10^{11}$ Pa - elastic modulus of the bars of the system;
$L_{AD} = 1.0$ m - length of the left span of the horizontal bar;
$b_{AD} = 1.0$ m - side of the cross-section of the left span of the horizontal bar;
$L_{AB} = 4.0$ m - length of the right span of the horizontal bar;
$b_{AB} = 4.0$ m - side of the cross-section of the right span of the horizontal bar;
$L_{AC} = 1.0$ m - length of the upper part of the vertical bar;
$b_{AC} = 1.0$ m - side of the cross-section of the upper part of the vertical bar;
$L_{AE} = 2.0$ m - length of the lower part of the vertical bar;
$b_{AE} = 2.0$ m - side of the cross-section of the lower part of the vertical bar;
$F = 1.0 \cdot 10^5$ N - value of the vertical concentrated force;
$p = 1.0 \cdot 10^3$ N/m - value of the vertical uniformly distributed load.

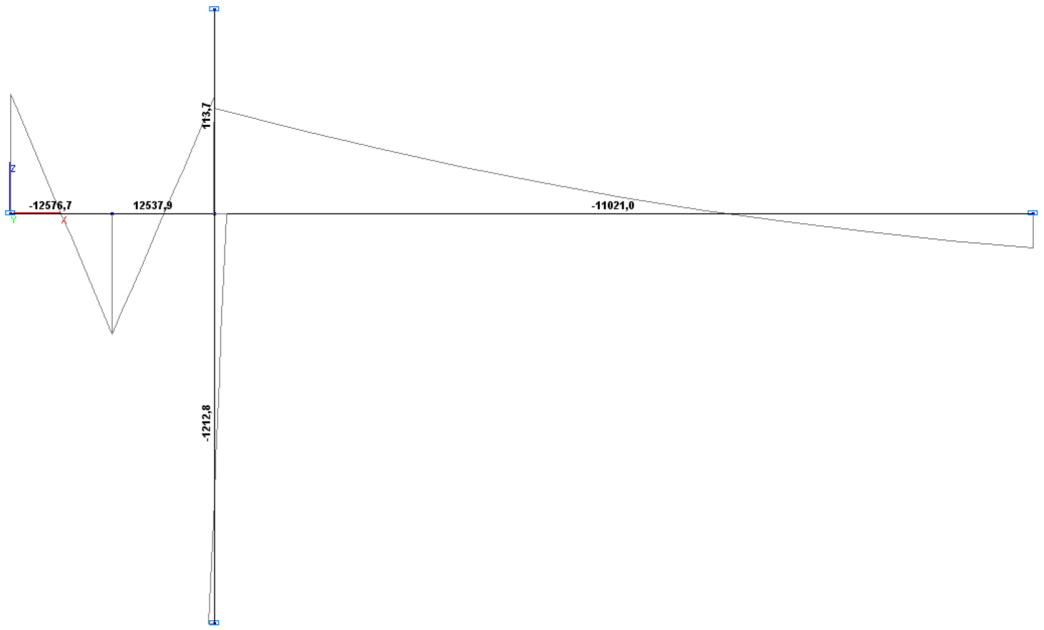
Finite element model: Design model – plane frame, 5 bar elements of type 10. Boundary conditions are provided by imposing constraints in the directions of the degrees of freedom X, Z for the simply supported node (point C) and in the directions of the degrees of freedom X, Z, UY for the clamped nodes (points E, D, B). Number of nodes in the design model – 6.

Results in SCAD



Design and deformed models

Values of rotation angles UY (rad)

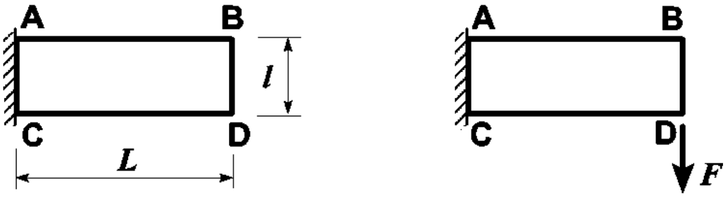


Values of bending moments M (N·m)

Comparison of solutions:

Parameter	Theory	SCAD	Deviations, %
Rotation angle UY (point A), rad	$-2.2712 \cdot 10^{-1}$	$-2.2740 \cdot 10^{-1}$	0.12
Bending moment *M* (bar AD), N·m	-12348.6	-12347.5	0.01
Bending moment *M* (bar AB), N·m	-11023.7	-11021.0	0.02
Bending moment *M* (bar AC), N·m	113.6	113.7	0.09
Bending moment *M* (bar AE), N·m	-1211.3	-1212.8	0.12

Cantilever Frame Subjected to a Concentrated Force



Objective: Determination of the stress-strain state of a cantilever frame subjected to a concentrated force.

Initial data file: **SSLL05_v11.3.SPR**

Problem formulation: The cantilever frame consists of two horizontal bars of the same length L, clamped on the left (points A, C) and joined by a vertical bar of the length l on the right (points B, D). Horizontal bars have considerable tensile/compressive stiffness, a vertical bar has both considerable tensile/compressive and bending stiffness. A vertical concentrated force F is applied in the joint between the lower horizontal bar and the vertical bar (point D). Determine the vertical displacements Z in the joints between the horizontal bars and the vertical bar (points B, D), as well as the bending moments M_y, shear forces Q_z and longitudinal forces N_x in the clamped nodes of the horizontal bars (points A, C).

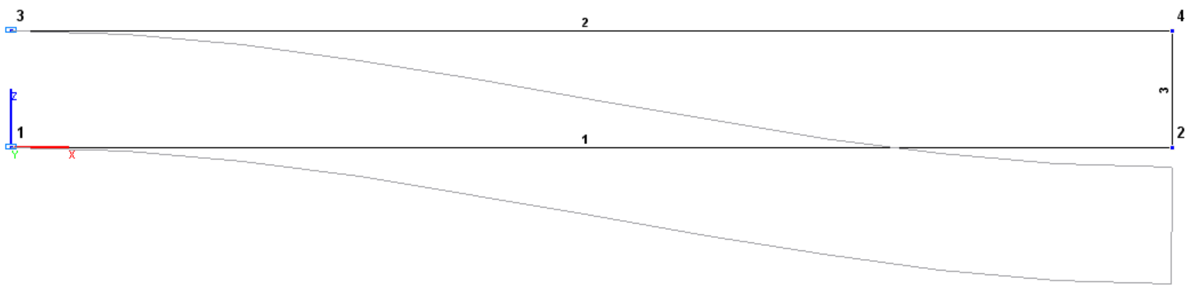
References: A. Campa, R. Chappert et R. Picand, La mecanique par les problemes, fasc. 4: Resistance des materiaux, Paris, Foucher, 1987.

Initial data:

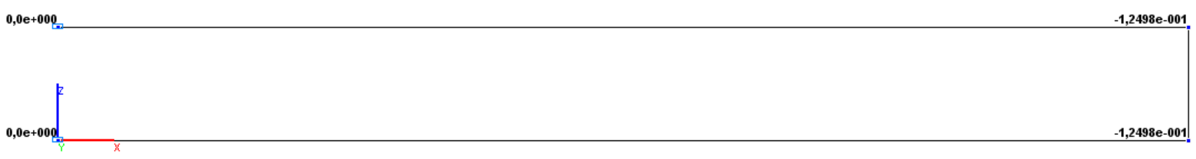
$E = 2.0\cdot10^{11}$ Pa	- elastic modulus of the horizontal bars;
L = 2.0 m	- length of the horizontal bars;
l = 0.2 m	- length of the vertical bar;
$I_z = 4/3\cdot10^{-8}$ m^4	- cross-sectional moment of inertia of the horizontal bars;
$F = 1.0\cdot10^3$ N	- value of the vertical concentrated force.

Finite element model: Design model – plane frame, 3 bar elements of type 10. Boundary conditions are provided by imposing constraints in the directions of the degrees of freedom X, Z, UY (points A, C). Tensile/compressive stiffness (E·A) of horizontal and vertical bars is taken as $1.0\cdot10^{12}$ N, bending stiffness of the vertical bar (E·I) is taken as $1.0\cdot10^{12}$ N·m^2. Number of nodes in the design model – 4.

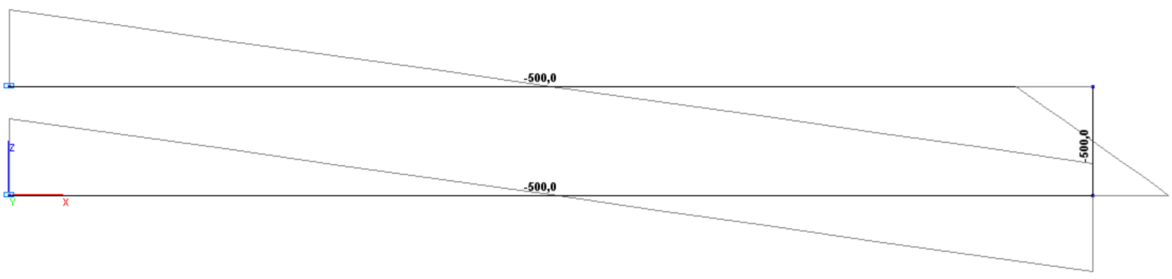
Results in SCAD



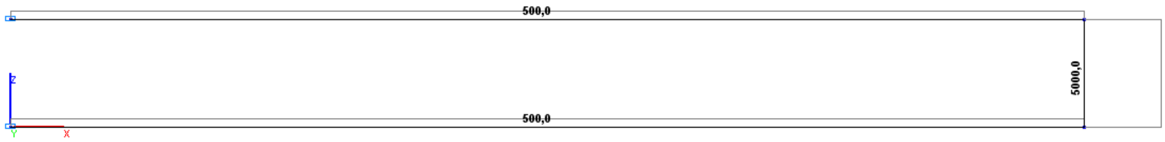
Design and deformed models



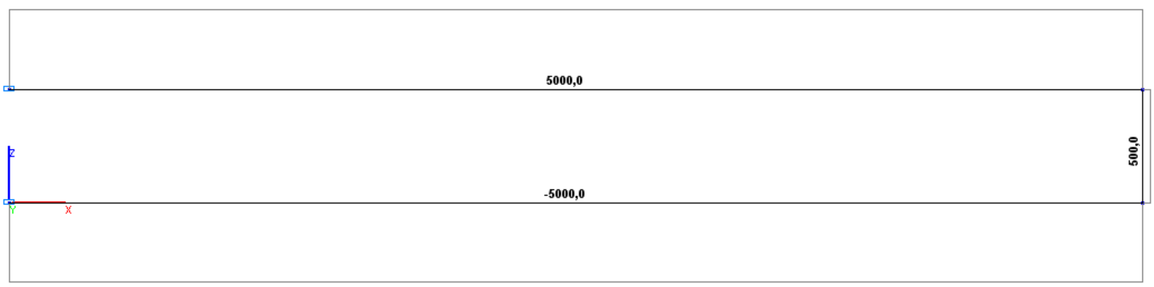
Values of vertical displacements Z (m)



Bending moment diagram M_y (kN·m)



Shear force diagram Q_z (kN)

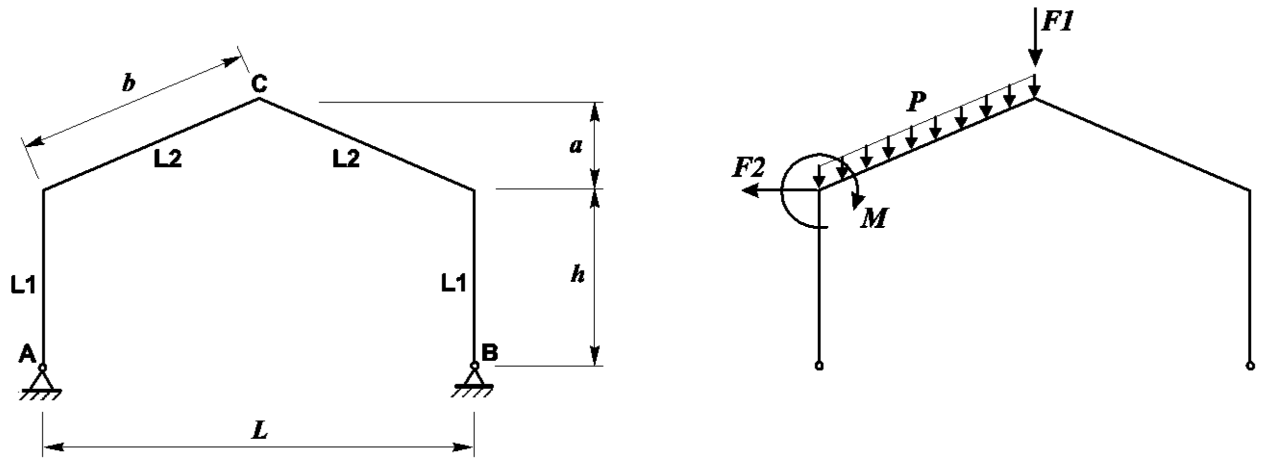


Longitudinal force diagram N_x (kN)

Comparison of solutions:

Parameter	**Theory**	**SCAD**	**Deviations, %**
Vertical displacement Z (point B), m	$-1.2500 \cdot 10^{-1}$	$-1.2498 \cdot 10^{-1}$	0.02
Vertical displacement Z (point D), m	$-1.2500 \cdot 10^{-1}$	$-1.2498 \cdot 10^{-1}$	0.02
Bending moment M_y (point A), N·m	-500.0	-500.0	0.00
Bending moment M_y (point C), N·m	-500.0	-500.0	0.00
Shear force Q_z (point A), N	500.0	500.0	0.00
Shear force Q_z (point C), N	500.0	500.0	0.00
Shear force N_x (point A), N	5000.0	5000.0	0.00
Shear force N_x (point C), N	-5000.0	-5000.0	0.00

Single-Span Simply Supported Plane Frame with a Dual-Pitched Girder Subjected to a Vertical Uniformly Distributed Load, Concentrated Vertical and Horizontal Forces and a Bending Moment



Objective: Determination of the stress-strain state of a single-span simply supported plane frame with a dual-pitched girder subjected to a vertical uniformly distributed load, concentrated vertical and horizontal forces and a bending moment.

Initial data file: **SSLL14_v11.3.spr**

Problem formulation: The single-span simply supported frame with a rigid connection between the dual-pitched girder and the columns is subjected to a vertical load P_{zx} uniformly distributed along the length of the left half-span of the girder 0.5·L, concentrated vertical force F1 in the ridge joint (point C), concentrated horizontal force F2 and bending moment *M* in the joint between the girder and the left column. Determine the vertical displacement Z in the ridge joint (point C), longitudinal *N* and shear *Q* force in the support node of the left column (point A).

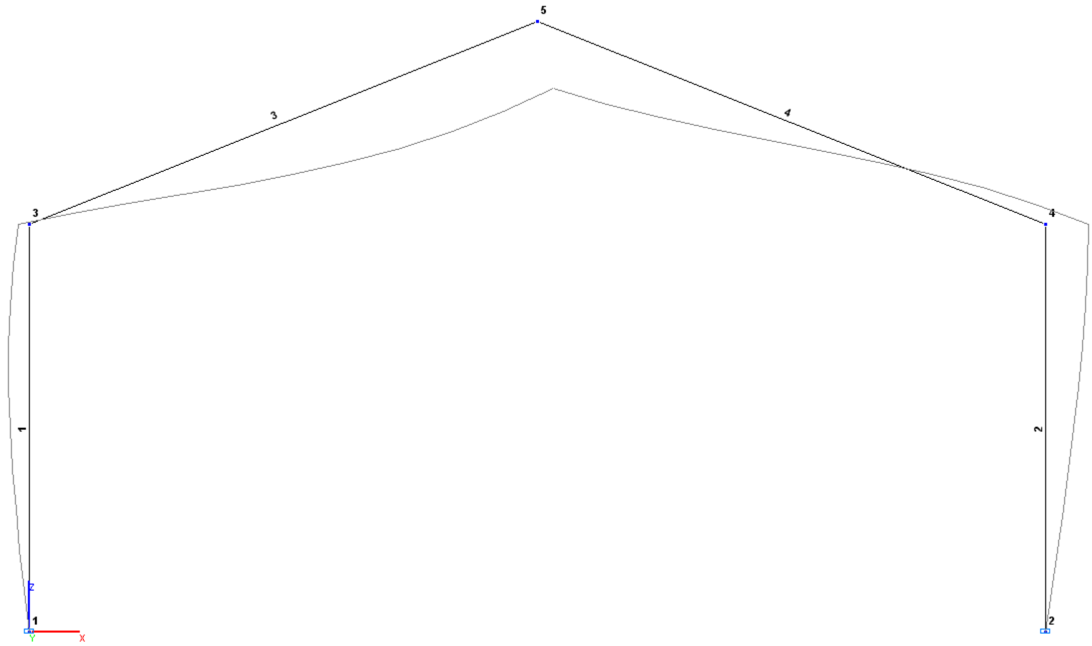
References: J.C. Bianchi, Rapport de la SOCOTEC, Paris, non publie, 1964.

Initial data:

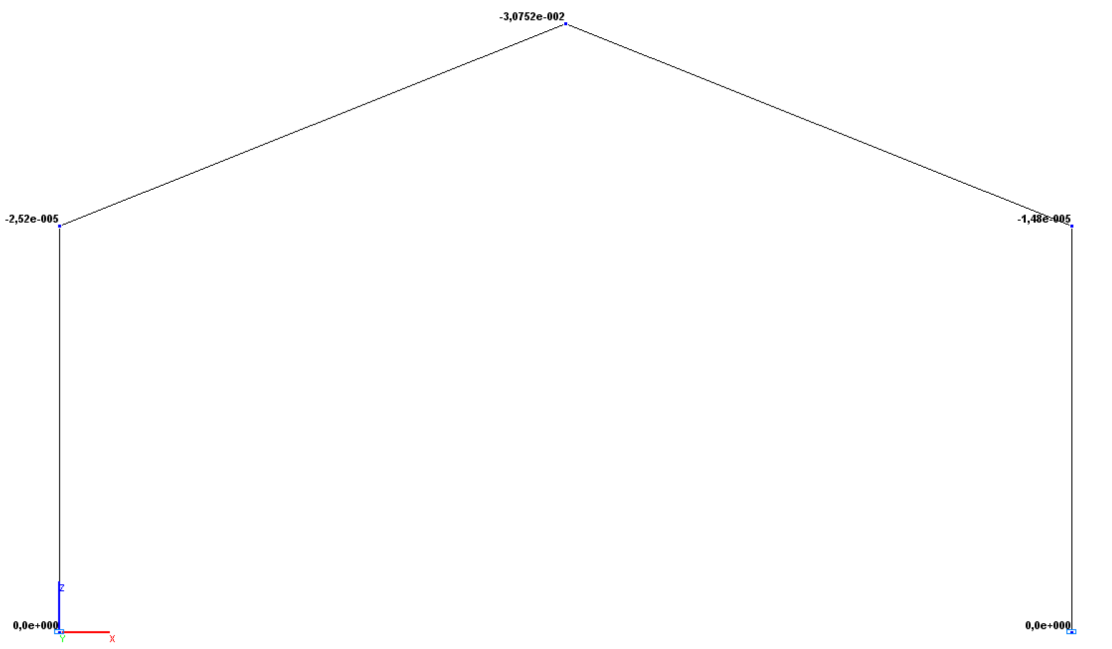
Material:	
$E = 2.1 \cdot 10^{11}$ Pa	- elastic modulus;
Columns L1:	
h = 8.0 m	- height;
$EA_1 = 1.0 \cdot 10^{10}$ N	- axial stiffness;
$EI_1 = 2.1 \cdot 10^{11} \cdot 5.0 \cdot 10^{-4} = 10.5 \cdot 10^7$ N·m^2	- bending stiffness;
Girder L2:	
L = 20.0 m	- span length;
a = 4.0 m	- rise;
$b = ((0.5 \cdot 20.0)^2 + 4.0^2)^{0.5}$	- length of the slope;
$EA_2 = 1.0 \cdot 10^{10}$ N	- axial stiffness;
$EI_2 = 2.1 \cdot 10^{11} \cdot 2.5 \cdot 10^{-4} = 5.25 \cdot 10^7$ N·m^2	- bending stiffness;
Loads and actions:	
$P_{zx} = 3.0 \cdot 10^3$ N/m	- vertical load uniformly distributed along the length of the left half-span of the girder 0.5·L;
$P_z = 3.0 \cdot 10^3 \cdot 0.5 \cdot 20.0/((0.5 \cdot 20.0)^2 + 4.0^2)^{0.5} = 2.78543 \cdot 10^3$ N/m	- the same load distributed along the length of the left slope of the girder b;
$F1 = 2.0 \cdot 10^4$ N	- concentrated vertical force in the ridge joint;
$F2 = 1.0 \cdot 10^4$ N	- concentrated horizontal force in the joint between the girder and the left column;
$M = 1.0 \cdot 10^5$ N·m	- concentrated bending moment in the joint between the girder and the left column.

Finite element model: Design model – plane frame, girder – 2 elements of type 2, columns – 2 elements of type 2. Boundary conditions are provided by imposing constraints in the directions of the degrees of freedom X, Z for pinned support nodes. Number of nodes in the design model – 5.

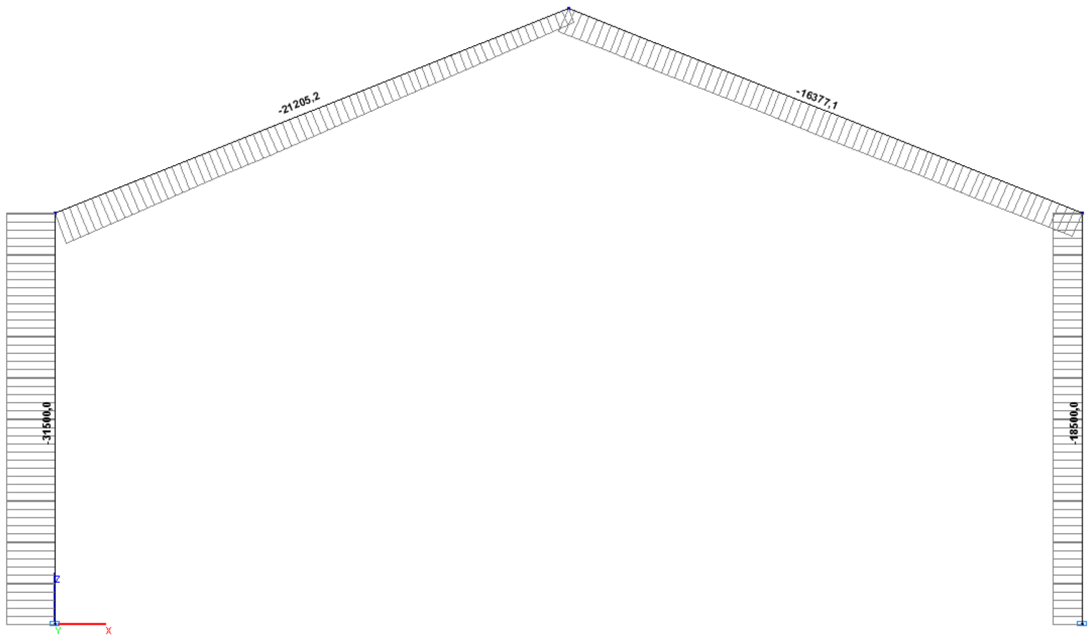
Results in SCAD



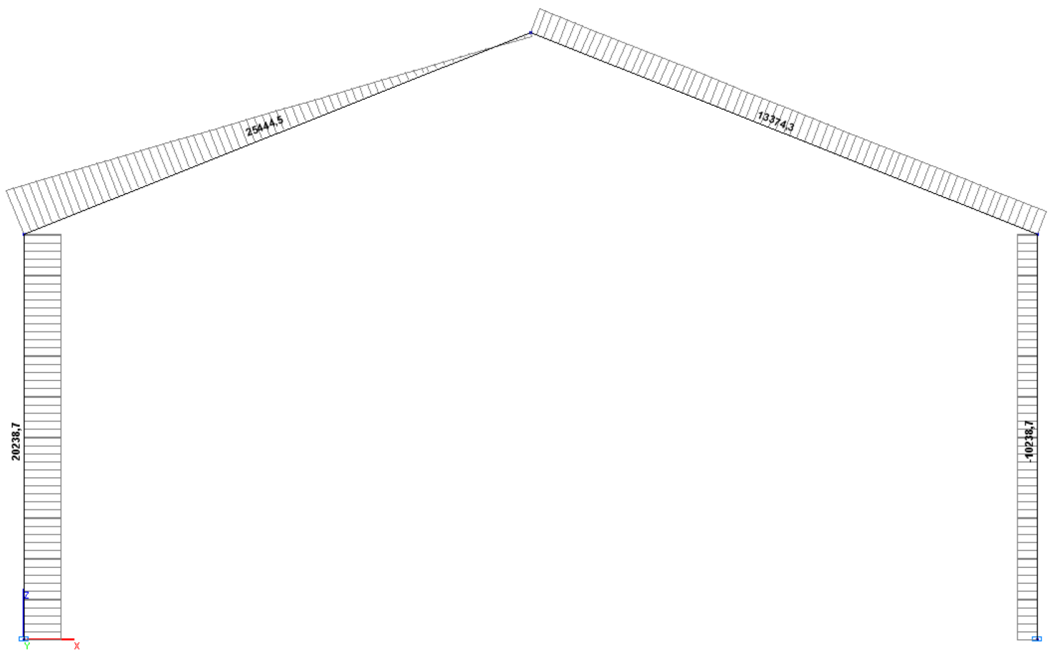
Design and deformed models



Values of vertical displacements Z (m)



Values of longitudinal forces N (N)

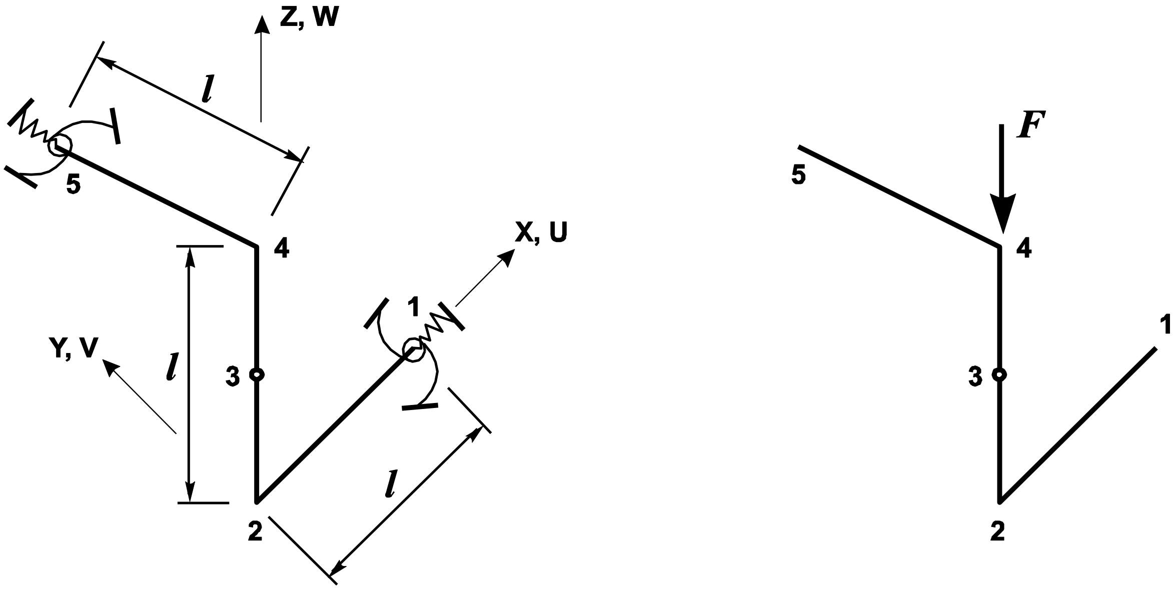


Values of shear forces Q (N)

Comparison of solutions:

Parameter	Theory	SCAD	Deviations, %
Vertical displacement Z_C, m	$-3.0720 \cdot 10^{-2}$	$-3.0752 \cdot 10^{-2}$	0.10
Longitudinal force N_A, N	-31.500	-31.500	0.00
Shear force N_A, N	20239.4	20238.7	0.00

Spatial Bar System with Elastic Constraints Subjected to a Concentrated Force



Objective: Determination of the stress-strain state of the spatial bar system with elastic constraints subjected to a concentrated force.

Initial data file: **SSLL04_v11.3.SPR**

Problem formulation: The spatial system consists of four bars connected in series. End bars lie orthogonally in parallel horizontal planes, intermediate bars are vertical and are connected by hinges relatively to the angular degrees of freedom (point 3). There are rigid constraints of linear and angular degrees of freedom in the plane of the cross-section of the respective end bar and elastic constraints of linear and angular degrees of freedom out of the plane of the cross-section of the respective end bar on both ends of the spatial system (points 1,5). A vertical concentrated force *F* is applied in the joint between the upper horizontal and vertical bars (point 4). Determine the vertical displacement Z for the joint between vertical bars (point 3), horizontal displacement Y along the upper end bar and the rotation angle UX in the vertical plane containing this bar for the upper constraint of the spatial system (point 5), as well as the torque and bending moments M_x, M_y, M_z for the upper and lower constraints of the spatial system (points 1, 5).

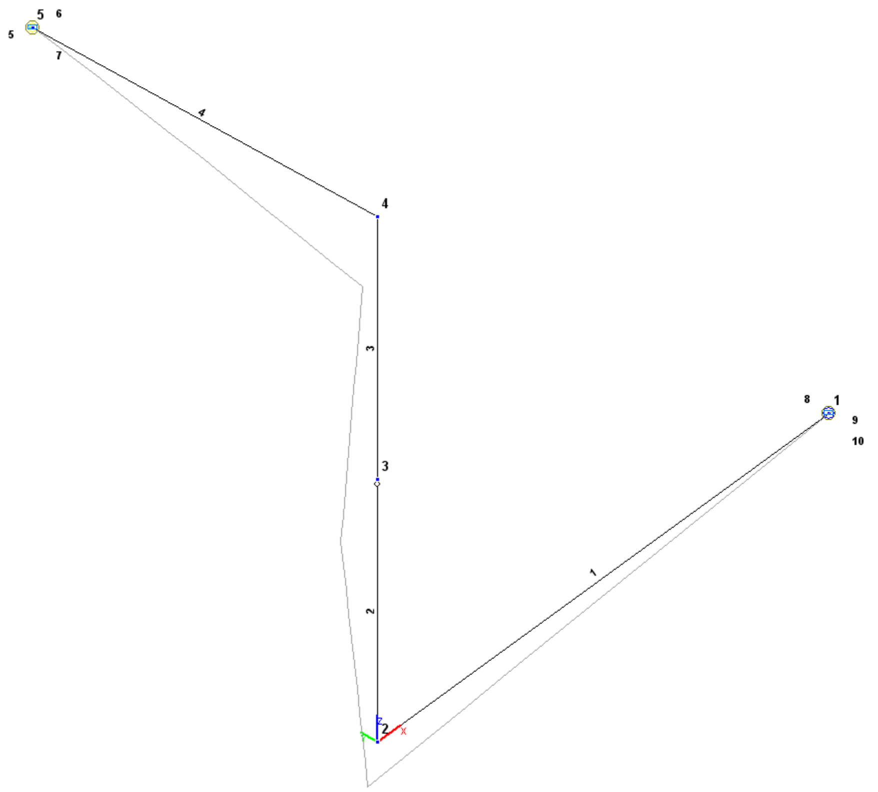
References: M. Laredo, Resistance des materiaux, Paris, Dunod, 1970, p. 165.

Initial data:

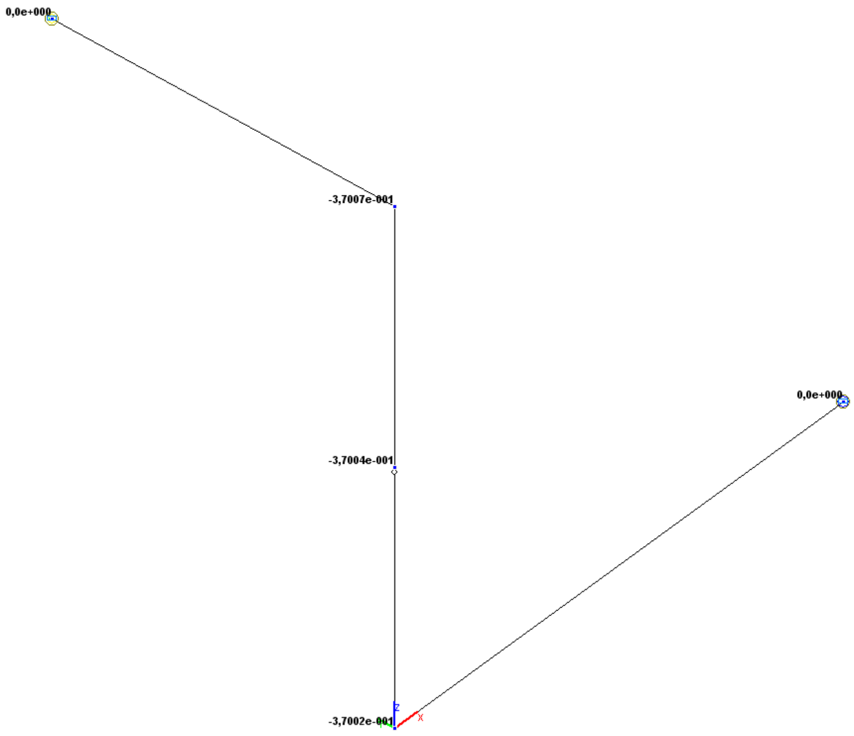
$E = 2.1 \cdot 10^{11}$ Pa	- elastic modulus,
$G = 0.7875 \cdot 10^{11}$ Pa	- shear modulus,
$l = 2.0$ m	- length of the horizontal bars;
$0.5\, l = 1.0$ m	- length of the vertical bars;
$A = 1.0 \cdot 10^{-3}$ m^2	- cross-sectional area of the bars;
$I_x = 2 \cdot 10^{-6}$ m^4	- moment of inertia in the plane of the cross-section of the bars (torsion);
$I_y = I_z = 2 \cdot 10^{-6}$ m^4	- moments of inertia out of the plane of the cross-section of the bars (bending);
$k = 5.25 \cdot 10^4$ N/m	- stiffness of constraints with respect to the linear degree of freedom;
$k_u = 5.25 \cdot 10^4$ N·m/rad	- stiffness of constraints with respect to the angular degrees of freedom;
$F = 1.0 \cdot 10^4$ N	- value of the vertical concentrated force.

Finite element model: Design model – general type system, 4 bar elements of type10. Boundary conditions are provided by imposing rigid constraints in the directions of the degrees of freedom X, Z, UY and constraints of finite rigidity in the directions of the degrees of freedom Y, UX, UZ (member type 51) – for the end of the upper bar of the spatial system (point 5); by imposing rigid constraints in the directions of the degrees of freedom Y, Z, UX and constraints of finite rigidity in the directions of the degrees of freedom X, UY, UZ (member type 51) – for the end of the lower bar of the spatial system (point 1). Number of nodes in the design model – 5.

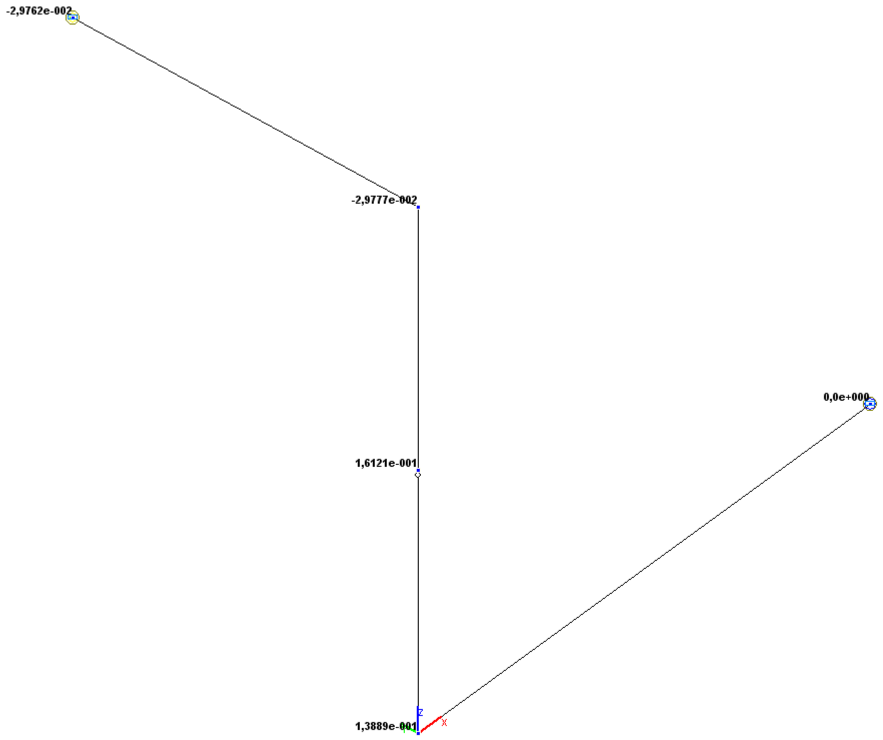
Results in SCAD



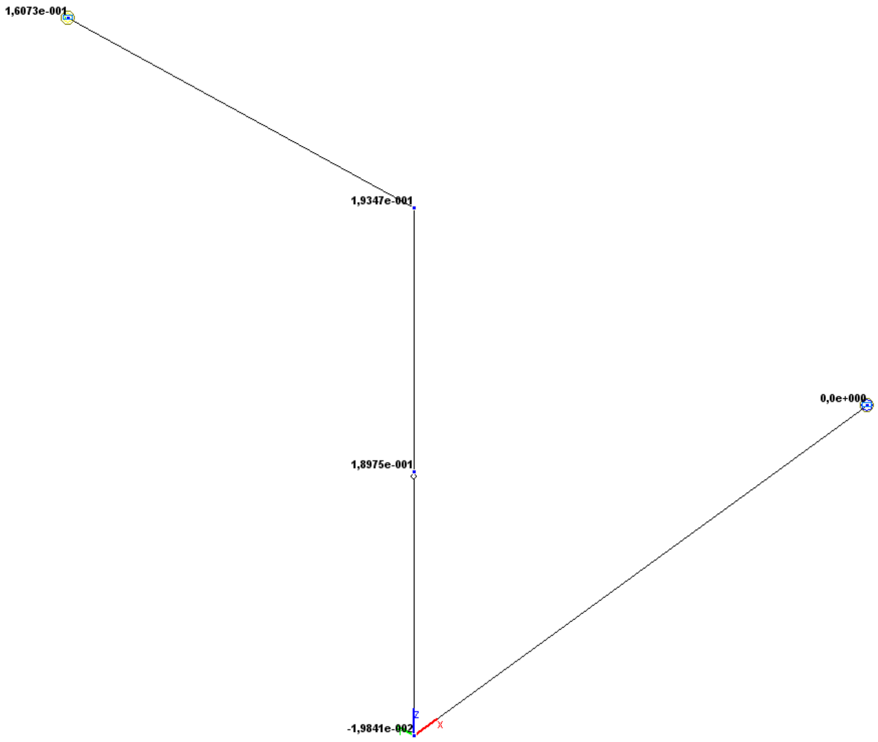
Design and deformed models



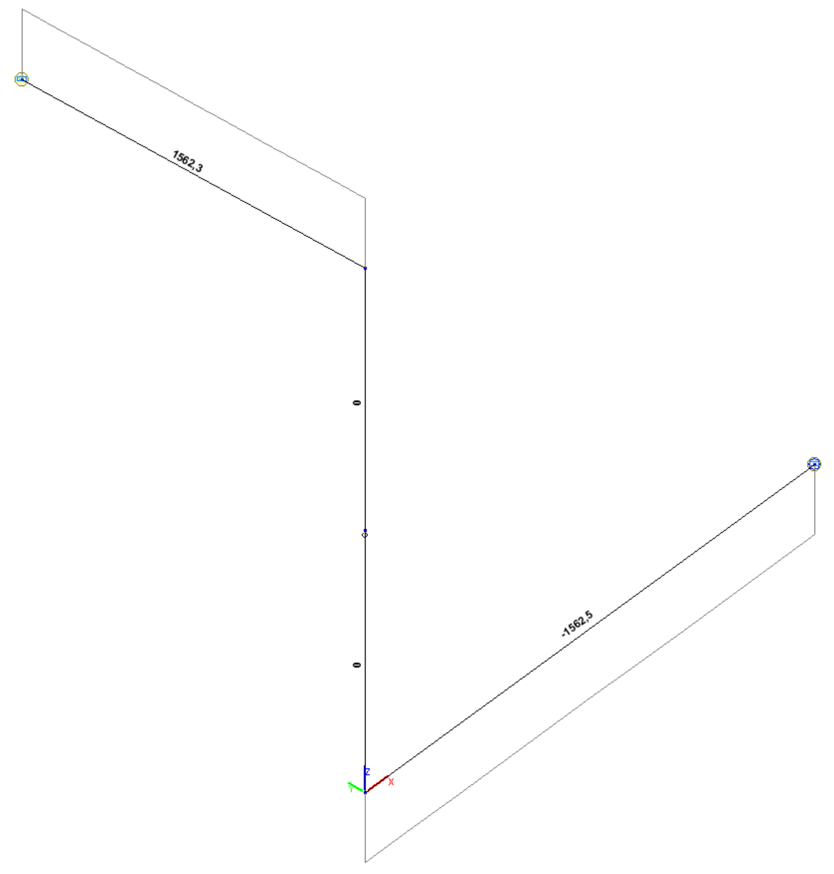
Values of vertical displacements Z (m)



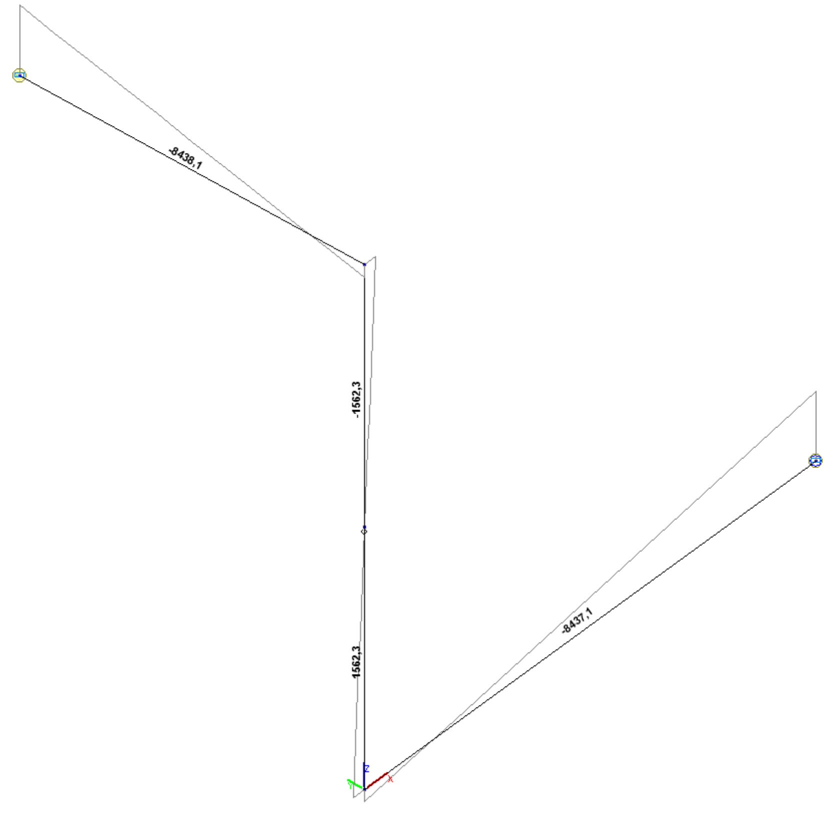
Values of horizontal displacements Y (m)



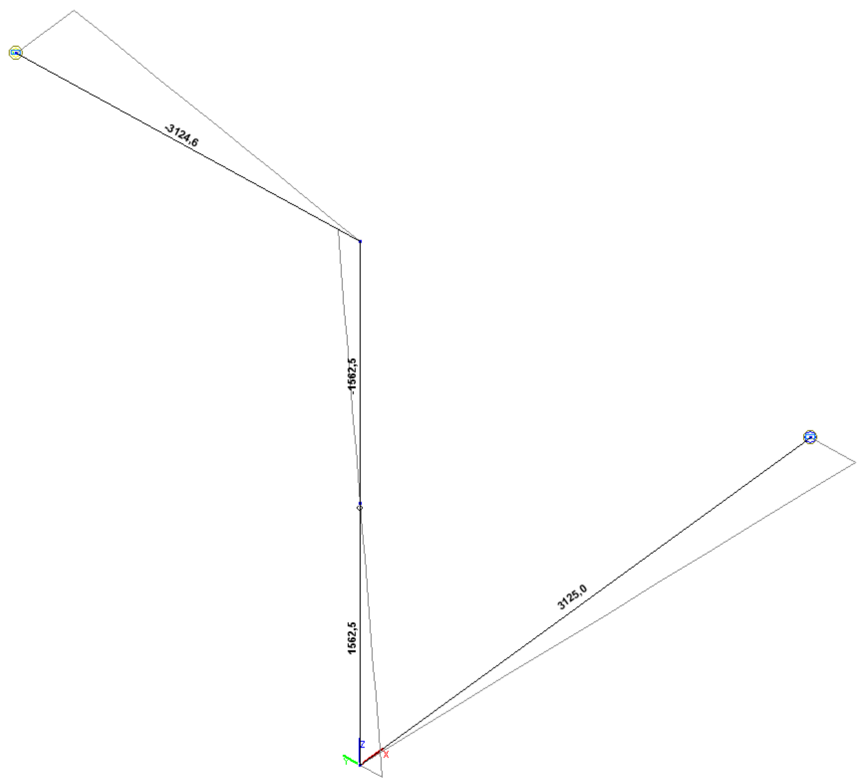
Values of rotation angles UX (rad)



*Torque diagram M_x (kN*m)*



*Bending moment diagram M_y (kN*m)*

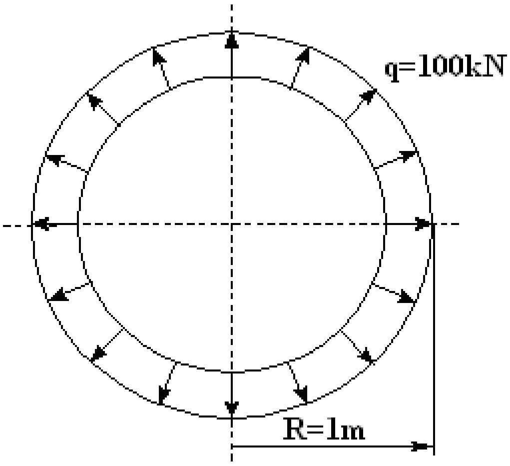


*Bending moment diagram M_z (kN*m)*

Comparison of solutions:

Parameter	**Theory**	**SCAD**	**Deviations, %**
Vertical displacement Z (point 3), m	$-3.7004 \cdot 10^{-1}$	$-3.7004 \cdot 10^{-1}$	0.00
Horizontal displacement Y (point 5), m	$-2.9762 \cdot 10^{-2}$	$-2.9762 \cdot 10^{-2}$	0.00
Rotation angle UX (point 5), rad	$1.6071 \cdot 10^{-1}$	$1.6073 \cdot 10^{-1}$	0.01
Torque M_x (point 5), N·m	1562.5	1562.3	0.01
Bending moment M_y (point 5), N·m	-8437.5	-8438.1	0.01
Bending moment M_z (point 5), N·m	-3125.0	3124.6	0.01
Torque M_x (point 1), N·m	-1562.5	-1562.5	0.00
Bending moment M_y (point 1), N·m	-8437.5	-8437.1	0.00
Bending moment M_z (point 1), N·m	3125.0	3125.0	0.00

Ring Subjected to a Distributed Load Acting in Its Plane



Objective: Analysis for bending in the ring plane under a concentrated force without taking into account the transverse shear deformations.

Initial data file: **4.7.SPR**

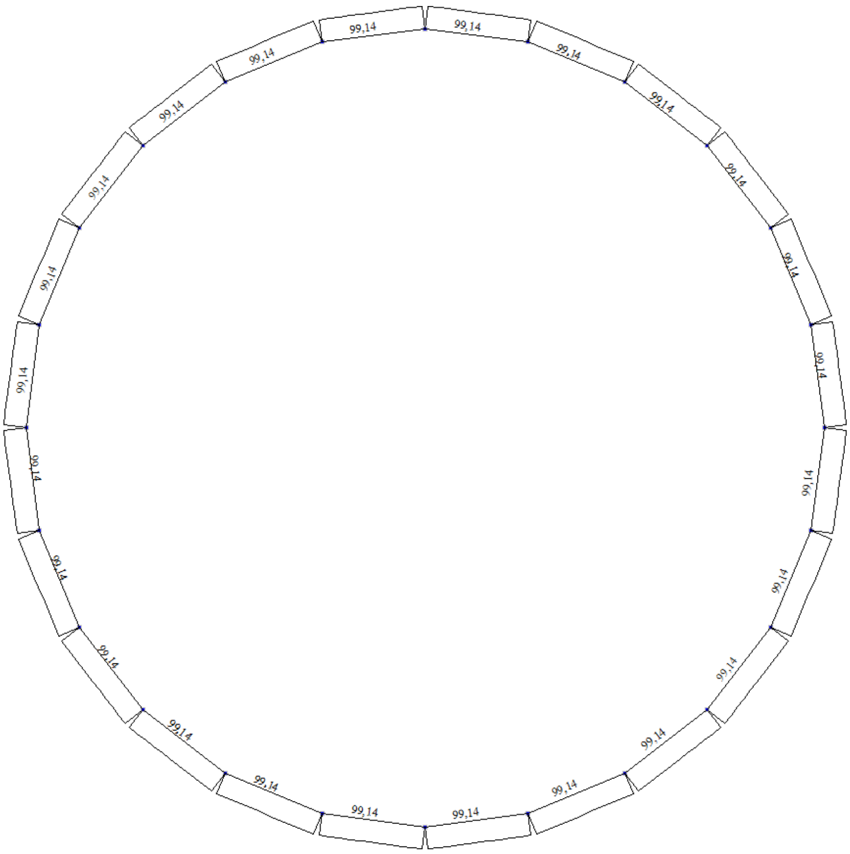
Problem formulation: The ring is subjected to a distributed load *q* acting in its plane. Determine: the normal force in the ring section *N* and the change in the ring diameter δ.

References: G.S. Pisarenko, A.P. Yakovlev, V.V. Matveev, Handbook on Strength of Materials. — Kiev: Naukova Dumka, 1988.

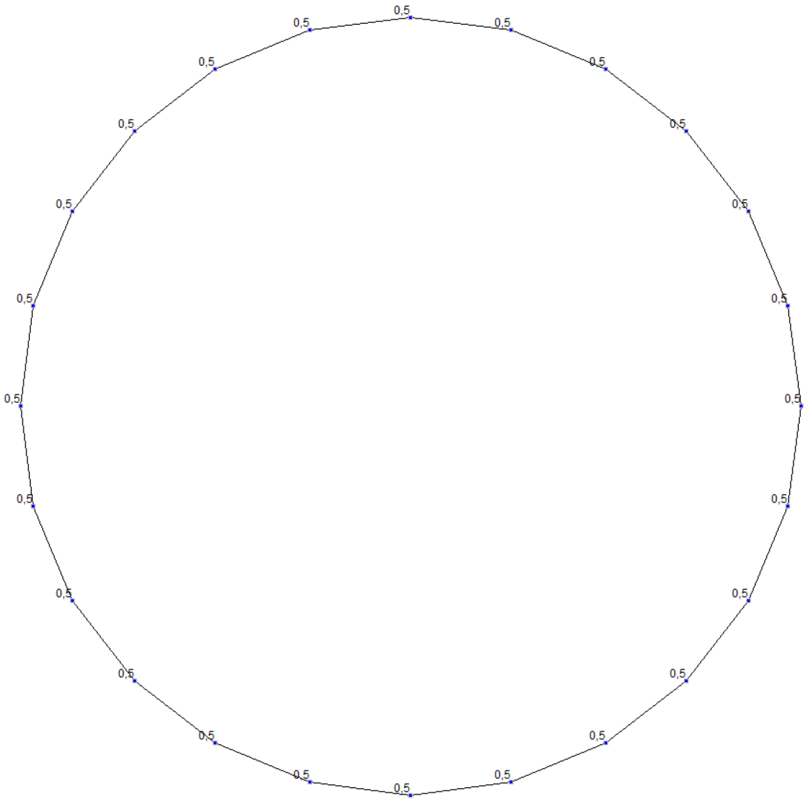
Initial data:
$E = 2.0 \cdot 10^{11}$ Pa - elastic modulus,
μ = 0.3 - Poisson's ratio,
R= 1 m - ring radius;
$F = 0{,}001\ m^2$ - cross-sectional area;
q = 100 kN/m - value of the distributed load.

Finite element model: Design model – general type system, 72 bar elements of type 10, 72 nodes.

Results in SCAD



Normal force diagram N (kN)



Values of displacements δ (mm)

Comparison of solutions:

Parameter	Theory	SCAD	Deviations, %
Change in the ring diameter δ, mm	0.50	0.50	0.00
Normal force in the ring section N, kN	100.00	99.14	0.86

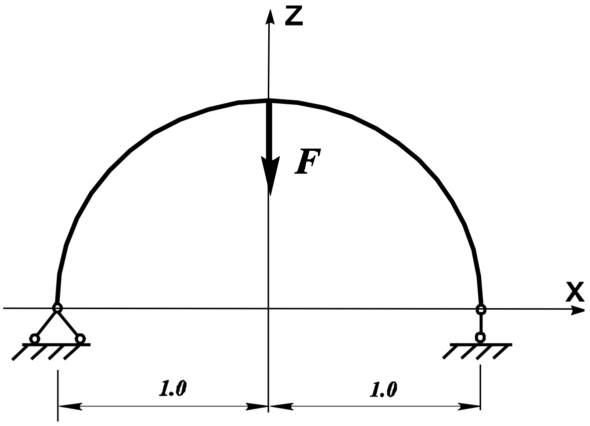
Notes: In the analytical solution, the change in the ring diameter is determined according to the following formulas (“Handbook on Strength of Materials” p. 384) :

$$\delta = \frac{q \cdot R^2}{E \cdot F}.$$

Normal force in the ring section:

$$N = q \cdot R.$$

Simply Supported Semicircular Arch of Constant Cross-Section Subjected to a Concentrated Force Acting in Its Plane



Objective**:** Determination of the strain state of a simply supported semicircular arch of constant cross-section subjected to a concentrated force acting in its plane.

Initial data file: **SSLL08_v11.3.SPR**

Problem formulation: The semicircular arch of constant cross-section with pinned and roller supports subjected to a concentrated force *F* acting in its plane at the level of the key, directed downward along the normal to the longitudinal axis. Determine the deflection of the longitudinal axis of the arch Z, displacement of the roller support X and rotation angles of the support hinges UY.

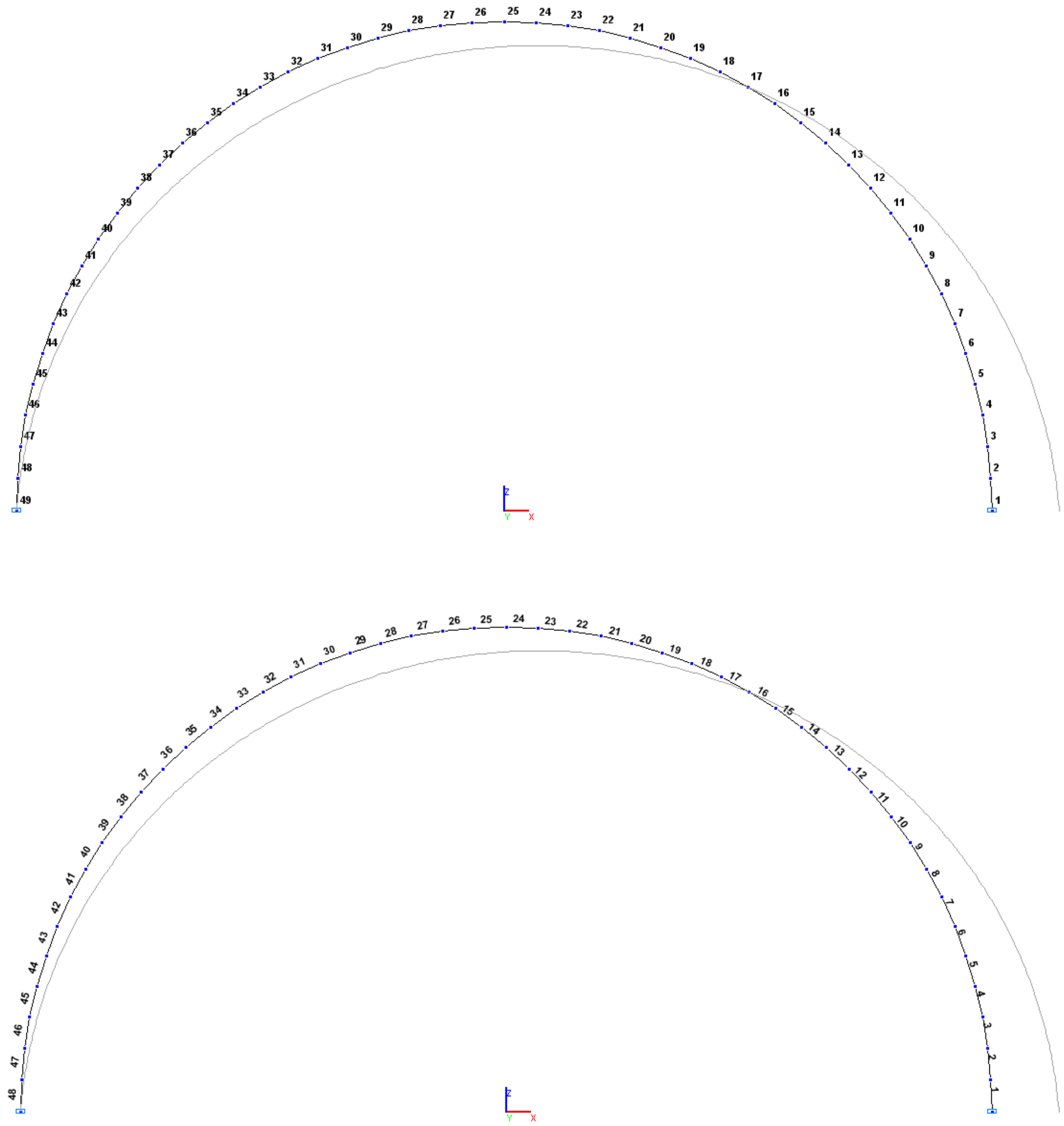
References: P. Dellus, Resistance de materiaux, Paris, Technique et Vulgarisation, 1958.

Initial data:

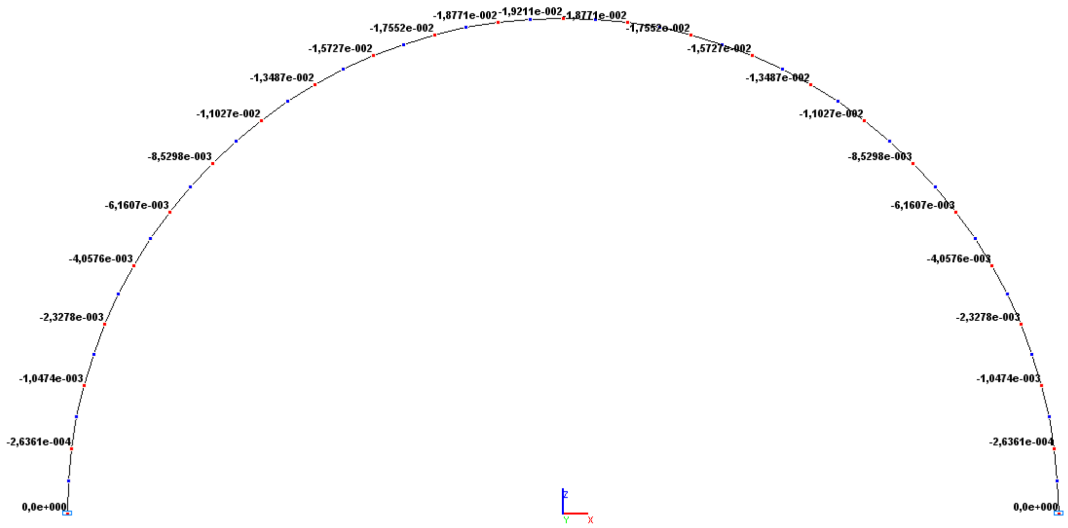
$E = 2.0 \cdot 10^{11}$ Pa	- elastic modulus of a semicircular arch;
r = 1.0 m	- arc radius of the longitudinal axis of the semicircular arch;
d_e = 0.020 m	- outer diameter of the ring cross-section of the arch;
d_i = 0.016 m	- inner diameter of the ring cross-section of the arch;
F = 100 N	- value of the concentrated force.

Finite element model: Design model – plane frame, 48 bar elements of type 10. Boundary conditions are provided by imposing constraints in the directions of the degrees of freedom X, Z – for the pinned support and Z – for the roller support. Number of nodes in the design model – 49.

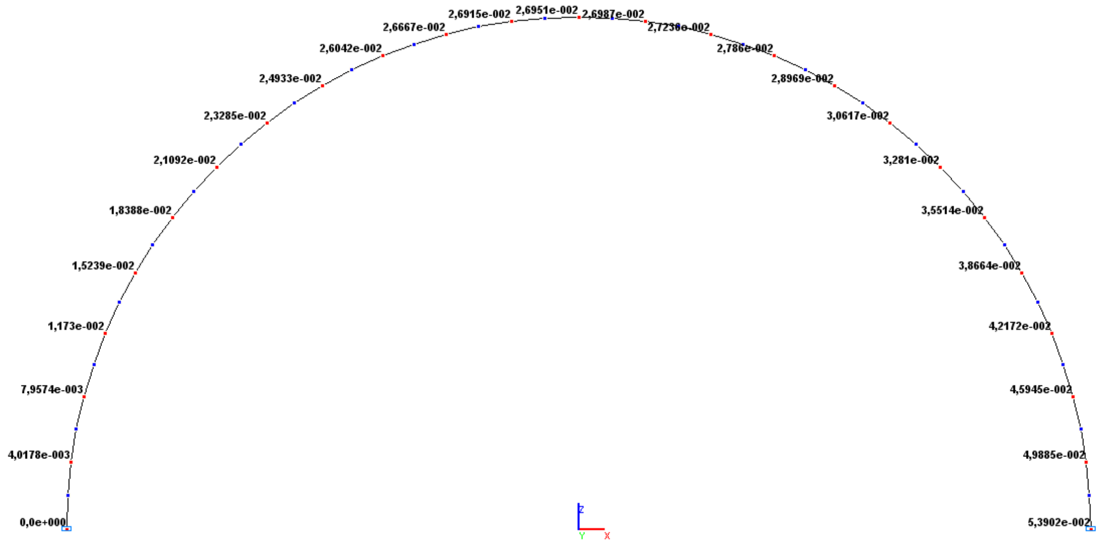
Results in SCAD



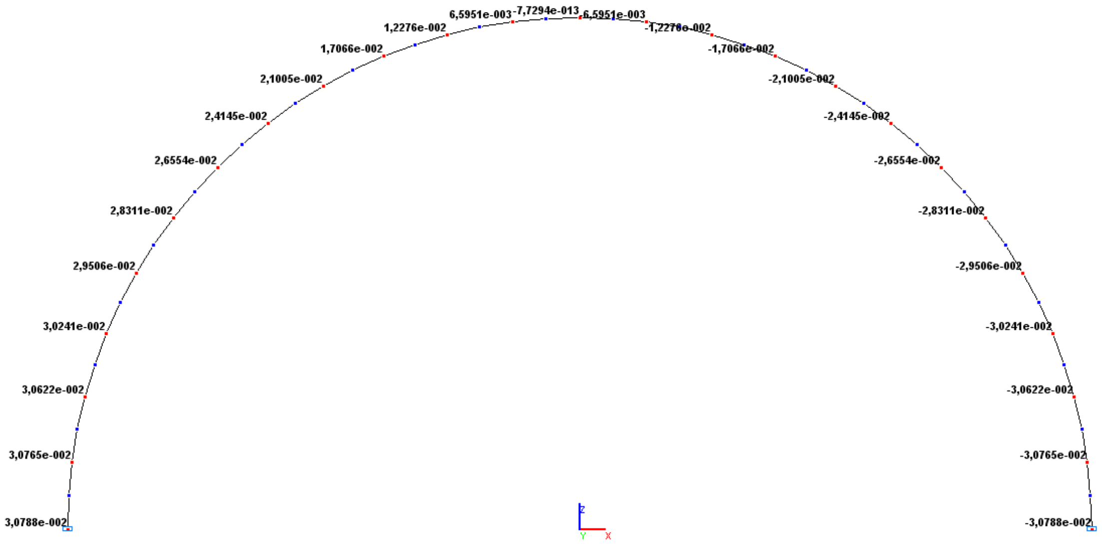
Design and deformed models



Values of vertical displacements Z (m)



Values of horizontal displacements X (m)



Values of rotation angles UY (rad)

Comparison of solutions:

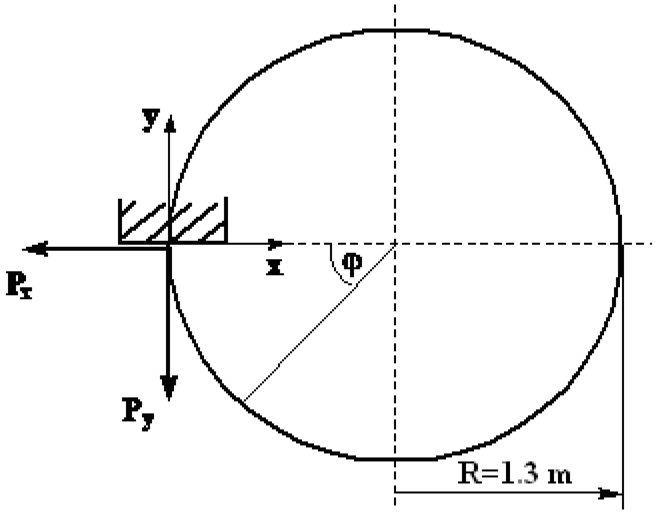
Parameter	**Theory**	**SCAD**	**Deviations, %**
Deflection of the longitudinal axis of the arch Z, m	$-1.9206 \cdot 10^{-2}$	$-1.9211 \cdot 10^{-2}$	0.03
Displacement of the roller support X, m	$5.3912 \cdot 10^{-2}$	$5.3902 \cdot 10^{-2}$	0.02
Rotation angle of the roller support UY, rad	$-3.0774 \cdot 10^{-2}$	$-3.0788 \cdot 10^{-2}$	0.05
Rotation angle of the pinned support UY, rad	$3.0774 \cdot 10^{-2}$	$3.0788 \cdot 10^{-2}$	0.05

Notes: In the analytical solution, the deflection of the longitudinal axis of the arch *Z*, the displacement of the roller support *X* and the rotation angles of the support hinges *UY* are determined according to the following formulas:

$$Z = \frac{\pi}{8} \cdot \frac{F \cdot r}{E \cdot A} + \left(\frac{3 \cdot \pi}{8} - 1 \right) \cdot \frac{F \cdot r^3}{E \cdot I}; \qquad X = \frac{1}{2} \cdot \frac{F \cdot r}{E \cdot A} - \frac{1}{2} \cdot \frac{F \cdot r^3}{E \cdot I}; \qquad UY = \pm \left(\frac{\pi}{4} - \frac{1}{2} \right) \frac{F \cdot r^2}{E \cdot I}, \text{ where:}$$

$$I = \frac{\pi \cdot d_e^{\,2}}{4} \cdot \left(1 - \left(\frac{d_i}{d_e} \right)^2 \right); \qquad I = \frac{\pi \cdot d_e^{\,4}}{64} \cdot \left(1 - \left(\frac{d_i}{d_e} \right)^4 \right).$$

Strain State of a Split Circular Ring Subjected to Two Mutually Perpendicular Forces P_x and P_y, Acting in the Plane of the Ring



Objective: Strain state of a split circular ring under bending in the plane without taking into account the transverse shear deformations.

Initial data file: **4.6.SPR**

Problem formulation: The split circular ring is subjected to two mutually perpendicular forces P_x and P_y, acting in the plane of the ring axes. Determine the strain state of the ring.

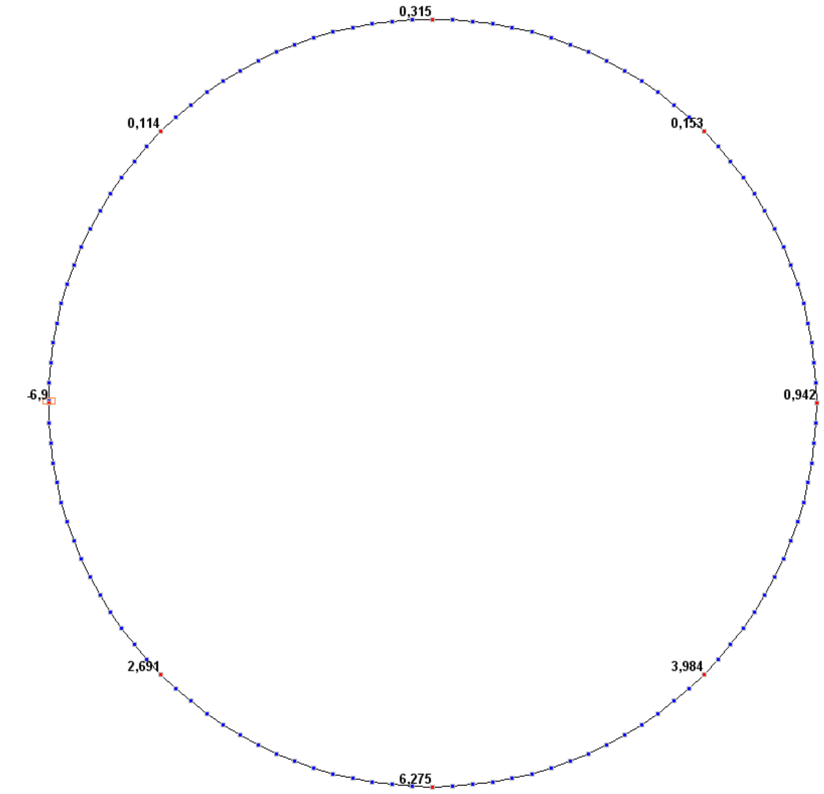
References: Strength Analysis in Mechanical Engineering / S. D. Ponomarev, V. L. Biderman, K. K. Likharev, et al., In three volumes. Volume 1. M.: Mashgiz, 1956.

Initial data:

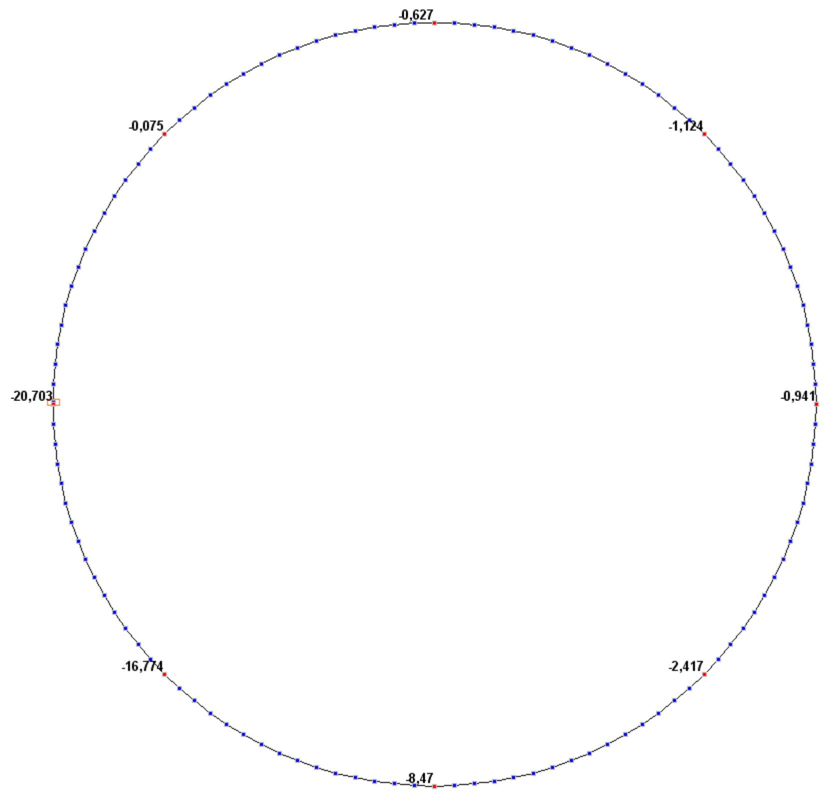
$E = 2.0 \cdot 10^{11}$ Pa	- elastic modulus;
$R = 1.3$ m	- radius of the ring axis;
$F = 1 \cdot 10^{-2}$ m^2	- cross-sectional area;
$I = 5 \cdot 10^{-6}$ m^4	- cross-sectional moment of inertia;
$P_x = P_y = 1$ kN	- value of the concentrated force.

Finite element model: Design model – plane model, 120 bar elements of type 2, 121 nodes.

Results in SCAD



Values of displacements u (mm)



Values of displacements v (mm)

Comparison of solutions:

Angle φ, degree	Displacements along the x axis			Displacements along the y axis		
	Theory	SCAD	Deviations, %	Theory	SCAD	Deviations, %
0	-6.902	-6.900	0.03	-20.706	-20.703	0.01
45	2.690	2.691	0.04	-16.777	-16.774	0.02
90	6.275	6.275	0.00	-8.472	-8.470	0.02
135	3.984	3.984	0.00	-2.419	-2.417	0.08
180	0.943	0.942	0.11	-0.943	-0.941	0.21
225	0.154	0.153	0.65	-1.125	-1.124	0.09
270	0.316	0.315	0.32	-0.627	-0.627	0.00
315	0.114	0.114	0.00	-0.074	-0.075	1.35
360	0.000	0.000	0.00	0.000	0.000	0.00

Notes: In the analytical solution the displacements of the points of the ring in the directions x and y are determined according to the following formulas:

$$u(\varphi) = \frac{P_x \cdot R^3}{E \cdot I} \cdot \beta_1(\varphi) + \frac{P_y \cdot R^3}{E \cdot I} \cdot \beta_2(\varphi),$$

where:

$$\beta_1(\varphi) = -0.5 \cdot (2 \cdot \pi - \varphi) - \sin(\varphi) + 0.5 \cdot \sin(\varphi) \cdot \cos(\varphi);$$

$$\beta_2(\varphi) = 1 + (2 \cdot \pi - \varphi) \cdot \sin(\varphi) - \cos(\varphi) + 0.5 \cdot \sin^2(\varphi);$$

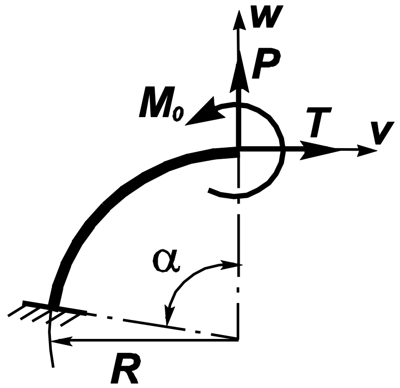
$$v(\varphi) = \frac{P_x \cdot R^3}{E \cdot I} \cdot \gamma_1(\varphi) + \frac{P_y \cdot R^3}{E \cdot I} \cdot \gamma_2(\varphi),$$

where:

$$\gamma_1(\varphi) = -1 + \cos(\varphi) + 0.5 \cdot \sin^2(\varphi),$$

$$\gamma_2(\varphi) = -0.5 \cdot (2 \cdot \pi - \varphi) - (2 \cdot \pi - \varphi) \cdot \cos(\varphi) - \sin(\varphi) - 0.5 \cdot \sin(\varphi) \cdot \cos(\varphi).$$

Cantilever Curved Beam with a Transverse Concentrated Force at Its Free End



Objective: Check of the accuracy of the determination of the displacement value for the free end of a beam in the direction of the concentrated force for models of different dimensions.

Initial data files:

File name	Description
4.38_c.SPR	Bar model
4.38_п.SPR	Shell element model
4.38_o.SPR	Solid element model

Problem formulation: The cantilever curved beam with a longitudinal circular axis having a length of the split ring and with a rectangular cross-section constant along the axis is subjected to a transverse concentrated force *P* applied at its free end. Determine the displacement of the free end of the beam *w* in the direction of the concentrated force.

References: Sacharov A., Altenbach J.: Finite Elements in Solid Mechanics. -Kiev: High School. 1982, Leipzig: Fahbuhferlag. 1982;
G.S. Pisarenko, A.P. Yakovlev, V.V. Matveev, Handbook on Strength of Materials. — Kiev: Naukova Dumka, 1975.

Initial data:
E = 100.0 kPa - elastic modulus;
ν = 0.0 - Poisson’s ratio;
R = 0.20 m - arc radius of the longitudinal axis of the cantilever curved beam;
α = 360º - central angle of the arc length of the longitudinal axis of the cantilever curved beam;
b = h = 0.01 m - cross-sectional dimensions of the cantilever curved beam;
P = 10^{-8} kN - value of the transverse concentrated force on the free end of the beam.

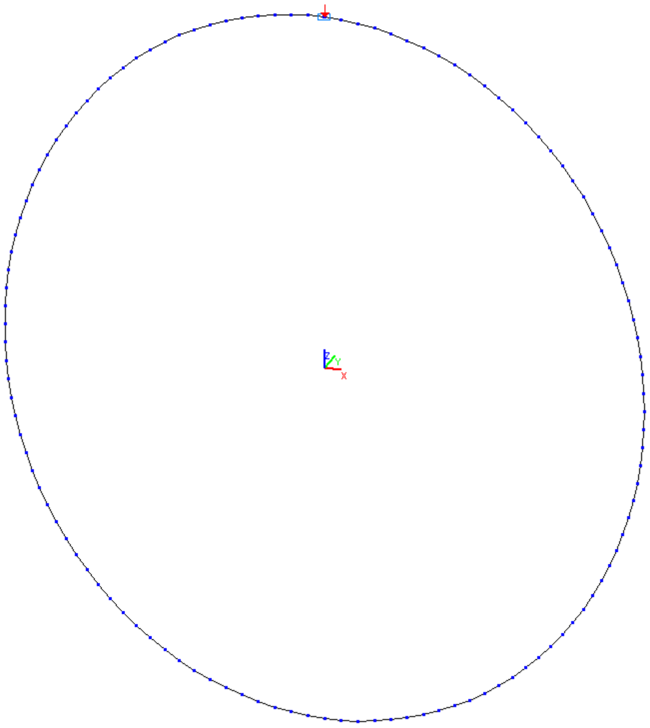
Finite element model: Design model – general type system. Three design models are considered:

Bar model (B), 120 elements of type 5, the spacing of the finite element mesh along the longitudinal axis is 3.0º, 121 nodes;

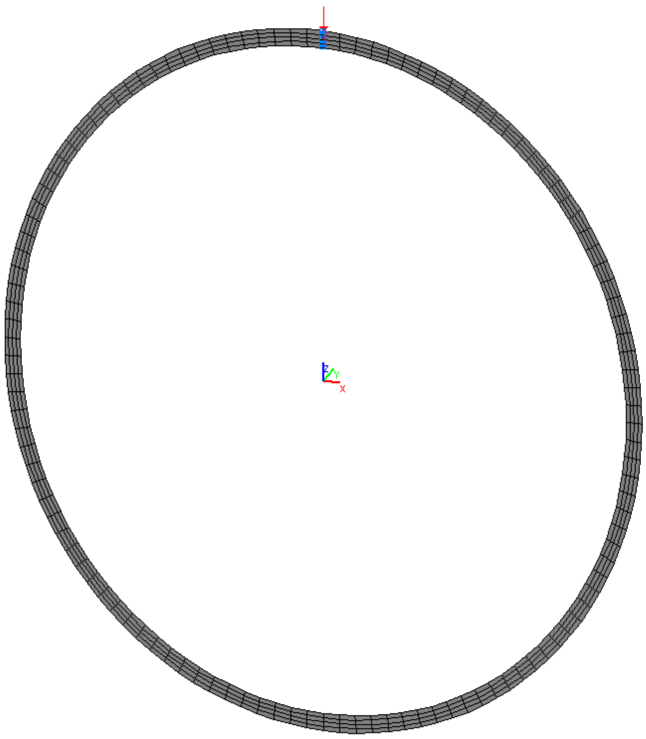
Shell element model (P), 480 eight-node elements of type 50, the spacing of the finite element mesh along the longitudinal axis is 3.0º, and along the height of the beam is 0.0025 m, 1689 nodes;

Solid element model (S), 1920 twelve-node elements of type 37, the spacing of the finite element mesh along the longitudinal axis is 3.0º, and along the height and width of the beam is 0.0025 m, 10865 nodes.

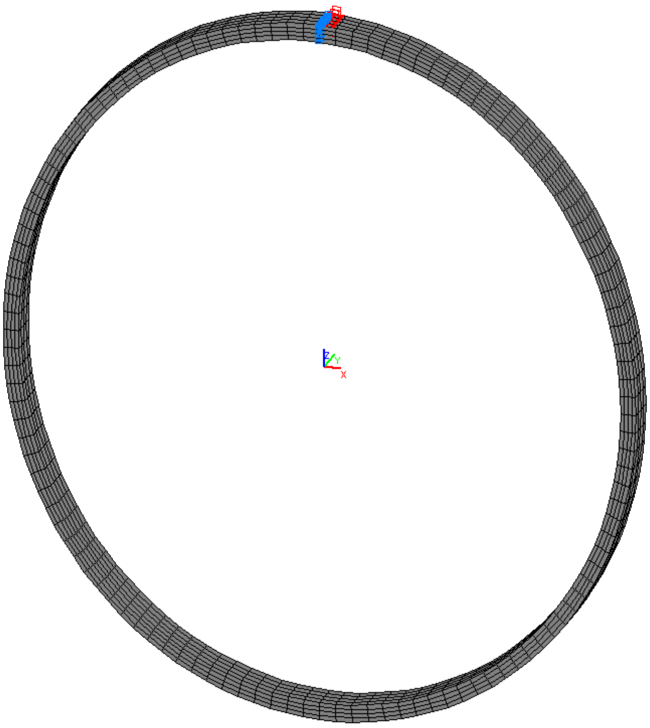
Results in SCAD



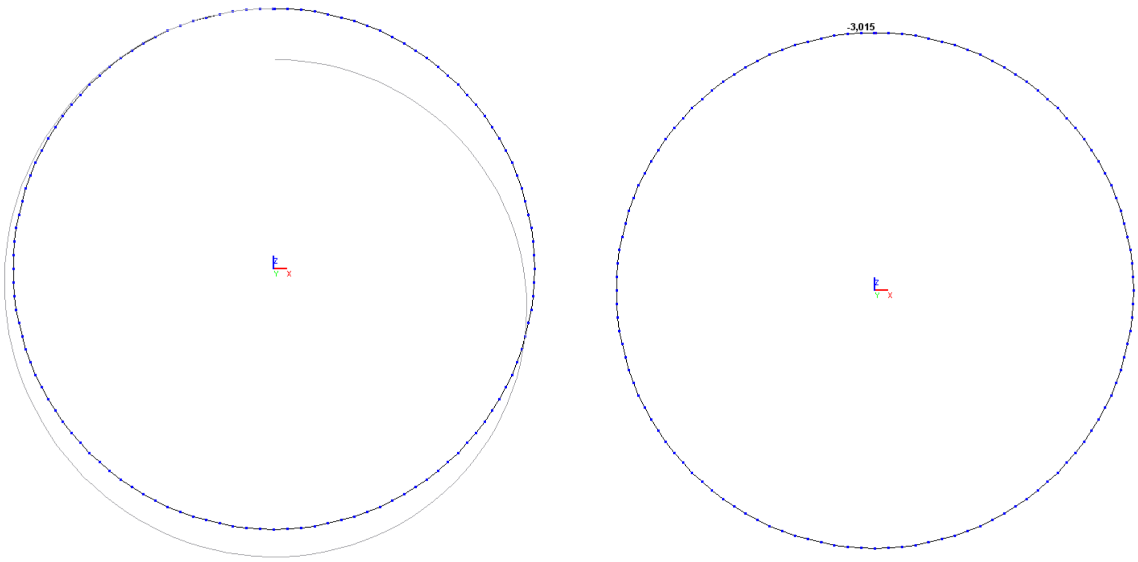
Design model. Bar model



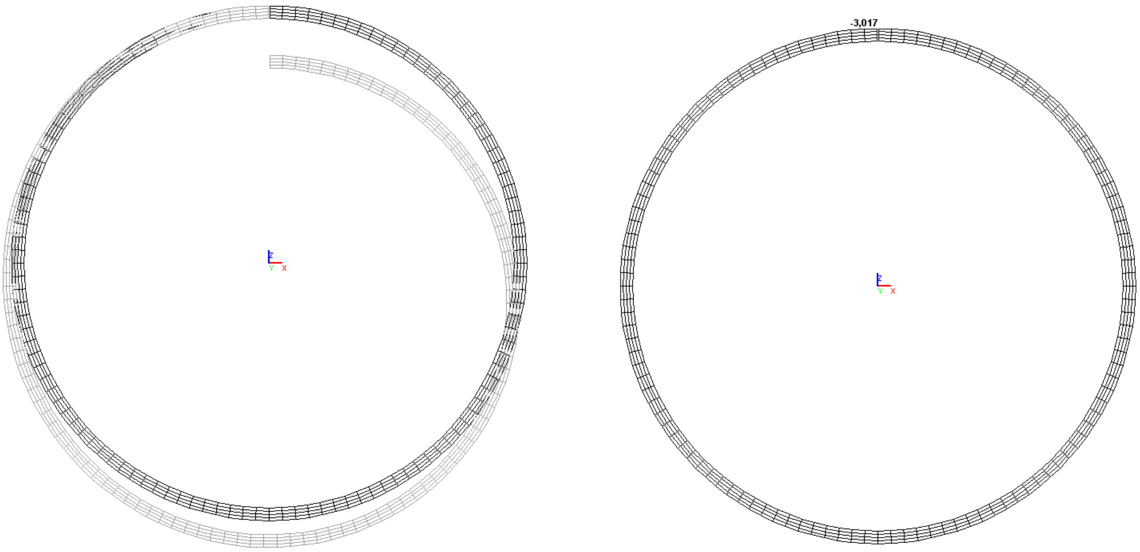
Design model. Shell element model



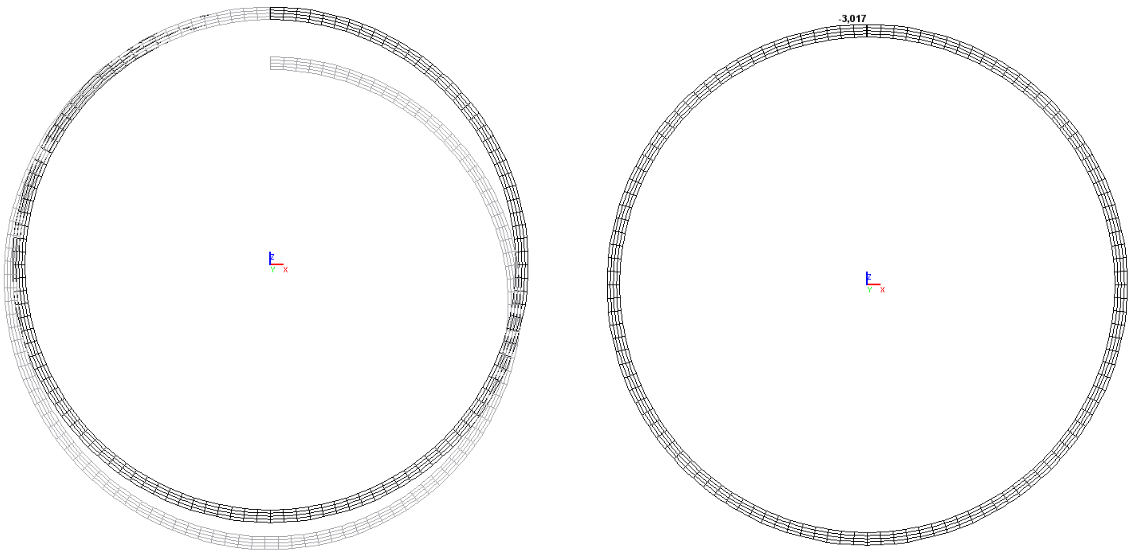
Design model. Solid element model



Deformed model and the values of the displacements of the free end of the beam w in the bar model (mm)



Deformed model and the values of the displacements of the free end of the beam w in the shell element model (mm)



Deformed model and the values of the displacements of the free end of the beam w in the solid element model (mm)

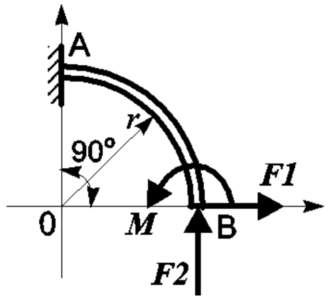
Comparison of solutions:

Model	Displacements w, mm	Deviations, %
Bar (B)	3.015	0.03
Shell element (P)	3.017	0.03
Solid element (S)	3.017	0.03
Theory	3.016	–

Notes: In the analytical solution the displacement of the free end of the beam *w* in the direction of the transverse concentrated force is determined according to the following formula (G.S. Pisarenko, A.P. Yakovlev, V.V. Matveev, Handbook on Strength of Materials. — Kiev: Naukova Dumka, 1975, p. 392):

$$w = \frac{12 \cdot P \cdot R^3}{E \cdot b \cdot h^3} \cdot \left(\frac{\alpha}{2} - \frac{sin(2 \cdot \alpha)}{4} \right).$$

Cantilever Circular Bar of Constant Cross-Section with Concentrated Forces and a Moment Acting in Its Plane at Its Free End



Objective: Determination of the strain state of a cantilever circular bar of constant cross-section with concentrated forces and a moment acting in its plane at its free end.

Initial data file:

File name	Description
SSLL06_вариант_1_v11.3.SPR	Design model – plane frame. Cantilever circular bar lies in the XOZ plane of the global coordinate system

Problem formulation: The cantilever circular bar of constant cross-section is subjected to concentrated horizontal (normal) F1 and vertical (tangential) F2 forces and a moment *M* acting in its plane and applied at its free end. Determine the horizontal X and vertical Z displacements , as well as the rotation angle UY of the free end of the bar (point B).

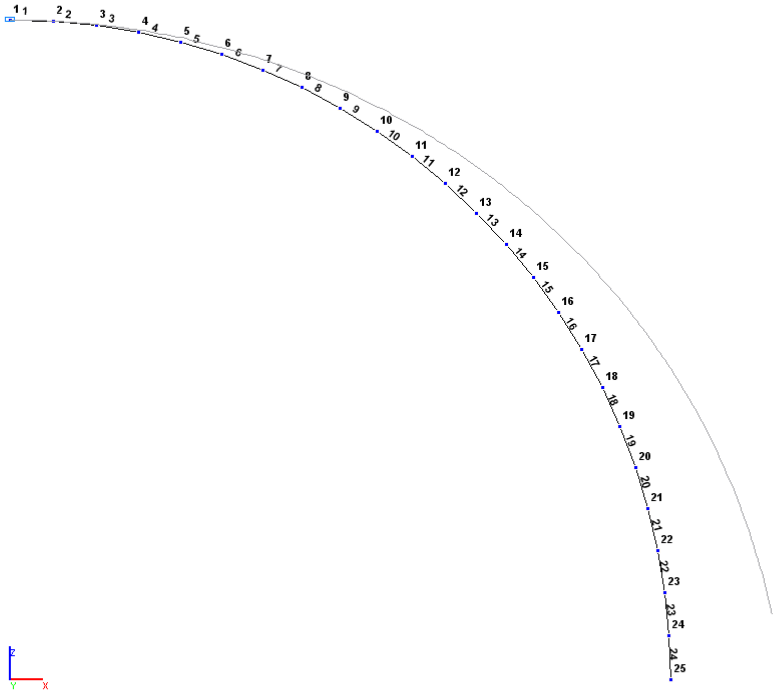
References: J.S. Przemieniecki, Theory of matrix structural analysis, New York, McGraw-Hill, 1968.

Initial data:

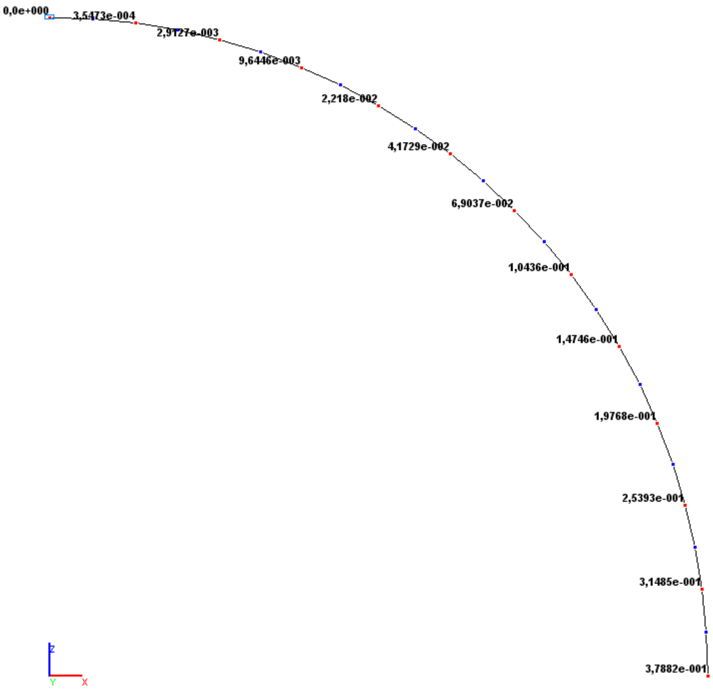
$E = 2.0 \cdot 10^{11}$ Pa - elastic modulus of the cantilever circular bar;
r = 3.0 m - arc radius of the longitudinal axis of the cantilever circular bar;
$\alpha = 90^o$ - central angle of the arc length of the longitudinal axis of the cantilever circular bar;
$d_e = 0.020$ m - outer diameter of the ring cross-section of the bar;
$d_i = 0.016$ m - inner diameter of the ring cross-section of the bar;
F1 = 10 N - value of the horizontal concentrated force;
F2 = 5 N - value of the vertical concentrated force;
M = 8 N·m - value of the concentrated moment.

Finite element model: Design model – plane frame, 24 bar elements of type 10. Boundary conditions are provided by imposing constraints in the directions of the degrees of freedom X, Z, UY (point A). Number of nodes in the design model – 25.

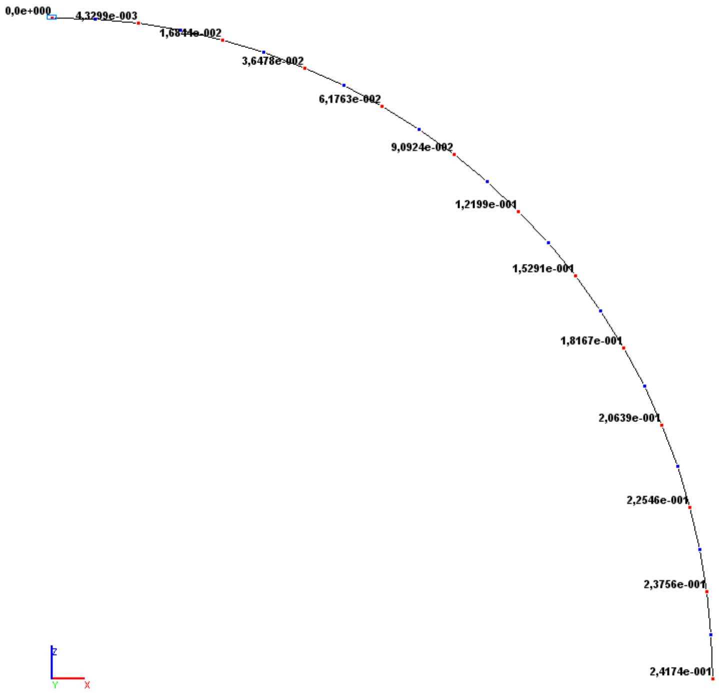
Results in SCAD



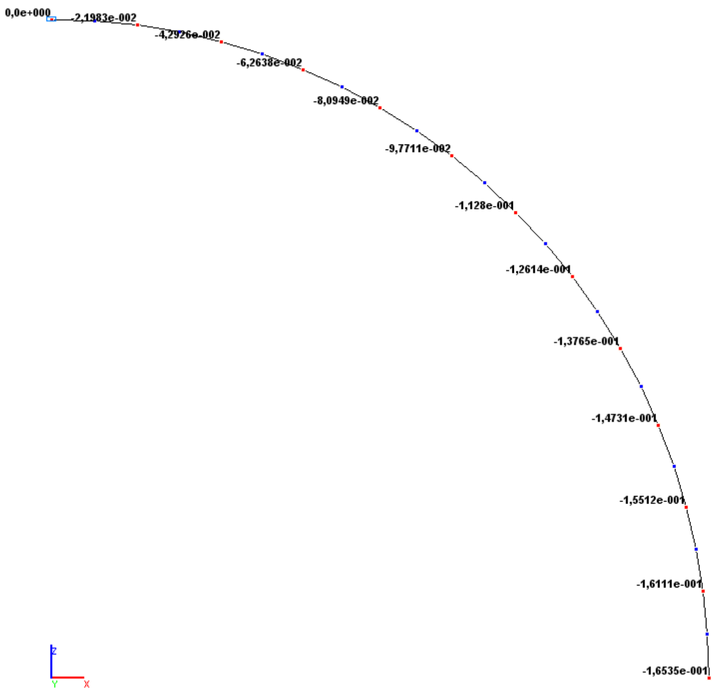
Design and deformed models



Values of horizontal displacements X (m)



Values of vertical displacements Z (m)



Values of rotation angles UY (rad)

Comparison of solutions:

Parameter	Theory	SCAD	Deviations, %
Horizontal displacement X (point B), m	$3.7908 \cdot 10^{-1}$	$3.7882 \cdot 10^{-1}$	0.07
Vertical displacement Z (point B), m	$2.4173 \cdot 10^{-1}$	$2.4174 \cdot 10^{-1}$	0.01
Rotation angle UY (point B), rad	$-1.6539 \cdot 10^{-1}$	$-1.6535 \cdot 10^{-1}$	0.02

Notes: In the analytical solution the horizontal X and vertical Z displacements, as well as the rotation angle UY of the free end of the bar are determined according to the following formulas:

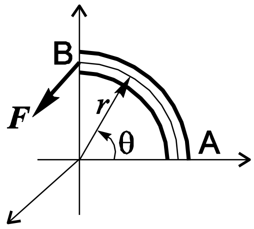
$$X = \frac{r^2}{E \cdot I} \cdot \left(M + F1 \cdot r \cdot \frac{\pi}{4} + F2 \cdot r \cdot \frac{1}{2} \right);$$

$$Z = \frac{r^2}{E \cdot I} \cdot \left(M \cdot \left(\frac{\pi}{2} - 1 \right) + F1 \cdot r \cdot \frac{1}{2} + F2 \cdot r \cdot \left(\frac{3 \cdot \pi}{4} - 2 \right) \right);$$

$$UY = -\frac{r}{E \cdot I} \cdot \left(M \cdot \frac{\pi}{2} + F1 \cdot r + F2 \cdot r \cdot \left(\frac{\pi}{2} - 1 \right) \right), \text{ where:}$$

$$I = \frac{\pi \cdot d_e^{4}}{64} \cdot \left(1 - \left(\frac{d_i}{d_e} \right)^4 \right).$$

Cantilever Circular Bar of Constant Cross-Section with a Concentrated Force out of Its Plane at Its Free End



Objective: Determination of the stress-strain state of a cantilever circular bar of constant cross-section with a concentrated force acting out of its plane at its free end.

Initial data file:

File name	Description
SSLL07_вариант_1_v11.3.SPR	Design model – general type system. Cantilever circular bar lies in the XOZ plane of the global coordinate system

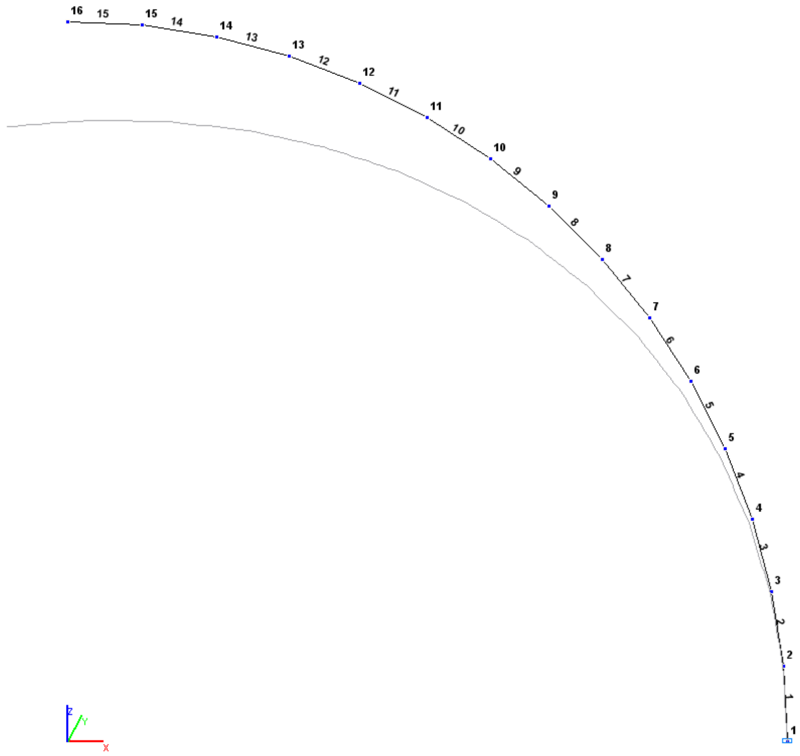
Problem formulation: The cantilever circular bar of constant cross-section is subjected to a concentrated force F acting in its plane and applied at its free end. Determine the displacement Y of the free end of the bar out of its plane (point B), as well as the torque M_x and out-of-plane bending moment M_z for the cross-section corresponding to the central angle θ from the clamped end.

References: S. Timoshenko, Strength of materials, Part 1: Elementary theory and problem, 3ed, 1955; R.J. Roark, Formulas for stress and strain, 4ed, New York, McGraw-Hill, 1965.

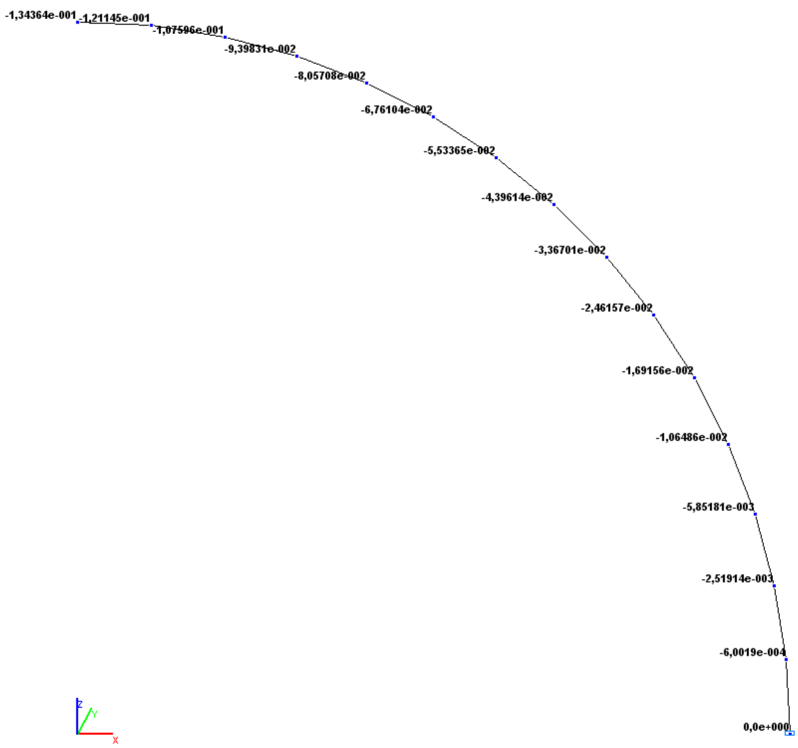
Initial data:
$E = 2.0 \cdot 10^{11}$ Pa - elastic modulus of the cantilever circular bar;
ν = 0.3 - Poisson’s ratio;
r = 1.0 m - arc radius of the longitudinal axis of the cantilever circular bar;
θ = 90º - central angle of the arc length of the longitudinal axis of the cantilever circular bar;
d_e = 0.020 m - outer diameter of the ring cross-section of the bar;
d_i = 0.016 m - inner diameter of the ring cross-section of the bar;
F = 100 N - value of the concentrated force.

Finite element model: Design model – general type system, 15 bar elements of type 10. Boundary conditions are provided by imposing constraints in the directions of the degrees of freedom X, Y, Z, UX, UY, UZ (point A). Number of nodes in the design model – 16.

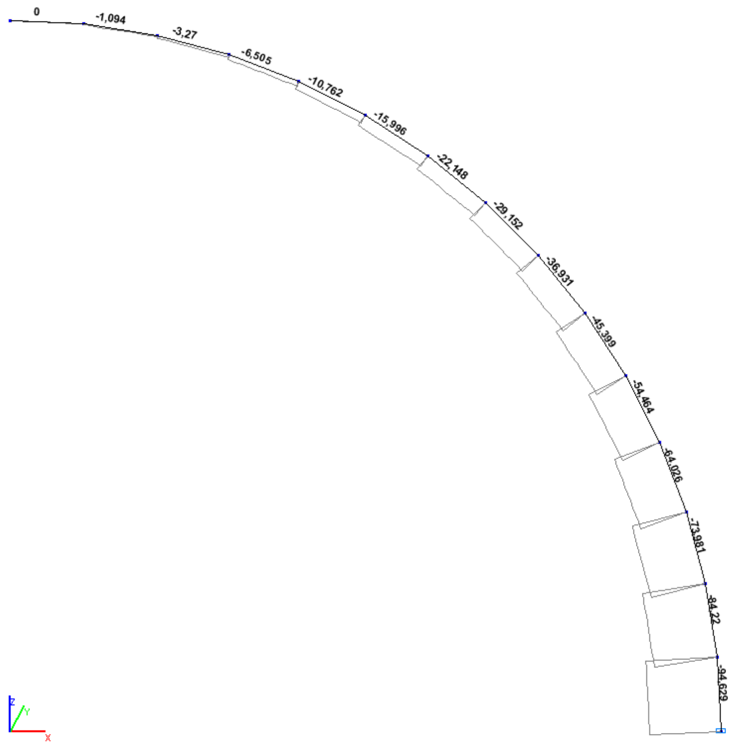
Results in SCAD



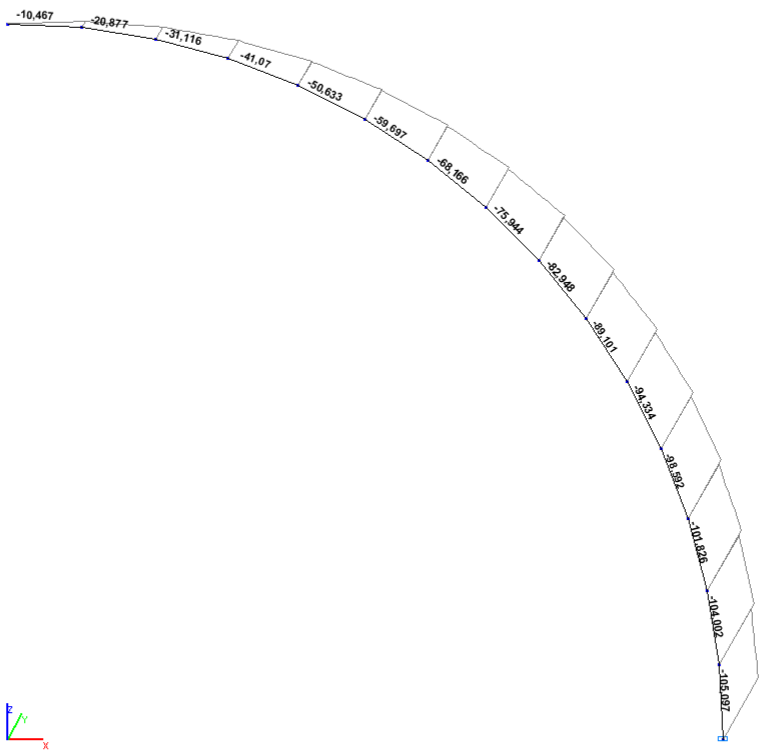
Design and deformed models



Values of displacements out of the plane of the bar Y (m)



Torque diagram M_x (kN·m)



Bending moment diagram out of the plane of the bar M_z (kN·m)

Comparison of solutions:

Parameter	Theory	SCAD	Deviations, %
Displacement out of the plane of the bar Y (point B), m	$-1.34462\cdot10^{-1}$	$-1.34364\cdot10^{-1}$	0.07
Torque M_x (θ = 15º), N·m	-74.118	-73.981	0.18
Bending moment out of the plane of the bar M_z (θ = 15º), N·m	-96.593	-96.593	0.00

***Notes*:** In the analytical solution the displacement Y of the free end of the bar out of its plane (point B), as well as the torque M_x and out-of-plane bending moment M_z for the cross-section corresponding to the central angle θ from the clamped end are determined according to the following formulas:

$$Y = \frac{F \cdot r^3}{E \cdot I} \cdot \left(\frac{1+3\cdot\lambda}{2} \cdot \frac{\pi}{2} - 2\cdot\lambda \right), \text{ where:}$$

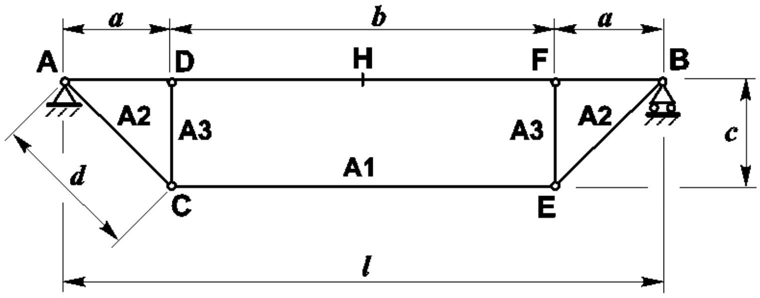
$$I_z = \frac{\pi \cdot d_e^{\,4}}{64} \cdot \left(1 - \left(\frac{d_i}{d_e} \right)^4 \right), \qquad \lambda = \frac{E \cdot I_z}{G \cdot I_x} = 1 + \nu$$

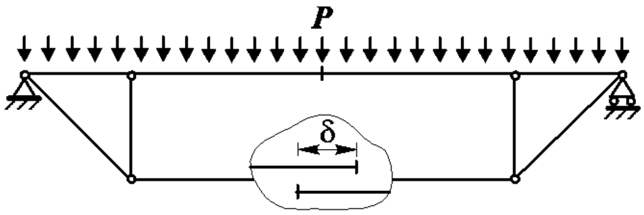
(for the ring cross-section);

$$M_x = F \cdot r \cdot cos(\theta);$$

$$M_z = F \cdot r \cdot (1 - sin(\theta)).$$

Single-Span Beam with a Prestressed Tie Subjected to a Uniformly Distributed Load





Objective: Determination of the stress-strain state of a beam with a tie taking into account the transverse shear deformations in the beam.

Initial data file: **SSLL13_v11.3.spr**

Problem formulation: The single-span beam with a tie tightened by the displacement value δ by the struts is subjected to a uniformly distributed load *q*. Determine the longitudinal force *N* in the tie CE, the bending moment *M* in the section of the stiffening beam H in the middle of its span, the vertical displacement z in the joint between the strut and the stiffening beam (point D).

References: M. Laredo, Resistence des materiaux, Paris, Dunod, 1970, p. 77.

Initial data:

Tie A1:

$EF = 9.450 \cdot 10^8$ N - axial stiffness;

Strut A2:

$EF = 7.308 \cdot 10^8$ N - axial stiffness;

Stiffening beam AB:

$EF = 3.1836 \cdot 10^9$ N - axial stiffness;

$EIy = 4.5654 \cdot 10^7$ N/m^2 - bending stiffness;

$GFy = 5.09376 \cdot 10^8$ N - shear stiffness;

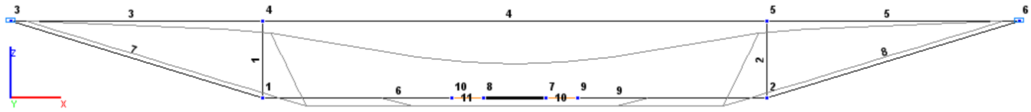
Loads and actions:

$\delta = 6.52 \cdot 10^{-3}$ m - displacement in the tie;

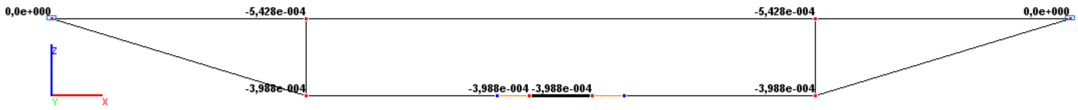
$P = 5.0 \cdot 10^4$ N/m - transverse uniformly distributed load on the stiffening beam.

Finite element model: Design model – plane frame, tie A1 – 4 elements of type 1, struts A2 – 2 elements of type 1, stiffening beam AB – 3 elements of type 2 taking into account the shear, elements modeling the prestressing of the tie in the CE section – 2 elements of type 154 with the axial stiffness $EF = 1.0 \cdot 10^{18}$ N. The tie in the CE section is represented by two elements of equal length increased with respect to half the length of the section by imposing rigid inserts in the longitudinal direction. The length of the elements is increased in order to separate their nodes near the symmetry axis of the structure at the prestressing stage. A null element is attached to each of these nodes, with the help of which they are displaced in the longitudinal direction. In order to prevent the dimensional instability of the system the displacements of nodes are combined by elements in the transverse direction by the degree of freedom Z in the section of the tie CE. Boundary conditions in the direction of the degree of freedom Z in the support nodes A and B are provided by imposing the respective rigid constraint. Number of nodes in the design model – 10.

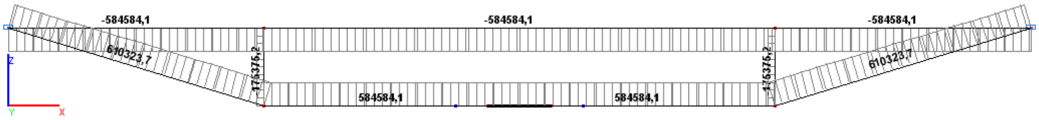
Results in SCAD



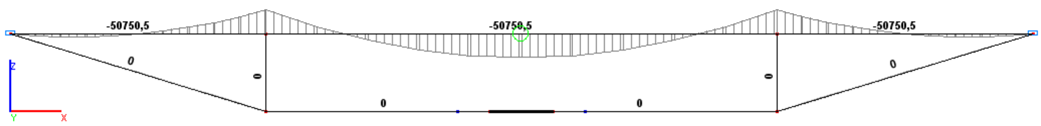
Design and deformed models

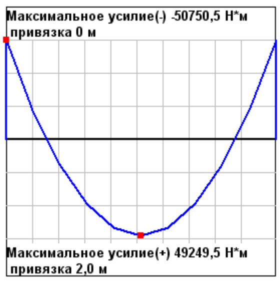


Values of vertical displacements Z (m)



Values of longitudinal forces N (N)



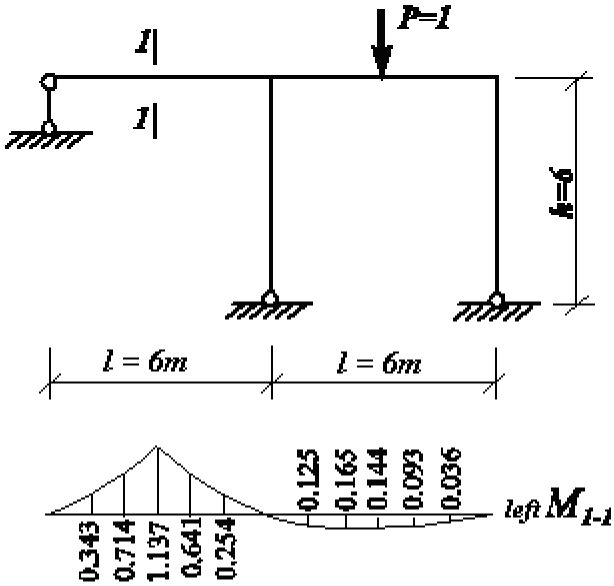


Values of bending moments M (N·m)

Comparison of solutions:

Parameter	Theory	SCAD	Deviations, %
Bending moment M_H, N·m	49249.5	49249.5	0.00
Longitudinal force N_{CE}, N	584584.0	584584.1	0.00
Vertical displacement Z_D, m	$-5.428 \cdot 10^{-4}$	$-5.428 \cdot 10^{-4}$	0.00

Two-Span Single-Storey Frame Subjected to a Constant Transverse Unit Force Moving Along the Girder Spans with a Small Speed. Plotting of Influence Lines of Internal Forces in the Frame Sections



Objective: Determination of the values of the bending moment in the section of the middle of the left girder span of a two-span single-storey frame depending on the position of a constant transverse unit force moving along the girder spans with a small speed.

Initial data file: **Influence_Line.SPR**

Problem formulation: The constant transverse unit force P moves along the girder of the two-span single-storey frame with a small speed. The girder is rigidly connected with the middle and right edge columns, which have pinned supports, and the end of its left span is simply supported. Determine the values of the bending moment in the section of the middle of the left girder span of the frame M_{1-1} depending on the position of the transverse force and plot the influence line.

References: A. F. Smirnov, A. V. Aleksandrov, B. Ya. Lashchenikov, N. N. Shaposhnikov, Structural Mechanics. Bar Systems, Moscow, Stroyizdat, 1981, p. 352-356.

Initial data:

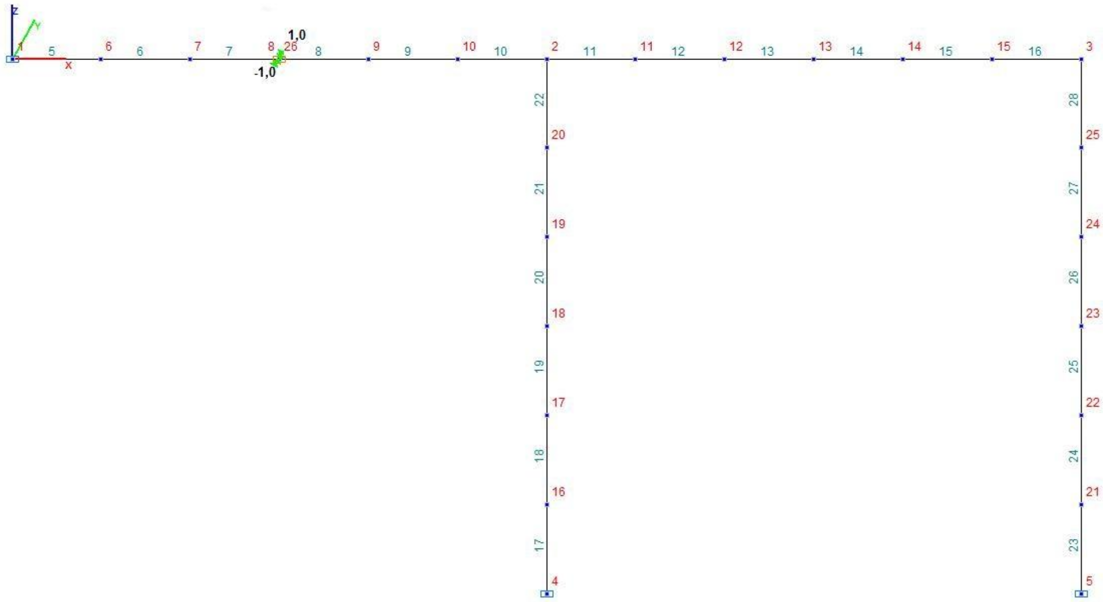
l = 6.0 m	- length of the girders of the frame;
h = 6.0 m	- height of the columns of the frame;
EA = $1.0 \cdot 10^6$ kN	- axial stiffness of the structural members of the frame;
EI = 83.3333 kN·m^2	- bending stiffness of the structural members of the frame;
P = 1.0 kN	- value of the transverse unit force.

Finite element model: Design model – plane frame, 24 elements of type 2. The spacing of the finite element mesh along the longitudinal axes of the structural elements (along the X1 axes of the local coordinate systems) is 1.0 m. Boundary conditions are provided by imposing constraints on the support nodes of the columns in the directions of the degrees of freedom X, Z and on the support node of the left girder span in the direction of the degree of freedom Z.
The problem is solved by the kinematic method:

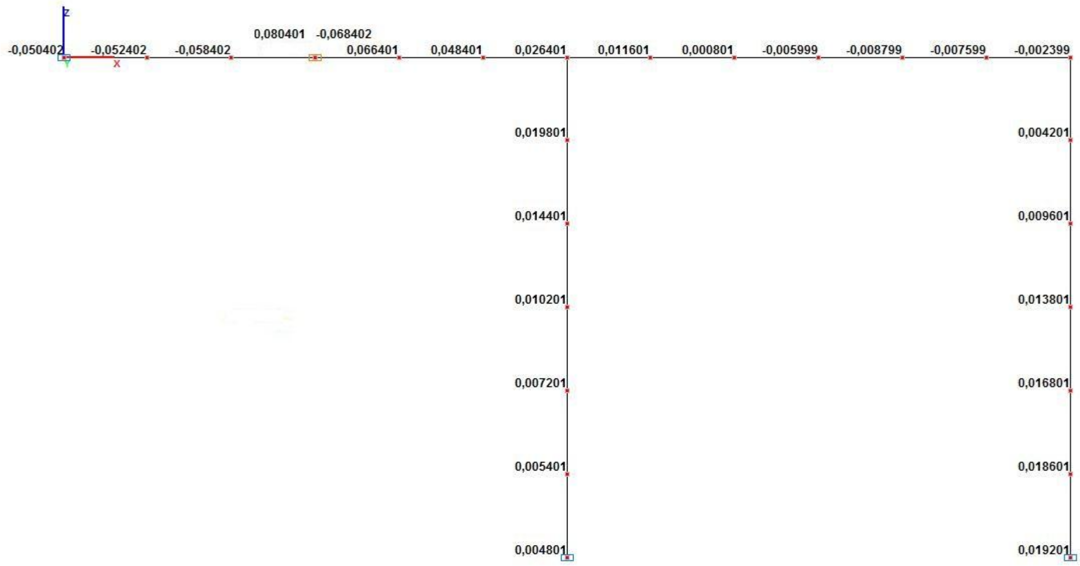
- the elements of the middle of the left girder span are divided with the formation of a pair of duplicate nodes each one belonging to one of these adjacent elements;
- the displacements of the pair of duplicate nodes are merged for all degrees of freedom except for UY;
- unit concentrated opposite bending moments M_y = 1.0 kN·m are applied to the nodes of the pair.

The result of the influence line of the bending moment in the section of the left frame span [nodes 26, 8] should be considered in the form of deformations according to the following formula: –Z/[UY26-UY8]/1000. It is necessary to divide the expression by 1000 if the dimension Z is given in mm.
Number of nodes in the design model – 26.

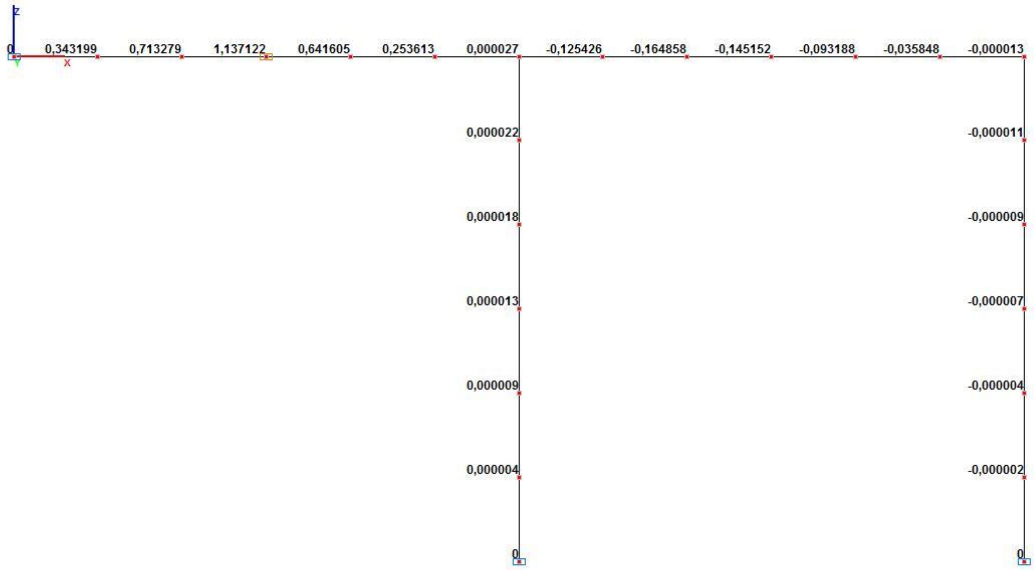
Results in SCAD



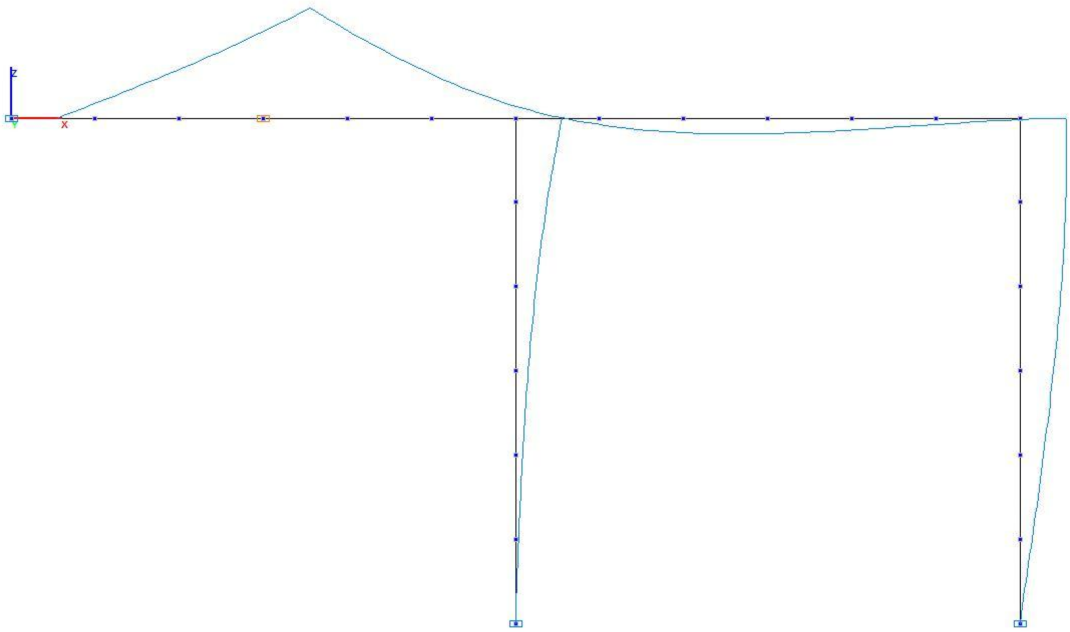
Design model



Values of rotation angles UY (rad)



Values of the bending moment in the section of the middle of the left girder span of the frame M_{1-1} (kN·m) depending on the position of the transverse force



Influence lines of the bending moment in the section of the middle of the left girder span of the frame M_{1-1}

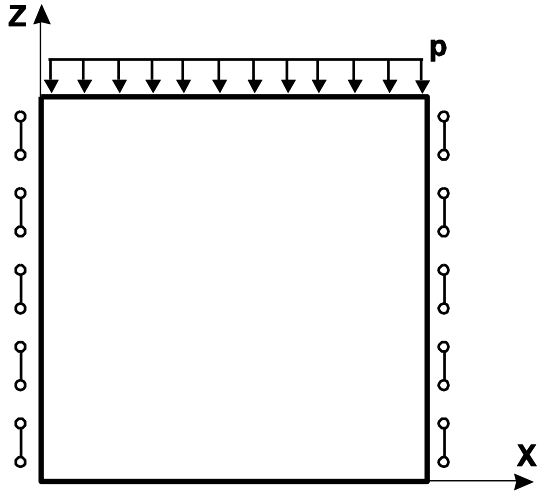
Comparison of solutions:

Values of the bending moment in the section of the middle of the left girder span of the frame M_{1-1} (kN·m) depending on the position of the transverse force

Position of the transverse force from the edge of the left span, m	Theory	SCAD	Deviation, %
0.00	0.000	0.000	0.00
1.00	0.343	0.343	0.00
2.00	0.714	0.713	0.14
3.00	1.137	1.137	0.00
4.00	0.641	0.642	0.16
5.00	0.254	0.254	0.00
6.00	0.000	0.000	0.00
7.00	-0.125	-0.125	0.00
8.00	-0.165	-0.165	0.00
9.00	-0.144	-0.145	0.69

Position of the transverse force from the edge of the left span, m	Theory	SCAD	Deviation, %
10.00	-0.093	-0.093	0.00
11.00	-0.036	-0.036	0.00
12.00	0.000	0.000	0.00

Bending of a Rectangular Deep Beam Rigidly Suspended along the Sides Subjected to a Uniformly Distributed Load Applied to Its Upper Side



Objective: Determination of the strain state of a rectangular deep beam rigidly suspended along the sides subjected to a uniformly distributed load applied to its upper side.

Initial data file: **KSLS01_v11.3.SPR**

Problem formulation: The uniformly distributed load *p* acting in the plane of the deep beam along the y axis is applied to the upper side of the rectangular deep beam rigidly suspended along the sides. Determine the components of the strain tensor in the Cartesian coordinates *u*(x,z) and *v*(x,z) for the midsurface of the deep beam in its plane.

References: A.S. Kalmanok, Analysis of Deep Beams, Moscow, Gosstroyizdat, 1956.

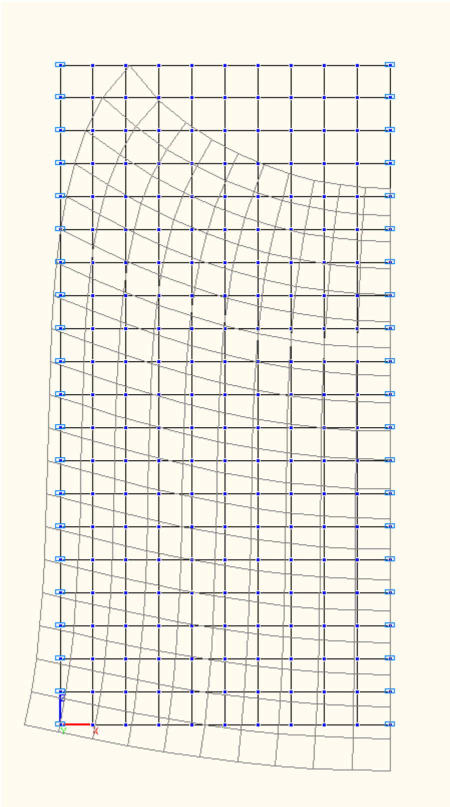
Initial data:

$E = 2.65 \cdot 10^6$ Pa	- elastic modulus;
$\nu = 0.15$	- Poisson's ratio;
h = 0.1 m	- thickness of the deep beam;
a = 1.6 m	- length of the deep beam span;
b = 1.6 m	- height of the deep beam;
p = 500.0 N/m	- uniformly distributed load.

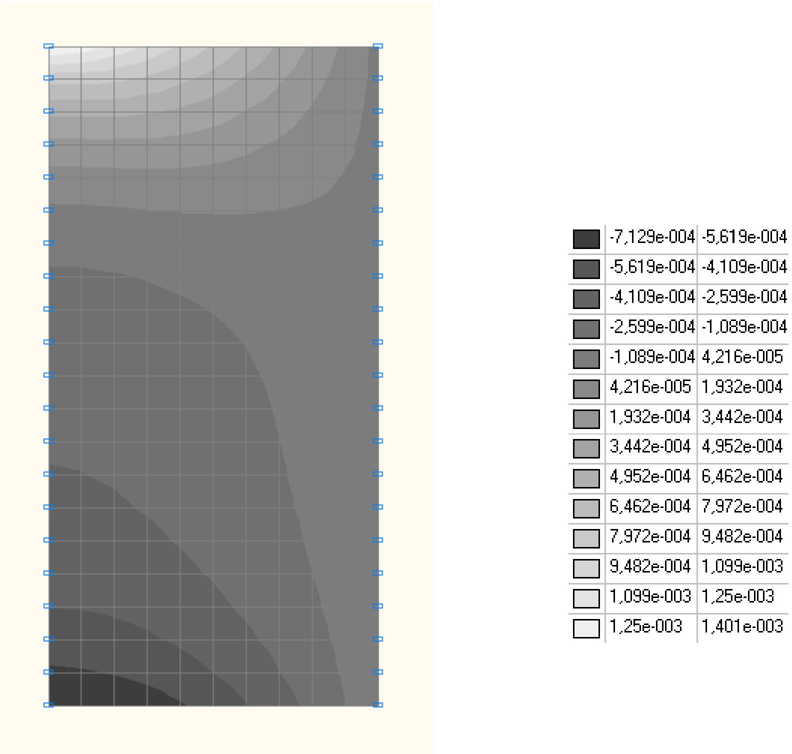
Finite element model: Design model – plane hinged bar system, 200 deep beam elements of type 21. The spacing of the finite element mesh along the x and z axes of the global coordinate system is 0.08 m. Boundary conditions are provided by imposing constraints in the direction of the degree of freedom Z for the side and in the direction of the degree of freedom X at the symmetry axis. Number of nodes in the design model – 231.

Results in SCAD

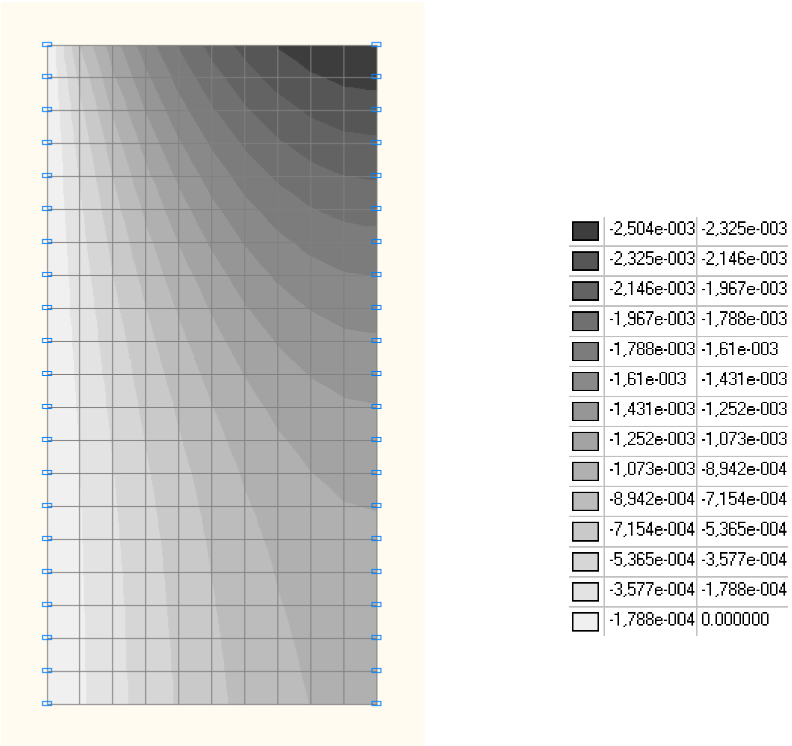
Design model



Deformed model



Values of displacements along the deep beam span u (m)



Values of displacements along the deep beam height v (m)

Comparison of solutions:

Coordinates		**Displacements *u*, m**			**Displacements *v*, m**		
x	**z**	**Theory**	**SCAD**	**Deviations, %**	**Theory**	**SCAD**	**Deviations, %**
0.0	0.0	$-0.719 \cdot 10^{-3}$	$-0.713 \cdot 10^{-3}$	0.83	$0.000 \cdot 10^{-3}$	$0.000 \cdot 10^{-3}$	–
0.0	0.8	$-0.220 \cdot 10^{-3}$	$-0.221 \cdot 10^{-3}$	0.45	$0.000 \cdot 10^{-3}$	$0.000 \cdot 10^{-3}$	–
0.0	1.6	$1.468 \cdot 10^{-3}$	$1.401 \cdot 10^{-3}$	4.56	$0.000 \cdot 10^{-3}$	$0.000 \cdot 10^{-3}$	–
0.4	0.0	$-0.508 \cdot 10^{-3}$	$-0.504 \cdot 10^{-3}$	0.79	$-0.672 \cdot 10^{-3}$	$-0.667 \cdot 10^{-3}$	0.74
0.4	0.8	$-0.148 \cdot 10^{-3}$	$-0.148 \cdot 10^{-3}$	0.00	$-0.950 \cdot 10^{-3}$	$-0.945 \cdot 10^{-3}$	0.53

Coordinates		Displacements *u*, m			Displacements *v*, m		
x	z	Theory	SCAD	Deviations, %	Theory	SCAD	Deviations, %
0.4	1.6	$0.780 \cdot 10^{-3}$	$0.778 \cdot 10^{-3}$	0.26	$-2.032 \cdot 10^{-3}$	$-2.027 \cdot 10^{-3}$	0.25
0.8	0.0	$0.000 \cdot 10^{-3}$	$0.000 \cdot 10^{-3}$	–	$-0.950 \cdot 10^{-3}$	$-0.943 \cdot 10^{-3}$	0.74
0.8	0.8	$0.000 \cdot 10^{-3}$	$0.000 \cdot 10^{-3}$	–	$-1.326 \cdot 10^{-3}$	$-1.320 \cdot 10^{-3}$	0.45
0.8	1.6	$0.000 \cdot 10^{-3}$	$0.000 \cdot 10^{-3}$	–	$-2.510 \cdot 10^{-3}$	$-2.504 \cdot 10^{-3}$	0.24

***Notes*:** In the analytical solution the components of the strain tensor in the Cartesian coordinates *u(x,z)* and *v(x,z)* for the midsurface of the deep beam in its plane can be calculated according to the following formulas:

$$u(x,z) = -\frac{p \cdot b}{E \cdot h} \cdot \sum_{m=1}^{m=\infty} \frac{a}{m \cdot \pi \cdot b} \cdot \left\{ \left[2 \cdot m \cdot \pi \cdot \frac{b}{a} \cdot sh\left(m \cdot \pi \cdot \frac{b}{a} \right) \right] \cdot \left[\left(-2 + (1+\nu) \cdot m \cdot \pi \cdot \frac{b}{a} \cdot cth\left(m \cdot \pi \cdot \frac{b}{a} \right) \right) \cdot sh\left(m \cdot \pi \cdot \frac{b-z}{a} \right) - \right.\right.$$

$$- (1+\nu) \cdot m \cdot \pi \cdot \frac{b-z}{a} \cdot ch\left(m \cdot \pi \cdot \frac{b-z}{a} \right) \Big] - \left[sh^2\left(m \cdot \pi \cdot \frac{b}{a} \right) + \left(m \cdot \pi \cdot \frac{b}{a} \right)^2 \right] \cdot \left[\left(-2 + (1+\nu) \cdot m \cdot \pi \cdot \frac{b}{a} \cdot cth\left(m \cdot \pi \cdot \frac{b}{a} \right) \right) \cdot sh\left(m \cdot \pi \cdot \frac{z}{a} \right) \right.$$

$$- (1+\nu) \cdot m \cdot \pi \cdot \frac{z}{a} \cdot ch\left(m \cdot \pi \cdot \frac{z}{a} \right) \Big] - \left[sh^2\left(m \cdot \pi \cdot \frac{b}{a} \right) - \left(m \cdot \pi \cdot \frac{b}{a} \right)^2 \right] \cdot \left[\left(2 \cdot \nu + (1+\nu) \cdot m \cdot \pi \cdot \frac{b}{a} \cdot cth\left(m \cdot \pi \cdot \frac{b}{a} \right) \right) \cdot sh\left(m \cdot \pi \cdot \frac{z}{a} \right) - \right.$$

$$\left.\left. - (1+\nu) \cdot m \cdot \pi \cdot \frac{z}{a} \cdot ch\left(m \cdot \pi \cdot \frac{z}{a} \right) \right] \right\} \cdot \frac{\left[1 + (-1)^{m+1} \right] \cdot cos\left(m \cdot \pi \cdot \frac{x}{a} \right)}{m \cdot \pi \cdot sh\left(m \cdot \pi \cdot \frac{b}{a} \right) \cdot \left[sh^2\left(m \cdot \pi \cdot \frac{b}{a} \right) - \left(m \cdot \pi \cdot \frac{b}{a} \right)^2 \right]}$$

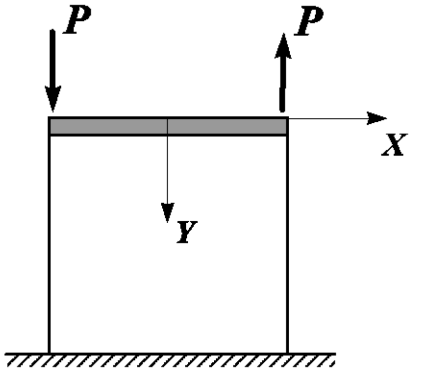
$$v(x,z) = \frac{p \cdot b}{E \cdot h} \cdot \sum_{m=1}^{m=\infty} \frac{a}{m \cdot \pi \cdot b} \cdot \left\{ \left[2 \cdot m \cdot \pi \cdot \frac{b}{a} \cdot sh\left(m \cdot \pi \cdot \frac{b}{a} \right) \right] \cdot \left[\left(1 - \nu + (1+\nu) \cdot m \cdot \pi \cdot \frac{b}{a} \cdot cth\left(m \cdot \pi \cdot \frac{b}{a} \right) \right) \cdot ch\left(m \cdot \pi \cdot \frac{b-z}{a} \right) - \right.\right.$$

$$- (1+\nu) \cdot m \cdot \pi \cdot \frac{b-z}{a} \cdot sh\left(m \cdot \pi \cdot \frac{b-z}{a} \right) \Big] + \left[sh^2\left(m \cdot \pi \cdot \frac{b}{a} \right) + \left(m \cdot \pi \cdot \frac{b}{a} \right)^2 \right] \cdot \left[\left(1 - \nu + (1+\nu) \cdot m \cdot \pi \cdot \frac{b}{a} \cdot cth\left(m \cdot \pi \cdot \frac{b}{a} \right) \right) \cdot ch\left(m \cdot \pi \cdot \frac{z}{a} \right) - \right.$$

$$- (1+\nu) \cdot m \cdot \pi \cdot \frac{z}{a} \cdot sh\left(m \cdot \pi \cdot \frac{z}{a} \right) \Big] + \left[sh^2\left(m \cdot \pi \cdot \frac{b}{a} \right) - \left(m \cdot \pi \cdot \frac{b}{a} \right)^2 \right] \cdot \left[\left(3 + \nu + (1+\nu) \cdot m \cdot \pi \cdot \frac{b}{a} \cdot cth\left(m \cdot \pi \cdot \frac{b}{a} \right) \right) \cdot ch\left(m \cdot \pi \cdot \frac{z}{a} \right) - \right.$$

$$\left.\left. - (1+\nu) \cdot m \cdot \pi \cdot \frac{z}{a} \cdot sh\left(m \cdot \pi \cdot \frac{z}{a} \right) \right] \right\} \cdot \frac{\left[1 + (-1)^{m+1} \right] \cdot sin\left(m \cdot \pi \cdot \frac{x}{a} \right)}{m \cdot \pi \cdot sh\left(m \cdot \pi \cdot \frac{b}{a} \right) \cdot \left[sh^2\left(m \cdot \pi \cdot \frac{b}{a} \right) - \left(m \cdot \pi \cdot \frac{b}{a} \right)^2 \right]}$$

Pure Bending of a Square Plate in the Plane Stress State Clamped on One Side and Simply Supported in the Center of the Opposite Side



Objective: Check of the equilibrium of the plate sections parallel to the support sides by the shear stresses.

Initial data files:

File name	Description
4.29_балка_КЭ_2.SPR	1 variant of the design model – support bar from elements of finite rigidity of type 2
4.29_балка_КЭ_100.SPR	2 variant of the design model – support bar from the rigid body element of type 100

Problem formulation: The square plate in the plane stress state clamped on one side and simply supported by a rigid bar on the opposite side is subjected to a pair of concentrated forces *P*, applied at the opposite ends of the bar and directed perpendicular to its axis. Check the equality of the values of the areas of shear stress diagrams τ for the plate sections parallel to the support sides and the values of the respective support reactions *H*.

References: Perelmuter A.V., Slivker V.I. Design models of structures and a possibility of their analysis. — Moscow: SCAD SOFT, 2011.

Initial data:

$E = 3.0 \cdot 10^5$ kPa	- elastic modulus;
ν = 0.25	- Poisson's ratio;
δ= 1.0 m	- thickness of the deep beam;
a = 16.0 m	- plate side;
P = 1000.0 kN	- concentrated force.

Finite element model: Two variants of the design model are considered.

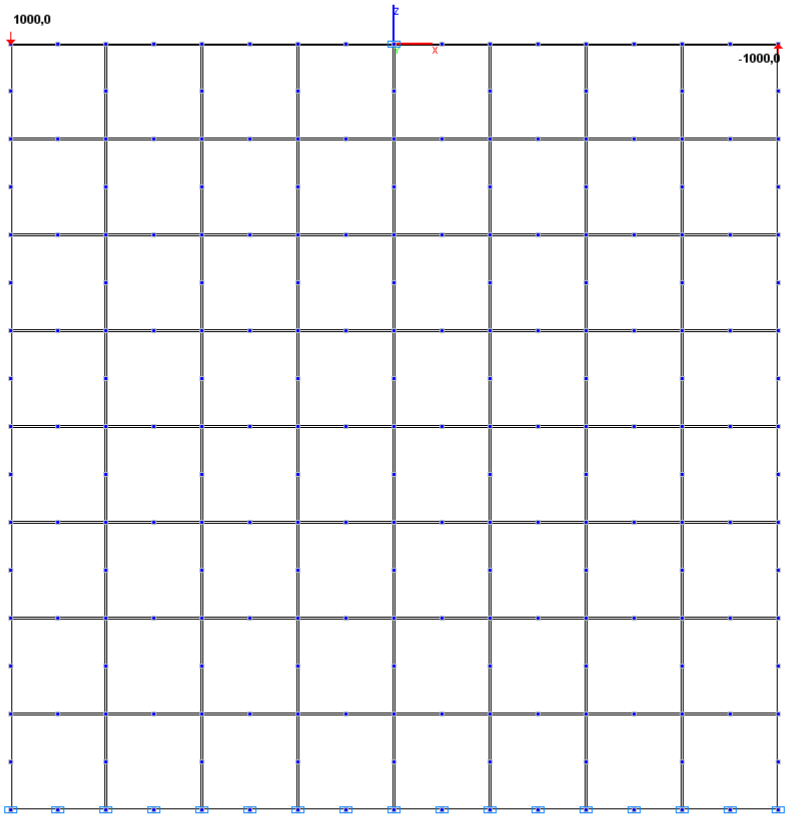
Variant 1:

Design model – plane frame, plate elements – 64 eight-node elements of type 30, bar elements – 16 elements of type 2 ($EA = 3.0 \cdot 10^{15}$ kN, $EI = 3.0 \cdot 10^{12}$ kN·m^2). The spacing of the finite element mesh in the directions parallel to the support sides is 1.0 m. Number of nodes in the design model – 225.

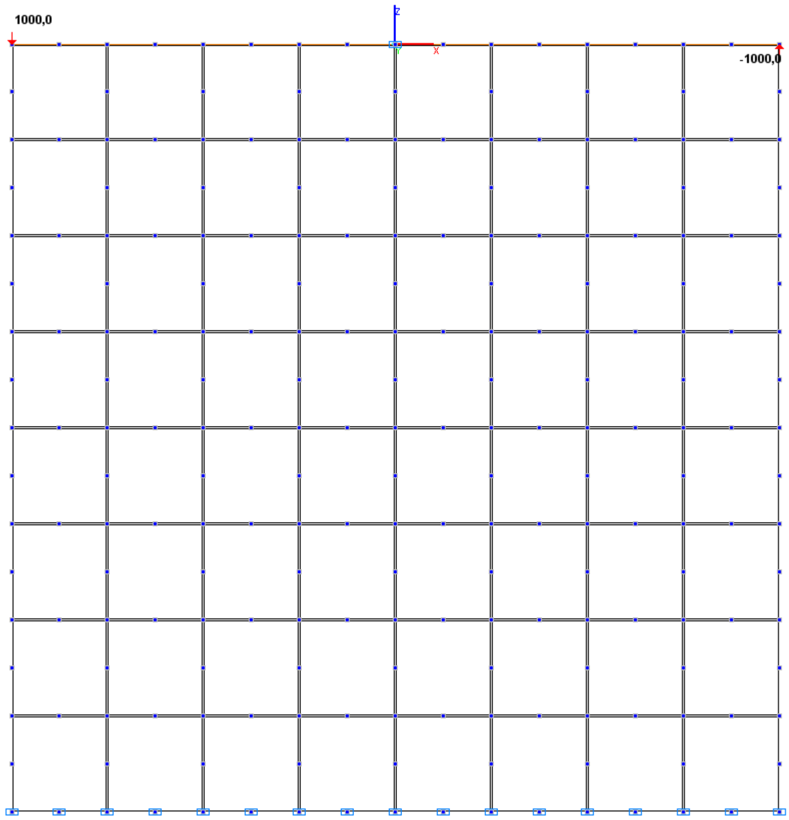
Variant 2:

Design model – plane frame, plate elements – 64 eight-node elements of type 30, bar elements – 1 element of type 100 (rigid body with a master node in the center of the simply supported side of the plate). The spacing of the finite element mesh in the directions parallel to the support sides is 1.0 m. Number of nodes in the design model – 225.

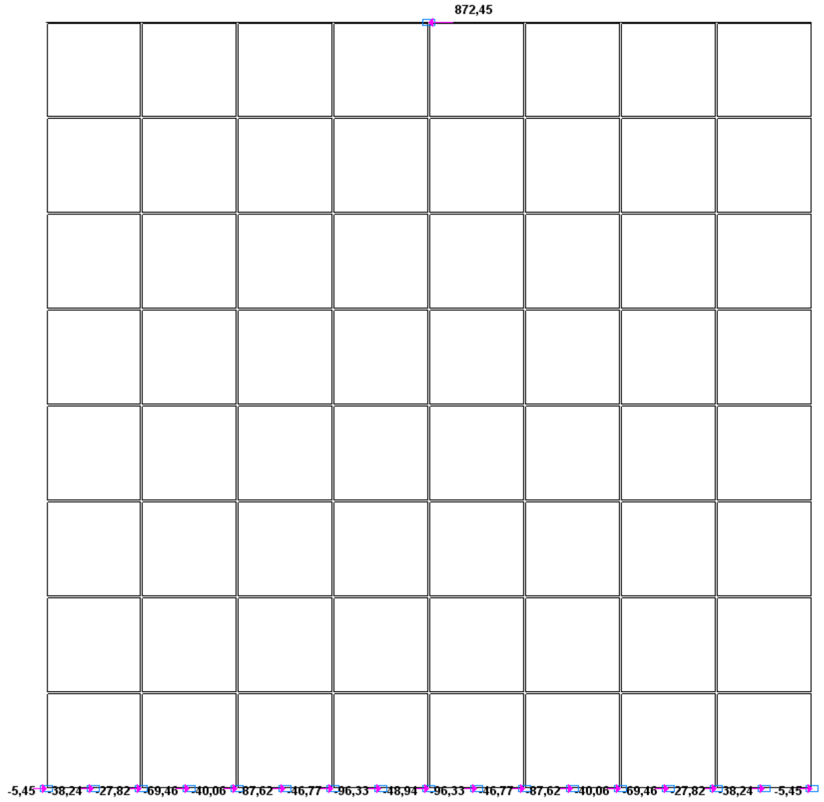
Results in SCAD



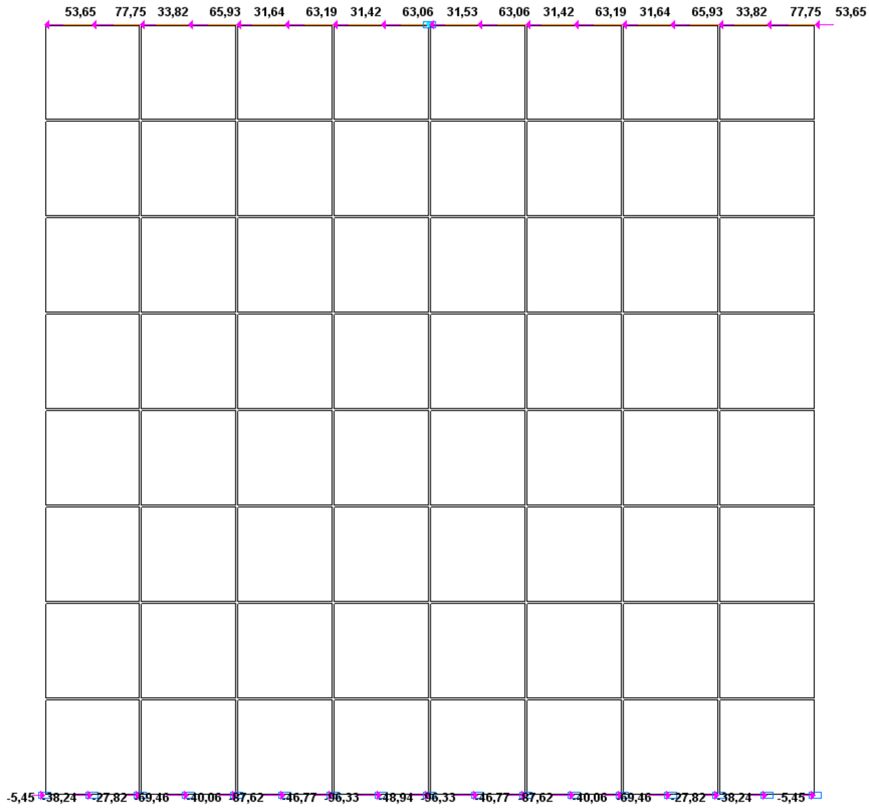
Design model. Variant 1



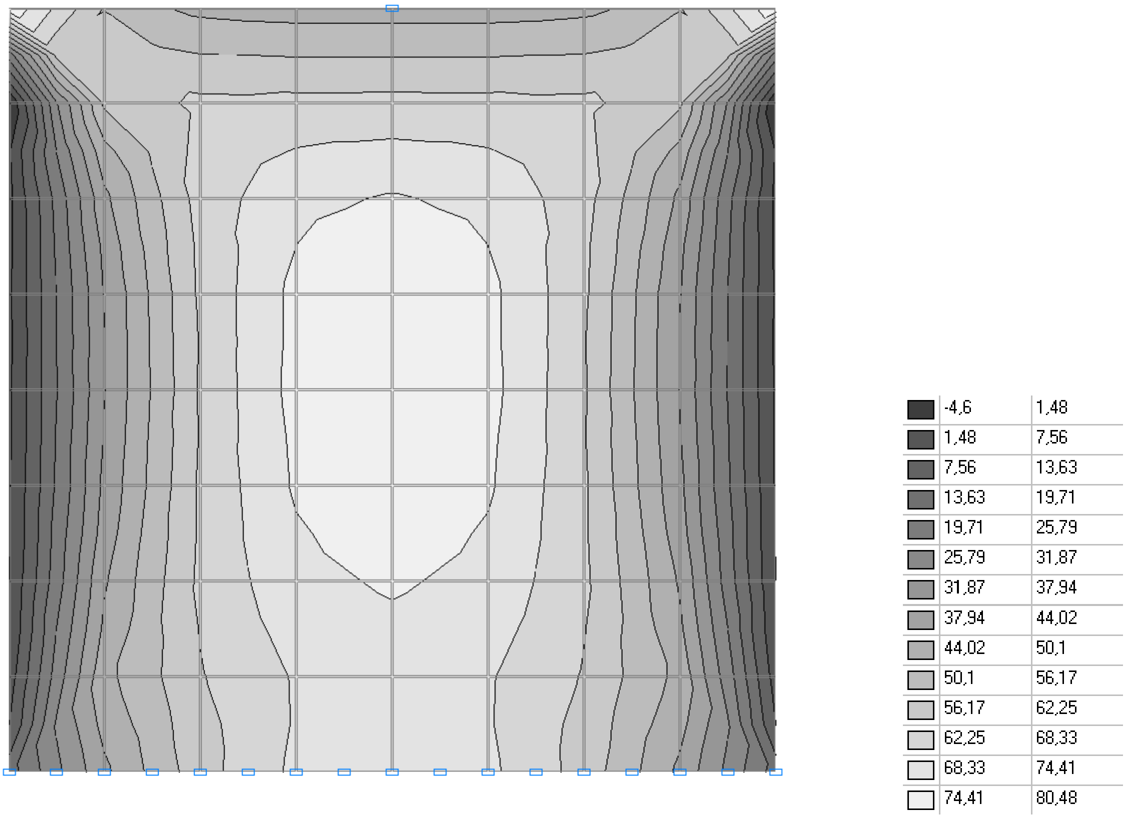
Design model. Variant 2



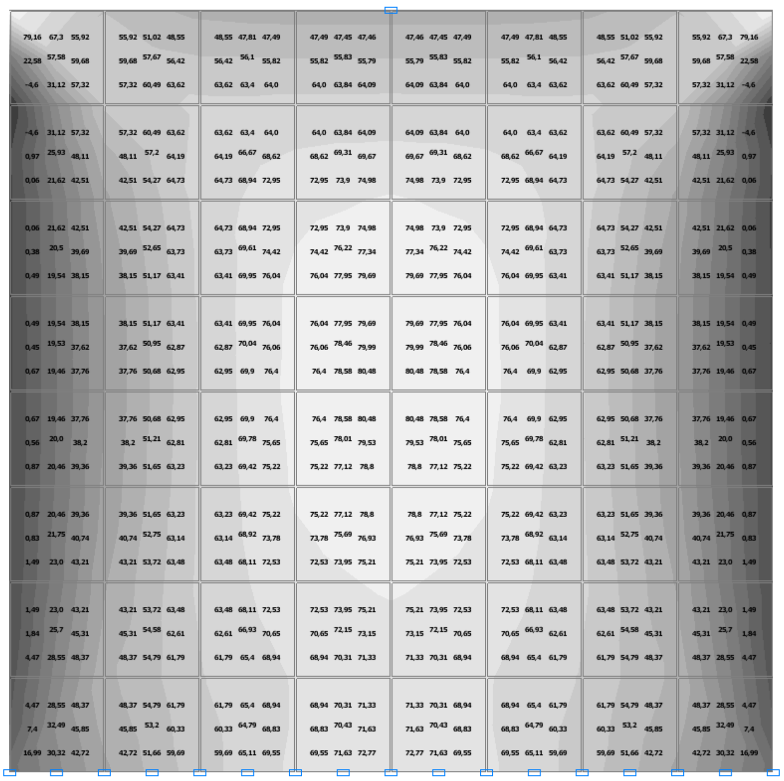
Values of support reactions H (kN) for the design model according to variant 1



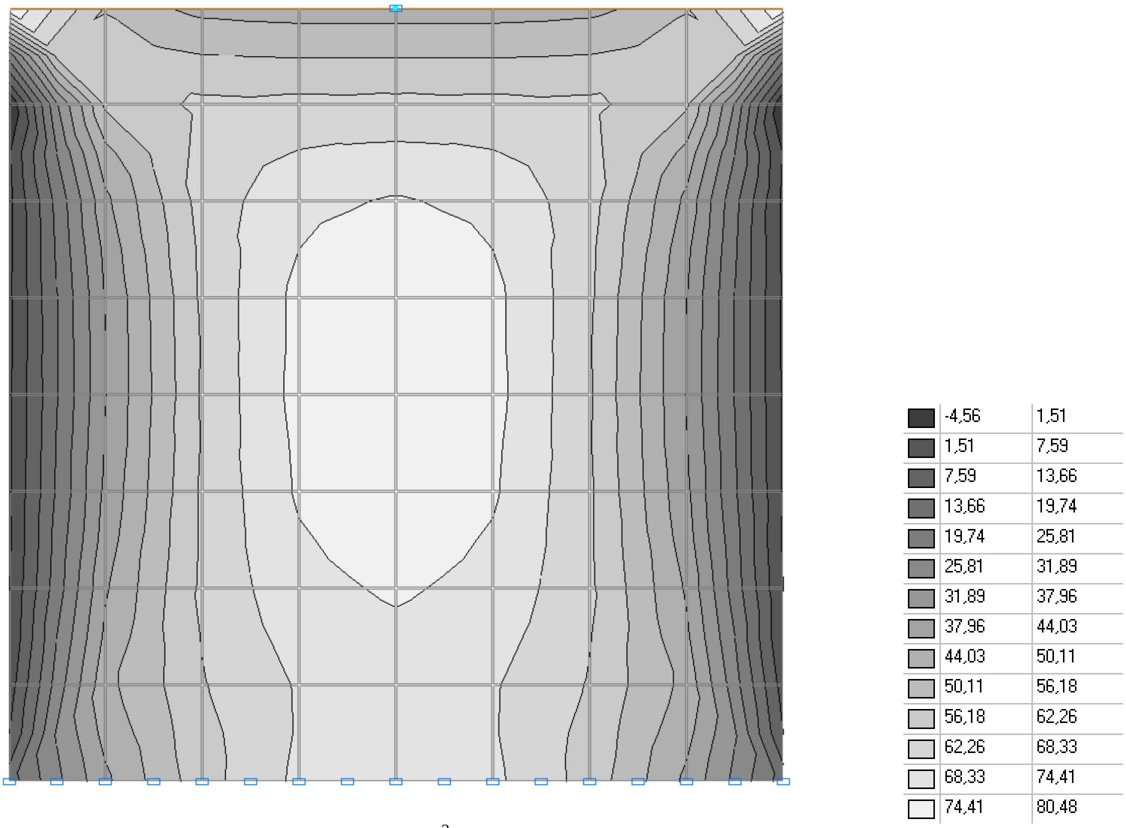
Values of support reactions H (kN) for the design model according to variant 2



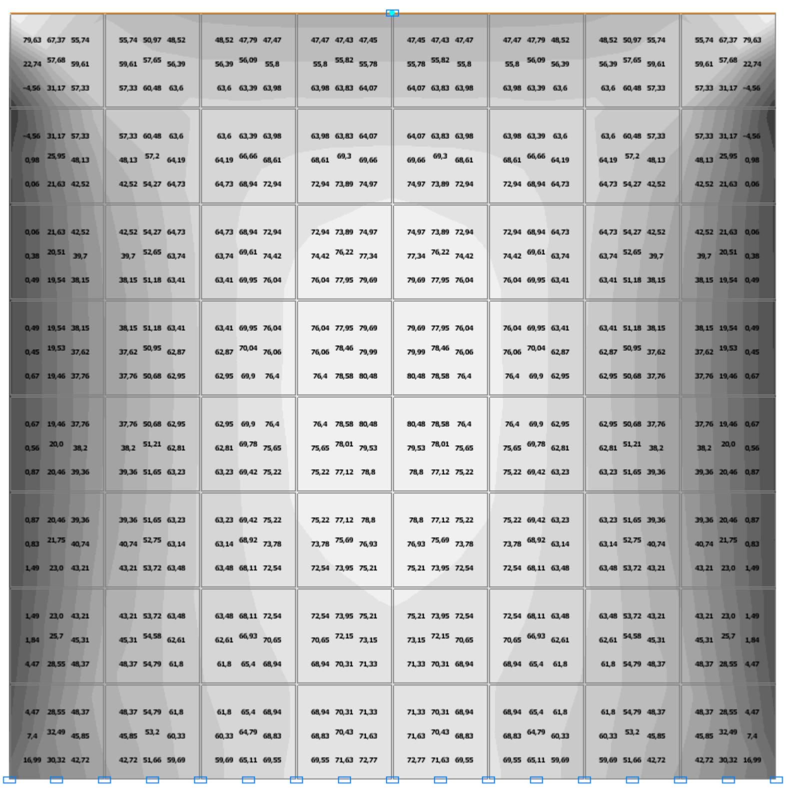
Isolines of stresses τ (kN/m²) for the design model according to variant 1



Values of stresses τ (kN/m²) for the design model according to variant 1



Isolines of stresses τ (kN/m²) for the design model according to variant 2



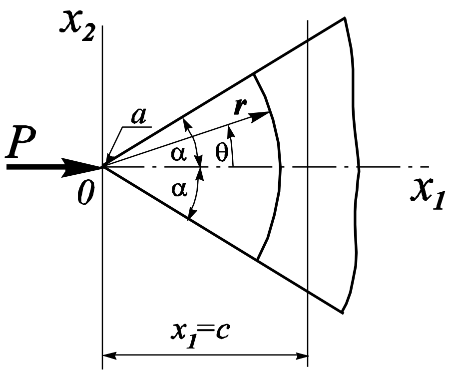
Values of stresses τ (kN/m²) for the design model according to variant 2

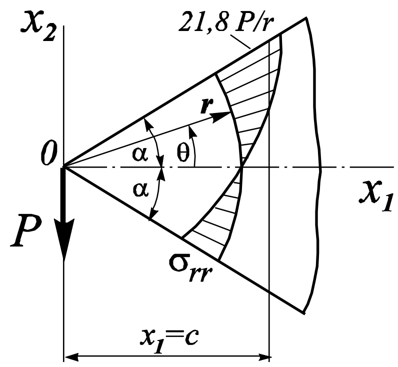
Comparison of solutions:

Comparison of the values of the areas of shear stress diagrams τ for the plate sections parallel to the support sides and located at the distance y from the simply supported side with the value of the support reaction *H* at the simply supported side.

Design model according to variant 1 **H = 872.45 kN**			**Design model according to variant 2** **H = 872.45 kN**		
y, m	$Q = \delta \cdot \int_0^a \tau \cdot dx$, **kN**	**Deviations, %**	**y, m**	$Q = \delta \cdot \int_0^a \tau \cdot dx$, **kN**	**Deviations, %**
0.0	857.71	1.69	0.0	857.67	1.69
2.0	867.07	0.62	2.0	867.06	0.62
4.0	872.87	0.05	4.0	872.88	0.05
6.0	872.60	0.02	6.0	872.61	0.02
8.0	872.59	0.02	8.0	872.59	0.02
10.0	872.61	0.02	10.0	872.62	0.02
12.0	872.70	0.03	12.0	872.71	0.03
14.0	872.10	0.04	14.0	872.11	0.04
16.0	871.11	0.15	16.0	871.12	0.15

Compression and Bending of a Symmetric Wedge by Concentrated Forces Applied to Its Vertex (Michell's Problem)





***Objective*:** Determination of the stress state of a symmetric wedge of unit thickness in polar coordinates subjected to compression and bending by concentrated forces applied to its vertex.

Initial data file: **4.22.SPR**

Problem formulation: The compressive force P_{x1} acting along the symmetry axis of the wedge OX1 and the bending force P_{x2}, which is a skew-symmetric load with respect to the symmetry axis of the wedge OX1, are applied to the vertex of the wedge of unit thickness. Determine the stress tensor components in polar coordinates σ_{rr}, $\sigma_{\theta\theta}$, $\sigma_{r\theta}$ at a radial distance r = 5.0 m from the vertex of the wedge.

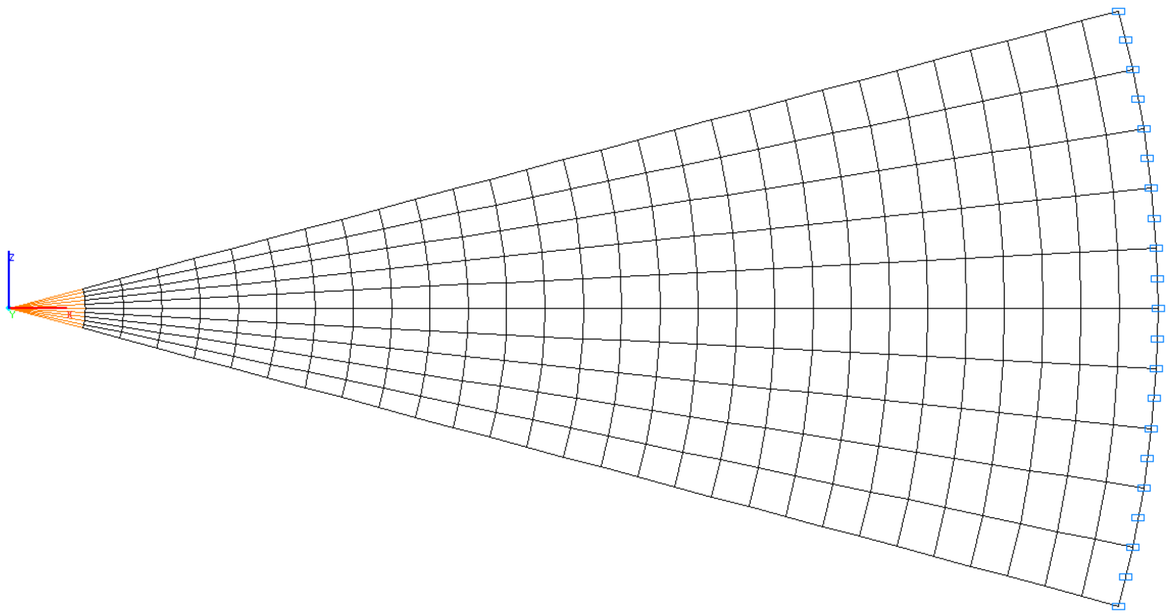
References: S.P. Demidov, Theory of Elasticity. — Moscow: High school, 1979.

Initial data:

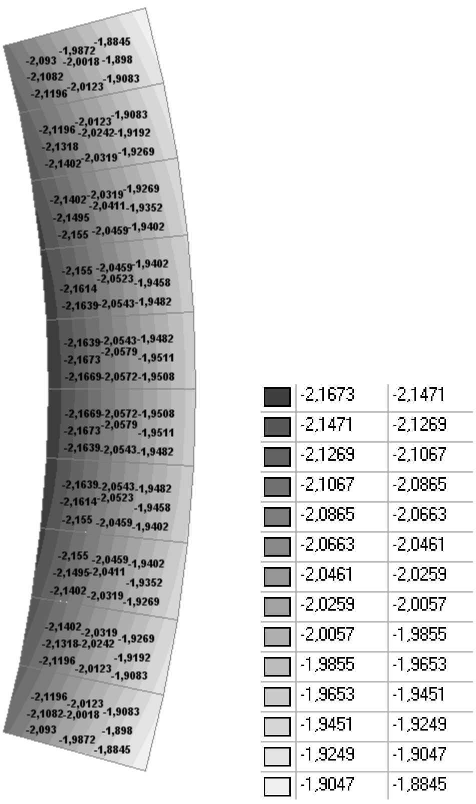
$E = 3.0\cdot10^7$ kPa	- elastic modulus;
$\mu = 0.2$	- Poisson's ratio;
h = 1.0 m	- thickness of the wedge;
$2\cdot\alpha = 30^o$	- apex angle of the wedge;
R = 15.0 m	- radius of the fixed end of the wedge;
$P_{x1} = -5.0$ kN	- concentrated force compressing the wedge (horizontal);
$P_{x2} = 5.0$ kN	- concentrated force bending the wedge (vertical).

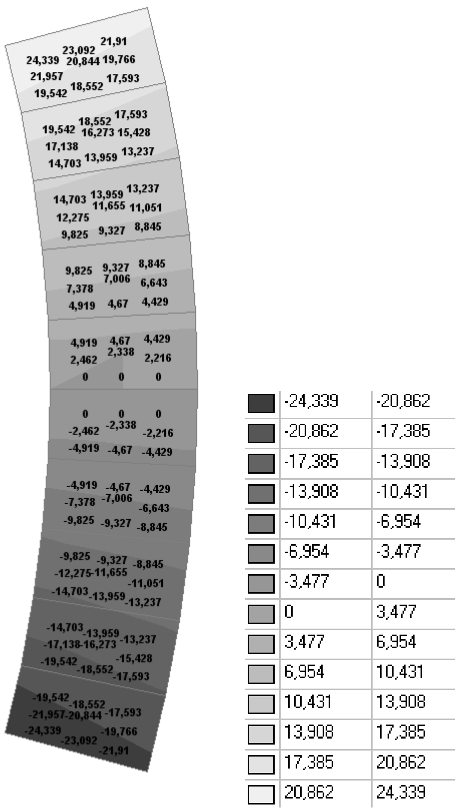
Finite element model: Design model – general type system, wedge elements– 280 eight-node elements of type 50. The spacing of the finite element mesh in the radial direction is 0.5 m, and in the tangential direction is 3º. The direction of the output of internal forces is radial tangential. Since in the case of a cylindrical surface of a small radius *a* the force P_{x1} at the vertex of the wedge cannot be represented as a resultant of stresses distributed according to the law of the analytical solution given below, the edge of the wedge is modeled by a rigid body with a master node at the vertex of the wedge and the slave nodes at the radial distance of *a* = 1.0 m from the vertex of the wedge (member type – 100). Since there are no forces distributed according to the law of the analytical solution at the fixed end of the wedge, in order to obtain an exact solution at the radial distance r = 5.0 m from the action of the force P_{x2} the radial distance to the fixed end is taken as R = 15.0 m. Number of nodes in the design model – 918.

Results in SCAD

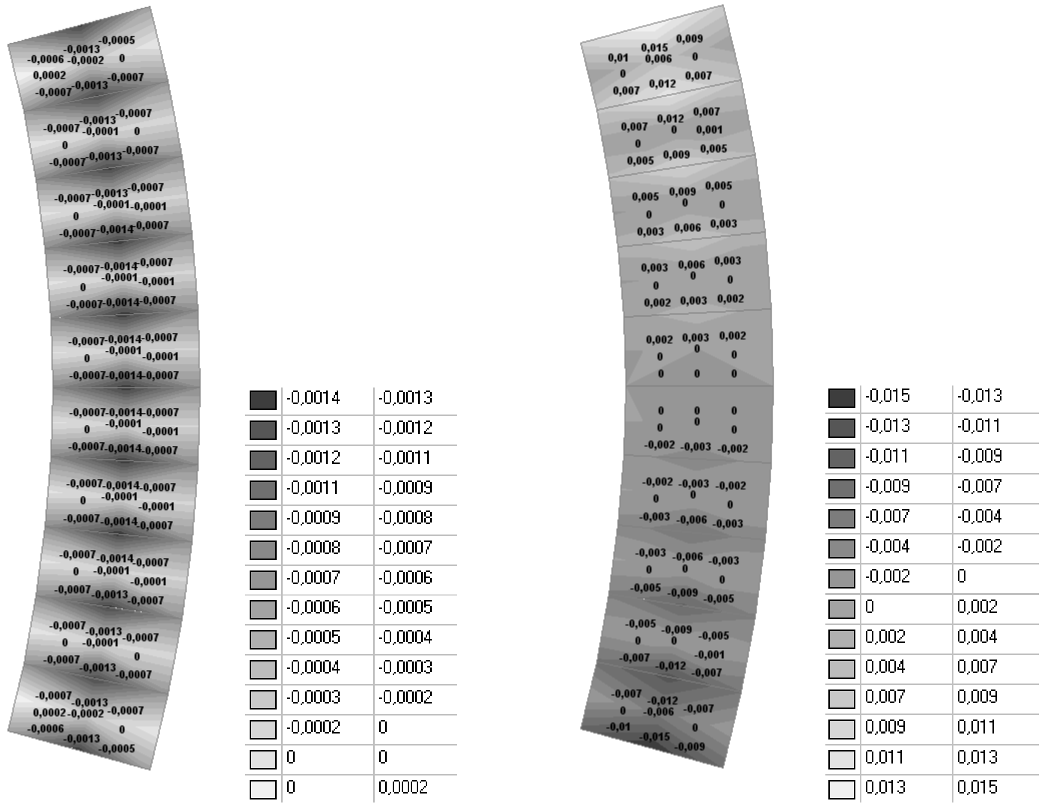


Design model

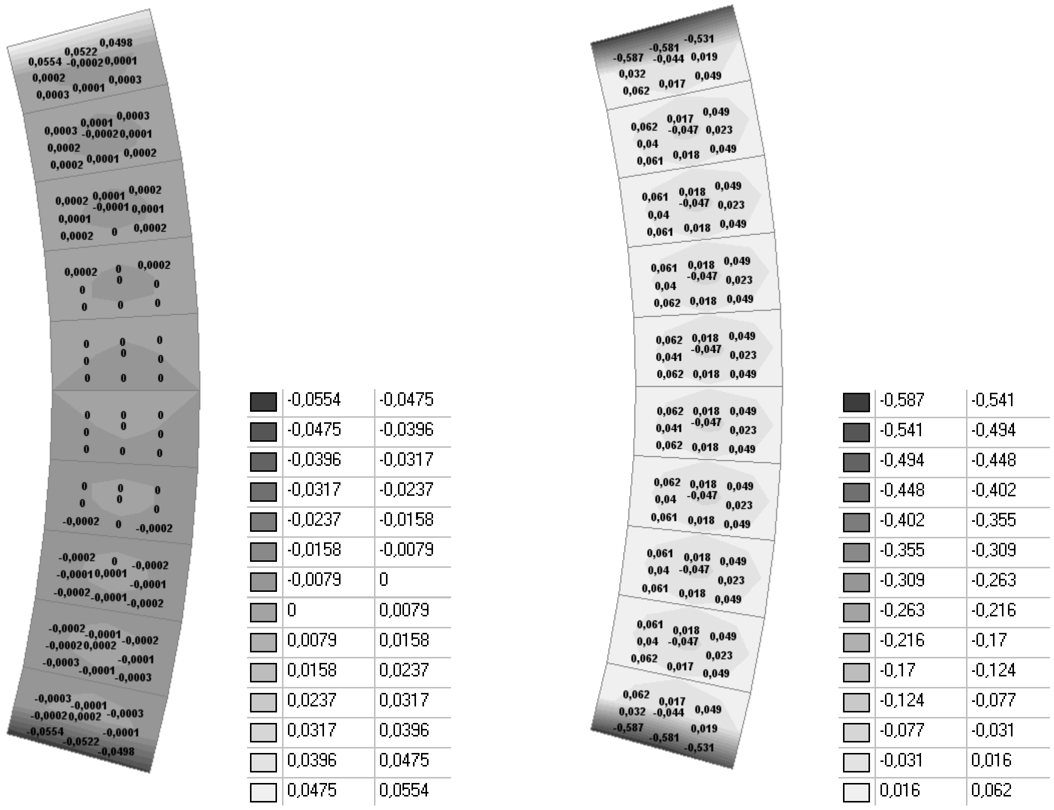




Values of stresses σ_{rr} (kN/m^2) under the compressive force P_{x1} and the bending force P_{x2}



Values of stresses $\sigma_{\theta\theta}$ (kN/m^2) under the compressive force P_{x1} and the bending force P_{x2}



Values of stresses $\sigma_{r\theta}$ (kN/m^2) under the compressive force P_{x1} and the bending force P_{x2}

Comparison of solutions:

Stress tensor components at a radial distance r = 5.0 m from the vertex of the wedge under the compressive force P_{x1}

Angle θ	**Stresses σ_{rr} (kN/m²)**		
	Theory	**SCAD**	**Deviations, %**
-15º	-1.8873	-1.8845	0.15
0º	-1.9539	-1.9508	0.16
+15º	-1.8873	-1.8845	0.15

Angle θ	**Stresses $\sigma_{\theta\theta}$ (kN/m²)**		
	Theory	**SCAD**	**Deviations, %**
-15º	0.0000	-0.0005	—
0º	0.0000	-0.0007	—
+15º	0.0000	-0.0005	—

Angle θ	**Stresses $\sigma_{r\theta}$ (kN/m²)**		
	Theory	**SCAD**	**Deviations, %**
-15º	0.0000	-0.0498	—
0º	0.0000	0.0000	—
+15º	0.0000	0.0498	—

Stress tensor components at a radial distance r = 5.0 m from the vertex of the wedge under the bending force P_{x2}

Angle θ	**Stresses σ_{rr} (kN/m²)**		
	Theory	**SCAD**	**Deviations, %**
-15º	-21.9350	-21.9098	0.11
0º	0.0000	0.0000	—
+15º	21.9350	21.9098	0.11

Angle θ	**Stresses $\sigma_{\theta\theta}$ (kN/m²)**		
	Theory	**SCAD**	**Deviations, %**
-15º	0.0000	-0.0086	—
0º	0.0000	0.0000	—
+15º	0.0000	0.0086	—

Angle θ	**Stresses $\sigma_{r\theta}$ (kN/m²)**		
	Theory	**SCAD**	**Deviations, %**
-15º	0.0000	0.5314	—
0º	0.0000	0.0494	—
+15º	0.0000	-05314	—

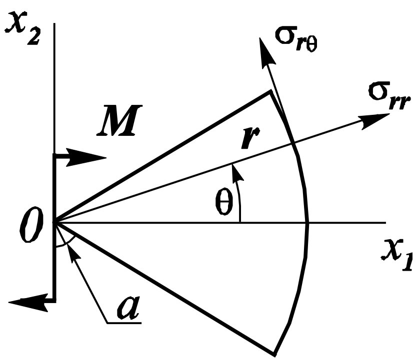
Notes: In the analytical solution the stresses σ_{rr}, $\sigma_{\theta\theta}$, $\sigma_{r\theta}$ in the body of the wedge subjected to the compressive force P_{x1} are determined according to the following formulas (S.P. Demidov, Theory of Elasticity. — Moscow: High school, 1979, p. 273):

$$\sigma_{rr} = \frac{2 \cdot P \cdot \cos\theta}{r \cdot (2 \cdot \alpha + \sin(2 \cdot \alpha))}; \qquad \sigma_{\theta\theta} = 0; \qquad \sigma_{r\theta} = 0.$$

In the analytical solution the stresses σ_{rr}, $\sigma_{\theta\theta}$, $\sigma_{r\theta}$ in the body of the wedge subjected to the bending force P_{x2} are determined according to the following formulas (S.P. Demidov, Theory of Elasticity. — Moscow: High school, 1979, p. 275):

$$\sigma_{rr} = \frac{2 \cdot P \cdot \sin\theta}{r \cdot (2 \cdot \alpha - \sin(2 \cdot \alpha))}; \qquad \sigma_{\theta\theta} = 0; \qquad \sigma_{r\theta} = 0.$$

Bending of a Symmetric Wedge by a Concentrated Moment Applied to Its Vertex (Inglis Problem)



Objective: Determination of the stress state of a symmetric wedge of unit thickness in polar coordinates subjected to bending by a concentrated moment applied to its vertex.

Initial data file: **4.23.SPR**

Problem formulation: The moment *M* acting in the plane of the wedge X_1OX_2 is applied to the vertex of the wedge of unit thickness. Determine the stress tensor components in polar coordinates σ_{rr}, $\sigma_{\theta\theta}$, $\sigma_{r\theta}$ at a radial distance r = 5.0 m from the vertex of the wedge.

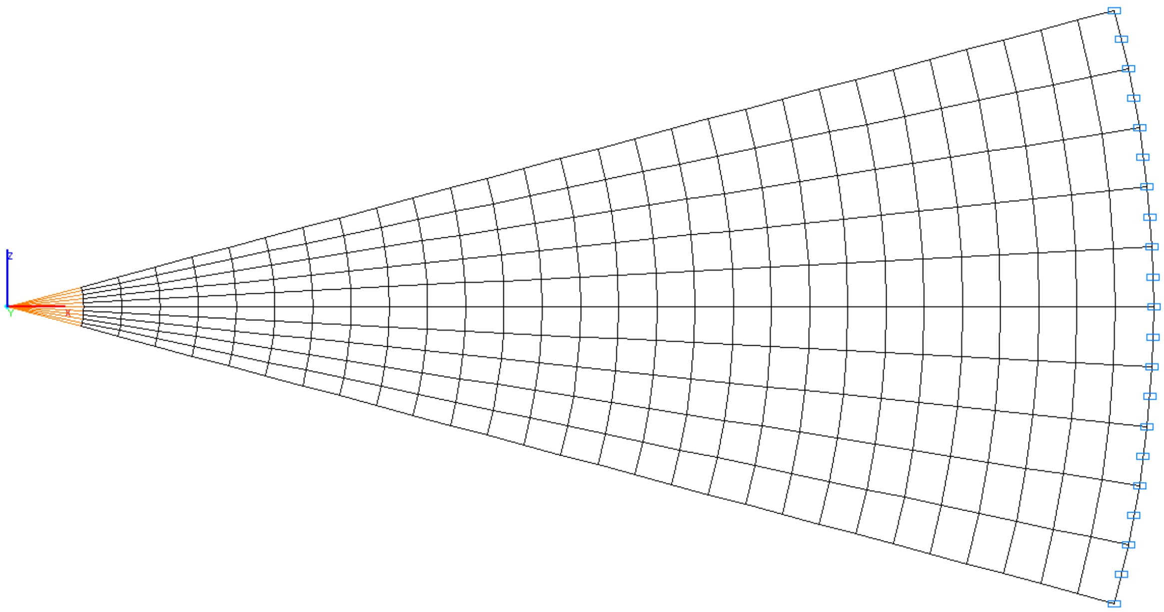
References: S.P. Demidov, Theory of Elasticity. — Moscow: High school, 1979.

Initial data:

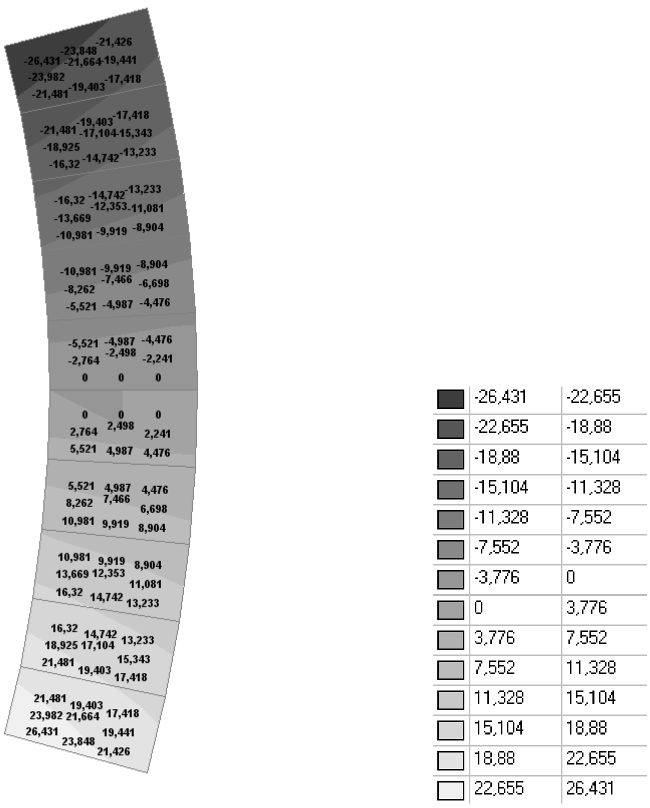
$E = 3.0 \cdot 10^7$ kPa	- elastic modulus;
$\mu = 0.2$	- Poisson’s ratio;
h = 1.0 m	- thickness of the wedge;
$2 \cdot \alpha = 30^o$	- apex angle of the wedge;
R = 15.0 m	- radius of the fixed end of the wedge;
M = -25.0 kN	- concentrated moment bending the wedge.

Finite element model: Design model – general type system, wedge elements – 280 eight-node elements of type 50. The spacing of the finite element mesh in the radial direction is 0.5 m, and in the tangential direction is 3º. The direction of the output of internal forces is radial tangential. Since in the case of a cylindrical surface of a small radius *a* the moment *M* at the vertex of the wedge cannot be represented as a resultant of stresses distributed according to the law of the analytical solution given below, the edge of the wedge is modeled by a rigid body with a master node at the vertex of the wedge and the slave nodes at the radial distance of *a* = 1.0 m from the vertex of the wedge (member type – 100). Since there are no forces distributed according to the law of the analytical solution at the fixed end of the wedge, in order to obtain an exact solution at the radial distance r = 5.0 m from the action of the moment *M* the radial distance to the fixed end is taken as R = 15.0 m. Number of nodes in the design model – 918.

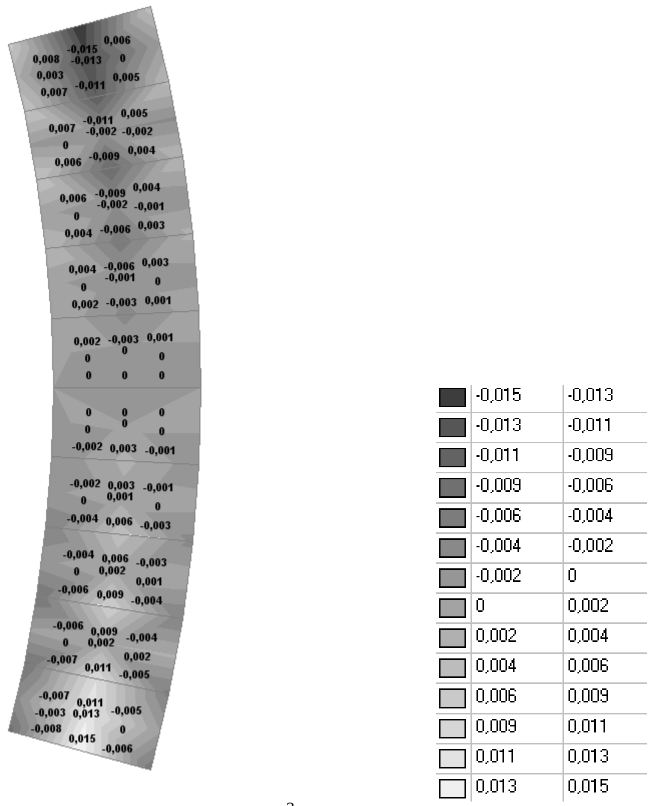
Results in SCAD



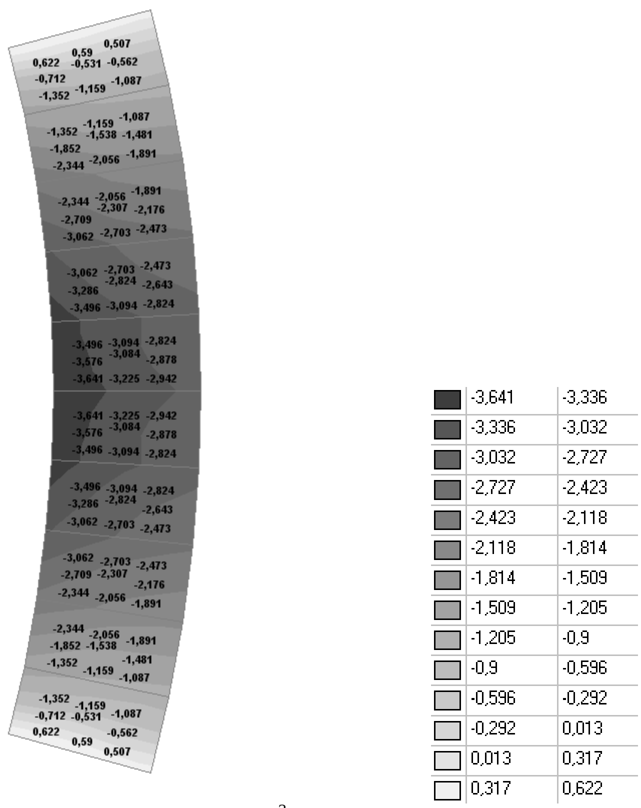
Design model



Values of stresses σ_{rr} (kN/m^2) under the bending moment M



Values of stresses $\sigma_{\theta\theta}$ (kN/m^2) under the bending moment M



Values of stresses $\sigma_{r\theta}$ (kN/m^2) under the bending moment M

Comparison of solutions:

Stress tensor components at a radial distance r = 5.0 m from the wedge top under the bending moment *M*.

Angle θ	Stresses σ_{rr} (kN/m^2)		
	Theory	**SCAD**	**Deviations, %**
-15º	21.4822	21.4264	0.26
0º	0.0000	0.0000	—
+15º	-21.4822	-21.4264	0.26

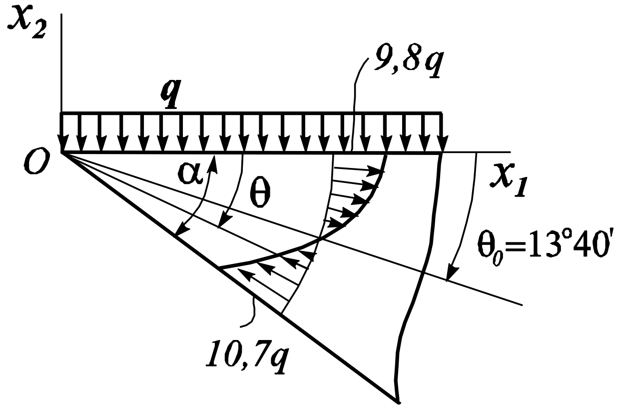
Angle θ	Stresses $\sigma_{\theta\theta}$ (kN/m^2)		
	Theory	**SCAD**	**Deviations, %**
-15º	0.0000	-0.0059	—
0º	0.0000	0.0000	—
+15º	0.0000	0.0059	—

Angle θ	Stresses $\sigma_{r\theta}$ (kN/m^2)		
	Theory	**SCAD**	**Deviations, %**
-15º	0.0000	0.5071	—
0º	-2.8781	-2.9418	2.21
+15º	0.0000	0.5071	—

Notes: In the analytical solution the stresses σ_{rr}, $\sigma_{\theta\theta}$, $\sigma_{r\theta}$ in the body of the wedge subjected to the bending moment *M* are determined according to the following formulas (S.P. Demidov, Theory of Elasticity. — Moscow: High school, 1979, p. 276):

$$\sigma_{rr} = -\frac{2 \cdot M \cdot sin(2 \cdot \theta)}{r^2 \cdot (2 \cdot \alpha - tg(2 \cdot \alpha)) \cdot cos(2 \cdot \alpha)}; \qquad \sigma_{\theta\theta} = 0; \qquad \sigma_{r\theta} = \frac{M \cdot (cos(2 \cdot \alpha) - cos(2 \cdot \theta))}{r^2 \cdot (2 \cdot \alpha - tg(2 \cdot \alpha)) \cdot cos(2 \cdot \alpha)}.$$

Bending of a Symmetric Wedge by a Uniformly Distributed Load Applied to the Surface of One of the Faces of the Wedge (Levi Problem)



Objective: Determination of the stress state of a symmetric wedge of unit thickness in polar coordinates subjected to bending by a uniformly distributed load applied to the surface of one of the faces of the wedge.

Initial data file: **4.24.SPR**

Problem formulation: The uniformly distributed load *q* acting in the plane of the wedge along the Ox_2 axis is applied to the surface of one of the faces of the wedge of unit thickness. Determine the stress tensor components in polar coordinates σ_{rr}, $\sigma_{\theta\theta}$, $\sigma_{r\theta}$ at a radial distance r = 5.0 m from the vertex of the wedge.

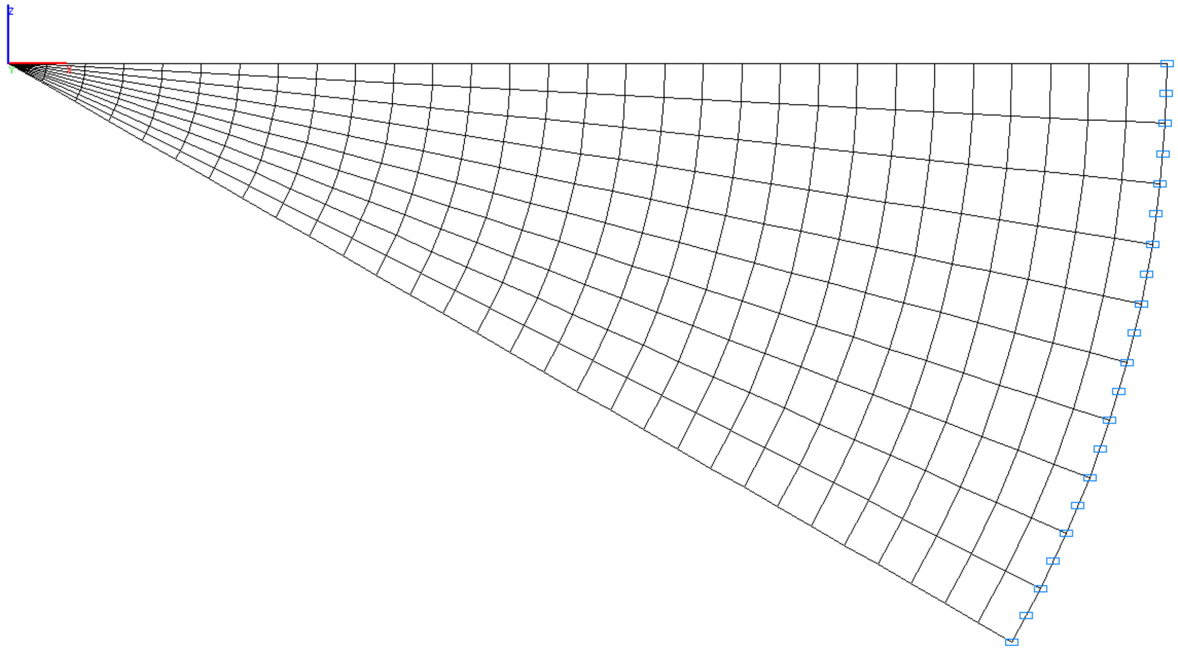
References: S.P. Demidov, Theory of Elasticity. — Moscow: High school, 1979.

Initial data:

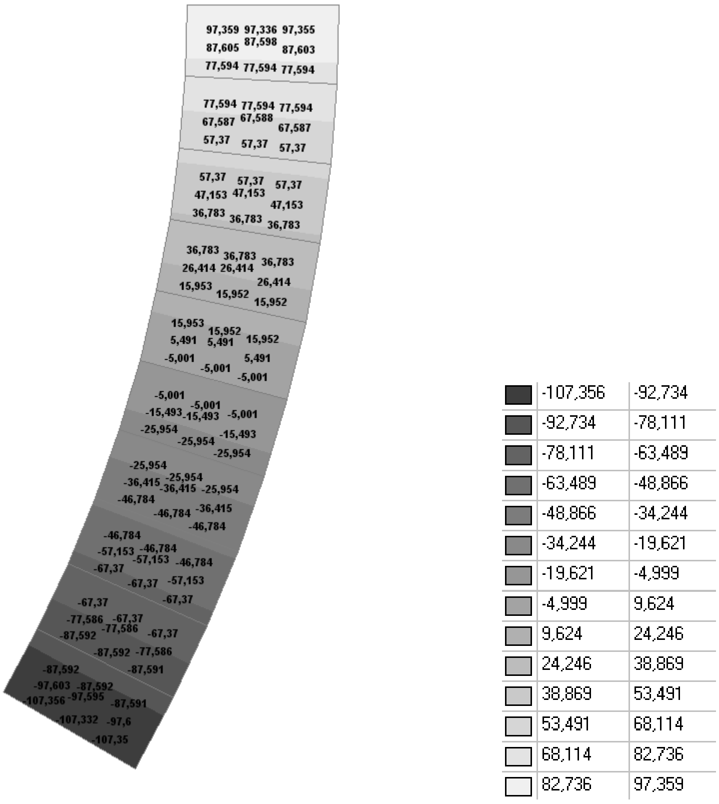
$E = 3.0 \cdot 10^7$ kPa	- elastic modulus;
$\mu = 0.2$	- Poisson's ratio;
h = 1.0 m	- thickness of the wedge;
$\alpha = 30^o$	- apex angle of the wedge;
R = 15.0 m	- radius of the fixed end of the wedge;
q = 10.0 kN/m	- uniformly distributed load bending the wedge.

Finite element model: Design model – general type system, wedge elements – 290 eight-node elements of type 50 and 10 six-node elements of type 45. The spacing of the finite element mesh in the radial direction is 0.5 m, and in the tangential direction is 3º. The direction of the output of internal forces is radial tangential. Since there are no forces distributed according to the law of the analytical solution at the fixed end of the wedge, in order to obtain an exact solution at the radial distance r = 5.0 m from the action of the uniformly distributed load *q* the radial distance to the fixed end is taken as R = 15.0 м. Number of nodes in the design model – 961.

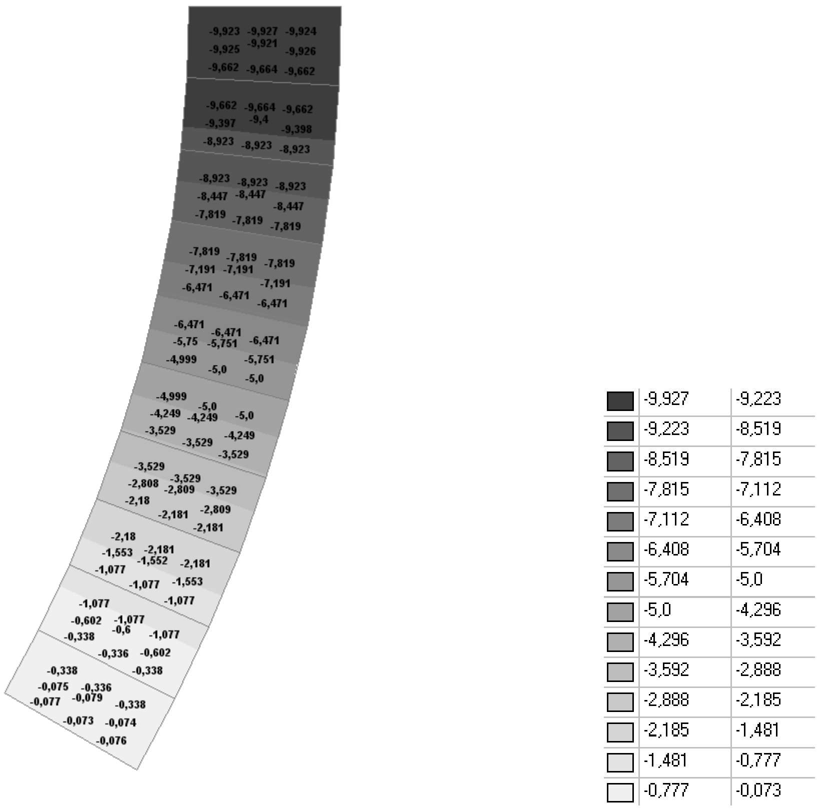
Results in SCAD



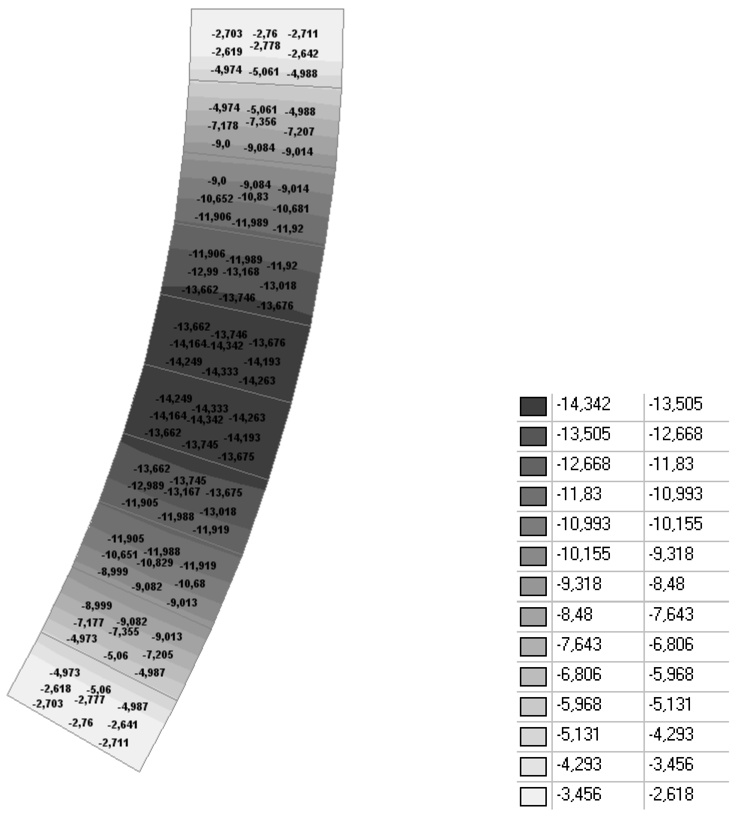
Design model



Values of stresses σ_{rr} (kN/m^2) under the action of the uniformly distributed load q



Values of stresses $\sigma_{\theta\theta}$ (kN/m^2) under the action of the uniformly distributed load q



Values of stresses $\sigma_{r\theta}$ (kN/m^2) under the action of the uniformly distributed load q

Comparison of solutions:

Stress tensor components at a radial distance r = 5.0 m from the vertex of the wedge under the uniformly distributed load *q*.

Angle θ	Stresses σ_{rr} (kN/m^2)		Stresses $\sigma_{\theta\theta}$ (kN/m^2)		Stresses $\sigma_{r\theta}$ (kN/m^2)	
	Theory	SCAD	Theory	SCAD	Theory	SCAD
0º	97.4110	97.3548	-10.0000	-9.9243	0.0000	-2.7111
15º	-5.0000	-5.0011	-5.0000	-5.0000	-14.3903	-14.2629
30º	-107.4110	-107.3501	0.0000	-0.0757	0.0000	-2.7108

Notes: In the analytical solution the stresses σ_{rr}, $\sigma_{\theta\theta}$, $\sigma_{r\theta}$ in the body of the wedge subjected to the uniformly distributed load *q* are determined according to the following formulas (S.P. Demidov, Theory of Elasticity. — Moscow: High school, 1979, p. 276):

$$\sigma_{rr} = \frac{q}{2 \cdot K} \cdot \left[2 \cdot \alpha - 2 \cdot \theta - \left(1 - \cos(2 \cdot \theta)\right) \cdot tg\alpha - \sin(2 \cdot \theta)\right];$$

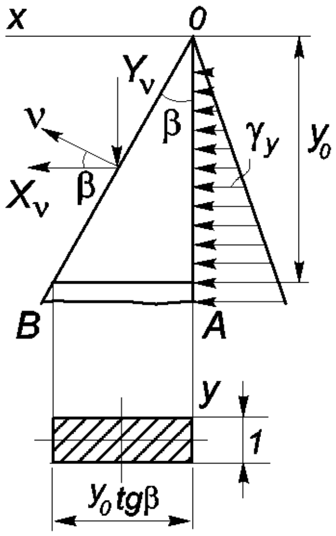
$$\sigma_{\theta\theta} = \frac{q}{2 \cdot K} \cdot \left[2 \cdot \alpha - 2 \cdot \theta - \left(1 + \cos(2 \cdot \theta)\right) \cdot tg\alpha + \sin(2 \cdot \theta)\right];$$

$$\sigma_{r\theta} = \frac{q}{2 \cdot K} \cdot \left[1 - tg\alpha \cdot \sin(2 \cdot \theta) - \cos(2 \cdot \theta)\right],$$

where:

$$K = tg\alpha - \alpha .$$

Triangular Dam Subjected to Its Self-Weight and Hydrostatic Pressure



Objective: Determination of the stress state of a triangular dam of unit thickness in Cartesian coordinates subjected to its self-weight and hydrostatic pressure.

Initial data file: **4.25.SPR**

Problem formulation: A horizontal load distributed according to the linear law with a unit volume weight γ acting in the plane of the dam is applied to the surface of the vertical face of the triangular dam of unit thickness. The dam is also subjected to the self-weight γ_1. Determine the stress tensor components in Cartesian coordinates σ_x, σ_y, τ_{xy} in the horizontal section of the dam located at the depth of y_0 = 5.0 m from the top of the dam.

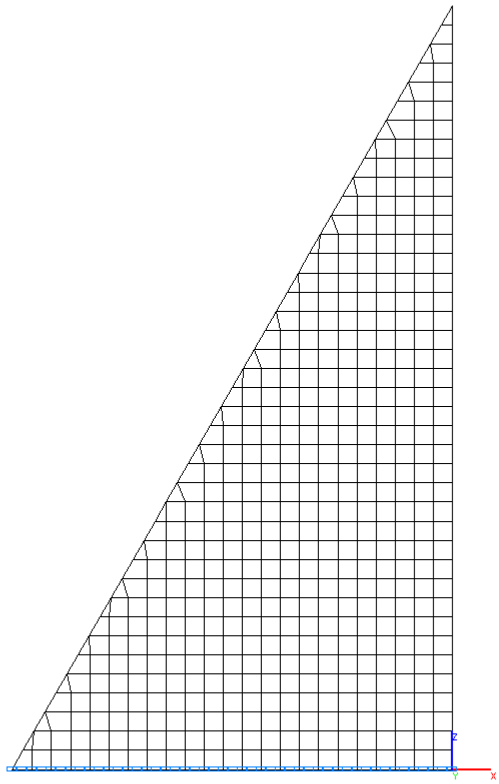
References: V. I. Samul, Fundamentals of the Elasticity and Plasticity Theory. — Moscow: High school, 1982.

Initial data:

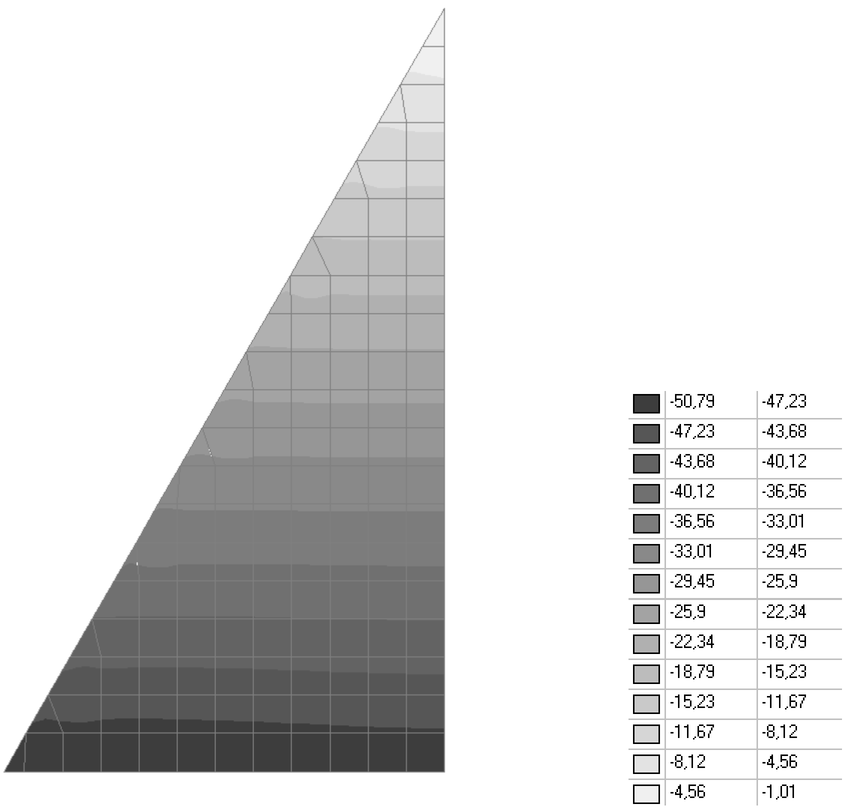
$E = 3.0 \cdot 10^7$ kPa	- elastic modulus of the dam material;
$\mu = 0.2$	- Poisson's ratio of the dam material;
h = 1.0 m	- thickness of the dam;
$\beta = 30^o$	- apex angle of the dam;
H = 15.0 m	- height of the dam;
$\gamma = 10.0$ kN/m^3	- specific weight of liquid;
$\gamma_1 = 20.0$ kN/m^3	- specific weight of the dam material.

Finite element model: Design model – plane frame, plate elements – 452 eight-node elements of type 30 and 23 six-node elements of type 25. The spacing of the finite element mesh in the horizontal OX and vertical OY directions is 0.25 m. The direction of the output of internal forces is along the OX and OY axes of the global coordinate system. Since there are no forces distributed according to the law of the analytical solution at the fixed end of the dam, in order to obtain an exact solution at the depth y_0 = 5.0 m from the top of the dam under the self-weight and the hydrostatic pressure, the height of the dam to the fixed end is taken as H = 15.0 m. Number of nodes in the design model – 1506.

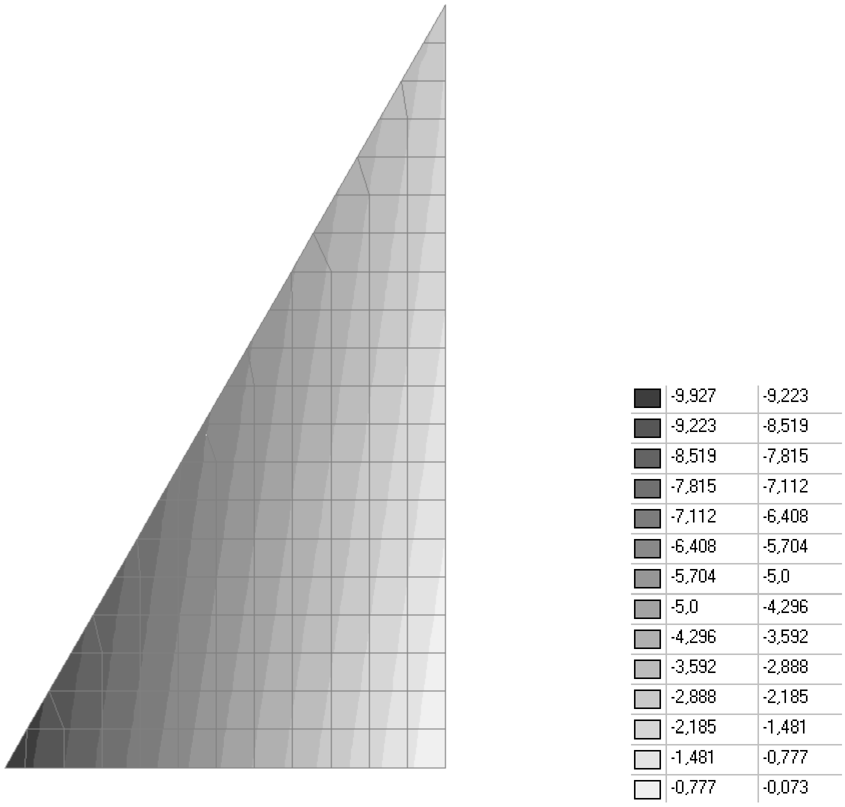
Results in SCAD



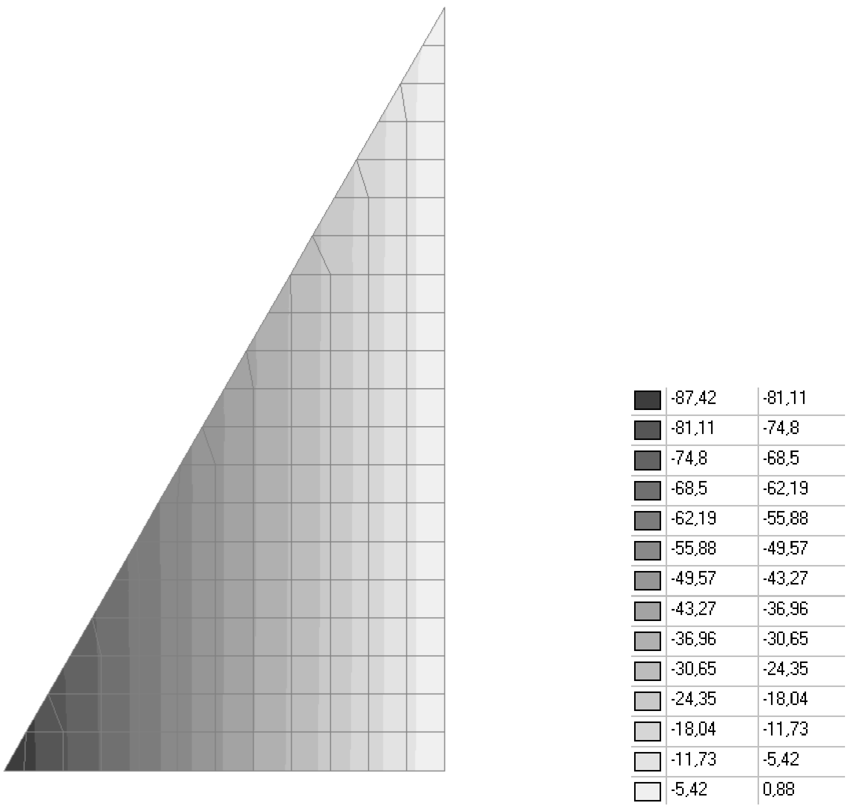
Design model



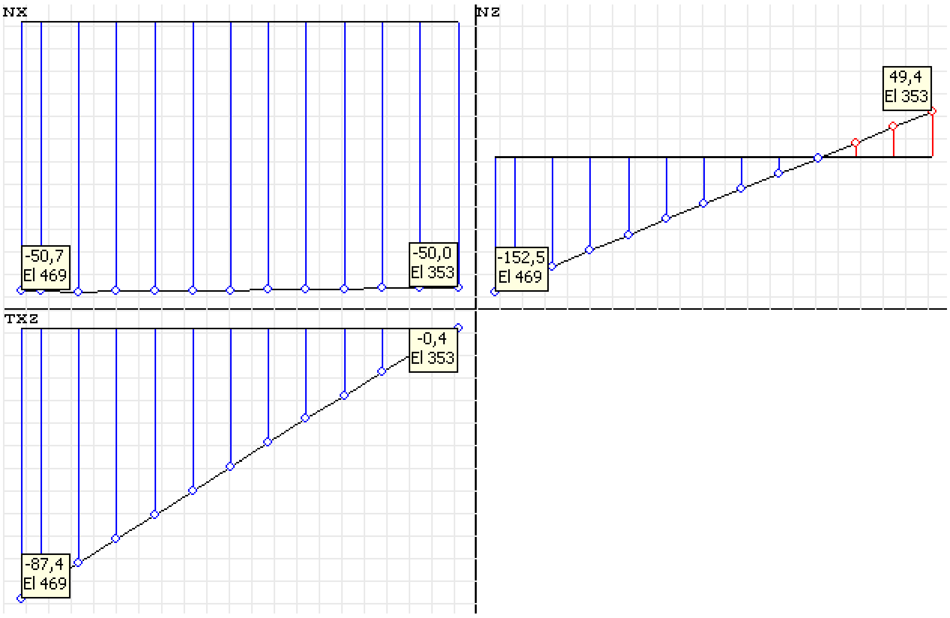
Values of stresses σ_x (kN/m^2)



Values of stresses σ_y (kN/m^2)



Values of stresses τ_{xy} (kN/m^2)



Values of stresses σ_x, σ_y, τ_{xy} (kN/m^2) in the horizontal section of the dam located at the depth of y_0 = 5.0 m from the top of the dam

Comparison of solutions:

Stress tensor components in Cartesian coordinates σ_x, σ_y, τ_{xy} in the horizontal section of the dam located at the depth of y_0 = 5.0 m from the top of the dam.

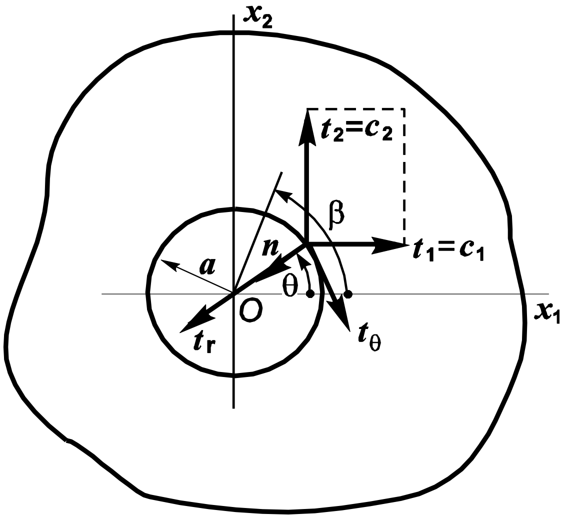
Parameter	**On the inclined face of the dam (x = y_0·tgβ = 2.8868 m)**		
	Theory	**SCAD**	**Deviations, %**
σ_x (kN/m^2)	-50.00	-50.69	1.38
σ_y (kN/m^2)	-150.00	-152.55	1.70
τ_{xy} (kN/m^2)	-86.60	-87.42	0.95

Parameter	**On the vertical face of the dam (x = 0.0000 m)**		
	Theory	**SCAD**	**Deviations, %**
σ_x (kN/m^2)	-50.00	-50.00	0.00
σ_y (kN/m^2)	50.00	49.43	1.14
τ_{xy} (kN/m^2)	0.00	-0.43	—

Notes: In the analytical solution the stresses σ_x, σ_y, τ_{xy} in the body of the dam subjected to its self-weight and hydrostatic pressure are determined according to the following formulas (V. I. Samul, Fundamentals of the Elasticity and Plasticity Theory. — Moscow: High school, 1982, p. 77):

$$\sigma_x = -\gamma \cdot y\,; \qquad \sigma_y = \left(\frac{\gamma_1}{tg\beta} - \frac{2\cdot\gamma}{tg^3\beta}\right)\cdot x + \left(\frac{\gamma}{tg^2\beta} - \gamma_1\right)\cdot y\,; \qquad \tau_{xy} = -\frac{\gamma\cdot x}{tg^2\beta}\,.$$

Plane Subjected to a Concentrated Moment and a Concentrated Force



Objective: Determination of the stress state of a plane of unit thickness in polar coordinates subjected to a concentrated moment and a concentrated force.

Initial data file: **4.26.SPR**

Problem formulation: The concentrated moment *M* and the concentrated force P_1 acting along the Ox_1 axis are applied in the origin of the plane of the unit thickness. Determine the stress tensor components in polar coordinates σ_{rr}, $\sigma_{\theta\theta}$, $\sigma_{r\theta}$ at different radial distances r from the origin of the plane at the angle to the Ox_1 axis $\theta = 0º$.

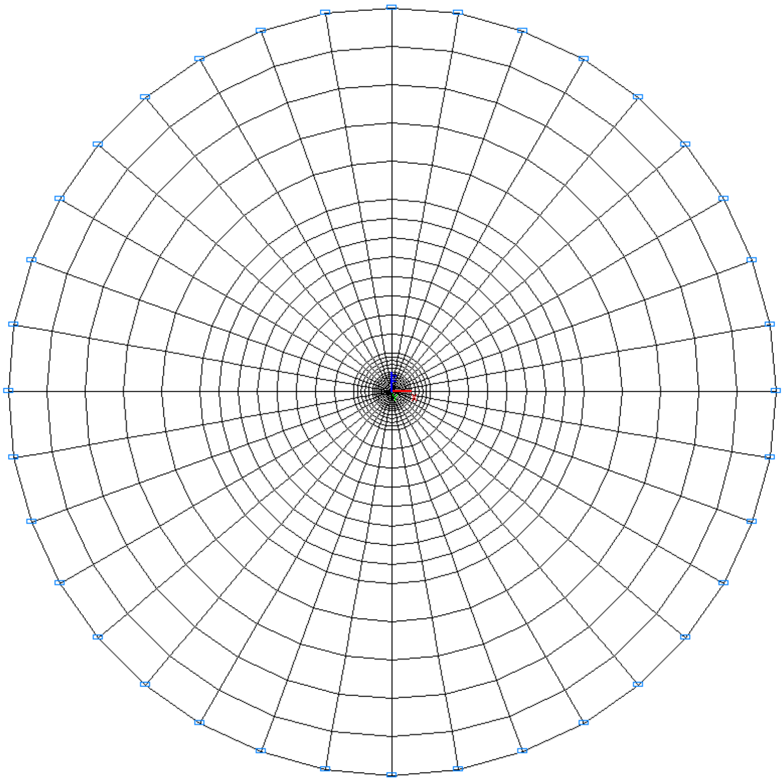
References: S.P. Demidov, Theory of Elasticity. — Moscow: High school, 1979.

Initial data:

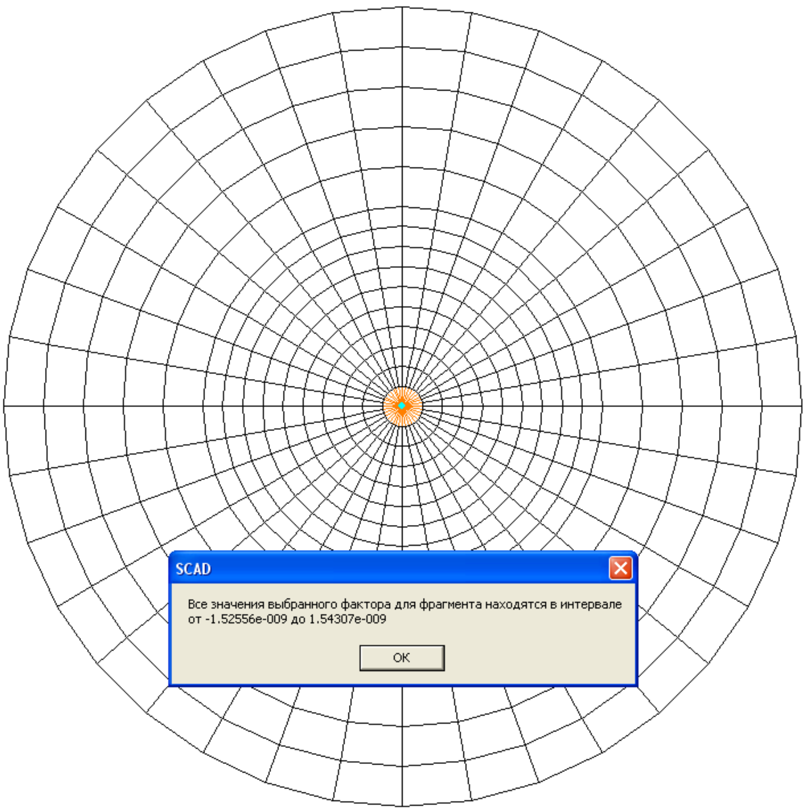
$E = 3.0 \cdot 10^7$ kPa	- elastic modulus;
$\nu = 0.2$	- Poisson's ratio;
h = 1.0 m	- thickness of the plane;
R = 10.0 m	- radius bounding the area of the plane along the fixed edge;
M = 100.0 kN·m	- concentrated moment acting in the plane;
P_1 = 100.0 kN	- concentrated force acting in the plane along the OX1 axis.
P_2 = 0.0 kN	- concentrated force acting in the plane along the OX2 axis.

Finite element model: Design model – plane frame, plate elements – 972 eight-node elements of type 30. The spacing of the finite element mesh in the radial direction from r = 0.00 m to r = 0.50 m is 0.05 m, from r = 0.50 m to r = 1.00 m is 0.10 m, from r = 1.00 m to r = 5.00 m is 0.50 m, from r = 5.00 m to r = 10.00 m is 1.00 m, and in the tangential direction the spacing is 10º. The direction of the output of internal forces is radial tangential. A concentrated moment *M* and a concentrated force P_1 in the vicinity of their application point on the cylindrical surface of a small radius *a* cannot be represented as a resultant of stresses distributed according to the laws of the analytical solution given below. Therefore, the area of the plane bounded by this cylindrical surface is modeled by a rigid body with a master node at the point of the application of concentrated forces and the slave nodes at the radial distance of *a* = 0.05 m from it (member type – 100). In order to exclude the effect of the boundary conditions on the accuracy of the solution, the radial distance to the fixed edge of the plane is taken as R = 10.0 m. Number of nodes in the design model – 2989.

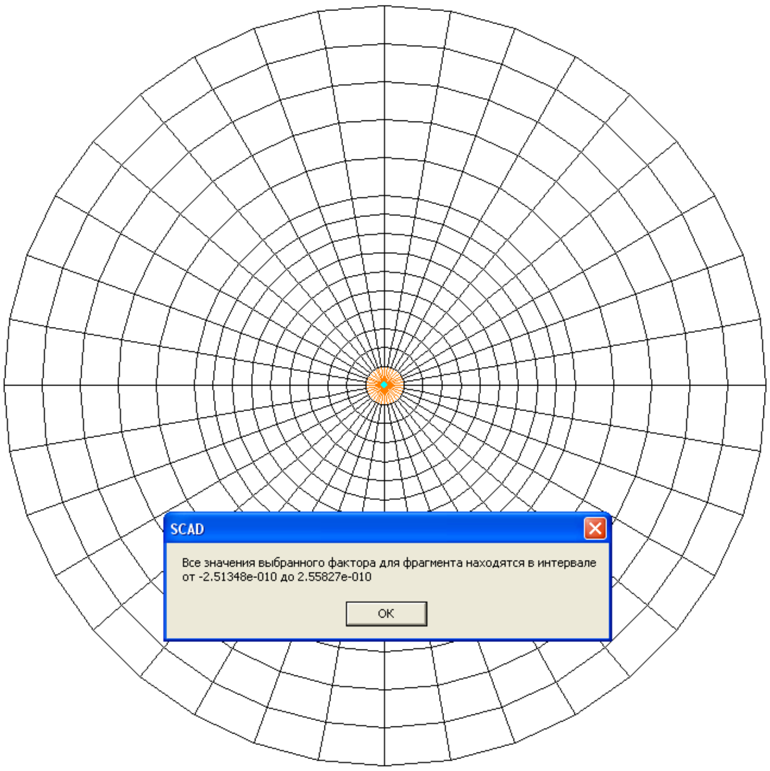
Results in SCAD



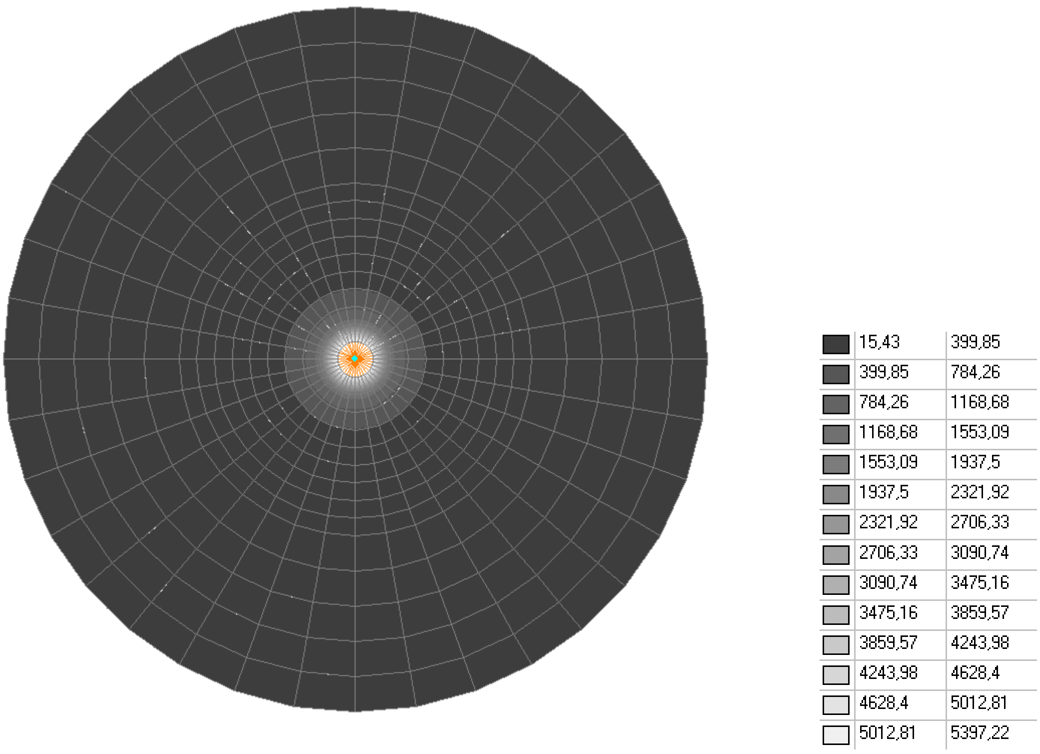
Design model



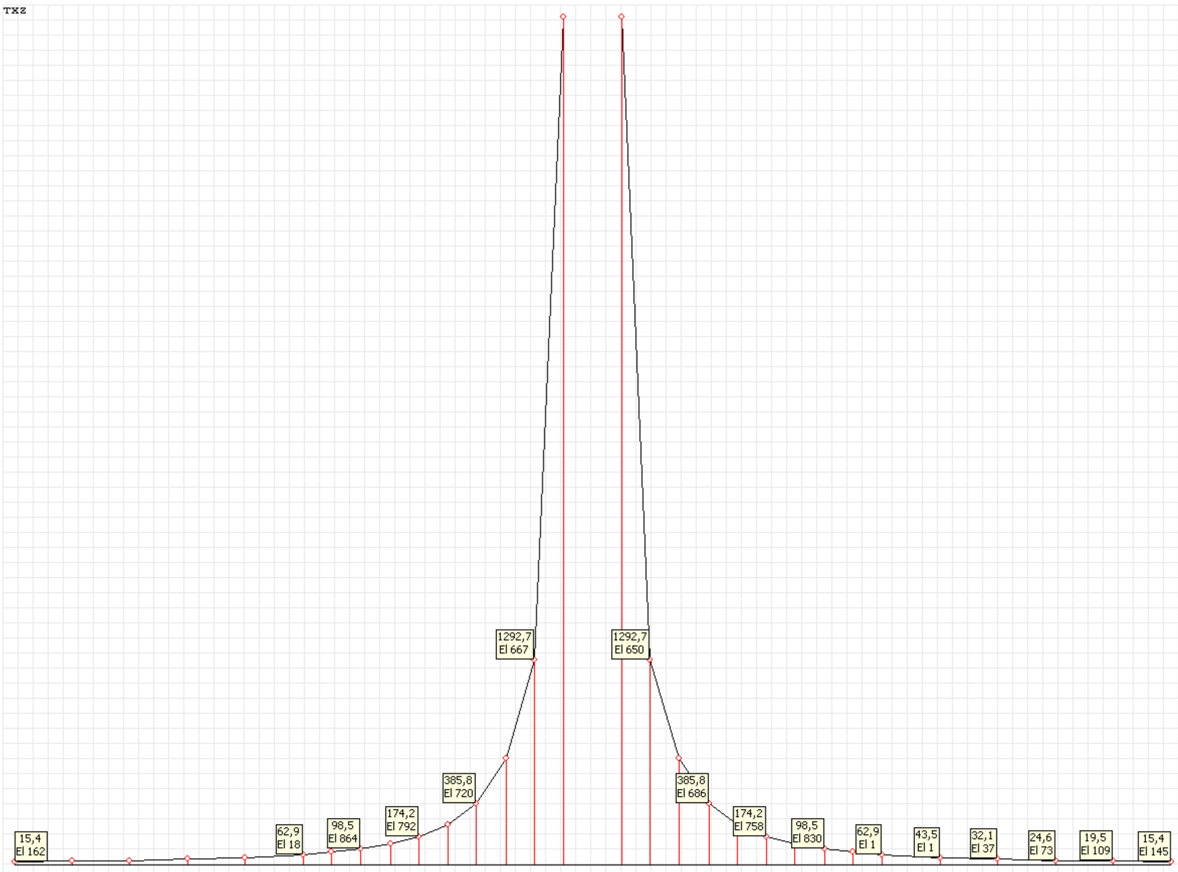
Values of stresses σ_{rr} (kN/m^2) under the concentrated moment M



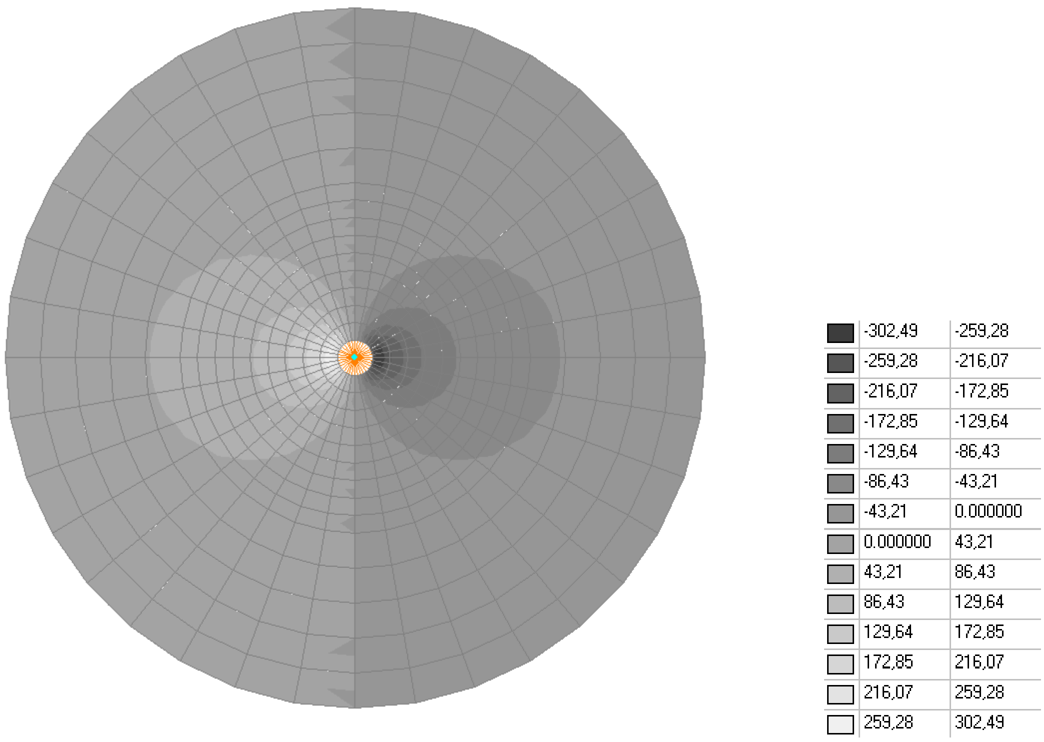
Values of stresses $\sigma_{\theta\theta}$ (kN/m^2) under the concentrated moment M



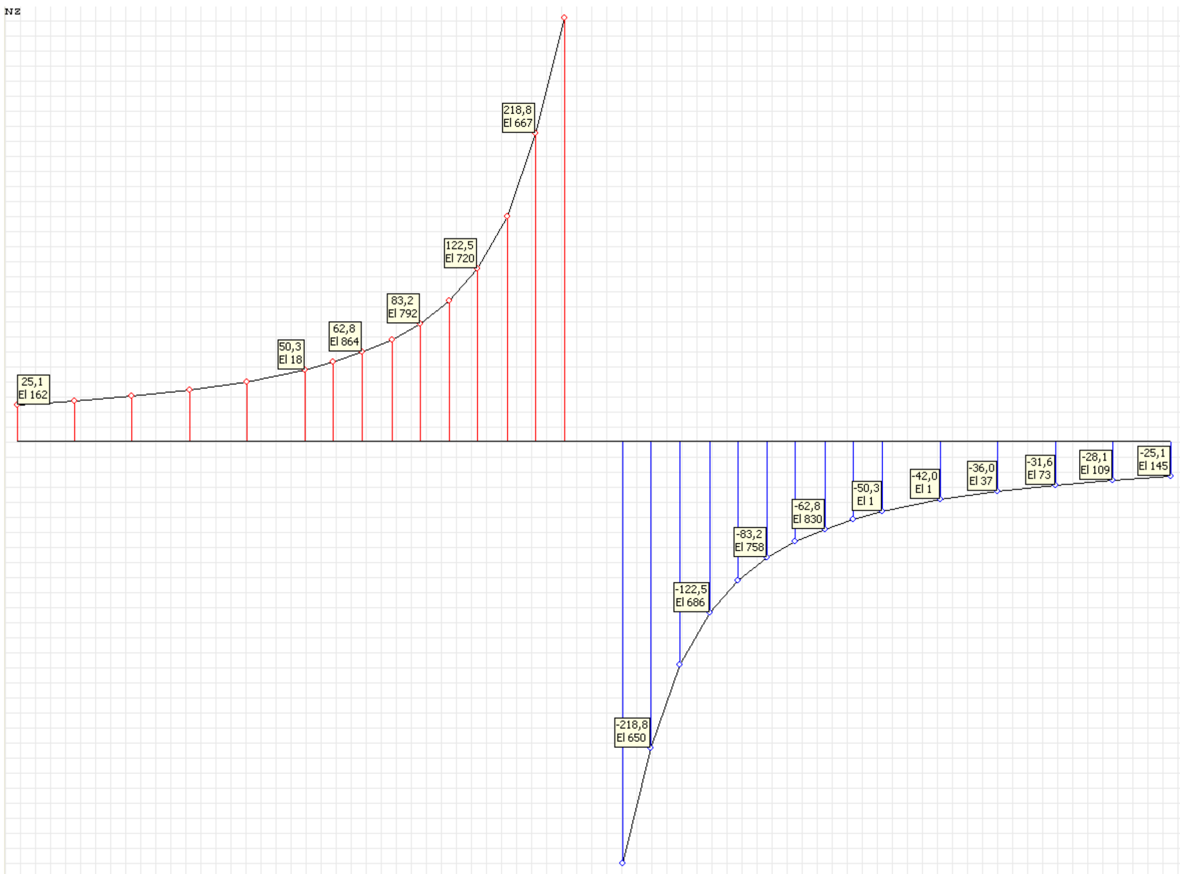
Values of stresses $\sigma_{r\theta}$ (kN/m^2) under the concentrated moment M



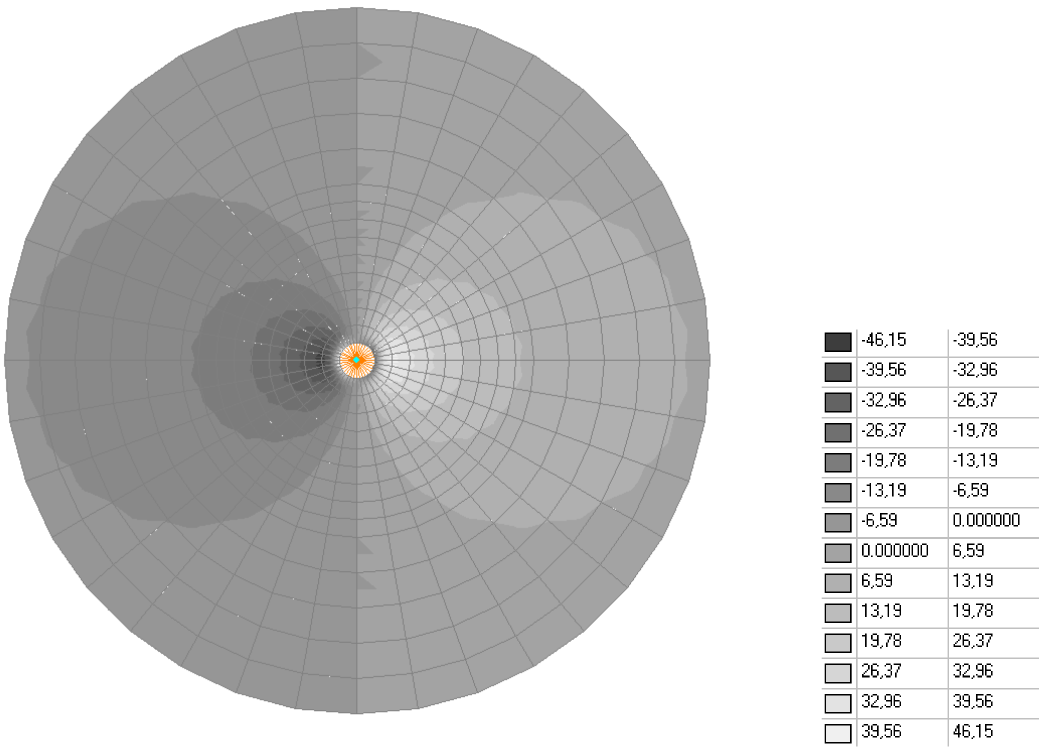
Stress diagram $\sigma_{r\theta}$ (kN/m^2) under the concentrated moment M
for the angle to the OX1 axis $\theta = 0^o$



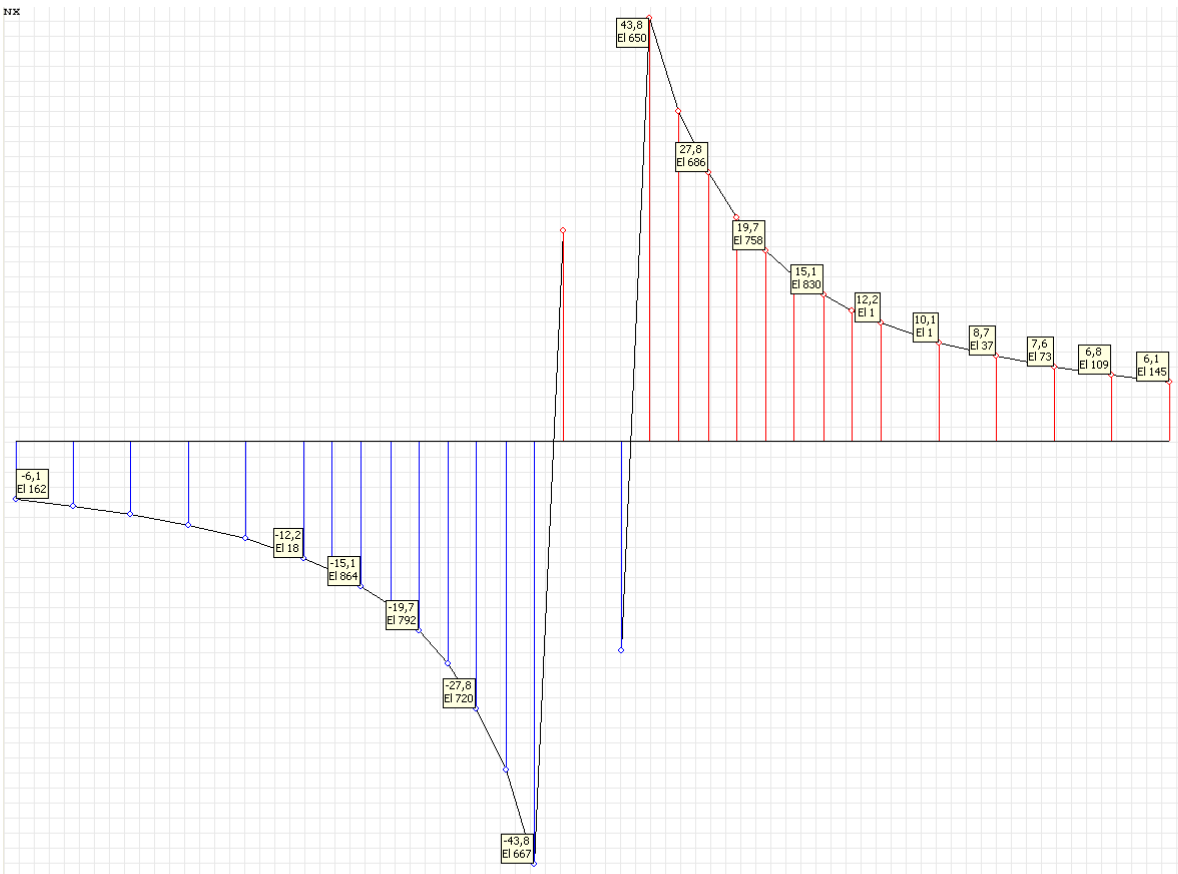
Values of stresses σ_{rr} (kN/m^2) under the concentrated force P_1



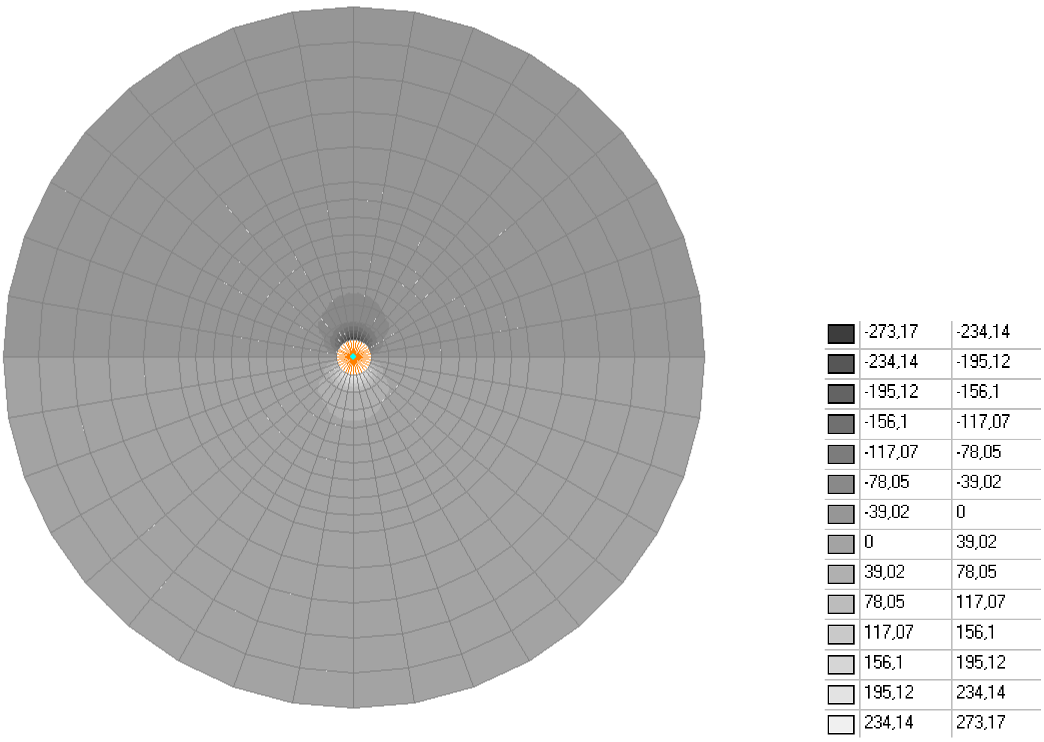
Stress diagram σ_{rr} (kN/m^2) under the concentrated force P_1
for the angle to the OX1 axis $\theta = 0^o$



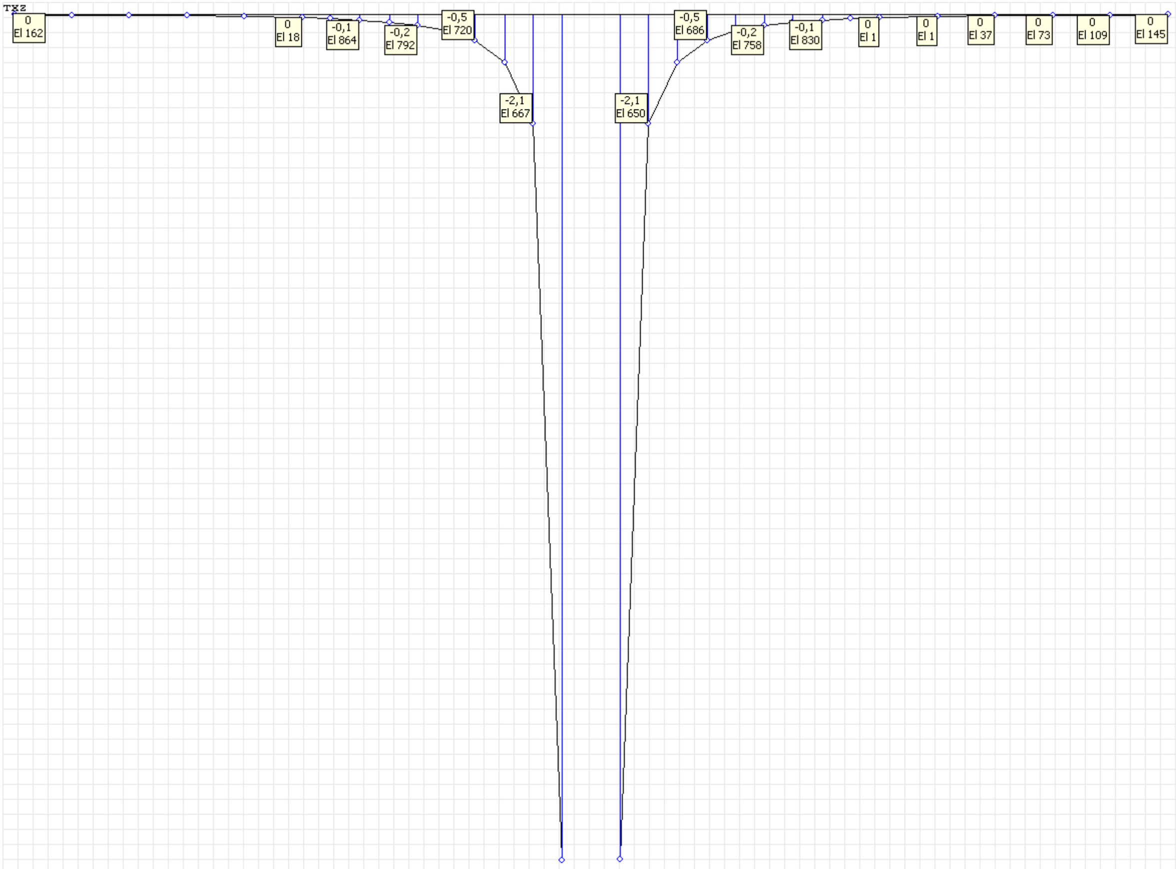
Values of stresses $\sigma_{\theta\theta}$ (kN/m^2) under the concentrated force P_1



Stress diagram $\sigma_{\theta\theta}$ (kN/m^2) under the concentrated force P_1
for the angle to the OX1 axis $\theta = 0^o$



Values of stresses $\sigma_{r\theta}$ (kN/m^2) under the concentrated force P_1



Stress diagram $\sigma_{r\theta}$ (kN/m^2) under the concentrated force P_1
for the angle to the OX1 axis $\theta = 0^o$

Comparison of solutions:

Stress tensor components for the angle to the Ox_1 axis $\theta = 0^o$ under the concentrated moment *M*

Radius r (m)	**Stresses σ_{rr} (kN/m^2)**		
	Theory	**SCAD**	**Deviations, %**
0.2	0.00	0.00	—
0.3	0.00	0.00	—
0.4	0.00	0.00	—
0.5	0.00	0.00	—
1.0	0.00	0.00	—
1.5	0.00	0.00	—
2.0	0.00	0.00	—
2.5	0.00	0.00	—
3.0	0.00	0.00	—

Radius r (m)	**Stresses $\sigma_{\theta\theta}$ (kN/m^2)**		
	Theory	**SCAD**	**Deviations, %**
0.2	0.00	0.00	—
0.3	0.00	0.00	—
0.4	0.00	0.00	—
0.5	0.00	0.00	—
1.0	0.00	0.00	—
1.5	0.00	0.00	—
2.0	0.00	0.00	—
2.5	0.00	0.00	—
3.0	0.00	0.00	—

Radius r (m)	Stresses $\sigma_{r\theta}$ (kN/m^2)		
	Theory	SCAD	Deviations, %
0.2	397.89	385.79	3.04
0.3	176.84	174.22	1.48
0.4	99.47	98.49	0.99
0.5	63.66	62.93	1.15
1.0	15.92	15.43	3.08
1.5	7.07	6.67	5.65
2.0	3.98	3.86	3.02
2.5	2.55	2.50	1.96
3.0	1.77	1.74	1.69

Stress tensor components for the angle to the Ox_1 axis Ox_1 $\theta = 0^o$ under the concentrated force P_1.

Radius r (m)	Stresses σ_{rr} (kN/m^2)		
	Theory	SCAD	Deviations, %
0.2	-127.32	-122.67	3.65
0.3	-84.88	-83.26	1.91
0.4	-63.66	-62.84	1.29
0.5	-50.93	-50.36	1.12
1.0	-25.46	-25.08	1.49
1.5	-16.98	-16.65	1.94
2.0	-12.73	-12.61	0.94
2.5	-10.19	-10.15	0.39
3.0	-8.49	-8.50	0.12

Radius r (m)	Stresses $\sigma_{\theta\theta}$ (kN/m^2)		
	Theory	SCAD	Deviations, %
0.2	31.83	27.80	12.66
0.3	21.22	19.69	7.21
0.4	15.92	15.08	5.28
0.5	12.73	12.16	4.48
1.0	6.37	6.09	4.40
1.5	4.24	3.96	6.60
2.0	3.18	2.92	8.18
2.5	2.55	2.27	10.98
3.0	2.12	1.82	14.15

Radius r (m)	Stresses $\sigma_{r\theta}$ (kN/m^2)		
	Theory	SCAD	Deviations, %
0.2	0.00	0.00	—
0.3	0.00	0.00	—
0.4	0.00	0.00	—
0.5	0.00	0.00	—
1.0	0.00	0.00	—
1.5	0.00	0.00	—
2.0	0.00	0.00	—
2.5	0.00	0.00	—
3.0	0.00	0.00	—

***Notes*:**

1. In the analytical solution the stresses σ_{rr}, $\sigma_{\theta\theta}$, $\sigma_{r\theta}$ in the plane under the concentrated moment are determined according to the following formulas (S.P. Demidov, Theory of Elasticity. — Moscow: High school, 1979, p. 299):

$$\sigma_{rr} = 0; \qquad \sigma_{\theta\theta} = 0; \qquad \sigma_{r\theta} = -\frac{M}{2 \cdot \pi \cdot r^2}.$$

In the analytical solution the stresses σ_{rr}, $\sigma_{\theta\theta}$, $\sigma_{r\theta}$ in the plane under the concentrated force are determined according to the following formulas (S.P. Demidov, Theory of Elasticity. — Moscow: High school, 1979, p. 300):

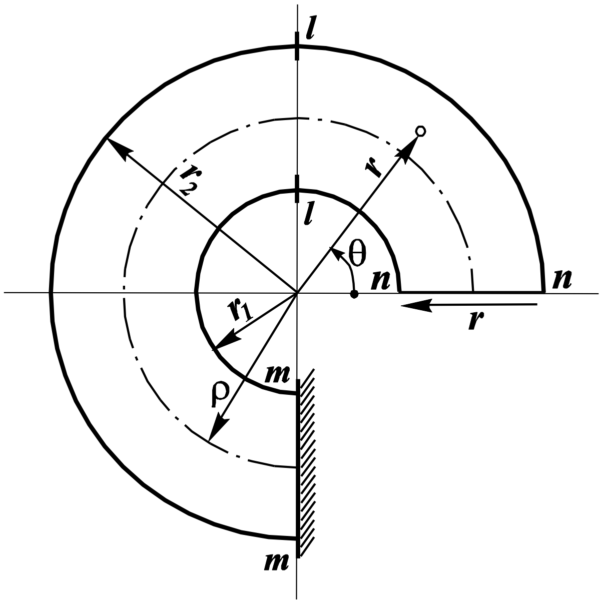
$$\sigma_{rr} = -\frac{3+\nu}{4\cdot\pi\cdot r}\cdot\left(P_1\cdot cos\,\theta + P_2\cdot sin\,\theta\right);$$

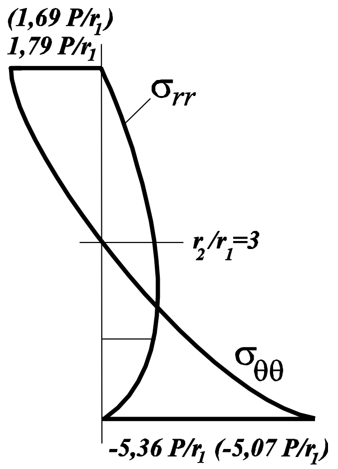
$$\sigma_{\theta\theta} = \frac{1-\nu}{4\cdot\pi\cdot r}\cdot\left(P_1\cdot cos\,\theta + P_2\cdot sin\,\theta\right);$$

$$\sigma_{r\theta} = \frac{1-\nu}{4\cdot\pi\cdot r}\cdot\left(P_1\cdot sin\,\theta - P_2\cdot cos\,\theta\right).$$

2. It is impossible to perform an accurate modeling of the problem considered in the source in SCAD, because an *infinite* plane is considered, and the solution has a *singularity*. Therefore, the verification matrix contains deviations from the theoretical solution in the point located at the distance of 1,5 m from the origin.

Bending of a Curved Beam of a Narrow Rectangular Cross-Section by a Force Applied to Its Free End (Golovin's Problem)





Objective: Determination of the stress state of a curved beam of a narrow rectangular cross-section subjected to bending by a concentrated force applied to its free end.

Initial data file: **4.21.SPR**

Problem formulation: A force P acting parallel to the edge in the plane of the circular axis of the beam is applied to the free end of the cantilever curved beam of the unit thickness. Determine the stress tensor components in polar coordinates σ_{rr}, $\sigma_{\theta\theta}$, $\sigma_{r\theta}$ for the beam cross-section at $\theta = 90^o$ to the edge of the free end of the beam (section n-n).

References: S.P. Demidov, Theory of Elasticity. — Moscow: High school, 1979.

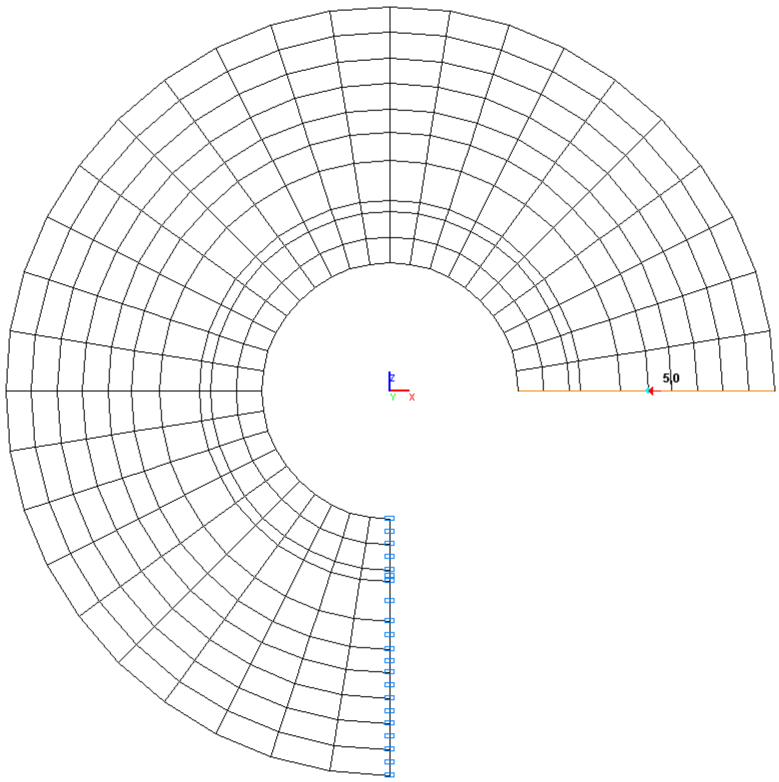
Initial data:

$E = 3.0 \cdot 10^7$ kPa	- elastic modulus;
$\mu = 0.2$	- Poisson's ratio;
h = 1.0 m	- thickness of the beam;
$r_1 = 5$ m	- inner radius of the beam;
$r_2 = 15$ m	- outer radius of the beam;
P = 5.0 kN	- concentrated force bending the beam (horizontal).

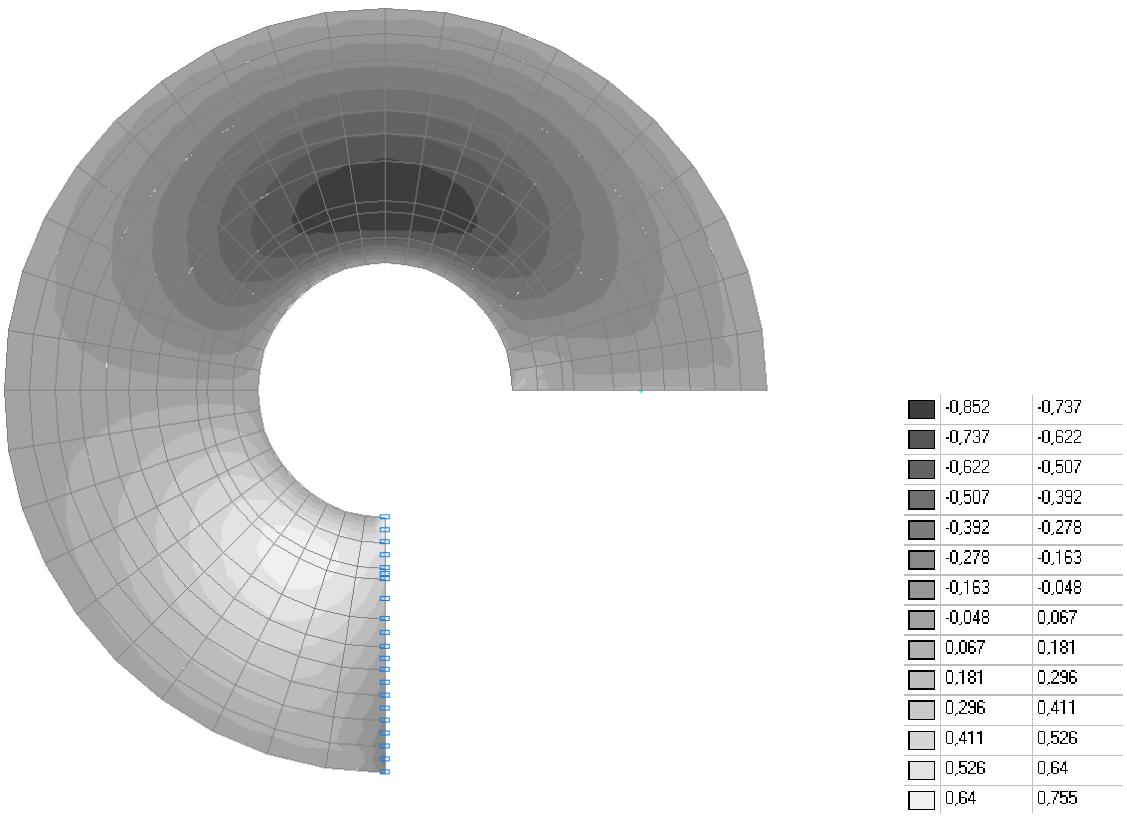
Constraints: full restraint of the nodes of the clamped edge of the beam (section m-m)

Finite element model: Design model – general type system, beam elements – 300 eight-node elements of type 50. The spacing of the finite element mesh in the radial direction is 1.0 m, and in the tangential direction is 9º. The direction of the output of internal forces is radial tangential. Since the boundary conditions at the end surface of the free end of the curved beam ($\theta = 0^o$) are given in integral form in the analytical solution, they are softened by introducing a rigid body (member type – 100), the nodes of which are located along the end surface. Number of nodes in the design model – 981.

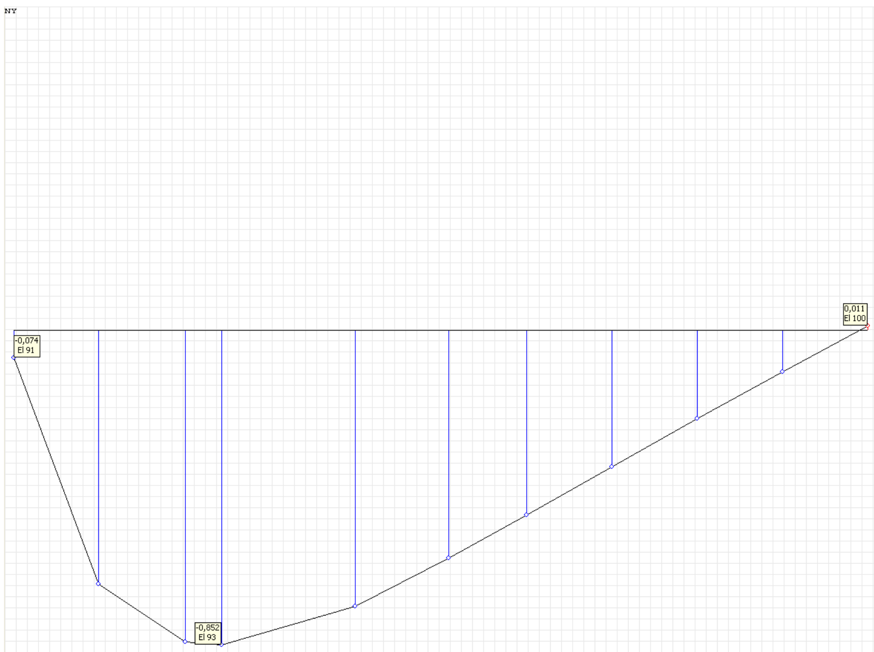
Results in SCAD



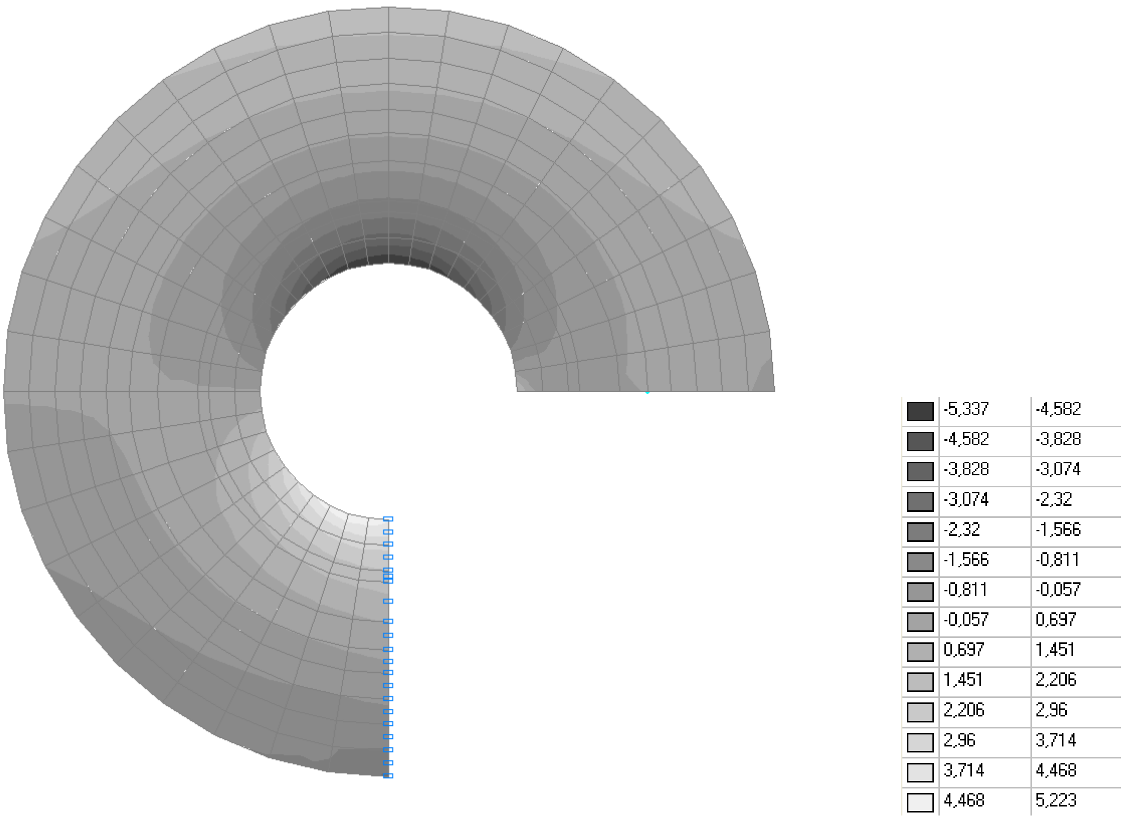
Design model



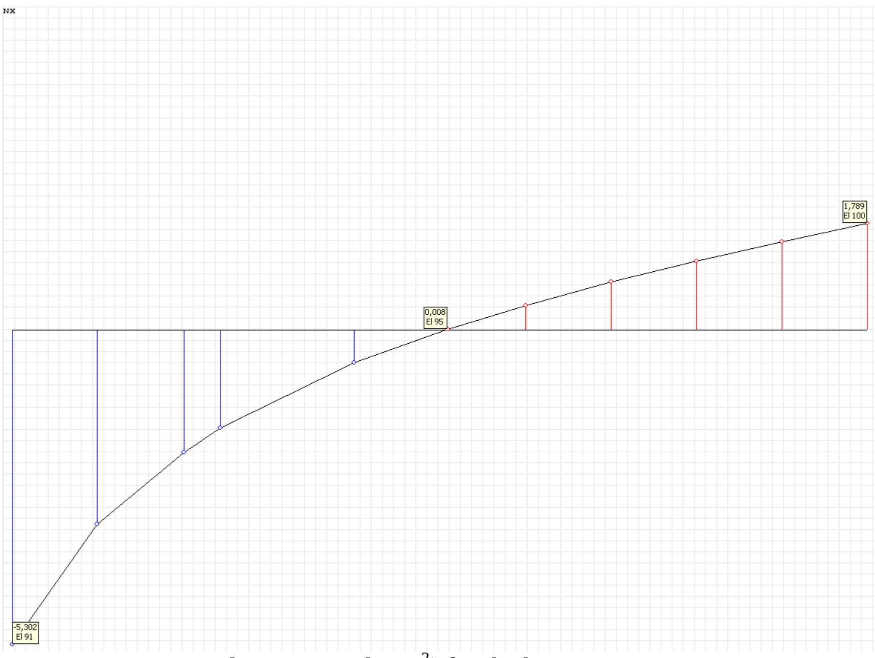
Values of stresses σ_{rr} (kN/m^2)



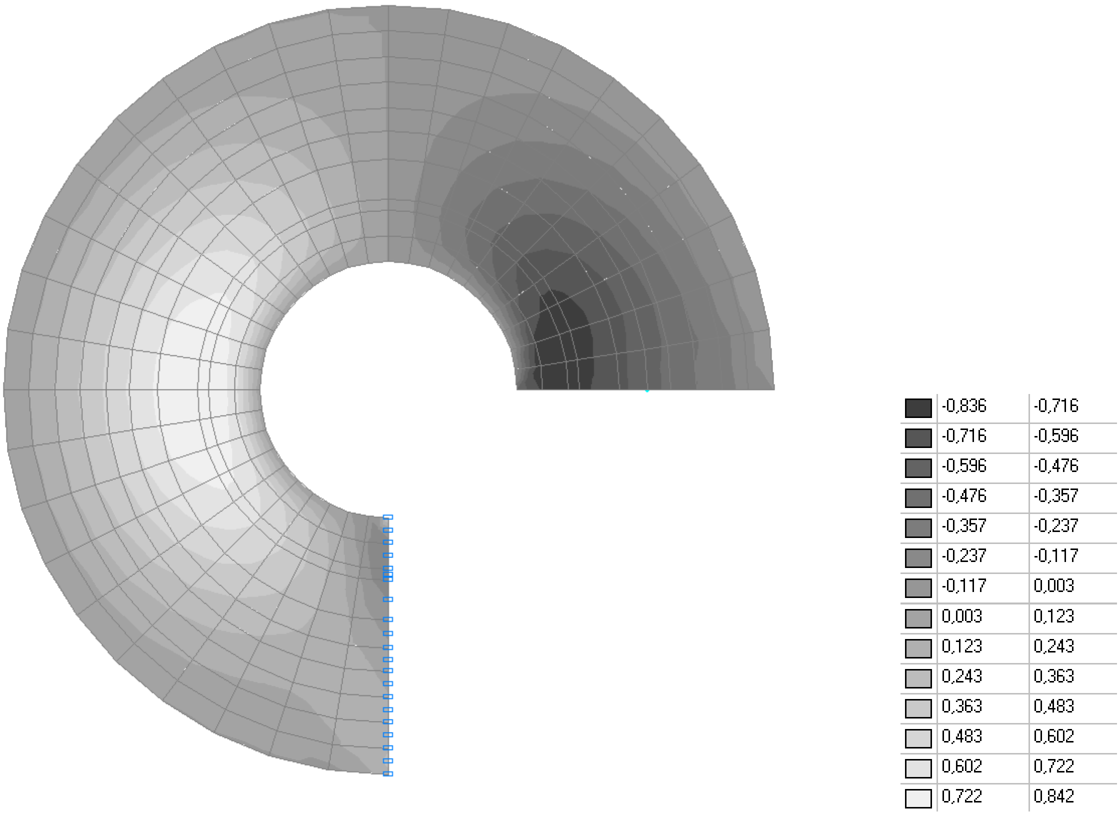
Stress diagram σ_{rr} (kN/m^2) for the beam cross-section at $\theta = 90^o$ to the edge of the free end of the beam (section n-n)



Values of stresses $\sigma_{\theta\theta}$ (kN/m^2)



Stress diagram $\sigma_{\theta\theta}$ (kN/m^2) for the beam cross-section
at $\theta = 90^o$ to the edge of the free end of the beam (section n-n)



Values of stresses $\sigma_{r\theta}$ (kN/m^2)

Comparison of solutions:

	Stresses σ_{rr} (kN/m^2)			Stresses $\sigma_{\theta\theta}$ (kN/m^2)		
	r = 5.0000 m	**r = 7.4349 m**	**r = 15.0000 m**	**r = 5.0000 m**	**r = 10.0876 m**	**r = 15.0000 m**
Theory	0.0000	-0.8375	0.0000	-5.3581	0.0000	1.7860
SCAD	-0.0744	-0.8515	0.0109	-5.3022	0.0078	1.7893
Deviations, %	–	1.67	–	1.04	–	0.18

Notes: In the analytical solution the stresses σ_{rr}, $\sigma_{\theta\theta}$, $\sigma_{r\theta}$ in the body of the cantilever curved beam subjected to the force *P* applied at its free end and directed parallel to its edge are determined according to the following formulas (S.P. Demidov, Theory of Elasticity. — Moscow: High school, 1979, p. 271):

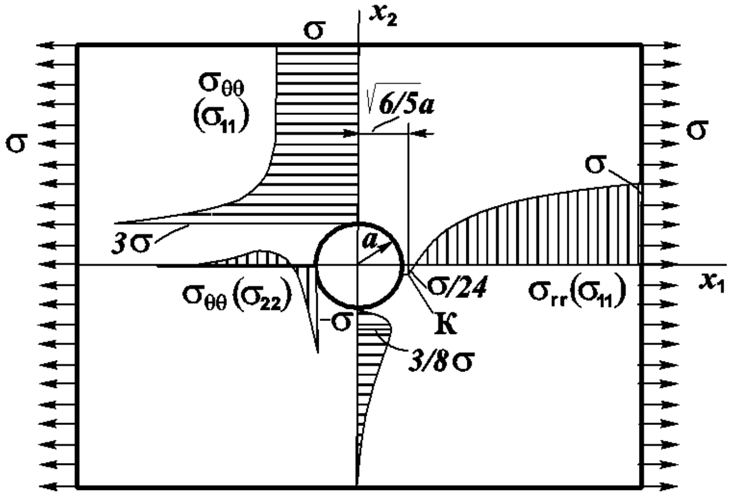
$$\sigma_{rr} = \frac{P}{K_0} \cdot \left(r - \frac{r_1^2 + r_2^2}{r} + \frac{r_1^2 \cdot r_2^2}{r^3} \right) \cdot \sin\theta \; ;$$

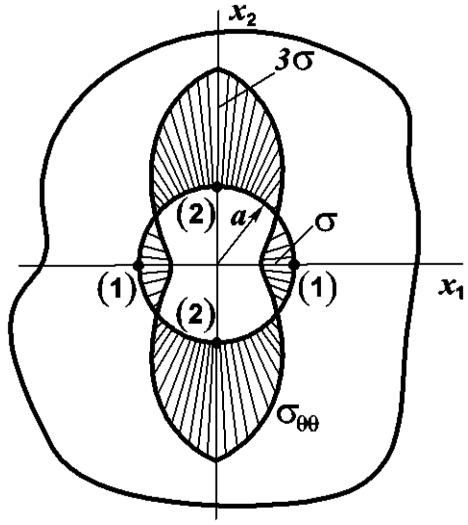
$$\sigma_{\theta\theta} = \frac{P}{K_0} \cdot \left(3 \cdot r - \frac{r_1^2 + r_2^2}{r} - \frac{r_1^2 \cdot r_2^2}{r^3} \right) \cdot \sin\theta \; ;$$

$$\sigma_{rr} = -\frac{P}{K_0} \cdot \left(r - \frac{r_1^2 + r_2^2}{r} + \frac{r_1^2 \cdot r_2^2}{r^3} \right) \cdot \cos\theta \; ;$$

$$K_0 = r_1^2 - r_2^2 + \left(r_1^2 + r_2^2 \right) \cdot \ln\left(r_2 / r_1 \right) .$$

Unilateral Tension of a Plate with a Small Circular Hole (Kirsch Problem)





Objective: Determination of the stress state of a plate of considerable width and unit thickness with a small circular hole in polar coordinates subjected to unilateral uniform tension.

Initial data files:

File name	Description
4.27_b_20_9_grad.SPR	1 variant of the design model – coarse FE mesh
4.27_b_60_4.5_grad.SPR	2 variant of the design model – fine FE mesh

Problem formulation: The square plate of considerable width and unit thickness with a small circular hole of radius *a* is subjected to unilateral uniform tension by stresses σ in the direction of the x_1 axis applied in its center. Determine the stress tensor components in polar coordinates σ_{rr}, $\sigma_{\theta\theta}$, $\sigma_{r\theta}$ at different radial distances r from the origin at the angles to the x_1 axis θ = 0º and θ = 90º.

References: S.P. Demidov, Theory of Elasticity. — Moscow: High school, 1979.

Initial data:

$E = 3.0 \cdot 10^7$ kPa - elastic modulus;
μ = 0.2 - Poisson's ratio;
h = 1.0 m - thickness of the plate;
a = 1.0 m - radius of the hole;
2·b = 20.0 m (60.0 m) - width of the plate;
σ = 100.0 kN/m - tensile stress in the direction of the x_1 axis.

Finite element model: Two variants of the design model are considered.

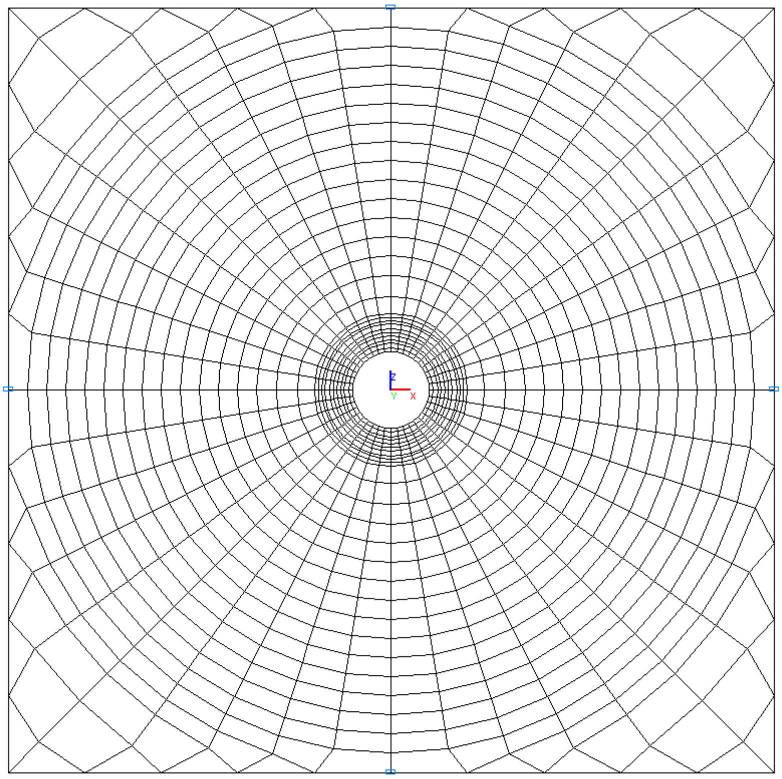
Variant 1:

Design model – plane frame, width of the plate 2·b = 20.0 m, plate elements – 1088 eight-node elements of type 30 and 32 six-node elements of type 25. The spacing of the finite element mesh in the radial direction from r = 1.00 m to r = 2.00 m is 0.10 m, from r = 2.00 m to r = 10.00 m is 0.50 m and in the tangential direction the spacing is 9º. The direction of the output of internal forces is radial tangential. Number of nodes in the design model – 3409.

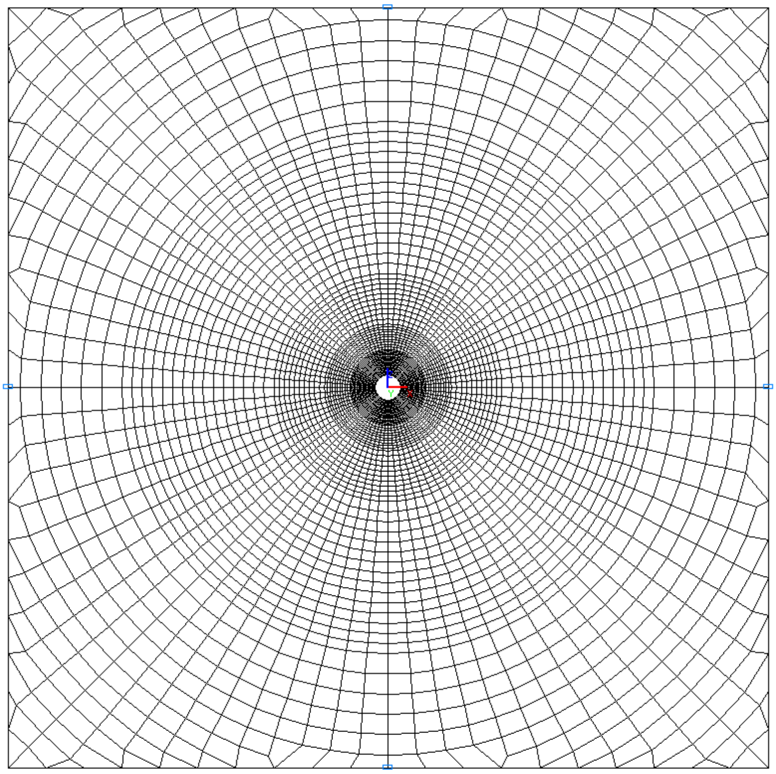
Variant 2:

Design model – plane frame, width of the plate 2·b = 60.0 m, plate elements – 5024 eight-node elements of type 30 and 40 six-node elements of type 25. The spacing of the finite element mesh in the radial direction from r = 1.00 m to r = 3.00 m is 0.10 m, from r = 3.00 m to r = 5.00 m is 0.20 m, from r = 5.00 m to r = 9.00 m is 0.40 m, from r = 9.00 m to r = 21.00 m is 0.80 m, from r = 21.00 m to r = 29.00 m is 1.60 m, and in the tangential direction the spacing is 4.5º. The direction of the output of internal forces is radial tangential. Number of nodes in the design model – 15312.

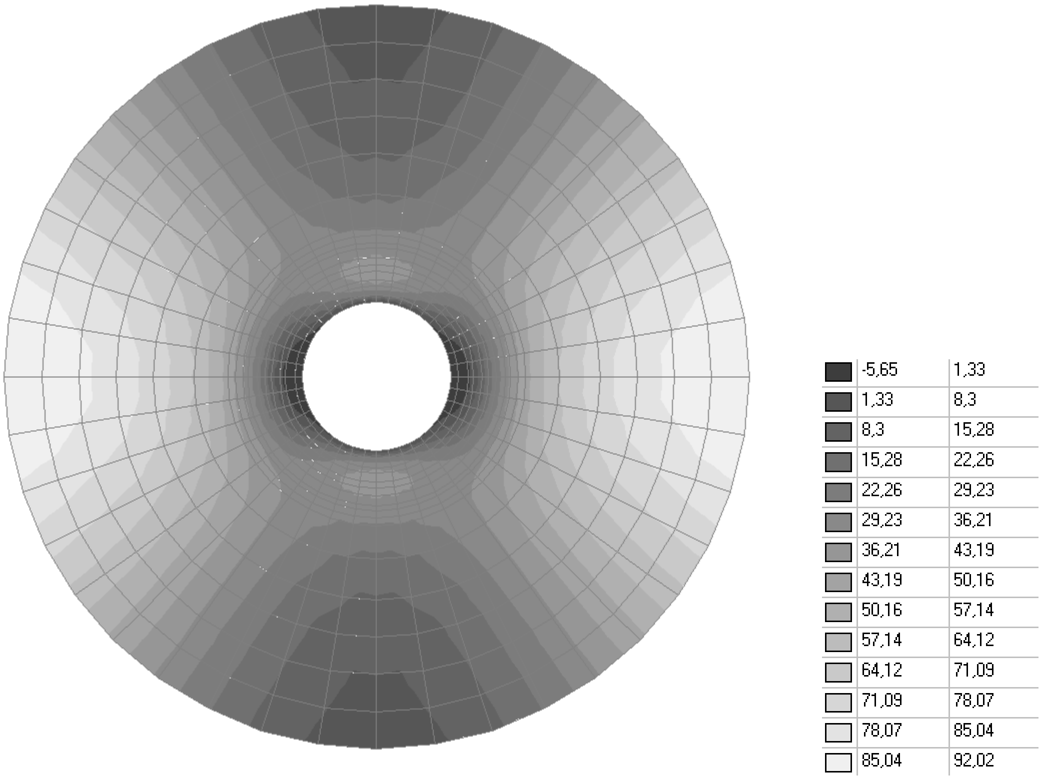
Results in SCAD



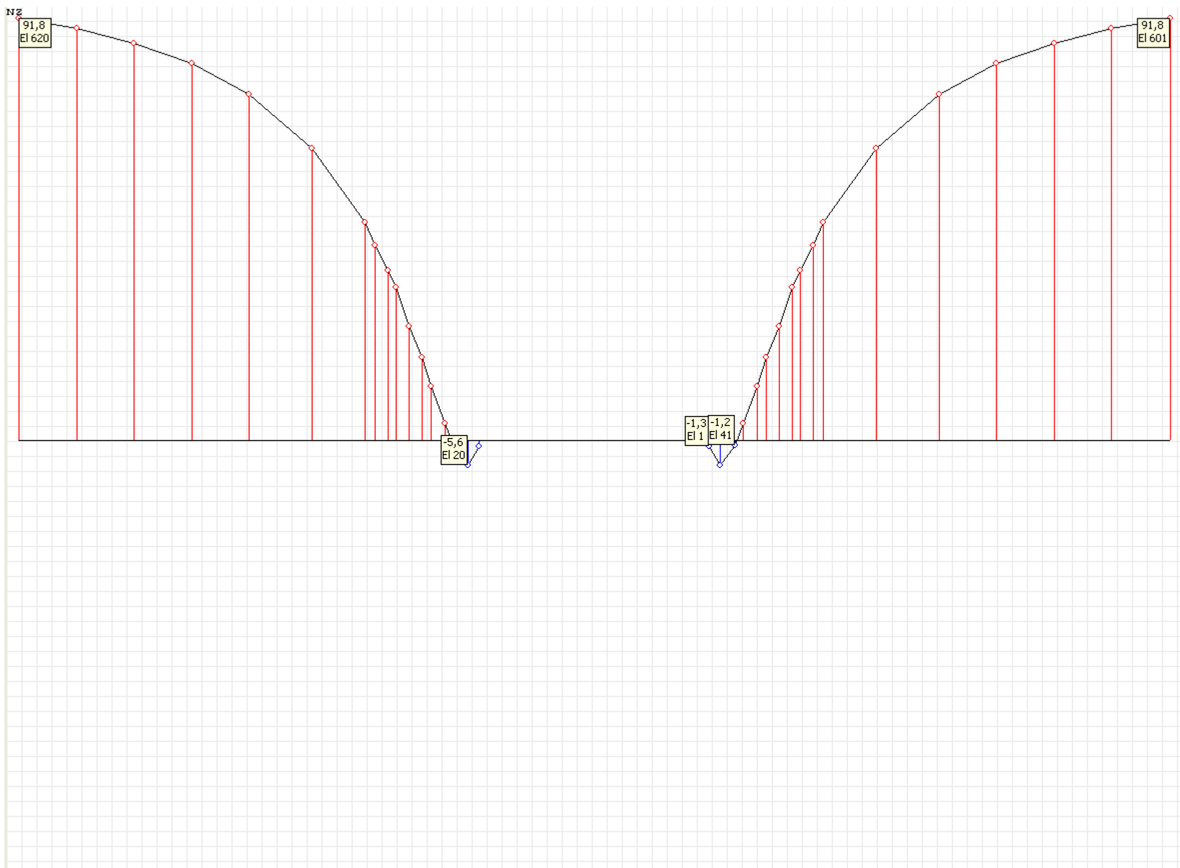
Design model. Variant 1



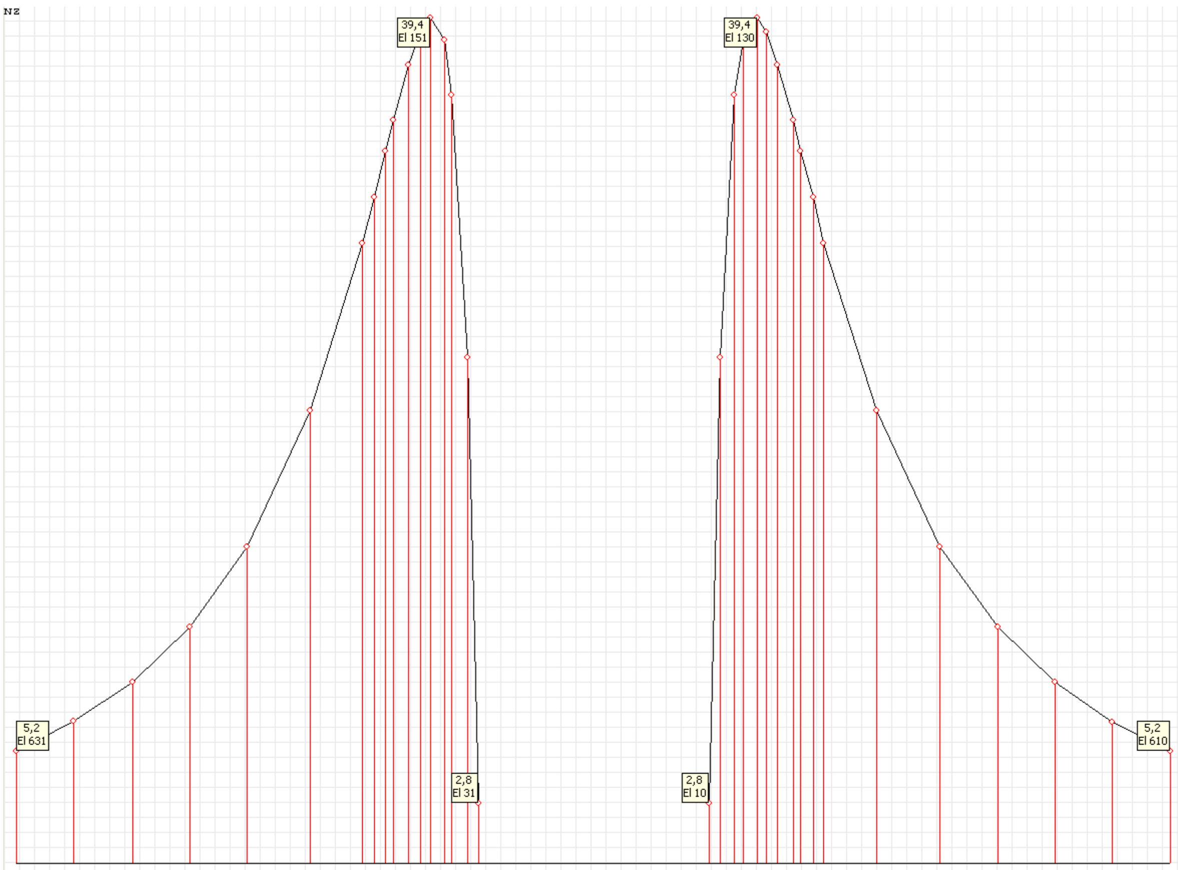
Design model. Variant 2



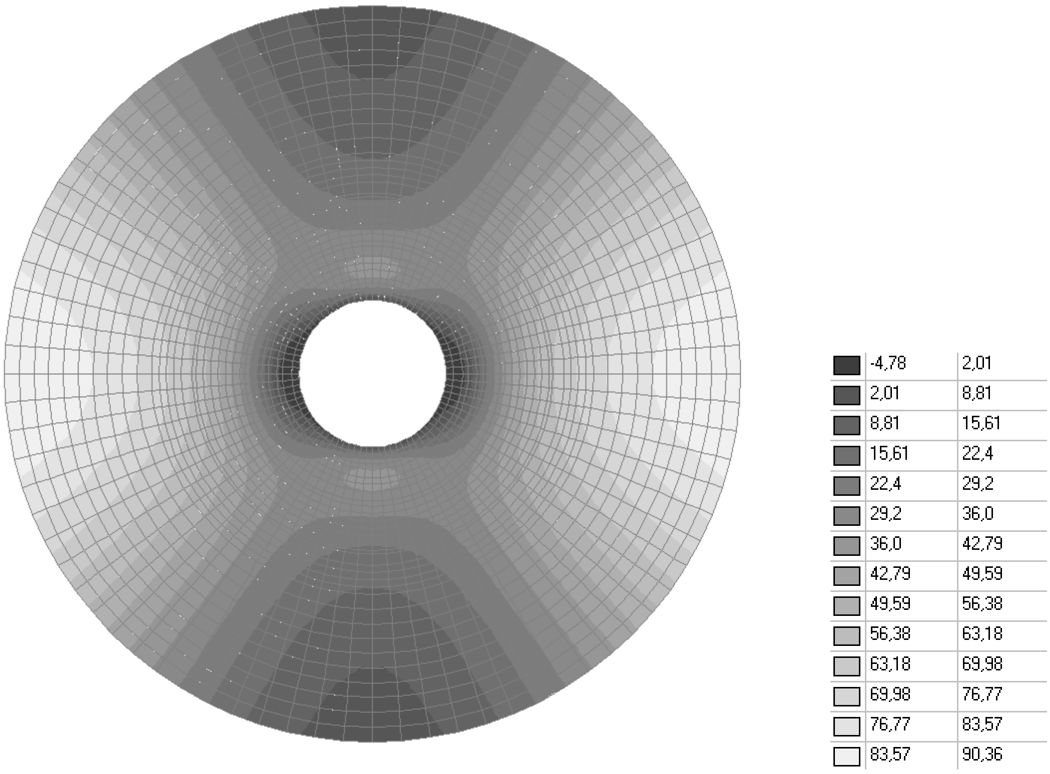
Values of stresses σ_{rr} (kN/m^2) for the design model according to variant 1



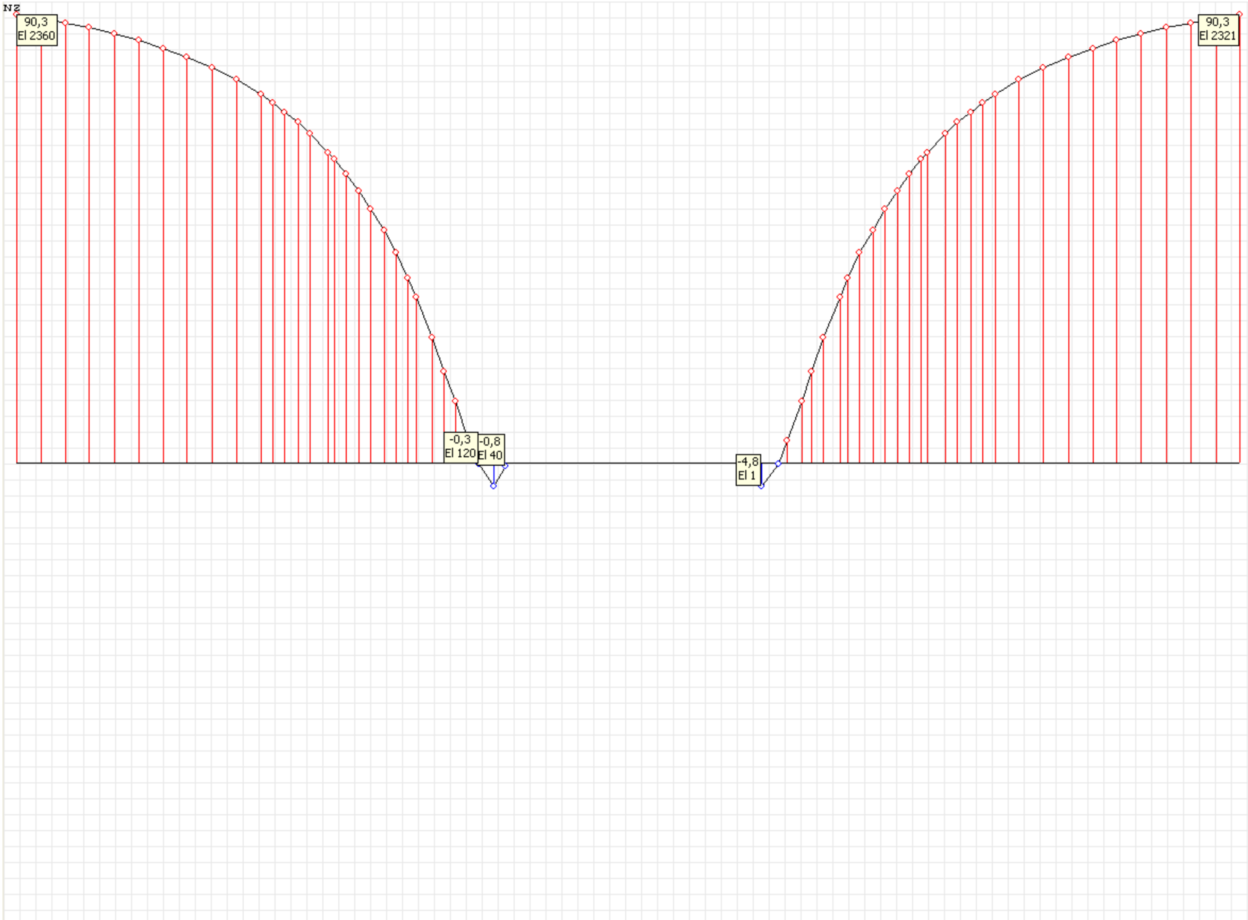
Stress diagram σ_{rr} (kN/m^2) at the angle to the Ox_1 axis $\theta = 0^o$ for the design model according to variant 1



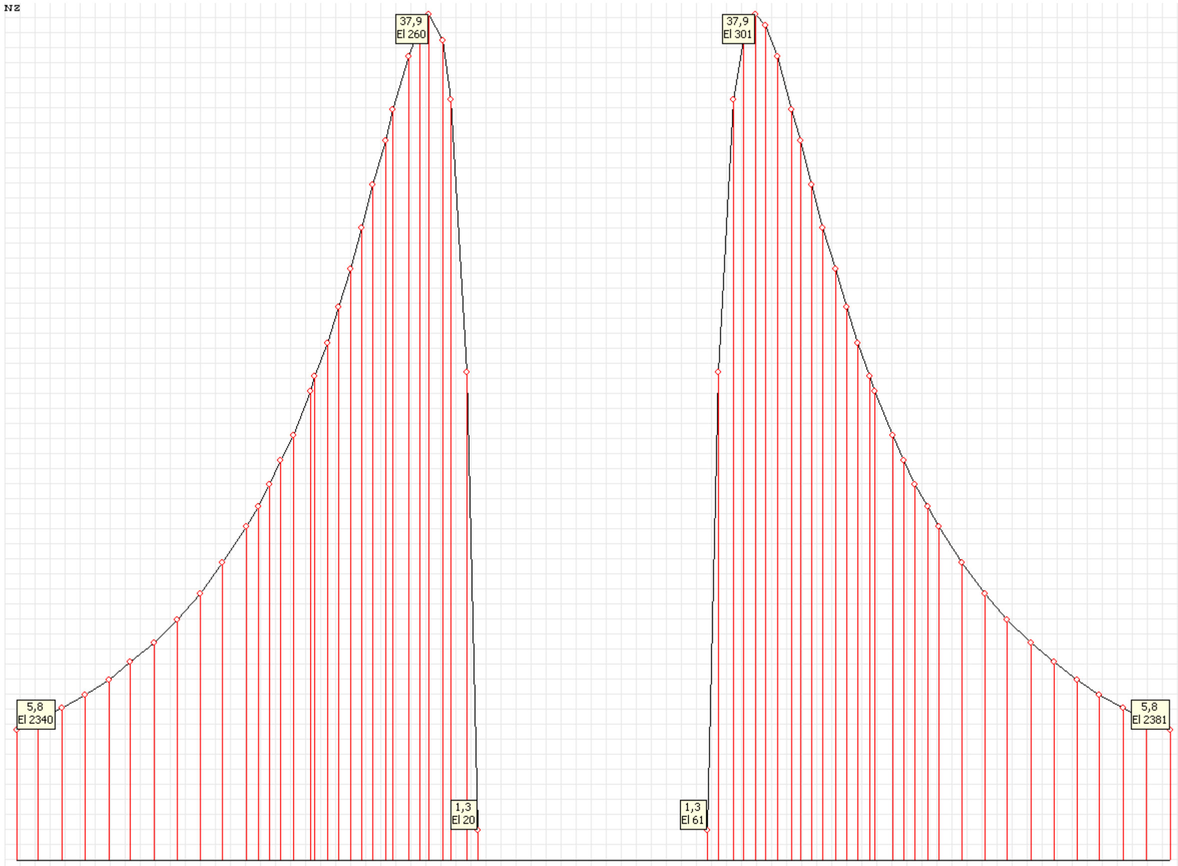
Stress diagram σ_{rr} (kN/m^2) at the angle to the Ox_1 axis $\theta = 90^o$ for the design model according to variant 1



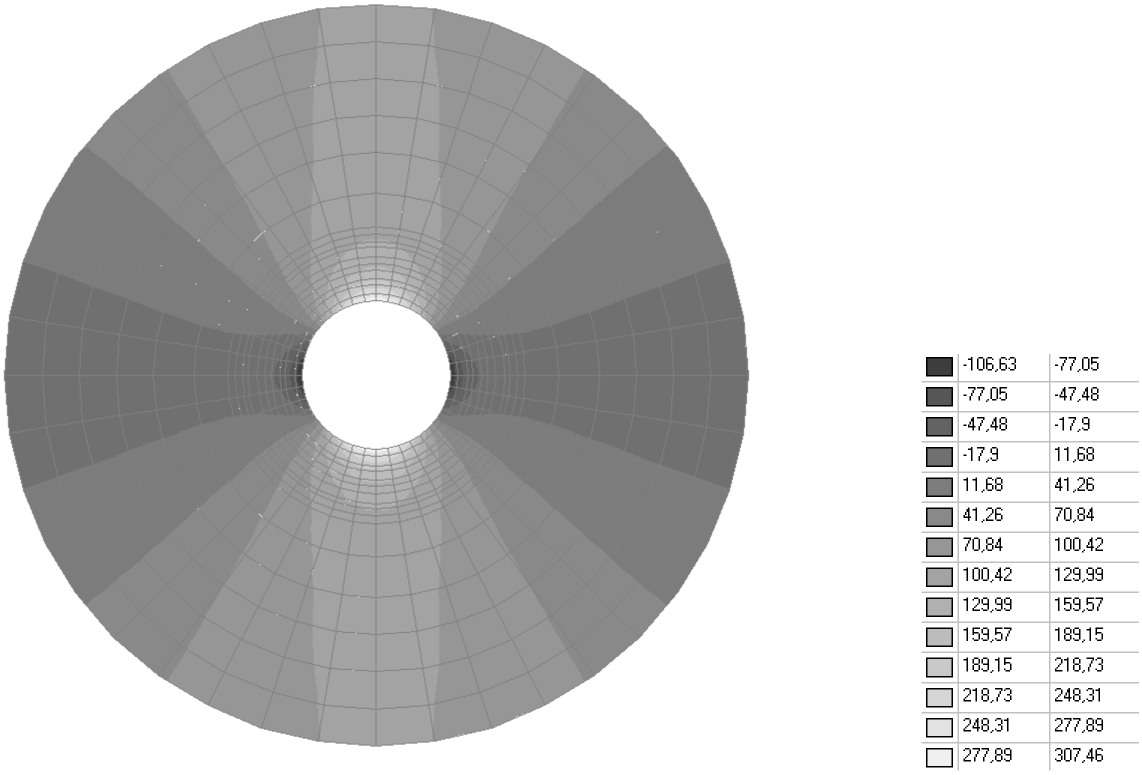
Values of stresses σ_{rr} (kN/m^2) for the design model according to variant 2



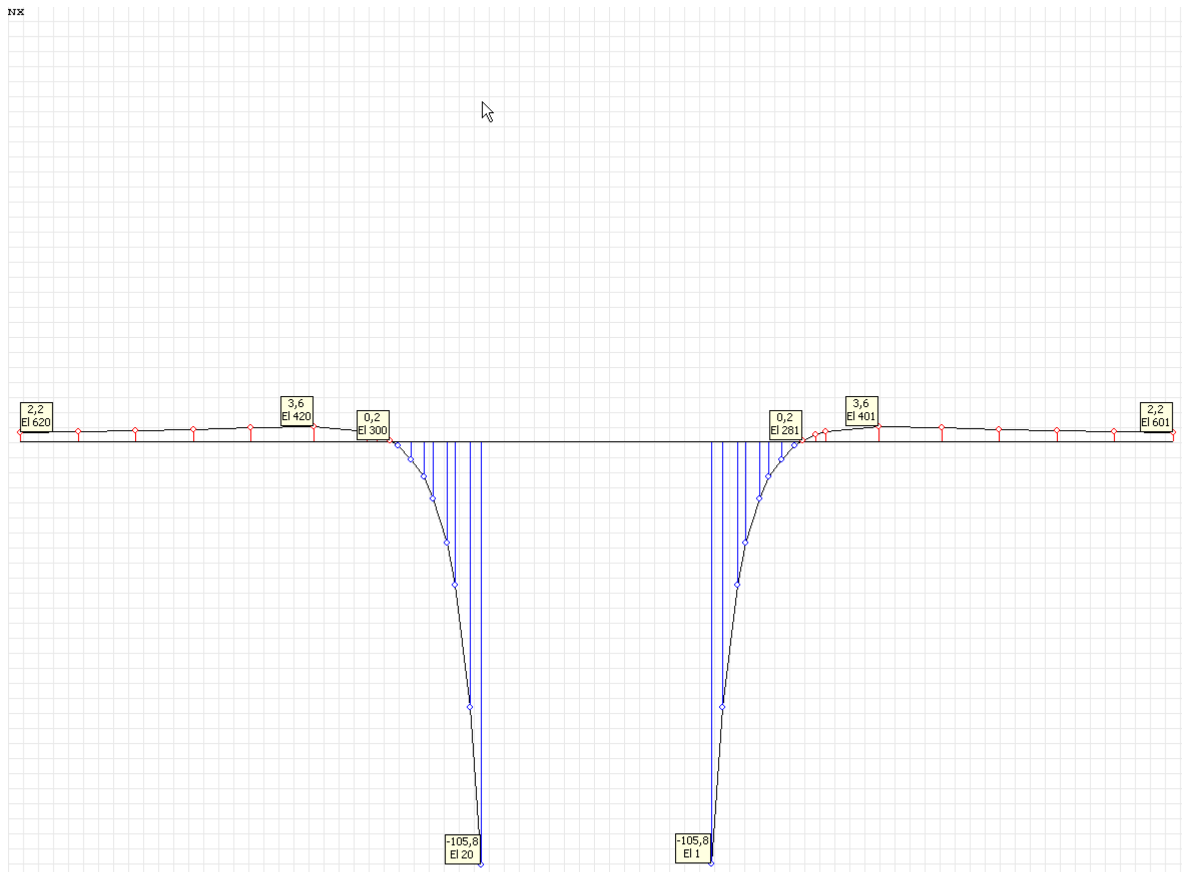
Stress diagram σ_{rr} (kN/m^2) at the angle to the Ox_1 axis $\theta = 0^o$ for the design model according to variant 2



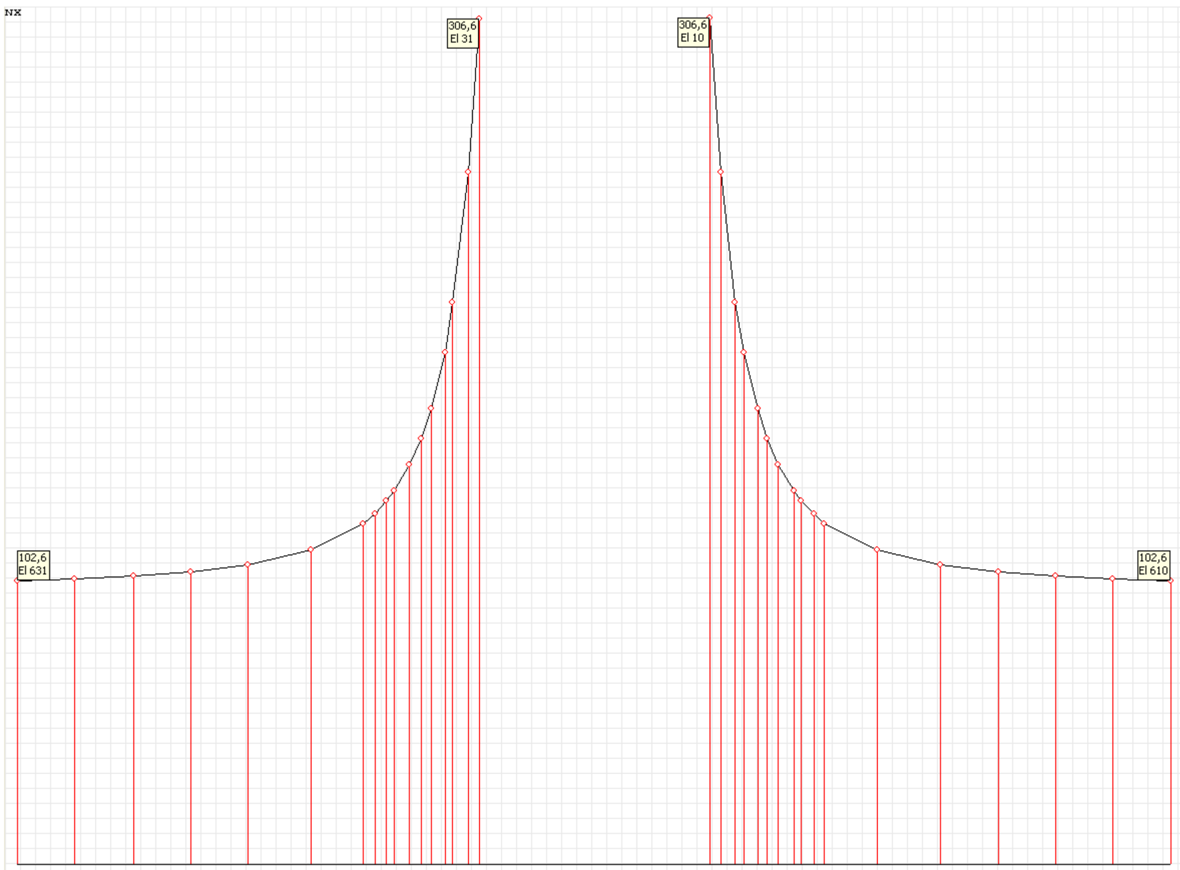
Stress diagram σ_{rr} (kN/m^2) at the angle to the Ox_1 axis $\theta = 90^o$ for the design model according to variant 2



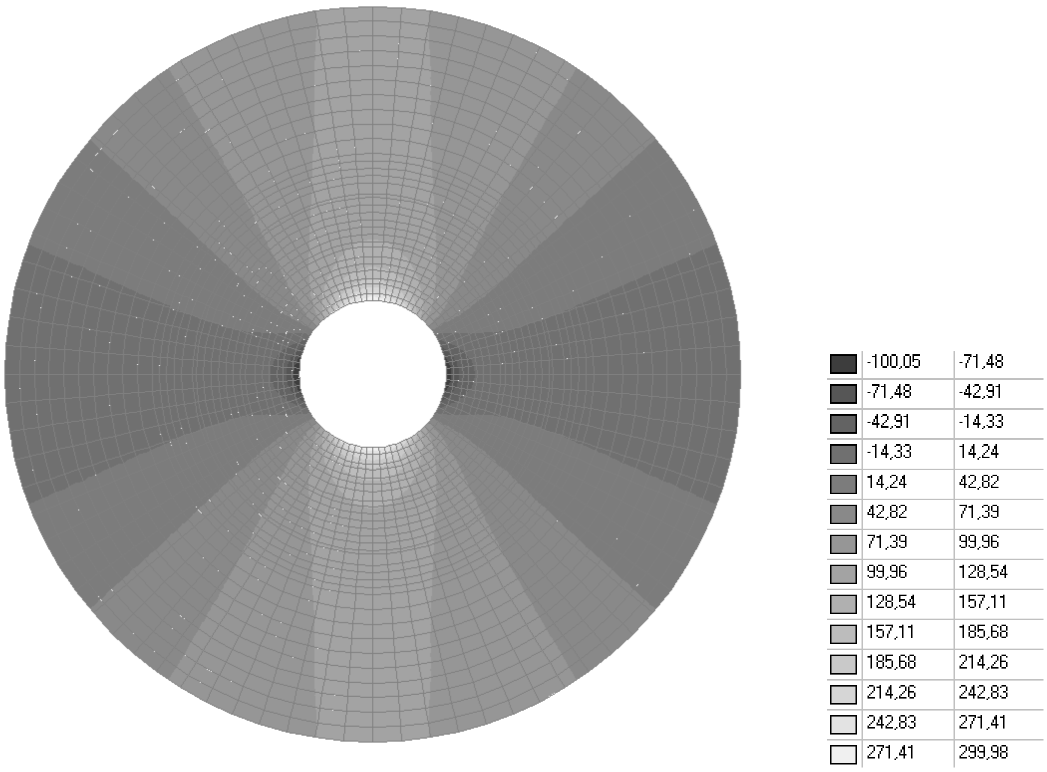
Values of stresses $\sigma_{\theta\theta}$ (kN/m^2) for the design model according to variant 1



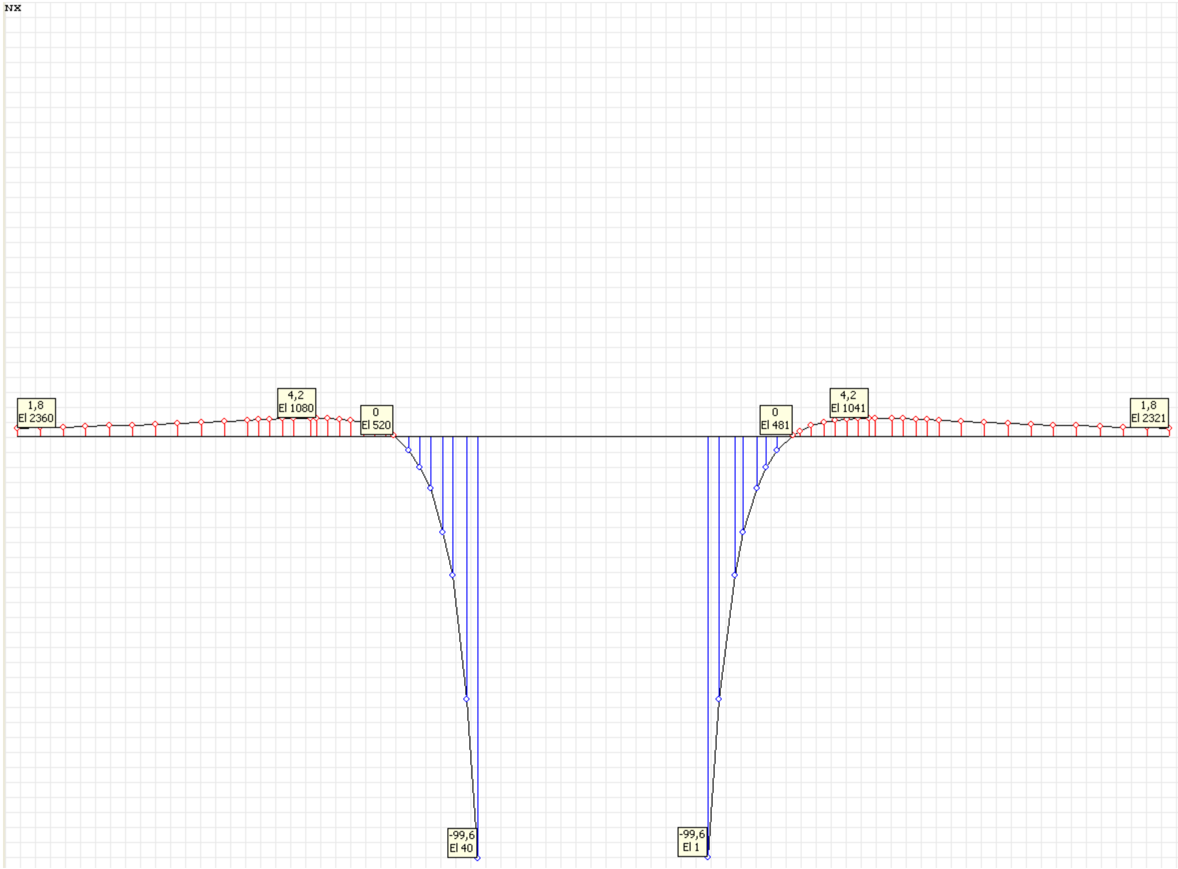
Stress diagram $\sigma_{\theta\theta}$ (kN/m^2) at the angle to the Ox_1 axis $\theta = 0^o$ for the design model according to variant 1



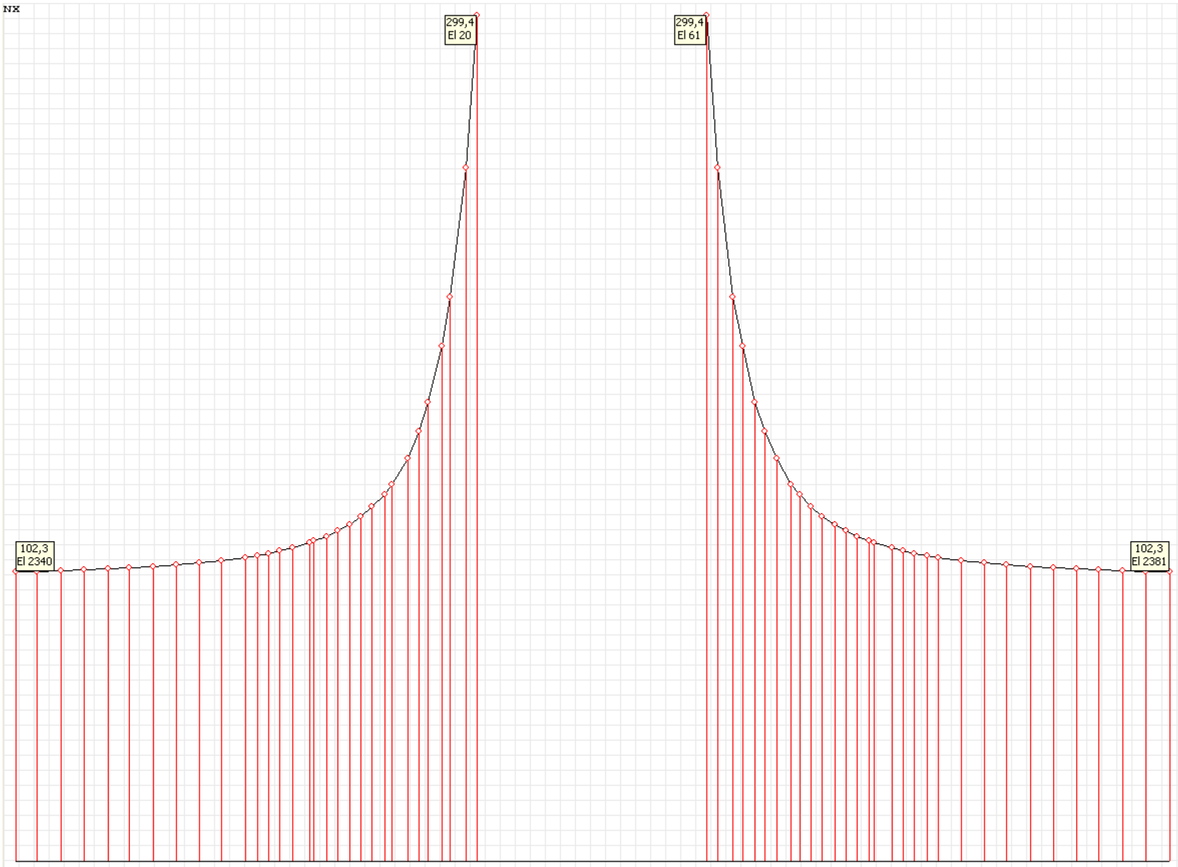
Stress diagram $\sigma_{\theta\theta}$ (kN/m^2) at the angle to the Ox_1 axis $\theta = 90^o$ for the design model according to variant 1



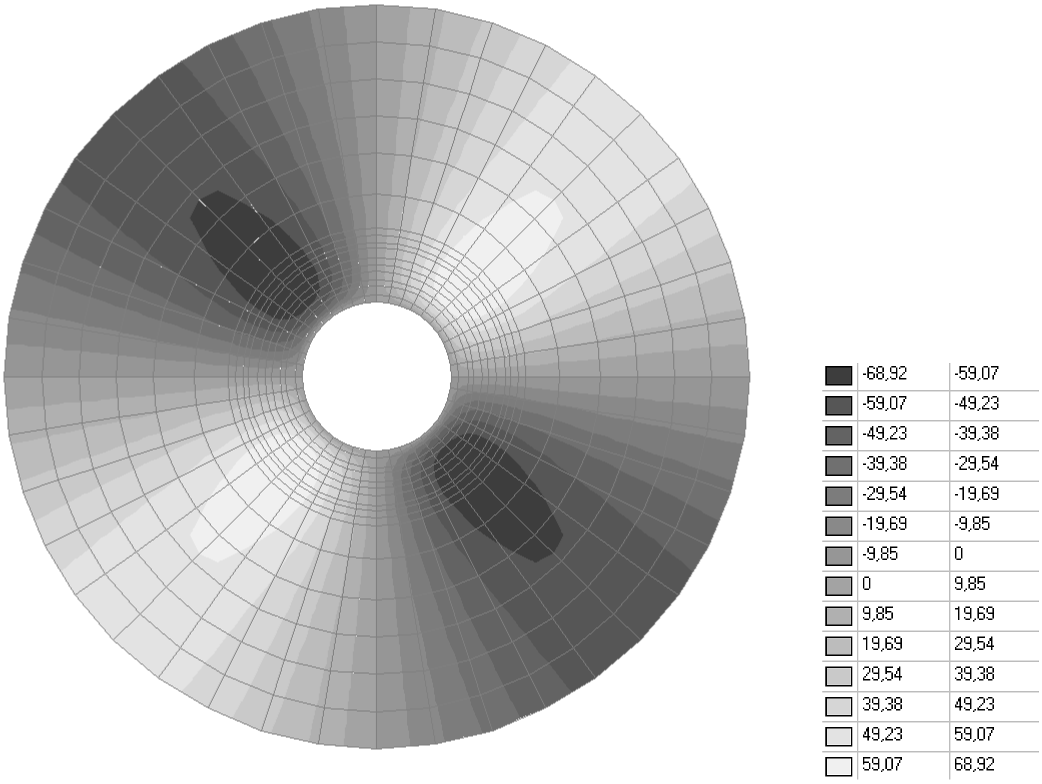
Values of stresses $\sigma_{\theta\theta}$ (kN/m^2) for the design model according to variant 2



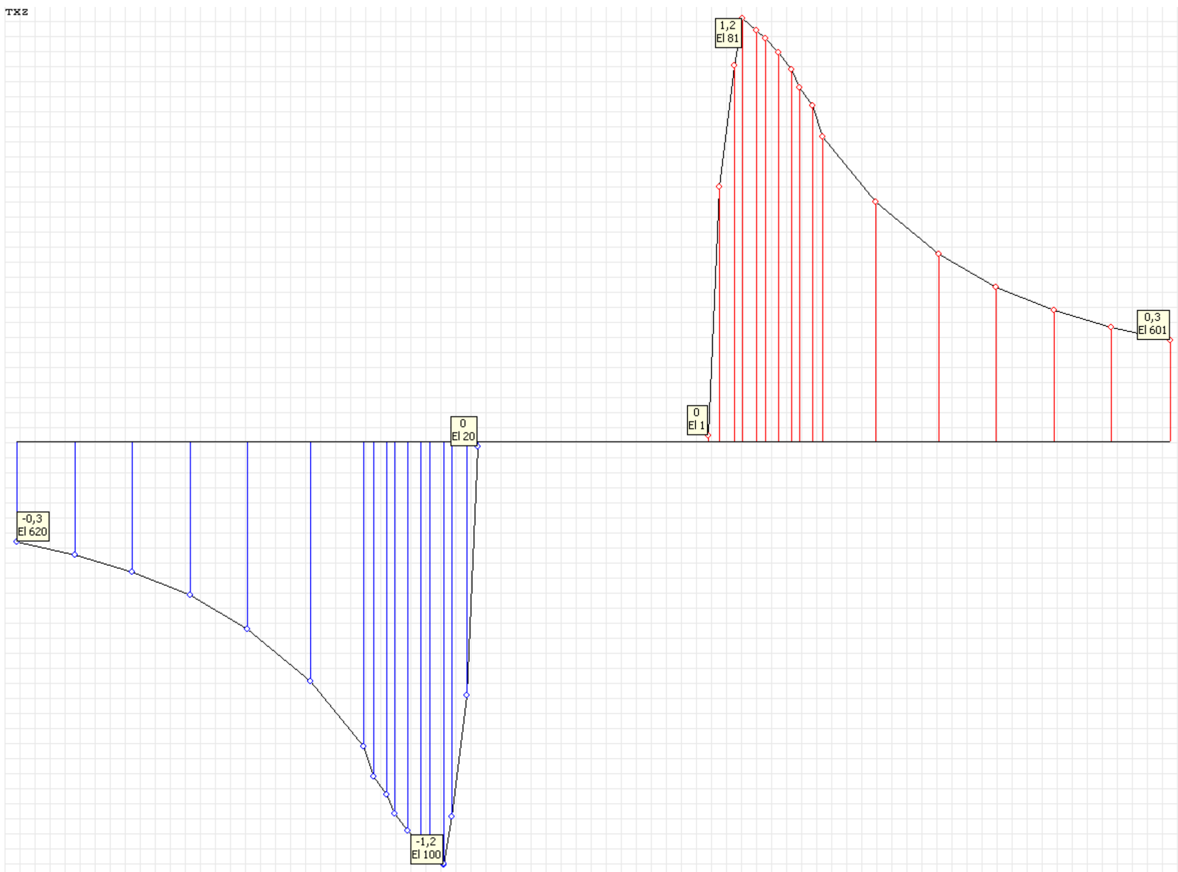
Stress diagram $\sigma_{\theta\theta}$ (kN/m^2) at the angle to the Ox_1 axis $\theta = 0^o$ for the design model according to variant 2



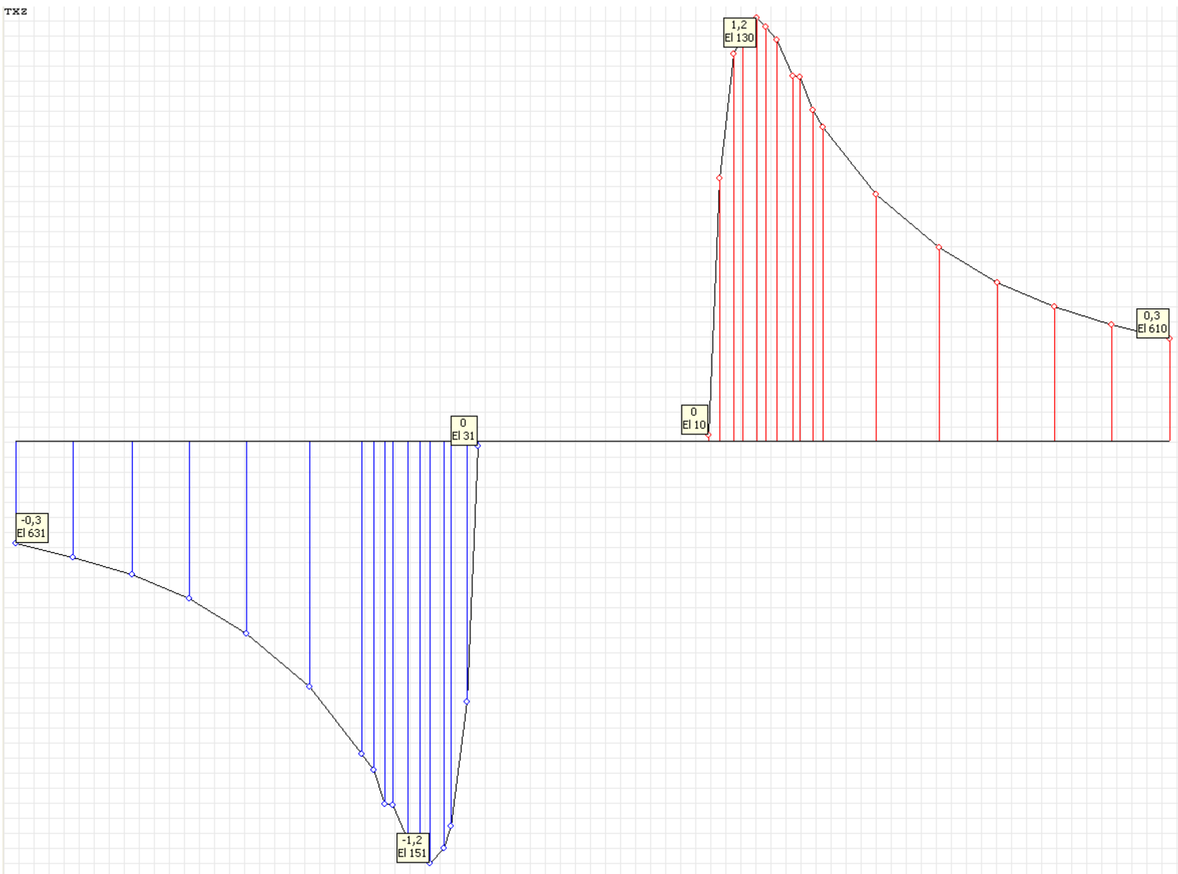
Stress diagram $\sigma_{\theta\theta}$ (kN/m^2) at the angle to the Ox_1 axis $\theta = 90^o$ for the design model according to variant 2



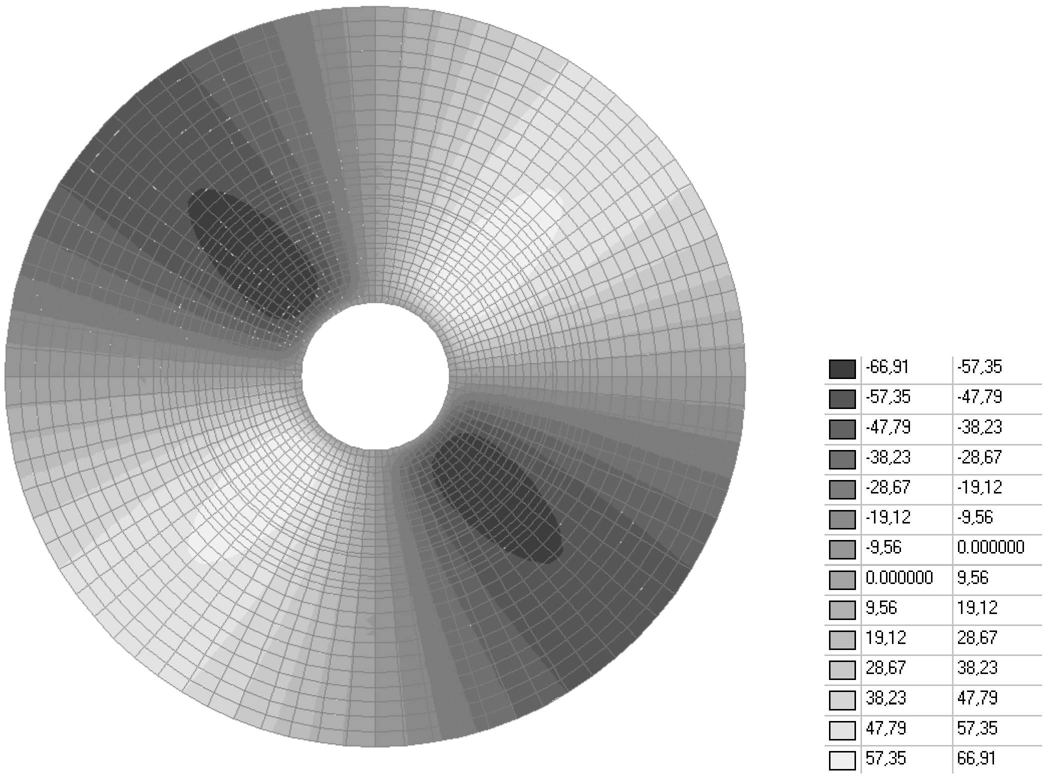
Values of stresses $\sigma_{r\theta}$ (kN/m^2) for the design model according to variant 1



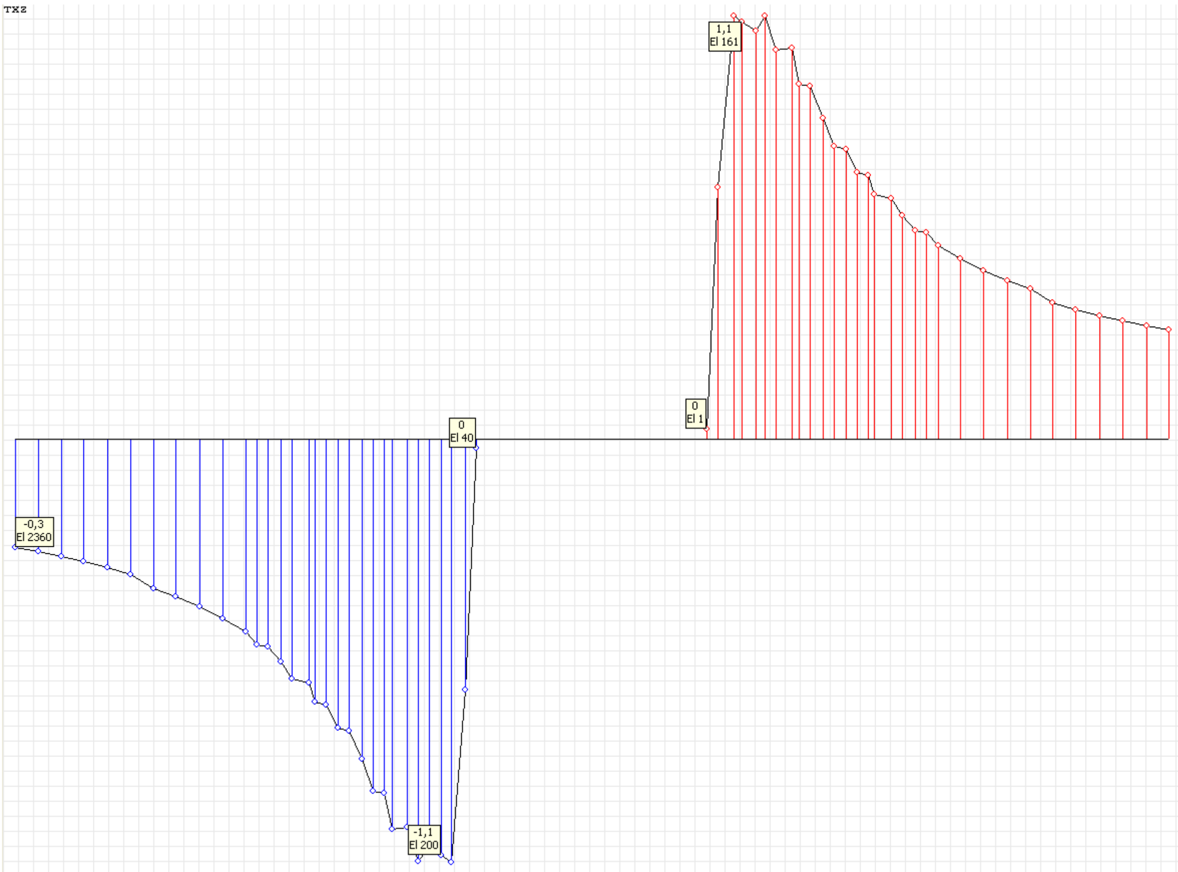
Stress diagram $\sigma_{r\theta}$ (kN/m^2) at the angle to the Ox_1 axis $\theta = 0^o$ for the design model according to variant 1



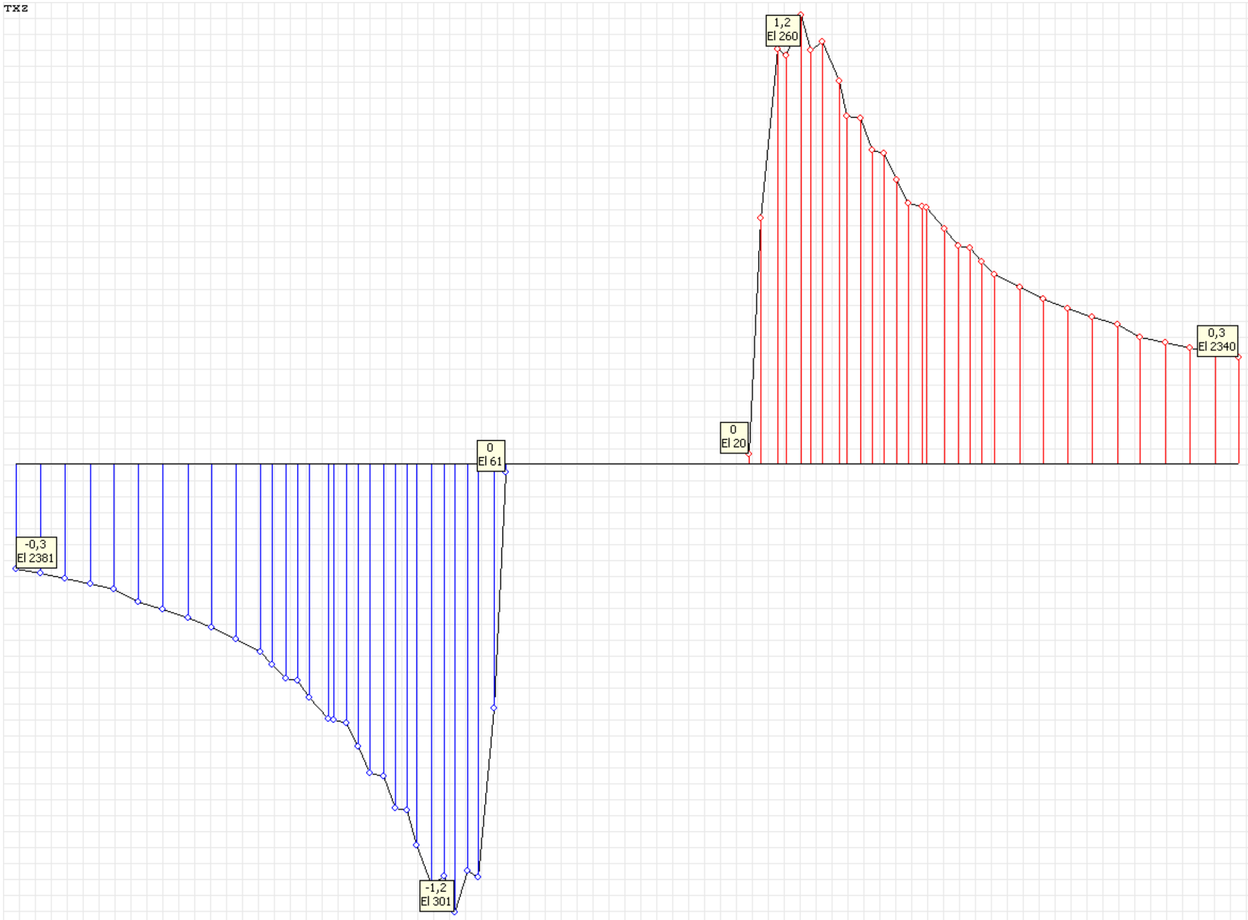
Stress diagram $\sigma_{r\theta}$ (kN/m^2) at the angle to the Ox$_1$ axis $\theta = 90^o$ for the design model according to variant 1



Values of stresses $\sigma_{r\theta}$ (kN/m^2) for the design model according to variant 2



Stress diagram $\sigma_{r\theta}$ (kN/m^2) at the angle to the Ox$_1$ axis $\theta = 0^o$ for the design model according to variant 2



Stress diagram $\sigma_{r\theta}$ (kN/m^2) at the angle to the Ox$_1$ axis $\theta = 90^o$ for the design model according to variant 2

Comparison of solutions:

Stress tensor components in polar coordinates σ_{rr}, $\sigma_{\theta\theta}$, $\sigma_{r\theta}$.

Solution	Stresses σ_{rr} (kN/m²)					Stresses $\sigma_{\theta\theta}$ (kN/m²)			
	θ = 0º			θ = 90º		θ = 0º			θ = 90º
	r = 1.000 m	r = (√6/5)·a = 1.095 m	r = (√3/2)·a = 1.225 m	r = 1.000 m	r = (√2)·a = 1.414 m	r = 1.000 m	r = (√3)·a = 1.732 m	r = (√6)·a = 2.449 m	r = 1.000 m
Theory	0.00	-σ/24 = -4.17	0.00	0.00	3·σ/8 = 37.50	-σ = -100.00	0.00	σ/24 = 4.17	3·σ = 300.00
SCAD, DM var.1	-1.32	-5.65	-1.26	2.77	39.43	-100.63	-1.18	3.56	307.46
Deviations, %	–	–	–	–	5.15	0.63	–	–	2.49
SCAD, DM var.2	-0.76	-4.78	-0.36	1.31	37.94	-100.05	-0.04	4.16	299.85
Deviations, %	–	–	–	–	1.17	0.05	–	–	0.05

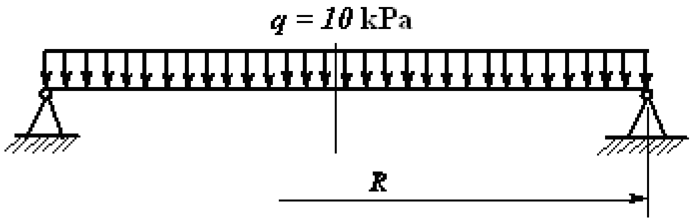
Notes: In the analytical solution the stresses σ_{rr}, $\sigma_{\theta\theta}$, $\sigma_{r\theta}$ in the plate with a small circular hole subjected to unilateral uniform tension are determined according to the following formulas (S.P. Demidov, Theory of Elasticity. — Moscow: High school, 1979, p. 302):

$$\sigma_{rr} = \frac{\sigma}{2} \cdot \left(1 - \frac{a^2}{r^2}\right) + \frac{\sigma}{2} \cdot \left(1 - 4 \cdot \frac{a^2}{r^2} + 3 \cdot \frac{a^4}{r^4}\right) \cdot cos(2 \cdot \theta);$$

$$\sigma_{\theta\theta} = \frac{\sigma}{2} \cdot \left(1 + \frac{a^2}{r^2}\right) - \frac{\sigma}{2} \cdot \left(1 + 3 \cdot \frac{a^4}{r^4}\right) \cdot cos(2 \cdot \theta);$$

$$\sigma_{r\theta} = -\frac{\sigma}{2} \cdot \left(1 + 2 \cdot \frac{a^2}{r^2} - 3 \cdot \frac{a^4}{r^4}\right) \cdot sin(2 \cdot \theta).$$

Stress-Strain State of a Simply Supported Circular Plate Subjected to a Uniformly Distributed Transverse Load



Objective: Determination of the stress-strain state of a simply supported circular plate of constant thickness subjected to a uniformly distributed transverse load.

Initial data file: **4.14.SPR**

Problem formulation: The simply supported circular plate of constant thickness is subjected to the uniformly distributed transverse load. Determine the deflection *w*, the radial slope *θ*, the radial M_r and tangential M_θ bending moments along the axis and along the external contour of the plate.

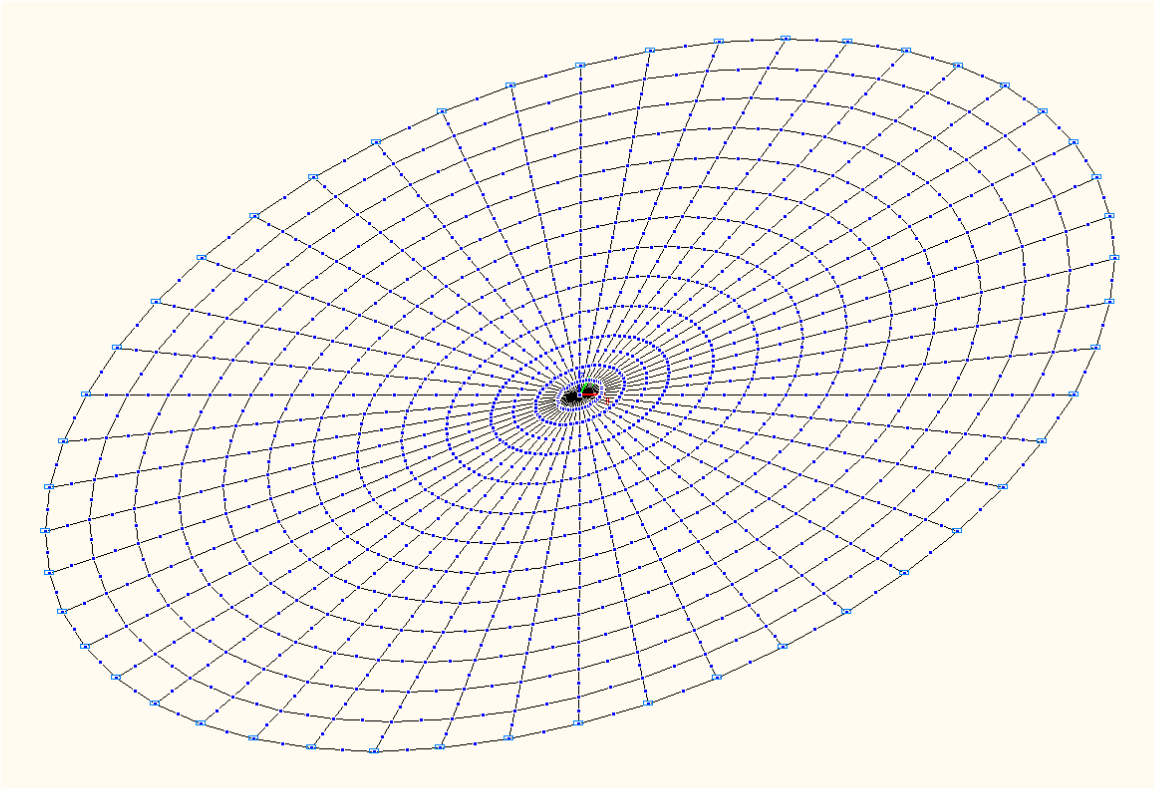
References: S.P. Timoshenko, Theory of Plates and Shells. — Moscow: OGIZ. Gostekhizdat, 1948.

Initial data:

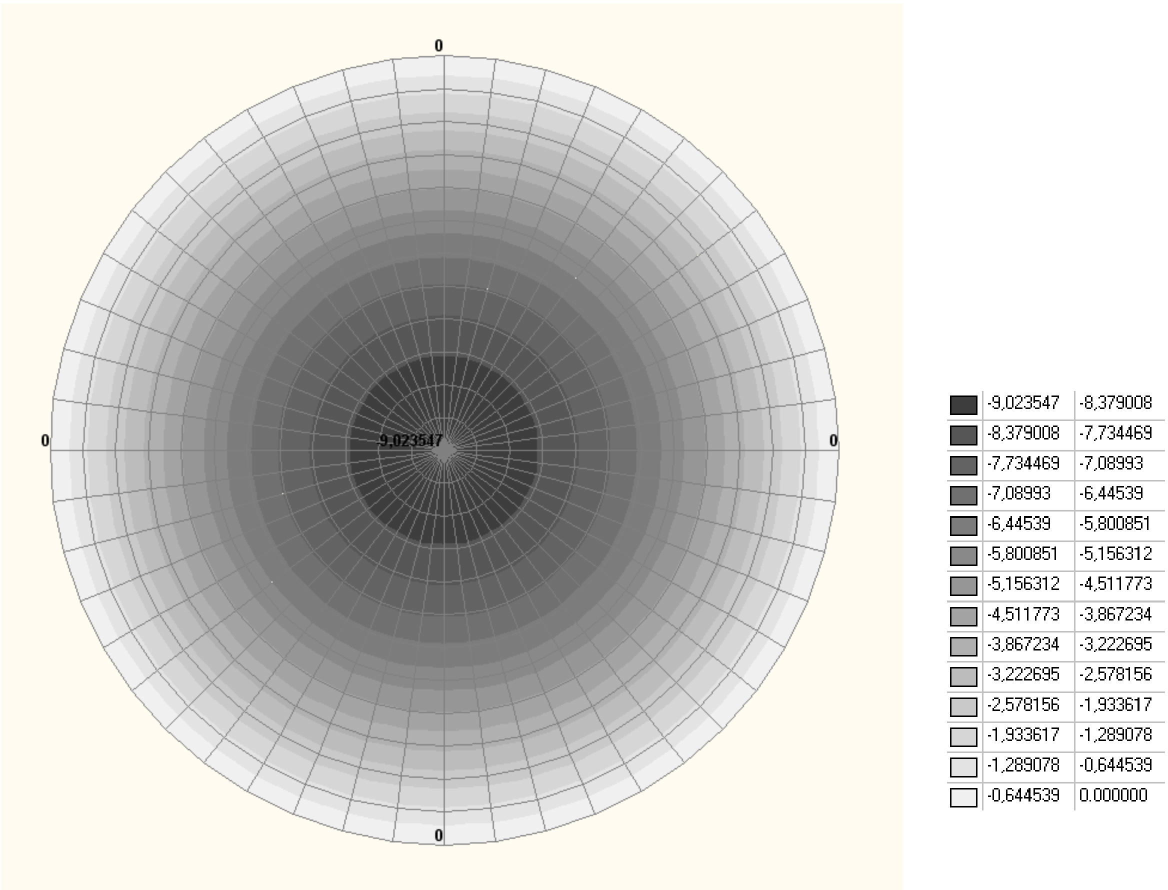
E = $2.0 \cdot 10^8$ kPa	- elastic modulus;
μ = 0.3	- Poisson's ratio;
R = 1.2 m	- outer radius of the plate;
h = $2.0 \cdot 10^{-2}$ m	- thickness of the plate;
q = 10 kPa	- uniformly distributed transverse load.

Finite element model: Design model – general type system, plate elements – 528 eight-node elements of type 50 and 48 six-node elements of type 45. The direction of the output of internal forces is radial tangential. Boundary conditions are provided by imposing constraints in the direction of the degree of freedom Z along the external contour of the plate. Number of nodes in the design model – 1729.

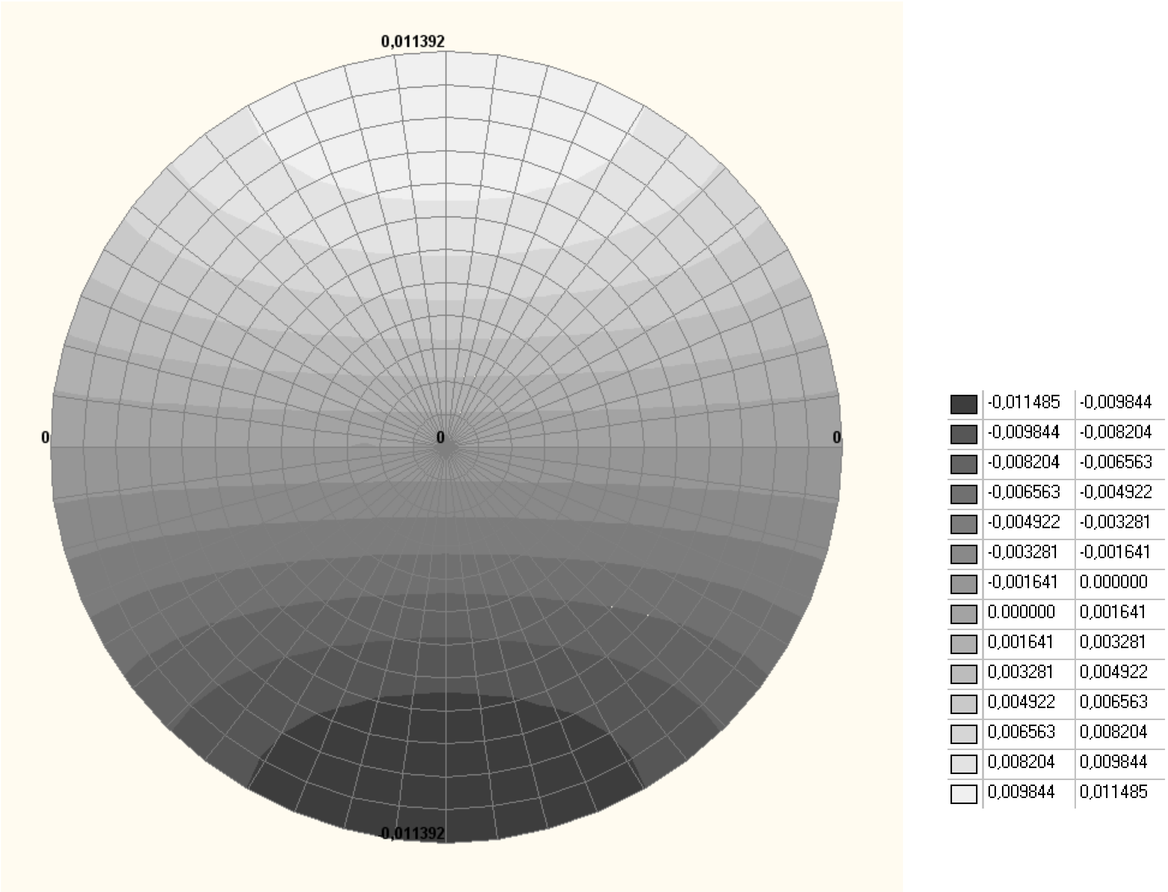
Results in SCAD



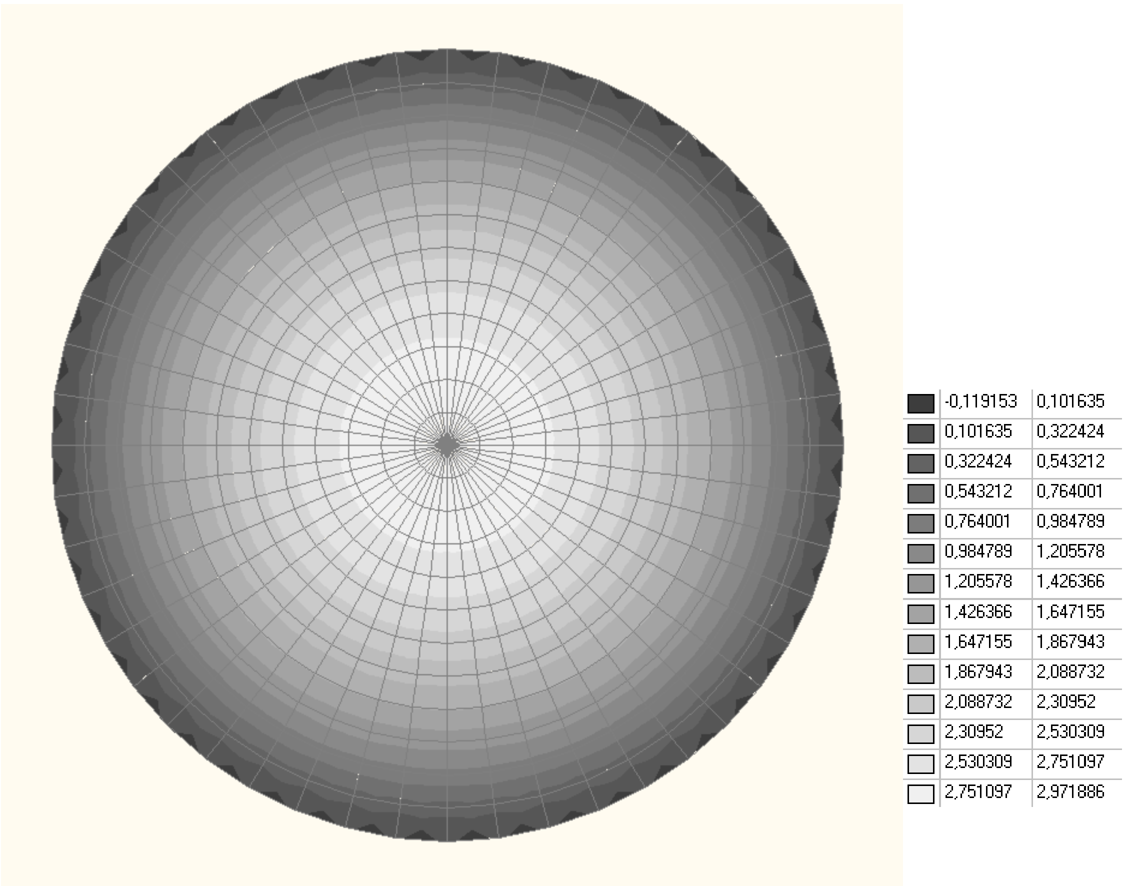
Design model



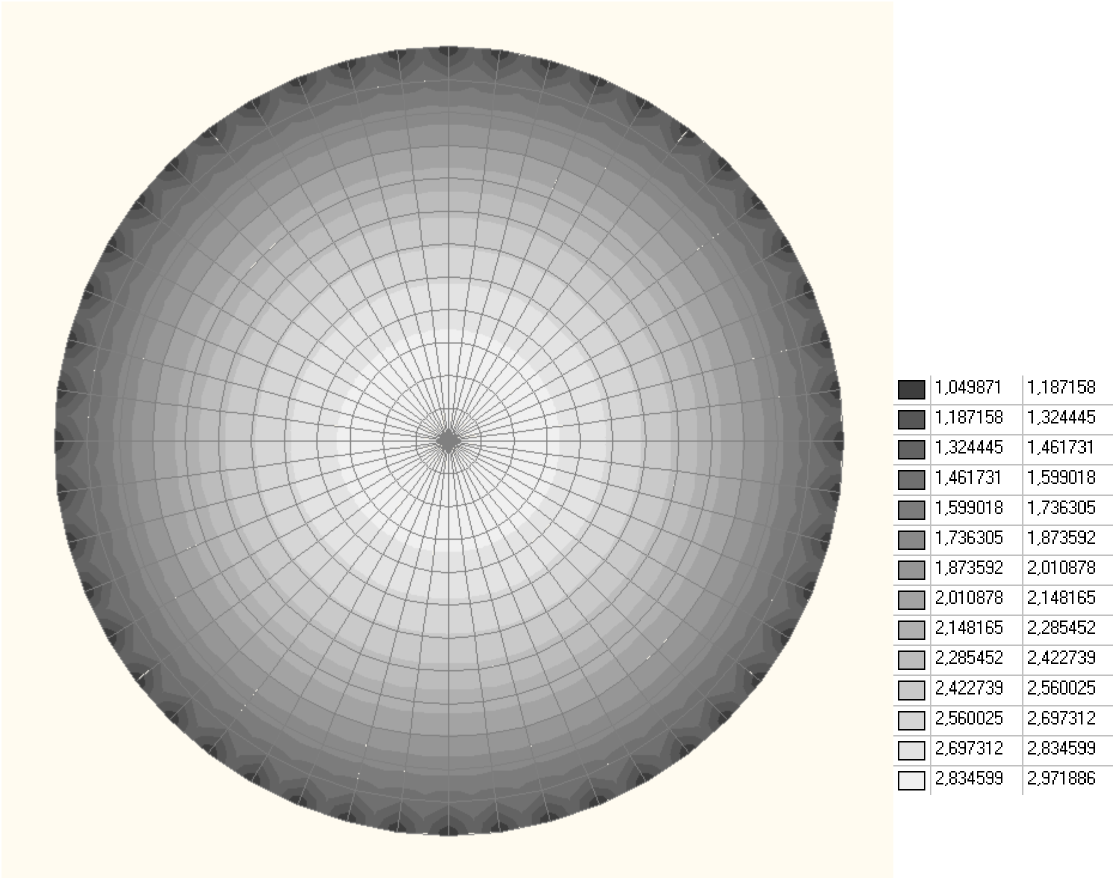
Values of deflections w (mm)



Values of radial slopes θ (rad)



Values of radial bending moments M_r (kN·m/m)



Values of tangential bending moments M_θ (kN·m/m)

Comparison of solutions:

Parameter	Along the axis of the plate			Along the external contour of the plate		
	Theory	**SCAD**	**Deviations, %**	**Theory**	**SCAD**	**Deviations, %**
w, mm	-9.015	-9.024	0.10	0.000	0.000	—
θ, rad	0.000000	0.000000	—	0.011340	0.011392	0.46
M_r, kN·m/m	2.970	2.972	0.07	0.000	0.063	—
M_θ, kN·m/m	2.970	2.972	0.07	1.260	1.226	2.70

Notes: . In the analytical solution the deflection *w*, the radial slope θ, the radial M_r and tangential M_θ bending moments along the axis of the plate can be determined according to the following formulas (S.P. Timoshenko, Theory of Plates and Shells. — Moscow: OGIZ. Gostekhizdat, 1948, p. 66):

$$w = -\frac{q \cdot R^4 \cdot (5+\mu)}{64 \cdot D \cdot (1+\mu)}$$, where:

$$D = \frac{E \cdot h^3}{12 \cdot (1-\mu^2)};$$

$$\theta = 0;$$

$$M_r = \frac{q \cdot R^2 \cdot (3+\mu)}{16};$$

$$M_\theta = \frac{q \cdot R^2 \cdot (3+\mu)}{16}.$$

In the analytical solution the deflection *w*, the radial slope θ, the radial M_r and tangential M_θ bending moments along the external contour of the plate can be determined according to the following formulas (S.P. Timoshenko, Theory of Plates and Shells. — Moscow: OGIZ. Gostekhizdat, 1948, p. 66):

$$w = 0;$$

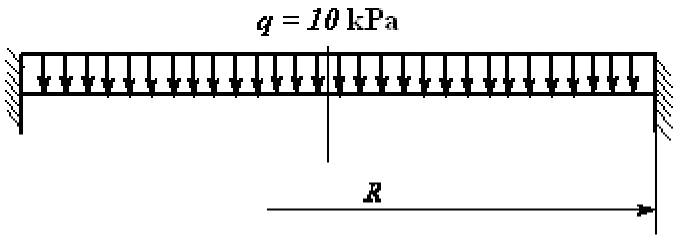
$$\theta = \frac{q \cdot R^3}{8 \cdot D \cdot (1+\mu)}, \text{ where:}$$

$$D = \frac{E \cdot h^3}{12 \cdot (1-\mu^2)};$$

$$M_r = 0;$$

$$M_\theta = \frac{q \cdot R^2 \cdot (1-\mu)}{8}.$$

Stress-Strain State of a Clamped Circular Plate Subjected to a Uniformly Distributed Transverse Load



Objective: Determination of the stress-strain state of a clamped circular plate of constant thickness subjected to a uniformly distributed transverse load.

Initial data file: **4.15.SPR**

Problem formulation: The clamped circular plate of constant thickness is subjected to the uniformly distributed transverse load. Determine the deflection w, the radial slope θ, the radial M_r and tangential M_θ bending moments along the axis and along the external contour of the plate.

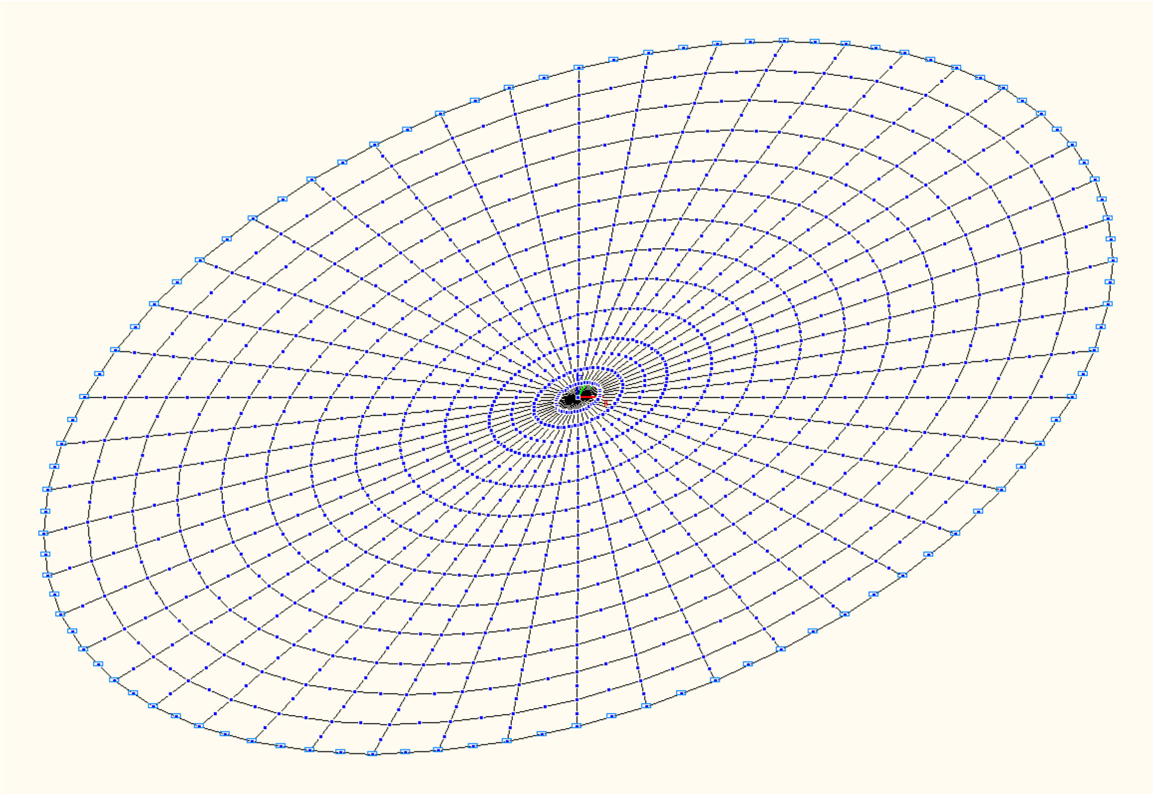
References: S.P. Timoshenko, Theory of Plates and Shells. — Moscow: OGIZ. Gostekhizdat, 1948.

Initial data:

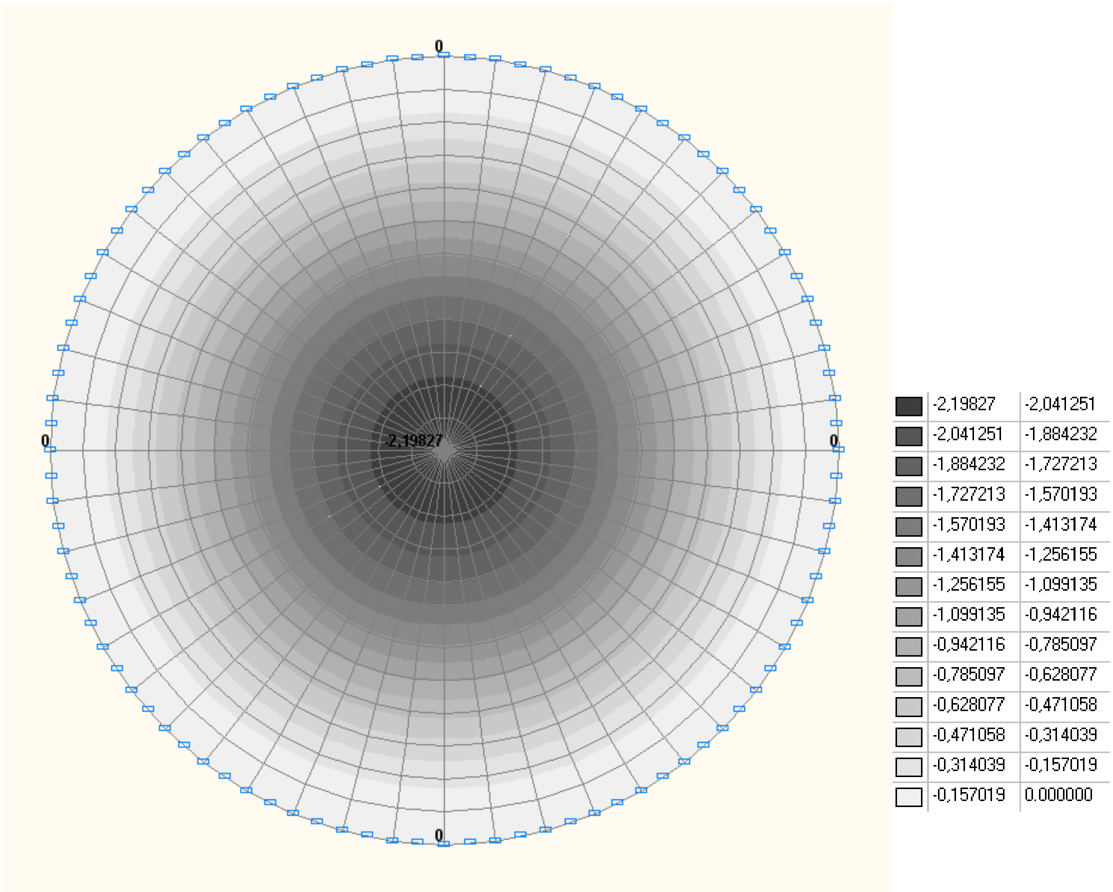
$E = 2.0 \cdot 10^8$ kPa	- elastic modulus;
$\mu = 0.3$	- Poisson's ratio;
R = 1.2 m	- outer radius of the plate;
$h = 2.0 \cdot 10^{-2}$ m	- thickness of the plate;
q = 10 kPa	- uniformly distributed transverse load.

Finite element model: Design model – general type system, plate elements – 528 eight-node elements of type 50 and 48 six-node elements of type 45. The direction of the output of internal forces is radial tangential. Boundary conditions are provided by imposing constraints in the directions of the degrees of freedom Z, UX, UY along the external contour of the plate. Number of nodes in the design model – 1729.

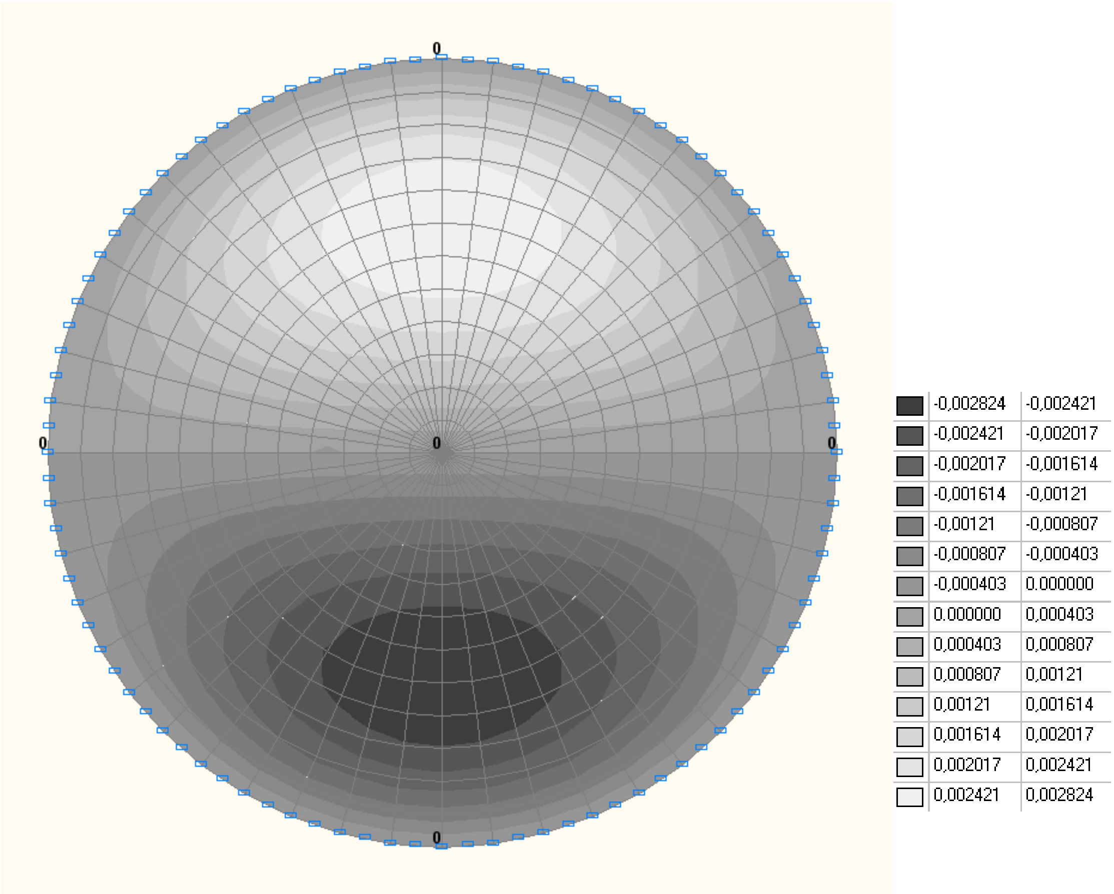
Results in SCAD



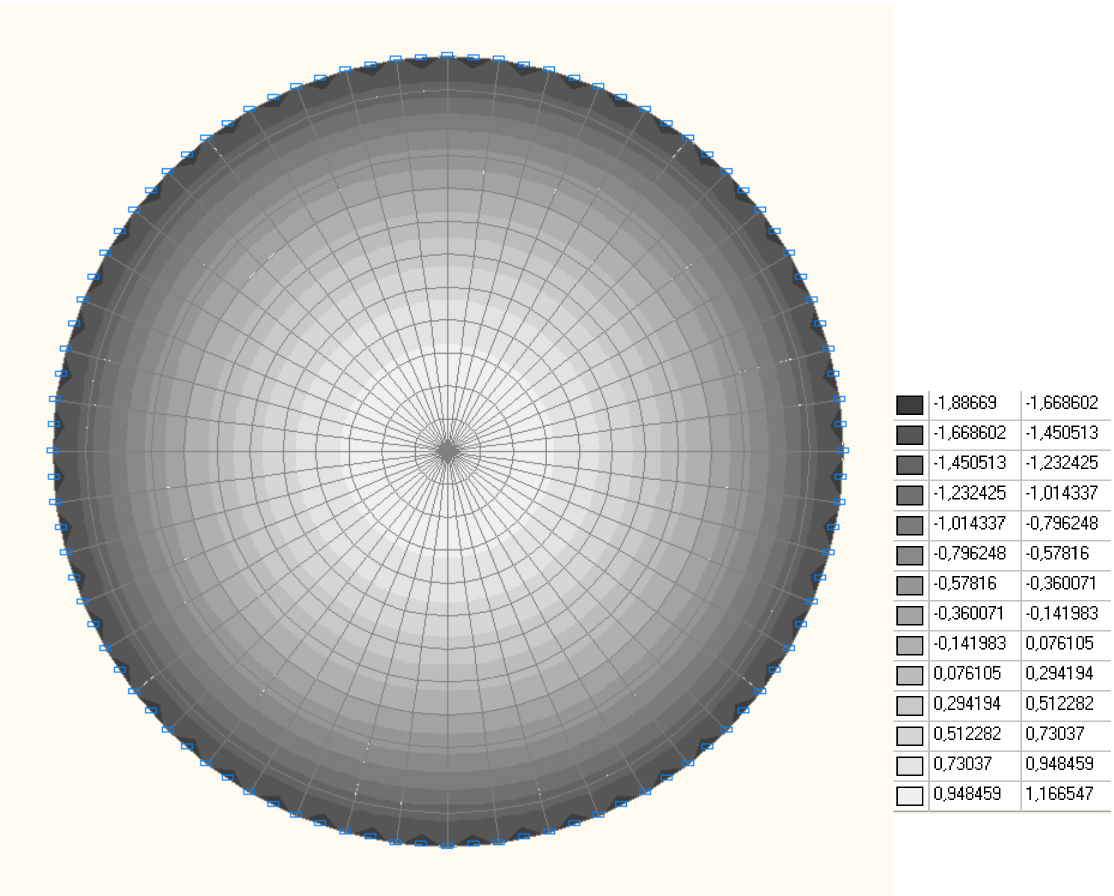
Design model



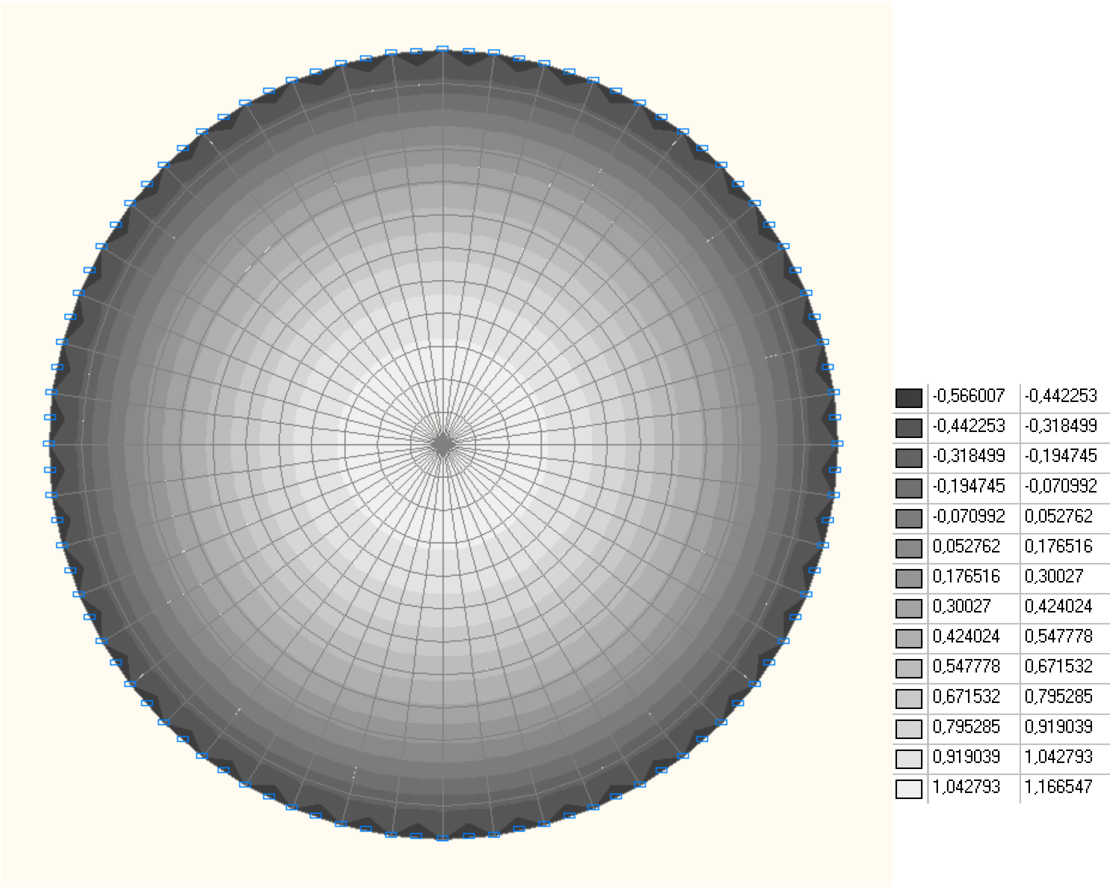
Values of deflections w (mm)



Values of radial slopes θ (rad)



Values of radial bending moments M_r (kN·m/m)



Values of tangential bending moments M_θ (kN·m/m)

Comparison of solutions:

Parameter	Along the axis of the plate			Along the external contour of the plate		
	Theory	**SCAD**	**Deviations, %**	**Theory**	**SCAD**	**Deviations, %**
w, mm	-2.211	-2.198	0.59	0.000	0.000	—
θ, rad	0.000000	0.000000	—	0.000000	0.000000	—
M_r, kN·m/m	1.170	1.167	0.26	-1.800	-1.736	3.56
M_θ, kN·m/m	1.170	1.167	0.26	-0.540	-0.505	6.48

Notes: In the analytical solution the deflection *w*, the radial slope θ, the radial M_r and tangential M_θ bending moments along the axis of the plate can be determined according to the following formulas (S.P. Timoshenko, Theory of Plates and Shells. — Moscow: OGIZ. Gostekhizdat, 1948, p. 65):

$$w = -\frac{q \cdot R^4}{64 \cdot D}, \text{ where:}$$

$$D = \frac{E \cdot h^3}{12 \cdot \left(1 - \mu^2\right)};$$

$$\theta = 0;$$

$$M_r = \frac{q \cdot R^2 \cdot (1 + \mu)}{16};$$

$$M_\theta = \frac{q \cdot R^2 \cdot (1 + \mu)}{16} .$$

In the analytical solution the deflection *w*, the radial slope *θ*, the radial M_r and tangential M_θ bending moments along the external contour of the plate can be determined according to the following formulas (S.P. Timoshenko, Theory of Plates and Shells. — Moscow: OGIZ. Gostekhizdat, 1948, p. 66):

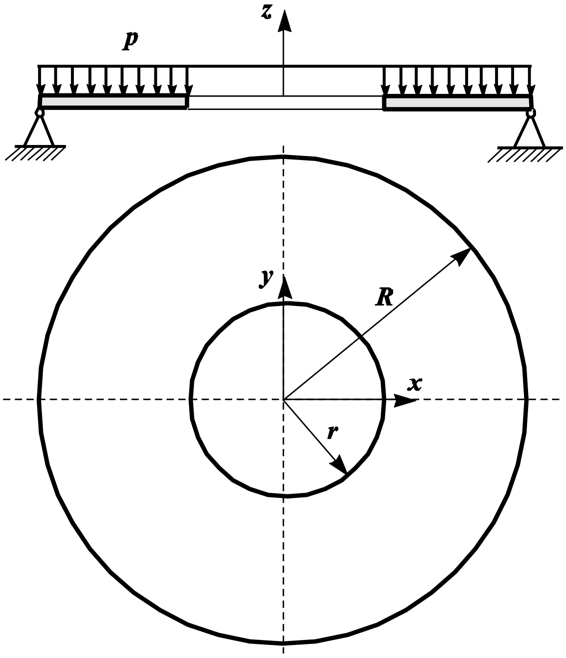
$$w = 0 ;$$

$$\theta = 0 ;$$

$$M_r = -\frac{q \cdot R^2}{8} ;$$

$$M_\theta = -\frac{q \cdot R^2 \cdot \mu}{8} .$$

Stress-Strain State of a Simply Supported Annular Plate Subjected to a Uniformly Distributed Transverse Load



Objective: Determination of the stress-strain state of a simply supported annular plate of constant thickness subjected to a uniformly distributed transverse load.

***Initial data file*: 4.16.SPR**

Problem formulation: The simply supported annular plate of constant thickness is subjected to the uniformly distributed transverse load. Determine the deflection *w*, the radial M_r and tangential M_θ bending moments along the internal and external contour of the plate.

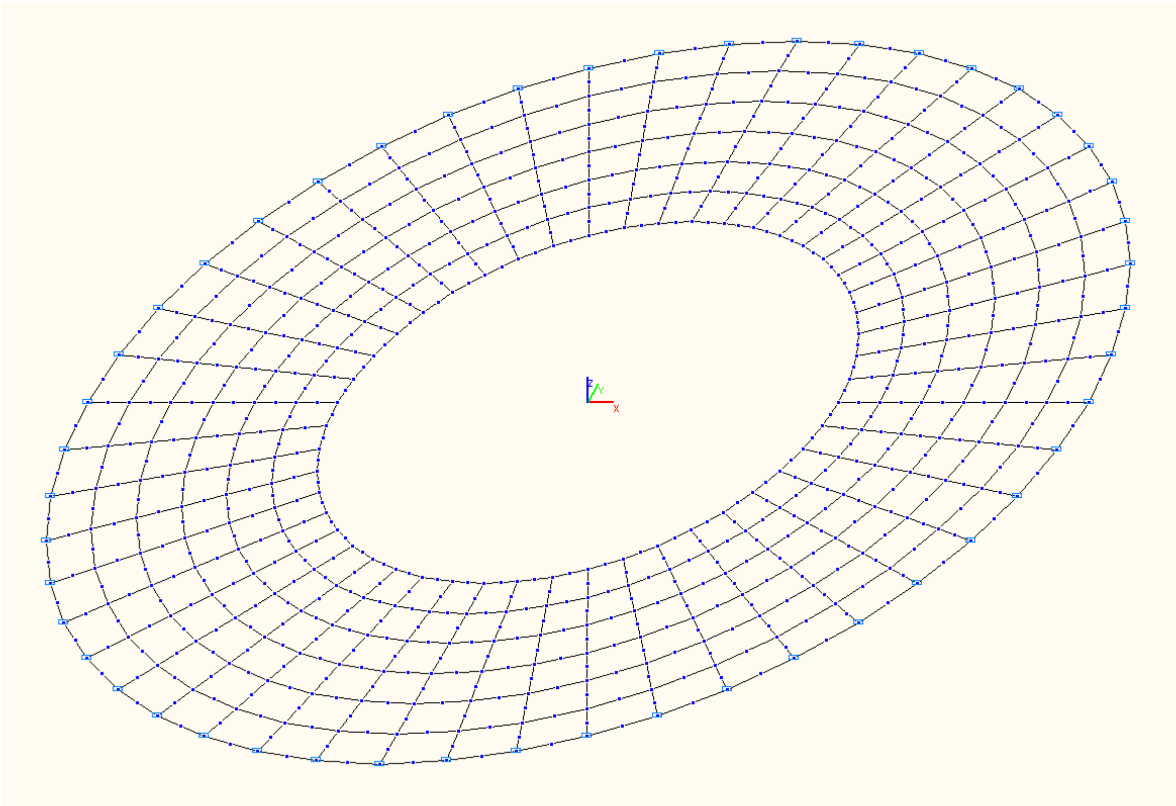
References: S.P. Timoshenko, Theory of Plates and Shells. — Moscow: OGIZ. Gostekhizdat, 1948.

Initial data:

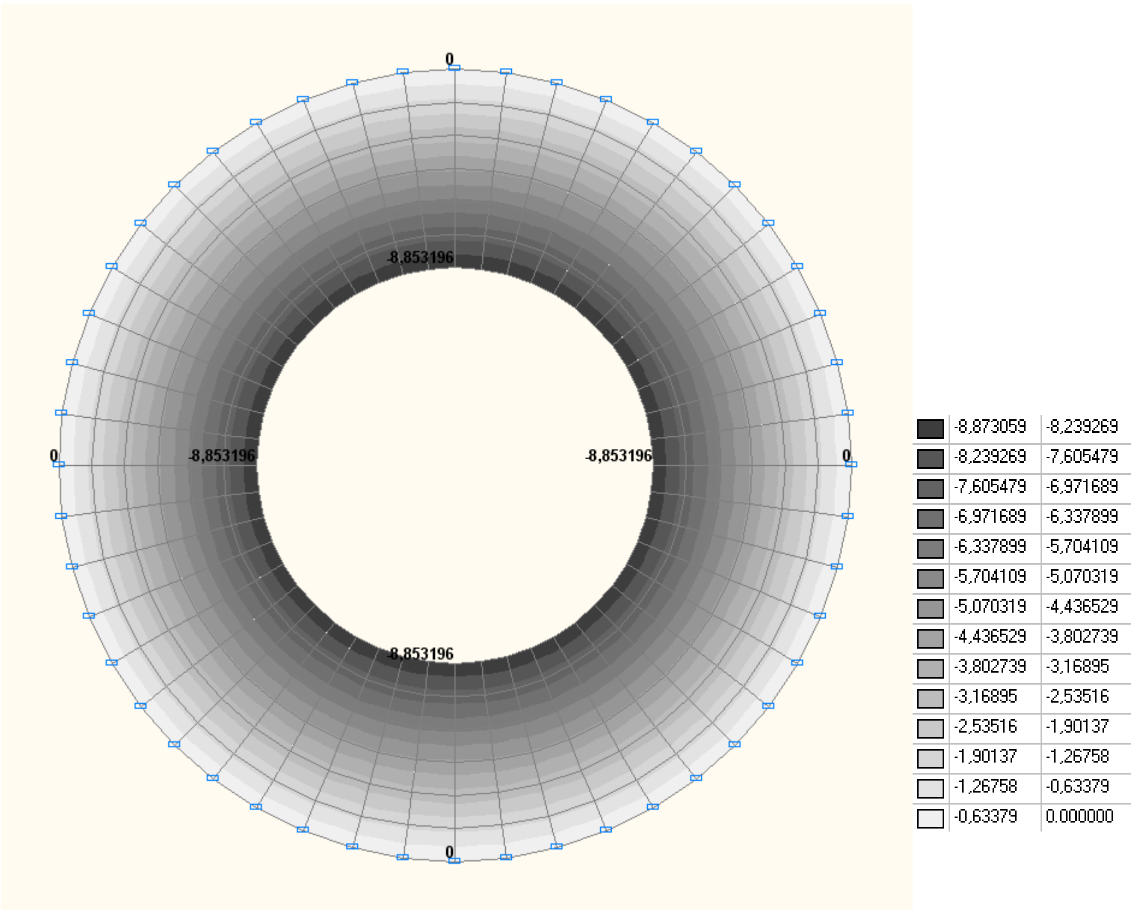
$E = 2.0 \cdot 10^8$ kPa	- elastic modulus;
$\mu = 0.3$	- Poisson’s ratio;
R = 1.2 m	- outer radius of the plate;
r = 0.6 m	- inner radius of the plate;
$h = 2.0 \cdot 10^{-2}$ m	- thickness of the plate;
p = 10 kPa	- uniformly distributed transverse load.

Finite element model: Design model – general type system, plate elements – 288 eight-node elements of type 50. The direction of the output of internal forces is radial tangential. Boundary conditions are provided by imposing constraints in the direction of the degree of freedom Z along the external contour of the plate. Number of nodes in the design model – 960.

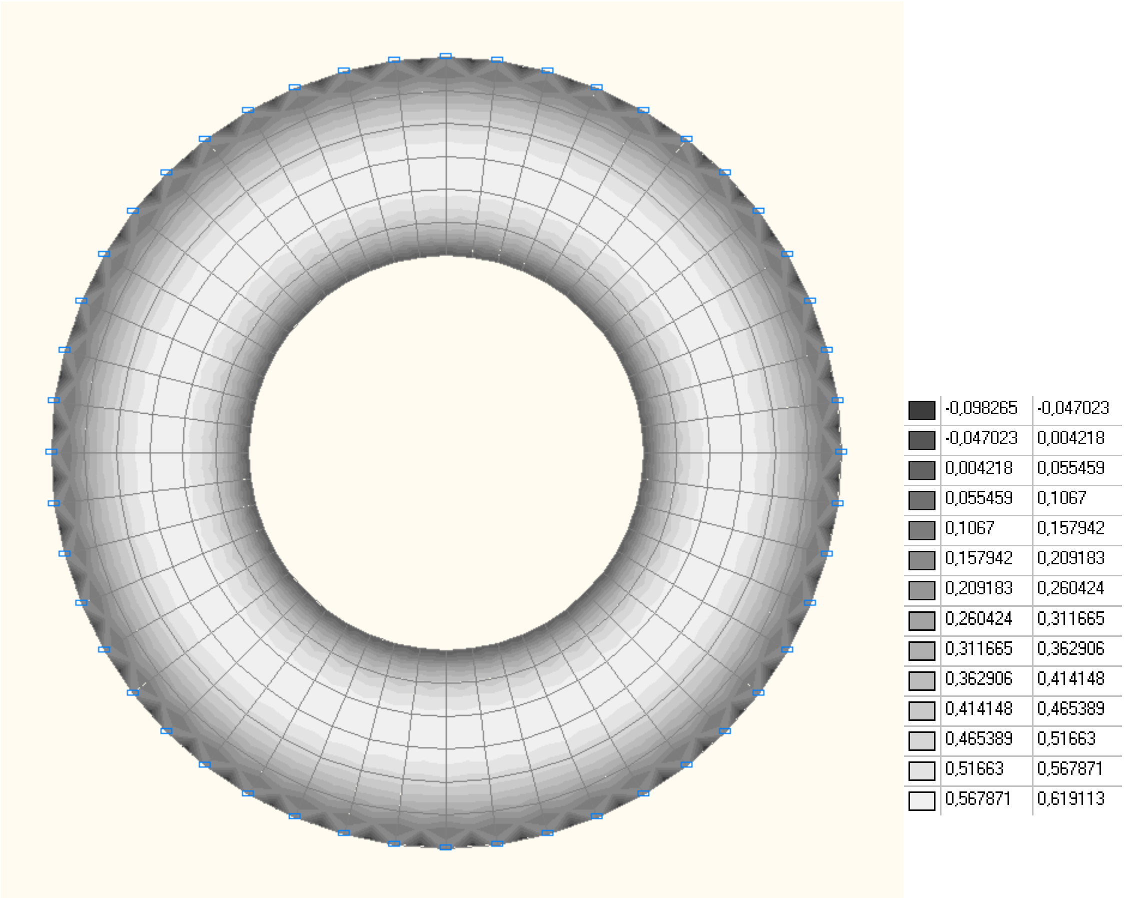
Results in SCAD



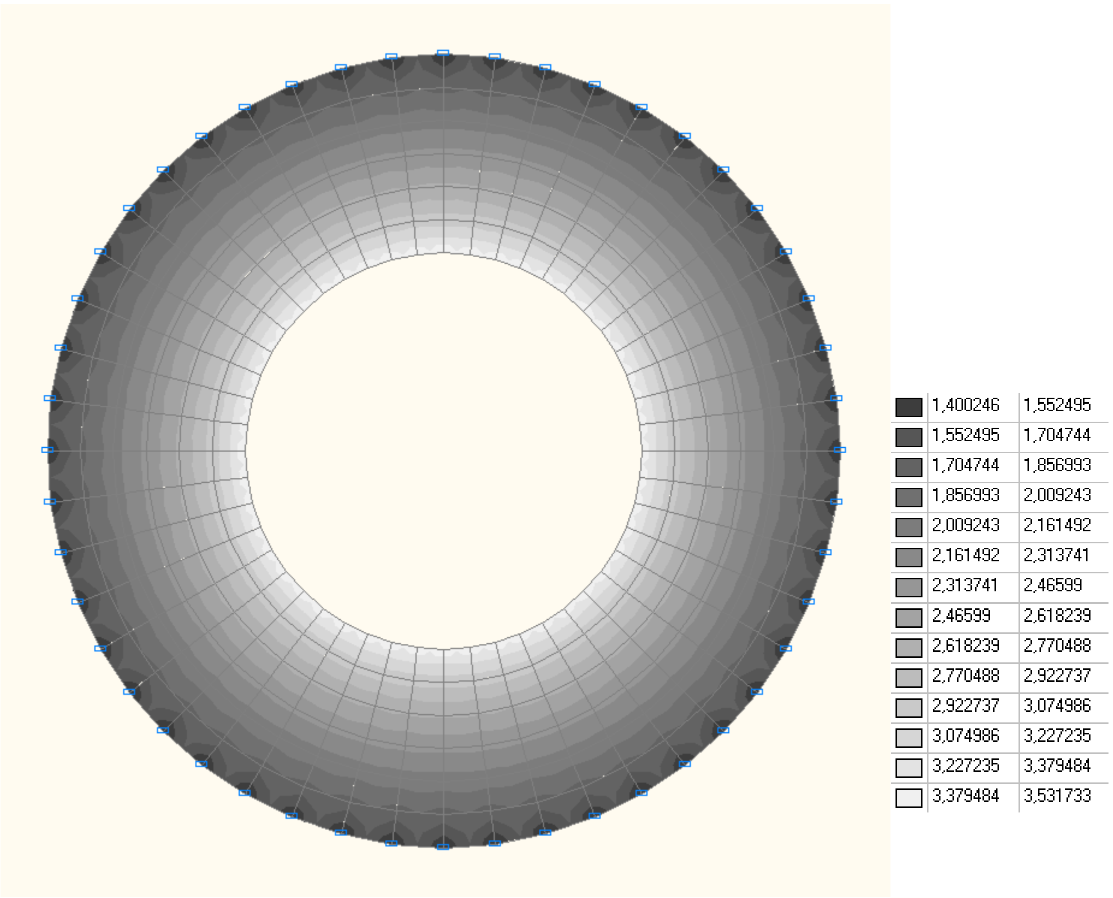
Design model



Values of deflections w (mm)



Values of radial bending moments M_r (kN·m/m)



Values of tangential bending moments M_θ (kN·m/m)

Comparison of solutions:

Parameter	Along the internal contour of the plate			Along the external contour of the plate		
	Theory	**SCAD**	**Deviations, %**	**Theory**	**SCAD**	**Deviations, %**
w, mm	-8.933	-8.863	0.78	0.000	0.000	–
M_r, kN·m/m	0.000	0.001	–	0.000	0.052	–
M_θ, kN·m/m	3.462	3.474	0.35	1.574	1.547	1.72

Notes: In the analytical solution the deflection w, the radial M_r and tangential M_θ bending moments along the internal contour of the plate can be determined according to the following formulas (S.P. Timoshenko, Theory of Plates and Shells. — Moscow: OGIZ. Gostekhizdat, 1948. p. 71):

$$w = -\frac{q}{64 \cdot D} \cdot \left[\frac{R^2 - r^2}{1+\mu} \cdot \left(R^2 \cdot (5+\mu) - r^2 \cdot (7 + 2 \cdot \mu) \right) - \frac{4 \cdot R^2 \cdot r^2}{1-\mu} \cdot ln\frac{r}{R} \cdot \left(3 + \mu + \frac{4 \cdot r^2}{R^2 - r^2} \cdot (1+\mu) \cdot ln\frac{r}{R} \right) \right],$$ where:

$$D = \frac{E \cdot h^3}{12 \cdot \left(1 - \mu^2\right)};$$

$$M_r = 0;$$

$$M_\theta = \frac{q}{8} \cdot \left[R^2 \cdot (3+\mu) - r^2 \cdot (1-\mu) + \frac{4 \cdot R^2 \cdot r^2}{R^2 - r^2} \cdot ln\frac{r}{R} \cdot (1+\mu) \right].$$

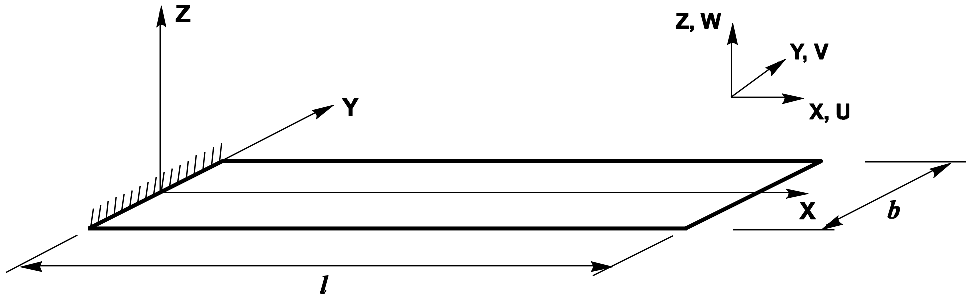
In the analytical solution the deflection w, the radial M_r and tangential M_θ bending moments along the external contour of the plate can be determined according to the following formulas (S.P. Timoshenko, Theory of Plates and Shells. — Moscow: OGIZ. Gostekhizdat, 1948. p. 71):

$$w = 0;$$

$$M_r = 0;$$

$$M_\theta = \frac{q}{8} \cdot \left[R^2 \cdot (1-\mu) + r^2 \cdot (1 + 3 \cdot \mu) + \frac{4 \cdot R^4}{R^2 - r^2} \cdot ln\frac{r}{R} \cdot (1+\mu) \right].$$

Rectangular Narrow Cantilever Plate Subjected to a Uniformly Distributed Transverse Load



Objective: Determination of the strain state of a rectangular narrow cantilever plate subjected to a uniformly distributed transverse load.

Initial data file: **SSLS01_v11.3.SPR**

Problem formulation: The rectangular narrow cantilever plate is subjected to the transverse load uniformly distributed over its area *P*. Determine the transverse displacement Z of the free edge of the plate.

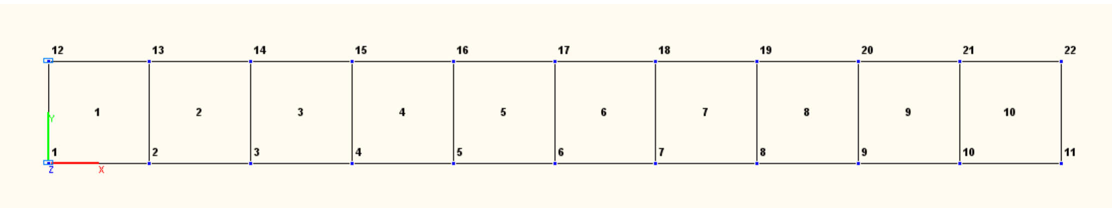
References: S. Timoshenko, Resistance des materiaux, t.1, Paris, Librairie Polytechnique Beranger, 1949.

Initial data:

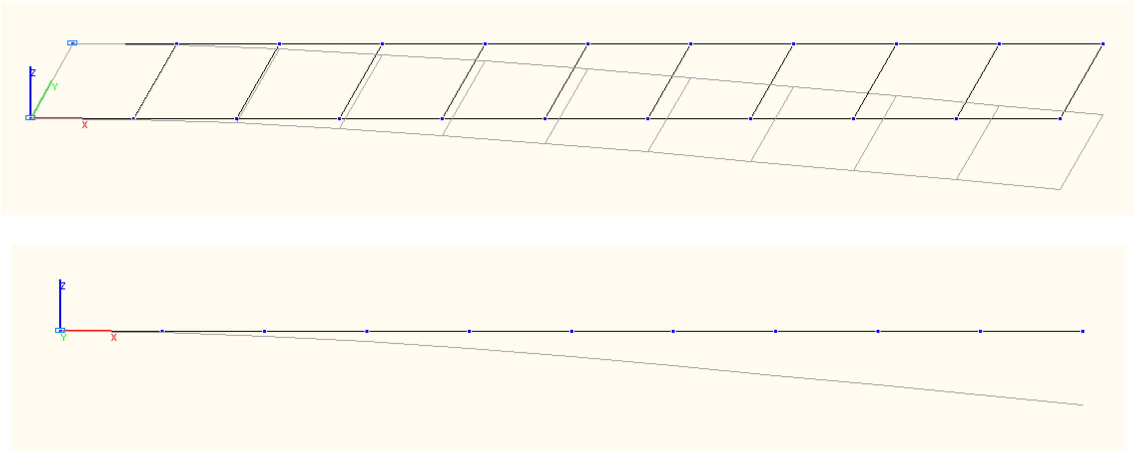
$E = 2.1{\cdot}10^{11}$ Pa	- elastic modulus;
$\nu = 0.0$	- Poisson's ratio;
$l = 1.0$ m	- length of the plate;
$b = 0.1$ m	- width of the plate;
$h = 0.005$ m	- thickness of the plate;
$P = 1.7{\cdot}10^3$ N/m^2	- value of the uniformly distributed transverse load.

Finite element model: Design model – grade beam / plate, 10 plate elements of type 11. Boundary conditions are provided by imposing constraints in the directions of the degrees of freedom Z, UX, UY for the clamped edge. Number of nodes in the design model – 22.

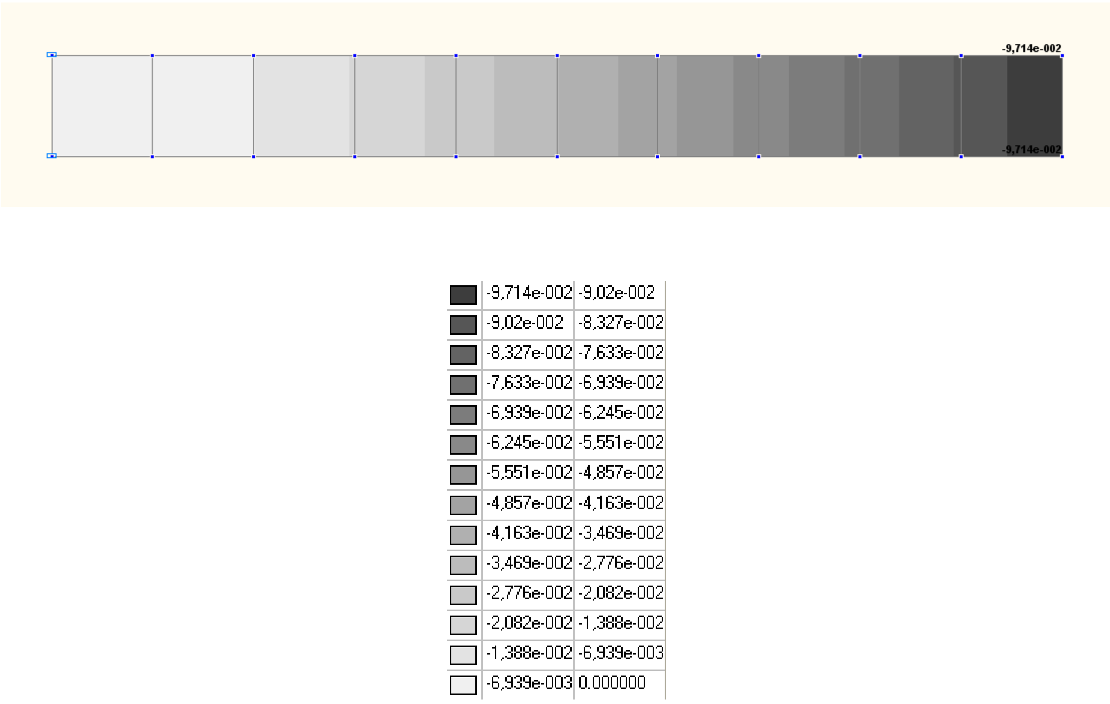
Results in SCAD



Design model



Deformed model



Values of transverse displacements Z (m)

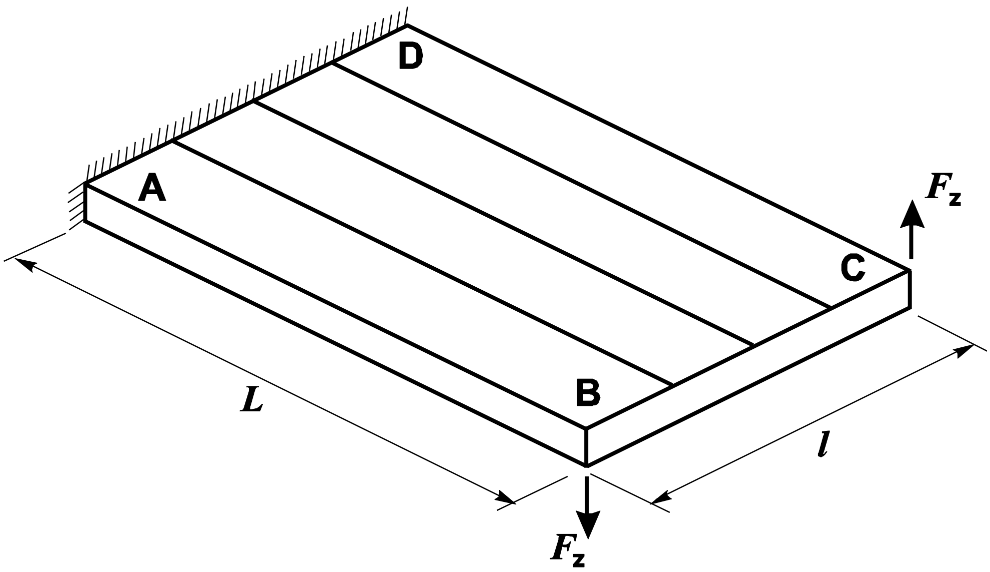
Comparison of solutions:

Parameter	Theory	SCAD	Deviations, %
Transverse displacement of the free edge Z, m	$-9.714 \cdot 10^{-2}$	$-9.714 \cdot 10^{-2}$	0.00

Notes: In the analytical solution the transverse displacement Z of the free edge of the plate is determined according to the following formula:

$$Z = \frac{3 \cdot P \cdot l^4}{2 \cdot E \cdot h^3}.$$

Torsion of a Rectangular Narrow Cantilever Plate by a Pair of Concentrated Forces



Objective: Determination of the strain state of a rectangular narrow cantilever plate subjected to a pair of transverse concentrated forces applied at the corners of its free edge.

Initial data file: **SSLS27_v11.3.SPR**

Problem formulation: The rectangular narrow cantilever plate is subjected to a pair of transverse concentrated forces F_z (points B, C) applied in the corners of the free edge. Determine: the transverse displacement Z of the corner of the free edge of the plate (point C).

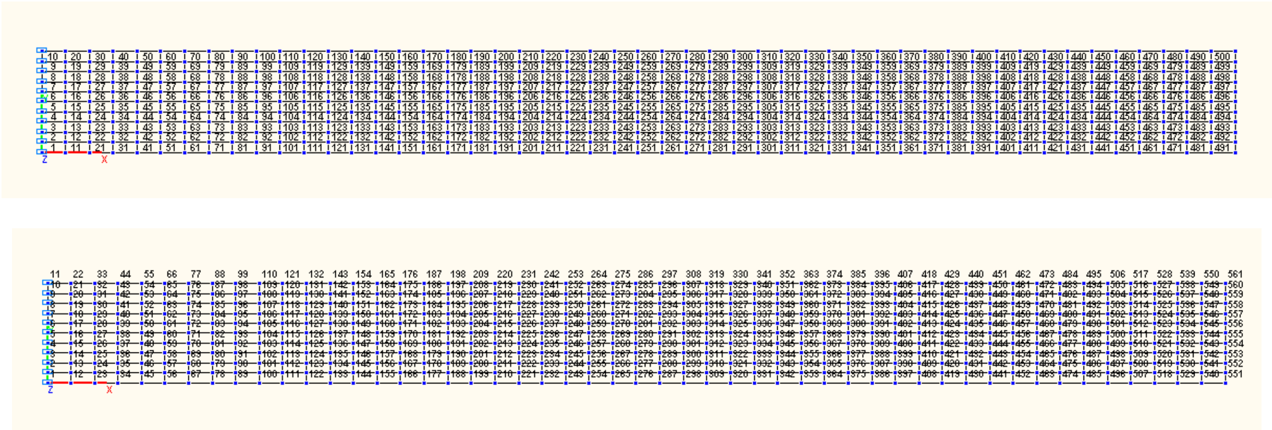
References: J. Robinson, Element evaluation. A set of assessment parts and standard tests, Proceeding of Finite Element Methods in the commercial environment, vol. 1, October 1978.
J.L. Batoz, M.B. Tahar, Evaluation of new quadrilateral thin plate boundary element, International Journal for numerical methods in engineering, vol. 18, Jon Wiley and Sons, 1982.

Initial data:

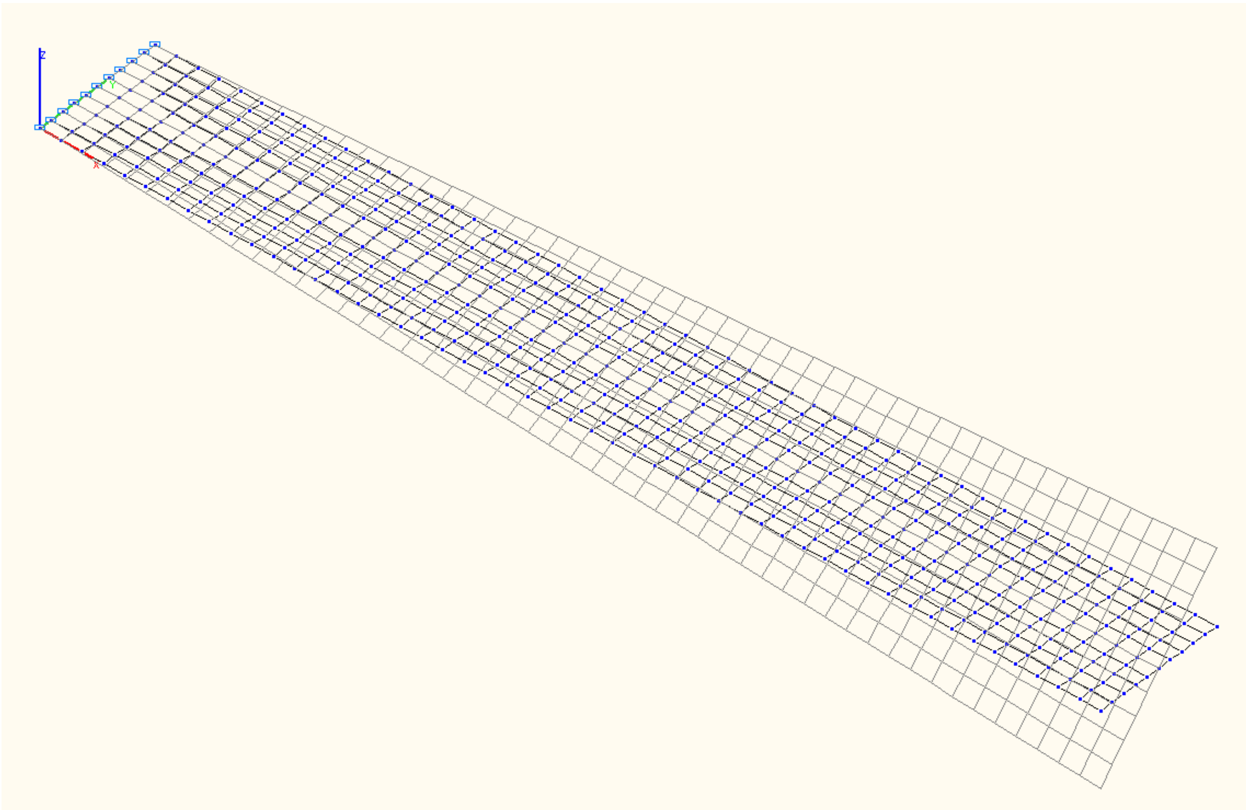
$E = 1.0 \cdot 10^7$ Pa	- elastic modulus,
$\nu = 0.25$	- Poisson's ratio,
l = 1.0 m	- width of the cantilever plate,
L = 12.0 m	- length of the cantilever plate,
h = 0.05 m	- thickness of the plate,
Fz, = 1.0 N	- value of the transverse concentrated force.

Finite element model: Design model – grade beam / plate, 500 plate elements of type 11. Boundary conditions are provided by imposing constraints in the directions of the degrees of freedom Z, UX, UY for the clamped edge (line AD). Number of nodes in the design model – 561.

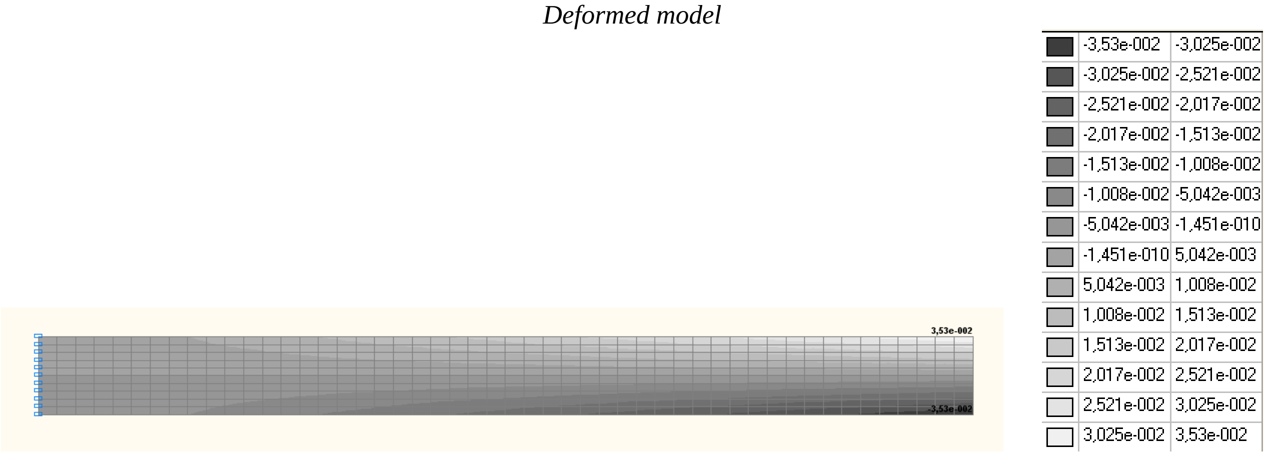
Results in SCAD



Design model



Deformed model

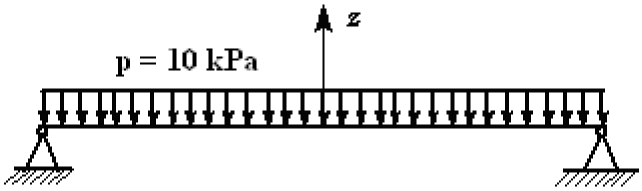


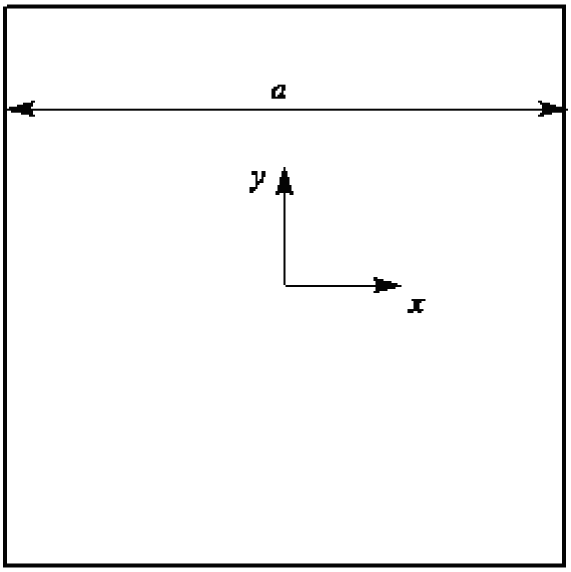
Values of transverse displacements Z (m)

Comparison of solutions:

Parameter	Theory	SCAD	Deviation, %
Transverse displacement Z of the corner of the free edge of the plate (point C), m	$3.537 \cdot 10^{-2}$	$3.530 \cdot 10^{-2}$	0.20

Square Plate Simply Supported along the Perimeter Subjected to a Uniformly Distributed Load





Objective: Determination of maximum displacements and bending moments in a square plate simply supported along the perimeter and subjected to a uniformly distributed load *p*.

Initial data file: **4.17.SPR**

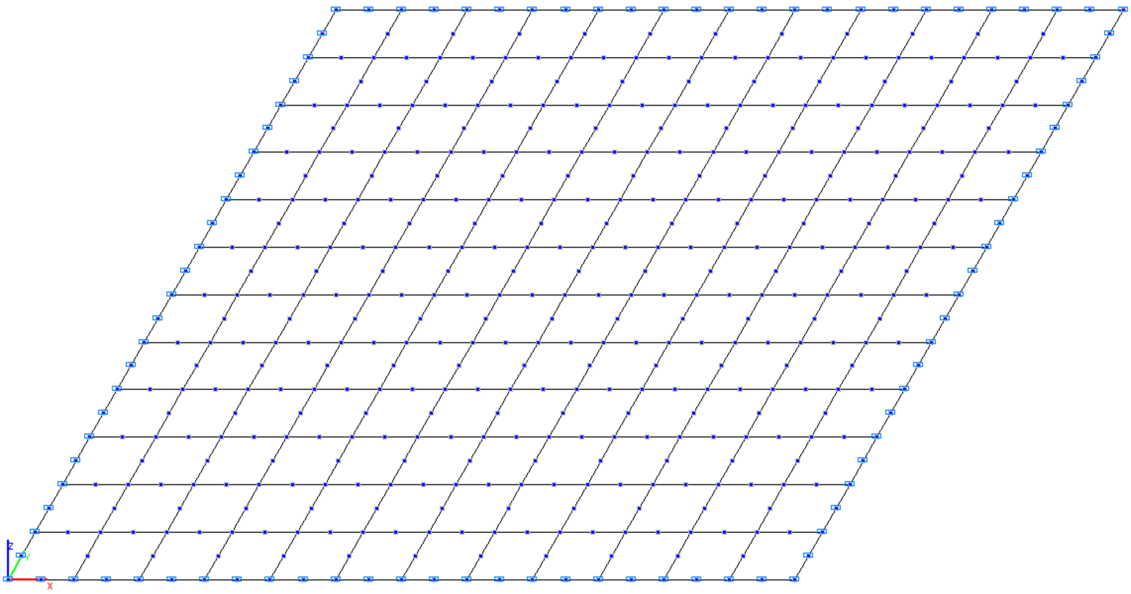
Problem formulation: The square isotropic plate of constant thickness is simply supported along the perimeter and subjected to the uniformly distributed load *p*. Determine: maximum displacements and bending moments.

References: Strength, Stability, Vibrations. Handbook in three volumes. Volume 1. Ed. I.A. Birger and Ya.G. Panovko. — M.: Mechanical engineering, 1968, p. 532-535

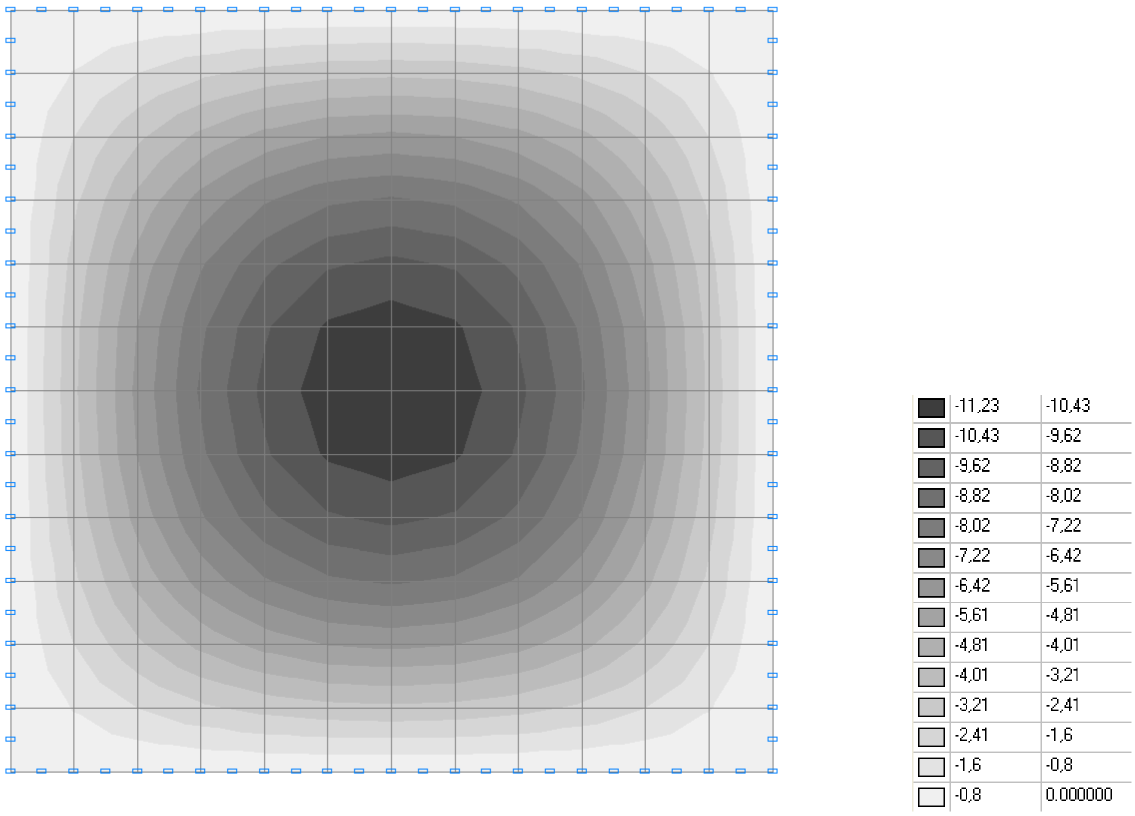
Initial data:
$E = 2.0 \cdot 10^8$ kPa - elastic modulus,
$\mu = 0.3$ - Poisson's ratio,
a = 1.5 m - size of the plate sides,
h = 0.01 m - thickness of the plate,
p = 10 kPa - normal pressure,
Constraints: hinge restraint of nodes along the contour out of the XOY plane (displacement *w* = 0)

Finite element model: Design model – grade beam, plate. Plate elements – 144 eight-node elements of type 20. Number of nodes in the design model – 481.

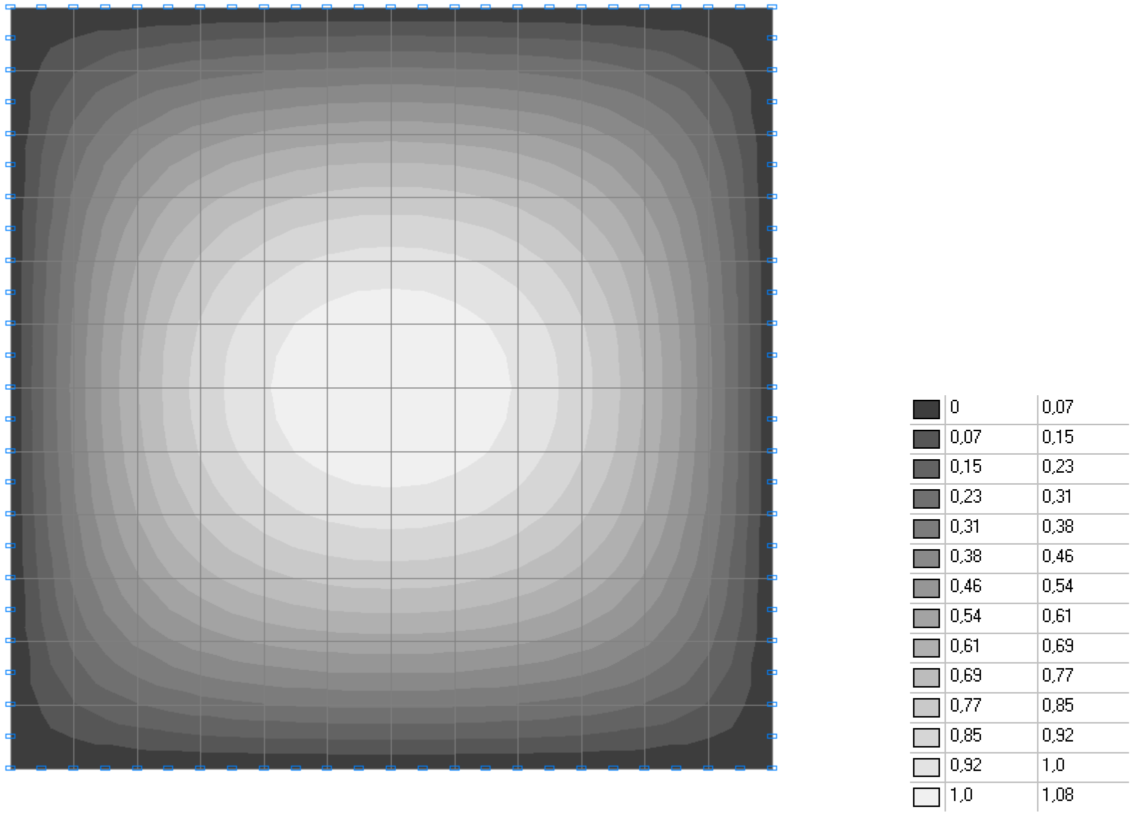
Results in SCAD



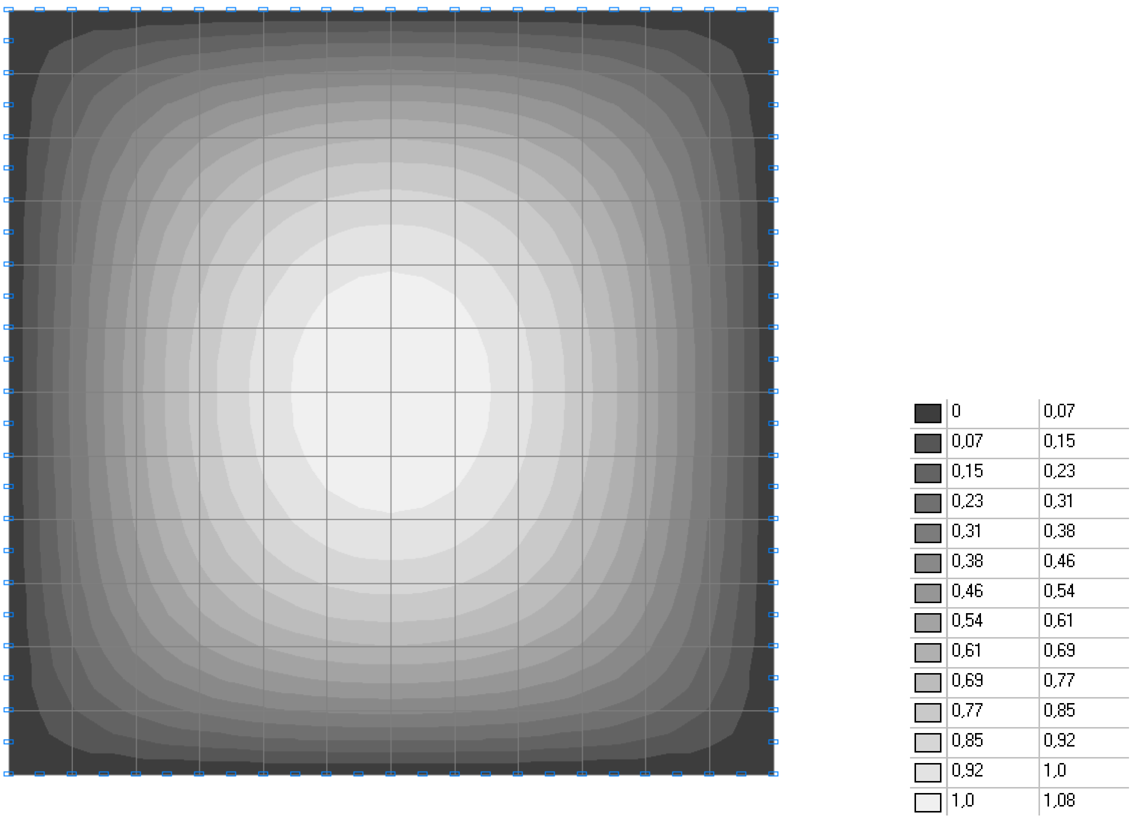
Design model



Values of displacements w (mm)



Values of bending moments M_x (kN·m/m)



Values of bending moments M_y (kN·m/m)

Comparison of solutions:

Parameter	Theory	SCAD	Deviations, %
Displacement in the center of the plate *w*, mm	11.22	11.23	0.09
Bending moment M_x, kN·m /m	1.078	1.077	0.09
Bending moment M_y, kN·m /m	1.078	1.077	0.09

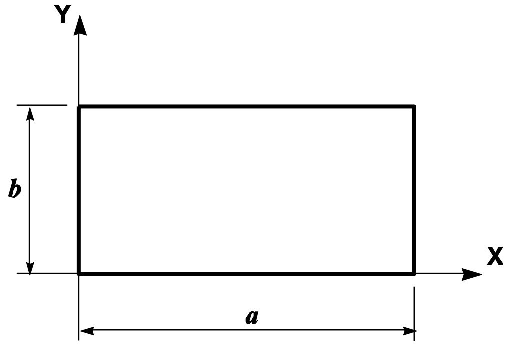
Notes: In the analytical solution the displacement *w* and the bending moments M_x and M_y in the center of the plate subjected to the uniformly distributed load are determined according to the following formulas (Handbook in three volumes. Volume 1. Ed. I.A. Birger and Ya.G. Panovko. — M.: Mechanical engineering, 1968, p. 532-535):

$$w = 0.00406 \cdot \frac{p \cdot a^4}{D}, \text{ where:}$$

$$D = \frac{E \cdot h^3}{12 \cdot \left(1 - \mu^2\right)};$$

$$M_x = M_y = 0.0479 \cdot p \cdot a^2.$$

Rectangular Plate Simply Supported along the Perimeter Subjected to a Uniformly Distributed Transverse Load



Objective: Determination of the stress-strain state of a rectangular plate simply supported along the perimeter and subjected to a uniformly distributed transverse load.

Initial data files:

File name	Description
SSLS24_b_1a_v11.3.SPR	Design model with the ratios of the sides of the plate b/a = 1.0
SSLS24_b_2a_v11.3.SPR	Design model with the ratios of the sides of the plate b/a = 2.0
SSLS24_b_5a_v11.3.SPR	Design model with the ratios of the sides of the plate b/a = 5.0

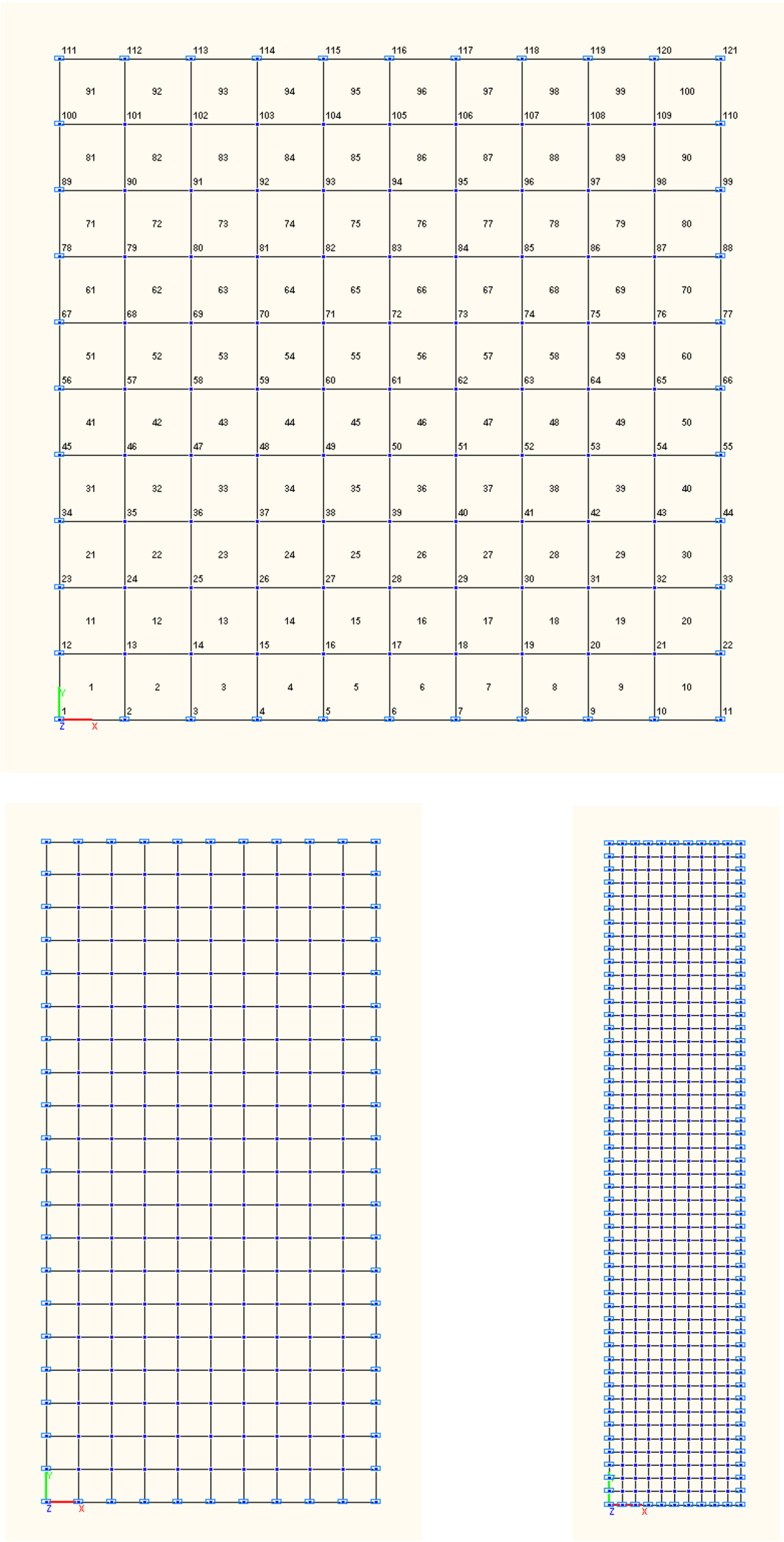
Problem formulation: The rectangular plate simply supported along the perimeter is subjected to the transverse load uniformly distributed over its area *p*. Determine the transverse displacement Z and bending moments M_x, M_y in the center of the plate for different ratios of its sides b/a.

References: S. Timoshenko, S. Woinowski, Theorie des plaques et des coques, Paris, Librairie Polytechnique Beranger, 1961.

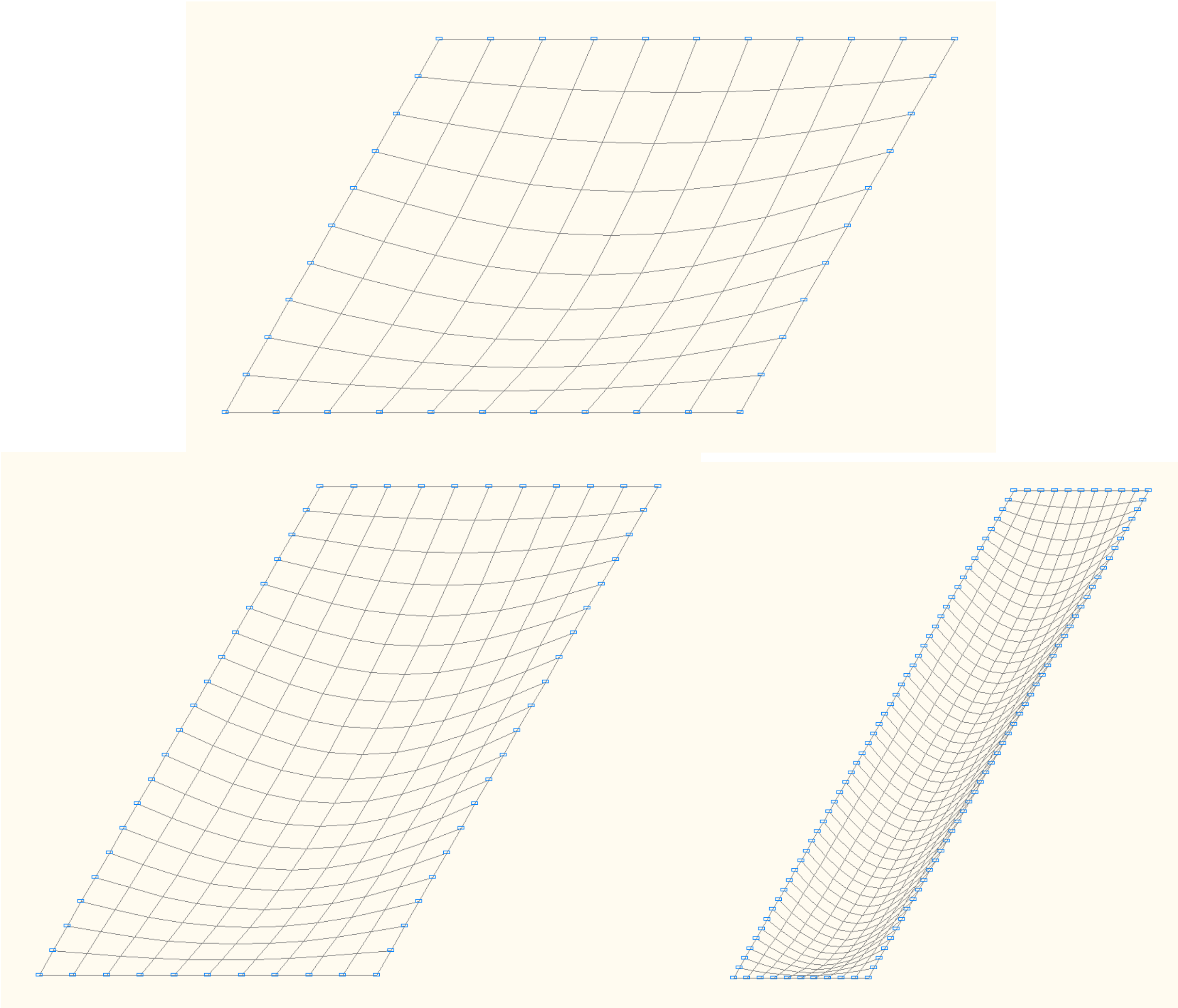
Initial data:
$E = 1.0 \cdot 10^7$ Pa - elastic modulus;
ν = 0.3 - Poisson's ratio;
h = 0.01 m - thickness of the plate;
a = 1.0 m - short side of the plate (along the X axis of the global coordinate system);
b = 1.0 m, 2.0 m, 5.0 m - long side of the plate (along the Y axis of the global coordinate system);
$p = 1.0$ N/m^2 - value of the uniformly distributed transverse load.

Finite element model: Three design models are considered.
Design model 1 (b/a = 1.0) – grade beam / plate, shell elements – 100 plate elements of type 20. Boundary conditions are provided by imposing constraints in the directions of the degrees of freedom Z, UY for the edges parallel to the X axis of the global coordinate system, and Z, UX for the edges parallel to the Y axis of the global coordinate system. Number of nodes in the design model – 121.
Design model 2 (b/a = 2.0) – grade beam / plate, shell elements – 200 plate elements of type 20. Boundary conditions are provided by imposing constraints in the directions of the degrees of freedom Z, UY for the edges parallel to the X axis of the global coordinate system, and Z, UX for the edges parallel to the Y axis of the global coordinate system. Number of nodes in the design model – 231.
Design model 3 (b/a = 5.0) – grade beam / plate, shell elements – 500 plate elements of type 20. Boundary conditions are provided by imposing constraints in the directions of the degrees of freedom Z, UY for the edges parallel to the X axis of the global coordinate system, and Z, UX for the edges parallel to the Y axis of the global coordinate system. Number of nodes in the design model – 561.

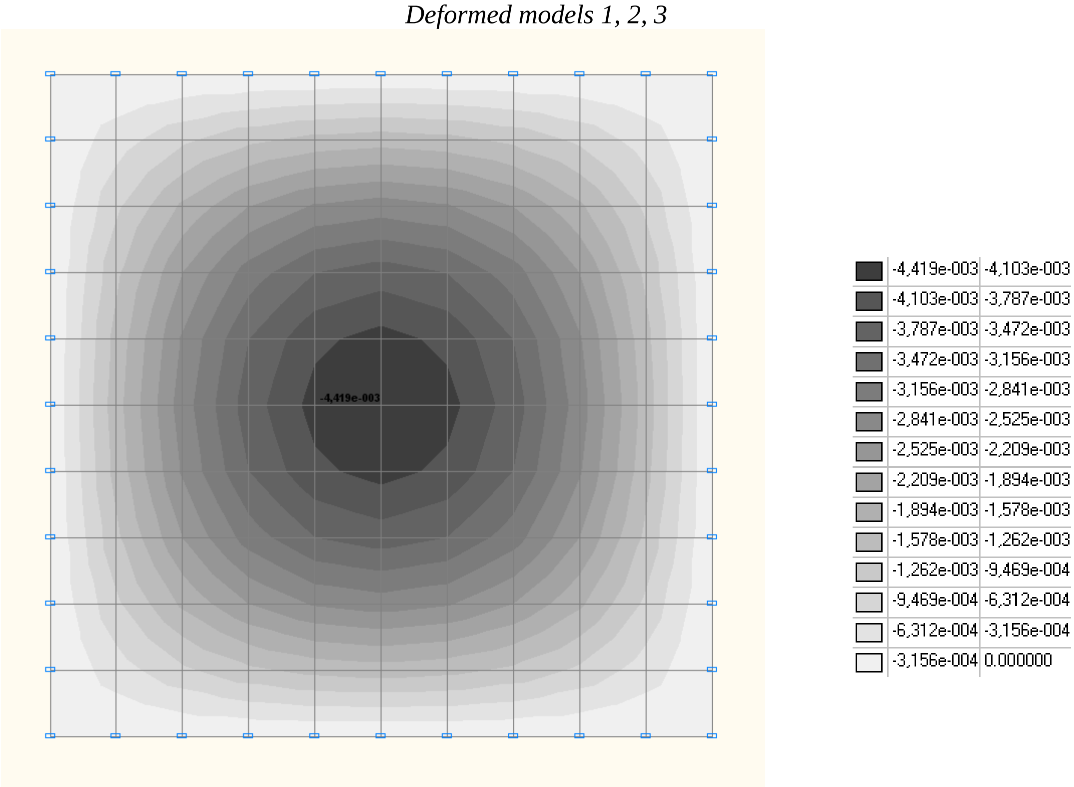
Results in SCAD



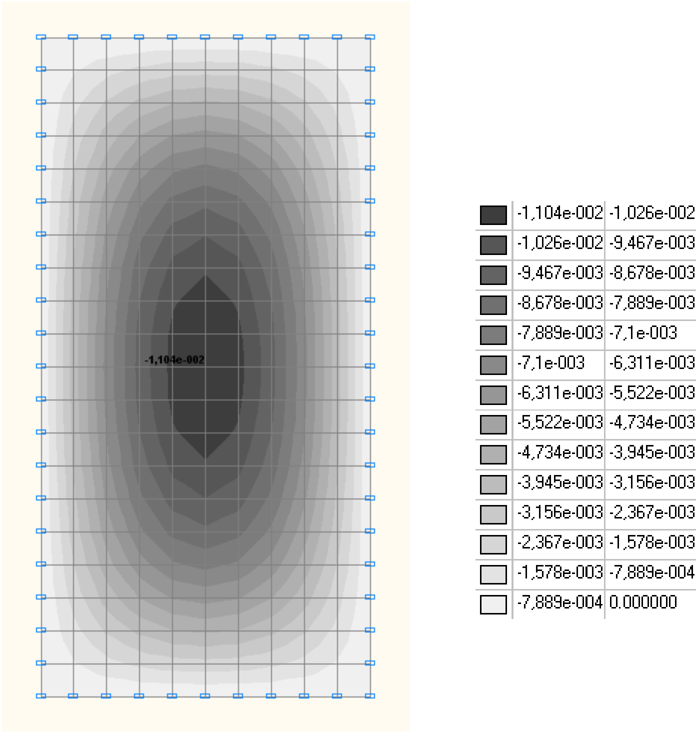
Design models 1, 2, 3



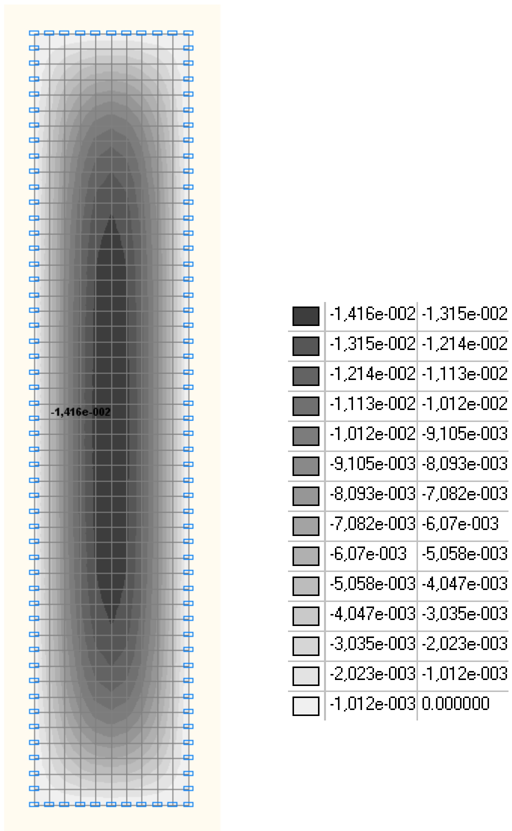
Deformed models 1, 2, 3



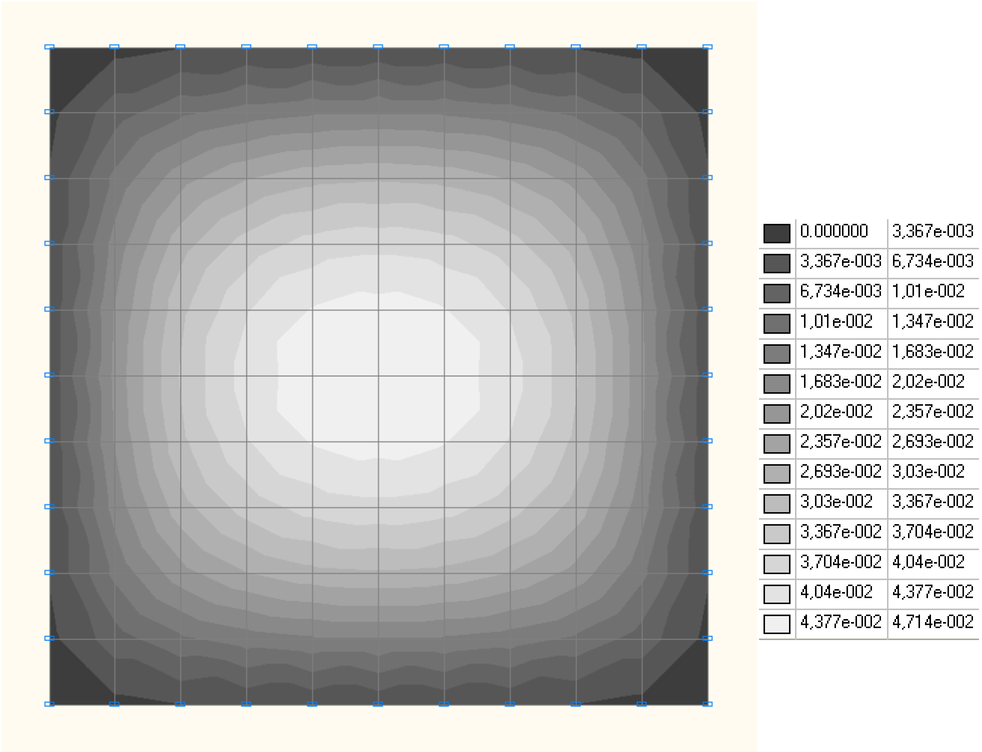
Values of transverse displacements Z (m)
for the design model 1



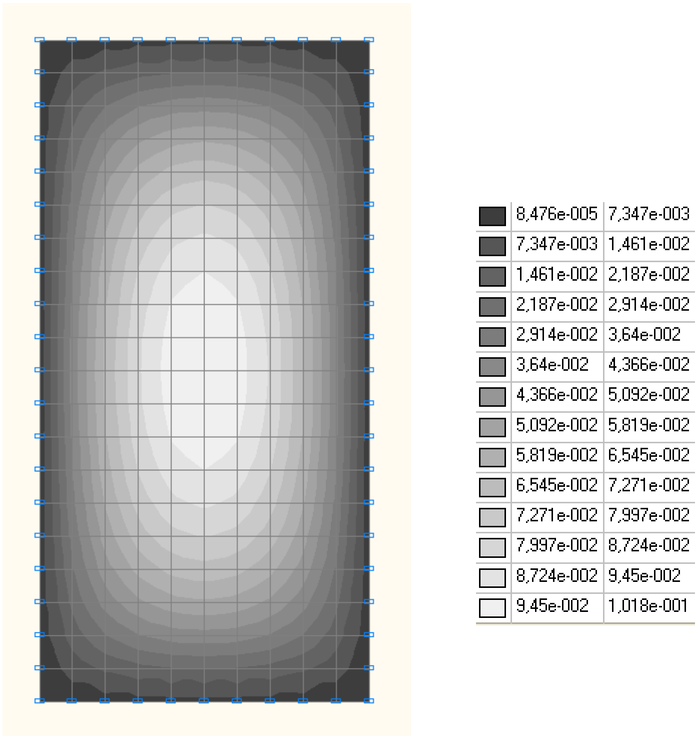
Values of transverse displacements Z (m)
for the design model 2



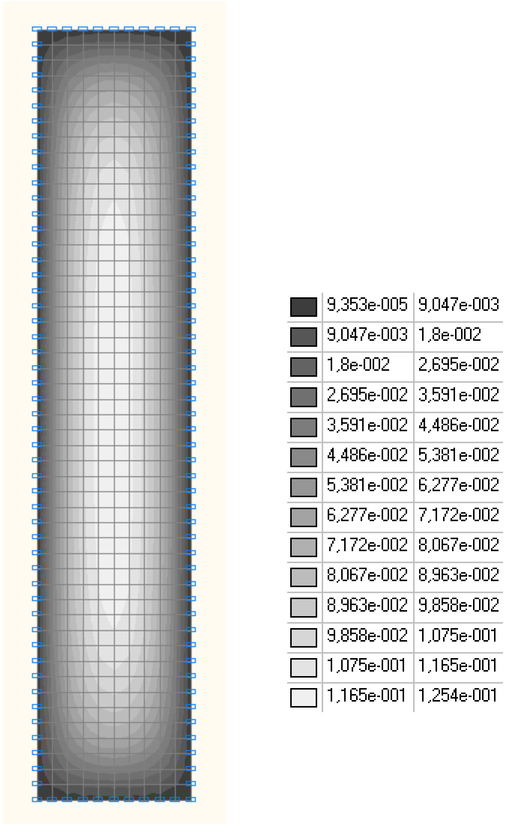
Values of transverse displacements Z (m)
for the design model 3



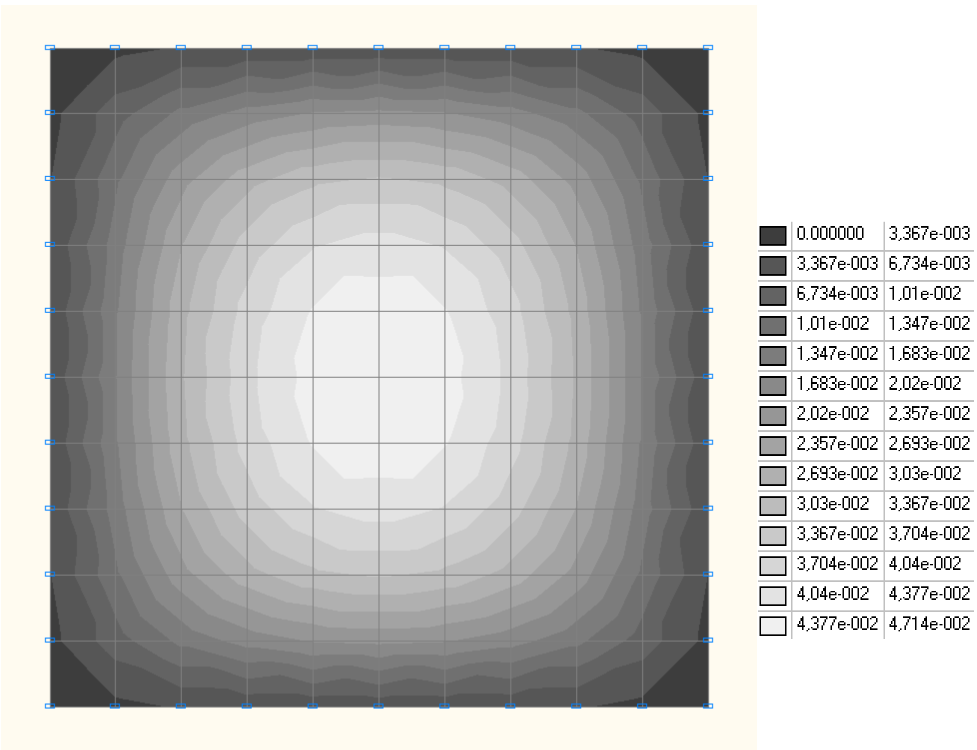
Values of bending moments M_x (N·m/m)
for the design model 1



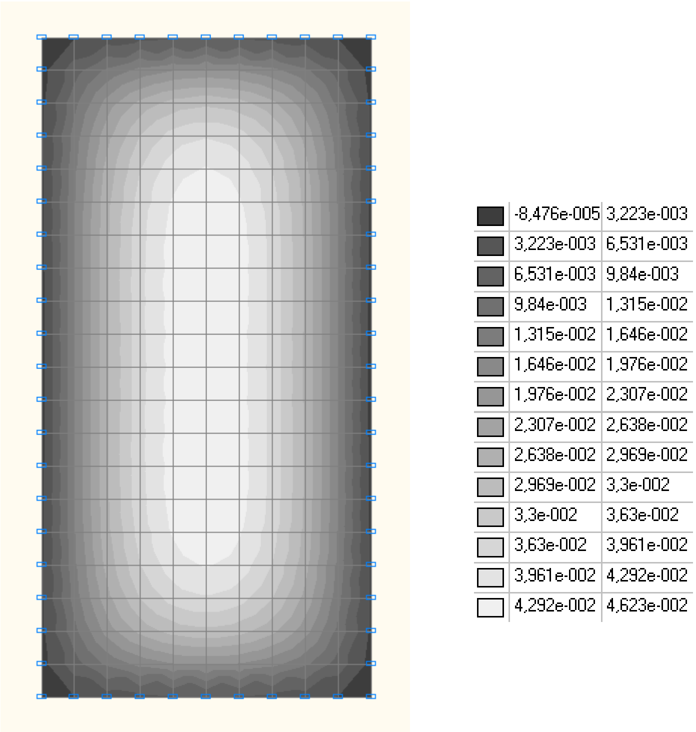
Values of bending moments M_x (N·m/m)
for the design model 2



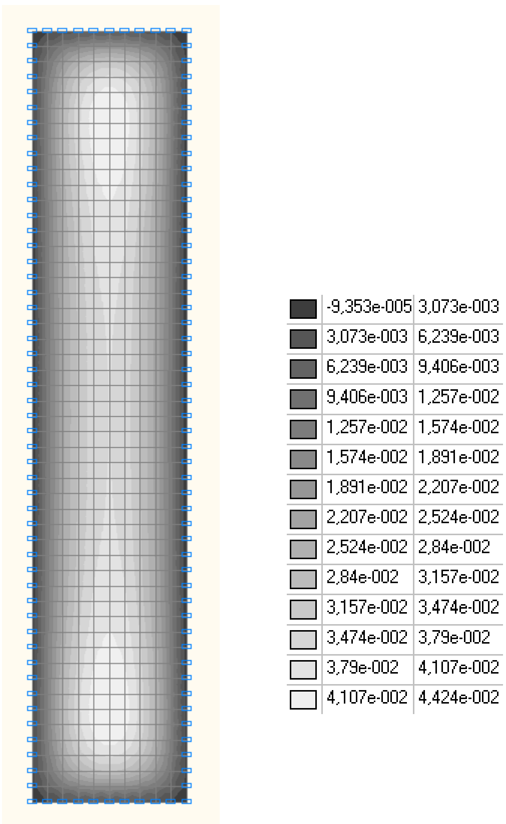
Values of bending moments M_x (N·m/m)
for the design model 3



Values of bending moments M_y (N·m/m)
for the design model 1



Values of bending moments M_y (N·m/m)
for the design model 2



Values of bending moments M_y (N·m/m)
for the design model 3

Comparison of solutions:

Design model 1 (b/a = 1.0)

Parameter	Theory	SCAD	Deviation, %
Transverse displacement Z in the center of the plate, m	$-4.436 \cdot 10^{-3}$	$-4.419 \cdot 10^{-3}$	0.38
Bending moments M_x in the center of the plate, N·m/m	$4.789 \cdot 10^{-2}$	$4.714 \cdot 10^{-2}$	1.57
Bending moments M_y in the center of the plate, N·m/m	$4.789 \cdot 10^{-2}$	$4.714 \cdot 10^{-2}$	1.57

Design model 2 (b/a = 2.0)

Parameter	Theory	SCAD	Deviation, %
Transverse displacement Z in the center of the plate, m	$-1.106 \cdot 10^{-2}$	$-1.104 \cdot 10^{-2}$	0.18
Bending moments M_x in the center of the plate, N·m/m	$1.017 \cdot 10^{-2}$	$1.018 \cdot 10^{-2}$	0.10
Bending moments M_y in the center of the plate, N·m/m	$4.635 \cdot 10^{-2}$	$4.607 \cdot 10^{-2}$	0.60

Design model 3 (b/a = 5.0)

Parameter	Theory	SCAD	Deviation, %
Transverse displacement Z in the center of the plate, m	$-1.416 \cdot 10^{-2}$	$-1.416 \cdot 10^{-2}$	0.00
Bending moments M_x in the center of the plate, N·m/m	$1.246 \cdot 10^{-1}$	$1.254 \cdot 10^{-1}$	0.64
Bending moments M_y in the center of the plate, N·m/m	$3.774 \cdot 10^{-2}$	$3.798 \cdot 10^{-2}$	0.64

Notes: In the analytical solution the transverse displacement Z and bending moments M_x, M_y in the center of the plate for different ratios of its sides b/a can be determined according to the following formulas:

$$Z = \alpha \cdot \frac{p \cdot a^4}{D}; \qquad M_x = \beta \cdot p \cdot a^2; \qquad M_y = \beta_1 \cdot p \cdot a^2,$$

where:

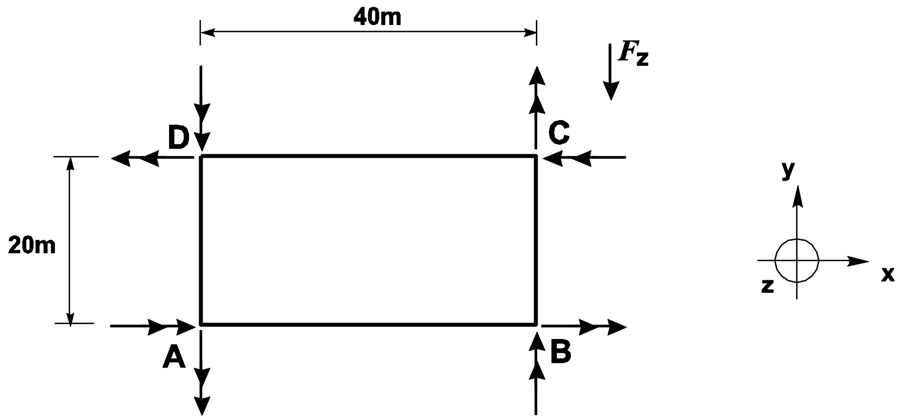
at $\frac{a}{b} = 1.0$ $\quad \alpha = 0.004062, \quad \beta = 0.047886, \quad \beta_1 = 0.047886,$

at $\frac{a}{b} = 2.0$ $\quad \alpha = 0.010129, \quad \beta = 0.101683, \quad \beta_1 = 0.046350,$

at $\frac{a}{b} = 5.0$ $\quad \alpha = 0.012971, \quad \beta = 0.124624, \quad \beta_1 = 0.037744,$

$$D = \frac{E \cdot h^3}{12 \cdot \left(1 - \mu^2\right)}.$$

Rectangular Plate Simply Supported at Three Vertices Subjected to a Concentrated Force and Concentrated Moments out of Its Plane



Objective: Determination of the strain state of a rectangular plate simply supported at three vertices and subjected to a concentrated force and concentrated moments out of its plane.

Initial data file: **SSLS26_v11.3.SPR**

Problem formulation: The rectangular plate simply supported at three vertices (points A, B, D) is subjected to a concentrated force F_z out of its plane applied to the free vertex (point C), and concentrated moments M_x and M_y, applied in pairs to all four vertices (points A, B, C, D) with unilateral bending in planes parallel to the adjacent sides. Determine the displacement Z of the free vertex (point C) out of the plane of the plate.

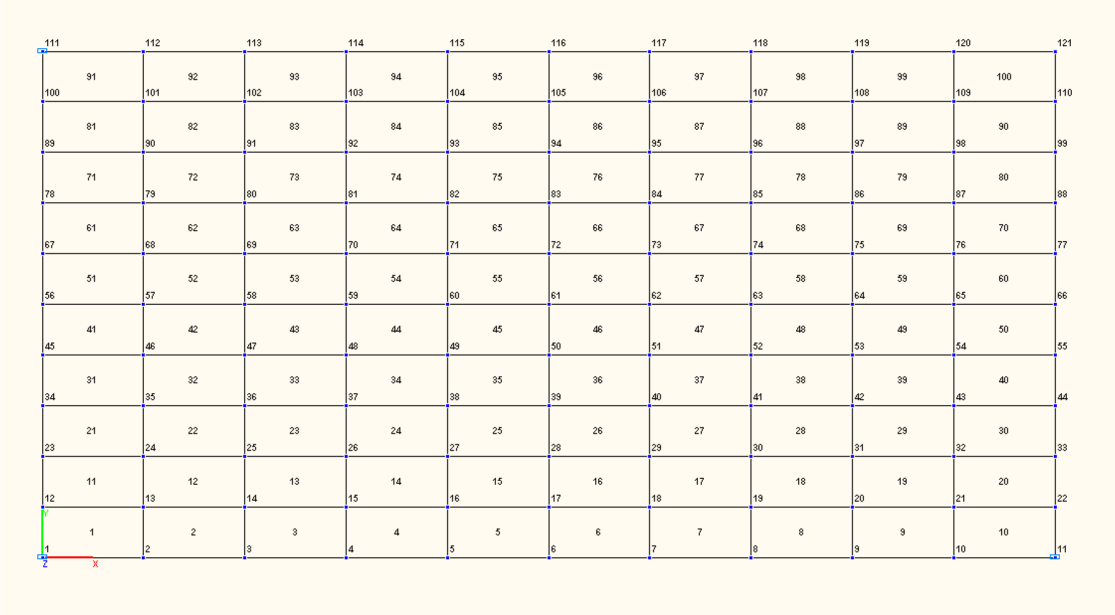
References: J.L. Batoz, An explicit formulation for an efficient triangular plate-bending element, International Journal for Numerical Methods in Engineering, vol.18, John Wiley and Sons, 1982.

Initial data:

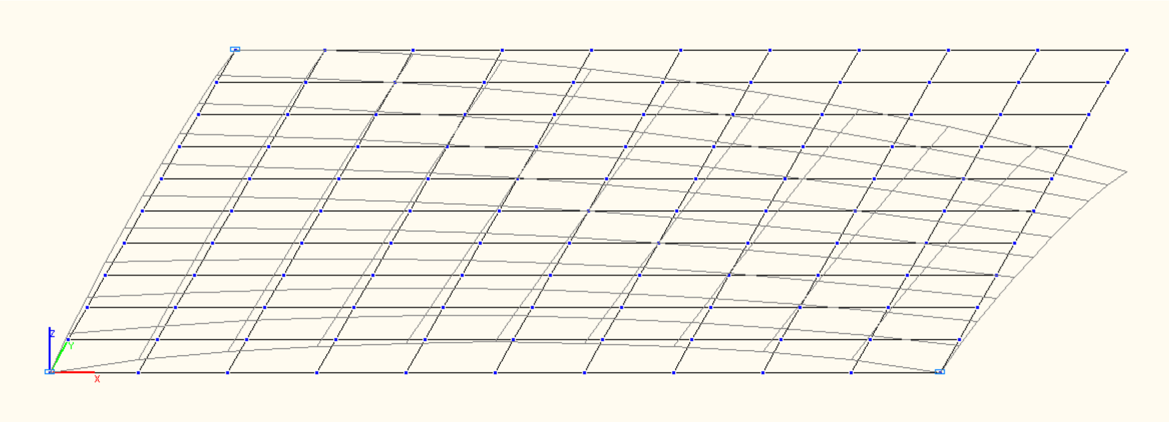
$E = 1.0 \cdot 10^3$ Pa	- elastic modulus;
ν = 0.3	- Poisson's ratio;
h = 1.0 m	- thickness of the plate;
a = 40.0 m	- long side of the plate (along the X axis of the global coordinate system);
b = 20.0 m	- short side of the plate (along the Y axis of the global coordinate system);
F_z = 2.0 N	- value of the transverse concentrated force;
M_x = 20.0 N·m	- value of the concentrated moments bending the plate along the short side (with respect to the X axis of the global coordinate system);
M_y = 10.0 N·m	- value of the concentrated moments bending the plate along the long side (with respect to the Y axis of the global coordinate system).

Finite element model: Design model – grade beam / plate, shell elements – 100 plate elements of type 20. Boundary conditions are provided by imposing constraints in the direction of the degree of freedom Z in the vertices of the plate on the X and Y axes of the global coordinate system (points A, B, C). Number of nodes in the design model – 121.

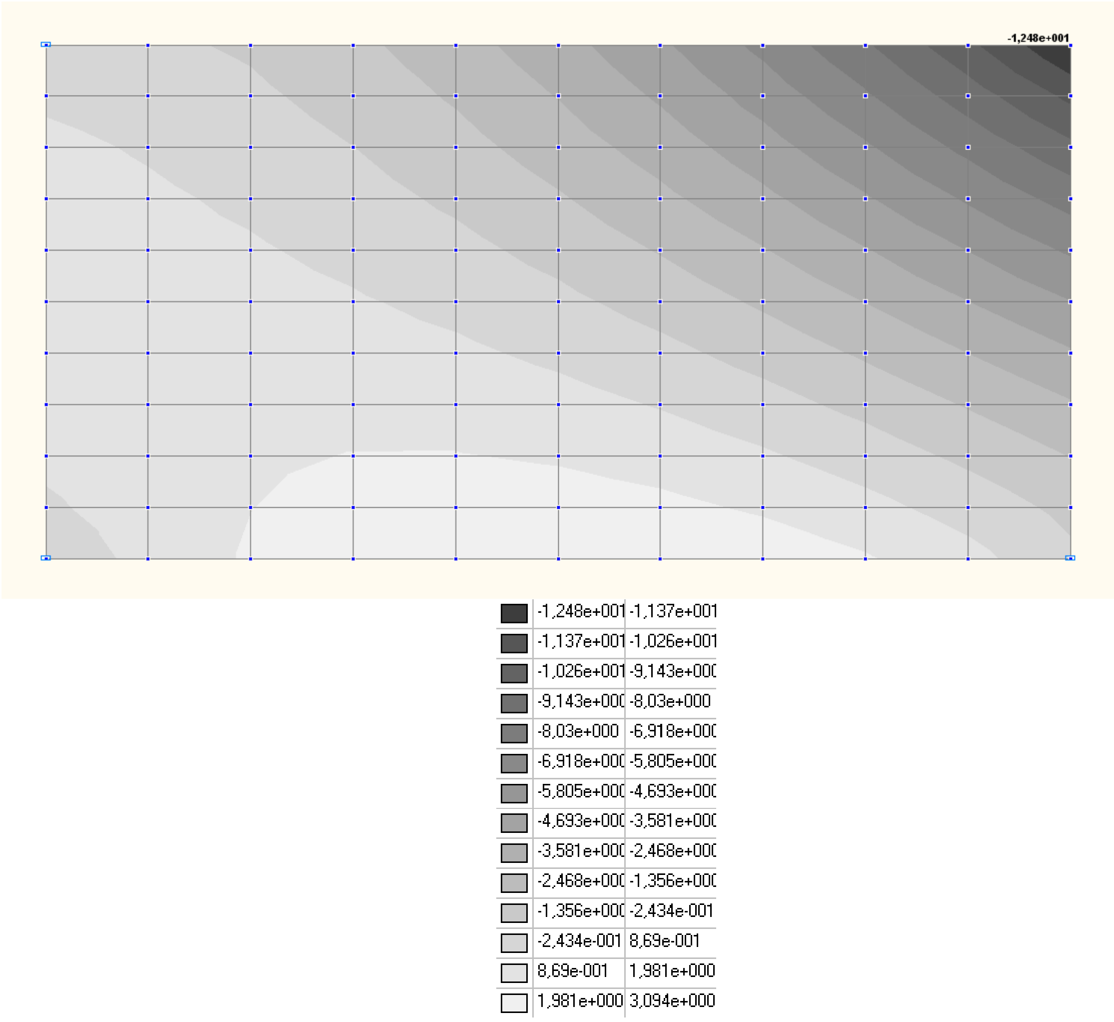
Results in SCAD



Design model



Deformed model

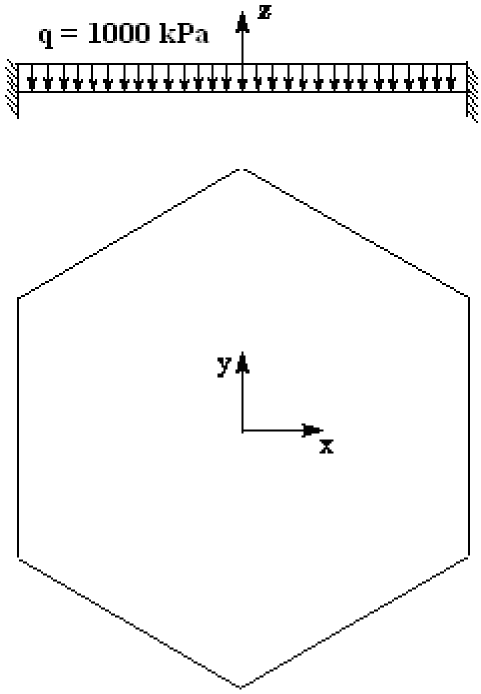


Values of transverse displacements Z (m)

Comparison of solutions:

Parameter	Theory	SCAD	Deviation, %
Displacement Z of the free vertex (point C), m	$-1.248 \cdot 10^{1}$	$-1.248 \cdot 10^{1}$	0.00

Stress-Strain State of a Clamped Hexagonal Plate Subjected to a Uniformly Distributed Load



Objective: Determination of displacements and bending moments in the center of a hexagonal plate clamped on all sides and subjected to a uniformly distributed load *q*.

***Initial data file*: 4.19.SPR**

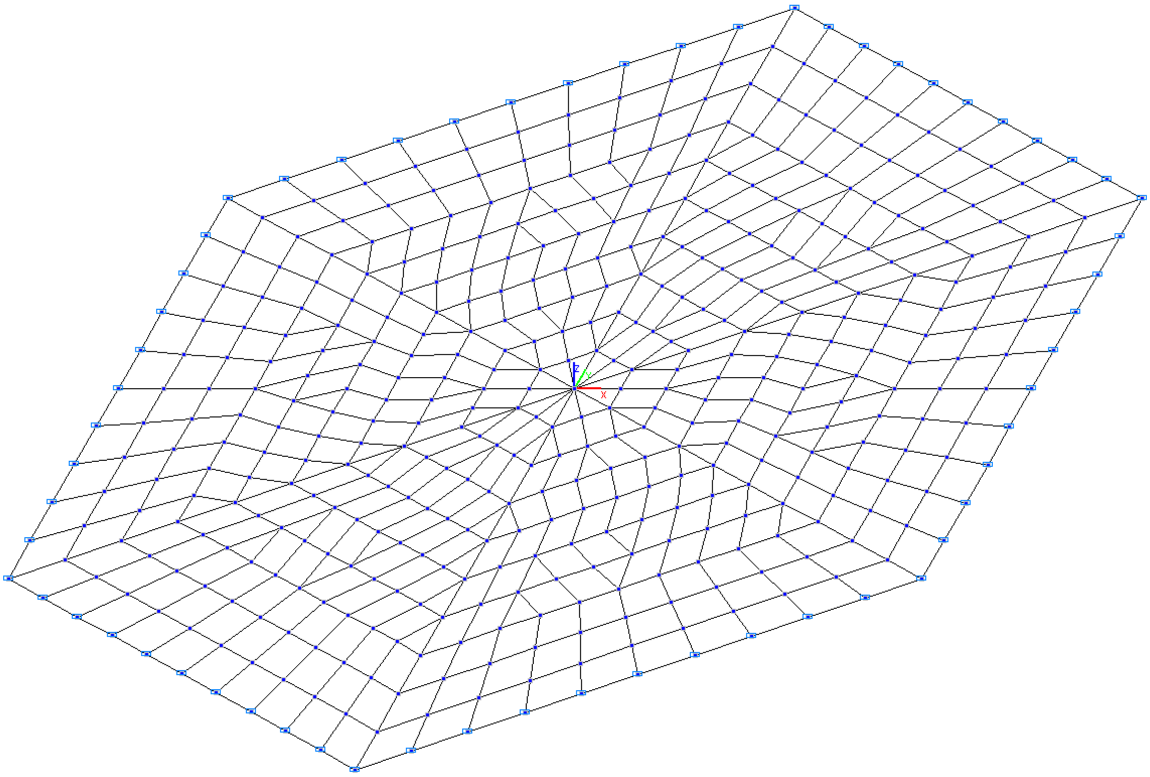
Problem formulation: The regular hexagonal plate of constant thickness clamped on all sides is subjected to normal pressure *q*. Determine: the axial displacement *w* and bending moments M_x, M_y in the center of the plate.

References: Vainberg D. V., Handbook on Strength, Stability and Oscillations of Plates. Kiev: Budivelnik, 1973.

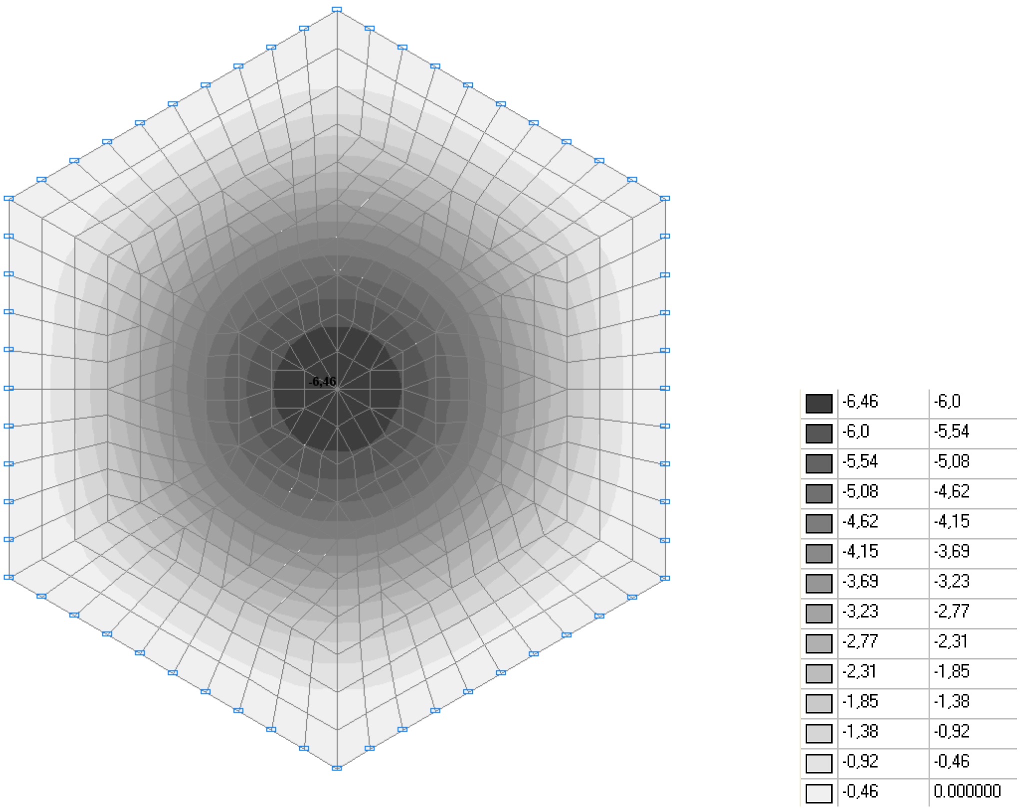
Initial data:
$E = 2.0 \cdot 10^8$ kPa - elastic modulus,
$\mu = 0.3$ - Poisson's ratio,
a = 0.134 m - side of the hexagonal plate,
h = 0.003 m - thickness of the plate,
q = 1000 kPa - normal pressure,
Constraints: rigid restraint of nodes along the contour (displacement $w = 0$)

Finite element model: Design model – general type system. Plate elements – 389 four-node elements of type 44 and 73 three-node elements of type 42. Number of nodes in the design model – 451.

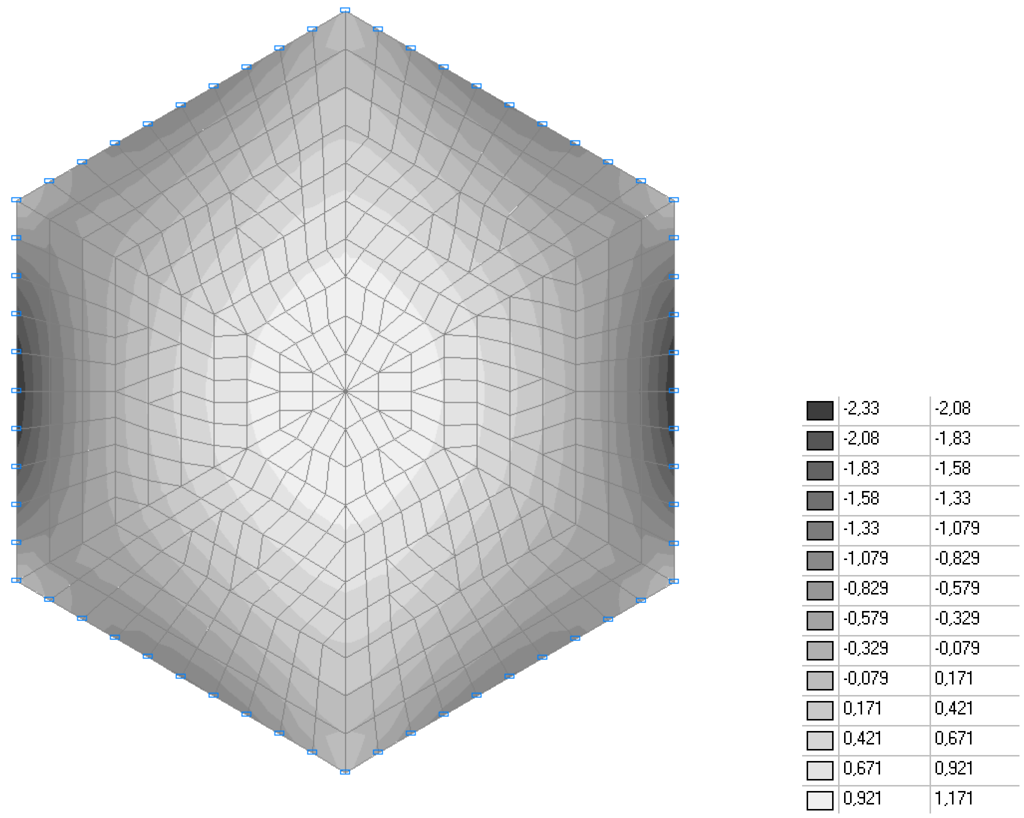
Results in SCAD



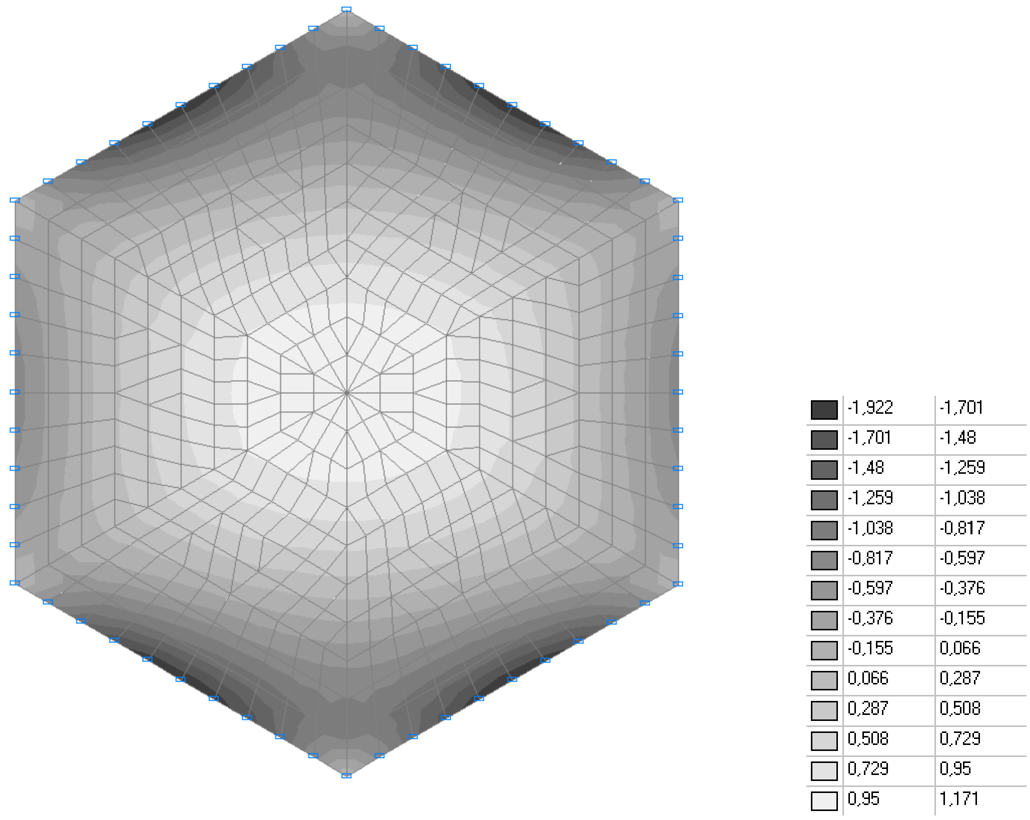
Design model



Values of displacements w (mm)



Values of bending moments M_x (kN·m/m)



Values of bending moments M_y (kN·m/m)

Comparison of solutions:

Parameter	Theory	SCAD	Deviations, %
Displacement in the center of the plate *w*, mm	6.51	6.46	0.77
Bending moment M_x, kN·m /m	1.163	1.171	0.69
Bending moment M_y, kN·m /m	1.163	1.171	0.69

Notes: In the analytical solution the displacement *w* and bending moments M_x and M_y in the center of the plate can be determined according to the following formulas (Vainberg D. V., Handbook on Strength, Stability and Oscillations of Plates. Kiev: Budivelnik, 1973):

$$w = 0.009979 \cdot \frac{q \cdot a^4}{D}$$, where:

$$D = \frac{E \cdot h^3}{12 \cdot \left(1 - \mu^2\right)};$$

$$M_x = M_y = 0.049835 \cdot \left(1 + \mu\right) \cdot q \cdot a^2 .$$

Clamped Rectangular Plate of Constant Thickness Subjected to Thermal Loading

Objective: Determine the bending moments and stresses in a rectangular plate clamped on all sides at the linear temperature variation across the thickness of the plate.

Initial data file: **4.20.SPR**

Problem formulation: The rectangular plate of constant thickness clamped on all sides is considered. The temperature is constant in the planes parallel to the midsurface and varies linearly across the thickness of the plate. Determine: the displacement *w*, bending moments M_x, M_y and maximum thermal stress σ.

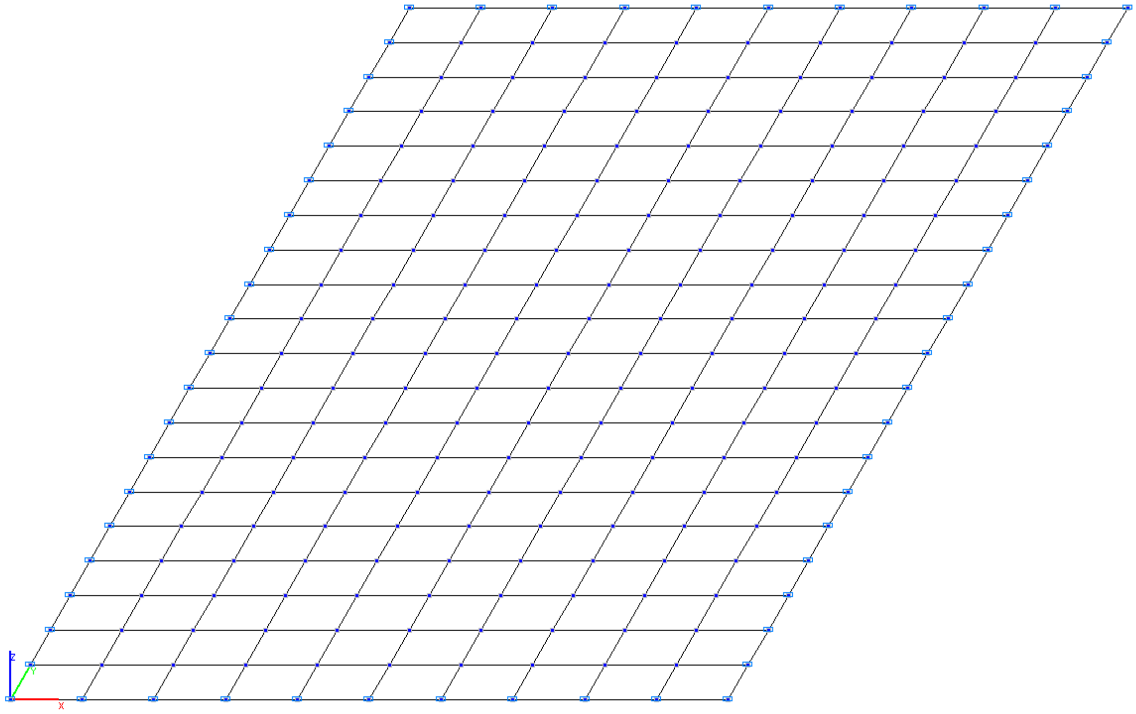
References: S. Timoshenko, S. Woinowsky-Krieger, Theory of Plates and Shells. — M.:Nauka, 1963.

Initial data:

$E = 2.0 \cdot 10^8$ kPa	- elastic modulus,
$\mu = 0.3$	- Poisson's ratio,
$a_x = 1.5$ m	- width of the plate,
$a_y = 2.5$ m	- length of the plate,
$h = 0.02$ m	- thickness of the plate,
$\alpha = 1.5 \cdot 10^{-5}$ $1/C^0$	- linear thermal expansion coefficient of the material,
$\Delta T = 20$ C^0	- temperature difference between the upper and lower surfaces of the plate

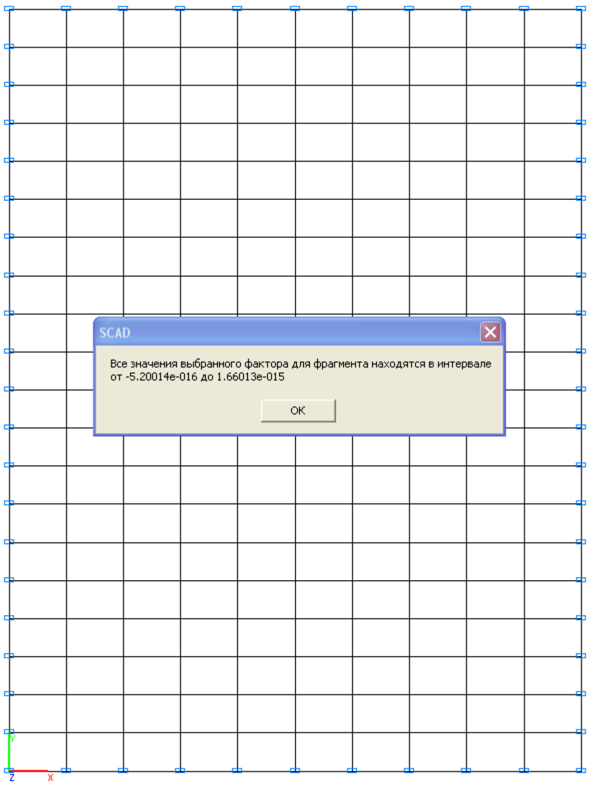
Constraints: rigid restraint of nodes along the contour (displacement $u=v=w=\theta_x=\theta_y=\theta_z=0$)

Finite element model: Design model – general type system. Plate elements – 200 four-node elements of type 41. Number of nodes in the design model – 231.

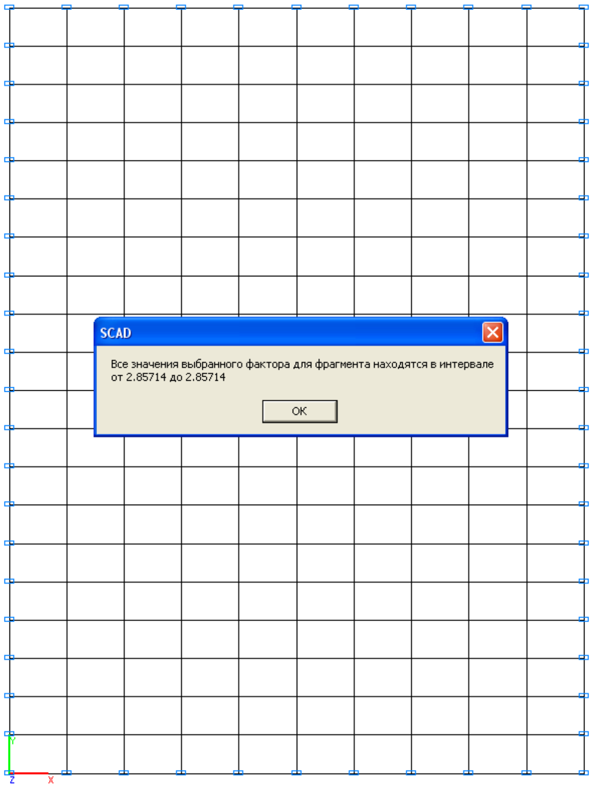


Design model

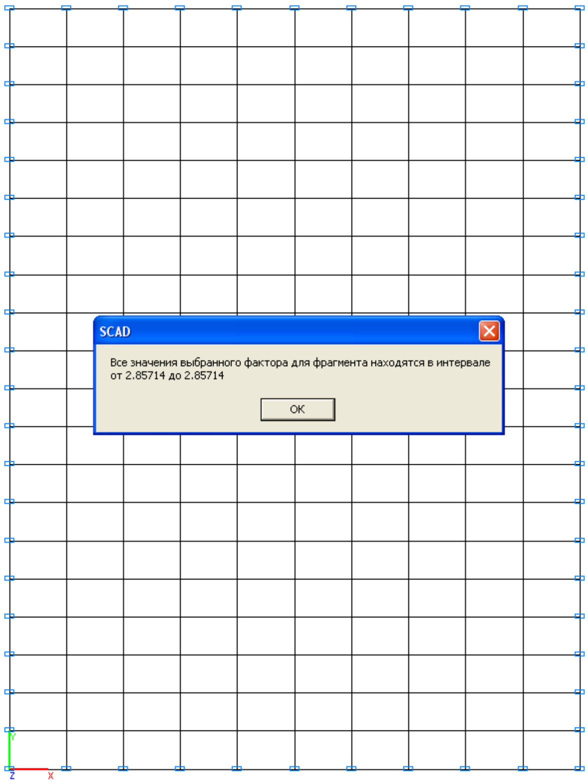
Results in SCAD



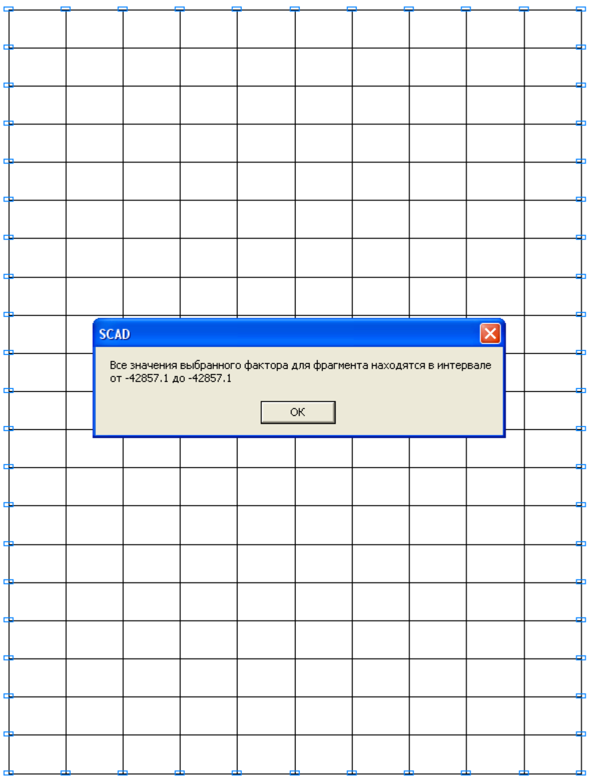
Values of displacements w (mm)



Values of bending moments M_x (kN·m/m)



Values of bending moments M_y (kN·m/m)



Values of stresses on the upper surface of the plate σ (kN/m^2)

Comparison of solutions:

Parameter	Theory	SCAD	Deviations, %
Displacement *w*, mm	0.00	0.00	—
Bending moments $M_x = M_y$, kN·m /m	2.857	2.857	0.00
Maximum thermal stress, kPa	42857	42857	0.00

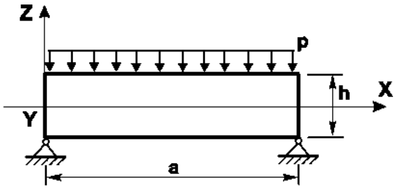
Notes: In the analytical solution the bending moments M_x, M_y and maximum thermal stress σ in the clamped plate subjected to the linear temperature variation across the thickness of the plate can be determined according to the following formulas (S. Timoshenko, S. Woinowsky-Krieger, Theory of Plates and Shells. — M.:Nauka, 1963, p. 64):

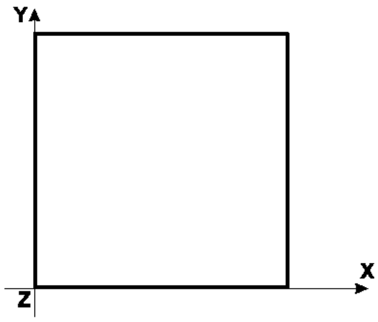
$$M_x = M_y = D \cdot \frac{\alpha \cdot \Delta T \cdot (1+\mu)}{h}, \text{ where:}$$

$$D = \frac{E \cdot h^3}{12 \cdot (1-\mu^2)},$$

$$\sigma = \frac{\alpha \cdot \Delta T \cdot E}{2 \cdot (1-\mu)}.$$

Simply Supported Thick Square Plate Subjected to a Uniformly Distributed Transverse Load





Objective:
Determination of the strain state of a simply supported thick square plate subjected to a uniformly distributed transverse load.

Initial data files:

File name	Description
толстая_плита_a_h_2.SPR	Design model for the plate side-to-thickness ratios a/h = 2.0
толстая_плита_a_h_4.SPR	Design model for the plate side-to-thickness ratios a/h = 4.0
толстая_плита_a_h_8.SPR	Design model for the plate side-to-thickness ratios a/h = 8.0

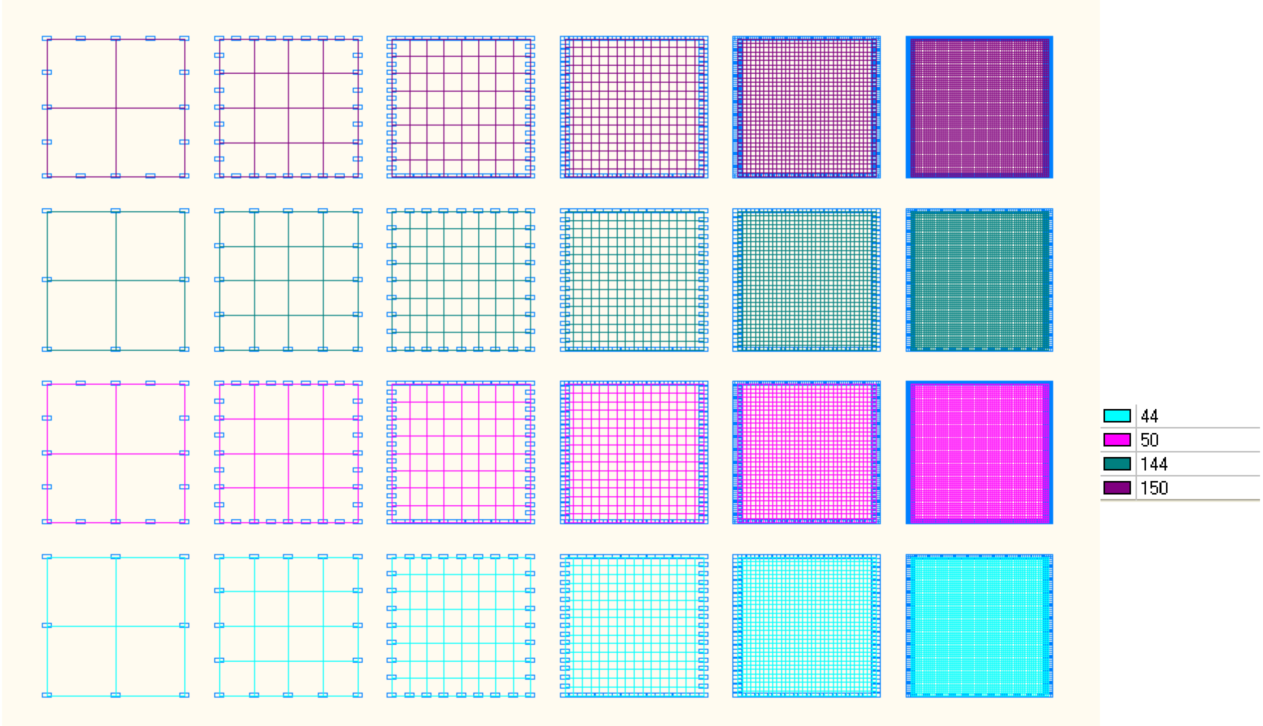
Problem formulation:
The simply supported thick square plate is subjected to a uniformly distributed transverse load *p*. Determine the deflection *w* in the center of the plate taking into account the transverse shear deformations.

References: L. G. Donnell, Beam, Plates, and Shells, Moscow, Nauka, 1982, p. 313-316.

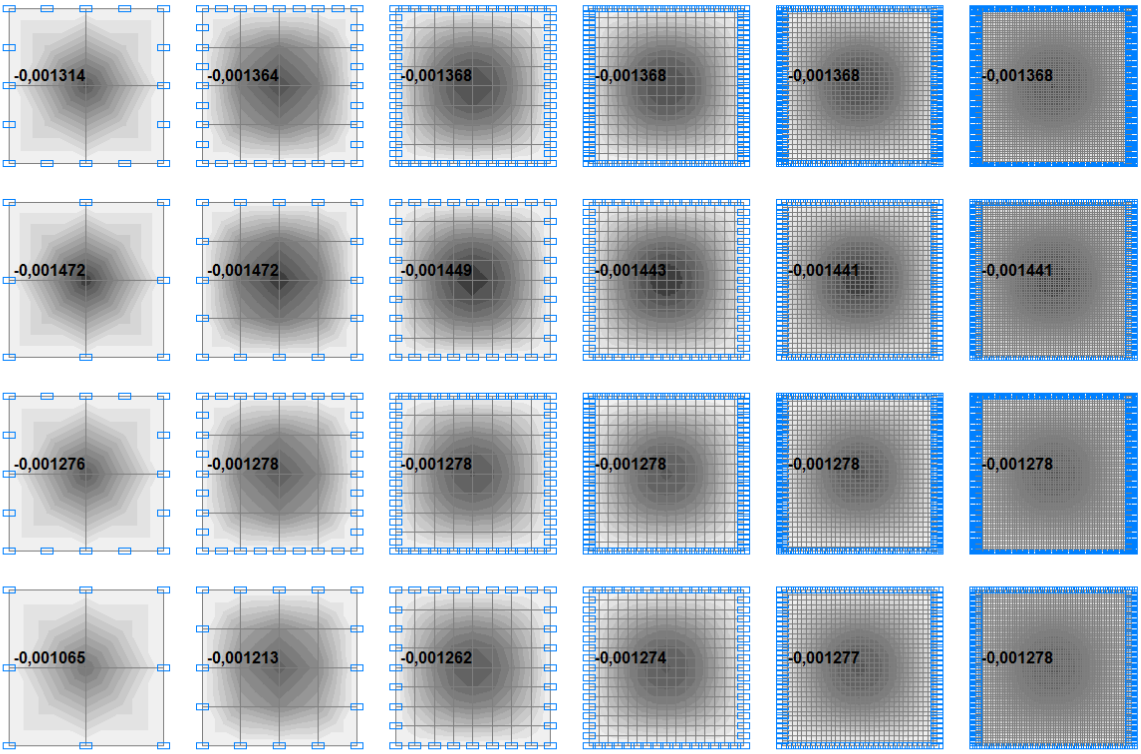
Initial data:
$E = 3.0 \cdot 10^7$ kPa - elastic modulus,
ν = 0.2 - Poisson's ratio,
h = 2.0; 4.0; 8.0 m - thickness of the plate;
a = 16.0 m - side of the plate;
$p = 100.0$ kN/m^2 - value of the uniformly distributed transverse load.

Finite element model:
Three design models for the following plate side-to-thickness ratios a/h = 8.0; 4.0; 2.0 are considered. Four variants of each model with the following types of finite elements are considered: 44, 50 – quadrangular four-node and eight-node thin shell elements for the calculation according to the Kirchhoff-Love theory; 144, 150 – quadrangular four-node and eight-node thick shell elements for the calculation according to the Reissner–Mindlin theory.
Design models are created for the following meshes: 2x2; 4x4; 8x8; 16x16; 32x32; 64x64.
Boundary conditions are provided by imposing constraints in the directions of the degrees of freedom X, Y, Z, UY for the edges along the X axis of the global coordinate system, and X, Y, Z, UX for the edges along the Y axis of the global coordinate system.

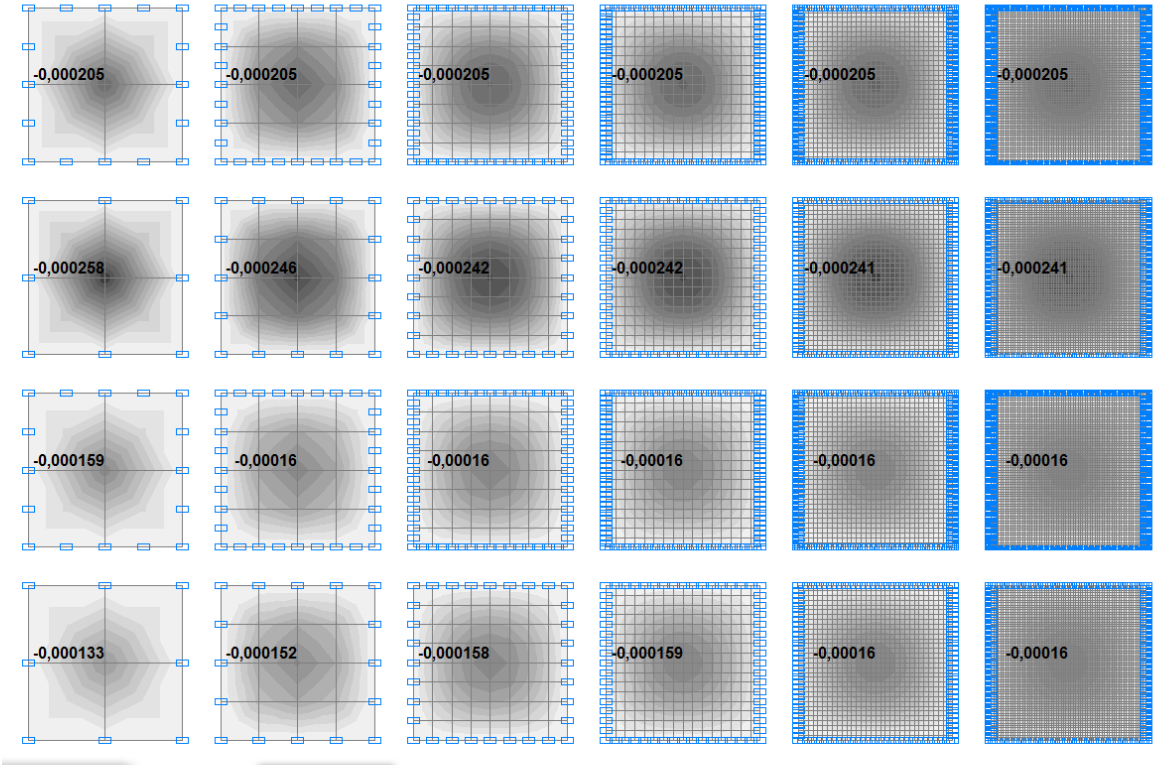
Results in SCAD



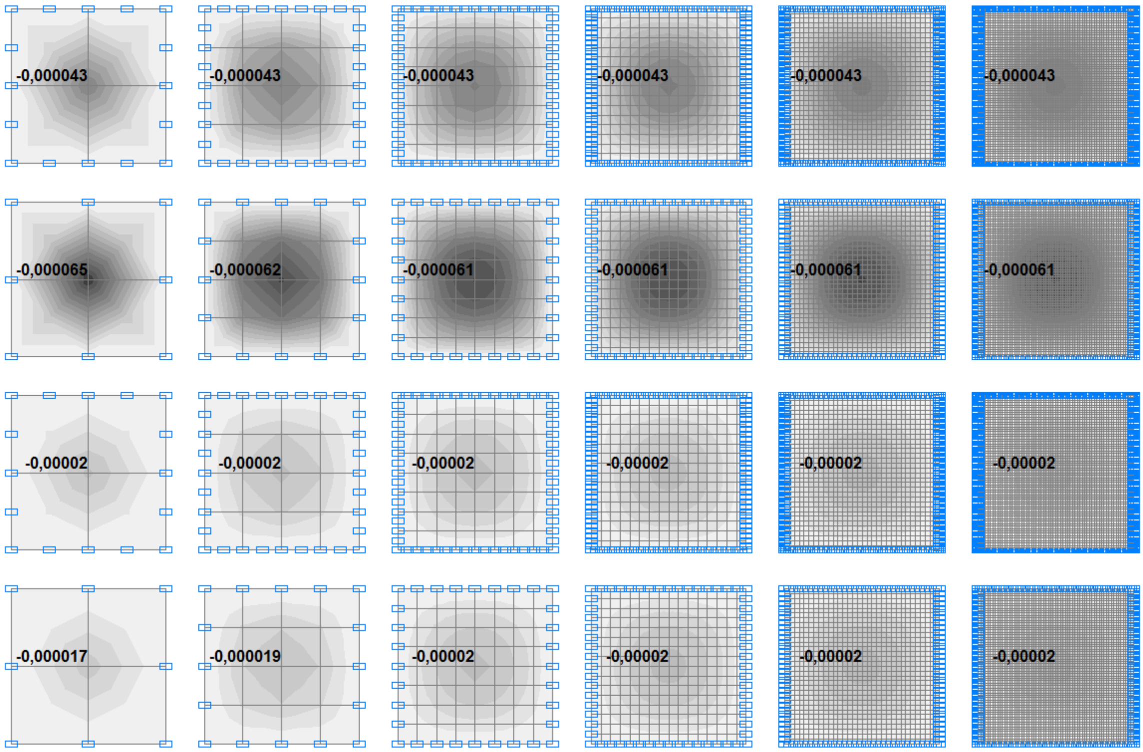
Design models



Deflections w of plates with the ratio a/h = 8.0, m



Deflections w of plates with the ratio a/h = 4.0, m



Deflections w of plates with the ratio a/h = 2.0, m

Comparison of solutions:

Deflections *w* in the center of the plates with the ratio a/h = 8.0, m

Member type	SCAD, mesh						Theory	Deviation
	2x2	4x4	8x8	16x16	32x32	64x64		
44	0.001065	0.001213	0.001262	0.001274	0.001277	0.001278	0.001278	0.00 %
50	0.001276	0.001278	0.001278	0.001278	0.001278	0.001278		0.00 %
144	0.001472	0.001472	0.001449	0.001443	0.001441	0.001441	0.001369	5.26 %
150	0.001314	0.001364	0.001368	0.001368	0.001368	0.001368		0.07 %

Deflections *w* in the center of the plates with the ratio a/h = 4.0, m

Member type	SCAD, mesh						Theory	Deviation
	2x2	4x4	8x8	16x16	32x32	64x64		
44	0.000133	0.000152	0.000158	0.000159	0.000160	0.000160	0.000160	0.00 %
50	0.000159	0.000160	0.000160	0.000160	0.000160	0.000160		0.00 %
144	0.000258	0.000246	0.000242	0.000242	0.000241	0.000241	0.000205	17.56 %
150	0.000205	0.000205	0.000205	0.000205	0.000205	0.000205		0.00 %

Deflections *w* in the center of the plates with the ratio a/h = 2.0, m

Member type	SCAD, mesh						Theory	Deviation
	2x2	4x4	8x8	16x16	32x32	64x64		
44	0.000017	0.000019	0.000020	0.000020	0.000020	0.000020	0.000020	0.00 %
50	0.000020	0.000020	0.000020	0.000020	0.000020	0.000020		0.00 %
144	0.000065	0.000062	0.000061	0.000061	0.000061	0.000061	0.000043	41.86 %
150	0.000043	0.000043	0.000043	0.000043	0.000043	0.000043		0.00 %

Notes: In the analytical solution the deflections w in the center of the plate are determined according to the following formulas:

without taking into account the transverse shear deformations

$$w=\frac{12\cdot\left(1-\nu^2\right)\cdot a^4\cdot p}{E\cdot h^3}\cdot\left[\frac{5}{384}-\sum_{m=1}^{\infty}\frac{sin\left(\frac{m\cdot\pi}{2}\right)\cdot\left(4+m\cdot\pi\cdot th\left(\frac{m\cdot\pi}{2}\right)\right)}{m^5\cdot\pi^5\cdot ch\left(\frac{m\cdot\pi}{2}\right)}\right] \text{ or}$$

$$w=\frac{192\cdot\left(1-\nu^2\right)\cdot a^4\cdot p}{\pi^6\cdot E\cdot h^3}\cdot\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left[\frac{1}{m\cdot n}\cdot\frac{1}{\left(m^2+n^2\right)^2}\cdot sin\left(\frac{m\cdot\pi}{2}\right)\cdot sin\left(\frac{n\cdot\pi}{2}\right)\right];$$

At $\nu = 0.2$ $\quad w\approx 0.004062\cdot\frac{a^4\cdot p}{D}$, where: $\quad D=\frac{E\cdot h^3}{12\cdot\left(1-\nu^2\right)}$.

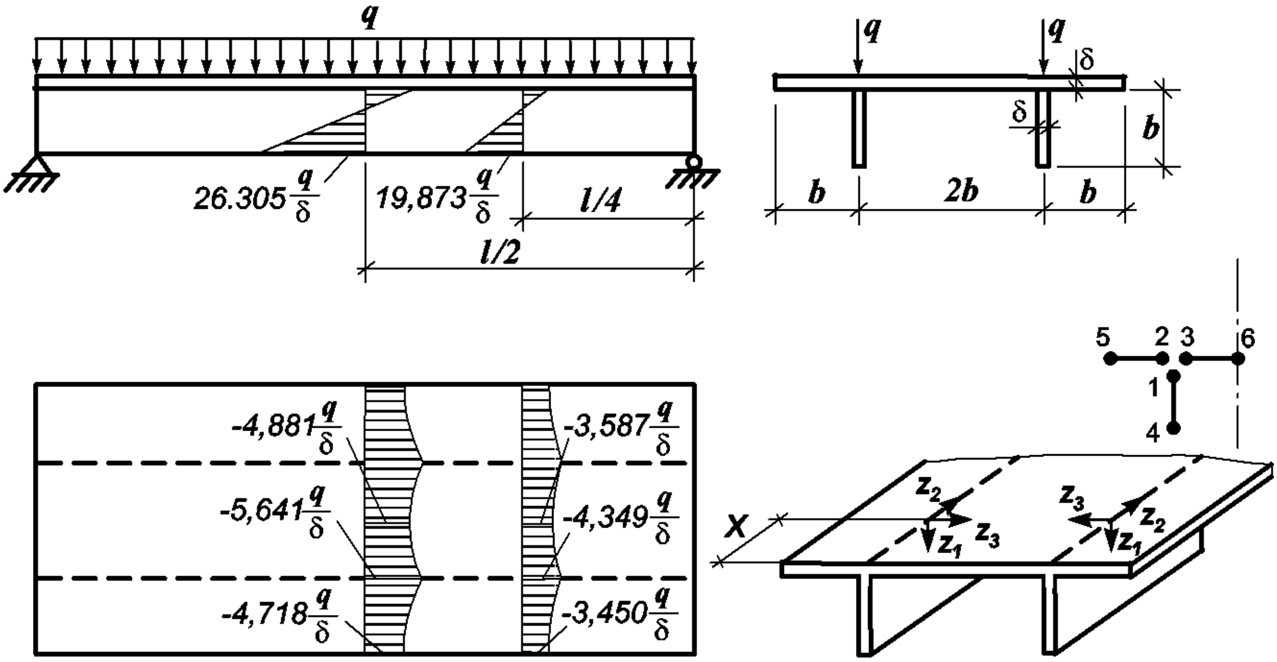
taking into account the transverse shear deformations

$$w=\frac{12\cdot\left(1-\nu^2\right)\cdot a^4\cdot p}{E\cdot h^3}\cdot\left[\frac{5}{384}-\sum_{m=1}^{\infty}\frac{sin\left(\frac{m\cdot\pi}{2}\right)\cdot\left(4+m\cdot\pi\cdot th\left(\frac{m\cdot\pi}{2}\right)\right)}{m^5\cdot\pi^5\cdot ch\left(\frac{m\cdot\pi}{2}\right)}+\frac{8-3\cdot\nu}{10\cdot(1-\nu)}\cdot\left(\frac{h}{a}\right)^2\cdot\left(\frac{1}{32}-\sum_{m=1}^{\infty}\frac{sin\left(\frac{m\cdot\pi}{2}\right)}{m^3\cdot\pi^3\cdot ch\left(\frac{m\cdot\pi}{2}\right)}\right)\right] \text{ or}$$

$$w=\frac{192\cdot\left(1-\nu^2\right)\cdot a^4\cdot p}{\pi^6\cdot E\cdot h^3}\cdot\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left[\frac{1}{m\cdot n}\cdot\frac{1}{\left(m^2+n^2\right)^2}\cdot\left[1+\frac{\pi^2\cdot h^2}{5\cdot(1-\nu)\cdot a^2}\cdot\left(m^2+n^2\right)\right]\cdot sin\left(\frac{m\cdot\pi}{2}\right)\cdot sin\left(\frac{n\cdot\pi}{2}\right)\right];$$

At $\nu = 0.2$ $\quad w\approx 0.004062\cdot\frac{a^4\cdot p}{D}\cdot\left[1+4.533786\cdot\left(\frac{h}{a}\right)^2\right]$, where: $\quad D=\frac{E\cdot h^3}{12\cdot\left(1-\nu^2\right)}$.

Two-Ribbed Beam Subjected to Uniformly Distributed Loads Applied in the Plane of the Ribs



Objective: Study of the distribution of the normal stresses in a two-ribbed beam subjected to uniformly distributed loads applied in the plane of the ribs.

Initial data file: **4.34.SPR**

Problem formulation: The two-ribbed beam simply supported by ideal end diaphragms rigid in their plane and compliant out of their plane is subjected to the loads *q* uniformly distributed along the line along the ribs and applied in their plane. Determine the normal stresses σ_{xi} acting along the beam in the elements of its structure in the points of the cross-section i = 1, 4, 5, 6 for the half (*l*/2) and quarter (*l*/4) of the beam span taking into account the following assumptions made when deriving the analytical solution:

- Bending deformations of the elements of the beam structure out of their plane are neglected;
- It is assumed that there are no displacements in the horizontal plane in the direction across the beam at the joints between the ribs and the flange;
- The difference between the stresses in the structural elements of the beam at the joints between the ribs and the flange is not taken into account.

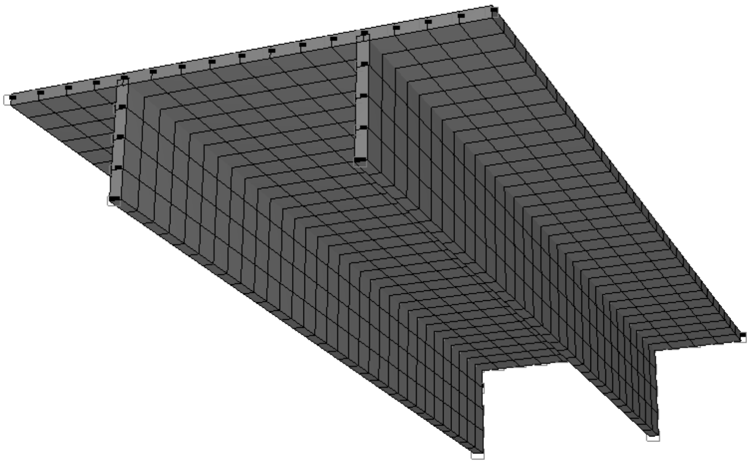
References: A. V. Aleksandrov, B. Ya. Lashchenikov, N. N. Shaposhnikov, Structural Mechanics. Thin-Walled Spatial Systems. — Moscow: Stroyizdat, 1983.

Initial data:

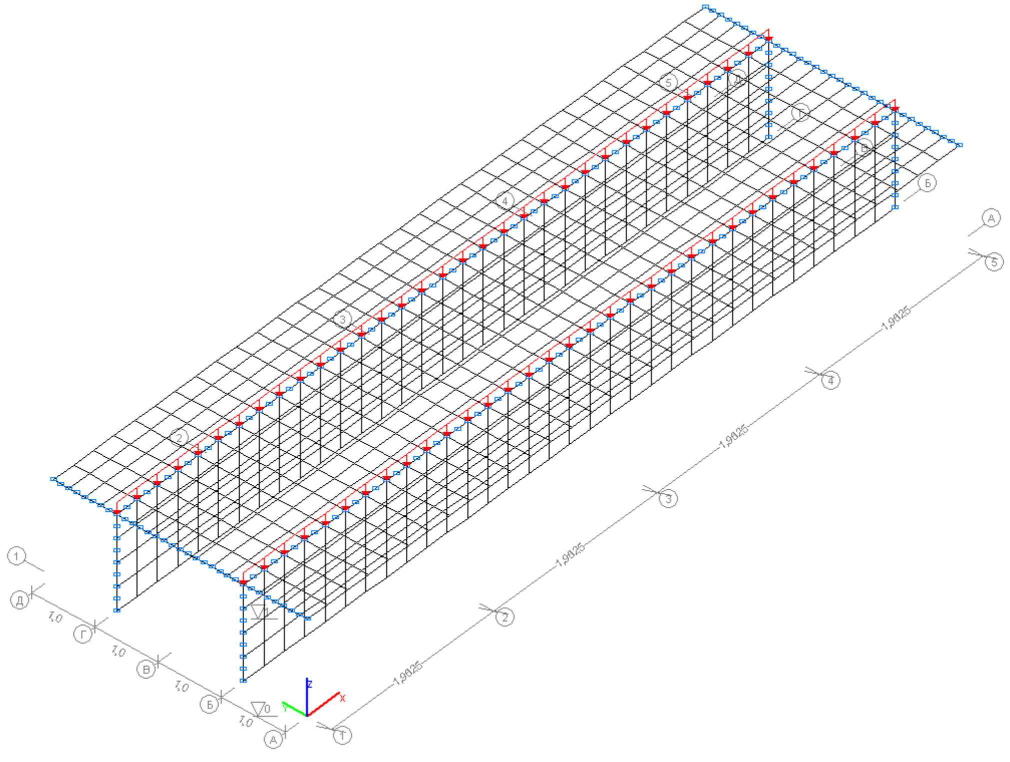
$E = 3\cdot10^7$ kPa	- elastic modulus;
$\mu = 0.15$	- Poisson's ratio;
$\delta = 0.1$ m	- thickness of the ribs and the flange;
b = 1.0 m	- height of the ribs;
2·b = 2.0 m	- distance between the ribs;
4·b = 4.0 m	- width of the flange;
l = 7.85·b = 7.85 m	- length of the beam;
q = 10.0 kN/m	- load uniformly distributed along the line along the ribs.

Finite element model: Design model – general type system, beam elements – 768 eight-node grade beam elements of type 27. The spacing of the finite element mesh in the direction across the beam is 0.25 m and in the direction along the beam is 0.2453125 m. The direction of the output of internal forces is along the OX axis of the global coordinate system. Number of nodes in the design model – 2417.

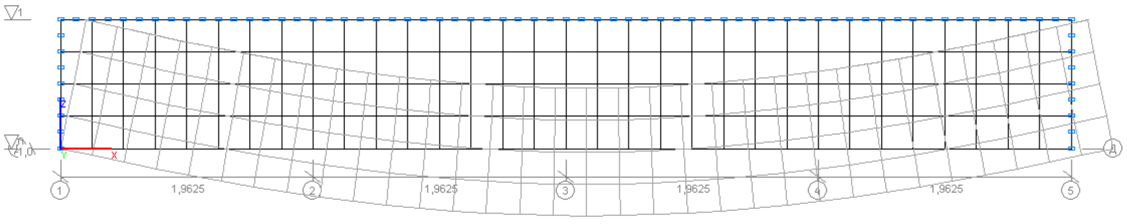
Results in SCAD



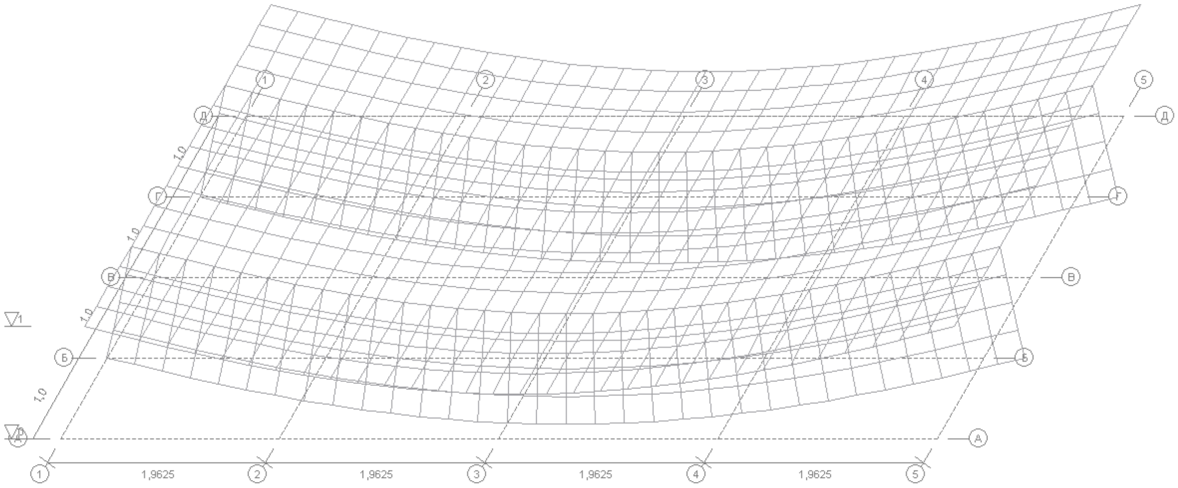
Design model



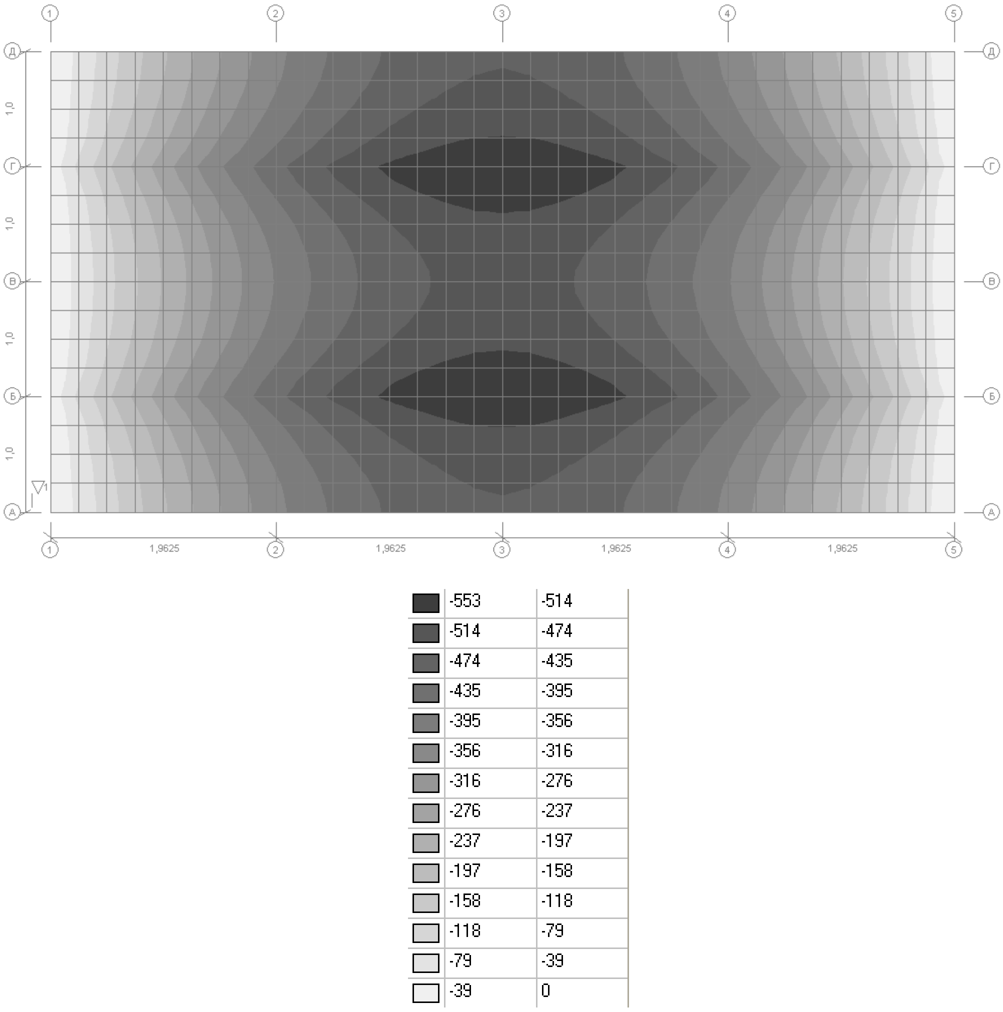
Design model



Deformed model



Deformed model



Values of the normal stresses in the beam flange σ_{xi} (kN/m^2)

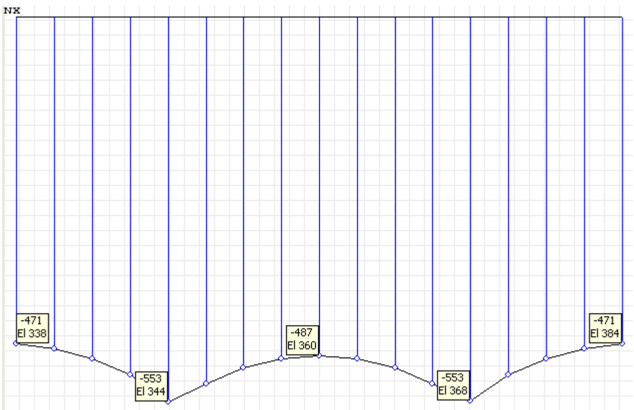


Diagram of the normal stresses in the beam flange σ_{xi} (kN/m2)
for the cross-section in the middle of the grade beam span l/2

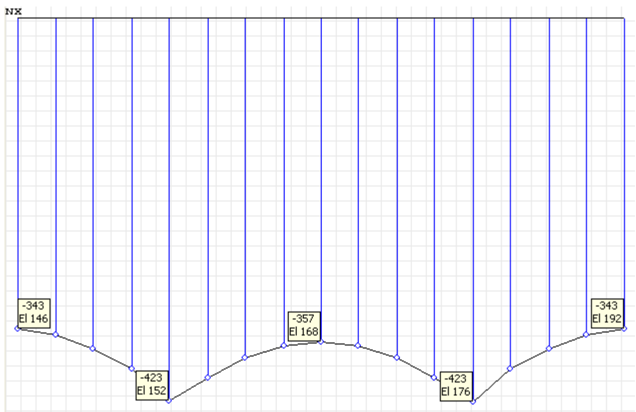
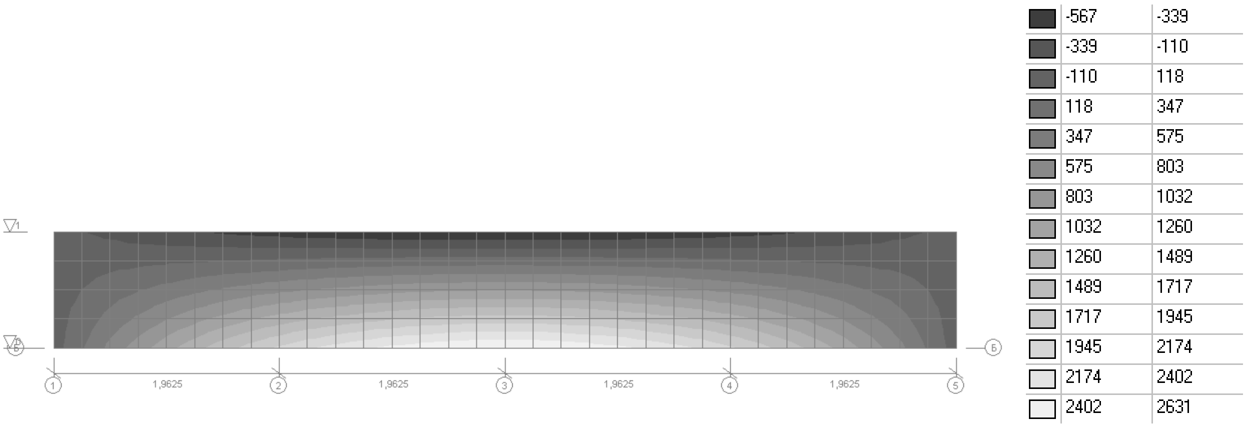


Diagram of the normal stresses in the beam flange σ_{xi} (kN/m2)
for the cross-section in the quarter of the grade beam span l/4



Values of the normal stresses in the beam rib σ_{xi} (kN/m2)

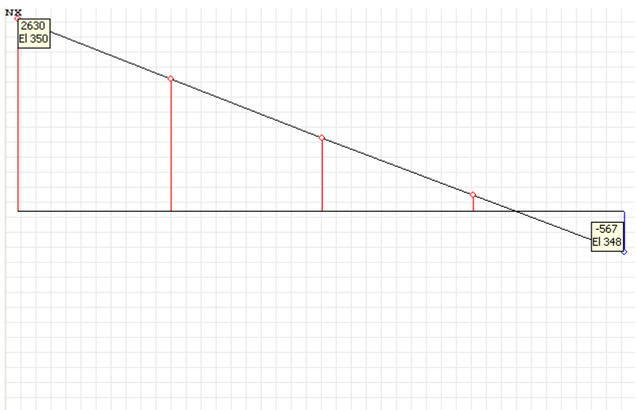


Diagram of the normal stresses in the beam rib σ_{xi} (kN/m2)
for the cross-section in the middle of the grade beam span l/2

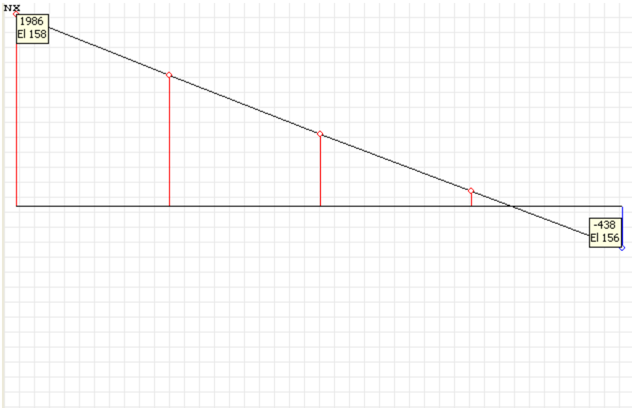


Diagram of the normal stresses in the beam rib σ_{xi} (kN/m2)
for the cross-section in the quarter of the grade beam span l/4

Comparison of solutions:

Normal stresses σ_{xi} (kN/m2) acting along the beam in the elements of its structure in the points of the cross-section i = 1, 4, 5, 6 for the half (*l*/2) and quarter (*l*/4) of the beam span

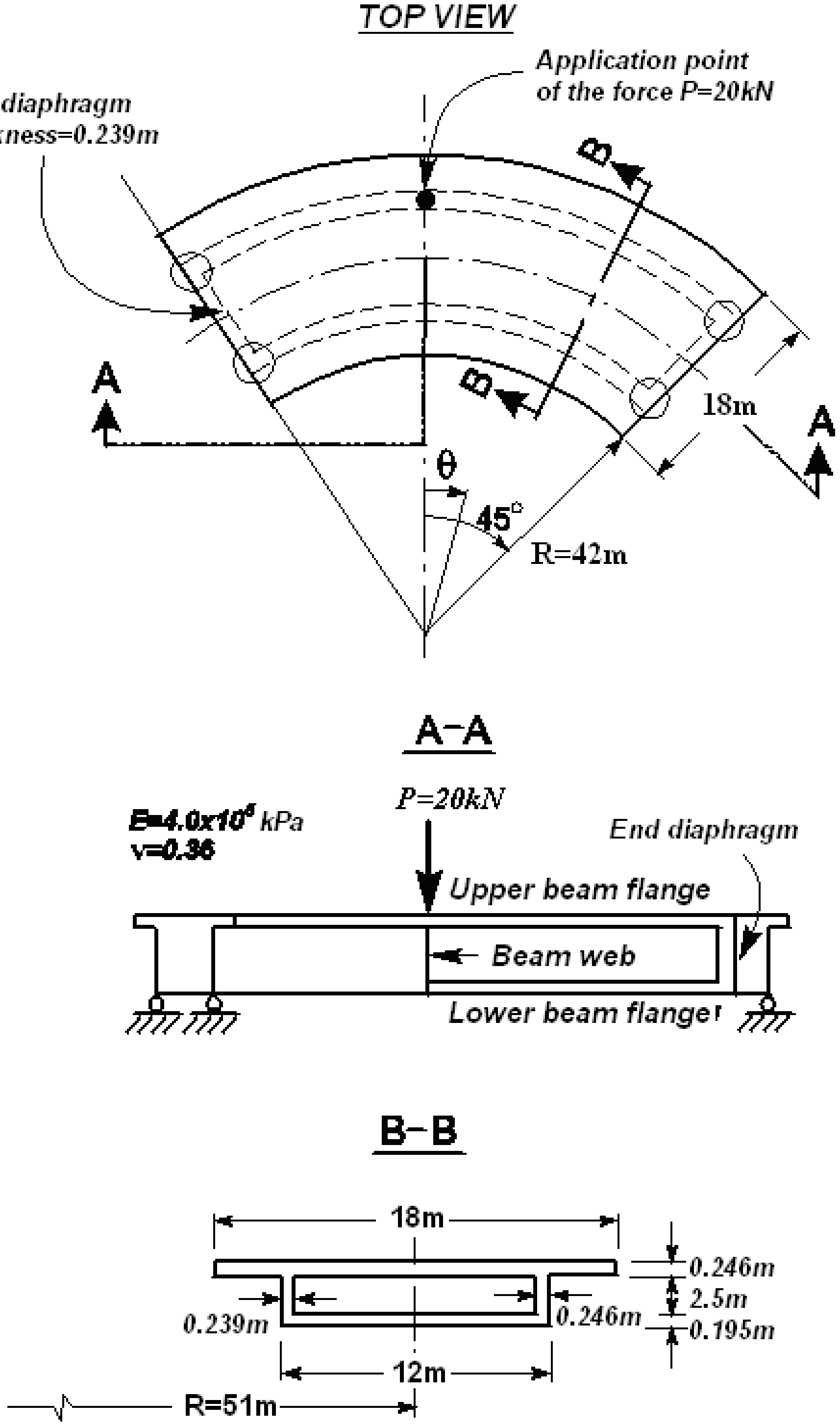
x, m	***l*/2 = 3.925**				***l*/4 = 1.9625**			
i	**1**	**4**	**5**	**6**	**1**	**4**	**5**	**6**
Theory	-564	2631	-472	-488	-435	1987	-345	-359
SCAD	-567	2631	-471	-487	-439	1989	-344	-358
Deviations, %	0.53	0.00	0.21	0.20	0.92	0.10	0.29	0.28

Notes: In the analytical solution the normal stresses σ_{xi} (kN/m2), acting along the beam in the elements of its structure in the points of the cross-section i = 1, 4, 5, 6 for the half (*l*/2) and quarter (*l*/4) of the beam span taking into account seven harmonics of unknown generalized displacements for $\mu = 0.15$ and $l = 7.85 \cdot b$ can be determined according to the following formulas (A. V. Aleksandrov, B. Ya. Lashchenikov, N. N. Shaposhnikov. Structural Mechanics. Thin-Walled Spatial Systems. — Moscow: Stroyizdat, 1983, p. 383):

$$\sigma_{x1}(l/2) = -5.641 \cdot \frac{q}{\delta}; \quad \sigma_{x4}(l/2) = 26.305 \cdot \frac{q}{\delta}; \quad \sigma_{x5}(l/2) = -4.718 \cdot \frac{q}{\delta}; \quad \sigma_{x6}(l/2) = -4.881 \cdot \frac{q}{\delta};$$

$$\sigma_{x1}(l/4) = -4.349 \cdot \frac{q}{\delta}; \quad \sigma_{x4}(l/4) = 19.873 \cdot \frac{q}{\delta}; \quad \sigma_{x5}(l/4) = -3.450 \cdot \frac{q}{\delta}; \quad \sigma_{x6}(l/4) = -3.587 \cdot \frac{q}{\delta};$$

Curved Hollow Section Beam of a Bridge Superstructure Subjected to a Concentrated Force



Objective: Study of the distribution of the tangential stresses and vertical displacements in a curved hollow section beam of a bridge superstructure subjected to a concentrated vertical force applied in the middle of the span above the outer web.

Initial data file: **4.35.SPR**

Problem formulation: The hollow section beam of a bridge superstructure with a longitudinal axis bent into a circular curve is simply supported by end diaphragms and subjected to a concentrated force *P* applied in the middle of the span above the outer web. Determine:

- distribution of the tangential stresses σ_x acting along the beam on the external surfaces and in the midplanes of the upper and lower flanges across the cross-section in the middle of the span;
- distribution of the tangential stresses σ_x, acting along the beam on the external surface of the lower flange along the longitudinal axis;
- distribution of the vertical displacements w across the lower faces of the outer and inner webs along the longitudinal axis.

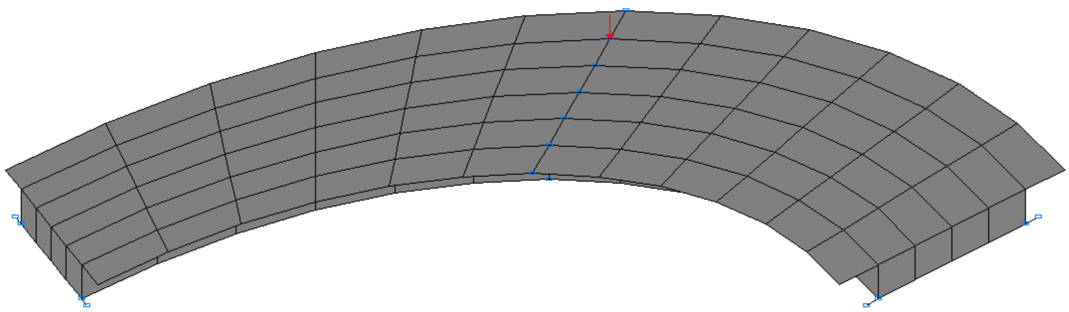
References: Worsak Kanok-Nukulchai, A simple and efficient finite element for general shell analysis, Int. J. num. meth. Engng, 14, 179-200 (1979); A.R.M. Fam and C. Turkstra, Model study of horizontally curved box girder, J. Engng Struct. Div., ASCE, 102, ST5, 1097-1108 (1976).

Initial data:

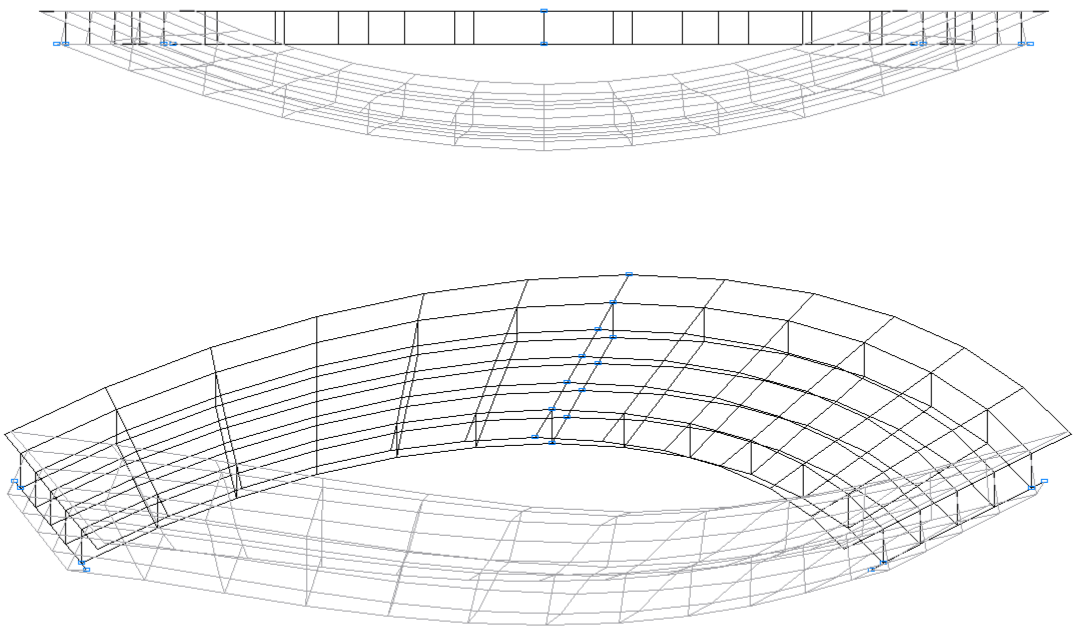
$E = 4.0 \cdot 10^5$ kPa	- elastic modulus;
$\nu = 0.36$	- Poisson's ratio;
R = 51.0 m	- radius of the longitudinal axis of the beam;
$\theta = 45º$	- central angle containing a half of the beam span;
P = 20 kN	- concentrated vertical force applied in the middle of the beam span above the outer web;
$b_{tf} = 18.0$ m	- width of the upper flange;
$t_{tf} = 0.246$ m	- thickness of the upper flange;
$b_{bf} = 12.0$ m	- width of the lower flange;
$t_{bf} = 0.195$ m	- thickness of the lower flange;
$h_{ew} = 2.5$ m	- height of the outer web along the inner flange surfaces;
$t_{ew} = 0.246$ m	- thickness of the outer web;
$h_{iw} = 2.5$ m	- height of the inner web along the inner flange surfaces;
$t_{iw} = 0.239$ m	- thickness of the inner web;
$t_{ed} = 0.239$ m	- thickness of the end diaphragm;

Finite element model: Design model – general type system, beam elements – 156 eight-node thick shell elements for the calculation according to the Reissner–Mindlin theory of type 150. The spacing of the finite element meshes of the upper and lower flanges in the radial direction is ~3.0 m and in the tangential direction is 7.5º. The spacing of the finite element meshes of the outer and inner webs in the vertical direction is ~2.7 m and in the tangential direction is 7.5º. The direction of the output of internal forces is radial tangential. Constraints providing simply supported conditions are installed in the vertical direction in the joints between the elements of the outer and inner webs and the elements of the end diaphragms and the lower flange. Constraints preventing the displacements of these joints in the horizontal plane in the radial direction are modeled by 4 bar elements of type 4 with the axial stiffness $EF = 4.0 \cdot 10^7$ kN and the end nodes constrained in all linear degrees of freedom. The dimensional stability of the design model in the tangential direction is provided by imposing constraints according to its symmetry conditions. Number of nodes in the design model – 466.

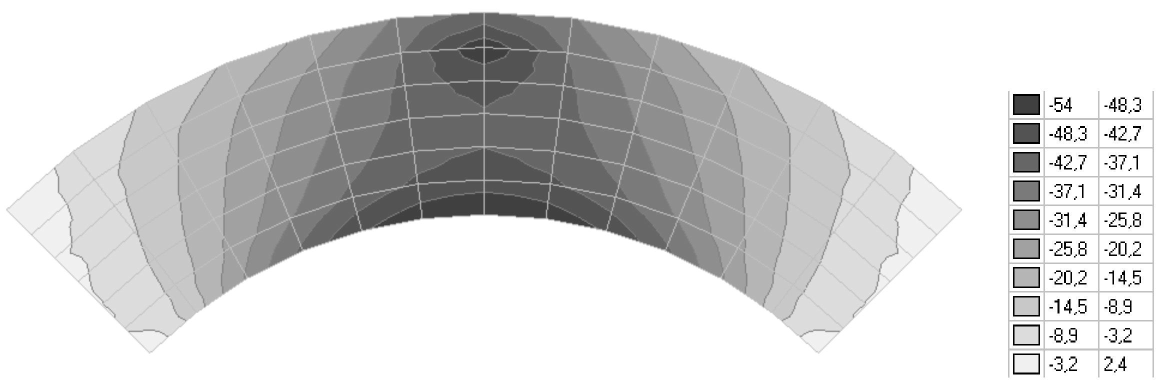
Results in SCAD



Design model



Deformed model



Values of tangential stresses σ_x, acting along the beam, in the midplane of the upper flange (kN/m^2)

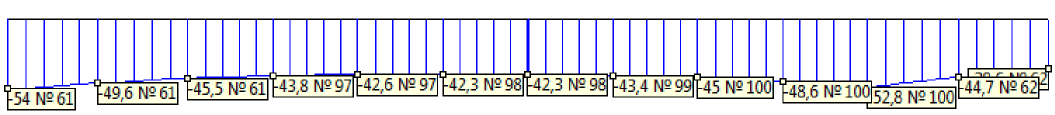
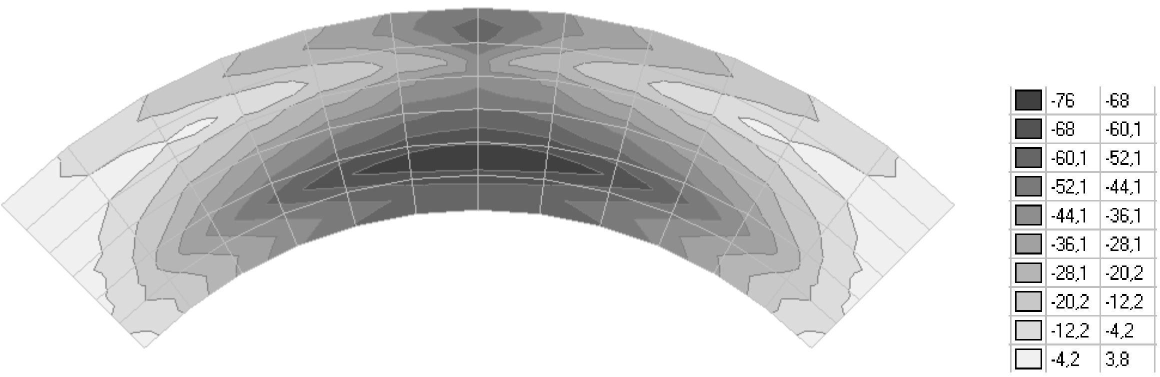


Diagram of the distribution of the tangential stresses σ_x, acting along the beam, in the midplane of the upper flange across the cross-section in the middle of the span (kN/m^2)



Values of tangential stresses σ_x, acting along the beam,
on the external surface of the upper flange (kN/m^2)

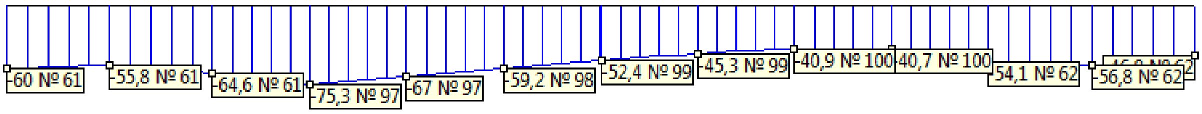
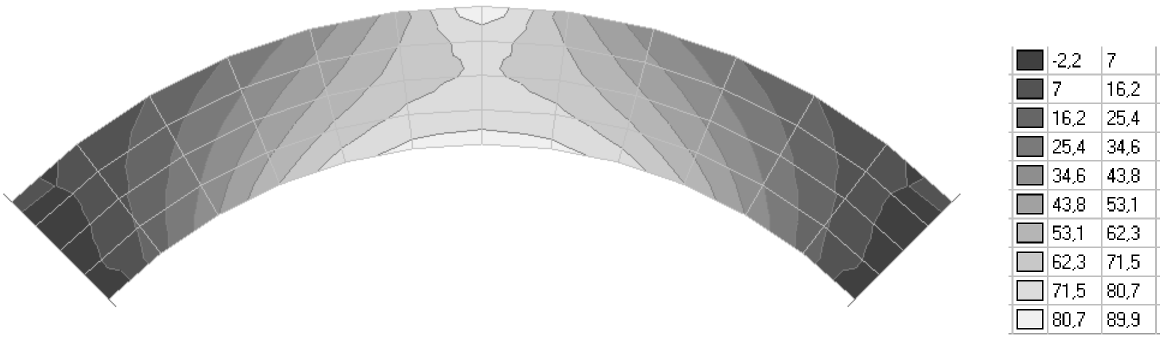


Diagram of the distribution of the tangential stresses σ_x, acting along the beam,
on the external surface of the upper flange across the cross-section in the middle of the span (kN/m^2)



Values of tangential stresses σ_x, acting along the beam,
in the midplane of the lower flange (kN/m^2)

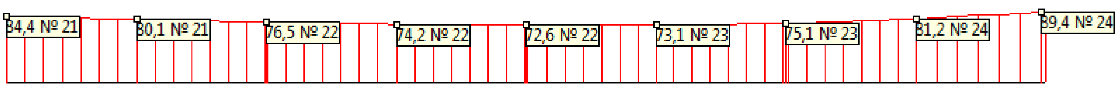
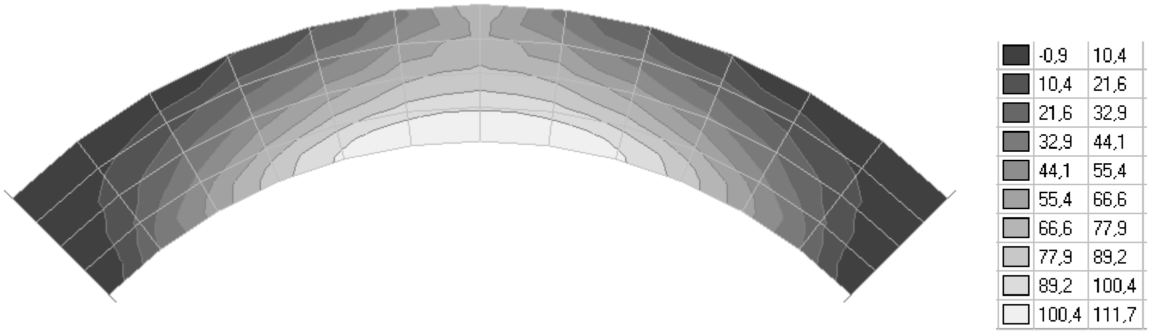


Diagram of the distribution of the tangential stresses σ_x, acting along the beam,
in the midplane of the lower flange across the cross-section in the middle of the span (kN/m^2)



Values of tangential stresses σ_x, acting along the beam, on the external surface of the lower flange (kN/m^2)

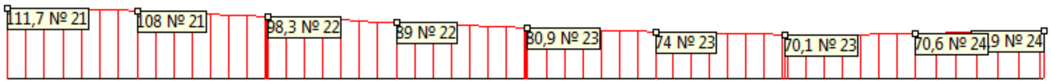
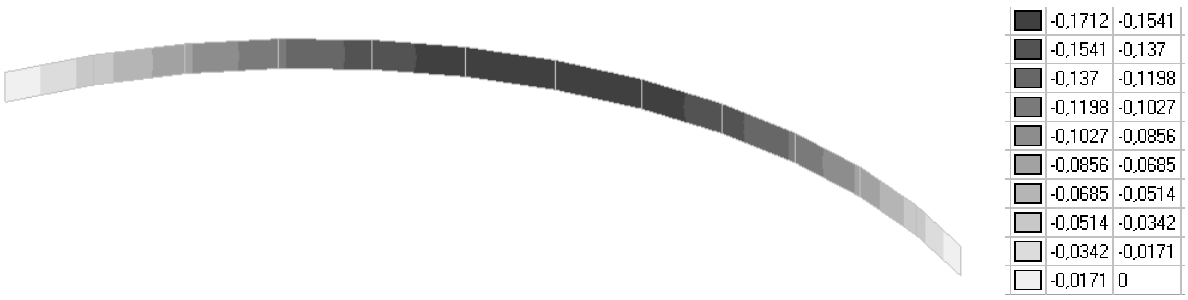
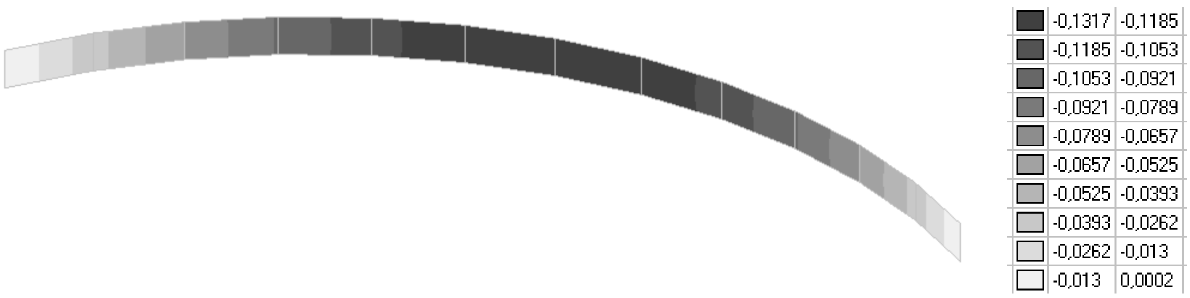


Diagram of the distribution of the tangential stresses σ_x, acting along the beam, on the external surface of the lower flange across the cross-section in the middle of the span (kN/m^2)

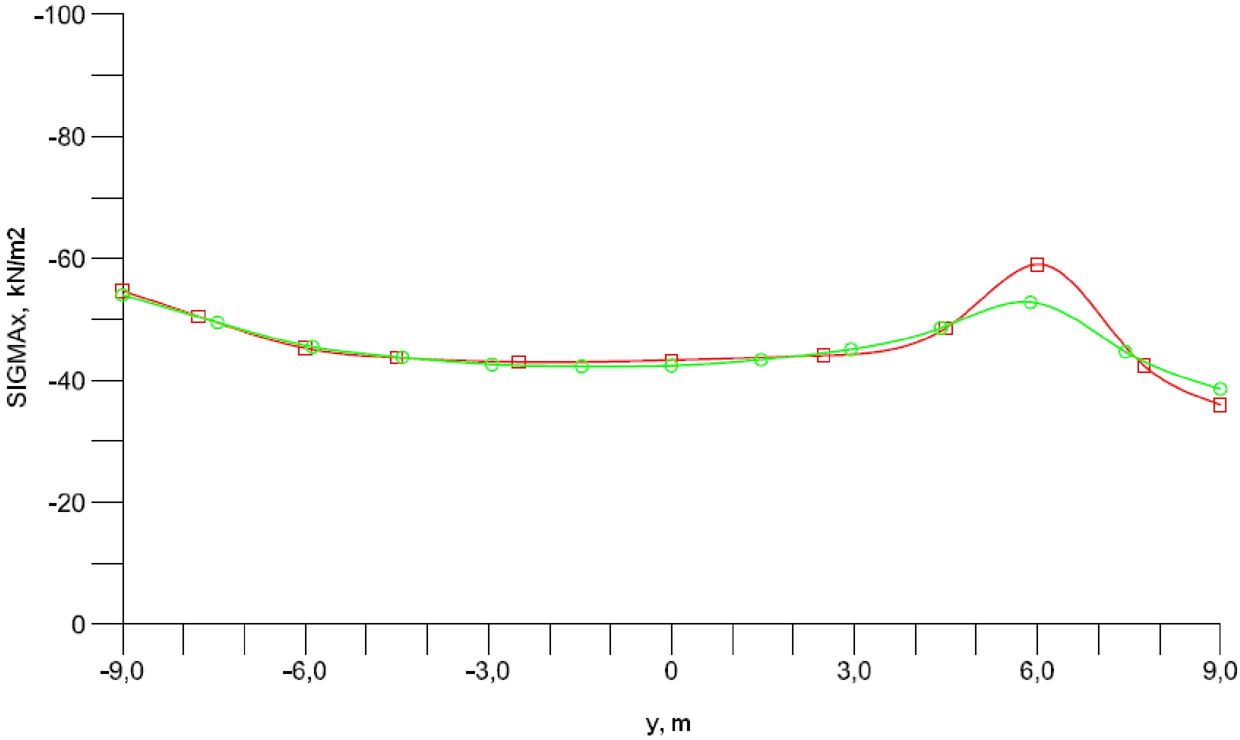


Values of vertical displacements w of the outer web (m)



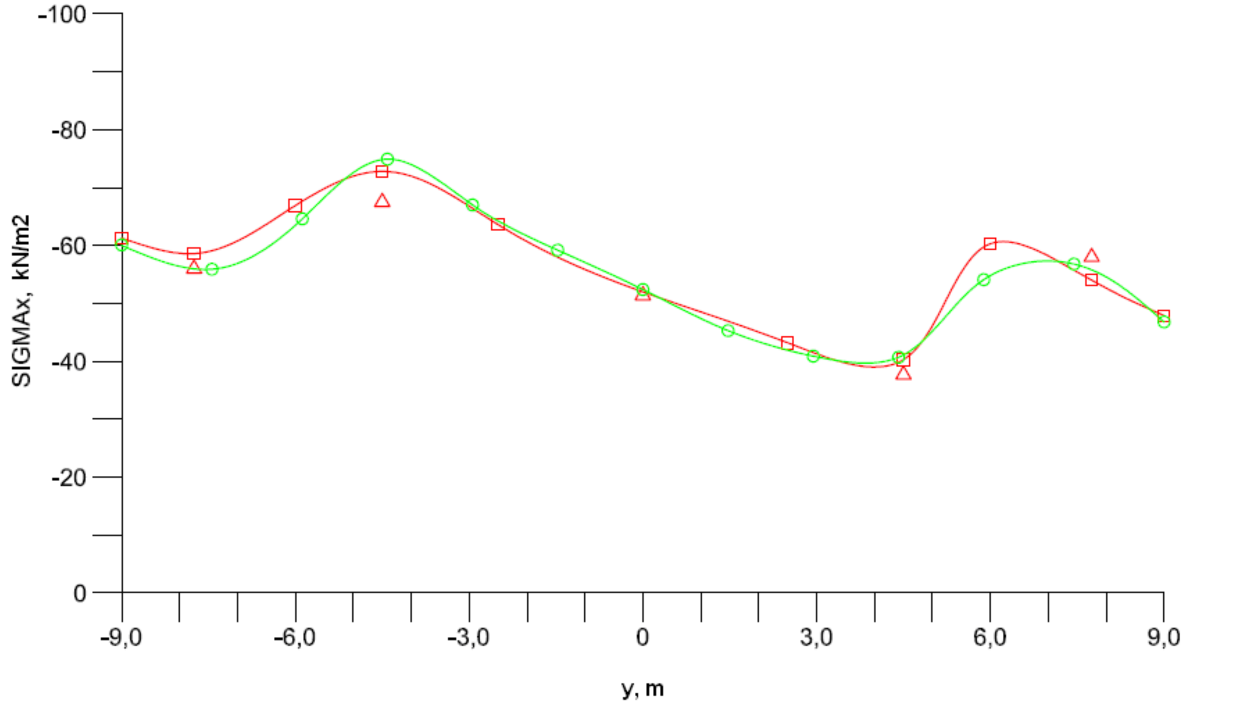
Values of vertical displacements w of the inner web (m)

Comparison of solutions:



Tangential stresses in the midplane of the upper flange in the middle of the span of the hollow section beam

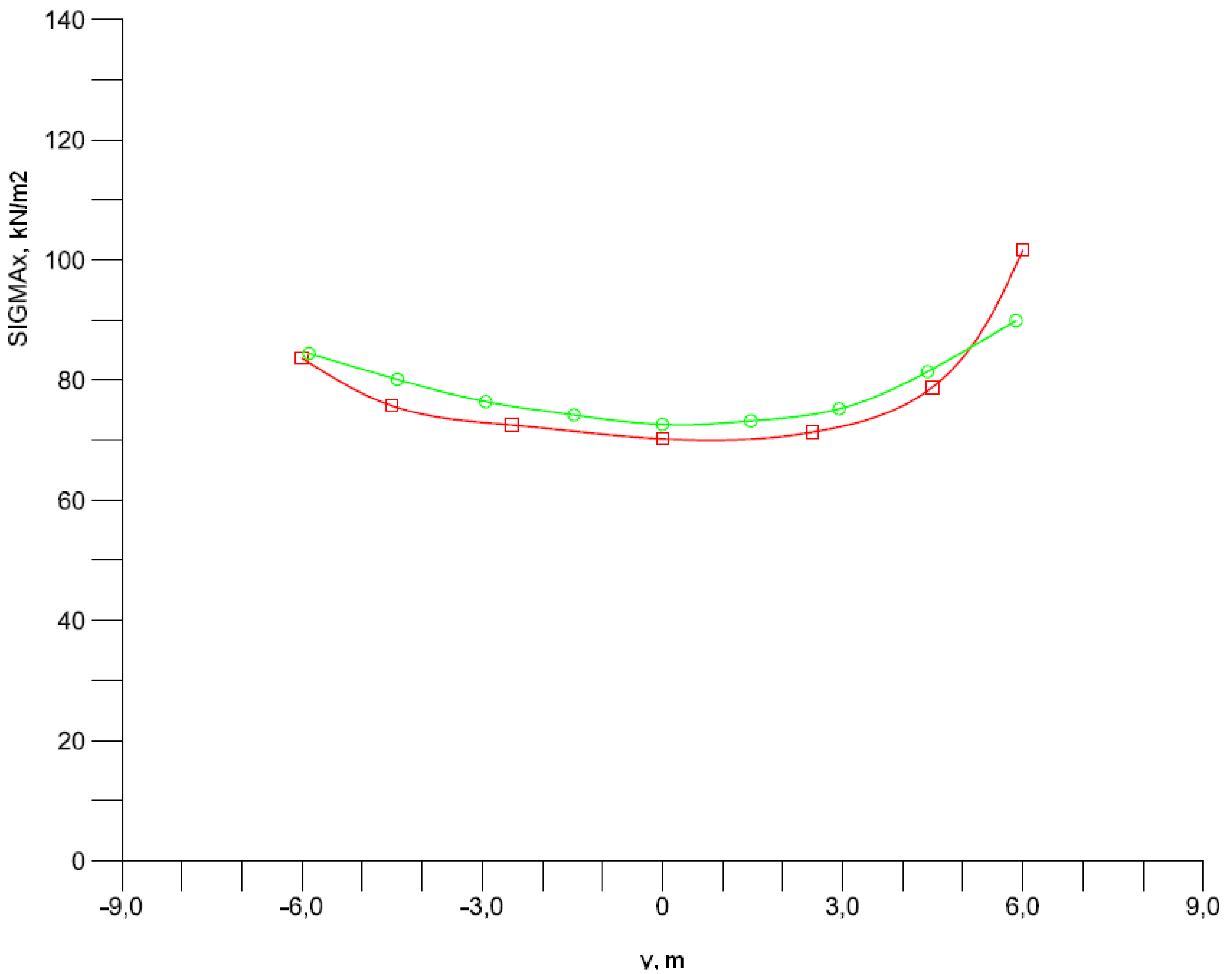
△ - results obtained experimentally (source)
□ - results obtained by calculation using high-order finite elements (source)
○ - results obtained by calculation (SCAD)



Tangential stresses on the external surface of the upper flange in the middle of the span of the hollow section beam

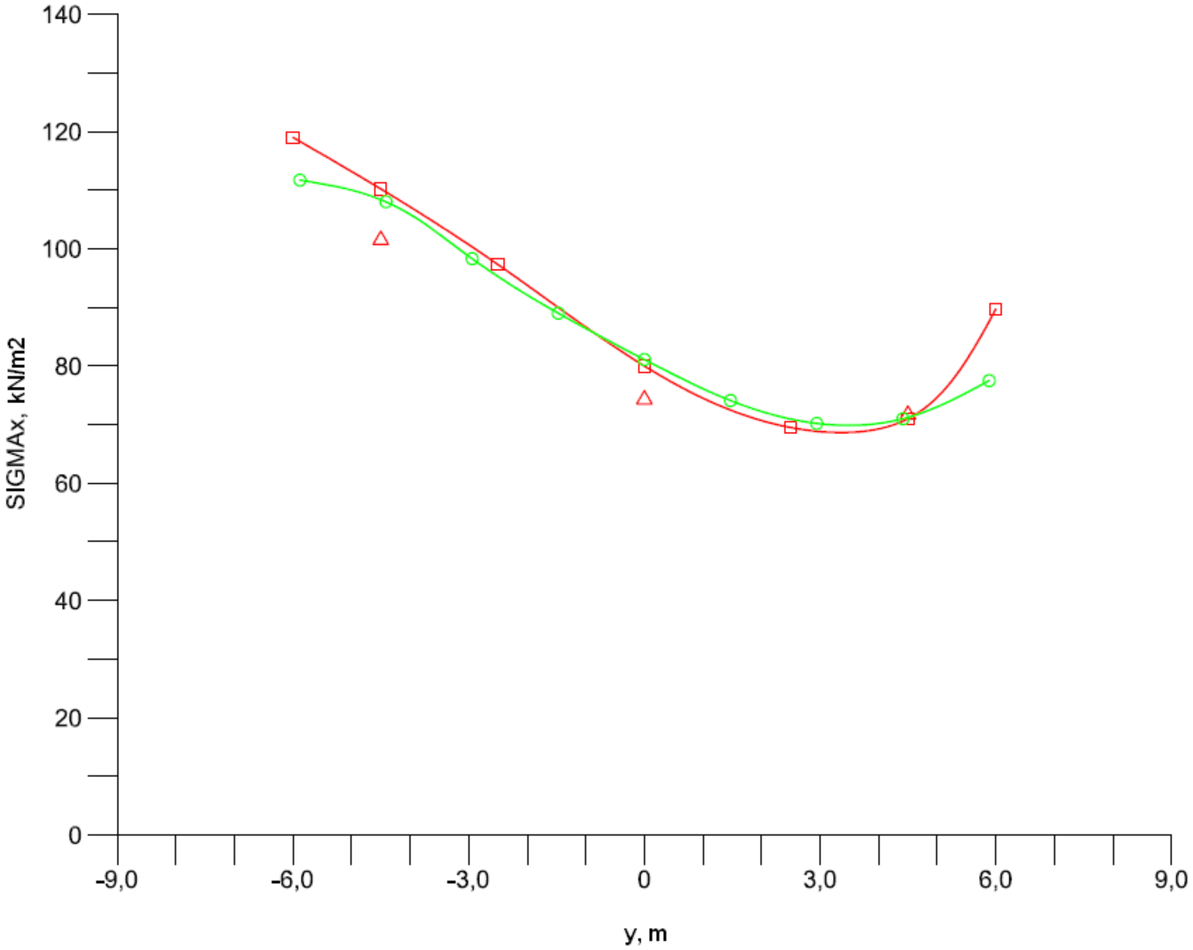
△ - results obtained experimentally (source)
□ - results obtained by calculation using high-order finite elements (source)
○ - results obtained by calculation (SCAD)

y	Experiment	SCAD	Deviations, %
-7.75	-56.0	-55.97	0.05
-4.50	-67.5	-74.82	10.84
0.00	-51.4	-52.40	1.95
4.50	-37.7	-41.10	9.02
7.75	-58.0	-55.79	3.81



Tangential stresses in the midplane of the lower flange in the middle of the span of the hollow section beam

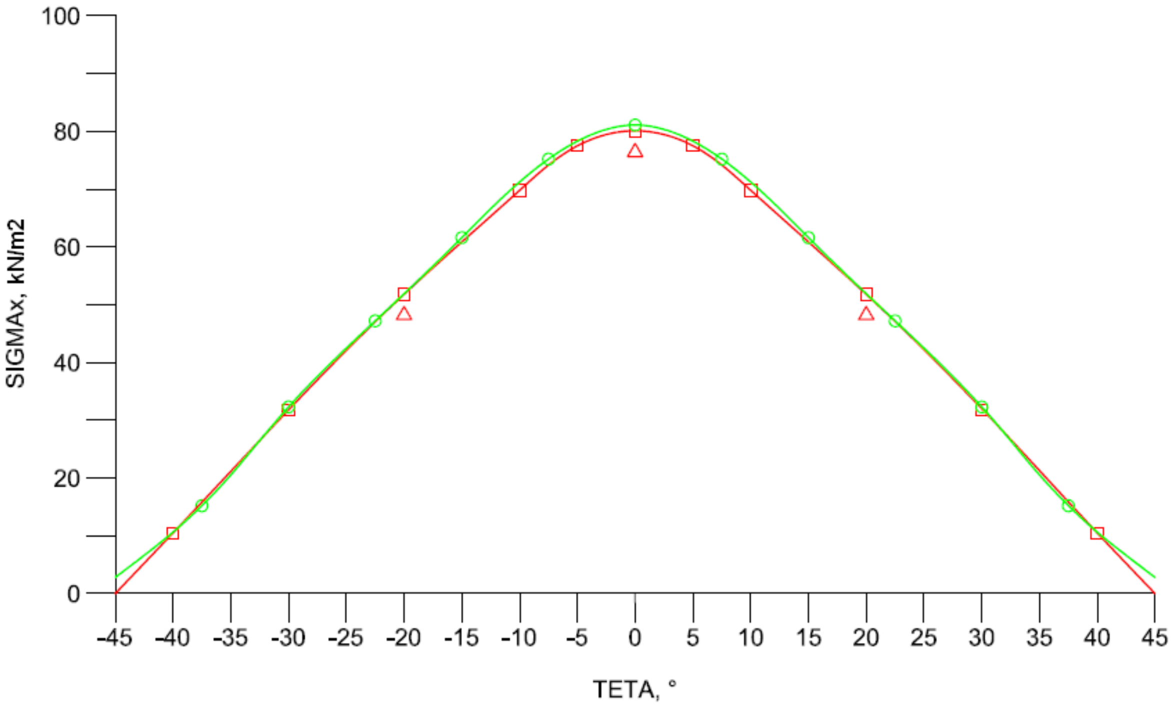
△ - results obtained experimentally (source)
□ - results obtained by calculation using high-order finite elements (source)
○ - results obtained by calculation (SCAD)



Tangential stresses on the external surface of the lower flange in the middle of the span of the hollow section beam

△ - results obtained experimentally (source)
□ - results obtained by calculation using high-order finite elements (source)
○ - results obtained by calculation (SCAD)

y	Experiment	SCAD	Deviations, %
-4.50	101.5	108.38	6.78
0.00	74.3	81.10	9.15
4.50	71.7	71.23	0.66



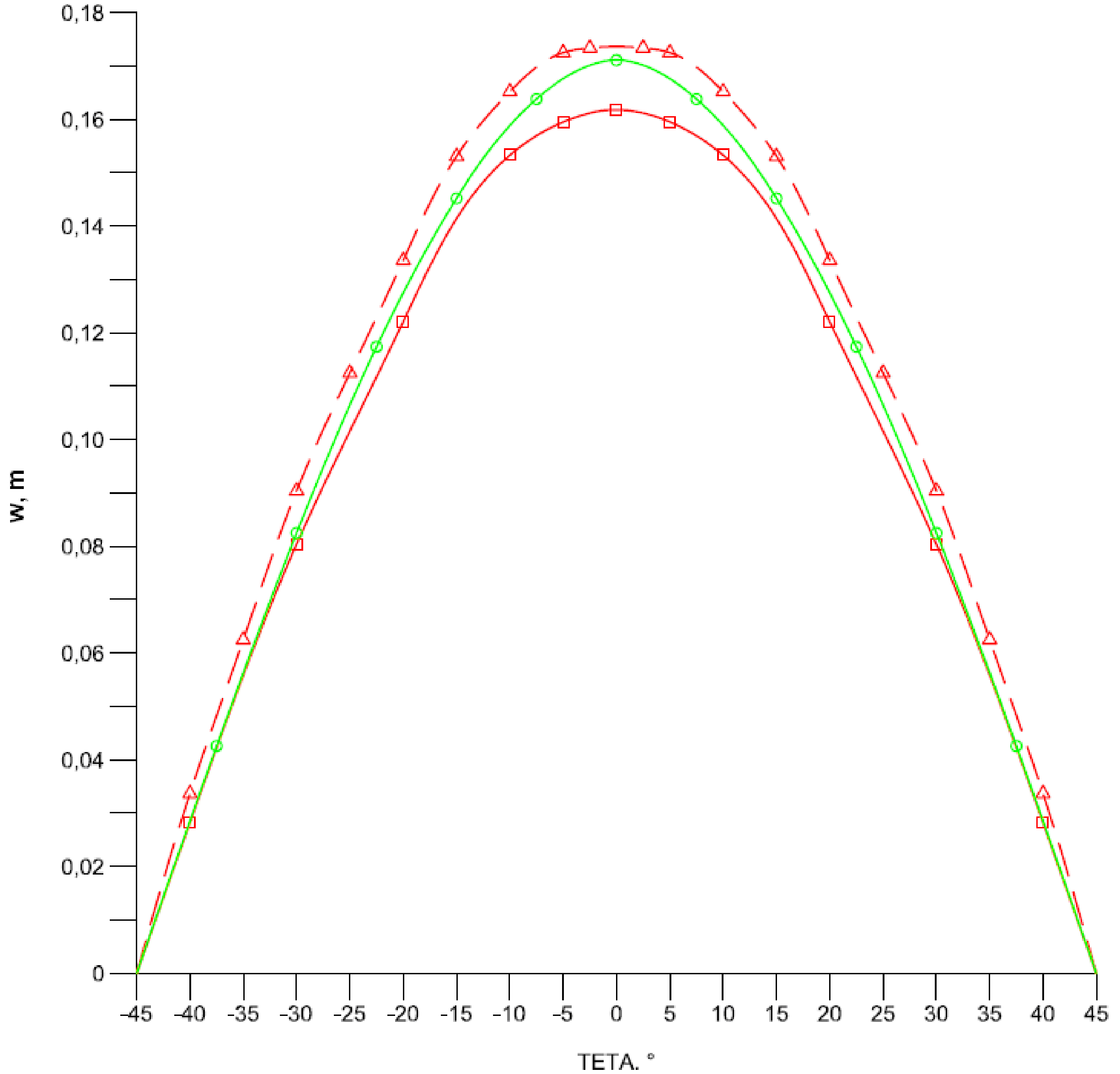
Tangential stresses on the external surface of the lower flange along the axis of the span of the hollow section beam

△ - results obtained experimentally (source)

□ - results obtained by calculation using high-order finite elements (source)

○ - results obtained by calculation (SCAD)

θ	Experiment	SCAD	Deviations, %
-20	48.2	51.91	7.70
0	76.4	81.10	6.15
20	48.2	51.91	7.70



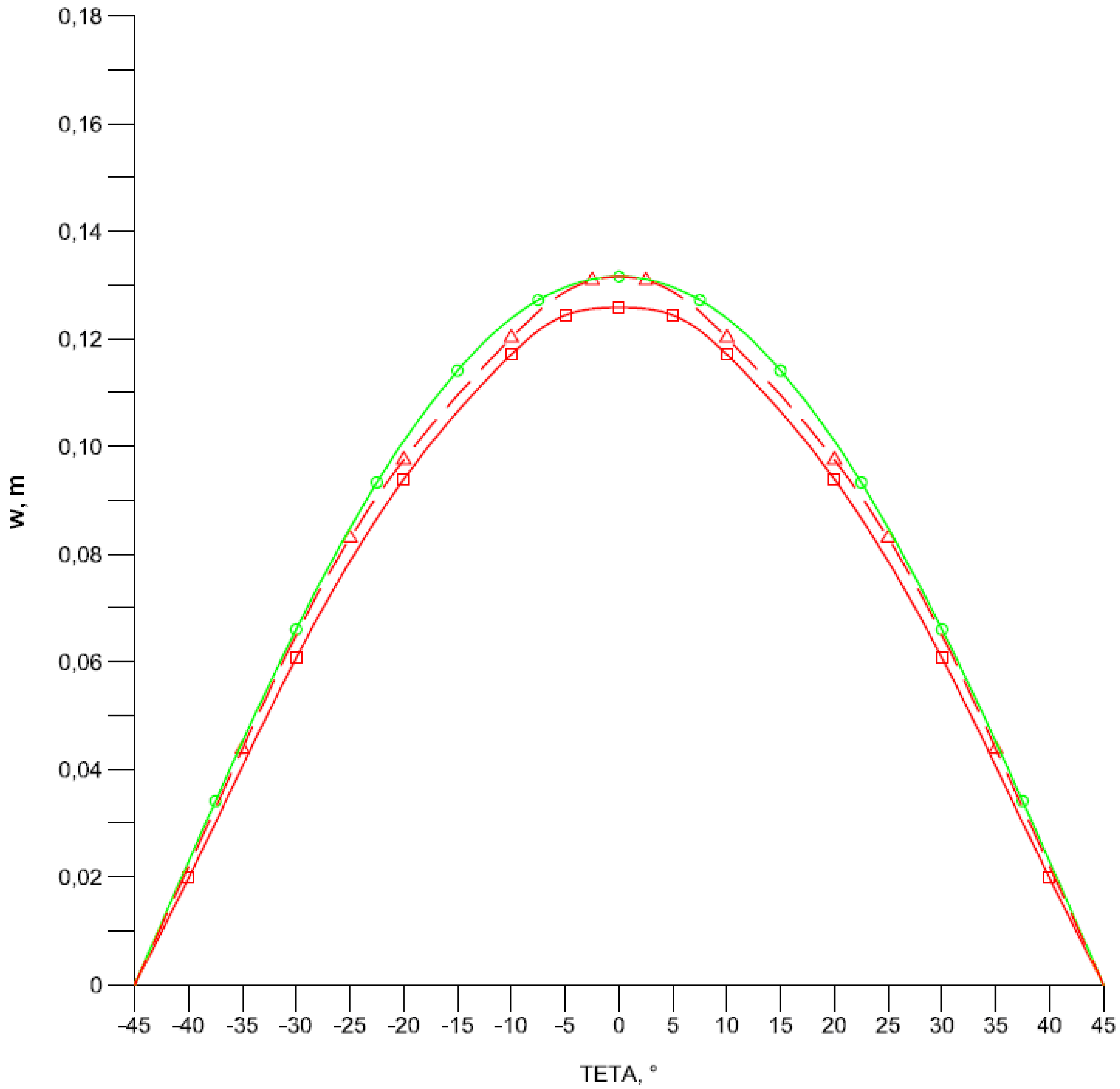
Vertical deflections along the lower face of the outer web along the axis of the span of the hollow section beam

△ - results obtained experimentally (source)

□ - results obtained by calculation using high-order finite elements (source)

○ - results obtained by calculation (SCAD)

θ	Experiment	SCAD	Deviations, %
-35	0.0625	0.05631	9.90
-30	0.0904	0.08250	8.74
-25	0.1125	0.10650	5.33
-20	0.1336	0.12750	4.57
-15	0.1531	0.14520	5.16
-10	0.1652	0.15887	3.83
-5	0.1725	0.16762	2.83
-2.5	0.1734	0.17016	1.87
2.5	0.1734	0.17016	1.87
5	0.1725	0.16762	2.83
10	0.1652	0.15887	3.83
15	0.1531	0.14520	5.16
20	0.1336	0.12750	4.57
25	0.1125	0.10650	5.33
30	0.0904	0.08250	8.74
35	0.0625	0.05631	9.90

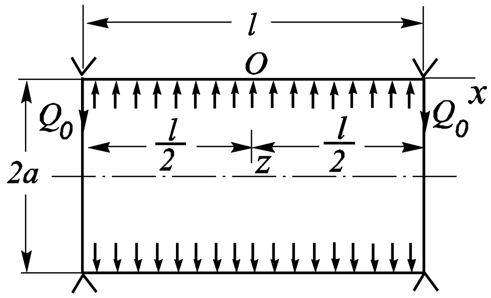


Vertical deflections along the lower face of the inner web
along the axis of the span of the hollow section beam

△ - results obtained experimentally (source)
□ - results obtained by calculation using high-order finite elements (source)
○ - results obtained by calculation (SCAD)

θ	Experiment	SCAD	Deviations, %
-35	0.0438	0.04510	2.97
-25	0.0830	0.08487	2.25
-20	0.0975	0.10101	3.60
-10	0.1202	0.12384	3.03
-2.5	0.1309	0.13108	0.14
2.5	0.1309	0.13108	0.14
10	0.1202	0.12384	3.03
20	0.0975	0.10101	3.60
25	0.0830	0.08487	2.25
35	0.0438	0.04510	2.97

Cylindrical Shell with Simply Supported Edges Subjected to Uniform Internal Pressure



Objective: Determination of the stress-strain state of a cylindrical shell with simply supported edges subjected to the internal pressure.

Initial data file: **4.31.SPR**

Problem formulation: The cylindrical thin-walled shell simply supported along the edges is subjected to uniform internal pressure *p*. Determine the bending moments and longitudinal forces acting on the midsurface of the shell in the meridian M_x, N_x and circumferential M_φ, N_φ directions, as well as the radial displacements *w* for the cross-section in the middle of the span.

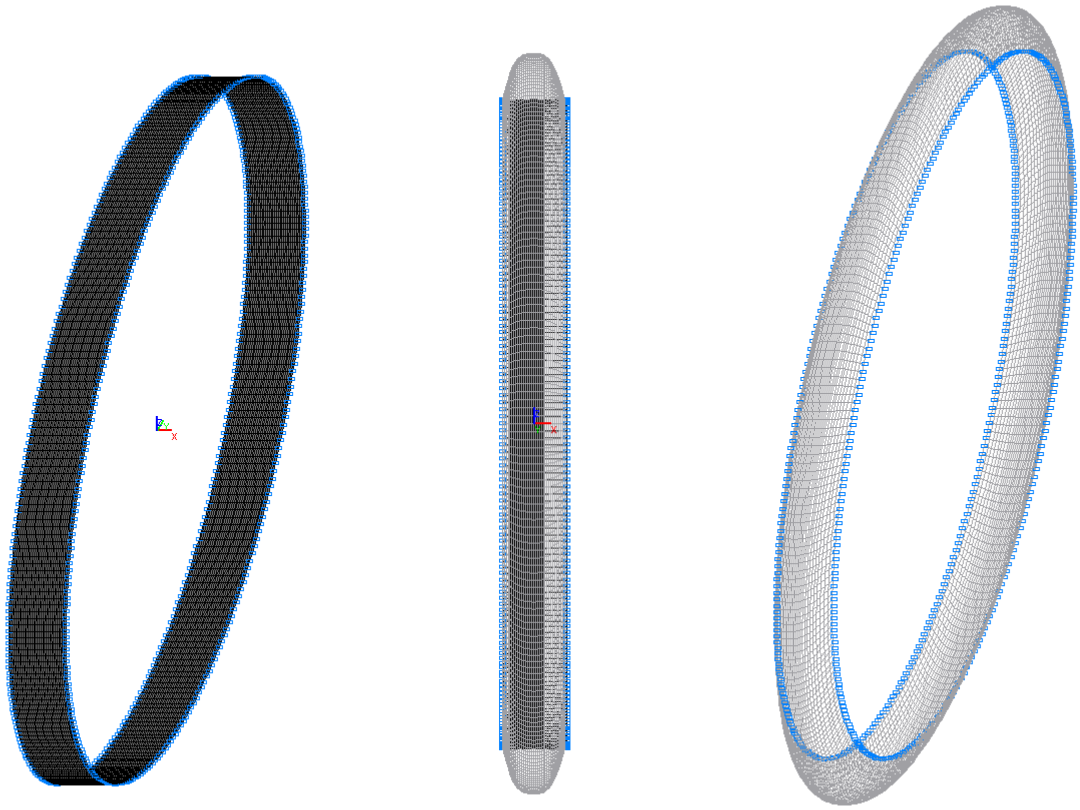
References: S.P. Timoshenko, Theory of Plates and Shells. — Moscow: OGIZ. Gostekhizdat, 1948.

Initial data:

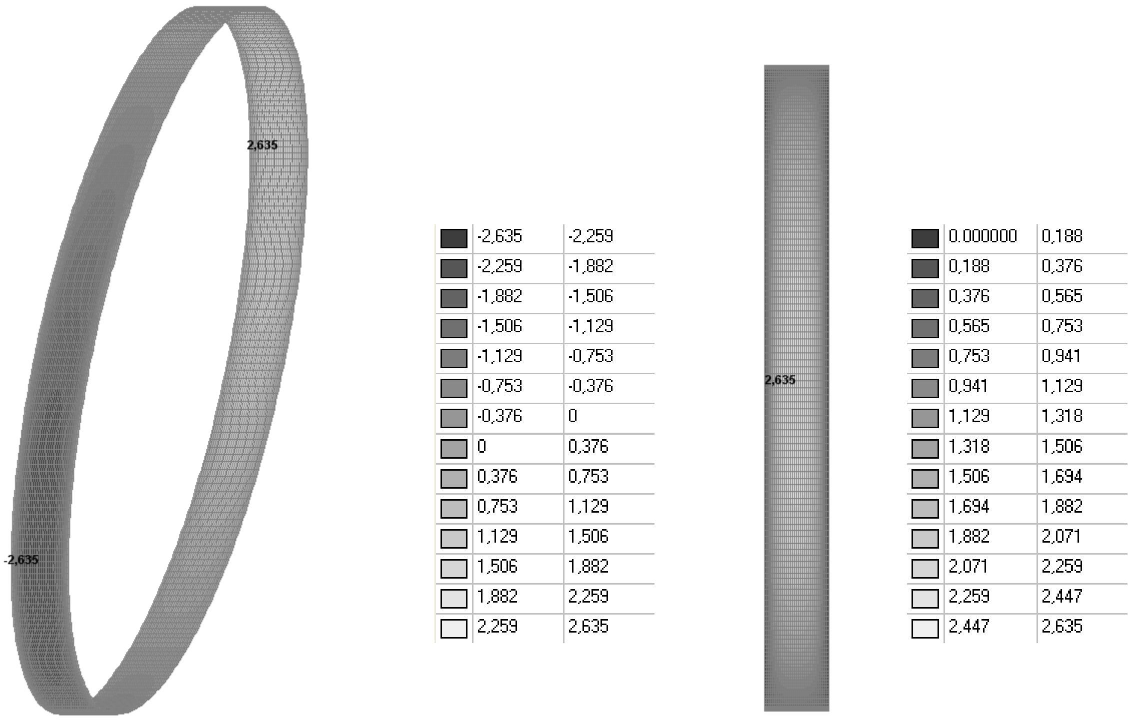
$E = 2.1 \cdot 10^8$ kPa	- elastic modulus;
ν = 0.3	- Poisson's ratio;
h = 0.02 m	- thickness of the shell;
a = 10.0 m	- radius of the midsurface of the shell;
l = 2.0 m	- length of the shell;
p = 10.0 kPa	- internal pressure.

Finite element model: Design model – general type system, shell elements – 9216 four-node elements of type 44. The spacing of the finite element mesh in the meridian direction is 0.0625 m and in the circumferential direction is 1.25º. Boundary conditions at the simply supported edges are provided by imposing constraints in the directions of the angular and linear displacements in their plane. Number of nodes in the design model – 9504.

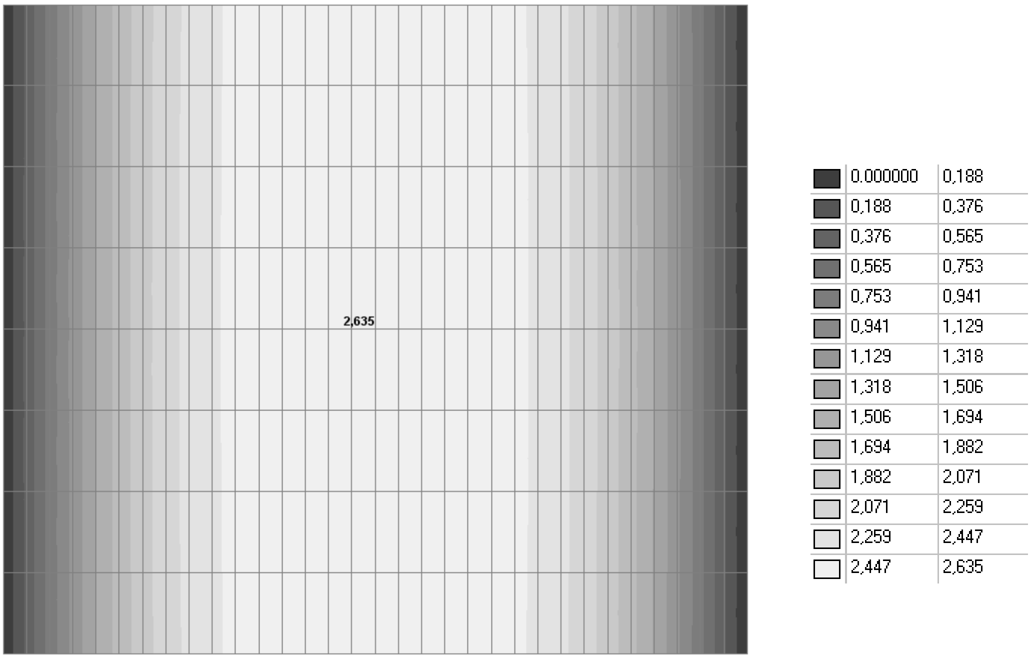
Results in SCAD



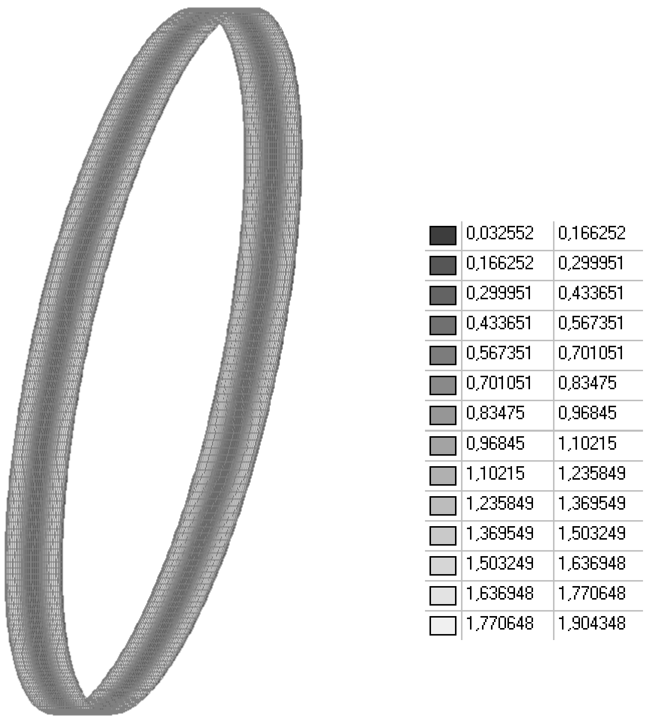
Design and deformed models



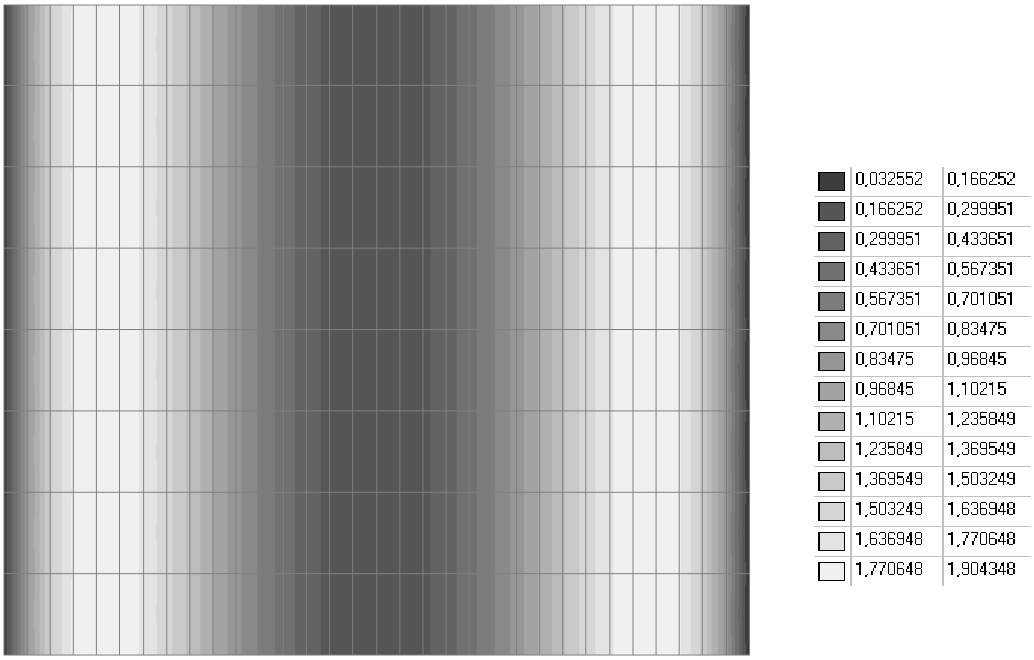
Values of radial displacements w (mm)



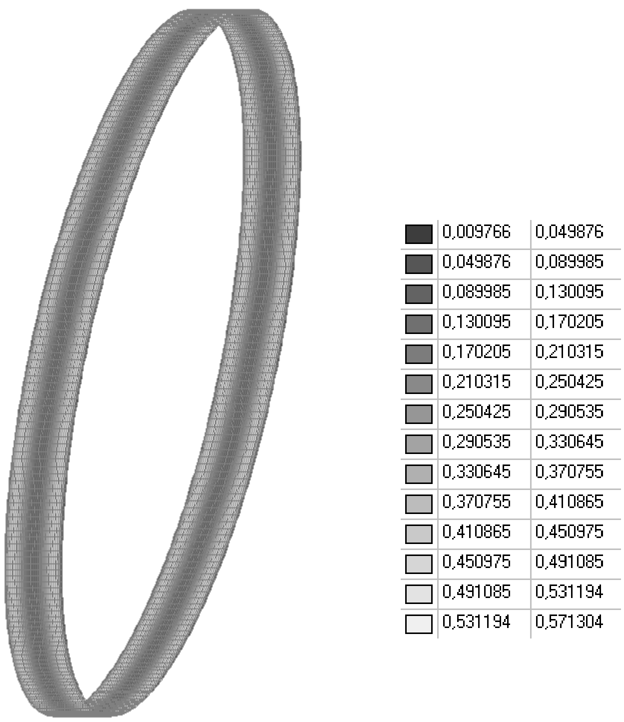
Values of radial displacements w (mm)
for the fragment of the model from the section in the area of the horizontal diameter with the central angle of 5.00º



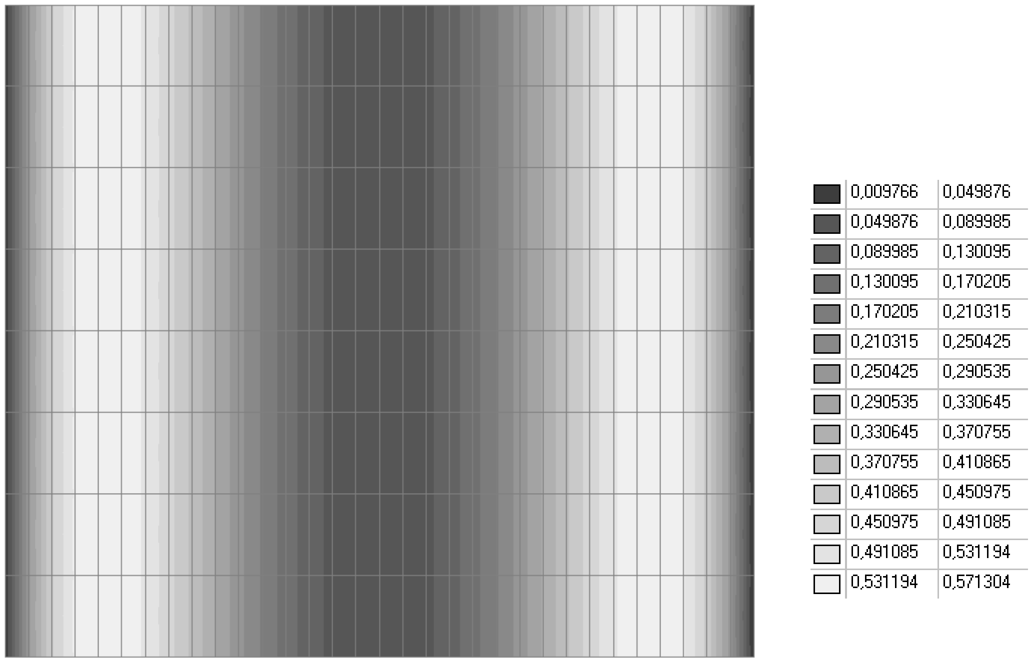
Values of bending moments acting on the midsurface of the shell
in the meridian direction M_x (kN·m/m)



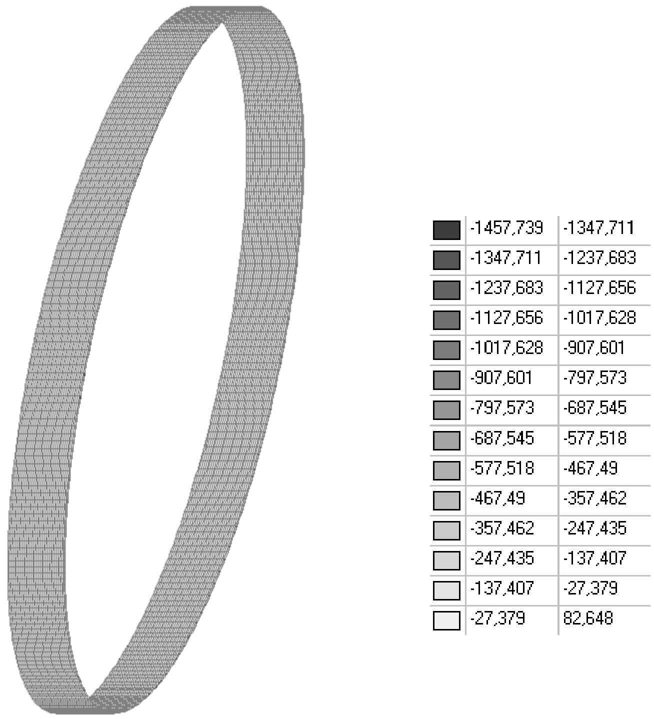
Values of bending moments acting on the midsurface of the shell
in the meridian direction M_x *(kN·m/m)*
for the fragment of the model from the section in the area of the horizontal diameter with the central angle of 5.00º



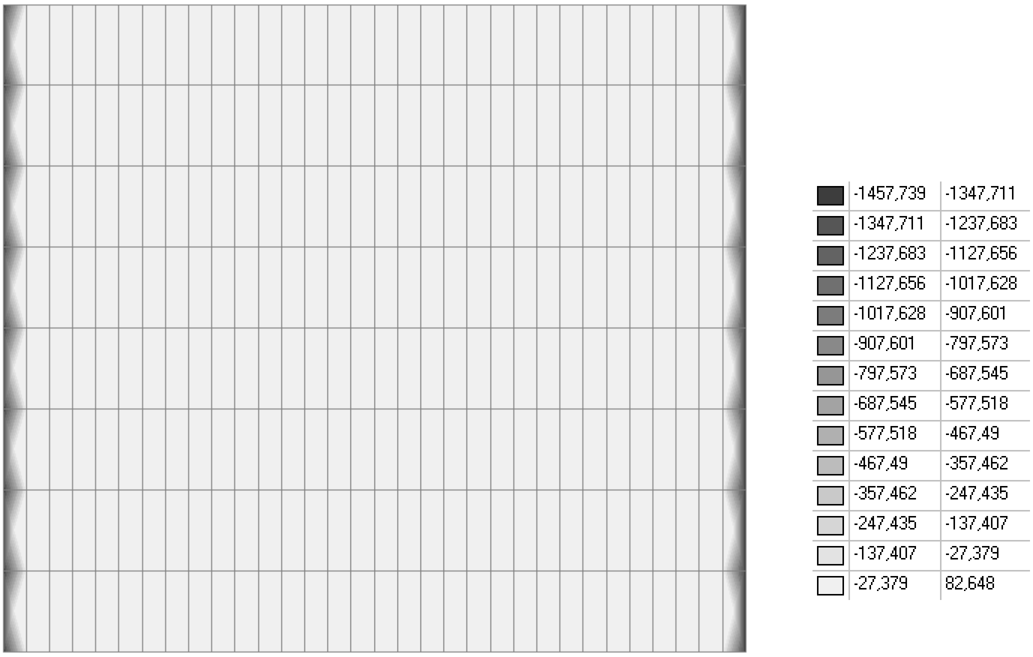
Values of bending moments acting on the midsurface of the shell
in the circumferential direction M_φ *(kN·m/m)*



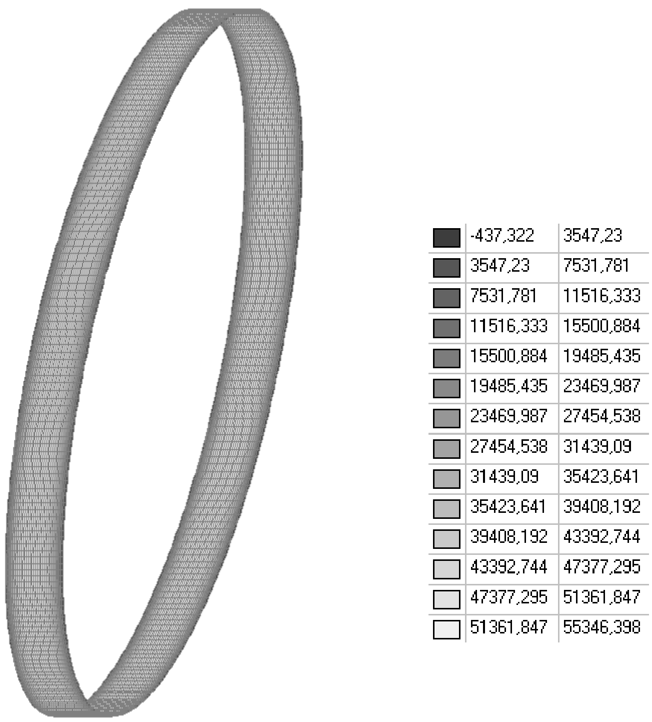
Values of bending moments acting on the midsurface of the shell
in the circumferential direction M_φ (kN·m/m)
for the fragment of the model from the section in the area of the horizontal diameter with the central angle of 5.00°



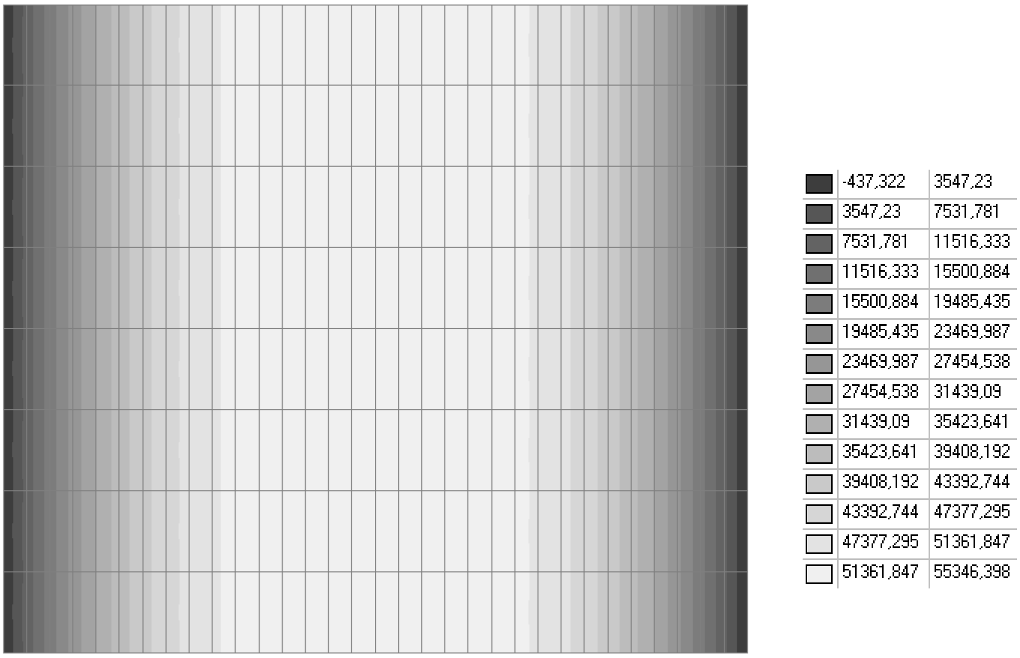
Values of longitudinal forces acting on the midsurface of the shell
in the meridian direction N_x (kN/m^2)



Values of longitudinal forces acting on the midsurface of the shell
in the meridian direction N_x (kN/m^2)
for the fragment of the model from the section in the area of the horizontal diameter with the central angle of 5.00^o



Values of longitudinal forces acting on the midsurface of the shell
in the circumferential direction N_φ (kN/m^2)



Values of longitudinal forces acting on the midsurface of the shell
in the circumferential direction N_φ (kN/m²)
for the fragment of the model from the section in the area of the horizontal diameter with the central angle of 5.00º

Comparison of solutions:

Parameter	Theory	SCAD	Deviations, %
$w(l/2)$, mm	2.640	2.635	0.19
$M_x(l/2)$, kN·m/m	0.178969	0.180453	0.83
$M_\varphi(l/2)$, kN·m/m	0.053691	0.054136	0.83
$N_x(l/2)$, kN/m	0.000	8.238·0.02 = 0.165	–
$N_\varphi(l/2)$, kN/m	1108.655	55346.398·0.02 = 1106.928	0.16

Notes: In the analytical solution the bending moments and longitudinal forces acting on the midsurface of the shell in the meridian M_x, N_x and circumferential M_φ, N_φ directions, as well as the radial displacements w for the cross-section in the middle of the span can be determined according to the following formulas (S.P. Timoshenko, Theory of Plates and Shells. — Moscow: OGIZ. Gostekhizdat, 1948, p. 377):

$$w = \frac{p \cdot l^4}{64 \cdot D \cdot \alpha^4} \cdot \left(1 - \frac{2 \cdot cos(\alpha) \cdot ch(\alpha)}{cos(2 \cdot \alpha) + ch(2 \cdot \alpha)}\right) = \frac{p \cdot a^2}{E \cdot h} \cdot \left(1 - \frac{2 \cdot cos(\alpha) \cdot ch(\alpha)}{cos(2 \cdot \alpha) + ch(2 \cdot \alpha)}\right);$$

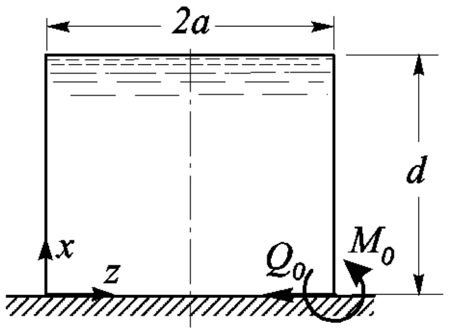
$$M_x = \frac{p \cdot l^2}{4 \cdot \alpha^2} \cdot \frac{sin(\alpha) \cdot sh(\alpha)}{cos(2 \cdot \alpha) + ch(2 \cdot \alpha)} = \frac{p \cdot a \cdot h}{\sqrt{3 \cdot (1 - \nu^2)}} \cdot \frac{sin(\alpha) \cdot sh(\alpha)}{cos(2 \cdot \alpha) + ch(2 \cdot \alpha)};$$

$$M_\varphi = \nu \cdot M_x = \frac{p \cdot a \cdot h \cdot \nu}{\sqrt{3 \cdot (1 - \nu^2)}} \cdot \frac{sin(\alpha) \cdot sh(\alpha)}{cos(2 \cdot \alpha) + ch(2 \cdot \alpha)};$$

$$N_x = 0; \qquad N_\varphi = -\frac{E \cdot h}{a} \cdot w = -p \cdot a \cdot \left(1 - \frac{2 \cdot cos(\alpha) \cdot ch(\alpha)}{cos(2 \cdot \alpha) + ch(2 \cdot \alpha)}\right), \text{ where:}$$

$$D = \frac{E \cdot h^3}{12 \cdot (1 - \nu^2)}, \qquad \beta = \sqrt[4]{\frac{3 \cdot (1 - \nu^2)}{a^2 \cdot h^2}}, \alpha = \frac{\beta \cdot l}{2}.$$

Cylindrical Vertical Tank with a Wall of Constant Thickness with a Flat Bottom Subjected to Internal Fluid Pressure



Objective: Determination of the stress-strain state of a cylindrical vertical tank with a wall of constant thickness clamped in a flat bottom subjected to internal fluid pressure which varies linearly with height.

Initial data file: **4.32.SPR**

Problem formulation: The cylindrical vertical tank with a wall of constant thickness is clamped in a flat bottom and subjected to internal pressure of the liquid with the specific weight γ. Determine the bending moments and longitudinal forces acting on the midsurface of the tank wall in the meridian M_x, N_x and in the circumferential $M_φ$, $N_φ$ directions, as well as the radial displacements *w* of the tank wall.

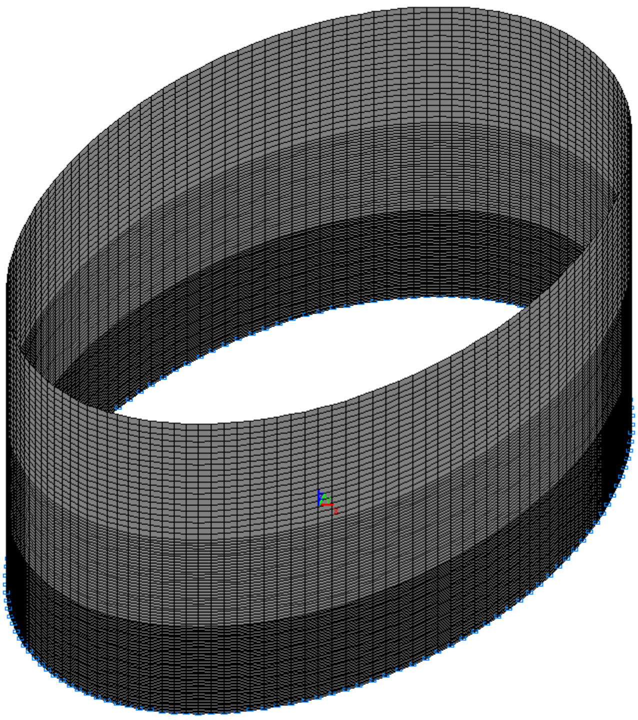
References: S.P. Timoshenko, Theory of Plates and Shells. — Moscow: OGIZ. Gostekhizdat, 1948.

Initial data:

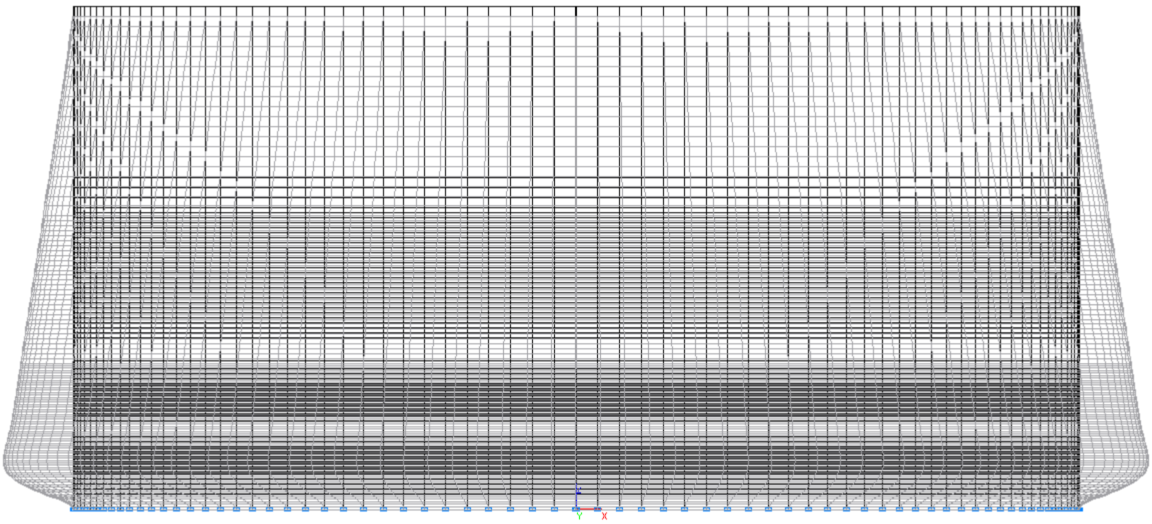
$E = 2.1 \cdot 10^8$ kPa	- elastic modulus;
ν = 0.3	- Poisson's ratio;
h = 0.01 m	- thickness of the tank wall;
a = 5.0 m	- radius of the midsurface of the tank wall;
d = 5.0 m	- height of the tank;
$γ = 10.0$ kN/m^3	- specific weight of the liquid in the tank.

Finite element model: Design model – general type system, shell elements – 15840 four-node elements of type 44. The spacing of the finite element mesh in the meridian direction is 0.025 m at the height x from the bottom from 0.0 m to 1.5 m; 0.050 m at the height x from the bottom from 1.5 m to 3.0 m; 0.100 m at the height x from the bottom from 3.0 m to 5.0 m; and in the circumferential direction the spacing is 2.5º. Boundary conditions at the clamping into the bottom are provided by imposing constraints in all directions of the angular and linear displacements. Number of nodes in the design model – 15984.

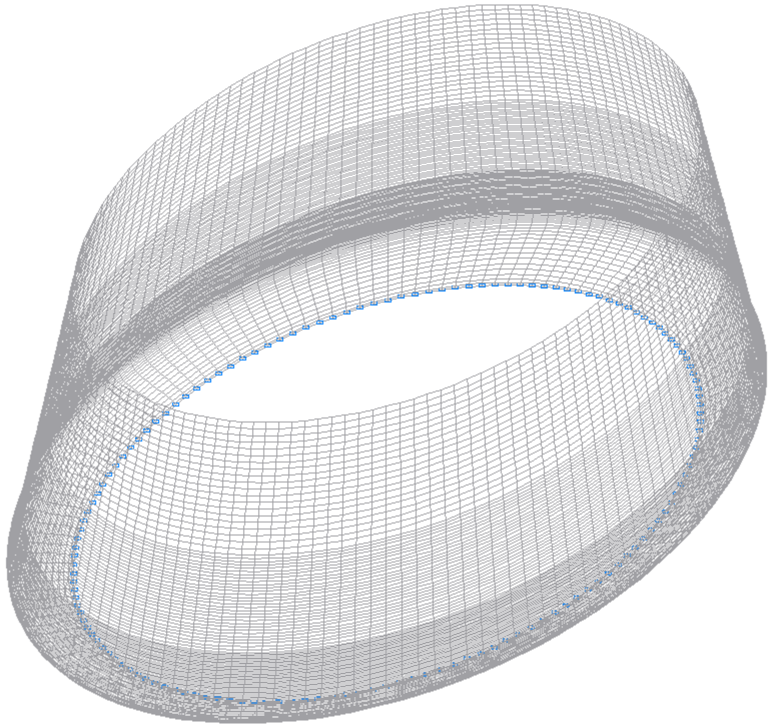
Results in SCAD



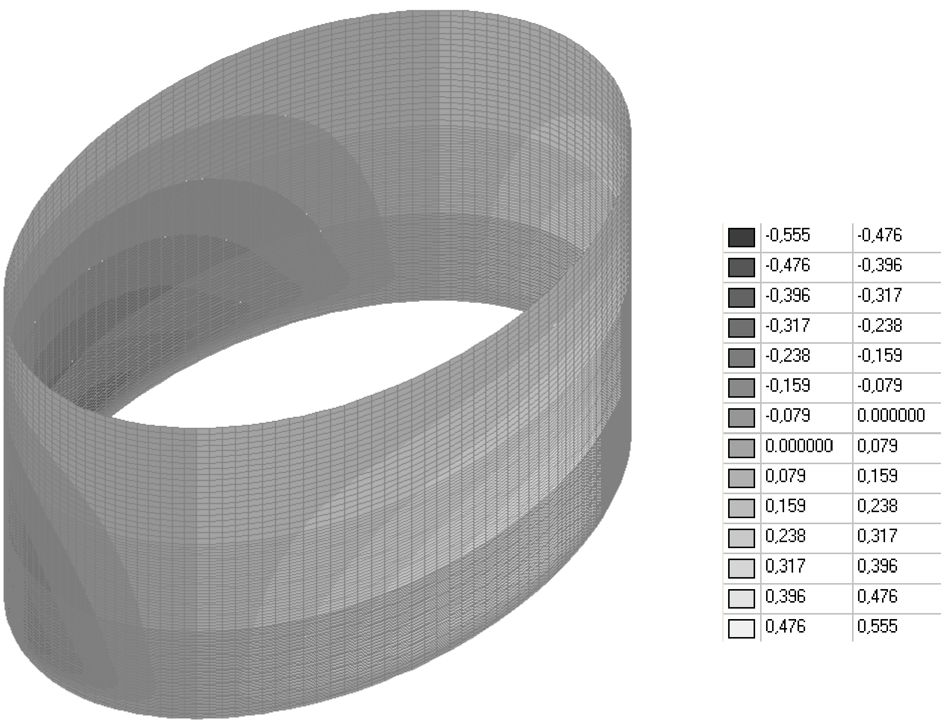
Design model



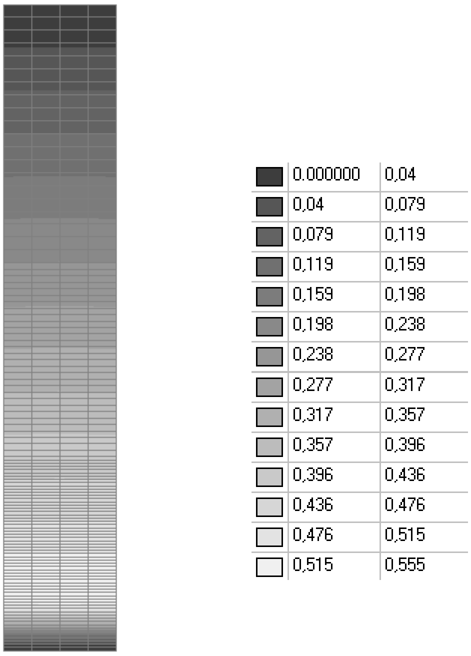
Deformed model



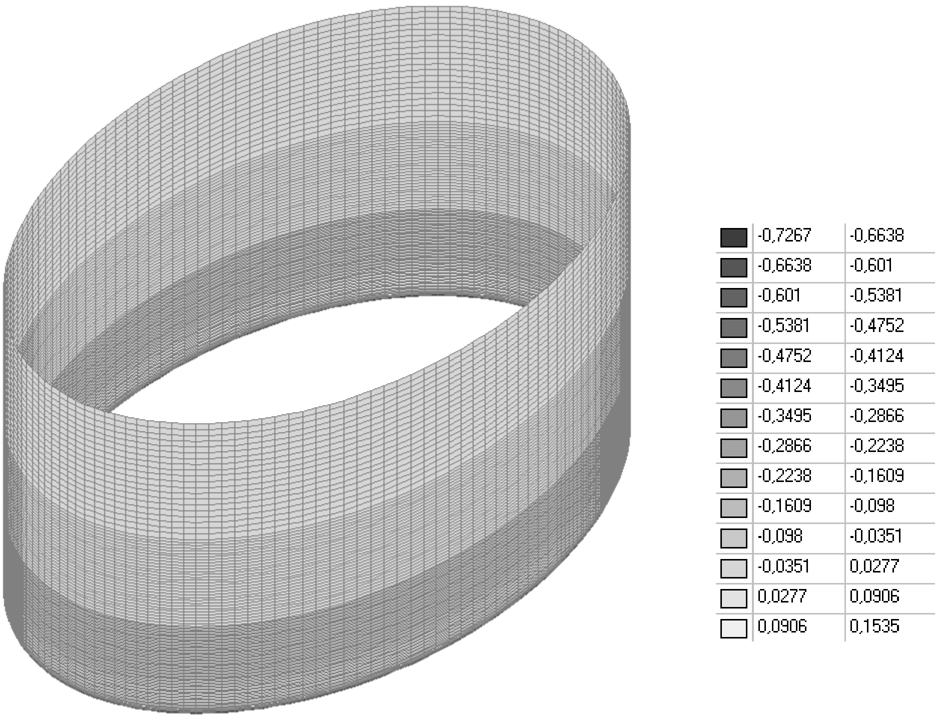
Deformed model



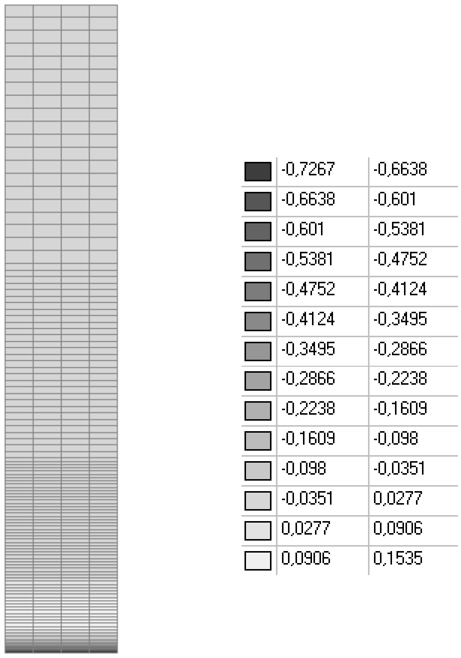
Values of radial displacements w (mm)



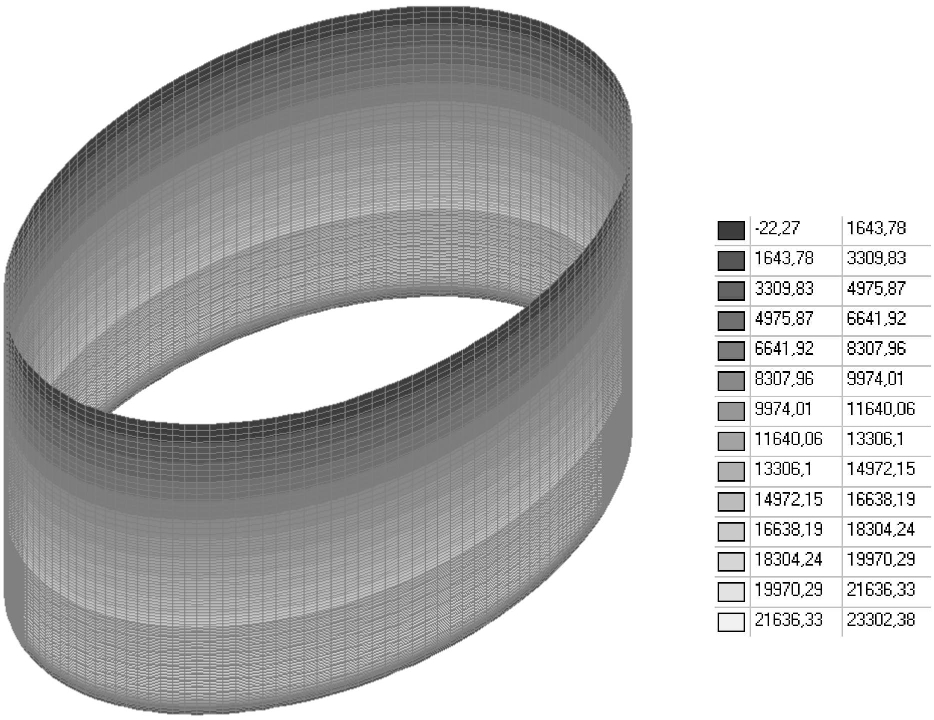
Values of radial displacements w (mm)
for the fragment of the model from the section with the central angle of 10.0º



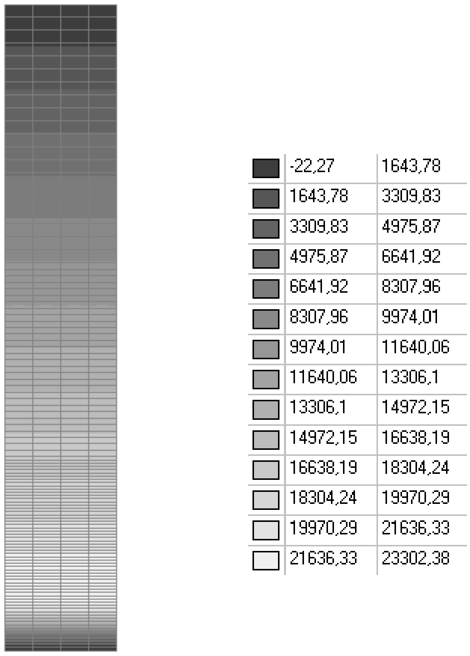
Values of bending moments acting on the midsurface of the tank wall
in the meridian direction M_x *(kN·m/m)*



Values of bending moments acting on the midsurface of the tank wall
in the meridian direction M_x (kN·m/m)
for the fragment of the model from the section with the central angle of 10.0º



Values of longitudinal forces acting on the midsurface of the tank wall
in the circumferential direction N_φ (kN/m^2)



Values of longitudinal forces acting on the midsurface of the tank wall in the circumferential direction N_φ (kN/m^2) for the fragment of the model from the section with the central angle of 10.0°

Comparison of solutions:

x, m	*w*, mm			M_x, kN·m/m			N_φ, kN/m		
	Theory	**SCAD**	**Deviations, %**	**Theory**	**SCAD**	**Deviations, %**	**Theory**	**SCAD**	**Deviations, %**
0.000	0.000	0.000	—	-0.7302	-0.7267	0.48	0.00	-22.27·0.01 = -0.22	—
0.025	0.011	0.011	0.00	-0.5321	-0.5253	1.28	4.52	432.34·0.01 = 4.32	4.42
0.050	0.039	0.039	0.00	-0.3644	-0.3564	2.20	16.33	1612.89·0.01 = 16.13	1.22
0.075	0.079	0.078	1.27	-0.2256	-0.2179	3.41	33.15	3285.96·0.01 = 32.86	0.87
0.100	0.126	0.125	0.79	-0.1134	-0.1069	5.73	53.08	5261.88·0.01 = 52.62	0.87
0.125	0.178	0.176	1.12	-0.0252	-0.0204	—	74.59	7388.51·0.01 = 73.89	0.94
0.150	0.230	0.227	1.30	0.0419	0.0448	—	96.46	9547.11·0.01 = 95.47	1.03
0.175	0.280	0.277	1.07	0.0907	0.0918	1.21	117.78	11648.07·0.01 = 116.48	1.10
0.200	0.328	0.324	1.22	0.1241	0.1235	0.48	137.88	13626.65·0.01 = 136.27	1.17
0.225	0.372	0.367	1.34	0.1448	0.1428	1.38	156.30	15439.08·0.01 = 154.39	1.22
0.250	0.411	0.406	1.22	0.1550	0.1520	1.94	172.76	17058.39·0.01 = 170.58	1.26
0.275	0.445	0.440	1.12	0.1572	0.1535	2.35	187.11	18471.73·0.01 = 184.72	1.28
0.300	0.475	0.468	1.47	0.1532	0.1491	2.68	199.32	19676.68·0.01 = 196.77	1.28
0.325	0.499	0.492	1.40	0.1447	0.1405	2.90	209.44	20678.89·0.01 = 206.79	1.27
0.350	0.518	0.512	1.16	0.1332	0.1291	3.08	217.60	21489.82·0.01 = 214.90	1.24
0.375	0.533	0.527	1.13	0.1198	0.1160	3.17	223.93	22124.83·0.01 = 221.25	1.20
0.400	0.544	0.538	1.10	0.1054	0.1021	3.13	228.64	22601.65·0.01 = 226.02	1.15
0.425	0.552	0.546	1.09	0.0909	0.0881	3.08	231.90	22939.13·0.01 = 229.39	1.08
0.450	0.557	0.551	1.08	0.0767	0.0745	2.87	233.93	23156.28·0.01 = 231.56	1.01
0.475	0.559	0.554	0.89	0.0633	0.0617	2.53	234.90	23271.56·0.01 =	0.93

x, m	*w*, mm			M_x, kN·m/m			N_φ, kN/m		
	Theory	SCAD	Deviations, %	Theory	SCAD	Deviations, %	Theory	SCAD	Deviations, %
								232.72	
0.500	0.560	0.555	0.89	0.0510	0.0500	1.96	235.01	23302.38·0.01 = 233.02	0.85
0.550	0.555	0.552	0.54	0.0303	0.0302	0.33	233.29	23172.87·0.01 = 231.73	0.67
0.600	0.547	0.545	0.37	0.0148	0.0155	4.73	229.89	22875.52·0.01 = 228.76	0.49
0.650	0.537	0.535	0.37	0.0043	0.0055	–	225.66	22490.85·0.01 = 224.91	0.33
0.700	0.527	0.526	0.19	-0.0022	-0.0008	–	221.17	22074.31·0.01 = 220.74	0.19
0.750	0.516	0.516	0.00	-0.0055	-0.0042	–	216.79	21660.60·0.01 = 216.61	0.08
0.800	0.506	0.506	0.00	-0.0067	-0.0055	–	212.70	21268.60·0.01 = 212.69	0.00
0.850	0.498	0.498	0.00	-0.0066	-0.0056	–	208.97	20906.05·0.01 = 209.06	0.04
0.900	0.490	0.490	0.00	-0.0057	-0.0049	–	205.59	20573.53·0.01 = 205.74	0.07
0.950	0.482	0.483	0.21	-0.0045	-0.0039	–	202.53	20267.56·0.01 = 202.68	0.07
1.000	0.475	0.476	0.21	-0.0032	-0.0028	–	199.71	19982.79·0.01 = 199.83	0.06

Notes: In the analytical solution the bending moments and longitudinal forces acting on the midsurface of the tank wall in the meridian M_x, N_x and circumferential M_φ, N_φ directions, as well as the radial displacements w of the tank wall can be determined according to the following formulas (S.P. Timoshenko, Theory of Plates and Shells. — Moscow: OGIZ. Gostekhizdat, 1948, p. 388):

$$w = \frac{\gamma \cdot a^2 \cdot d}{E \cdot h} \cdot \left(1 - \frac{x}{d} - e^{-\beta \cdot x} \cdot \left(cos(\beta \cdot x) + \left(1 - \frac{1}{\beta \cdot d}\right) \cdot sin(\beta \cdot x)\right)\right);$$

$$M_x = \frac{\gamma \cdot a \cdot d \cdot h}{\sqrt{12 \cdot (1-\nu^2)}} \cdot e^{-\beta \cdot x} \cdot \left(sin(\beta \cdot x) - \left(1 - \frac{1}{\beta \cdot d}\right) \cdot cos(\beta \cdot x)\right);$$

$$M_\varphi = \nu \cdot M_x = \frac{\gamma \cdot a \cdot d \cdot h \cdot \nu}{\sqrt{12 \cdot (1-\nu^2)}} \cdot e^{-\beta \cdot x} \cdot \left(sin(\beta \cdot x) - \left(1 - \frac{1}{\beta \cdot d}\right) \cdot cos(\beta \cdot x)\right);$$

$$N_x = 0; \qquad N_\varphi = \frac{E \cdot h}{a} \cdot w = \gamma \cdot a \cdot d \cdot \left(1 - \frac{x}{d} - e^{-\beta \cdot x} \cdot \left(cos(\beta \cdot x) + \left(1 - \frac{1}{\beta \cdot d}\right) \cdot sin(\beta \cdot x)\right)\right),$$ where:

$$\beta = \sqrt[4]{\frac{3 \cdot (1-\nu^2)}{a^2 \cdot h^2}}.$$

Cylindrical Shell with Free Edges at a Temperature Gradient across the Thickness (in the Radial Direction)

Objective: Determination of the stress-strain state of a cylindrical shell with free edges subjected to a temperature gradient across the thickness.

Initial data file: **4.33.SPR**

Problem formulation: The cylindrical thin-walled shell free from constraints is subjected to a temperature gradient across the thickness. The temperatures of the cylinder wall on its internal t_1 and external surfaces t_2 are constant. The temperature varies linearly across the thickness of the wall. Determine the stress tensor components on the internal and external surfaces of the shell in the meridian σ_x^{ext} (σ_x^{int}) and circumferential σ_φ^{ext} (σ_φ^{int}) directions, as well as the radial displacements *w*.

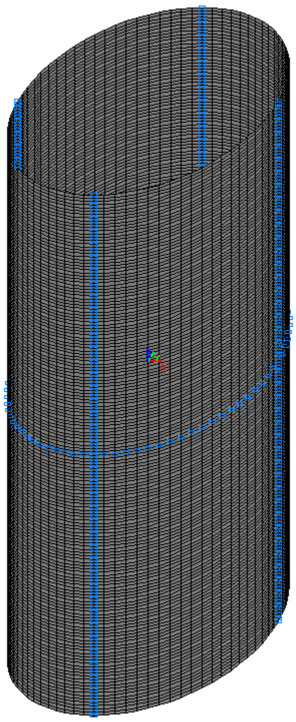
References: S.P. Timoshenko, Theory of Plates and Shells. — Moscow: OGIZ. Gostekhizdat, 1948.

Initial data:

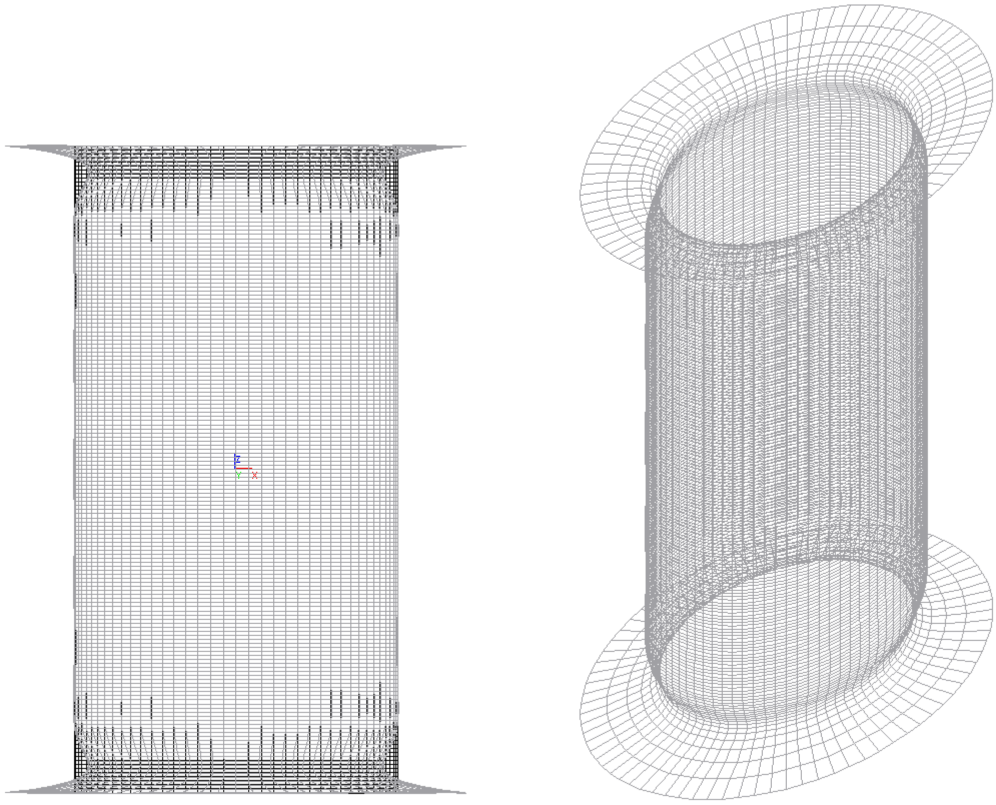
$E = 2.1 \cdot 10^8$ kPa	- elastic modulus;
$\nu = 0.3$	- Poisson's ratio;
h = 0.02 m	- thickness of the shell wall;
a = 1.0 m	- radius of the midsurface of the shell wall;
l = 4.0 m	- length of the shell;
$\alpha = 0.12 \cdot 10^{-4}$ 1/ºC	- linear expansion coefficient;
$t_1 = 20$ ºC	- temperature on the internal surface of the cylinder wall;
$t_2 = 0$ ºC	- temperature on the external surface of the cylinder wall.

Finite element model: Design model – general type system, shell elements – 12800 four-node elements of type 44. The spacing of the finite element mesh in the meridian direction is 0.025 m and in the circumferential direction is 4.5º. The dimensional stability of the design model is provided by imposing constraints according to its symmetry conditions. Number of nodes in the design model – 12880.

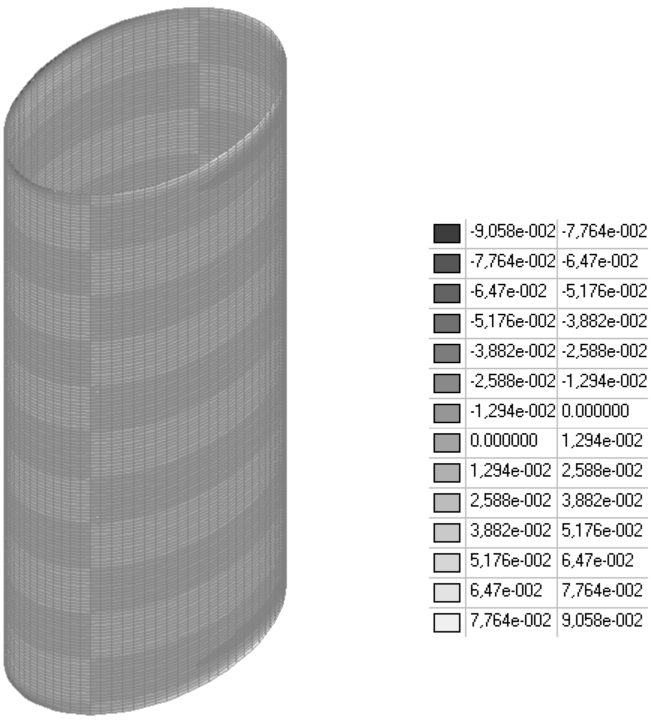
Results in SCAD



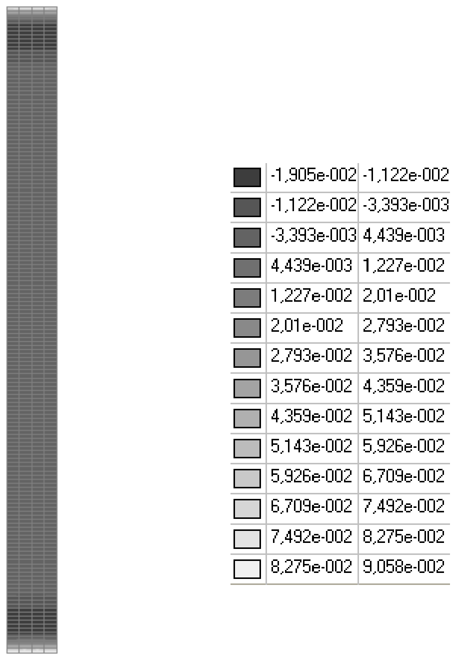
Design model



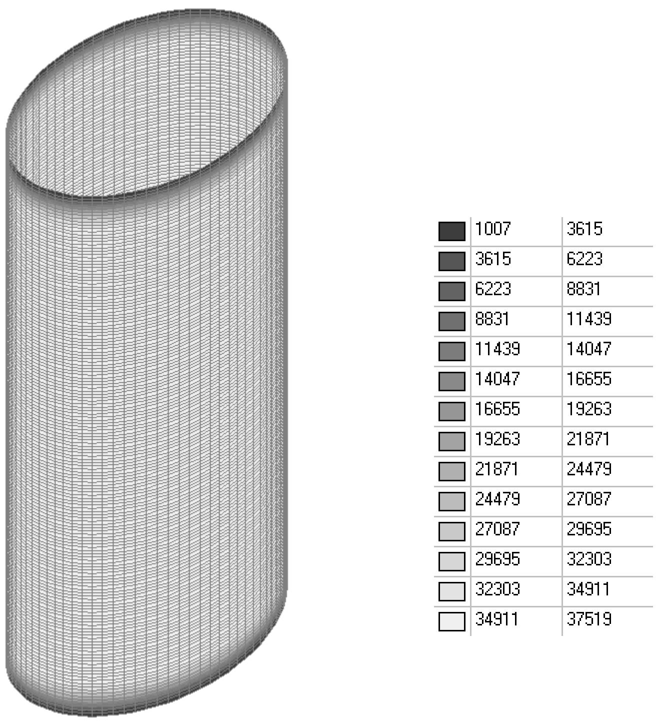
Deformed model



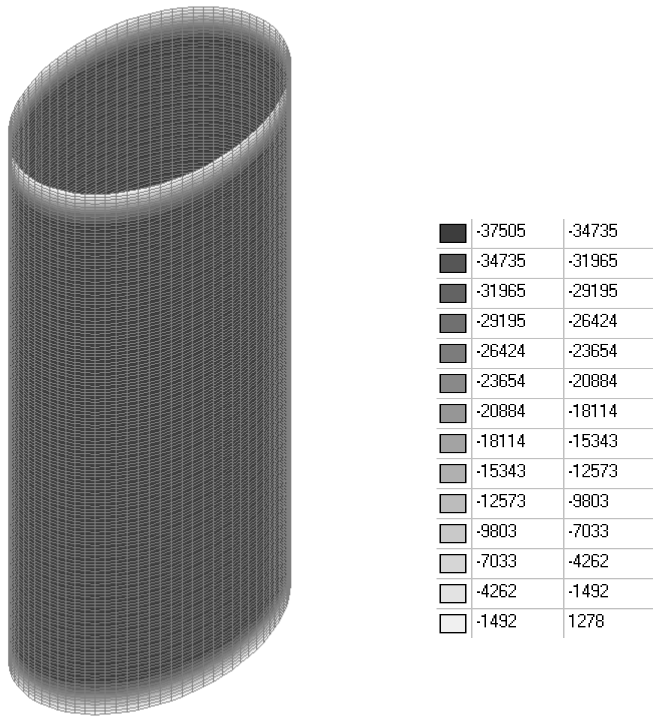
Values of radial displacements w (mm)



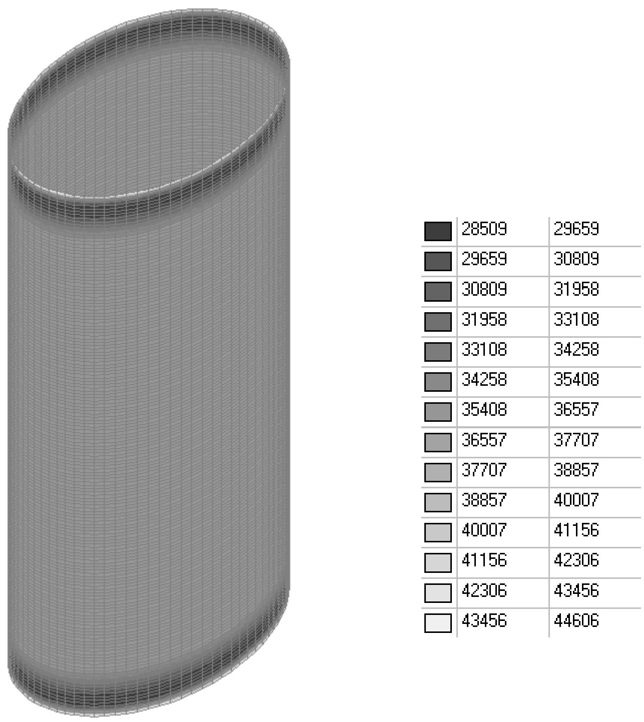
Values of radial displacements w (mm)
for the fragment of the model from the section with the central angle of 18.0º



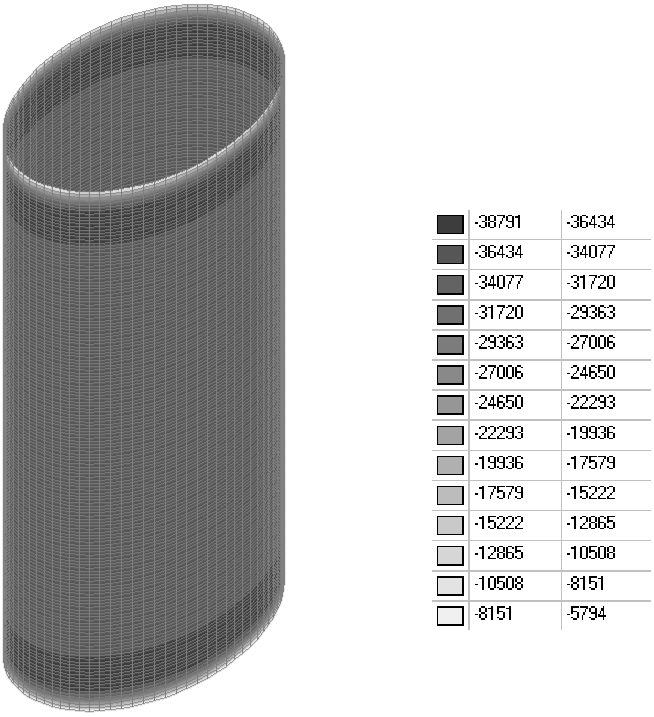
Values of stresses on the external surface of the shell
in the meridian direction σ_x^{ext} *(kN/m2)*



Values of stresses on the internal surface of the shell in the meridian direction σ_x^{int} *(kN/m2)*



Values of stresses on the external surface of the shell in the circumferential direction σ_φ^{ext} *(kN/m2)*



Values of stresses on the internal surface of the shell in the circumferential direction σ_φ^{int} (kN/m2)

Comparison of solutions:

x, m	***w*, mm**		
	Theory	**SCAD**	**Deviations, %**
0.200	$-18.61\cdot10^{-3}$	$-18.01\cdot10^{-3}$	3.22
0.250	$-13.71\cdot10^{-3}$	$-13.20\cdot10^{-3}$	3.72
0.300	$-8.14\cdot10^{-3}$	$-7.81\cdot10^{-3}$	4.05
0.350	$-3.76\cdot10^{-3}$	$-3.60\cdot10^{-3}$	4.26
0.400	$-1.01\cdot10^{-3}$	$-0.97\cdot10^{-3}$	3.96
0.450	$0.36\cdot10^{-3}$	$0.34\cdot10^{-3}$	5.56
0.500	$0.82\cdot10^{-3}$	$0.78\cdot10^{-3}$	4.88
0.550	$0.79\cdot10^{-3}$	$0.75\cdot10^{-3}$	5.06
0.600	$0.57\cdot10^{-3}$	$0.54\cdot10^{-3}$	5.26
0.650	$0.33\cdot10^{-3}$	$0.32\cdot10^{-3}$	3.03
0.700	$0.15\cdot10^{-3}$	$0.14\cdot10^{-3}$	6.67
0.750	$0.04\cdot10^{-3}$	$0.04\cdot10^{-3}$	0.00
0.800	$-0.02\cdot10^{-3}$	$-0.02\cdot10^{-3}$	–
0.850	$-0.04\cdot10^{-3}$	$-0.03\cdot10^{-3}$	–
0.900	$-0.03\cdot10^{-3}$	$-0.03\cdot10^{-3}$	–
0.950	$-0.02\cdot10^{-3}$	$-0.02\cdot10^{-3}$	–
1.000	$-0.01\cdot10^{-3}$	$-0.01\cdot10^{-3}$	–
1.100	0	0	–
1.200	0	0	–
1.300	0	0	–
1.400	0	0	–
1.500	0	0	–
1.600	0	0	–
1.700	0	0	–
1.800	0	0	–
1.900	0	0	–
2.000	0	0	–

x, m	**σ_x^{ext} (kN/m2)**			**σ_x^{int} (kN/m2)**		
	Theory	**SCAD**	**Deviations, %**	**Theory**	**SCAD**	**Deviations, %**
0.200	31761	32052	0.92	-31761	-32090	1.04
0.250	35560	35681	0.34	-35560	-35685	0.35
0.300	37206	37221	0.04	-37206	-37210	0.01
0.350	37553	37519	0.09	-37553	-37505	0.13

x, m	σ_x^{ext} (kN/m2)			σ_x^{int} (kN/m2)		
	Theory	SCAD	Deviations, %	Theory	SCAD	Deviations, %
0.400	37286	37241	0.12	-37286	-37229	0.15
0.450	36841	36804	0.10	-36841	-36796	0.12
0.500	36441	36418	0.06	-36441	-36414	0.07
0.550	36164	36154	0.03	-36164	-36152	0.03
0.600	36010	36007	0.01	-36010	-36007	0.01
0.650	35945	35947	0.01	-35945	-35947	0.01
0.700	35933	35936	0.01	-35933	-35937	0.01
0.750	35946	35949	0.01	-35946	-35949	0.01
0.800	35965	35967	0.01	-35965	-35968	0.01
0.850	35982	35983	0.00	-35982	-35983	0.00
0.900	35994	35994	0.00	-35994	-35994	0.00
0.950	36000	36000	0.00	-36000	-36000	0.00
1.000	36002	36002	0.00	-36002	-36002	0.00
1.100	36002	36002	0.00	-36002	-36002	0.00
1.200	36001	36001	0.00	-36001	-36001	0.00
1.300	36000	36000	0.00	-36000	-36000	0.00
1.400	36000	36000	0.00	-36000	-36000	0.00
1.500	36000	36000	0.00	-36000	-36000	0.00
1.600	36000	36000	0.00	-36000	-36000	0.00
1.700	36000	36000	0.00	-36000	-36000	0.00
1.800	36000	36000	0.00	-36000	-36000	0.00
1.900	36000	36000	0.00	-36000	-36000	0.00
2.000	36000	36000	0.00	-36000	-36000	0.00
0.000	45027	44606	0.93	-5373	-5794	7.84
0.025	37510	37025	1.29	-13846	-14584	5.33
0.050	32614	32413	0.62	-21047	-21639	2.81
0.075	29785	29786	0.00	-26849	-27290	1.64
0.100	28500	28633	0.47	-31284	-31586	0.97
0.150	28809	29047	0.83	-36646	-36735	0.24
0.200	30819	31034	0.70	-38637	-38608	0.08
0.250	32988	33133	0.44	-38748	-38677	0.18
0.300	34652	34726	0.21	-38072	-38003	0.18
0.350	35676	35700	0.07	-37256	-37208	0.13
0.400	36173	36169	0.01	-36598	-36572	0.07
0.450	36328	36313	0.04	-36176	-36167	0.02
0.500	36305	36289	0.04	-35960	-35961	0.00
0.550	36215	36203	0.03	-35883	-35888	0.01
0.600	36123	36116	0.02	-35883	-35888	0.01
0.650	36053	36050	0.01	-35914	-35918	0.01
0.700	36011	36011	0.00	-35949	-35951	0.01
0.750	35991	35992	0.00	-35976	-35977	0.00
0.800	35986	35987	0.00	-35993	-35994	0.00
0.850	35987	35988	0.00	-36002	-36002	0.00
0.900	35991	35992	0.00	-36005	-36005	0.00
0.950	35995	35995	0.00	-36005	-36005	0.00
1.000	35998	35998	0.00	-36004	-36003	0.00
1.100	36000	36000	0.00	-36001	-36001	0.00
1.200	36001	36000	0.00	-36000	-36000	0.00
1.300	36000	36000	0.00	-36000	-36000	0.00
1.400	36000	36000	0.00	-36000	-36000	0.00
1.500	36000	36000	0.00	-36000	-36000	0.00
1.600	36000	36000	0.00	-36000	-36000	0.00
1.700	36000	36000	0.00	-36000	-36000	0.00
1.800	36000	36000	0.00	-36000	-36000	0.00
1.900	36000	36000	0.00	-36000	-36000	0.00
2.000	36000	36000	0.00	-36000	-36000	0.00

x – ordinate along the axis of the cylindrical shell (meridian direction) measured from the free edge.

Notes: In the analytical solution the stresses on the internal and external surfaces of the shell in the meridian σ_x^{ext} (σ_x^{int}) and circumferential σ_φ^{ext} (σ_φ^{int}) directions, as well as the radial displacements *w* can be determined according to the following formulas (S.P. Timoshenko, Theory of Plates and Shells. — Moscow: OGIZ. Gostekhizdat, 1948, p. 399), which give a good approximation “at points at a considerable distance from the edges of the shell”:

$$w = 0.5 \cdot \alpha \cdot (t_1 - t_2) \cdot a \cdot \sqrt{\frac{1+\nu}{3 \cdot (1-\nu)}} \cdot e^{-\beta \cdot x} \cdot (sin(\beta \cdot x) - cos(\beta \cdot x));$$

$$\sigma_x^{ext} = \frac{E \cdot \alpha \cdot (t_1 - t_2)}{2 \cdot (1-\nu)} \cdot \left[-1 + e^{-\beta \cdot x} \cdot (cos(\beta \cdot x) + sin(\beta \cdot x))\right];$$

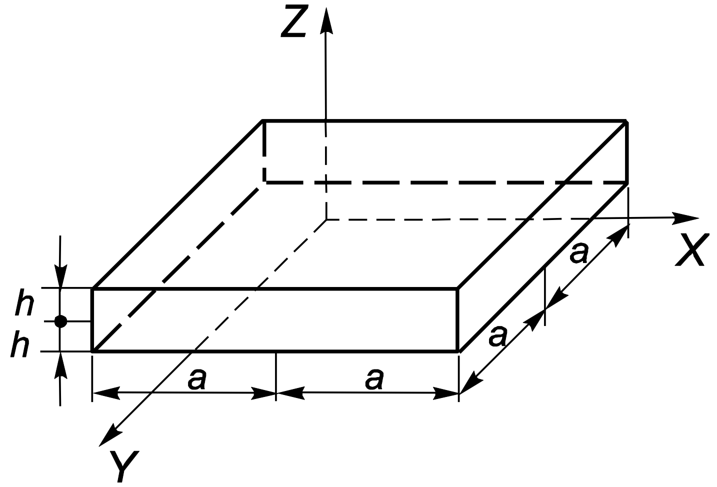
$$\sigma_x^{int} = \frac{E \cdot \alpha \cdot (t_1 - t_2)}{2 \cdot (1-\nu)} \cdot \left[1 - e^{-\beta \cdot x} \cdot (cos(\beta \cdot x) + sin(\beta \cdot x))\right];$$

$$\sigma_\varphi^{ext} = \frac{E \cdot \alpha \cdot (t_1 - t_2)}{2 \cdot (1-\nu)} \cdot \left[-1 + \nu \cdot e^{-\beta \cdot x} \cdot (cos(\beta \cdot x) + sin(\beta \cdot x)) - \sqrt{\frac{1-\nu^2}{3}} \cdot e^{-\beta \cdot x} \cdot (sin(\beta \cdot x) - cos(\beta \cdot x))\right];$$

$$\sigma_\varphi^{int} = \frac{E \cdot \alpha \cdot (t_1 - t_2)}{2 \cdot (1-\nu)} \cdot \left[1 - \nu \cdot e^{-\beta \cdot x} \cdot (cos(\beta \cdot x) + sin(\beta \cdot x)) - \sqrt{\frac{1-\nu^2}{3}} \cdot e^{-\beta \cdot x} \cdot (sin(\beta \cdot x) - cos(\beta \cdot x))\right],$$ where:

$$\beta = \sqrt[4]{\frac{3 \cdot (1-\nu^2)}{a^2 \cdot h^2}}.$$

Thick Square Slab Simply Supported along the Sides Subjected to a Transverse Load Distributed over the Upper Face According to the Cosine Law



Objective: Determination of the stress-strain state of a thick square slab simply supported along the sides subjected to a transverse load distributed over the upper face according to the cosine law in accordance with the spatial problem of the theory of elasticity.

SCAD version used: **21.1**
Initial data files:

File name	Description
4.36a_gamma_3.SPR	Design model for the slab thickness of 4 m (γ = a / h = 3)

Problem formulation: The thick square slab is simply supported along the sides and subjected to a transverse load distributed over the upper face according to the cosine law $q \cdot \cos((\pi \cdot x)/(2 \cdot a)) \cdot \cos((\pi \cdot y)/(2 \cdot a))$.
Determine:

- distribution of the horizontal normal stresses σ_x across the slab thickness z in its center (x = 0, y = 0);
- distribution of the horizontal tangential stresses τ_{xy} across the slab thickness z on its lateral edge (x = a, y = a);
- value of the vertical normal stresses σ_z in the center of the slab (x = 0, y = 0, z = 0);
- value of the vertical tangential stresses τ_{xz} in the center of the lateral face of the slab (x = a, y = 0, z = 0);
- distribution of the vertical displacements z across the slab thickness z in its center (x = 0, y = 0);
- distribution of the horizontal displacements x across the slab thickness in the center of its lateral face (x = a, y = 0, z = 0).

References: M.K. Usarov, *The problem of bending the thick orthotropic plate of three-dimensional formulation*, Magazine of Civil Engineering, 2011, No. 4, p. 40-47.

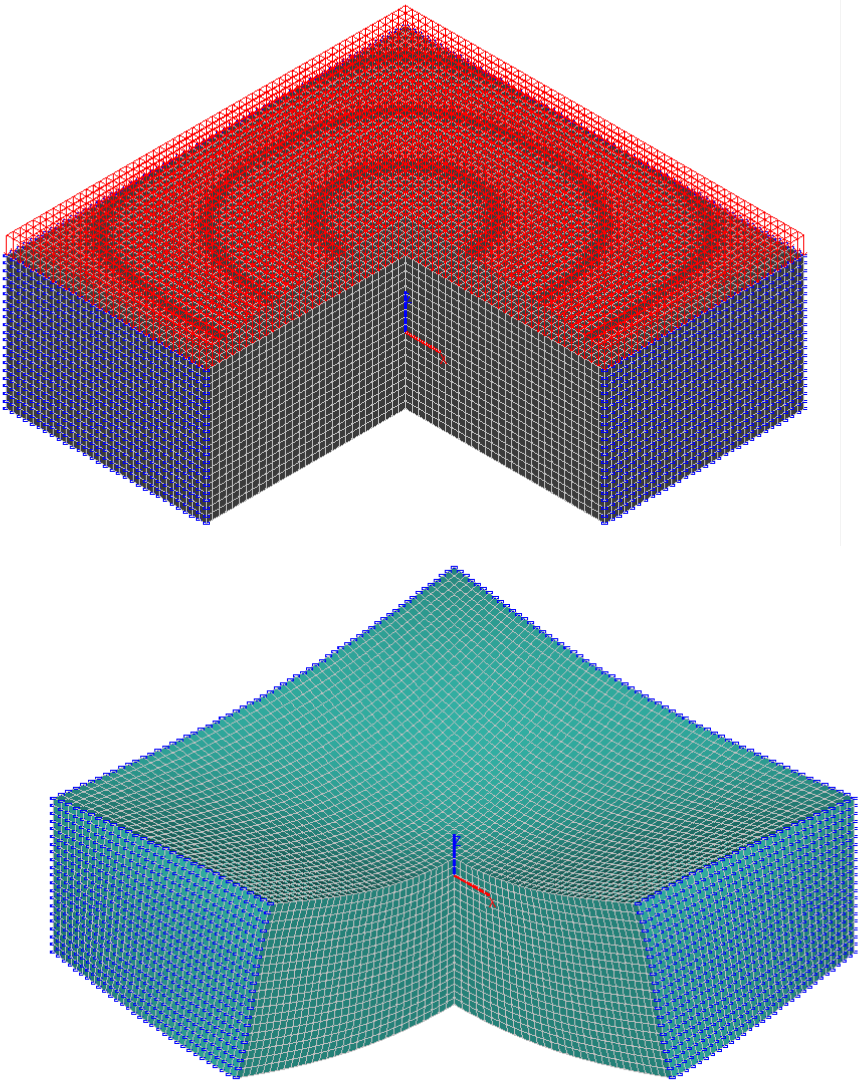
Initial data:

$E = 1.0 \cdot 10^5$ tf/m^2	- elastic modulus of the slab material;
$\upsilon = 0.3$	- Poisson’s ratio of the slab material;
$2 \cdot a = 30.0$ m	- side of the slab;
$2 \cdot h = 10.0$ m	- thickness of the slab;
$q = 10.0$ tf/m^2	- amplitude value of the transverse load distributed over the upper face of the slab according to the cosine law.

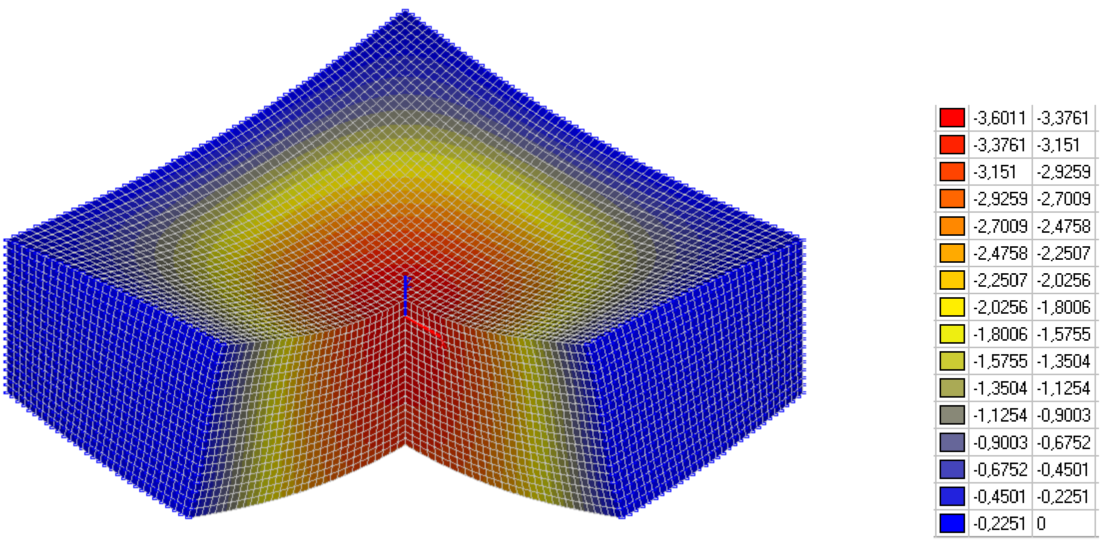
Finite element model: Design model – general type system, plate elements – 72000 solid eight-node isoparametric elements of type 36. The spacing of the finite element mesh of the slab in plan and along the thickness is 0.5 m. Internal forces are output along the axes of the global coordinate system. Constraints of the linear degrees of freedom Y, Z are installed in the nodes of the lateral faces of the slab x = ± a.

Constraints of the linear degrees of freedom X, Z are installed in the nodes of the lateral faces of the slab y = ± a b. Number of nodes in the design model – 78141.

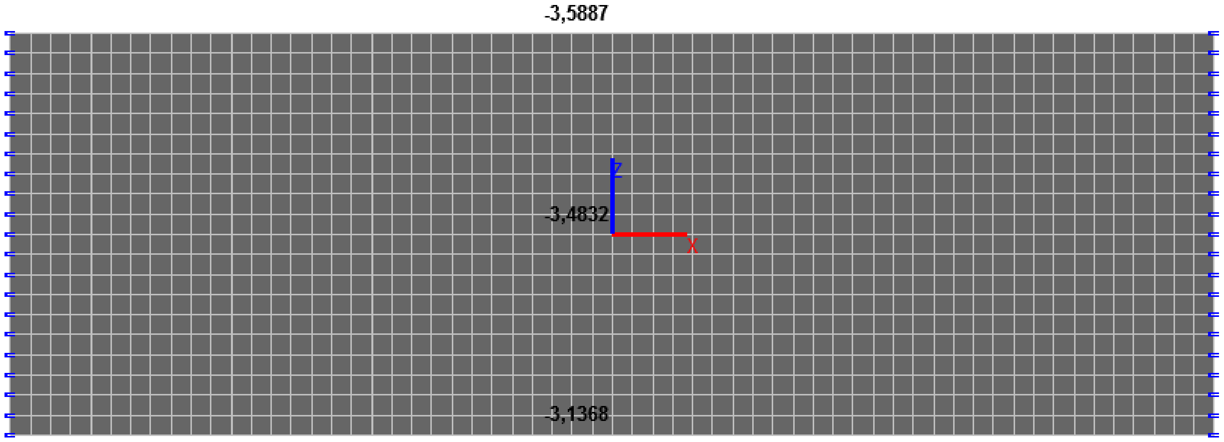
Results in SCAD



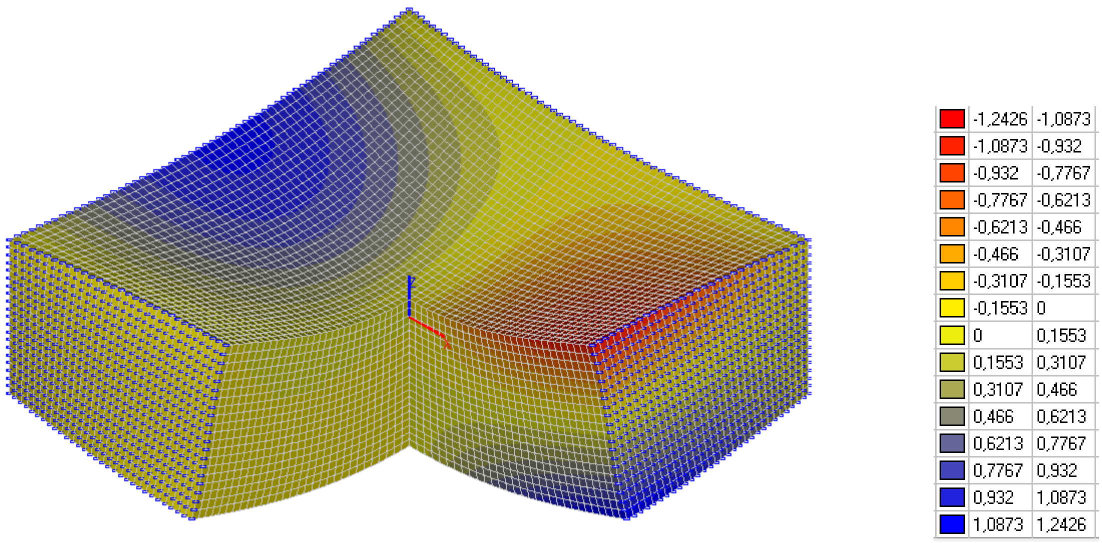
Design and deformed models



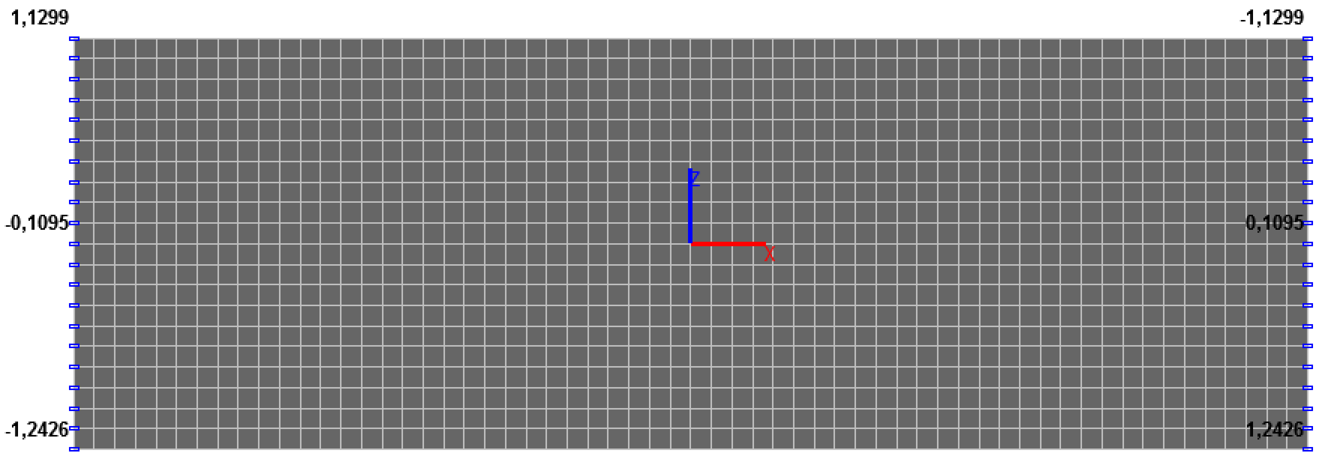
Values of vertical displacements z (mm)



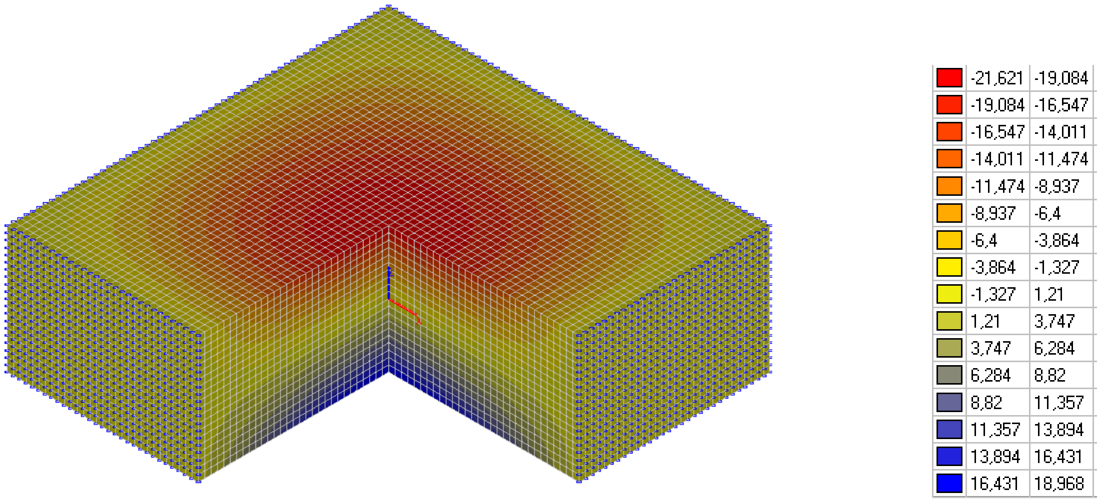
Values of vertical displacements z (mm) in the center of the slab (x = 0, y = 0)



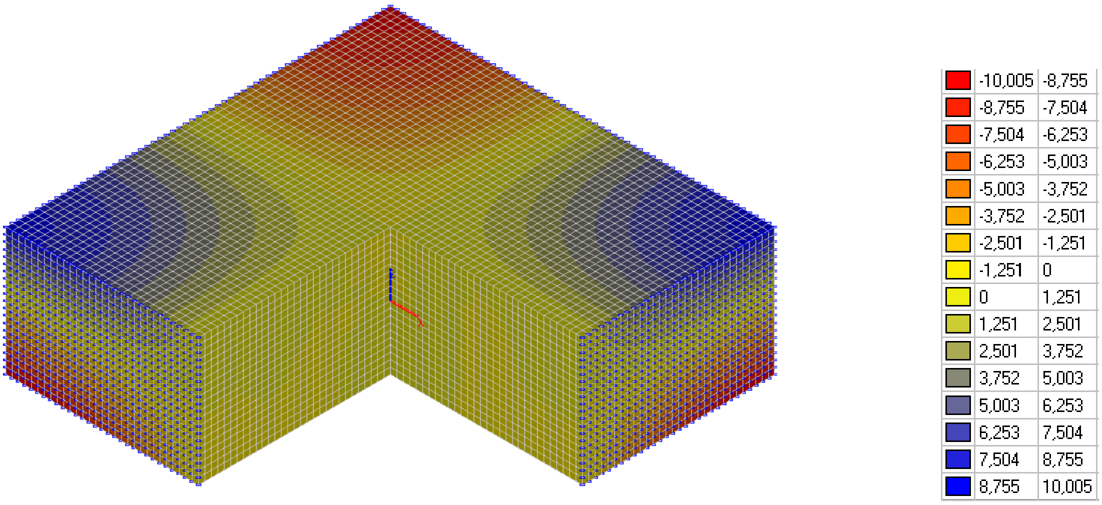
Values of horizontal displacements x (mm)



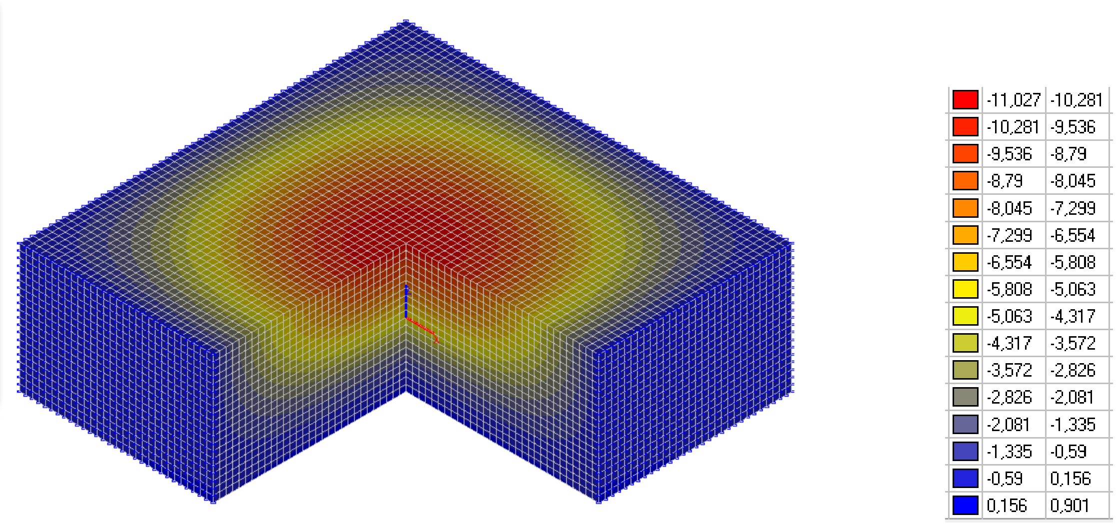
Values of horizontal displacements x (mm) in the middle of the lateral faces of the slab (x = ± a, y = 0, z = 0)



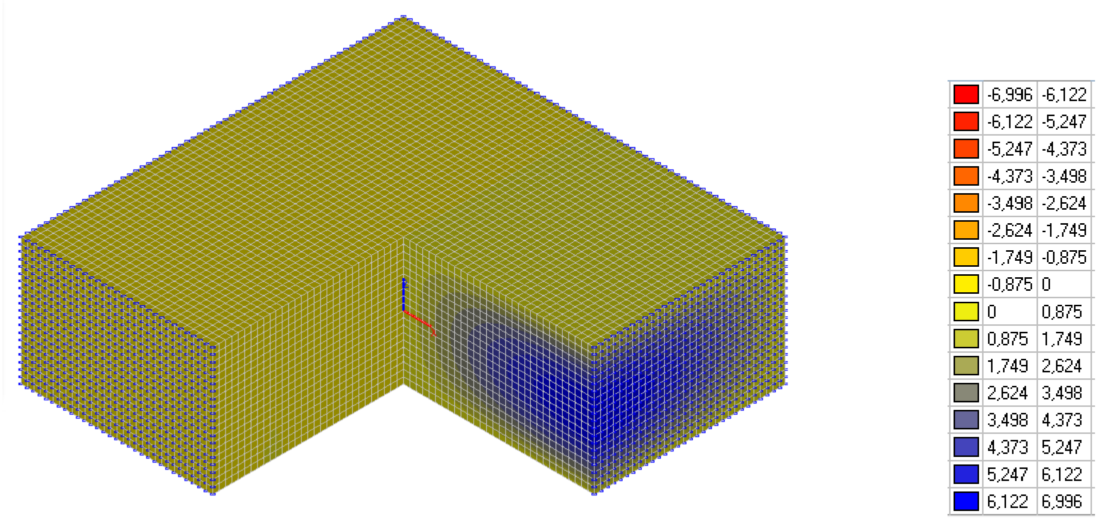
Values of horizontal normal stresses σ_x (tf/m^2)



Values of horizontal tangential stresses τ_{xy} (tf/m^2)



Values of vertical normal stresses σ_z (tf/m^2)



Values of vertical tangential stresses τ_{xz} (tf/m²)

Comparison of solutions:

z / h	σ_x , tf/m² (x = y = 0)			τ_{xy} , tf/m² (x = y = a)		
	Theory	SCAD	Deviations, %	Theory	SCAD	Deviations, %
1.0	-21.240	-21.591	1.65	9.129	9.098	0.34
0.0	-0.481	-0.479	0.42	-0.882	-0.881	0.11
-1.0	18.639	18.942	1.63	-10.036	-10.005	0.31

z / h	σ_z , tf/m² (x = y = 0)			τ_{xz} , tf/m² (x = a, y = 0)		
	Theory	SCAD	Deviations, %	Theory	SCAD	Deviations, %
0.0	-4.944	-4.939	0.10	7.023	6.996	0.38

z / h	z, mm (x = y = 0)			x, mm (x = a, y = 0)		
	Theory	SCAD	Deviations, %	Theory	SCAD	Deviations, %
1.0	-3.5963	-3.5887	0.21	-1.1333	-1.1299	0.30
0.0	-3.4906	-3.4832	0.21	0.1095	0.1095	0.00
-1.0	-3.1440	-3.1368	0.23	1.2459	1.2426	0.26

Notes: In the analytical solution the horizontal normal stresses σ_x across the slab thickness z in its center (x = 0, y = 0), horizontal tangential stresses τ_{xy} across the slab thickness z on its lateral edge (x = a, y = a), vertical normal stresses σ_z in the center of the slab (x = 0, y = 0, z = 0), vertical tangential stresses τ_{xz} in the center of the lateral face of the slab (x = a, y = 0, z = 0), vertical displacements z across the slab thickness z in its center (x = 0, y = 0), horizontal displacements x across the slab thickness in the center of its lateral face (x = a, y = 0, z = 0) for υ = 0.3 and γ = a / h = 3 can be determined according to the following formulas:

$\frac{z}{h} = 1.0:$ $\sigma_x = -2.1240 \cdot q$; $\tau_{xy} = -0.9129 \cdot q$;

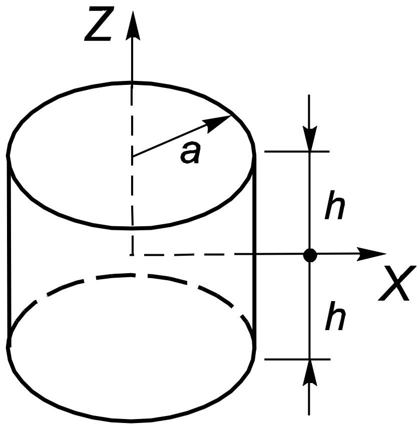
$z = -3596.3 \cdot \frac{q \cdot 2 \cdot h}{E}$; $x = -1133.3 \cdot \frac{q \cdot 2 \cdot h}{E}$;

$\frac{z}{h} = 0.0:$ $\sigma_x = -0.0481 \cdot q$; $\tau_{xy} = -0.0882 \cdot q$ $\sigma_z = -0.4944 \cdot q$;

$\tau_{xy} = 0.7023 \cdot q$ $z = -3490.6 \cdot \frac{q \cdot 2 \cdot h}{E}$; $x = 109.5 \cdot \frac{q \cdot 2 \cdot h}{E}$;

$\frac{z}{h} = -1.0:$ $\sigma_x = 1.8639 \cdot q$; $\tau_{xy} = 1.0036 \cdot q$; $z = -3144.0 \cdot \frac{q \cdot 2 \cdot h}{E}$; $x = 1245.9 \cdot \frac{q \cdot 2 \cdot h}{E}$

Thick Circular Slab Clamped along the Side Surface Subjected to a Load Uniformly Distributed over the Upper Face



Objective: Determination of the stress-strain state of a thick circular slab clamped along the side surface subjected to a load uniformly distributed over the upper face in accordance with the spatial problem of the theory of elasticity.

SCAD version usedInitial data files:

File name	Description
4.37_4m.SPR	Design model for the slab thickness of 4 m
4.37_6m.SPR	Design model for the slab thickness of 6 m

Problem formulation: The thick circular slab is clamped along the side surface and subjected to a load *q* uniformly distributed over the upper face. Determine:
distribution of the radial σ_r and vertical σ_z normal stresses across the slab thickness in its center ($r = 0$);
distribution of the vertical displacements w across the slab thickness in its center ($r = 0$).

References: Solyanik-Krassa K.V. Axisymmetric Problem of the Theory of Elasticity. – M.: Stroyizdat. 1987. p. 336.

Initial data:

$E = 1.0 \cdot 10^7$ kPa - elastic modulus;
$\mu = 0.25$ - Poisson's ratio;
$2 \cdot a = 20.0$ m - diameter of the slab;
$2 \cdot h = 4.0$ m; 6.0 m - thickness of the slab;
q = 10 kPa - load uniformly distributed over the upper face.

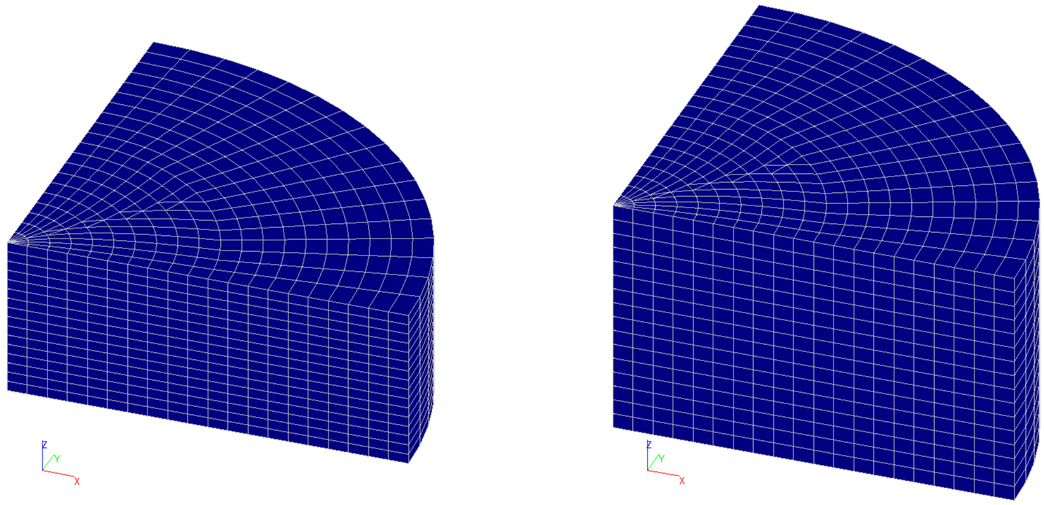
Finite element model

The spacing of the finite element mesh of the slab in plan in the radial direction is 0.5 m and there are 16 layers of finite elements along the thickness *(models 1x1).*

Elements of the design model:
4384 solid twenty-node isoparametric elements of type 37 (parallelepiped);
400 solid fifteen-node isoparametric elements of type 35 (triangular prism).

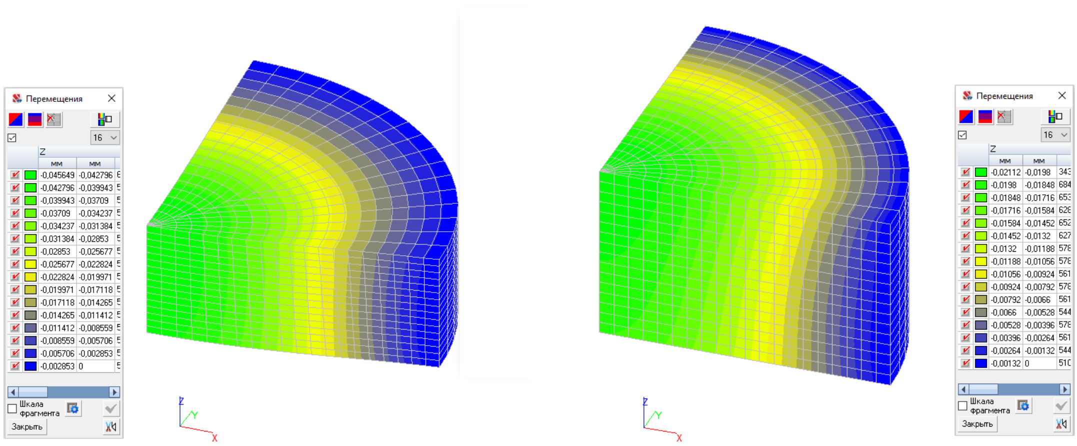
Number of nodes in the design model – 20866.

The calculation was performed taking into account the symmetry planes. The constraints were imposed:
on the side surface in the directions of all the linear degrees of freedom;
on the YOZ plane – along the *x* axis;
on the XOZ plane – along the *y* axis.



Design models of 4.0 m and 6.0 m thick slabs

Results in SCAD



Values of vertical displacements w (mm) in 4.0 m and 6.0 m thick slabs

Comparison of solutions:

Thickness	***Value***	***Point***	***Approximate theory***	***SCAD***	**Deviation (%)**
4m	***w(mm)***	(0,0,2)	-0.0436	-0.04538	4.08
		(0,0,0)	-0.0424	-0.0454	7.08
		(0,0,-2)	-0.0411	-0.04364	6.18
	$\sigma_r=\sigma_\theta$ ***(kPa)***	(0,0,2)	-34.51	-33.78	2.12
		(0,0,0)	-1.6667	-1.5547	6.72
		(0,0,-2)	31.1719	30.62	1.76
	σ_z ***(kPa)***	(0,0,2)	-10	-10.16	0.16
		(0,0,0)	-5	-5.07	0.14
		(0,0,-2)	0	-0.05	–
6m	***w***	(0,0,3)	-0.02097	-0.02112	0.72
		(0,0,0)	-0.01916	-0.01994	4.07
		(0,0,-3)	-0.01722	-0.01851	7.49
	$\sigma_r=\sigma_\theta$ ***(kPa)***	(0,0,3)	-18.2292	-18.51	1.54
		(0,0,0)	-1.6667	-1.5149	9.12
		(0,0,-3)	14.896	14.4884	2.74
	σ_z ***(kPa)***	(0,0,3)	-10	-9.797	2.03
		(0,0,0)	-5	-5.0569	1.14
		(0,0,-3)	0	0.043	–

Note 1: *The approximate analytical values were calculated according to the formulas given on pages 124-125 of "Solyanik-Krassa K.V. Axisymmetric Problem of the Theory of Elasticity. – M.: Stroyizdat. 1987."*

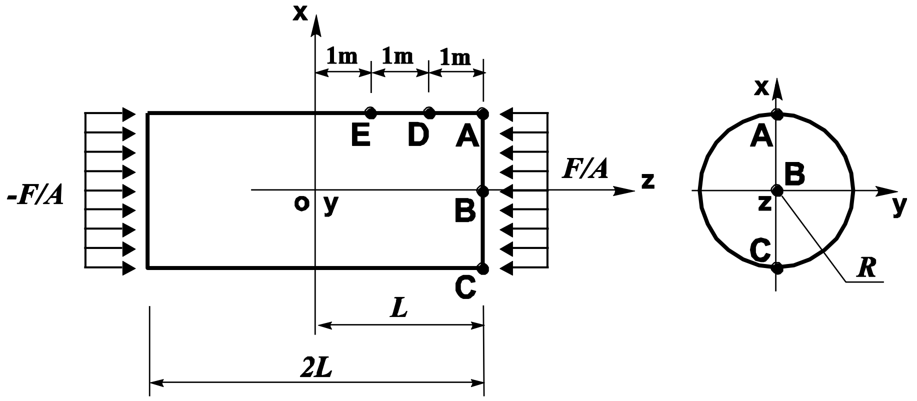
Note 2: The calculations were performed for meshes refined by a factor of 2 and 4 (*4x4 models*) to study the convergence of the method. The symmetry planes were taken into account. The maximum design model contained:
280576 solid twenty-node isoparametric elements of type 37 (parallelepiped);
25600 solid fifteen-node isoparametric elements of type 35 (triangular prism).

Number of nodes in the design model – 1222501.

Comparison of solutions:

Thickness	***Value***	***Point***	***SCAD***		**Deviation (%)**
			4x4	1x1	
4m	***w(mm)***	(0,0,2)	-0.04534	-0.04538	0.09
		(0,0,0)	-0.0454	-0.0454	–
		(0,0,-2)	-0.04374	-0.04364	0.23
	$\sigma_r=\sigma_\theta$ ***(kPa)***	(0,0,2)	-33.6603	-33.78	0.36
		(0,0,0)	-1.5683	-1.5547	0.87
		(0,0,-2)	30.527	30.62	0.30
	σ_z ***(kPa)***	(0,0,2)	-10.0062	-10.16	1.36
		(0,0,0)	-5.0037	-5.0742	1.41
		(0,0,-2)	0.00326	-0.05	–
6m	***w***	(0,0,3)	-0.02108	-0.02112	0.19
		(0,0,0)	-0.01995	-0.01994	0.05
		(0,0,-3)	-0.01852	-0.01851	0.05
	$\sigma_r=\sigma_\theta$ ***(kPa)***	(0,0,3)	-17.373	-17.557	1.06
		(0,0,0)	-1.5213	-1.5149	0.42
		(0,0,-3)	14.3485	14.4884	0.98
	σ_z ***(kPa)***	(0,0,3)	-10.0006	-9.797	2.03
		(0,0,0)	-5.0367	-5.0694	0.65
		(0,0,-3)	0.0028	0.0434	–

Cylindrical Body Free from Restraints Subjected to a Longitudinal Load Uniformly Distributed over the Edges



Objective: Determination of the strain state of a cylindrical body free from restraints subjected to a longitudinal load uniformly distributed over the edges.

Initial data file: **SSLV01_v11.5.SPR**

***Problem formulation*:** The cylindrical body free from restraints is subjected to a longitudinal load uniformly distributed over the edges F/A. Determine the meridional ΔL and radial ΔR displacements of the points E, D, A (C) of the side surface of the cylinder at the distances from its transverse symmetry plane along the generatrix $L/3$, $2 \cdot L/3$, L respectively, as well as the point B of the center of its edge surface.

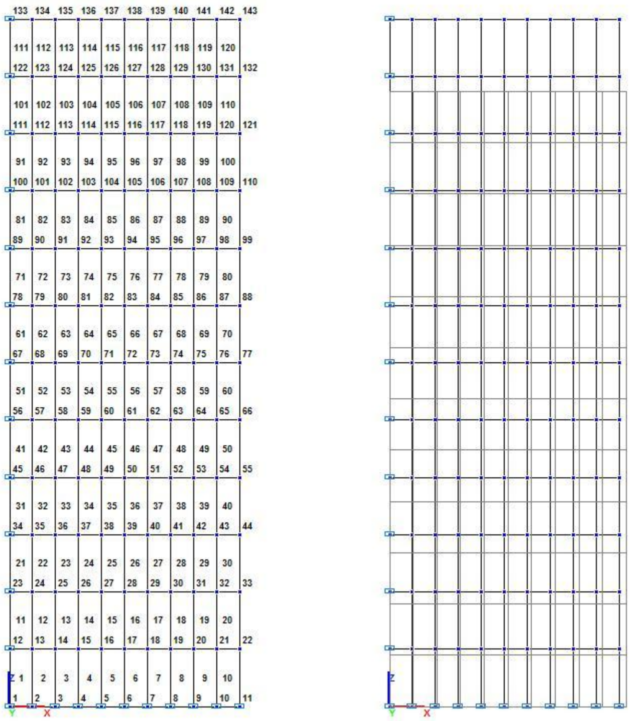
References: P. Germain, Introduction a la mecanique des milieux continus, Paris, Masson, 1986.

Initial data:

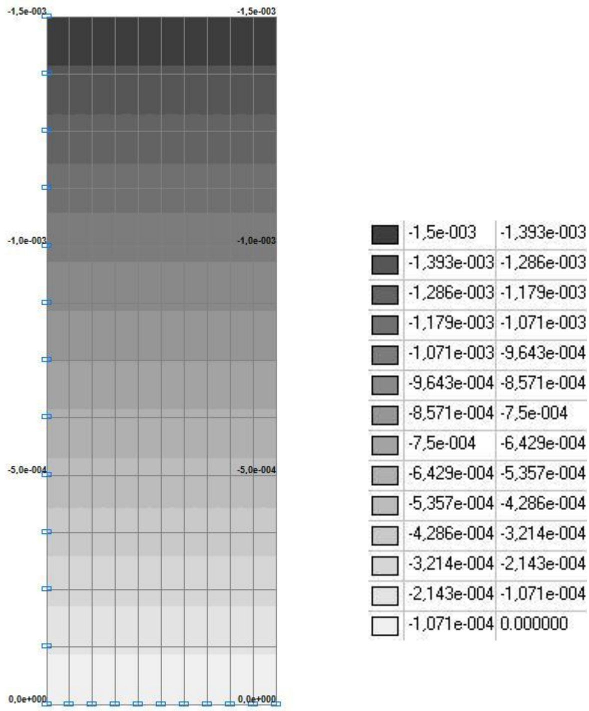
$E = 2.0 \cdot 10^5$ Pa	- elastic modulus;
$\nu = 0.3$	- Poisson’s ratio;
R = 1.0 m	- radius of the cylinder;
$L = 4.0$ m	- length of the cylinder;
$F/A = 1.0 \cdot 10^2$ Pa	- load uniformly distributed over the edges.

Finite element model: Design model – axisymmetric problem, axisymmetric elements – 120 shell elements of type 61. The spacing of the finite element mesh in the meridian direction is 0.25 m and in the radial direction is 0.10 m. The dimensional stability of the design model is provided by imposing constraints according to its symmetry conditions. Number of nodes in the design model – 143.

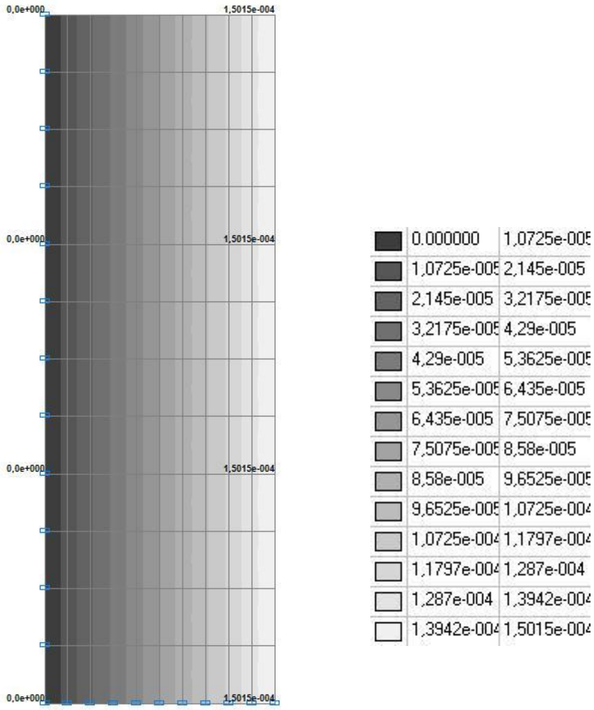
Results in SCAD



Design and deformed models



Values of meridional displacements Z (ΔL) m



Values of radial displacements X (ΔR) m

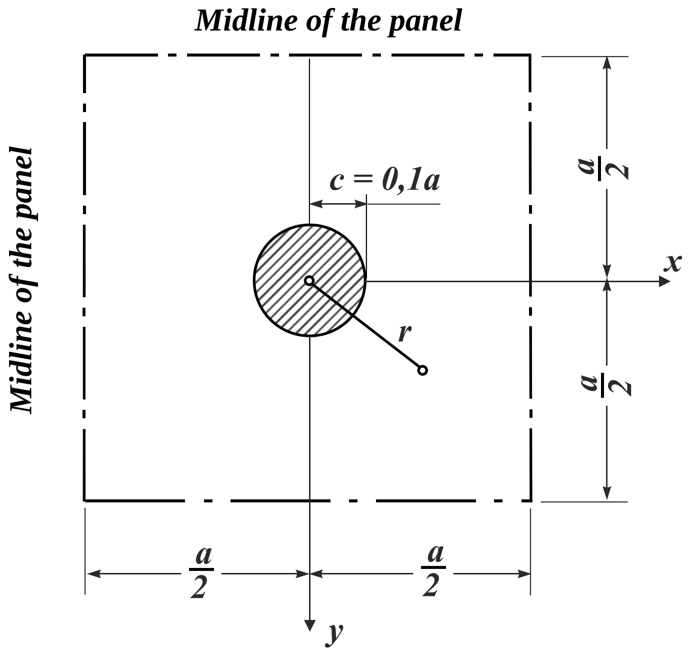
Comparison of solutions:

Parameter	**Theory**	**SCAD**	**Deviations, %**
Meridional displacement Δ*L* (point E), m	$-0.500\cdot10^{-3}$	$-0.500\cdot10^{-3}$	0.00
Radial displacement ΔR (point E), m	$-0.150\cdot10^{-3}$	$-0.150\cdot10^{-3}$	0.00
Meridional displacement Δ*L* (point D), m	$-1.000\cdot10^{-3}$	$-1.000\cdot10^{-3}$	0.00
Radial displacement ΔR (point D), m	$-0.150\cdot10^{-3}$	$-0.150\cdot10^{-3}$	0.00
Meridional displacement Δ*L* (points A and C), m	$-1.500\cdot10^{-3}$	$-1.500\cdot10^{-3}$	0.00
Radial displacement ΔR (points A and C), m	$-0.150\cdot10^{-3}$	$-0.150\cdot10^{-3}$	0.00
Meridional displacement Δ*L* (point B), m	$-1.500\cdot10^{-3}$	$-1.500\cdot10^{-3}$	0.00
Radial displacement ΔR (point B), m	$0.000\cdot10^{-3}$	$0.000\cdot10^{-3}$	0.00

Notes: In the analytical solution the meridional Δ*L* and radial ΔR displacements can be determined according to the following formulas:

$$\Delta L = \frac{P \cdot X}{E}; \qquad \Delta R = \frac{\nu \cdot P \cdot R}{E}.$$

Square Panel of a Flat Slab Rigidly Connected to a Column of a Circular Cross-Section Subjected to a Uniformly Distributed Transverse Load



Objective:
Determine the bending moments in the characteristic points of a square panel of a flat slab rigidly connected to a column of a circular cross-section subjected to a uniformly distributed transverse load.

Initial data file: **Flate_plate_Circular_column.spr**

Problem formulation:
The square panel of a flat slab rigidly connected to a column of a circular cross-section is subjected to a uniformly distributed transverse load *q*. Determine the bending moments M_x, M_y in the characteristic points of the square panel of the flat slab.

References:
S. Timoshenko, S. Woinowsky-Krieger, Theory of Plates and Shells, Moscow, Book House "LIBROKOM", 2009, p. 287-289.

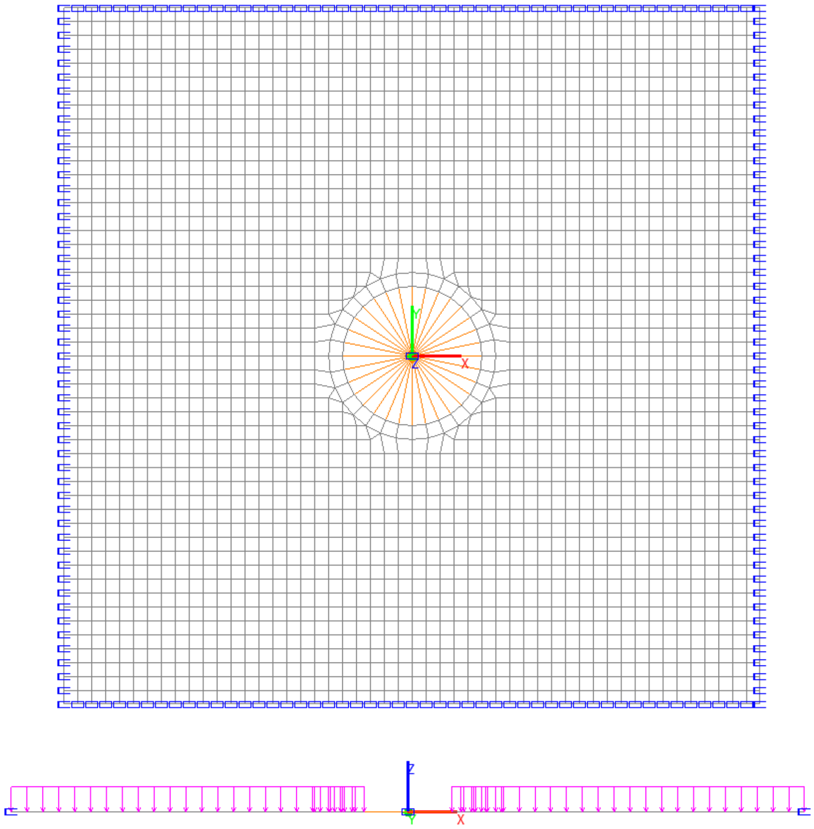
Initial data:

$E = 3.0 \cdot 10^7$ N/m^2	- elastic modulus,
ν = 0.2	- Poisson's ratio,
h = 0.1 m	- thickness of the panel of the flat slab;
a = 2.5 m	- side of the panel of the flat slab;
c = 0.1·a = 0.25 m	- radius of the column cross-section;
q = 100.0 N/m^2	- value of the uniformly distributed transverse load.

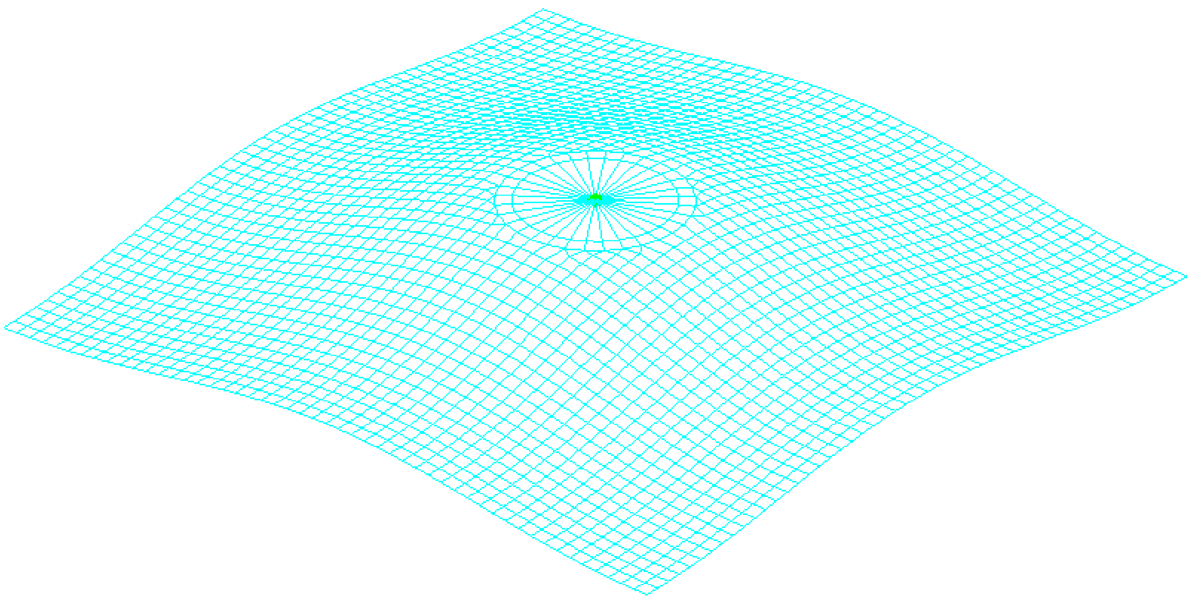
Finite element model:
Design model – grade beam, plate; elements of the panel of the flat slab – 2412 quadrangular four-node thin plate elements for the calculation according to the Kirchhoff-Love theory of type 20 and 16 triangular three-node thin plate elements for the calculation according to the Kirchhoff-Love theory of type 15; element of the column cross-section – 1 rigid body element of type 100. The spacing of the finite element mesh of the panel of the flat slab in the directions of the axes of the global coordinate system is 0.05 m except for the support contour where the spacing of the finite element mesh in the radial direction is 0.05 m and in the circumferential direction is 11.25°. Internal forces are output along the axes of the global coordinate system. Boundary conditions are provided by imposing constraints in the directions of the

degrees of freedom UX for the edges of the panel parallel to the X axis of the global coordinate system, and UY for the edges of the panel parallel to the Y axis of the global coordinate system. The master node of the rigid body of the column is in the center of its cross-section and is restrained in the direction of the degree of freedom Z. Number of nodes in the design model – 2537.

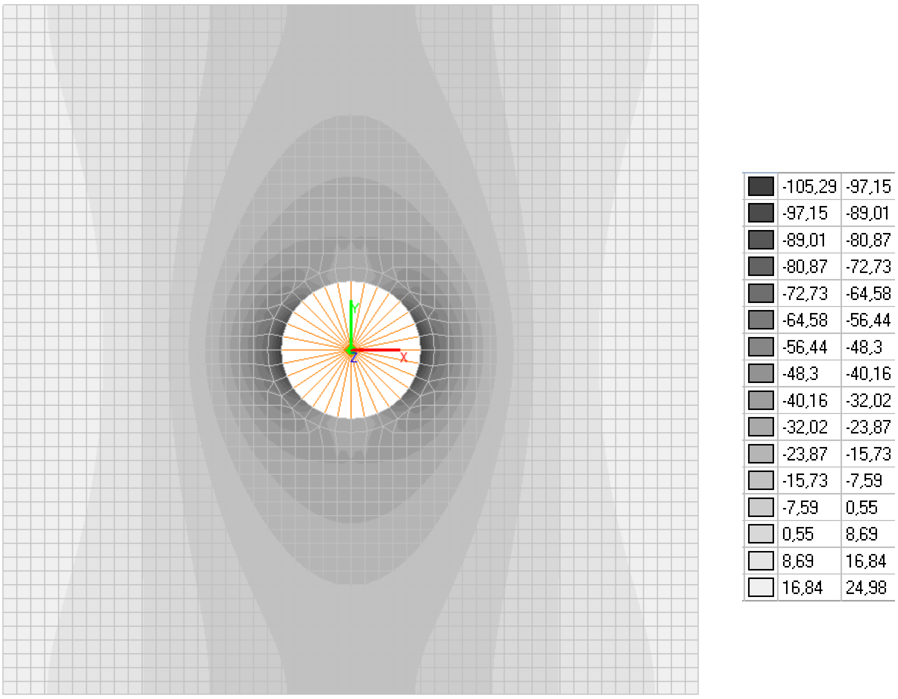
Results in SCAD:



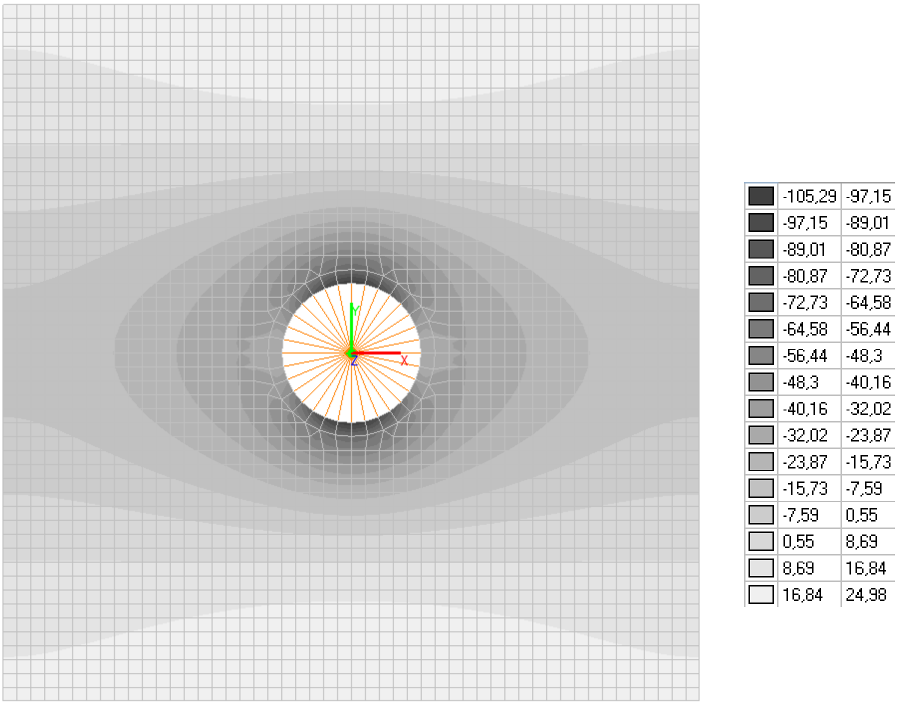
Design model



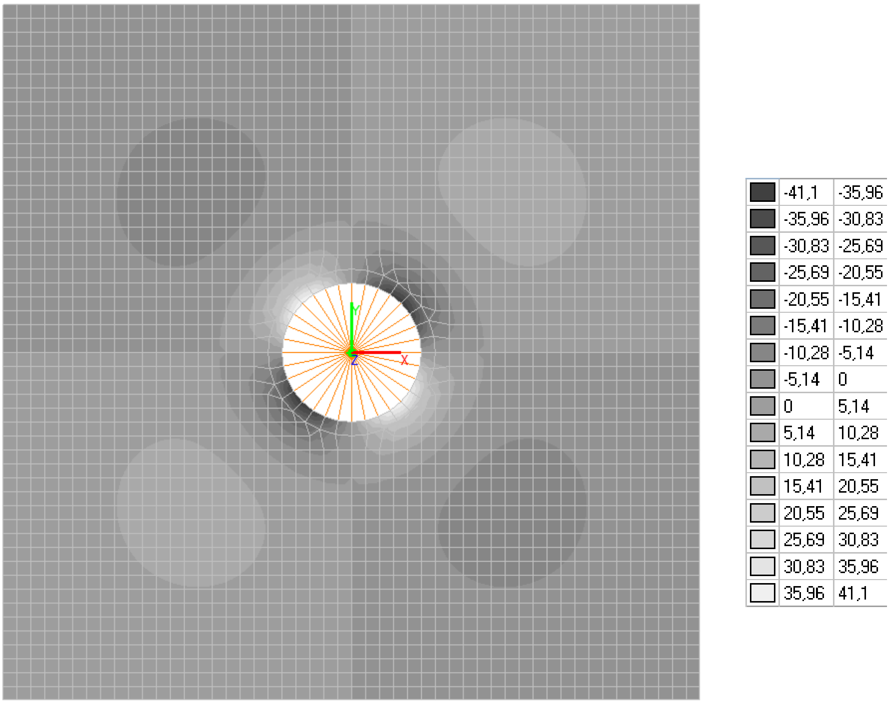
Deformed model



Bending moments M_x, N· m/m



Bending moments M_y, N· m/m



Bending moments M_{xy}, N· m/m

Comparison of solutions:

Bending moment	**Panel point**	**Theory**	**SCAD**	**Deviations, %**
$M_x = M_y$	$x = a/2$, $y = a/2$	18.2500	17.8300	2.30
M_x	$x = a/2$, $y = 0$	24.9375	24.9800	0.17
M_y	$x = a/2$, $y = 0$	-10.0625	-10.1400	0.77
M_x	$x = c$, $y = 0$	-105.1250	-105.2900	0.16

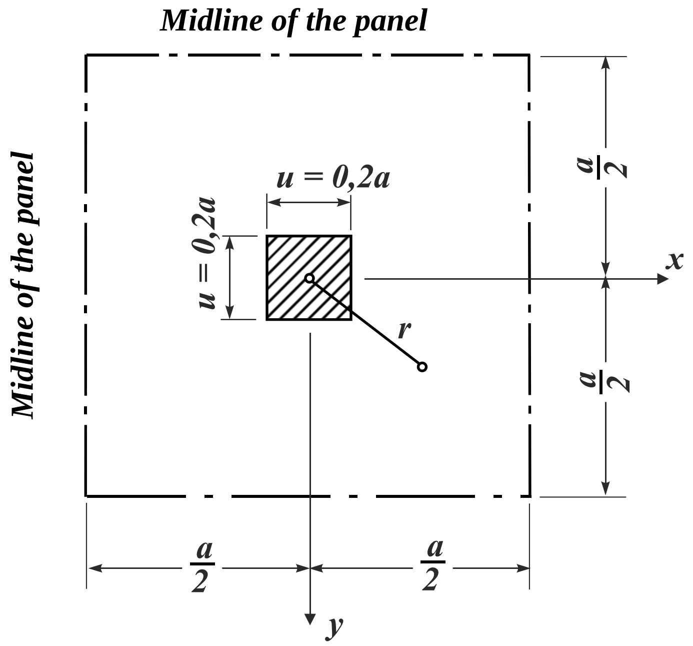
Notes: In the analytical solution the bending moments M_x, M_y in the characteristic points of the square panel of the flat slab are determined according to the following formulas:

$$M=\beta \cdot q \cdot a^2 .$$

The coefficients β for the calculation of bending moments at $c = 0.1 \cdot a$ and $\upsilon = 0.2$

Bending moment	**Panel point**	**β**
$M_x = M_y$	$x = a/2$, $y = a/2$	0.0292
M_x	$x = a/2$, $y = 0$	0.0399
M_y	$x = a/2$, $y = 0$	-0.0161
M_x	$x = c$, $y = 0$	-0.1682

Square Panel of a Flat Slab Rigidly Connected to a Column of a Square Cross-Section Subjected to a Uniformly Distributed Transverse Load



Objective:
Determination of the bending moments in the characteristic points of a square panel of a flat slab rigidly connected to a column of a square cross-section subjected to a uniformly distributed transverse load.

Initial data file: **Flate_plate_Square_column.spr**

Problem formulation:
The square panel of a flat slab rigidly connected to a column of a square cross-section is subjected to a uniformly distributed transverse load *q*. Determine the bending moments M_x, M_y in the characteristic points of the square panel of the flat slab.

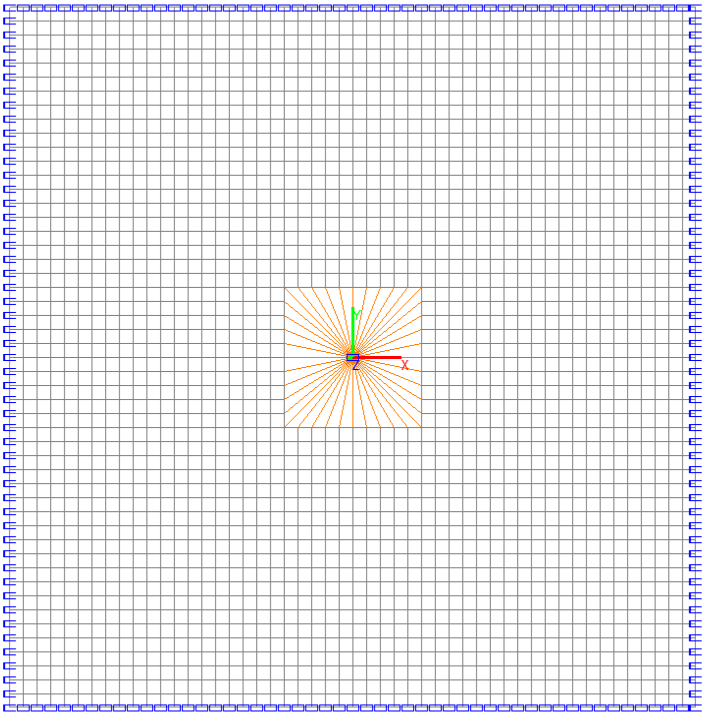
References:
S. Timoshenko, S. Woinowsky-Krieger, Theory of Plates and Shells, Moscow, Book House "LIBROKOM", 2009, p. 287-289.

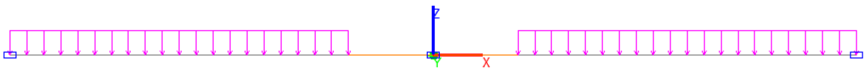
Initial data:

$E = 3.0 \cdot 10^7$ N/m^2	- elastic modulus,
ν = 0.2	- Poisson’s ratio,
h = 0.1 m	- thickness of the panel of the flat slab;
a = 2.5 m	- side of the panel of the flat slab;
u = 0.2·a = 0.5 m	- side of the column cross-section;
q = 100.0 N/m^2	- value of the uniformly distributed transverse load.

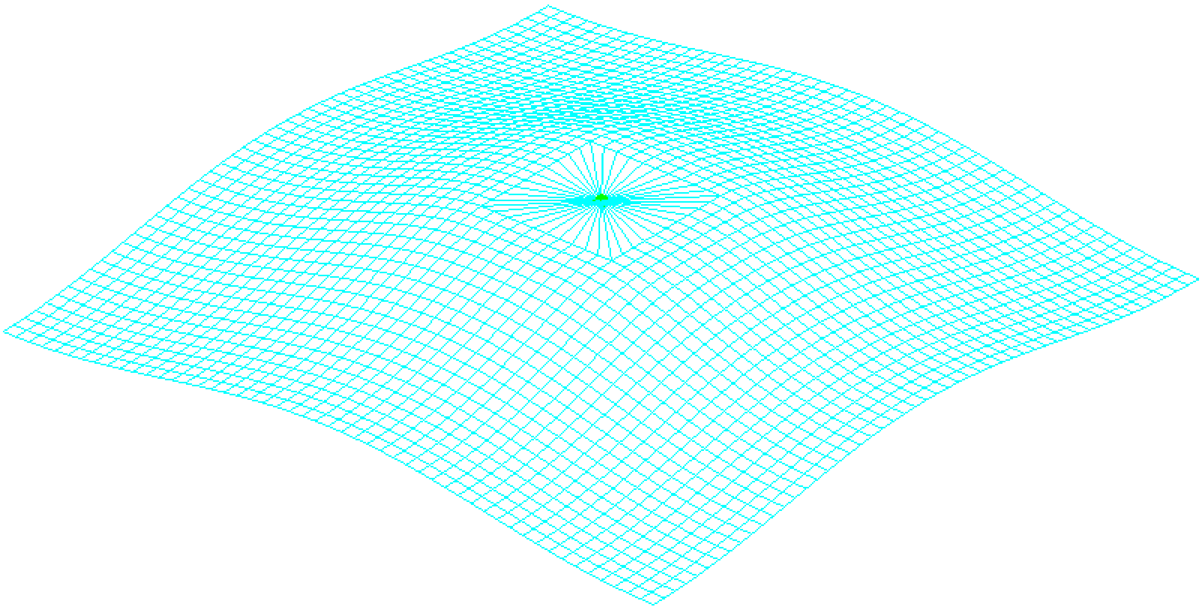
Finite element model:
Design model – grade beam, plate; elements of the panel of the flat slab – 2400 quadrangular four-node thin plate elements for the calculation according to the Kirchhoff-Love theory of type 20; element of the column cross-section – 1 rigid body element of type 100. The spacing of the finite element mesh of the panel of the flat slab in the directions of the axes of the global coordinate system is 0.05 m. Internal forces are output along the axes of the global coordinate system. Boundary conditions are provided by imposing constraints in the directions of the degrees of freedom UX for the edges of the panel parallel to the X axis of the global coordinate system, and UY for the edges of the panel parallel to the Y axis of the global coordinate system. The master node of the rigid body of the column is in the center of its cross-section and is restrained in the direction of the degree of freedom Z. Number of nodes in the design model – 2521.

Results in SCAD:

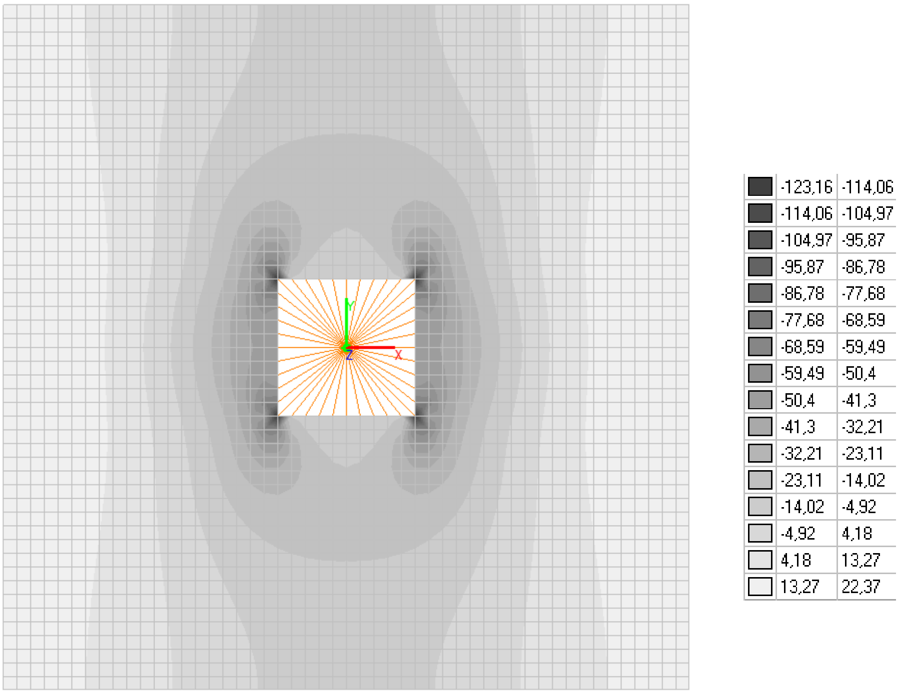




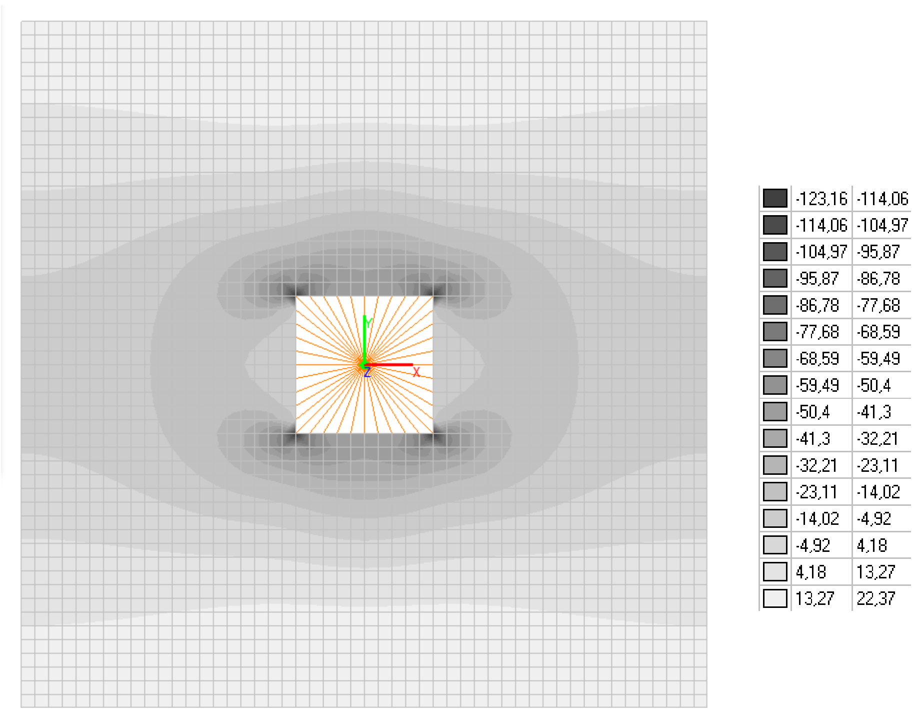
Design model



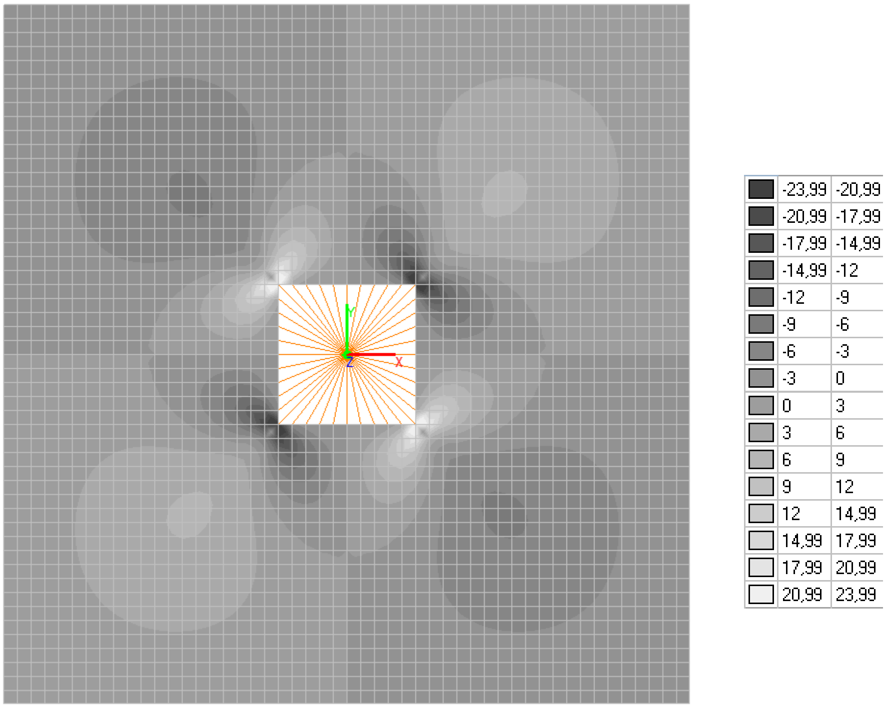
Deformed model



Bending moments M_x*, N· m/m*



Bending moments M_y*, N· m/m*



Bending moments M_{xy}, N m/m

Comparison of solutions:

Bending moment	**Panel point**	**Theory**	**SCAD**	**Deviations**
$M_x = M_y$	$x = a/2, y = a/2$	16.500	16.620	0.73
M_x	$x = a/2, y = 0$	21.750	22.370	2.85
M_y	$x = a/2, y = 0$	-9.125	-8.770	3.89
M_x	$x = u/2, y = 0$	-39.125	-43.210	9.45
M_x	$x = u/2, y = u/2$	-∞	-123.16	—

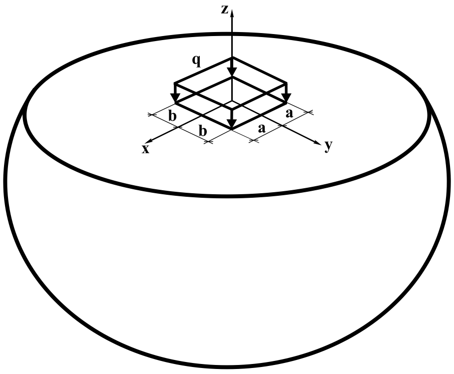
Notes: In the analytical solution the bending moments M_x, M_y in the characteristic points of the square panel of the flat slab are determined according to the following formulas:

$$M = \beta \cdot q \cdot a^2 .$$

Approximate values of the coefficients β for the calculation of bending moments at $u = 0.2 \cdot a$ and $\upsilon = 0.2$

Bending moment	**Panel point**	**β**
$M_x = M_y$	$x = a/2, y = a/2$	0.0264
M_x	$x = a/2, y = 0$	0.0348
M_y	$x = a/2, y = 0$	-0.0146
M_x	$x = u/2, y = 0$	-0.0626
M_x	$x = u/2, y = u/2$	-∞

Elastic Half-Space Subjected to a Transverse Load Uniformly Distributed over a Rectangular Surface. Love's Problem



Objective: Determination of the stress-strain state of the elastic half-space subjected to a transverse load uniformly distributed over a rectangular surface in accordance with the spatial problem of the theory of elasticity.

***Initial data files*: Lave.SPR**

Problem formulation: The elastic half-space is subjected to the transverse load q uniformly distributed over a rectangular surface. Determine:

- distribution of the normal stresses σ_x, σ_y, σ_z across the half-space;
- distribution of the tangential stresses τ_{xy}, τ_{xz}, τ_{yz} across the half-space;
- distribution of the displacements u, v, w across the half-space.

References: Z.G. Ter-Martirosyan, Soil Mechanics, Moscow, MGSU Publishing House of the Association of Construction Institutions of Higher Education, 2009, p. 204;
V.A. Florin, Fundamentals of Soil Mechanics, Volume 1, Leningrad, State Publishing House of Literature on Construction, Architecture and Building Materials, 1959, p. 123;
V.A. Florin, Fundamentals of Soil Mechanics, Volume 2, Leningrad, State Publishing House of Literature on Construction, Architecture and Building Materials, 1959, p. 24.

Initial data:

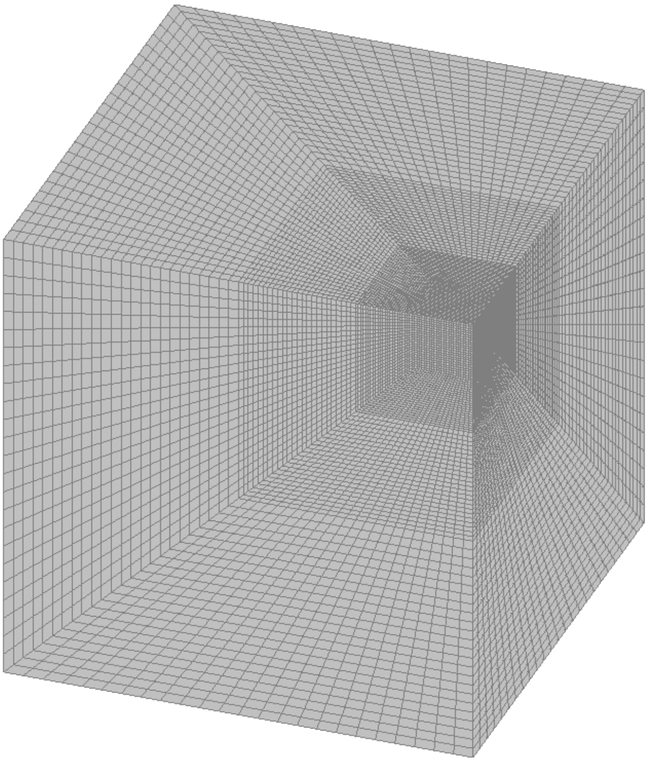
$E = 30000$ kN/m^2	- elastic modulus of the half-space;
$\mu = 0.3$	- Poisson's ratio;
$a = b = 2.0$ m	- length of the half of the side of a rectangular loaded surface;
$q = 100$ kN/m^2	- transverse load q uniformly distributed over a rectangular surface.

Finite element model: 96×96×48 m parallelepiped is analyzed. The design model (quarter of the parallelepiped cut off by the symmetry planes XOZ and YOZ) – general type system, elements of the elastic half-space – 138253 20-node isoparametric solid elements of type 37. The spacing of the initial finite element mesh of the half-space in the load application area in plan and along the depth is 0,25 m. The sizes of the finite elements increase with the distance from the load application area.
Internal forces are output along the axes of the global coordinate system. Upper faces of the elements of the half-space boundary are subjected to the surface transverse load within the following dimensions in plan 2a×2b = 4.0 м×4.0 m.

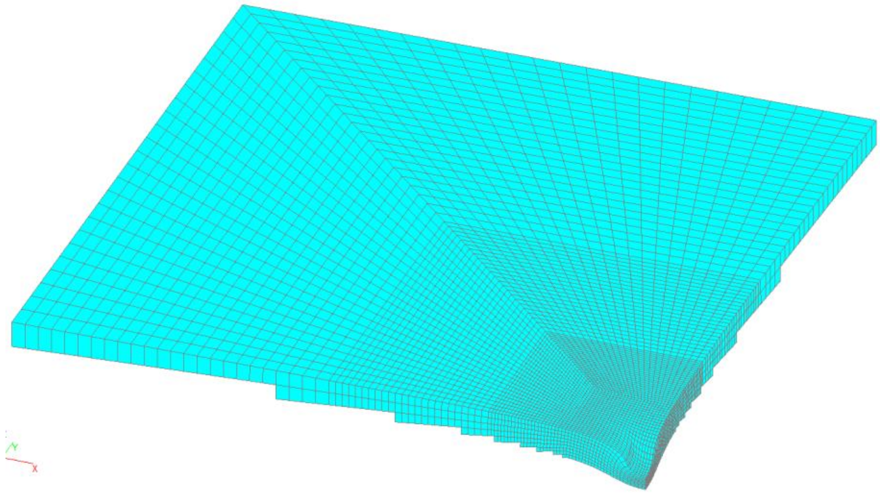
The boundary conditions were defined as follows: normal displacements on the lower and side surfaces are restrained.

Number of nodes in the design model – 573985.

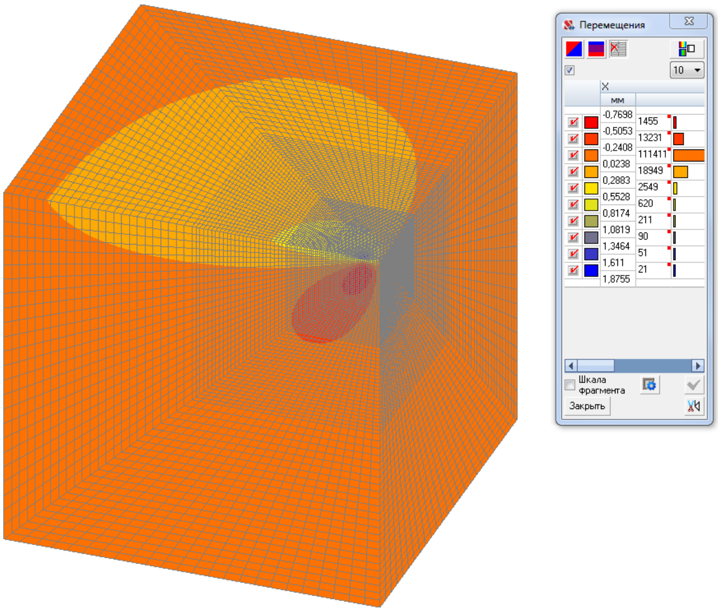
Results in SCAD



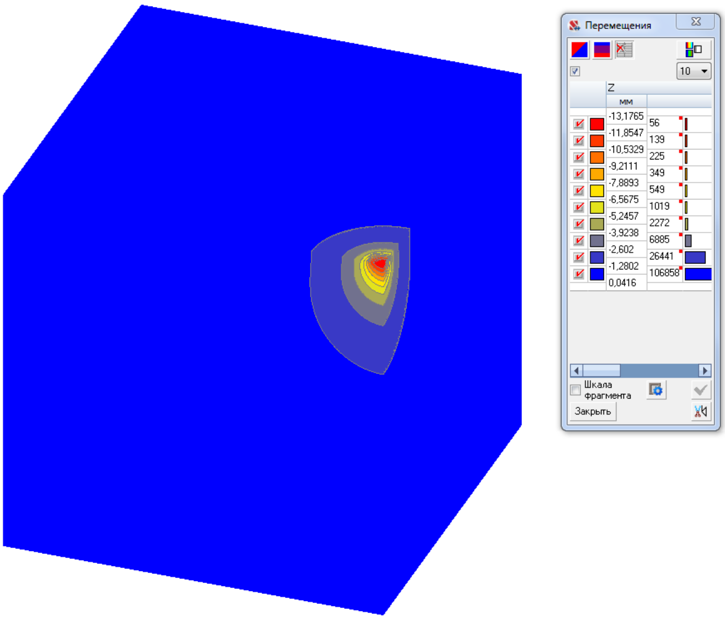
Design model



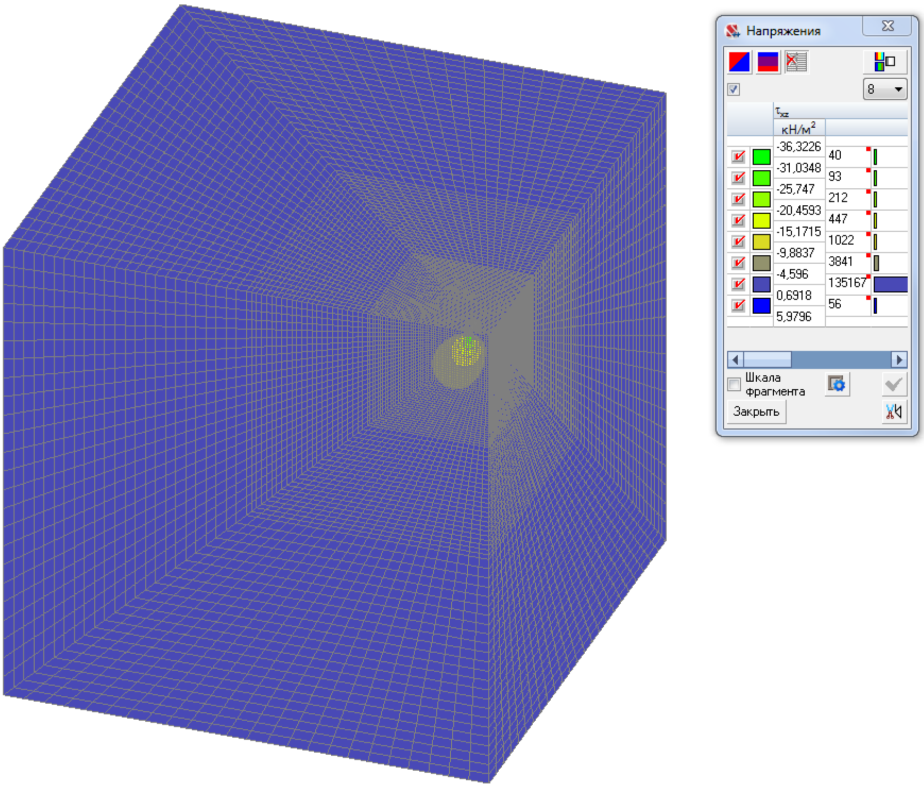
Deformed model of the surface of the elastic half-space



Values of horizontal displacements u (mm)



Values of vertical displacements w (mm)



Values of vertical τ_{xz} tangential stresses (t/m^2)

Comparison of solutions:

Solution of the Love's problem by 20-node isoparametric elements (mm, kN/m^2)

Point	Parameter		Theory	SCAD	Deviations, %
(0,0,0) Node 1	w	mm	-13,616	-13,177	3,2
	$\sigma_x=\sigma_y$	kN/m^2	-80,0	-79,919	0,10
	σ_z	kN/m^2	-100,0	-100,079	0,09
(0,0,-2) Node 10005	w	mm	-9,017	-8,574	4,91
	$\sigma_x=\sigma_y$	kN/m^2	-8,29	-8,189	1,21
	σ_z	kN/m^2	-70,09	-70,109	0,03
(-2,2,-2) Node 10213	u=v	mm	0,488	0,492	0,82
	w	mm	-5,704	-5,262	7,75
	$\sigma_x=\sigma_y$	kN/m^2	-7,56	-7,496	0,85
	σ_z	kN/m^2	-23,25	-23,267	0,07
	τ_{xy}	kN/m^2	-5,27	-5,288	0,34
	$\tau_{xz}=\tau_{yz}$	kN/m^2	12,11	12,166	0,46

Notes: In the analytical solution the distribution of the normal stresses σ_x, σ_y, σ_z, tangential stresses τ_{xy}, τ_{xz}, τ_{yz} and displacements u, v, w across the half-space is determined according to the following formulas:

$$\sigma_x(x,y,z)=\frac{q}{2\cdot\pi}\cdot\left\{-(1-2\cdot\mu)\cdot arctg\left[\frac{(x-a)\cdot(y-b)}{(x-a)^2+z^2-z\cdot\sqrt{(x-a)^2-(y-b)^2+z^2}}\right]+\right.$$

$$+2\cdot\mu\cdot arctg\left[\frac{(x-a)\cdot(y-b)}{z\cdot\sqrt{(x-a)^2+(y-b)^2+z^2}}\right]-\frac{z\cdot(x-a)\cdot(y-b)}{\left[(x-a)^2+z^2\right]\cdot\sqrt{(x-a)^2+(y-b)^2+z^2}}+$$

$$+(1-2\cdot\mu)\cdot arctg\left[\frac{(x-a)\cdot(y+b)}{(x-a)^2+z^2-z\cdot\sqrt{(x-a)^2+(y+b)^2+z^2}}\right]-$$

$$-2\cdot\mu\cdot arctg\left[\frac{(x-a)\cdot(y+b)}{z\cdot\sqrt{(x-a)^2+(y+b)^2+z^2}}\right]+\frac{z\cdot(x-a)\cdot(y+b)}{\left[(x-a)^2+z^2\right]\cdot\sqrt{(x-a)^2+(y+b)^2+z^2}}+$$

$$+(1-2\cdot\mu)\cdot arctg\left[\frac{(x+a)\cdot(y-b)}{(x+a)^2+z^2-z\cdot\sqrt{(x+a)^2+(y-b)^2+z^2}}\right]-$$

$$-2\cdot\mu\cdot arctg\left[\frac{(x+a)\cdot(y-b)}{z\cdot\sqrt{(x+a)^2+(y-b)^2+z^2}}\right]+\frac{z\cdot(x+a)\cdot(y-b)}{\left[(x+a)^2+z^2\right]\cdot\sqrt{(x+a)^2+(y-b)^2+z^2}}-$$

$$-(1-2\cdot\mu)\cdot arctg\left[\frac{(x+a)\cdot(y+b)}{(x+a)^2+z^2-z\cdot\sqrt{(x+a)^2+(y+b)^2+z^2}}\right]+$$

$$\left.+2\cdot\mu\cdot arctg\left[\frac{(x+a)\cdot(y+b)}{z\cdot\sqrt{(x+a)^2+(y+b)^2+z^2}}\right]-\frac{z\cdot(x+a)\cdot(y+b)}{\left[(x+a)^2+z^2\right]\cdot\sqrt{(x+a)^2+(y+b)^2+z^2}}\right\}.$$

$$\sigma_y(x,y,z)=\frac{q}{2\cdot\pi}\cdot\left\{-(1-2\cdot\mu)\cdot arctg\left[\frac{(x-a)\cdot(y-b)}{(y-b)^2+z^2-z\cdot\sqrt{(x-a)^2-(y-b)^2+z^2}}\right]+\right.$$

$$+2\cdot\mu\cdot arctg\left[\frac{(x-a)\cdot(y-b)}{z\cdot\sqrt{(x-a)^2+(y-b)^2+z^2}}\right]-\frac{z\cdot(x-a)\cdot(y-b)}{\left[(y-b)^2+z^2\right]\cdot\sqrt{(x-a)^2+(y-b)^2+z^2}}+$$

$$+(1-2\cdot\mu)\cdot arctg\left[\frac{(x-a)\cdot(y+b)}{(y+b)^2+z^2-z\cdot\sqrt{(x-a)^2+(y+b)^2+z^2}}\right]+$$

$$+\frac{z\cdot(x-a)\cdot(y+b)}{\left[(y+b)^2+z^2\right]\cdot\sqrt{(x-a)^2+(y+b)^2+z^2}}+$$

$$+(1-2\cdot\mu)\cdot arctg\left[\frac{(x+a)\cdot(y-b)}{(y-b)^2+z^2-z\cdot\sqrt{(x+a)^2+(y-b)^2+z^2}}\right]-$$

$$+2\cdot\mu\cdot arctg\left[\frac{(x+a)\cdot(y-b)}{z\cdot\sqrt{(x+a)^2+(y-b)^2+z^2}}\right]+\frac{z\cdot(x+a)\cdot(y-b)}{\left[(y-b)^2+z^2\right]\cdot\sqrt{(x+a)^2+(y-b)^2+z^2}}-$$

$$+(1-2\cdot\mu)\cdot arctg\left[\frac{(x+a)\cdot(y+b)}{(y+b)^2+z^2-z\cdot\sqrt{(x+a)^2+(y+b)^2+z^2}}\right]+$$

$$\left.+2\cdot\mu\cdot arctg\left[\frac{(x+a)\cdot(y+b)}{z\cdot\sqrt{(x+a)^2+(y+b)^2+z^2}}\right]-\frac{z\cdot(x+a)\cdot(y+b)}{\left[(y+b)^2+z^2\right]\cdot\sqrt{(x+a)^2+(y+b)^2+z^2}}\right\}.$$

$$\sigma_z(x,y,z)=\frac{q}{2\cdot\pi}\cdot\left\{\frac{z\cdot(x-a)\cdot(y-b)\cdot\left[(x-a)^2+(y-b)^2+2\cdot z^2\right]}{\left[(x-a)^2+z^2\right]\cdot\left[(y-b)^2+z^2\right]\cdot\sqrt{(x-a)^2+(y-b)^2+z^2}}+\right.$$

$$+arctg\left[\frac{(x-a)\cdot(y-b)}{z\cdot\sqrt{(x-a)^2+(y-b)^2+z^2}}\right]-$$

$$-\frac{z\cdot(x-a)\cdot(y+b)\cdot\left[(x-a)^2+(y+b)^2+2\cdot z^2\right]}{\left[(x-a)^2+z^2\right]\cdot\left[(y+b)^2+z^2\right]\cdot\sqrt{(x-a)^2+(y+b)^2+z^2}}-$$

$$-arctg\left[\frac{(x-a)\cdot(y+b)}{z\cdot\sqrt{(x-a)^2+(y+b)^2+z^2}}\right]-$$

$$-\frac{z\cdot(x+a)\cdot(y-b)\cdot\left[(x+a)^2+(y-b)^2+2\cdot z^2\right]}{\left[(x+a)^2+z^2\right]\cdot\left[(y-b)^2+z^2\right]\cdot\sqrt{(x+a)^2+(y-b)^2+z^2}}-$$

$$-arctg\left[\frac{(x+a)\cdot(y-b)}{z\cdot\sqrt{(x+a)^2+(y-b)^2+z^2}}\right]+$$

$$+\frac{z\cdot(x+a)\cdot(y+b)\cdot\left[(x+a)^2+(y+b)^2+2\cdot z^2\right]}{\left[(x+a)^2+z^2\right]\cdot\left[(y+b)^2+z^2\right]\cdot\sqrt{(x+a)^2+(y+b)^2+z^2}}+$$

$$\left.+arctg\left[\frac{(x+a)\cdot(y+b)}{z\cdot\sqrt{(x+a)^2+(y+b)^2+z^2}}\right]\right\}.$$

$$\tau_{xy}(x,y,z)=\frac{q}{2\cdot\pi}\cdot\left\{-(1-2\cdot\mu)\cdot ln\left[\sqrt{(x-a)^2+(y-b)^2+z^2}-z\right]+\right.$$

$$+\frac{z}{\sqrt{(x-a)^2+(y-b)^2+z^2}}+$$

$$+(1-2\cdot\mu)\cdot ln\left[\sqrt{(x-a)^2+(y+b)^2+z^2}-z\right]-$$

$$-\frac{z}{\sqrt{(x-a)^2+(y+b)^2+z^2}}+$$

$$+(1-2\cdot\mu)\cdot ln\left[\sqrt{(x+a)^2+(y-b)^2+z^2}-z\right]-$$

$$-\frac{z}{\sqrt{(x+a)^2+(y-b)^2+z^2}}-$$

$$-(1-2\cdot\mu)\cdot ln\left[\sqrt{(x+a)^2+(y+b)^2+z^2}-z\right]+$$

$$\left.+\frac{z}{\sqrt{(x+a)^2+(y+b)^2+z^2}}\right\}.$$

$$\tau_{xz}(x,y,z)=\frac{q}{2\cdot\pi}\cdot\left\{-\frac{z^2\cdot(y-b)}{\left[(x-a)^2+z^2\right]\cdot\sqrt{(x-a)^2+(y-b)^2+z^2}}+\right.$$

$$+\frac{z^2\cdot(y+b)}{\left[(x-a)^2+z^2\right]\cdot\sqrt{(x-a)^2+(y+b)^2+z^2}}+$$

$$+\frac{z^2\cdot(y-b)}{\left[(x+a)^2+z^2\right]\cdot\sqrt{(x+a)^2+(y-b)^2+z^2}}-$$

$$\left.-\frac{z^2\cdot(y+b)}{\left[(x+a)^2+z^2\right]\cdot\sqrt{(x+a)^2+(y+b)^2+z^2}}\right\}.$$

$$\tau_{yz}(x,y,z)=\frac{q}{2\cdot\pi}\cdot\left\{-\frac{z^2\cdot(x-a)}{\left[(y-b)^2+z^2\right]\cdot\sqrt{(x-a)^2+(y-b)^2+z^2}}+\right.$$

$$+\frac{z^2\cdot(x+a)}{\left[(y-b)^2+z^2\right]\cdot\sqrt{(x+a)^2+(y+b)^2+z^2}}+$$

$$+\frac{z^2\cdot(x-a)}{\left[(y+b)^2+z^2\right]\cdot\sqrt{(x-a)^2+(y+b)^2+z^2}}-$$

$$\left.-\frac{z^2\cdot(x+a)}{\left[(y+b)^2+z^2\right]\cdot\sqrt{(x+a)^2+(y+b)^2+z^2}}\right\}.$$

$$u(x,y,z)=\frac{q\cdot(1+\mu)}{2\cdot\pi\cdot E}\cdot\left\{2\cdot(1-\mu)\cdot z\cdot ln\left[\sqrt{(x-a)^2+(y-b)^2+z^2}+(y-b)\right]-\right.$$

$$-(1-2\cdot\mu)\cdot(y-b)\cdot ln\left[\sqrt{(x-a)^2+(y-b)^2+z^2}-z\right]-$$

$$-(1-2\cdot\mu)\cdot(x-a)\cdot arctg\left(\frac{y-b}{x-a}\right)-$$

$$-(1-2\cdot\mu)\cdot(x-a)\cdot arctg\left[\frac{z\cdot(y-b)}{(x-a)\cdot\sqrt{(x-a)^2+(y-b)^2+z^2}}\right]-$$

$$-2\cdot(1-\mu)\cdot z\cdot ln\left[\sqrt{(x-a)^2+(y+b)^2+z^2}+(y+b)\right]+$$

$$+(1-2\cdot\mu)\cdot(y+b)\cdot ln\left[\sqrt{(x-a)^2+(y+b)^2+z^2}-z\right]+$$

$$+(1-2\cdot\mu)\cdot(x-a)\cdot arctg\left(\frac{y+b}{x-a}\right)+$$

$$+(1-2\cdot\mu)\cdot(x-a)\cdot arctg\left[\frac{z\cdot(y+b)}{(x-a)\cdot\sqrt{(x-a)^2+(y+b)^2+z^2}}\right]-$$

$$-2\cdot(1-\mu)\cdot z\cdot ln\left[\sqrt{(x+a)^2+(y-b)^2+z^2}+(y-b)\right]+$$

$$+(1-2\cdot\mu)\cdot(y-b)\cdot ln\left[\sqrt{(x+a)^2+(y-b)^2+z^2}-z\right]+$$

$$+(1-2\cdot\mu)\cdot(x+a)\cdot arctg\left(\frac{y-b}{x+a}\right)+$$

$$+(1-2\cdot\mu)\cdot(x+a)\cdot arctg\left[\frac{z\cdot(y-b)}{(x+a)\cdot\sqrt{(x+a)^2+(y-b)^2+z^2}}\right]+$$

$$+2\cdot(1-\mu)\cdot z\cdot ln\left[\sqrt{(x+a)^2+(y+b)^2+z^2}+(y+b)\right]-$$

$$-(1-2\cdot\mu)\cdot(y+b)\cdot ln\left[\sqrt{(x+a)^2+(y+b)^2+z^2}-z\right]-$$

$$-(1-2\cdot\mu)\cdot(x+a)\cdot arctg\left(\frac{y+b}{x+a}\right)-$$

$$\left.-(1-2\cdot\mu)\cdot(x+a)\cdot arctg\left[\frac{z\cdot(y+b)}{(x+a)\cdot\sqrt{(x+a)^2+(y+b)^2+z^2}}\right]\right\}.$$

$$v(x,y,z)=\frac{q\cdot(1+\mu)}{2\cdot\pi\cdot E}\cdot\left\{2\cdot(1-\mu)\cdot z\cdot ln\left[\sqrt{(x-a)^2+(y-b)^2+z^2}+(x-a)\right]-\right.$$

$$-(1-2\cdot\mu)\cdot(x-a)\cdot ln\left[\sqrt{(x-a)^2+(y-b)^2+z^2}-z\right]-$$

$$-(1-2\cdot\mu)\cdot(y-b)\cdot arctg\left(\frac{x-a}{y-b}\right)-$$

$$-(1-2\cdot\mu)\cdot(y-b)\cdot arctg\left[\frac{z\cdot(x-a)}{(y-b)\cdot\sqrt{(x-a)^2+(y-b)^2+z^2}}\right]-$$

$$-2\cdot(1-\mu)\cdot z\cdot ln\left[\sqrt{(x-a)^2+(y+b)^2+z^2}+(x-a)\right]+$$

$$+(1-2\cdot\mu)\cdot(x-a)\cdot ln\left[\sqrt{(x-a)^2+(y+b)^2+z^2}-z\right]+$$

$$+(1-2\cdot\mu)\cdot(y+b)\cdot arctg\left(\frac{x-a}{y+b}\right)+$$

$$+(1-2\cdot\mu)\cdot(y+b)\cdot arctg\left[\frac{z\cdot(x-a)}{(y+b)\cdot\sqrt{(x-a)^2+(y+b)^2+z^2}}\right]-$$

$$-2\cdot(1-\mu)\cdot z\cdot ln\left[\sqrt{(x+a)^2+(y-b)^2+z^2}+(x+a)\right]+$$

$$+(1-2\cdot\mu)\cdot(x+a)\cdot ln\left[\sqrt{(x+a)^2+(y-b)^2+z^2}-z\right]+$$

$$+(1-2\cdot\mu)\cdot(y-b)\cdot arctg\left(\frac{x+a}{y-b}\right)+$$

$$+(1-2\cdot\mu)\cdot(y-b)\cdot arctg\left[\frac{z\cdot(x+a)}{(y-b)\cdot\sqrt{(x+a)^2+(y-b)^2+z^2}}\right]+$$

$$+2\cdot(1-\mu)\cdot z\cdot ln\left[\sqrt{(x+a)^2+(y+b)^2+z^2}+(x+a)\right]-$$

$$-(1-2\cdot\mu)\cdot(x+a)\cdot ln\left[\sqrt{(x+a)^2+(y+b)^2+z^2}-z\right]-$$

$$-(1-2\cdot\mu)\cdot(y+b)\cdot arctg\left(\frac{x+a}{y+b}\right)-$$

$$\left.-(1-2\cdot\mu)\cdot(y+b)\cdot arctg\left[\frac{z\cdot(x+a)}{(y+b)\cdot\sqrt{(x+a)^2+(y+b)^2+z^2}}\right]\right\}.$$

$$w(x,y,z)=\frac{q\cdot(1+\mu)}{2\cdot\pi\cdot E}\cdot\Bigg\{-2\cdot(1-\mu)\cdot(x-a)\cdot ln\left[\sqrt{(x-a)^2+(y-b)^2+z^2}+(y-b)\right]-$$

$$-2\cdot(1-\mu)\cdot(y-b)\cdot ln\left[\sqrt{(x-a)^2+(y-b)^2+z^2}+(x-a)\right]+$$

$$+(1-2\cdot\mu)\cdot z\cdot arctg\left[\frac{(x-a)\cdot(y-b)}{z\cdot\sqrt{(x-a)^2+(y-b)^2+z^2}}\right]+$$

$$+2\cdot(1-\mu)\cdot(x-a)\cdot ln\left[\sqrt{(x-a)^2+(y+b)^2+z^2}+(y+b)\right]+$$

$$+2\cdot(1-\mu)\cdot(y+b)\cdot ln\left[\sqrt{(x-a)^2+(y+b)^2+z^2}+(x-a)\right]-$$

$$-(1-2\cdot\mu)\cdot z\cdot arctg\left[\frac{(x-a)\cdot(y+b)}{z\cdot\sqrt{(x-a)^2+(y+b)^2+z^2}}\right]+$$

$$+2\cdot(1-\mu)\cdot(x+a)\cdot ln\left[\sqrt{(x+a)^2+(y-b)^2+z^2}+(y-b)\right]+$$

$$+2\cdot(1-\mu)\cdot(y-b)\cdot ln\left[\sqrt{(x+a)^2+(y-b)^2+z^2}+(x+a)\right]-$$

$$-(1-2\cdot\mu)\cdot z\cdot arctg\left[\frac{(x+a)\cdot(y-b)}{z\cdot\sqrt{(x+a)^2+(y-b)^2+z^2}}\right]-$$

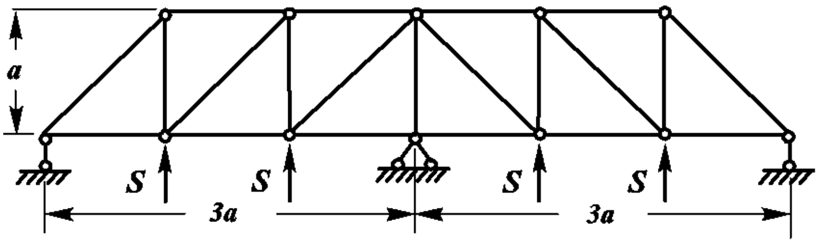
$$-2\cdot(1-\mu)\cdot(x+a)\cdot ln\left[\sqrt{(x+a)^2+(y+b)^2+z^2}+(y+b)\right]-$$

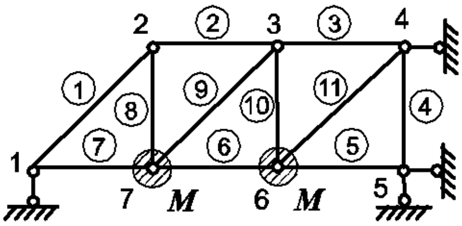
$$-2\cdot(1-\mu)\cdot(y+b)\cdot ln\left[\sqrt{(x+a)^2+(y+b)^2+z^2}+(x+a)\right]+$$

$$+(1-2\cdot\mu)\cdot z\cdot arctg\left[\frac{(x+a)\cdot(y+b)}{z\cdot\sqrt{(x+a)^2+(y+b)^2+z^2}}\right]\Bigg\}.$$

Linear Dynamics

Plane Truss Subjected to Instantaneous Pulses Concentrated in Non-Supporting Nodes of the Bottom Chord





***Objective*:** Determination of the strain state of a plane truss subjected to instantaneous pulses concentrated in non-supporting nodes of the bottom chord.

Initial data files: **5.11.SPR, График_5.11.txt**

***Problem formulation*:** The plane two-span truss with parallel chords and a diagonal lattice with three panels of equal length in each span is supported by the bottom chord. Masses M are concentrated and concentrated instantaneous transverse pulses S are applied in the intermediate (non-supporting) nodes of the bottom chord. Determine the natural oscillation modes and natural frequencies ω of the plane truss, as well as the transverse displacements of the nodal masses Z with time.

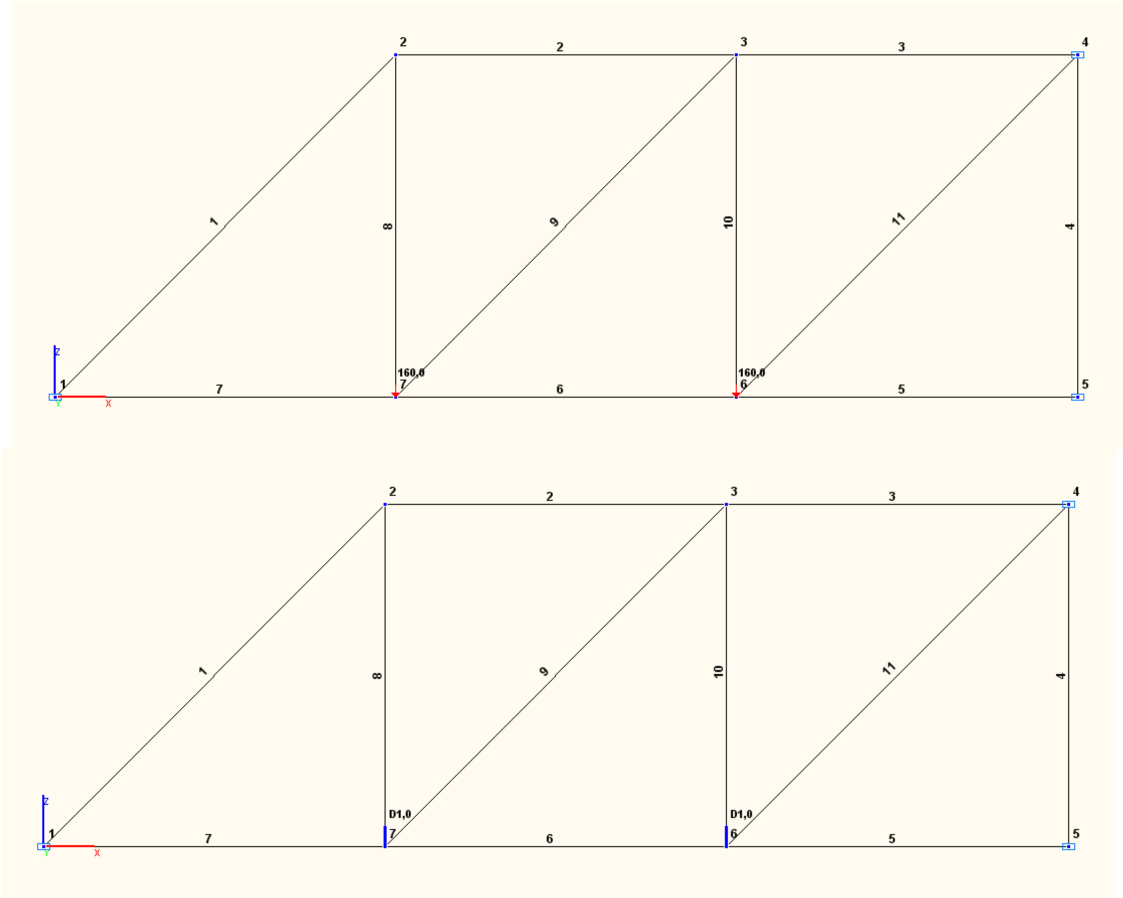
***References*:** Rabinovich I.M., Sinitsyn A.P., Luzhin O.V., Terenin V.M., Analysis of Structures Subject to Pulse Actions, Moscow, Stroyizdat, 1970, p. 153.

***Initial data*:**

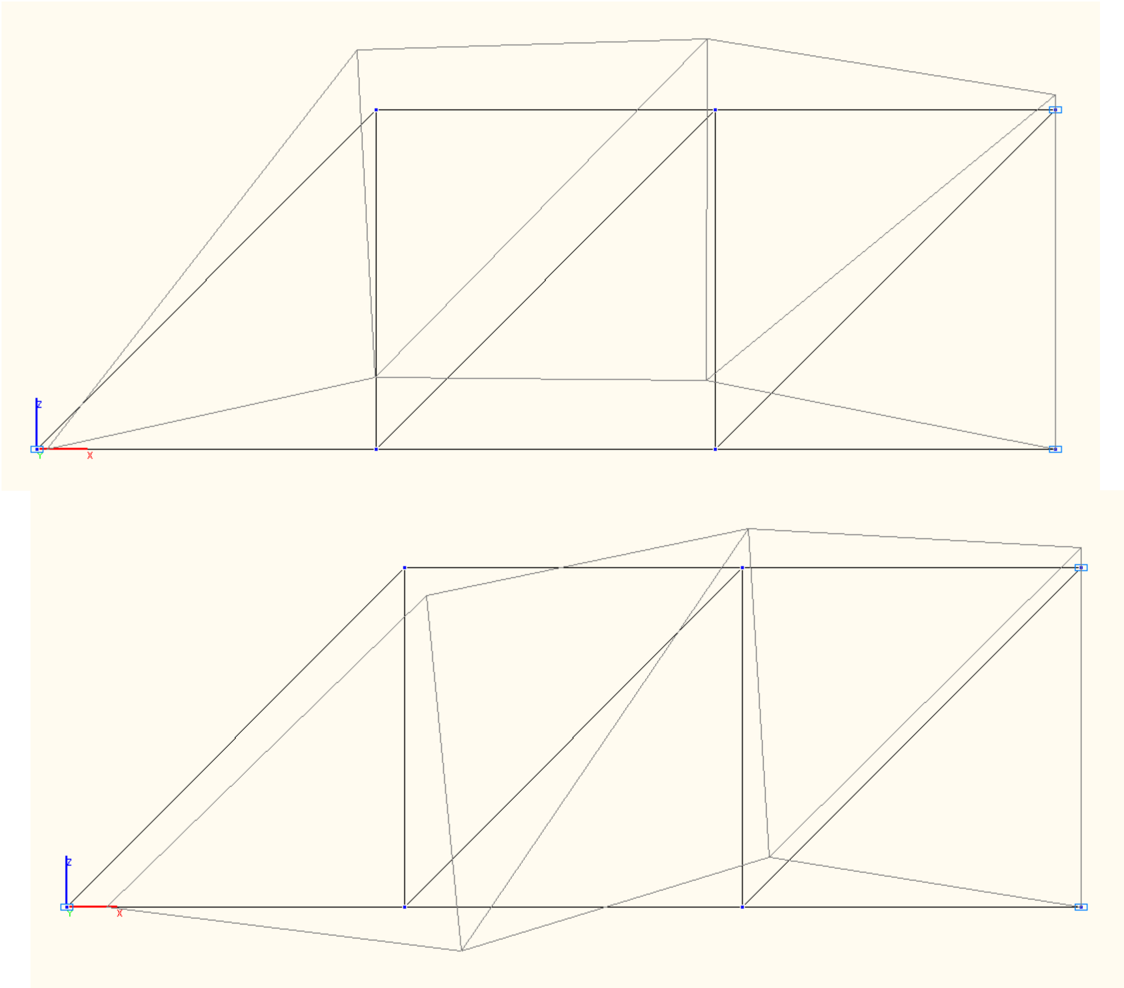
$E = 2.0 \cdot 10^8$ tf/m^2	- elastic modulus;
$F = 1 \cdot 10^{-2}$ m^2	- cross-sectional area of the truss elements except for the column above the middle support;
$2 \cdot F = 2 \cdot 10^{-2}$ m^2	- cross-sectional area of the column above the middle support;
a = 2.0 m	- height of the truss and length of the truss panel;
M = 16.0 tf·s^2/m	- value of the concentrated masses in the intermediate (non-supporting) nodes of the truss bottom chord;
S = 4.0· tf·s	- value of the concentrated instantaneous transverse pulses applied in the intermediate (non-supporting) nodes of the truss bottom chord;
g = 10.00 m/s^2	- gravitational acceleration.

***Finite element model*:** Since the structure and the applied loads are symmetric, only a half of the truss is considered with the restraints of the bottom and top chords on the symmetry axis in the longitudinal (horizontal) direction (degree of freedom X) and halving the stiffness of the column above the middle support. Design model – plane hinged bar system, 11 bar elements of type 1. Boundary conditions of the support nodes of the truss bottom chord are provided by imposing constraints in the direction of the degree of freedom Z. The concentrated masses are specified by transforming the static nodal loads *M·g*.
The calculation is performed in two stages: first the natural oscillation modes and natural frequencies ω are determined by the modal analysis, and then the transverse displacements of the nodal masses Z with time are determined by the direct integration of the equations of motion method. The action of the concentrated instantaneous transverse pulses is described by the graph of the load variation with time and is given in the form of nodal forces acting along the Z axis of the global coordinate system with the scale factor of 1.0 and the delay time 0.0 s. Intervals between the time points of the load variation graph are equal to Δt_{int} = 0.00001 s and correspond to the integration step. When plotting the graph the pulse action is taken with a linear shape function, force value P = 400000 tf and duration Δt_{int} = 0.00001 s. The duration of the process is equal to t = 0.12 s, which roughly corresponds to twice the value of the fundamental period of oscillations $4 \cdot \pi/\omega_1$. Critical damping ratios for the 1-st and 2-nd natural frequencies are taken with the minimum value ξ = 0.0001. The conversion factor for the added static loading is equal to *k* = 0.981 (mass generation). Number of nodes in the design model – 7. The determination of the natural oscillation modes and natural frequencies is performed by the method of subspace iteration. The matrix of concentrated masses is used in the calculation.

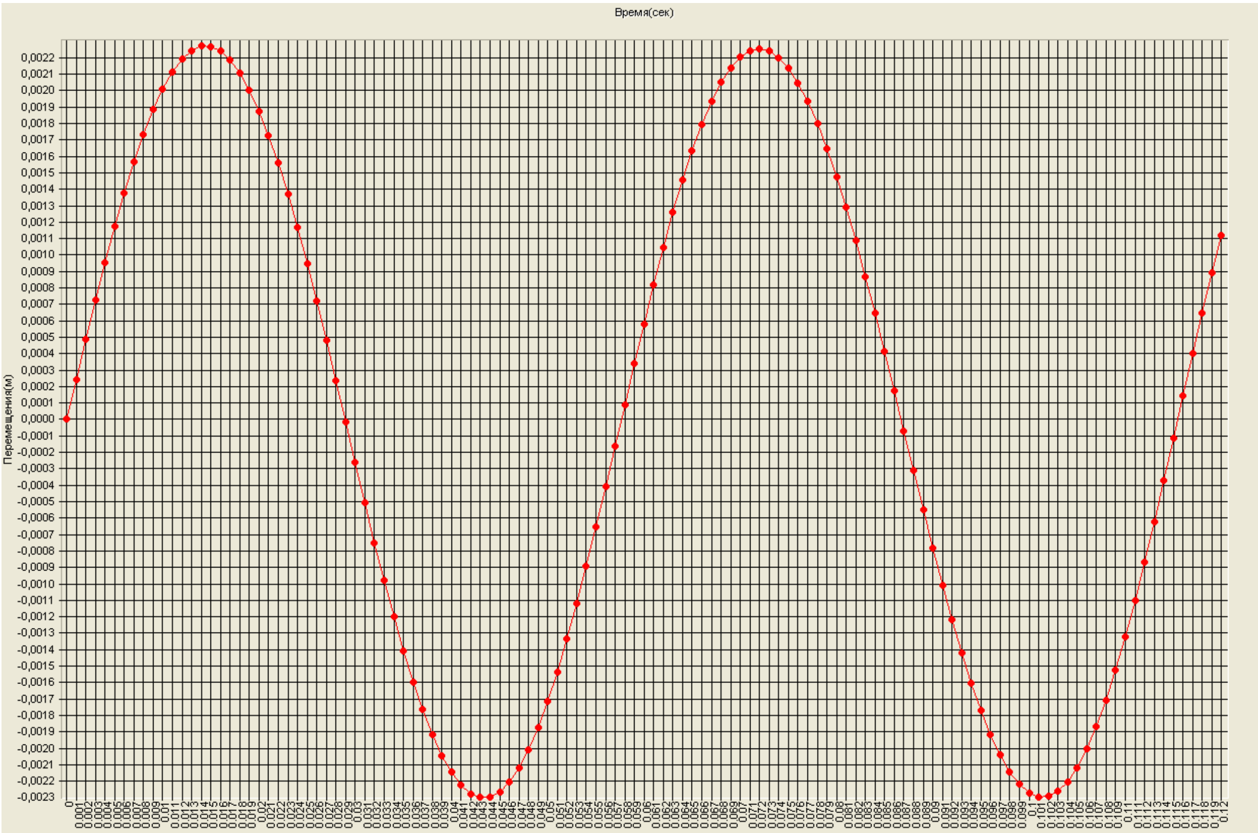
Results in SCAD



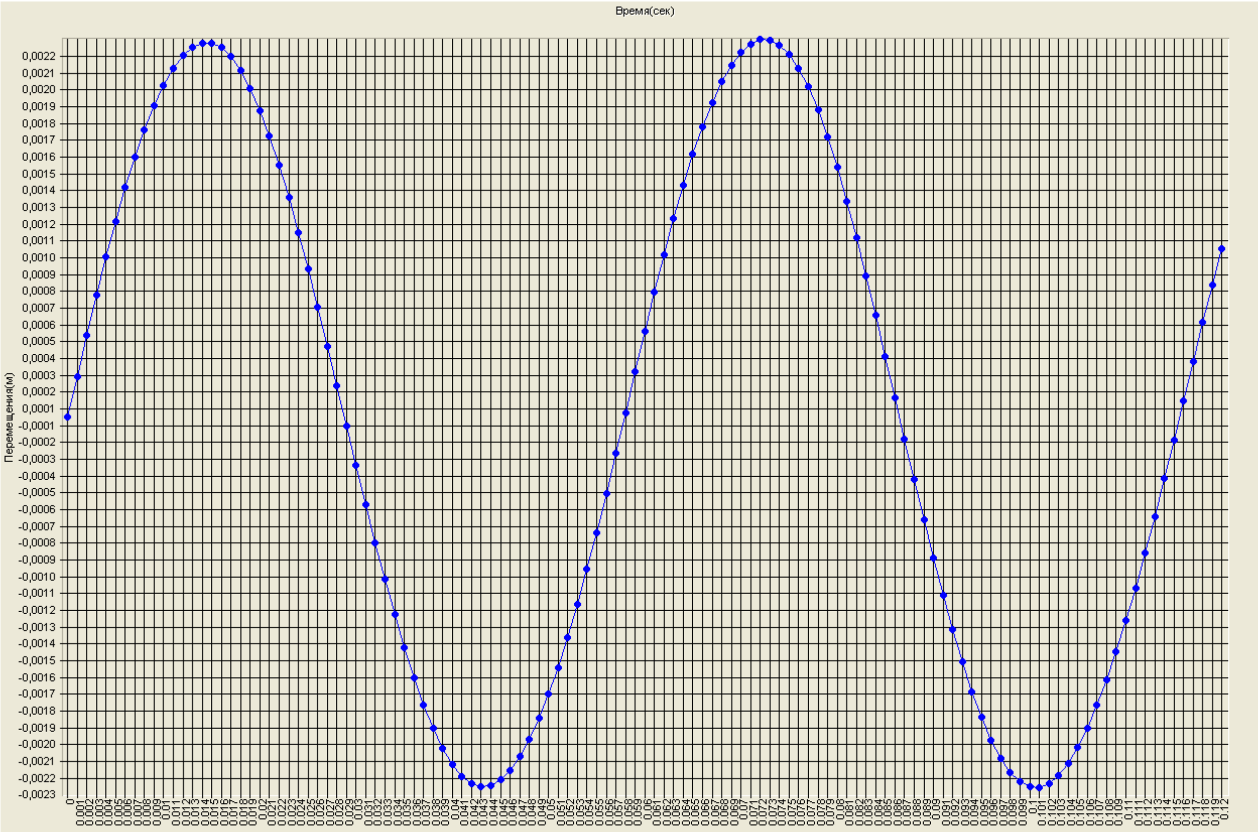
Design model



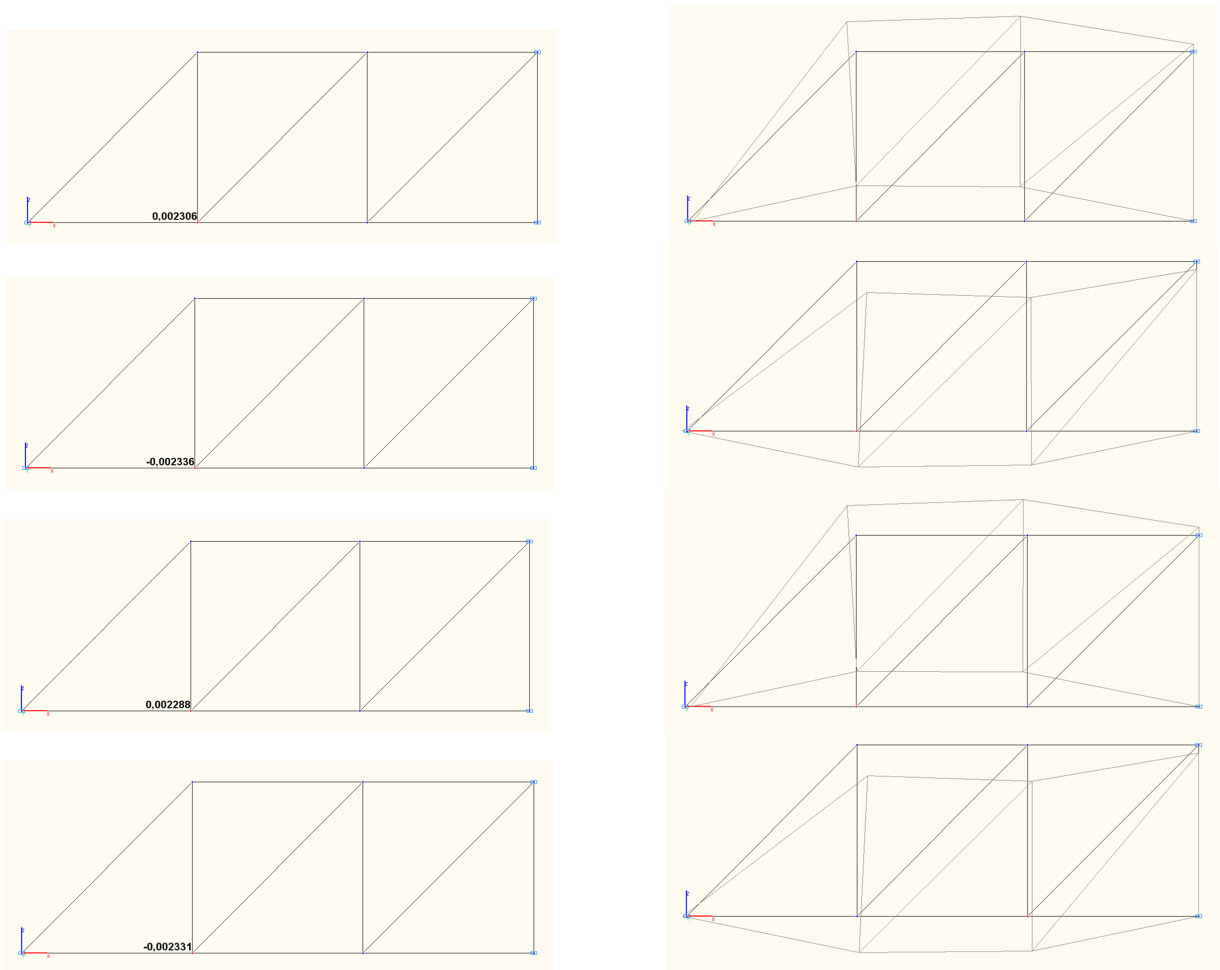
1-st and 2-nd natural oscillation modes (symmetric)



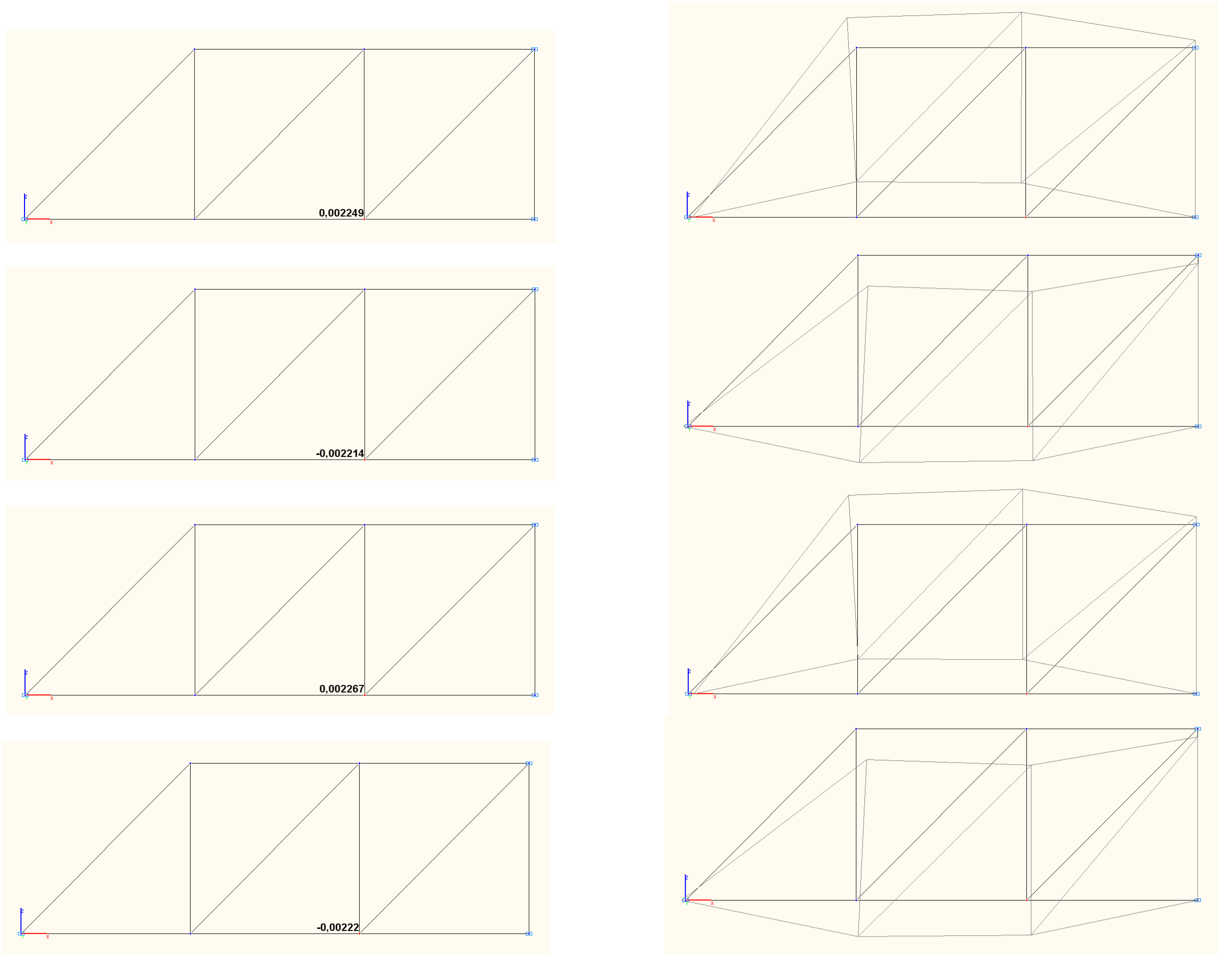
Graph of the variation of the transverse displacements of the nodal mass Z7 (closest to the end support) with time (m)



Graph of the variation of the transverse displacements of the nodal mass Z6 (closest to the middle support) with time (m)



Amplitude values of the transverse displacements of the nodal mass Z7 (m) and the deformed models at the respective time points



Amplitude values of the transverse displacements of the nodal mass Z6 (m)
and the deformed models at the respective time points

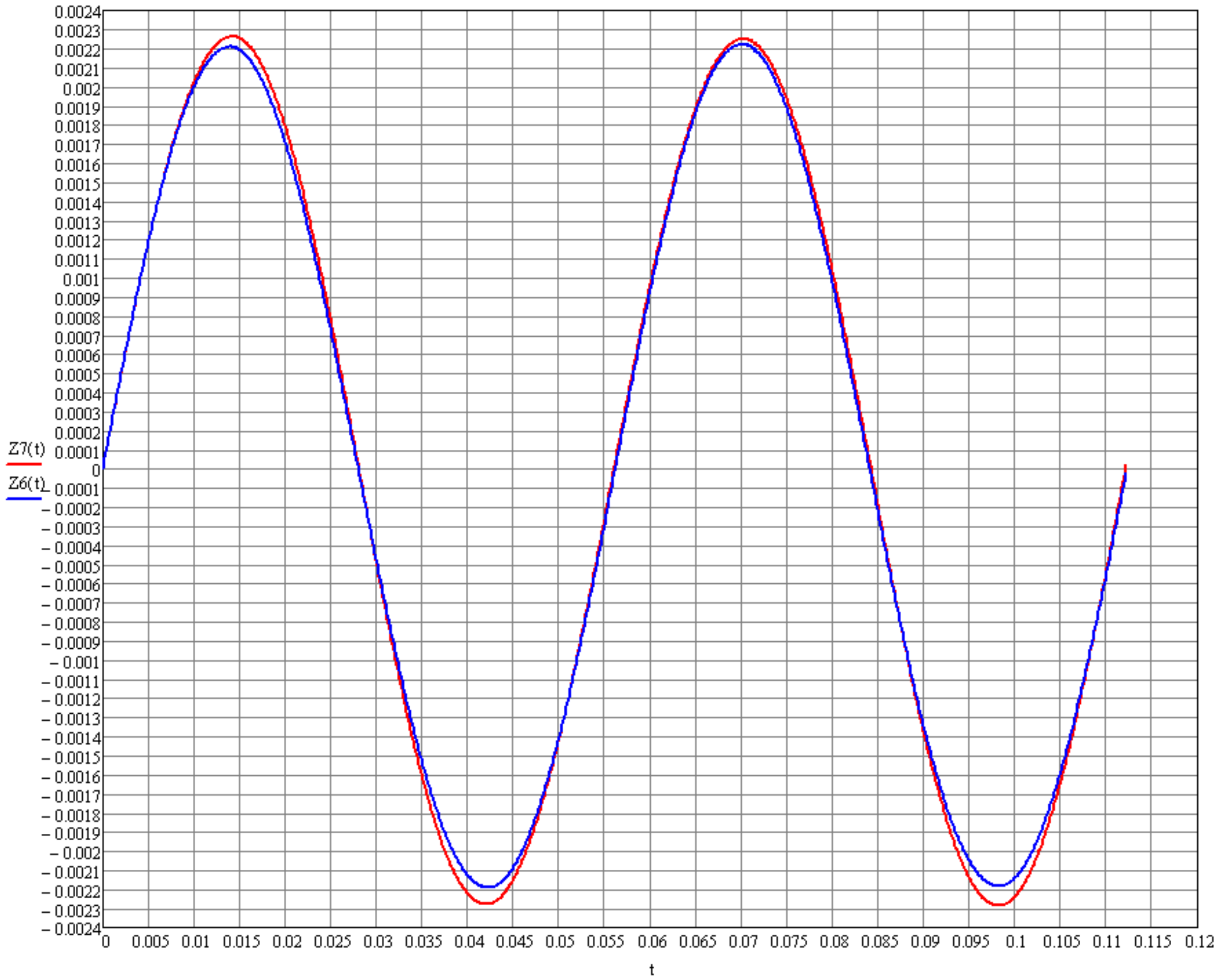
Comparison of solutions:

Natural frequencies ω, rad/s

Oscillation mode	Theory	SCAD	Deviations, %
1	112.0	108.8	2.86
2	208.0	197.4	5.10

Amplitude values of the transverse displacements of the nodal masses Z

Nodal mass	Theory		SCAD		
	Time, s	Displacement, m	Time, s	Displacement, m	Deviations, %
7	0.0142	0.002264	0.0144	0.002306	1.86
7	0.0420	-0.002280	0.0433	-0.002336	2.46
7	0.0702	0.002251	0.0720	0.002288	1.64
7	0.0982	-0.002287	0.1014	-0.002331	1.92
6	0.0139	0.002209	0.0145	0.002249	1.81
6	0.0422	-0.002192	0.0432	-0.002214	1.00
6	0.0701	0.002222	0.0724	0.002267	2.03
6	0.0982	-0.002185	0.1006	-0.002220	1.60



Graphs of the variation of the transverse displacements of the nodal masses Z7 and Z6 with time according to the theoretical solution (m)

Notes: In addition to taking the symmetry into account the following assumptions were made when deriving the analytical solution:

- the displacement of masses in the longitudinal (horizontal) direction is neglected;
- the difference between the mutual transverse (vertical) displacements of the lower and upper nodes of each vertical of the truss is neglected, and the masses are concentrated only in the lower nodes.

In the analytical solution the natural frequencies of oscillations ω of the plane truss are determined according to the following formulas:

$$\omega_7 = 0.448 \cdot \sqrt{\frac{E \cdot F}{a \cdot M}}\,; \qquad \omega_6 = 0.832 \cdot \sqrt{\frac{E \cdot F}{a \cdot M}}\,.$$

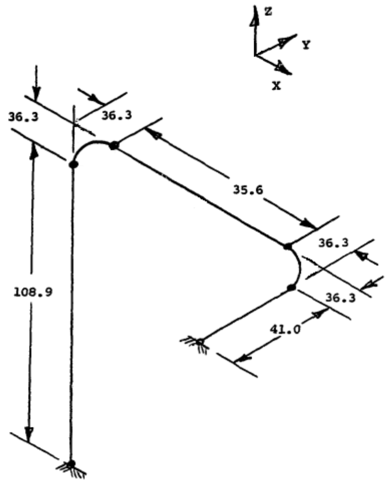
In the analytical solution the transverse displacements of the nodal masses of the plane truss Z with time are determined according to the following formulas:

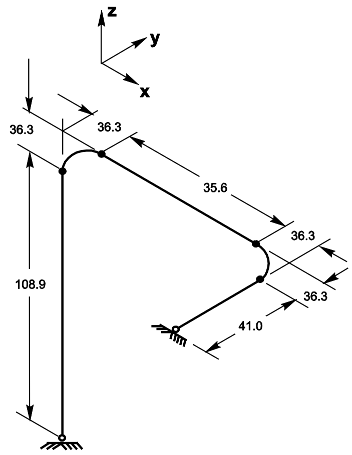
$$Z_7 = 1 \cdot \frac{1.016 \cdot S}{0.448 \cdot M} \cdot \sqrt{\frac{a \cdot M}{E \cdot F}} \cdot \sin(\omega_1 \cdot t) - 1 \cdot \frac{0.016 \cdot S}{0.832 \cdot M} \cdot \sqrt{\frac{a \cdot M}{E \cdot F}} \cdot \sin(\omega_2 \cdot t);$$

$$Z_6 = 0.972 \cdot \frac{1.016 \cdot S}{0.448 \cdot M} \cdot \sqrt{\frac{a \cdot M}{E \cdot F}} \cdot \sin(\omega_1 \cdot t) + 1.028 \cdot \frac{0.016 \cdot S}{0.832 \cdot M} \cdot \sqrt{\frac{a \cdot M}{E \cdot F}} \cdot \sin(\omega_2 \cdot t).$$

The deviations from the theory for the natural frequencies of oscillations are due to the fact that the “manual” calculation in the source is performed with significant errors.

Natural Oscillations of a Spatial Pipeline Clamped at the Edges (Hougaard's Problem)





Objective: Modal analysis of a spatial pipeline clamped at the edges.

***Initial data file*:** **5.1.SPR**

Problem formulation: Determine the natural oscillation modes and natural frequencies *f* of the spatial steel pipeline composed of three mutually orthogonal straight segments connected in series by fittings, clamped at the edges and filled with water.

References: William Hovgaard, Stresses in Three-dimensional Pipe Bends. Transactions of ASME, vol. 57, FSP 75-12, 1935.

Initial data:

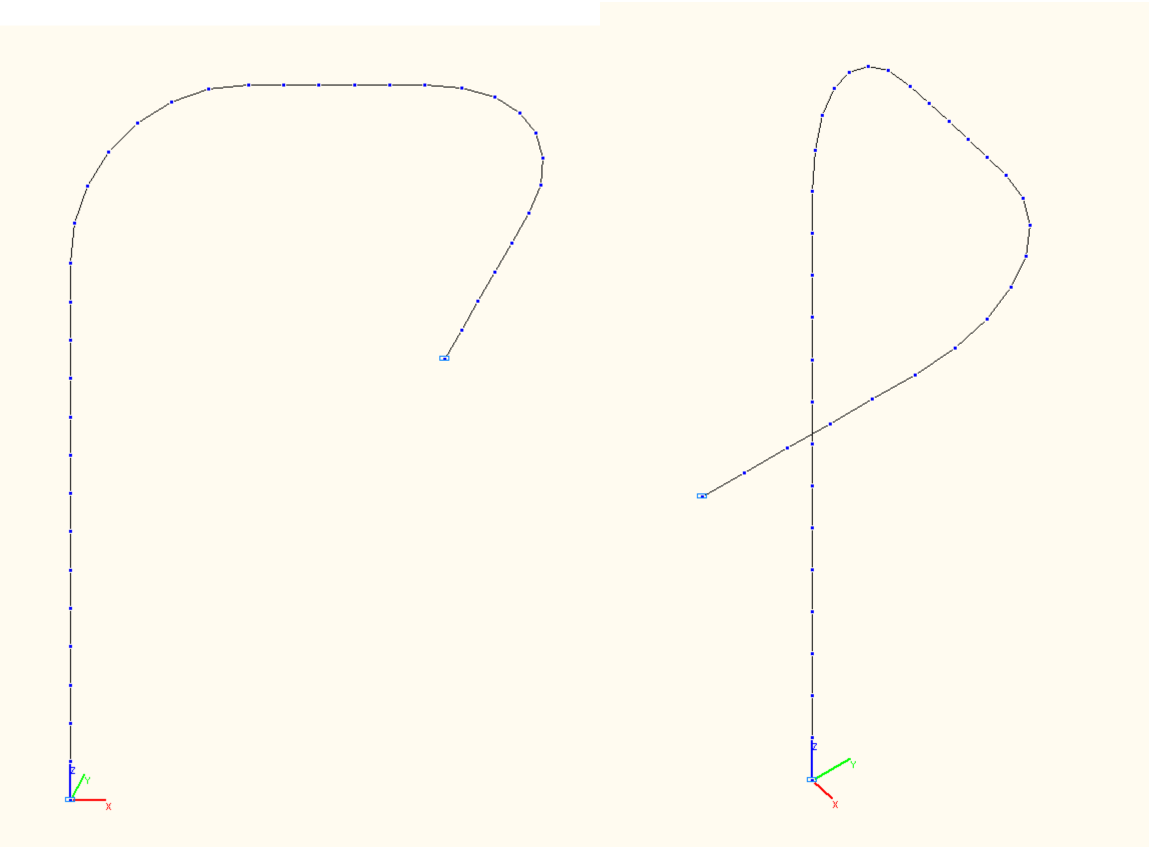
$E = 24.0\cdot10^6$ psi $= 1.654740\cdot10^8$ kPa	- elastic modulus;
$\nu = 0.3$	- Poisson's ratio;
$D_e = 7.288$ in $= 0.185115$ m	- outer diameter of the pipe cross-section;
$t = 0.241$ in $= 0.006121$ m	- thickness of the pipe cross-section;
$\rho_s = 0.283$ lb/in^3 $= 7.833$ t/m^3	- density of the pipe material (steel);
$\rho_w = 0.036$ lb/in^3 $= 0.996$ t/m^3	- density of the filling material (water);
$L_{str1} = 108.9$ in $= 2.766$ m	- length of the first straight section of the pipeline;
$L_{str2} = 35.6$ in $= 0.904$ m	- length of the second straight section of the pipeline;
$L_{str3} = 41.0$ in $= 1.041$ m	- length of the third straight section of the pipeline;
$R_{elb} = 36.3$ in $= 0.922$ m	- radius of the axis of the pipeline fittings;

stiffness properties and masses:

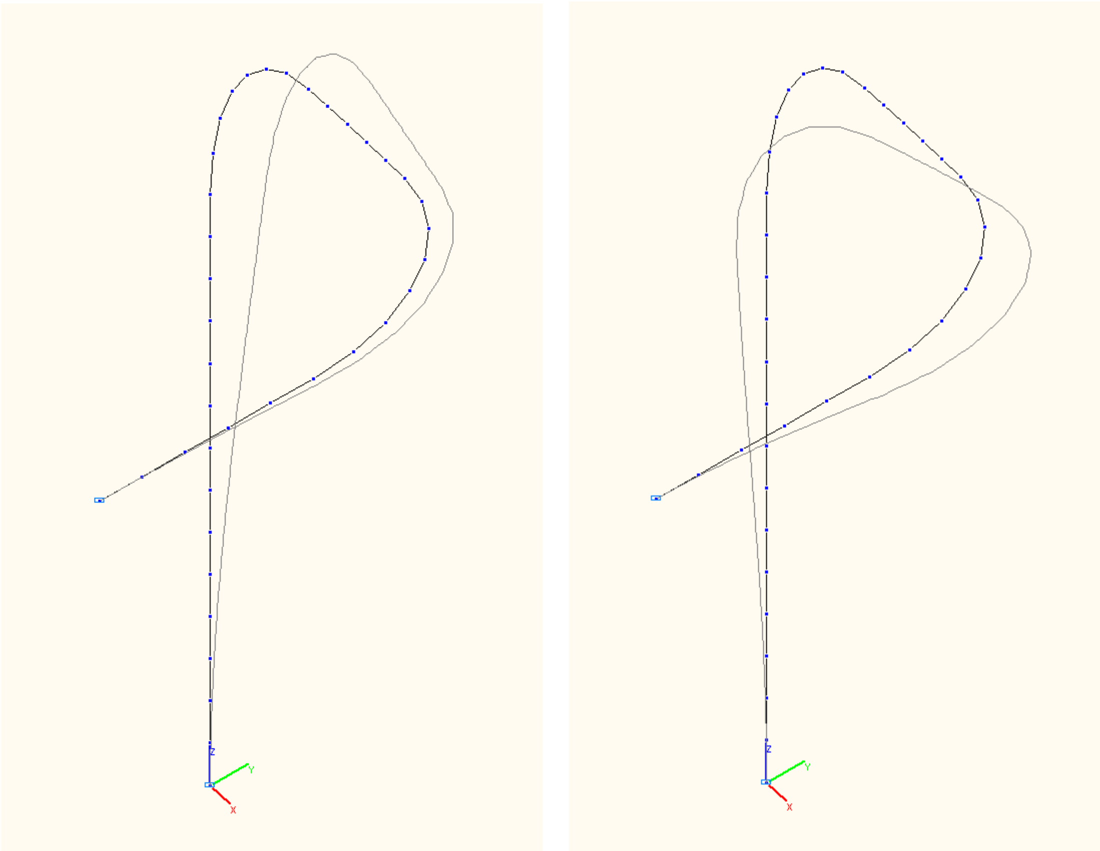
$EA = E\cdot(\pi\cdot D_e^2/4)\cdot(1-(1-2\cdot t/D_e)^2) = 569598$ kN	- axial stiffness of the pipe cross-section;
$EI_{b,str} = E\cdot(\pi\cdot D_e^4/64)\cdot(1-(1-2\cdot t/D_e)^4) = 2283.81$ kN·m	- bending stiffness of the cross-section of the straight segment of the pipe;
$EI_{b,elb} = E\cdot(\pi\cdot D_e^4/64)\cdot(1-(1-2\cdot t/D_e)^4)/k = 995.824$ kN·m	- bending stiffness of the cross-section of the pipe fitting (taking into account the flattening), where:
$k = (10+12\cdot\lambda^2)/\ (1+12\cdot\lambda^2) = 2.293391$	- Von Karman coefficient of flexibility,
$\lambda = t\cdot R_{elb}/((D_e-2\cdot t)/4) = 0.704654$	- geometric parameter;
$GI_t = (E/(2\cdot(1+\nu))\ \cdot(\pi\cdot D_e^4/32)\cdot(1-(1-2\cdot t/D_e)^4) = 1756.78$ kN·m	- torsional stiffness of the pipe cross-section;
$m = \cdot(\pi\cdot D_e^2/4)\cdot(\rho_s -(\ \rho_s- \rho_w)(1-2\cdot t/D_e)^2)\cdot g = 0.4938$ kN/m	- linear static load from the weight of the pipe filled with water.

Finite element model: Design model – general type system, pipeline elements – 38 bar elements of type 5. The spacing of the finite element mesh in the longitudinal direction (along the X1 axis of the local coordinate system) is ≈ 0.2 m. Boundary conditions are provided by imposing constraints in the directions of the degrees of freedom X, Y, Z, UX, UY, UZ for the end nodes of the pipeline. The distributed mass is specified by transforming the static load from the weight of the pipe filled with water, *m*. Number of nodes in the design model – 39. The determination of the natural oscillation modes and natural frequencies is performed by the method of subspace iteration. The matrix of concentrated masses is used in the calculation.

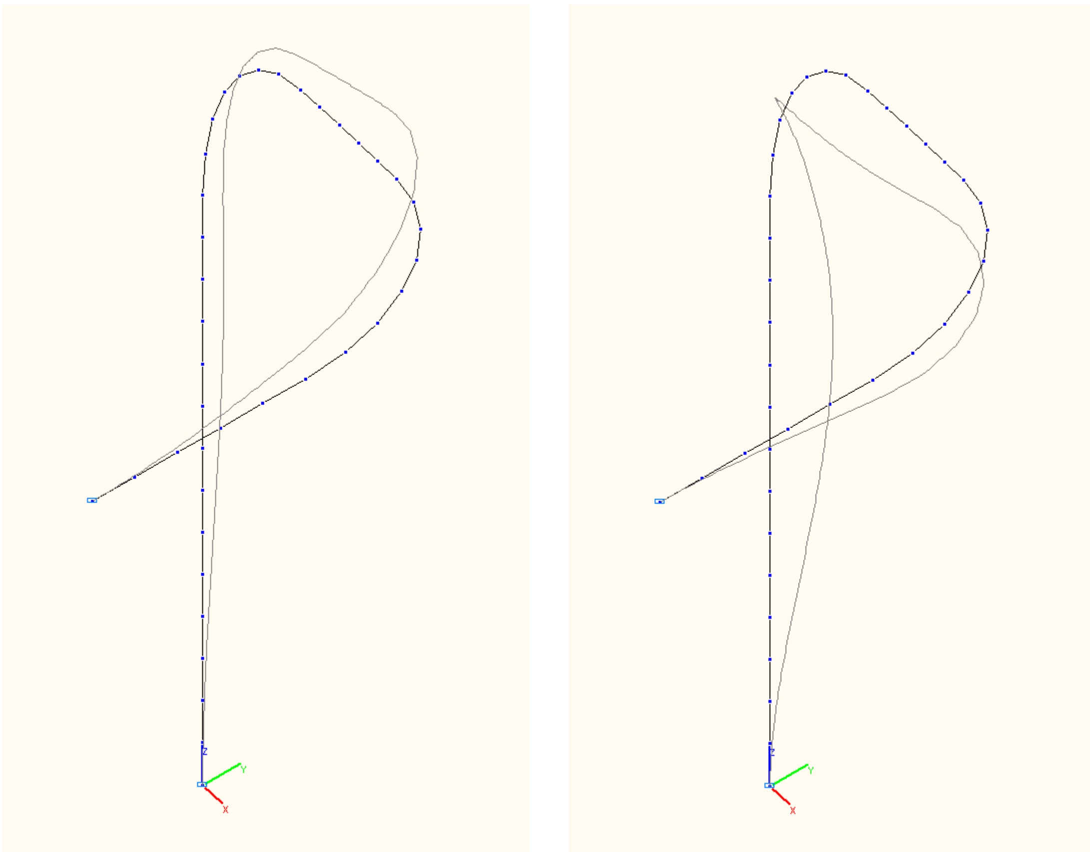
Results in SCAD



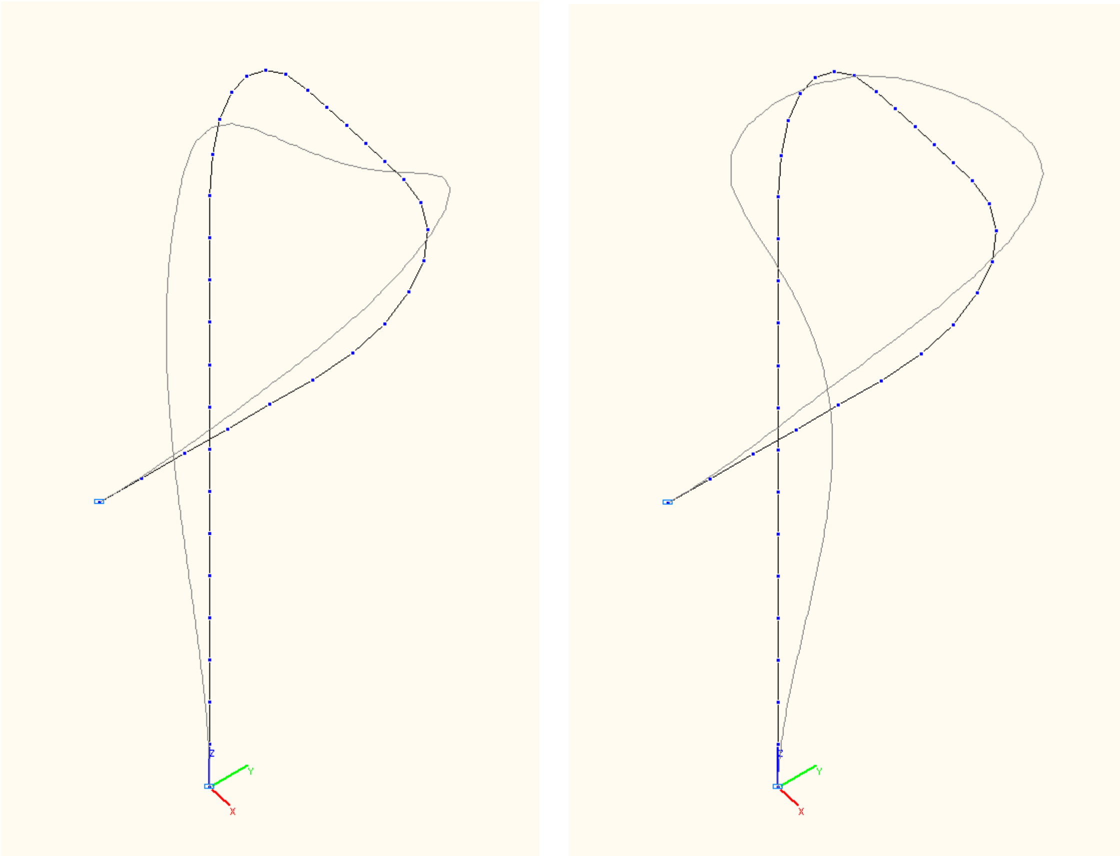
Design model



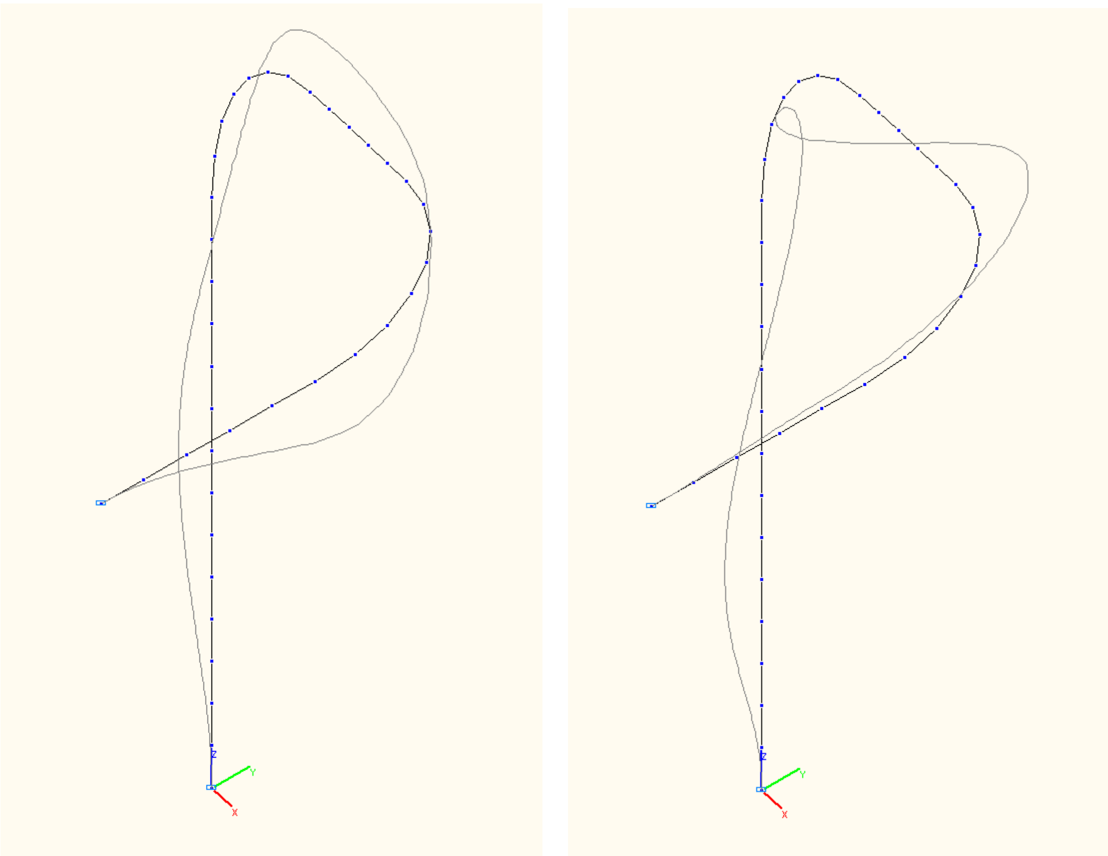
1-st and 2-nd natural oscillation modes



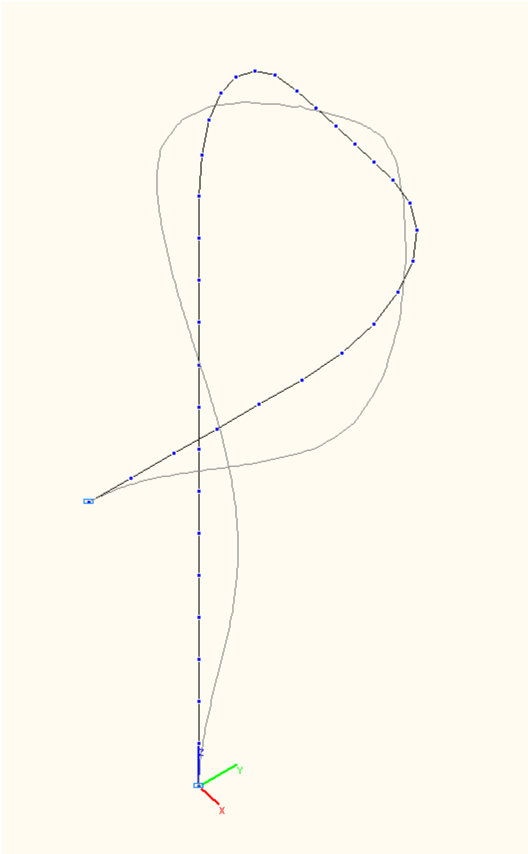
3-rd and 4-th natural oscillation modes



5-th and 6-th natural oscillation modes



7-th and 8-th natural oscillation modes



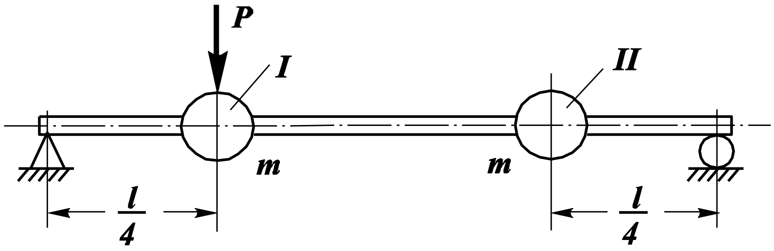
9-th natural oscillation mode

Comparison of solutions:

Natural frequencies *f*, Hz

Oscillation mode	**Theory**	**SCAD**	**Deviations, %**
1	10.18	10.01	1.67
2	19.54	19.29	1.28
3	25.47	24.55	3.61
4	48.09	46.79	2.70
5	52.86	50.77	3.95
6	75.94	82.21	8.26
7	80.11	84.29	5.22
8	122.34	126.58	3.47
9	123.15	128.51	4.35

Simply Supported Weightless Beam with Two Concentrated Masses and Transverse Sudden Constant Load Applied to One of Them



Objective: Determination of the stress-strain state of a simply supported weightless beam with two concentrated masses and transverse sudden constant load applied to one of them.

Initial data files: **5.12_Sudd_L.SPR, График_5.12_Sudd_L.txt**

Problem formulation: Two identical loads of mass *m* are attached to the simply supported beam of constant cross-section at a quarter span distance from each support. The mass of the beam is neglected in comparison with the masses of the loads. The force *P* is applied to one of the masses at the initial time and remains constant. Determine the natural oscillation modes and natural frequencies p of the simply supported beam, as well as the deflections η and bending moments *M* in the cross-sections of the beam with the attached masses with time.

References: S.D. Ponomarev, V.L. Biederman, K.K. Likharev, V.M. Makushin, N.N. Malinin, V.I. Feodos'yev, Fundamentals of Modern Methods for Strength Analysis in Mechanical Engineering. Dynamic Analysis. Stability. Creep. Moscow, Mashgiz, 1952, p.150.

Initial data:

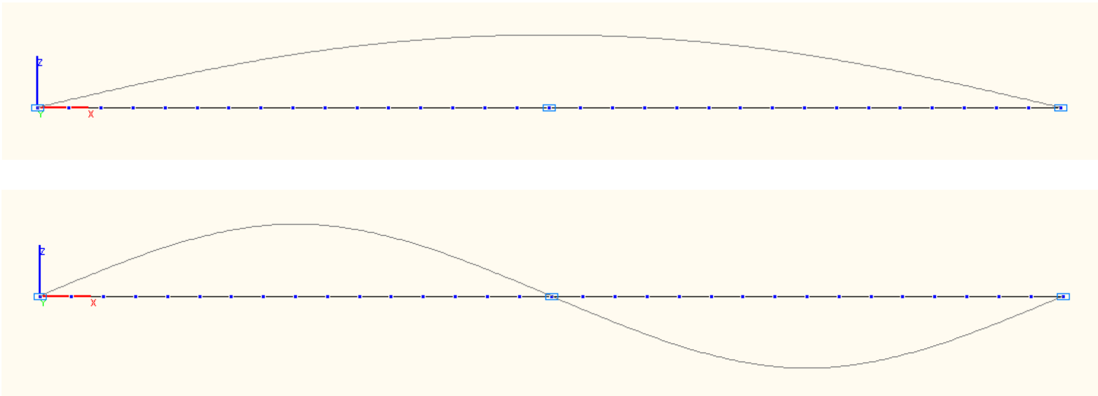
$E = 3.0 \cdot 10^6$ tf/m^2	- elastic modulus;
$\nu = 0.2$	- Poisson's ratio;
b = 0.4 m	- width of the rectangular cross-section of the beam;
h = 0.8 m	- height of the rectangular cross-section of the beam;
l = 8.0 m	- beam span length;
$m = 3.0$ tf·s^2/m	- value of the concentrated masses attached to the beam;
P = 76.8 tf	- value of the transverse sudden constant force applied to one of the masses;
g = 10.00 m/d^2	- gravitational acceleration;
$I = b \cdot h^3/12 = 0.017067$	- cross-sectional moment of inertia of the beam.

Finite element model: Design model – plane frame, 32 bar elements of type 2. Boundary conditions of the simply supported ends of the beam are provided by imposing constraints in the direction of the degree of freedom Z. The dimensional stability of the design model is provided by imposing a constraint in the node on the symmetry axis of the beam in the direction of the degree of freedom X. The concentrated masses are specified by transforming the static nodal loads $m \cdot g$.
The calculation is performed in two stages: first the natural oscillation modes and natural frequencies *p* are determined by the modal analysis, and then the deflections η and bending moments *M* in the cross-sections of the beam with the attached masses with time are determined by the direct integration of the equations of motion method. The action of the transverse sudden constant force is described by the graph of the load variation with time and is given in the form of a nodal force acting along the Z axis of the global coordinate system with the scale factor of 1.0 and the delay time 0.0 s. Intervals between the time points of the load variation graph are equal to Δt_{int} = 0.001571 c (T_1/100) and correspond to the integration step. When plotting the graph, the action of the transverse sudden constant force is taken as P = 76.8 tf at all time points $n \cdot \Delta t_{int}$. The duration of the process is equal to t = 0.3142 s, which corresponds to twice the value of the fundamental period of oscillations $2 \cdot T_1$. Critical damping ratios for the 1-st and 2-nd natural frequencies are taken with the minimum value ξ = 0.0001. The conversion factor for the added static loading is equal to k = 0.981 (mass generation). Number of nodes in the design model – 33. The modal integration method is used

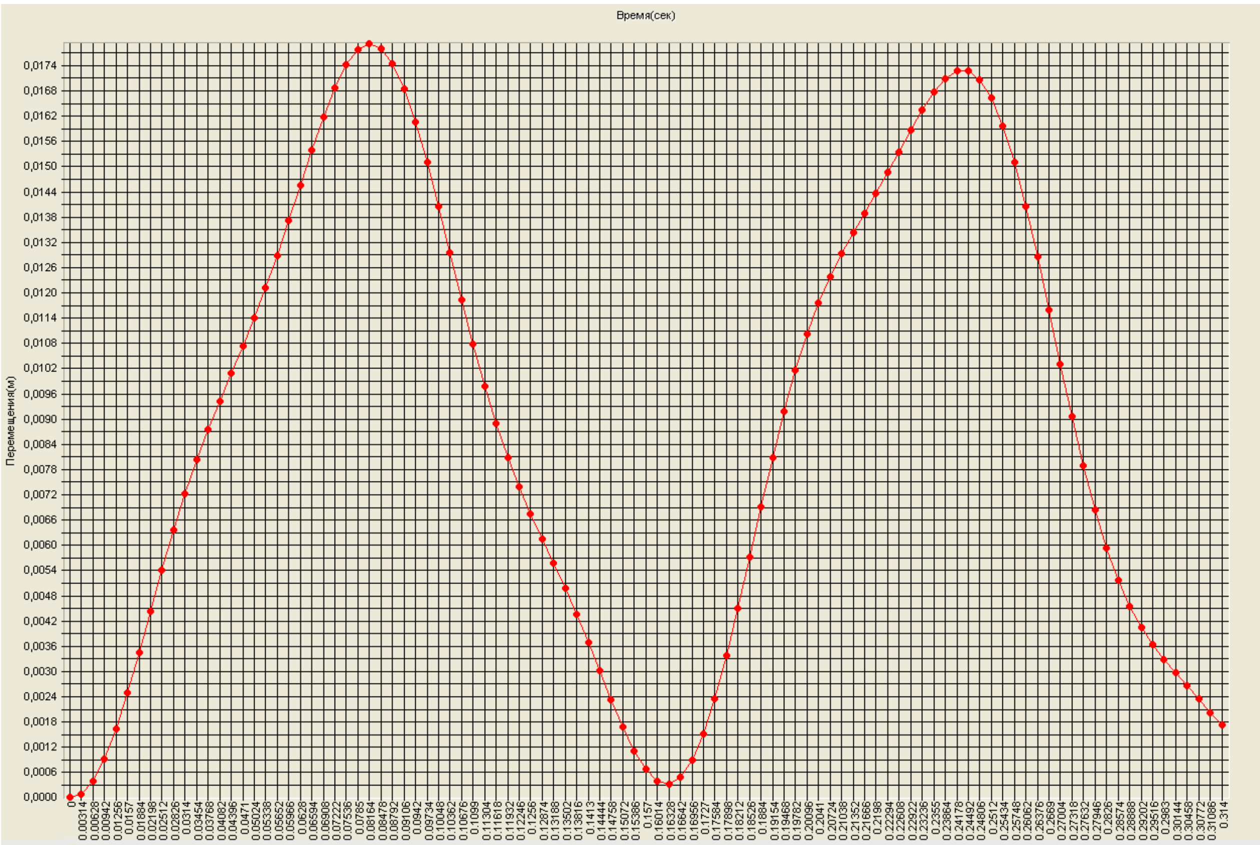
in the calculation. The determination of the natural oscillation modes and natural frequencies is performed by the method of subspace iteration. The matrix of concentrated masses is used in the calculation.

Results in SCAD

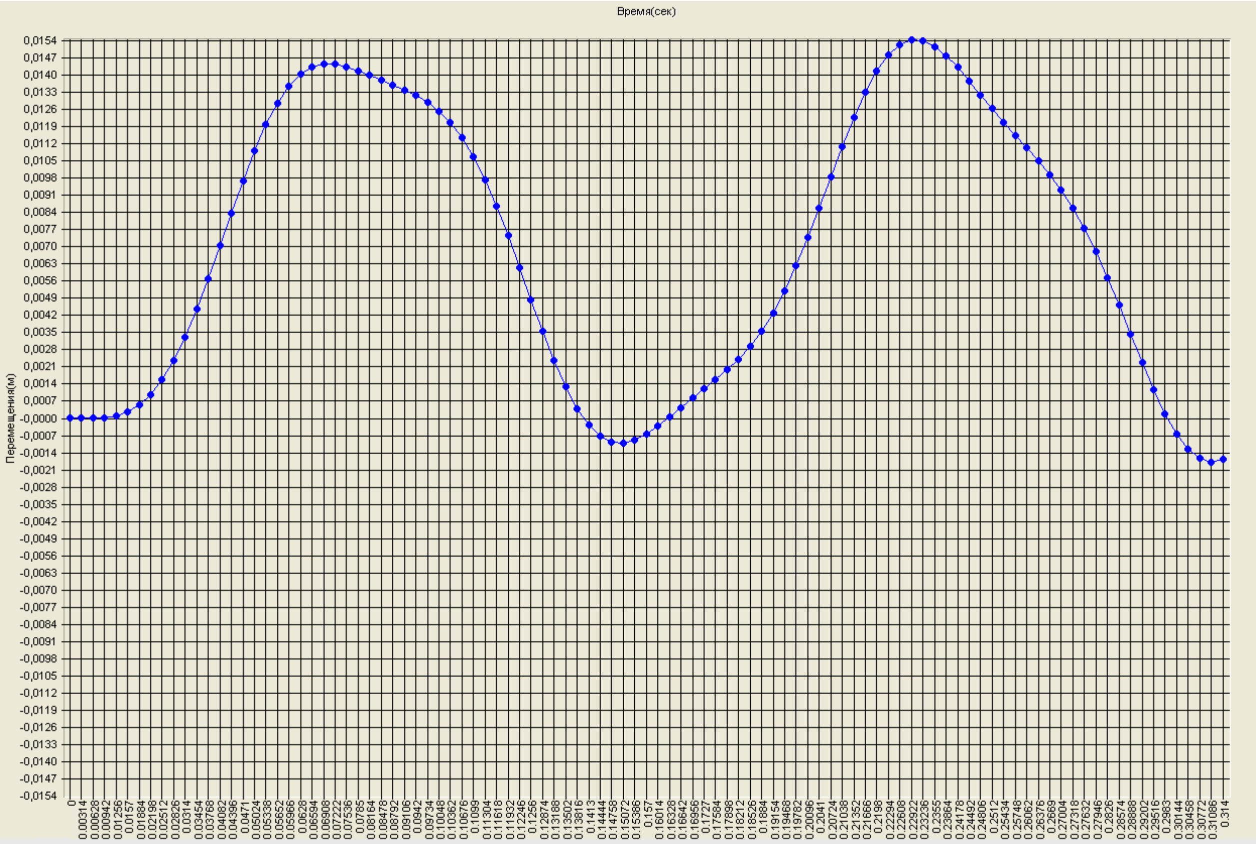
Design model



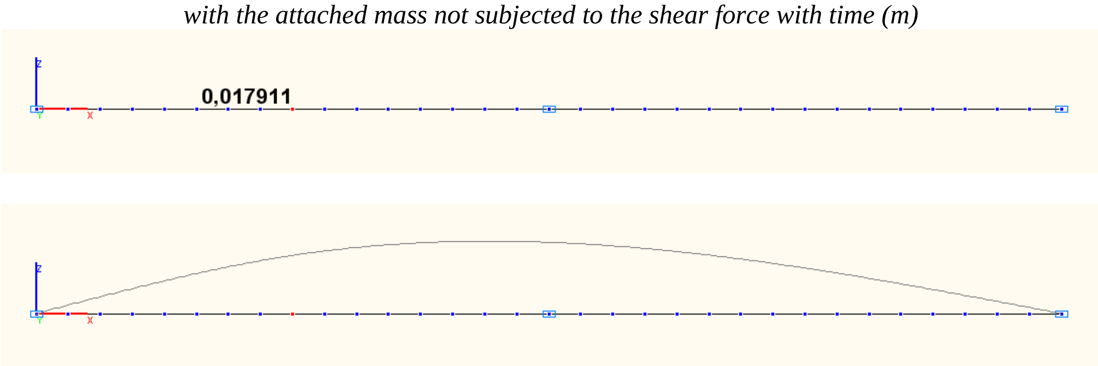
1-st and 2-nd natural oscillation modes



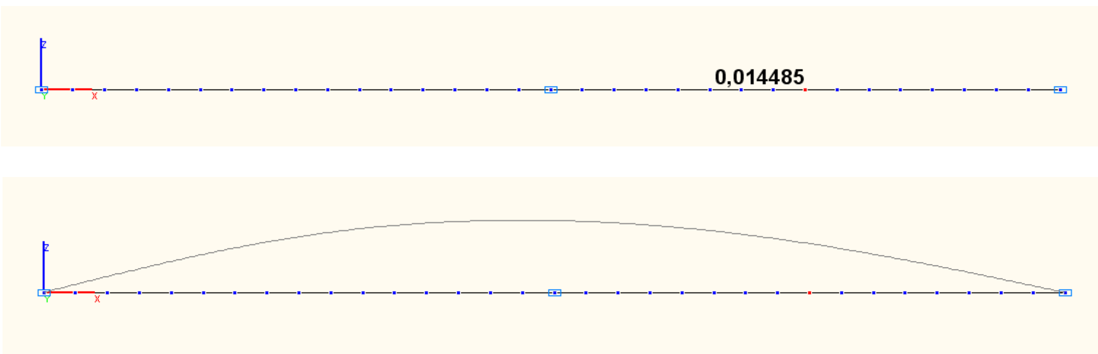
Graph of the variation of the deflection η_1 in the cross-section of the beam with the attached mass subjected to the shear force with time (m)



Graph of the variation of the deflection η_2 in the cross-section of the beam with the attached mass not subjected to the shear force with time (m)

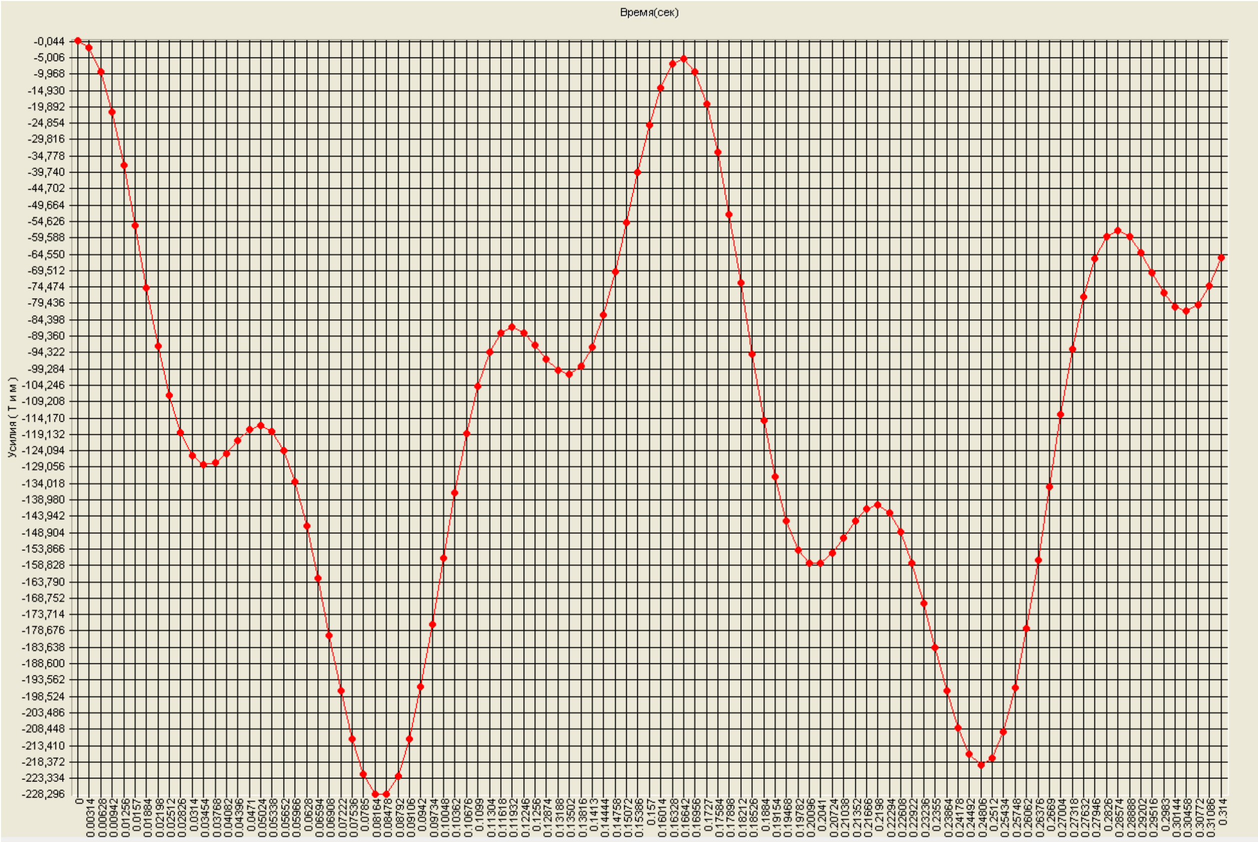


Amplitude value of the deflection η_1 in the cross-section of the beam with the attached mass subjected to the shear force and the deformed model at the respective time point (m)

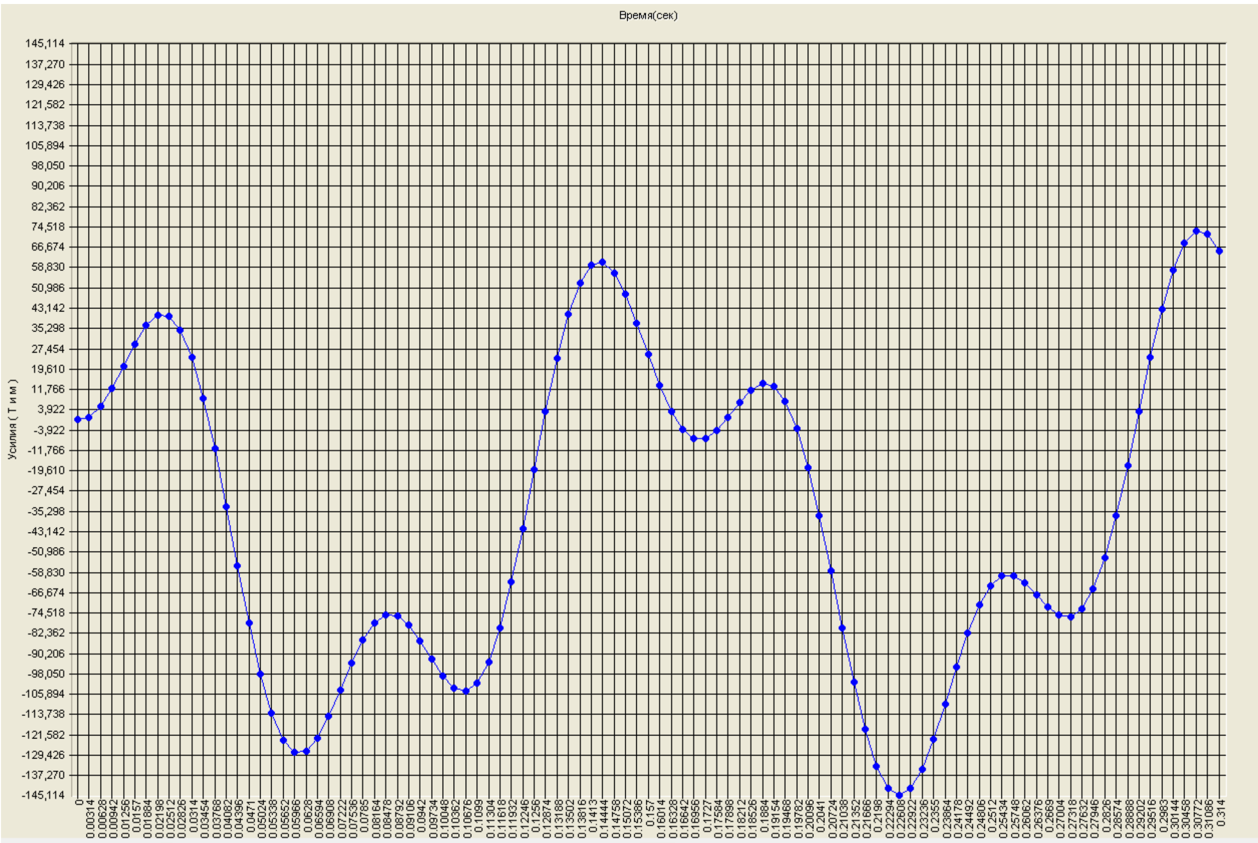


Amplitude value of the deflection η_2 in the cross-section of the beam

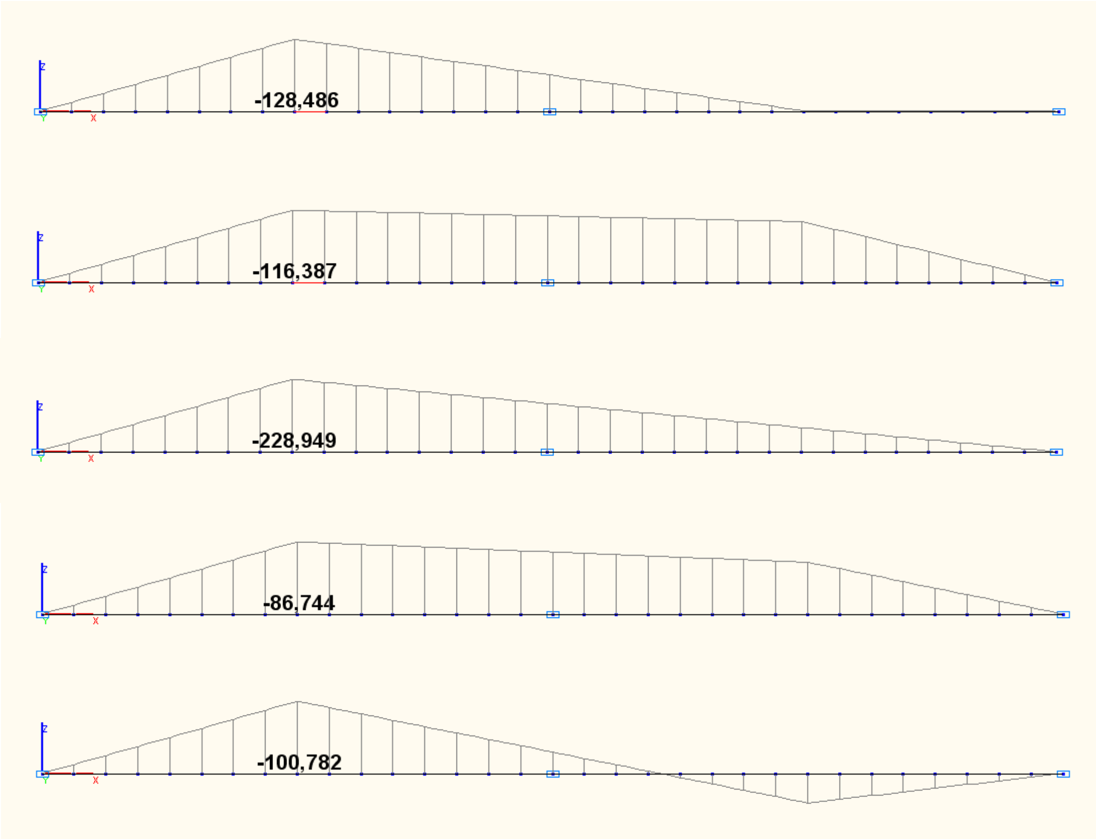
with the attached mass not subjected to the shear force
and the deformed model at the respective time point (m)



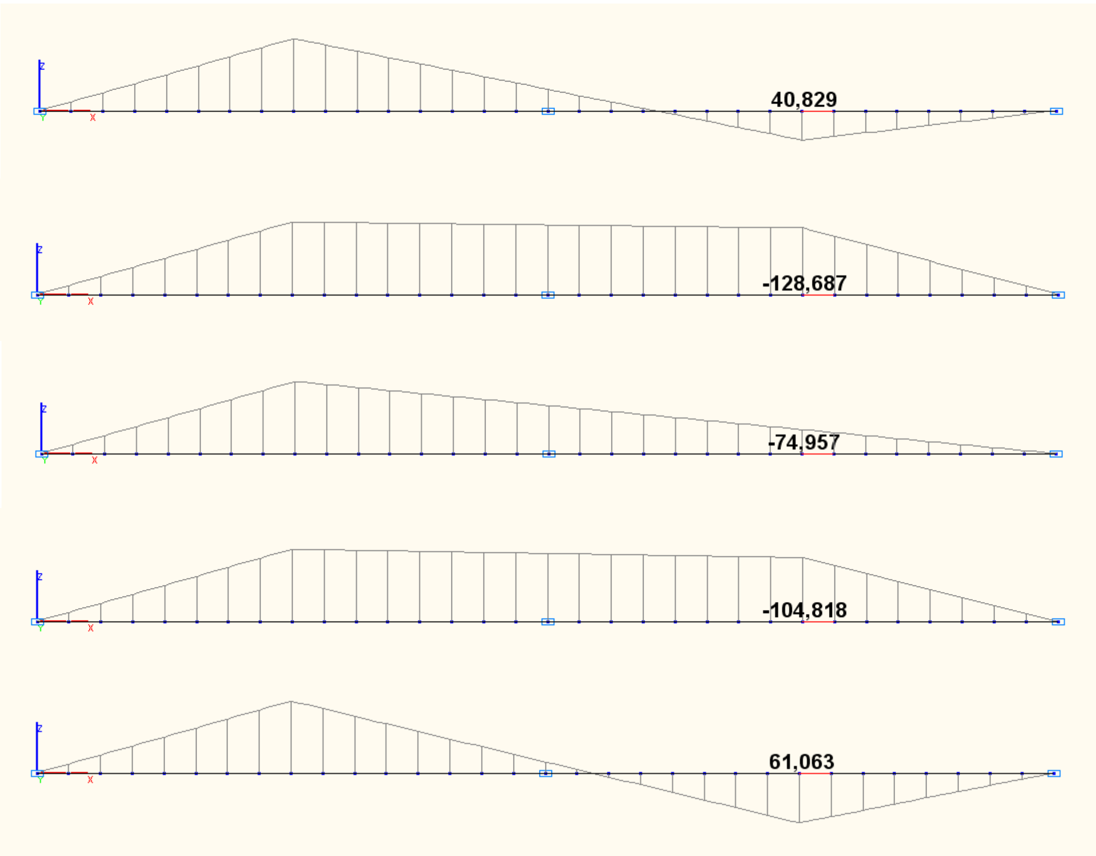
Graph of the variation of the bending moment M_1 in the cross-section of the beam
with the attached mass subjected to the shear force, with time (tm·m)



Graph of the variation of the bending moment M_2 in the cross-section of the beam *with the attached mass not subjected to the shear force, with time (tm·m)*

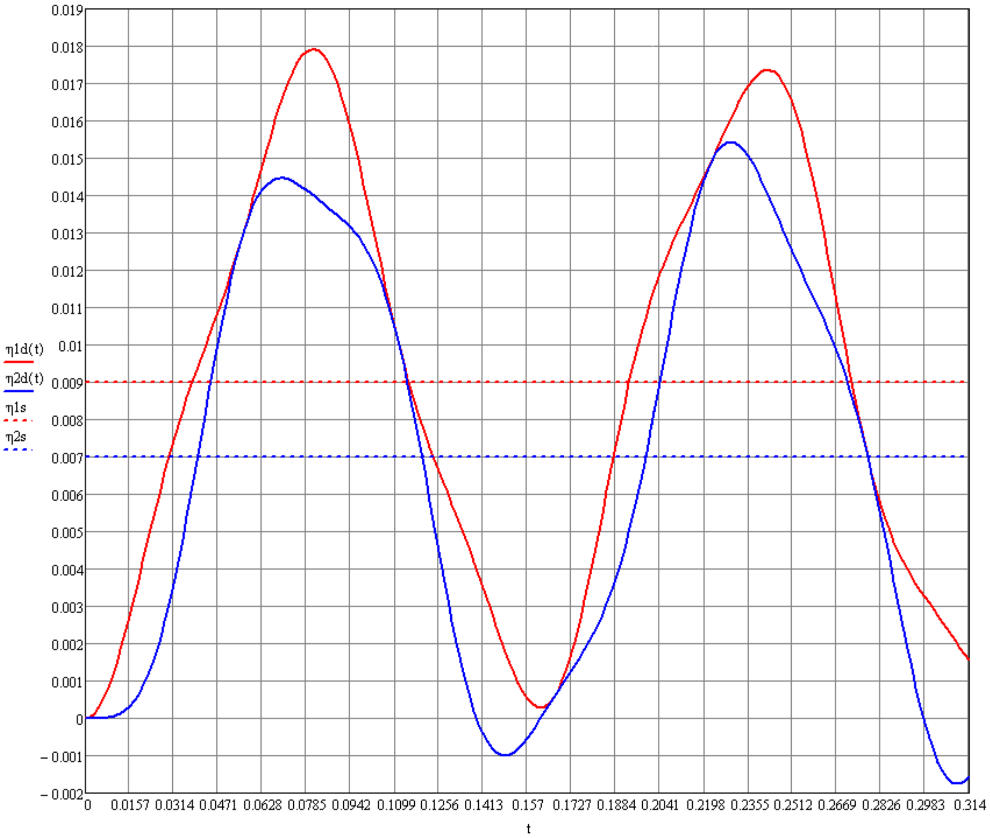


Amplitude values of the bending moment M_1 in the cross-section of the beam with the attached mass subjected to the shear force (tm·m)



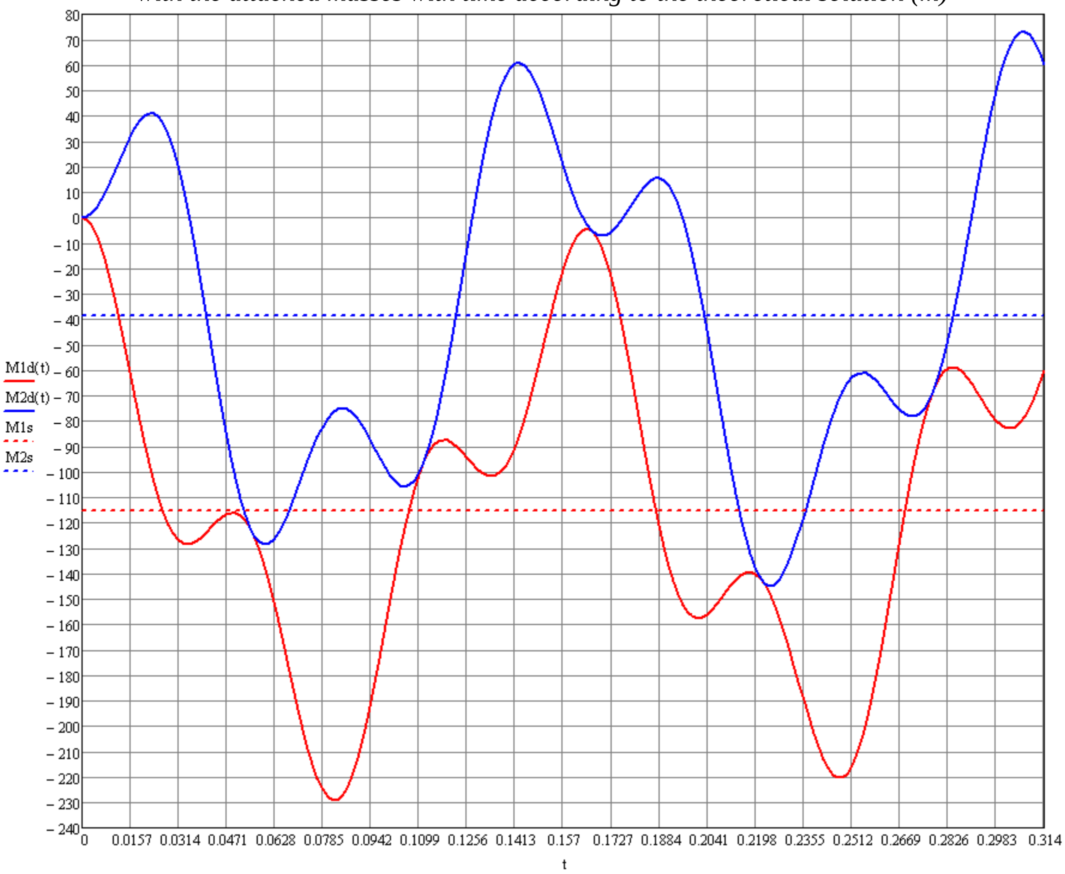
Amplitude values of the bending moment M_2 in the cross-section of the beam with the attached mass not subjected to the shear force (tm·m)

Comparison of solutions:



The dashed lines show the values of static deflections

Graphs of the variation of the deflections η_1 and η_2 in the cross-sections of the beam with the attached masses with time according to the theoretical solution (m)



The dashed lines show the values of static bending moments

Graphs of the variation of the bending moments M_1 and M_2 in the cross-sections of the beam with the attached masses with time according to the theoretical solution (tf·m)

Natural frequencies p, rad/s

Oscillation mode	Theory	SCAD	Deviations, %
1	40.000	40.000	0.00
2	113.137	113.137	0.00

Amplitude value of the deflections η in the cross-sections of the beam with the attached masses, m

Nodal mass	Theory		SCAD		
	Time, s	Deflection, m	Time, s	Deflection, m	Deviations, %
1	0.0809	0.017928	0.0817	0.017911	0.09
2	0.0695	0.014474	0.0707	0.014485	0.08

Amplitude value of the bending moments M in the cross-sections of the beam with the attached masses, tf·m

Nodal mass	Theory		SCAD		
	Time, s	Bending moment, tf·m	Time, s	Bending moment, tf·m	Deviations, %
1	0.0346	-128.426	0.0361	-128.486	0.05
1	0.0493	-115.960	0.0503	-116.387	0.37
1	0.0824	-229.286	0.0833	-228.949	0.15
1	0.1180	-87.419	0.1194	-86.744	0.77
1	0.1334	-101.705	0.1351	-100.782	0.91
2	0.0226	+41.120	0.0236	+40.829	0.71
2	0.0599	-128.638	0.0613	-128.687	0.04
2	0.0849	-74.952	0.0864	-74.957	0.01
2	0.1052	-105.748	0.1068	-104.818	0.88
2	0.1423	+60.864	0.1430	+61.063	0.33

Notes: In the analytical solution the natural frequencies of oscillations p of the simply supported beam are determined according to the following formulas:

$$p_1 = \sqrt{\frac{48 \cdot E \cdot I}{m \cdot l^3}}; \qquad p_2 = \sqrt{\frac{384 \cdot E \cdot I}{m \cdot l^3}}.$$

In the analytical solution the deflections η in the cross-sections of the beam with the attached masses with time are determined according to the following formulas:

$$\eta_1(t) = \frac{P \cdot l^3}{768 \cdot E \cdot I} \cdot \left[8 \cdot (1 - cos(p_1 \cdot t)) + (1 - cos(p_2 \cdot t))\right];$$

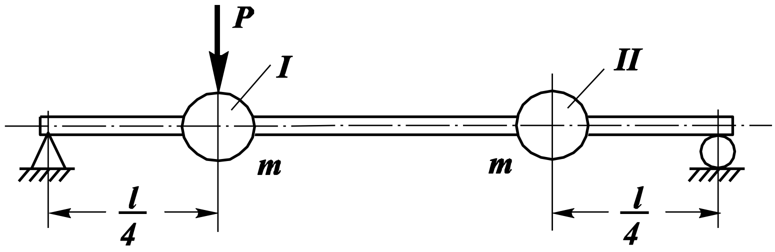
$$\eta_2(t) = \frac{P \cdot l^3}{768 \cdot E \cdot I} \cdot \left[8 \cdot (1 - cos(p_1 \cdot t)) - (1 - cos(p_2 \cdot t))\right].$$

In the analytical solution the bending moments M in the cross-sections of the beam with the attached masses with time are determined according to the following formulas:

$$M_1(t) = -\frac{P \cdot l}{16} \cdot \left[2 \cdot (1 - cos(p_1 \cdot t)) + (1 - cos(p_2 \cdot t))\right];$$

$$M_2(t) = -\frac{P \cdot l}{16} \cdot \left[2 \cdot (1 - cos(p_1 \cdot t)) - (1 - cos(p_2 \cdot t))\right].$$

Simply Supported Weightless Beam with Two Concentrated Masses and Transverse Harmonic Exciting Force Applied to One of Them



Objective: Determination of the strain state of a simply supported weightless beam with two concentrated masses subjected to a transverse harmonic exciting force applied to one of them.

Initial data files: **5.12_Harm_L.SPR**
График_5.12_Harm_L_Forc_Freq_1.txt
График_5.12_Harm_L_Forc_Freq_2.txt
График_5.12_Harm_L_Forc_Freq_3.txt
График_5.12_Harm_L_Forc_Freq_4.txt
График_5.12_Harm_L_Forc_Freq_5.txt
График_5.12_Harm_L_Forc_Freq_6.txt
График_5.12_Harm_L_Forc_Freq_7.txt

Problem formulation: Two identical loads of mass *m* are attached to the simply supported beam of constant cross-section at a quarter span distance from each support. The mass of the beam is neglected in comparison with the masses of the loads. The force P_0 is applied to one of the masses at the initial time and varies harmonically with the frequency ω. Determine the natural oscillation modes and natural frequencies *p* of the simply supported beam, as well as the deflections η in the cross-sections of the beam with the attached masses with time.

References: S.D. Ponomarev, V.L. Biederman, K.K. Likharev, V.M. Makushin, N.N. Malinin, V.I. Feodos'yev, Fundamentals of Modern Methods for Strength Analysis in Mechanical Engineering. Dynamic Analysis. Stability. Creep. Moscow, Mashgiz, 1952, p. 153.

Initial data:

$E = 3.0\cdot10^6$ tf/m^2	- elastic modulus;
$\nu = 0.2$	- Poisson's ratio;
b = 0.4 m	- width of the rectangular cross-section of the beam;
h = 0.8 m	- height of the rectangular cross-section of the beam;
l = 8.0 m	- beam span length;
$m = 3.0$ tf·s^2/m	- value of the concentrated masses attached to the beam;
$P_0 = 76.8$ tf	- amplitude value of the harmonic exciting force applied to one of the masses;
$g = 10.00$ m/s^2	- gravitational acceleration;
$I = b\cdot h^3/12 = 0.017067$	- cross-sectional moment of inertia of the beam

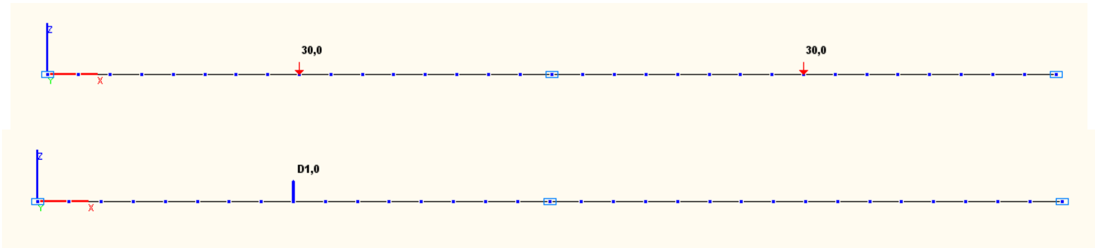
The following values of frequencies of the harmonic exciting force ω_i depending on the values of natural frequencies of the beam p_i are considered:

$\omega_j = 0.5\cdot p_1;\ 0.95\cdot p_1;\ 1.05\cdot p_1;\ 0.5\cdot(p_1+ p_2);\ 0.95\cdot p_2;\ 1.05\cdot p_2;\ 1.5\cdot p_2.$

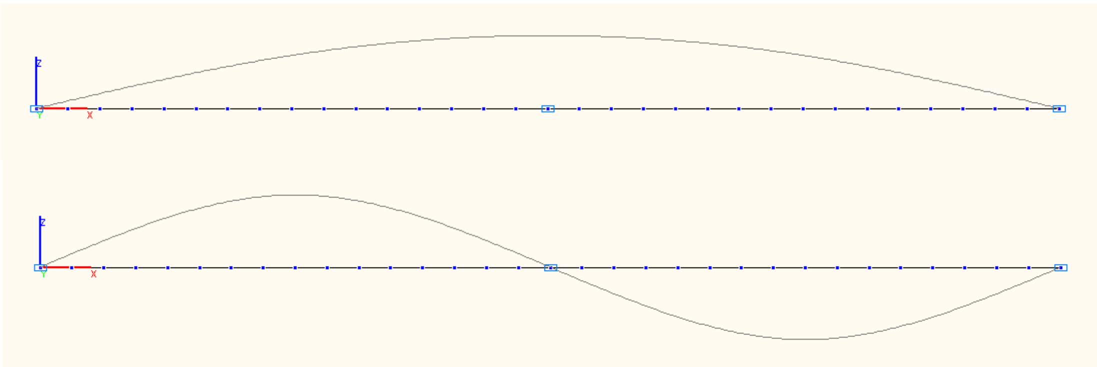
Finite element model: Design model – plane frame, 32 bar elements of type 2. Boundary conditions of the simply supported ends of the beam are provided by imposing constraints in the direction of the degree of freedom Z. The dimensional stability of the design model is provided by imposing a constraint in the node of the cross-section along the symmetry axis of the beam in the direction of the degree of freedom X. The concentrated masses are specified by transforming the static nodal loads *m·g*.
The calculation is performed in two stages: first the natural oscillation modes and natural frequencies *p* are determined by the modal analysis, and then the deflections η in the cross-sections of the beam with the

attached masses with time are determined by the direct integration of the equations of motion method. The action of the transverse harmonic exciting force is described by the graph of the load variation with time and is given in the form of a nodal force acting along the Z axis of the global coordinate system with the scale factor of 1.0 and the delay time 0.0 s. Intervals between the time points of the load variation graph are equal to $\Delta t_{int} = T_j/100$, where T_j – period of the harmonic exciting force, and correspond to the integration step. When plotting the graph, the action of the transverse harmonic exciting force is taken as $P_n = P_0 \cdot \cos(\omega_j \cdot n \cdot \Delta t_{int})$ at the time points $n \cdot \Delta t_{int}$. The duration of the process is equal to $t = 2 \cdot T_j$. Critical damping ratios for the 1-st and 2-nd natural frequencies are taken with the minimum value $\xi = 0.0001$. The conversion factor for the added static loading is equal to k = 0.981 (mass generation). Number of nodes in the design model – 33. The modal integration method is used in the calculation. The determination of the natural oscillation modes and natural frequencies is performed by the method of subspace iteration. The matrix of concentrated masses is used in the calculation.

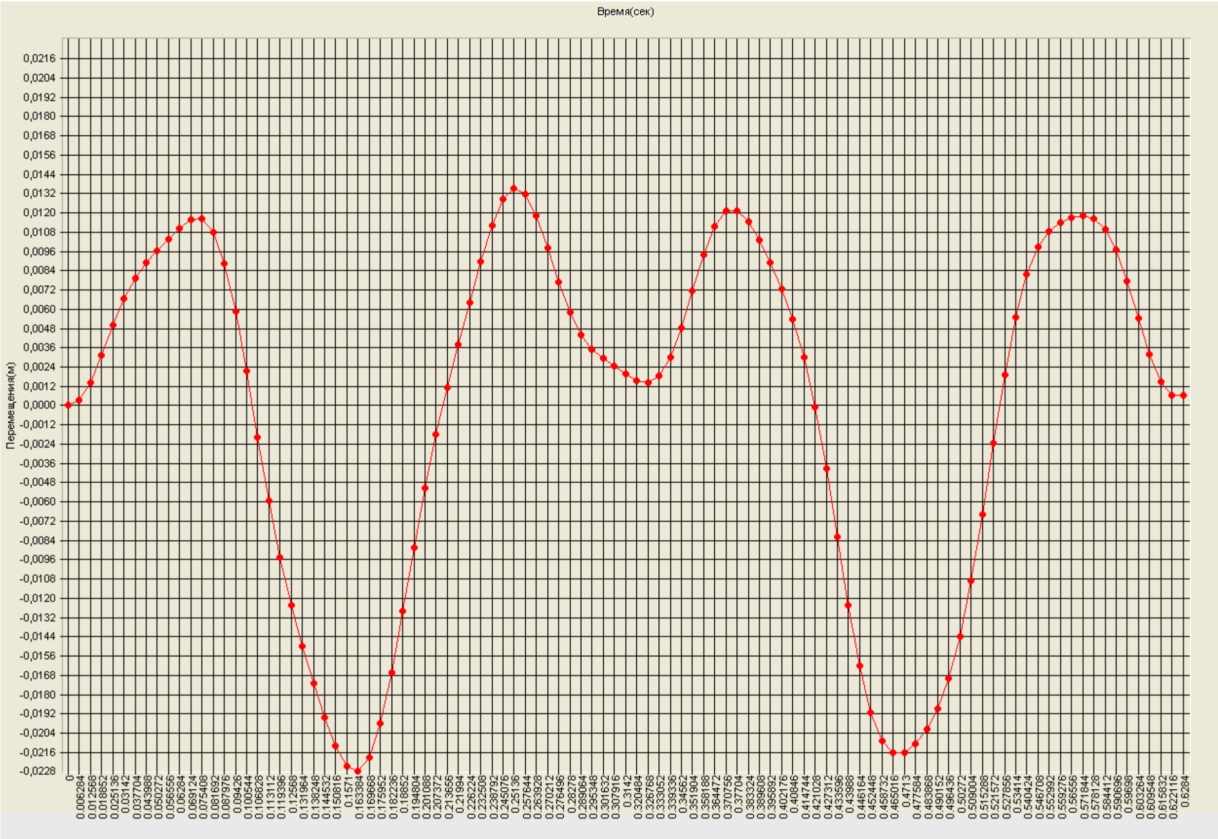
Results in SCAD



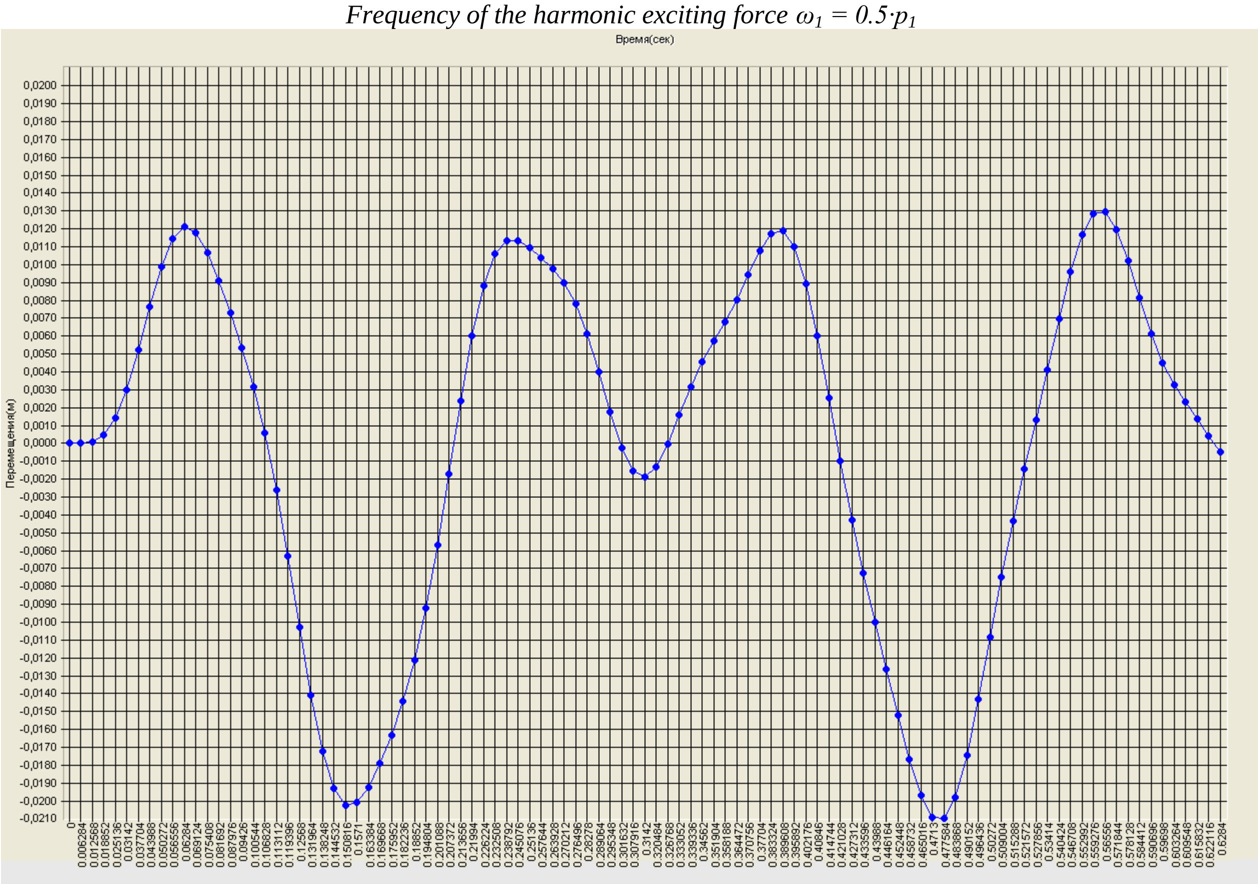
Design model



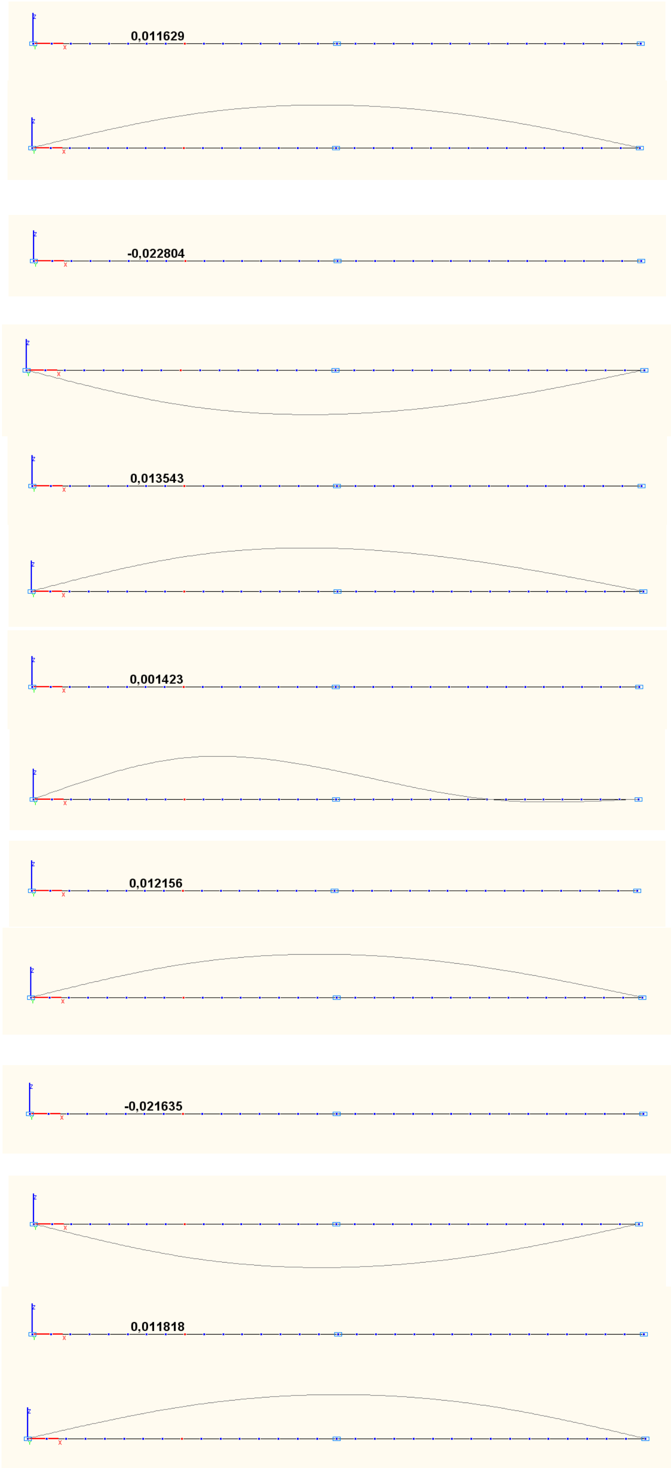
1-st and 2-nd natural oscillation modes



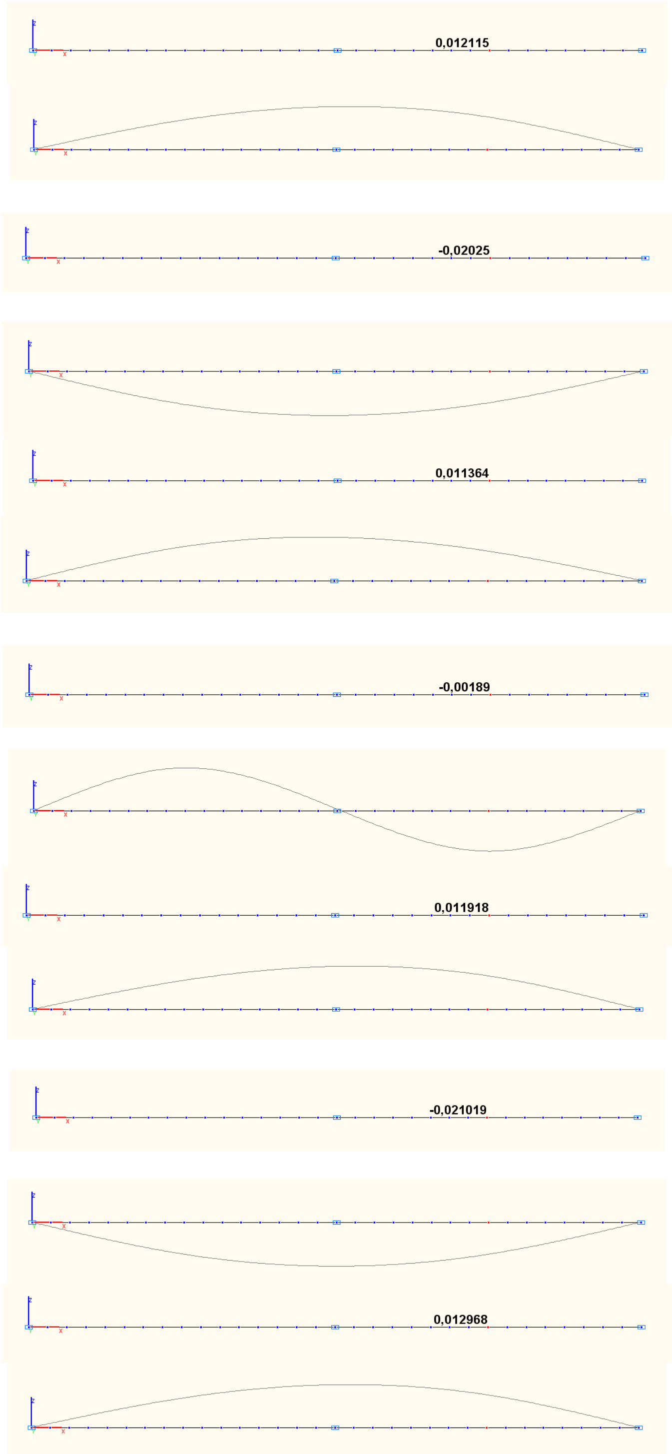
Graph of the variation of the deflection η_1 in the cross-section of the beam with the attached mass subjected to the shear force with time (m).
Frequency of the harmonic exciting force $\omega_1 = 0.5 \cdot p_1$



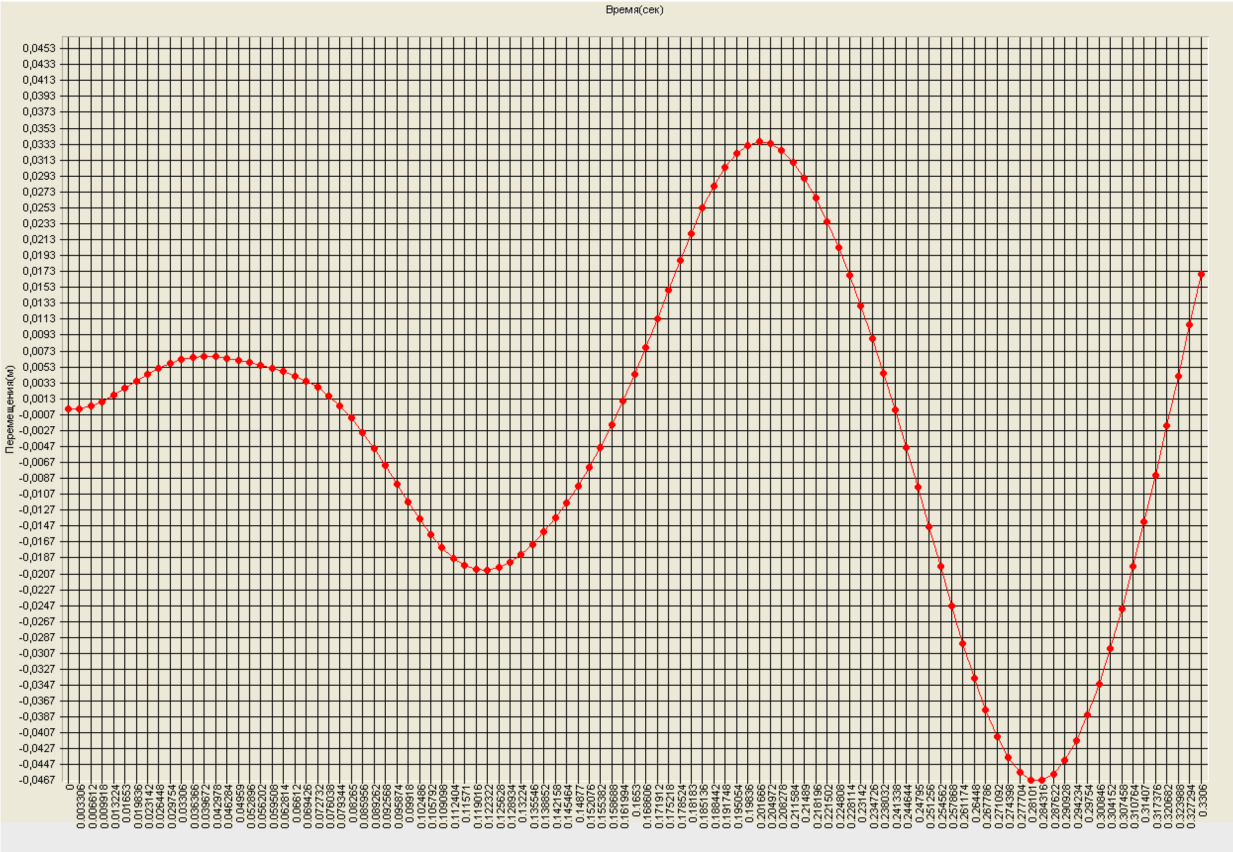
Graph of the variation of the deflection η_2 in the cross-section of the beam with the attached mass not subjected to the shear force with time (m).
Frequency of the harmonic exciting force $\omega_1 = 0.5 \cdot p_1$



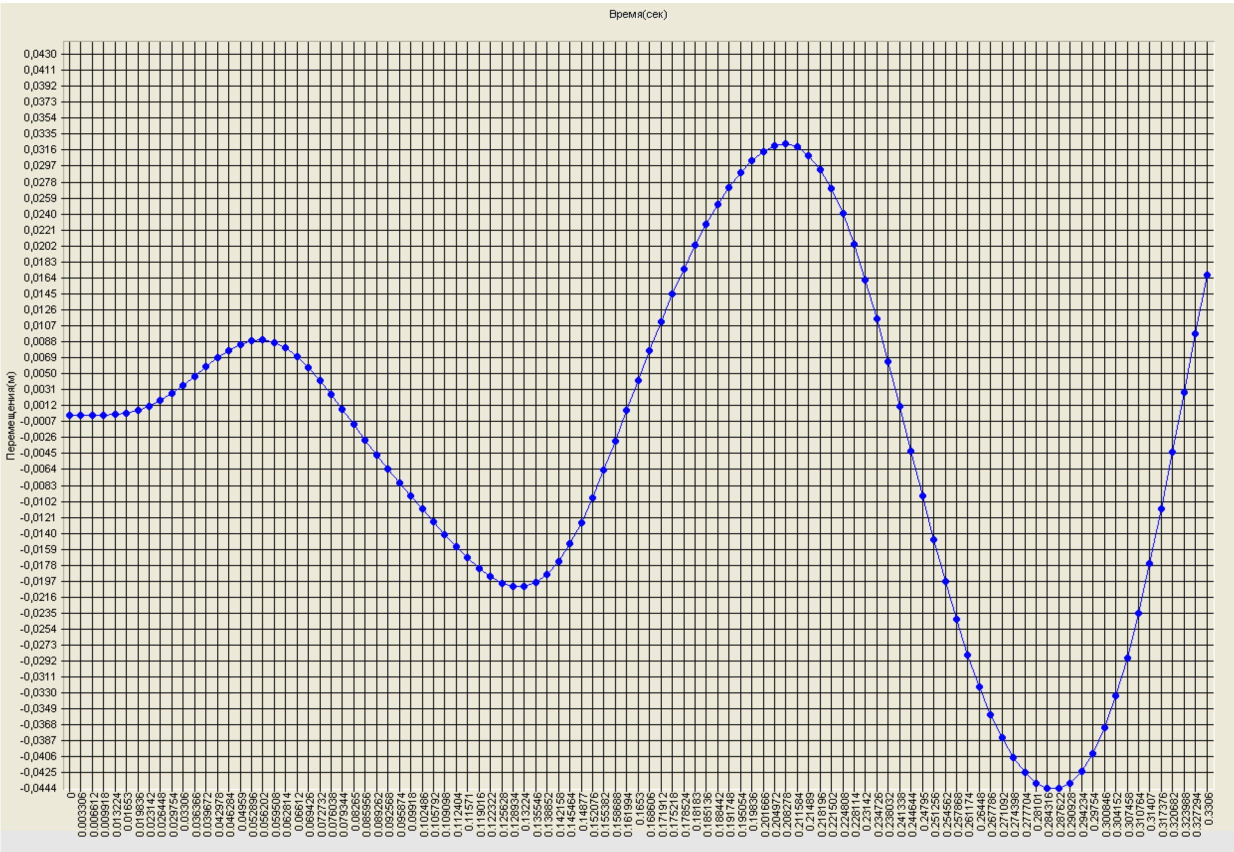
Amplitude values of the deflection η_1 in the cross-section of the beam with the attached mass subjected to the shear force and the deformed models at the respective time points (m). Frequency of the harmonic exciting force $\omega_1 = 0.5 \cdot p_1$



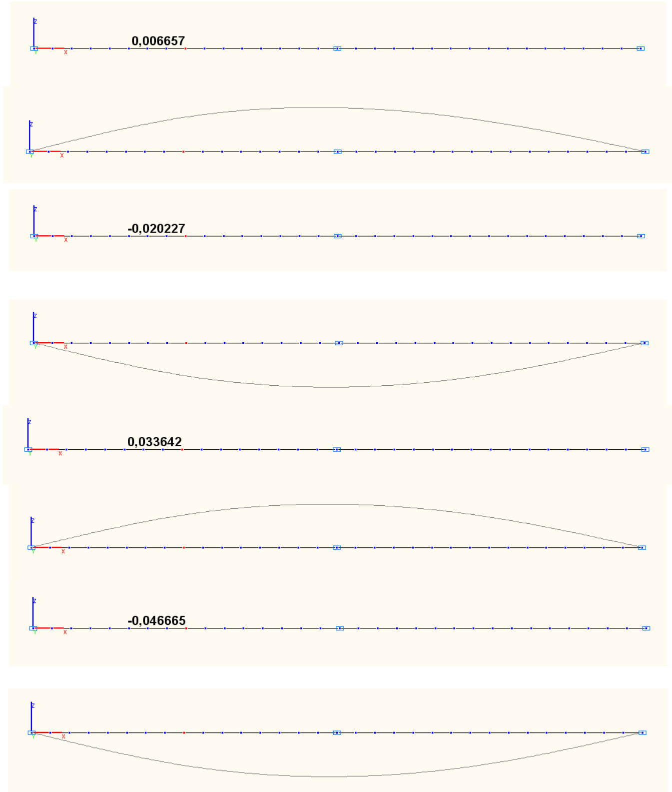
Amplitude values of the deflection η_2 in the cross-section of the beam with the attached mass not subjected to the shear force and the deformed models at the respective time points (m). Frequency of the harmonic exciting force $\omega_1 = 0.5 \cdot p_1$



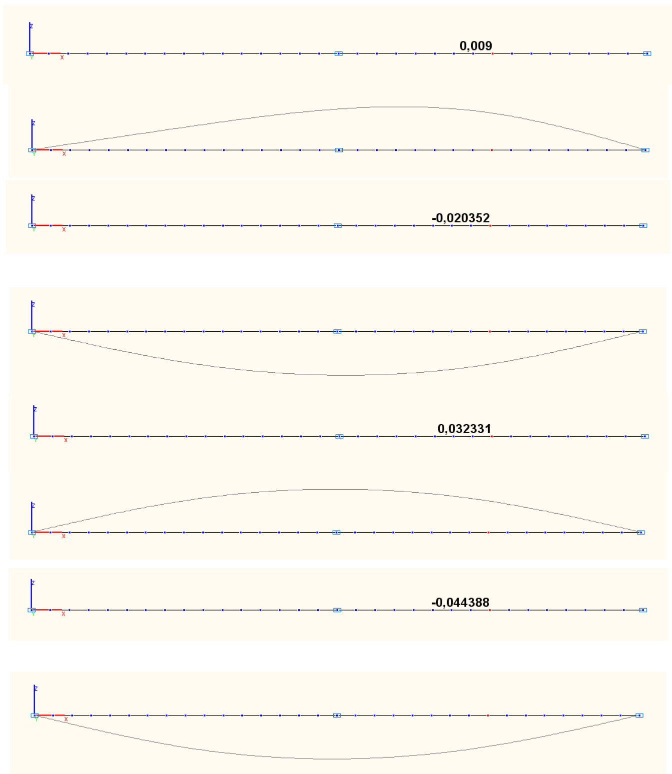
Amplitude value of the deflection η_1 in the cross-section of the beam with the attached mass subjected to the shear force and the deformed model at the respective time point (m). Frequency of the harmonic exciting force $\omega_2 = 0.95{\cdot}p_1$



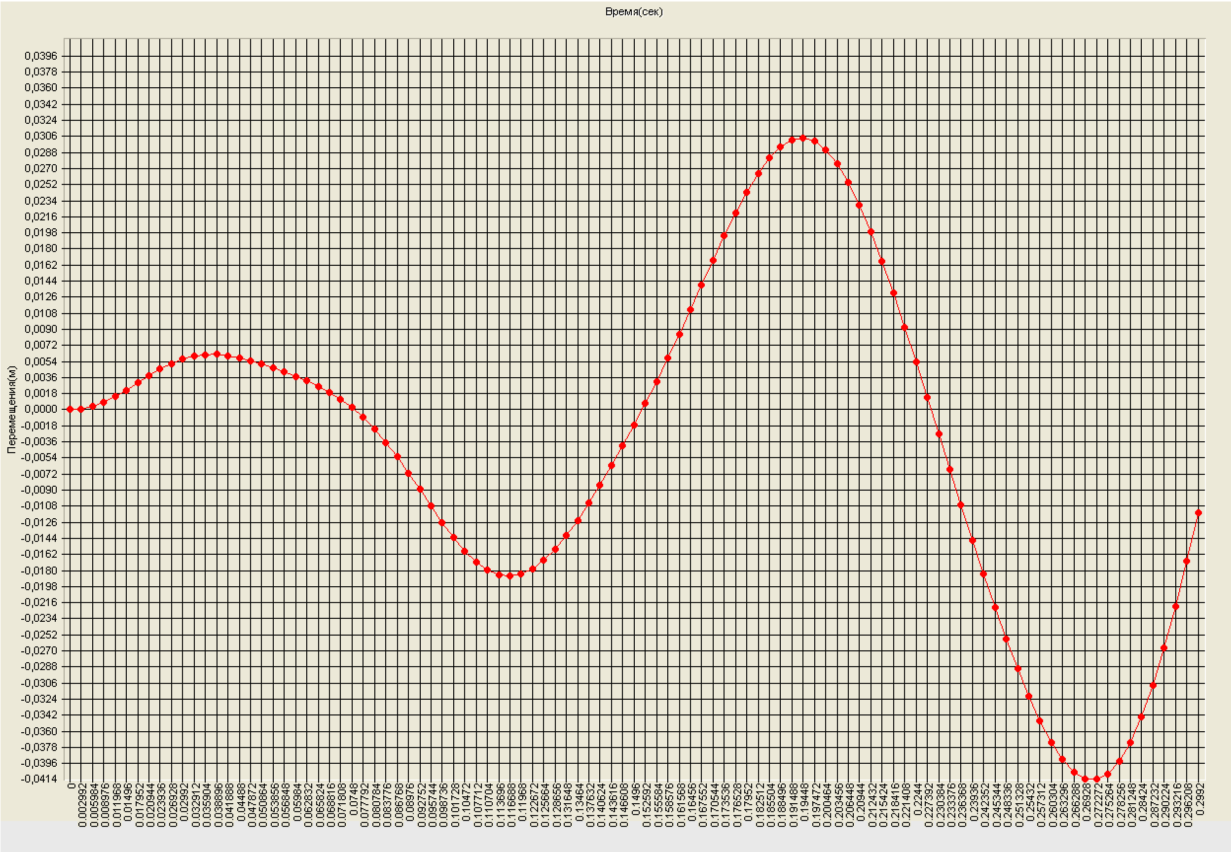
Graph of the variation of the deflection η_2 in the cross-section of the beam with the attached mass not subjected to the shear force with time (m). Frequency of the harmonic exciting force $\omega_2 = 0.95{\cdot}p_1$



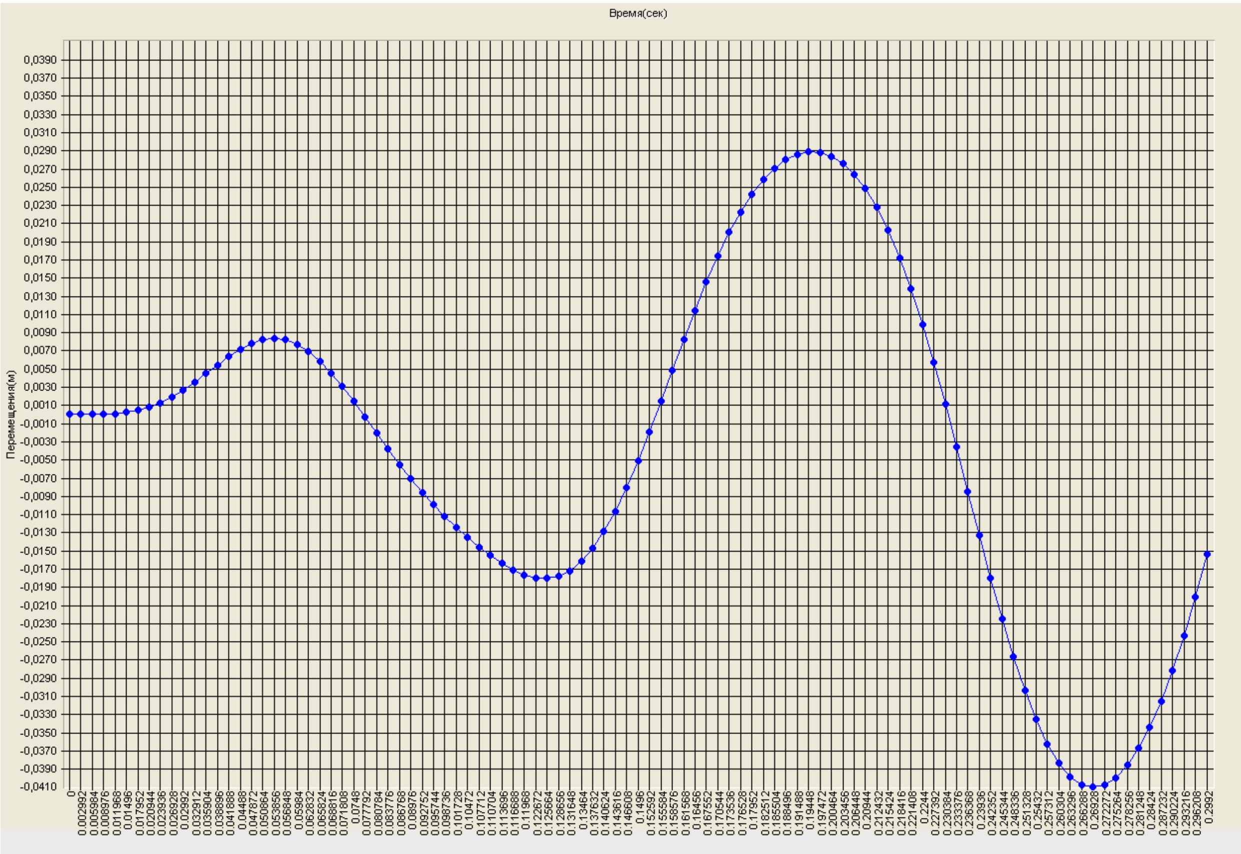
Amplitude values of the deflection η_1 in the cross-section of the beam with the attached mass subjected to the shear force and the deformed models at the respective time points (m). Frequency of the harmonic exciting force $\omega_2 = 0.95 \cdot p_1$



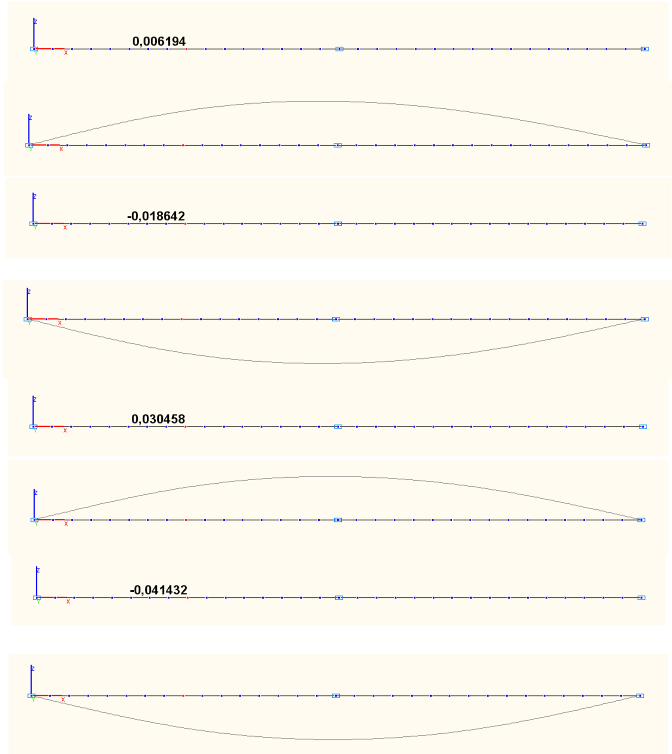
Amplitude values of the deflection η_2 in the cross-section of the beam with the attached mass not subjected to the shear force and the deformed models at the respective time points (m). Frequency of the harmonic exciting force $\omega_2 = 0.95 \cdot p_1$



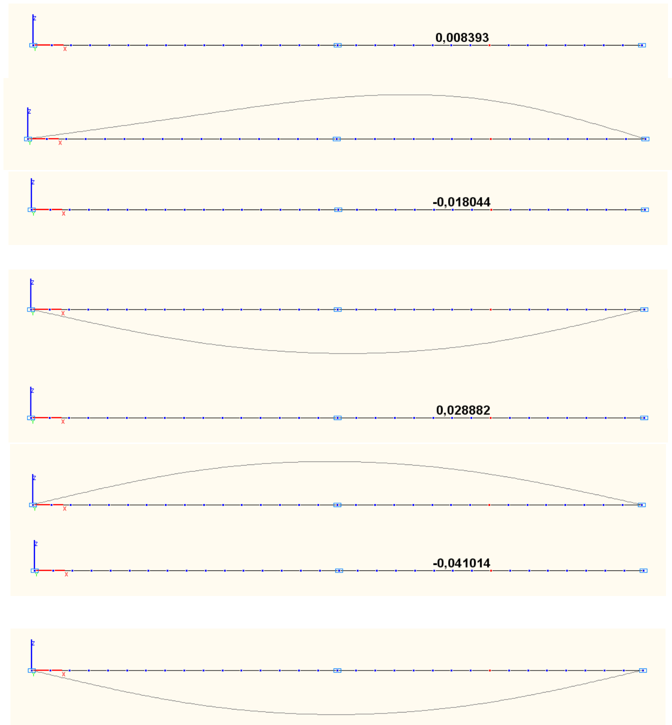
Amplitude value of the deflection η_1 in the cross-section of the beam with the attached mass subjected to the shear force and the deformed model at the respective time point (m). Frequency of the harmonic exciting force $\omega_3 = 1.05 \cdot p_1$



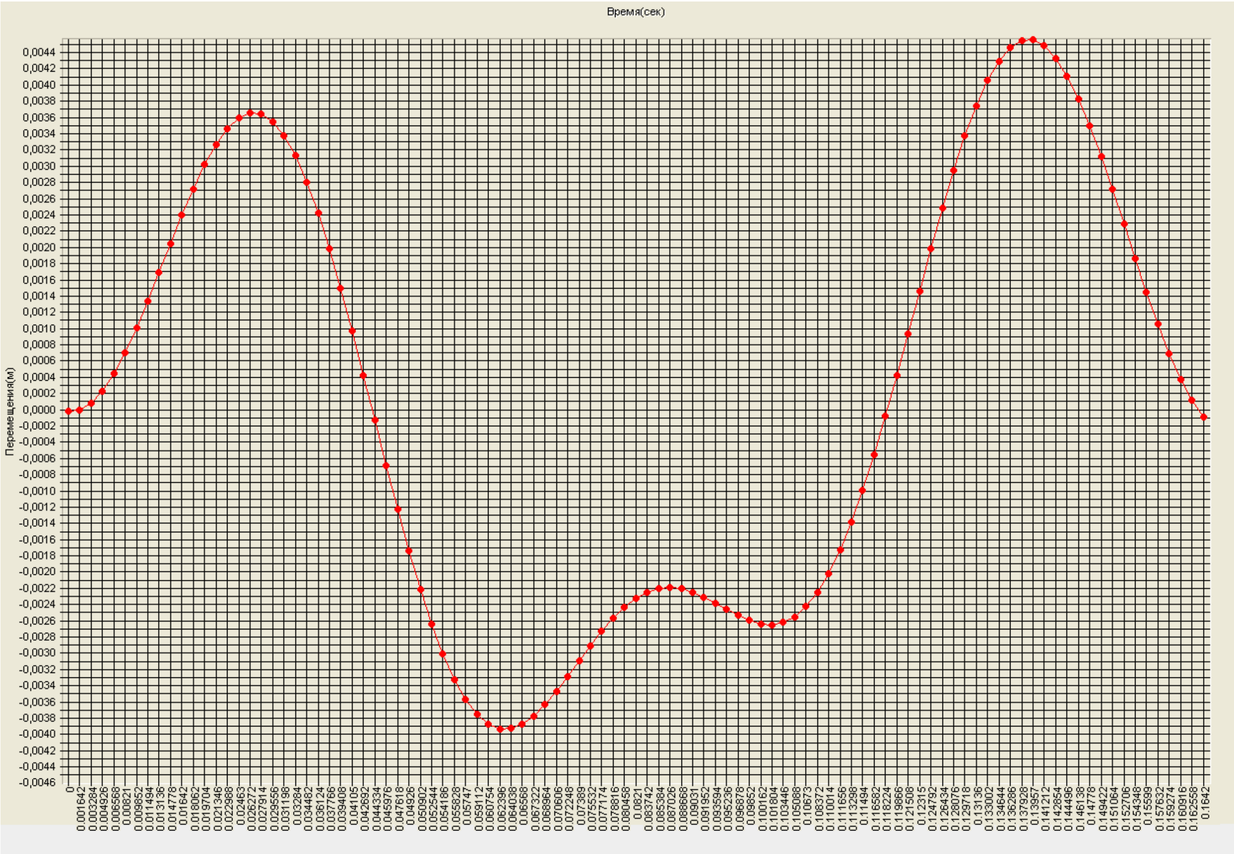
Graph of the variation of the deflection η_2 in the cross-section of the beam with the attached mass not subjected to the shear force, with time (m). Frequency of the harmonic exciting force $\omega_3 = 1.05 \cdot p_1$



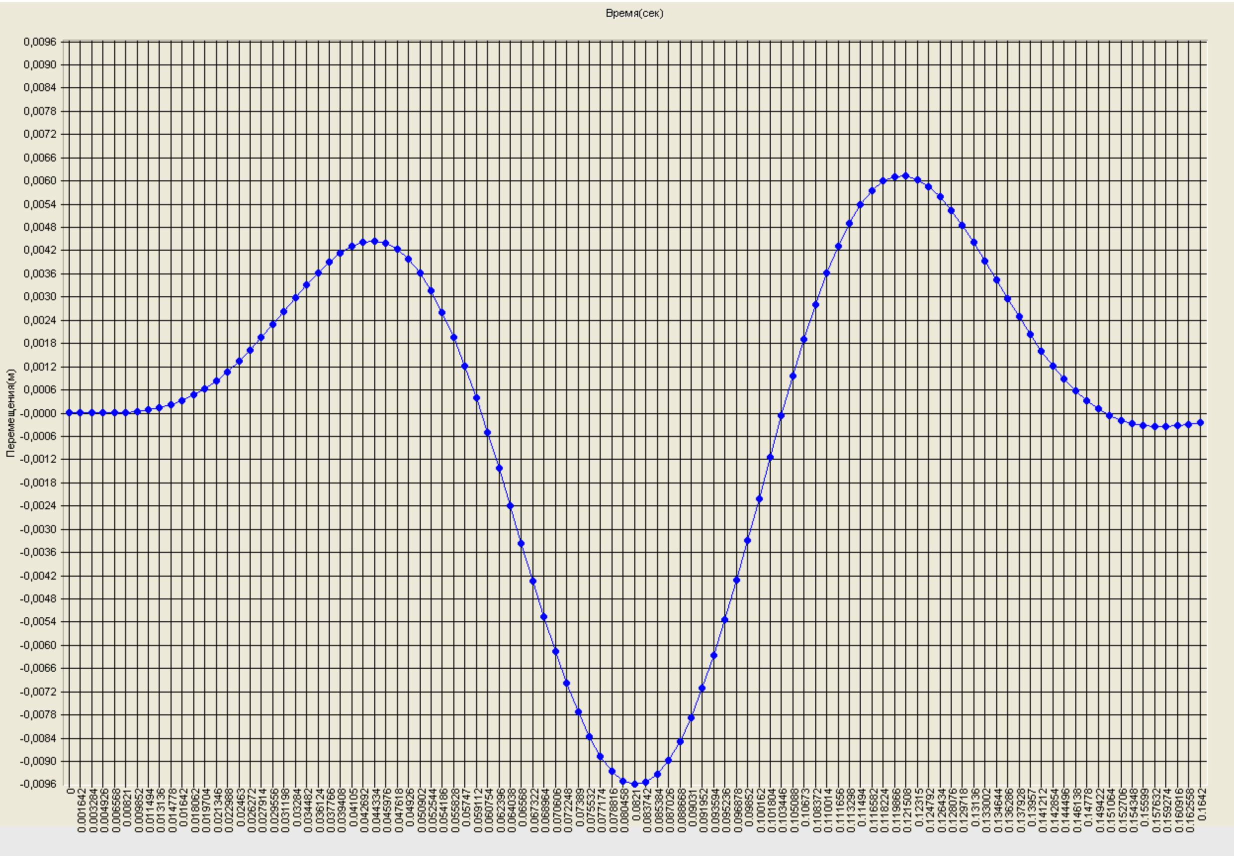
Amplitude values of the deflection η_1 in the cross-section of the beam with the attached mass subjected to the shear force and the deformed models at the respective time points (m). Frequency of the harmonic exciting force $\omega_3 = 1.05 \cdot p_1$



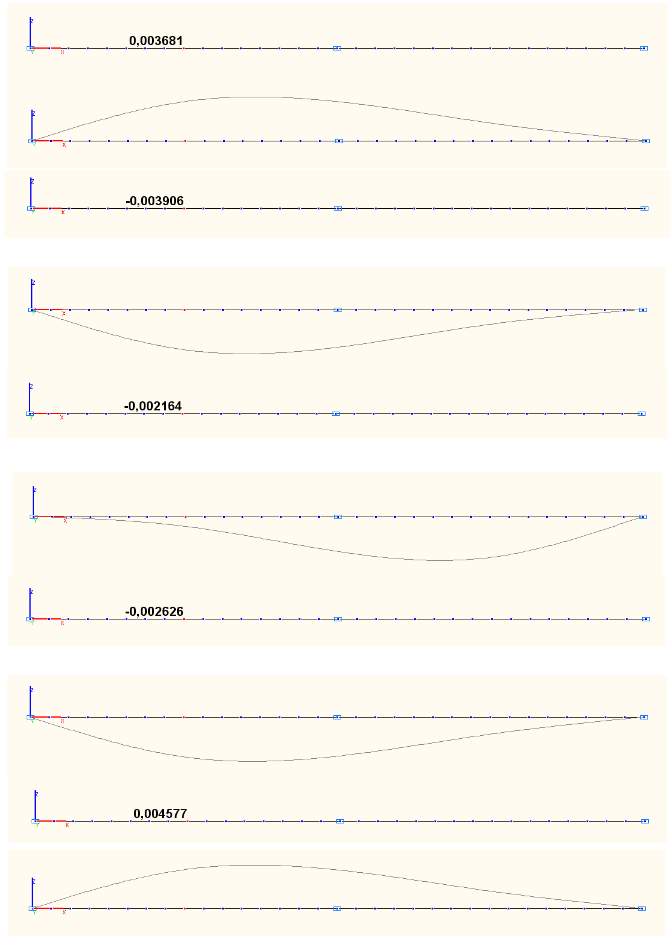
Amplitude values of the deflection η_2 in the cross-section of the beam with the attached mass not subjected to the shear force and the deformed models at the respective time points (m). Frequency of the harmonic exciting force $\omega_3 = 1.05 \cdot p_1$



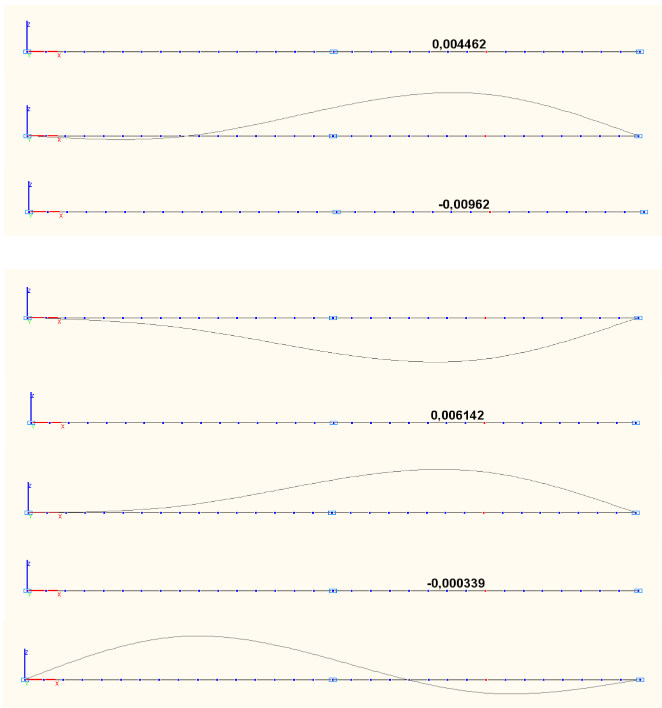
Amplitude value of the deflection η_1 in the cross-section of the beam with the attached mass subjected to the shear force and the deformed model at the respective time point (m). Frequency of the harmonic exciting force $\omega_4 = 0.5\cdot(p_1 + p_2)$



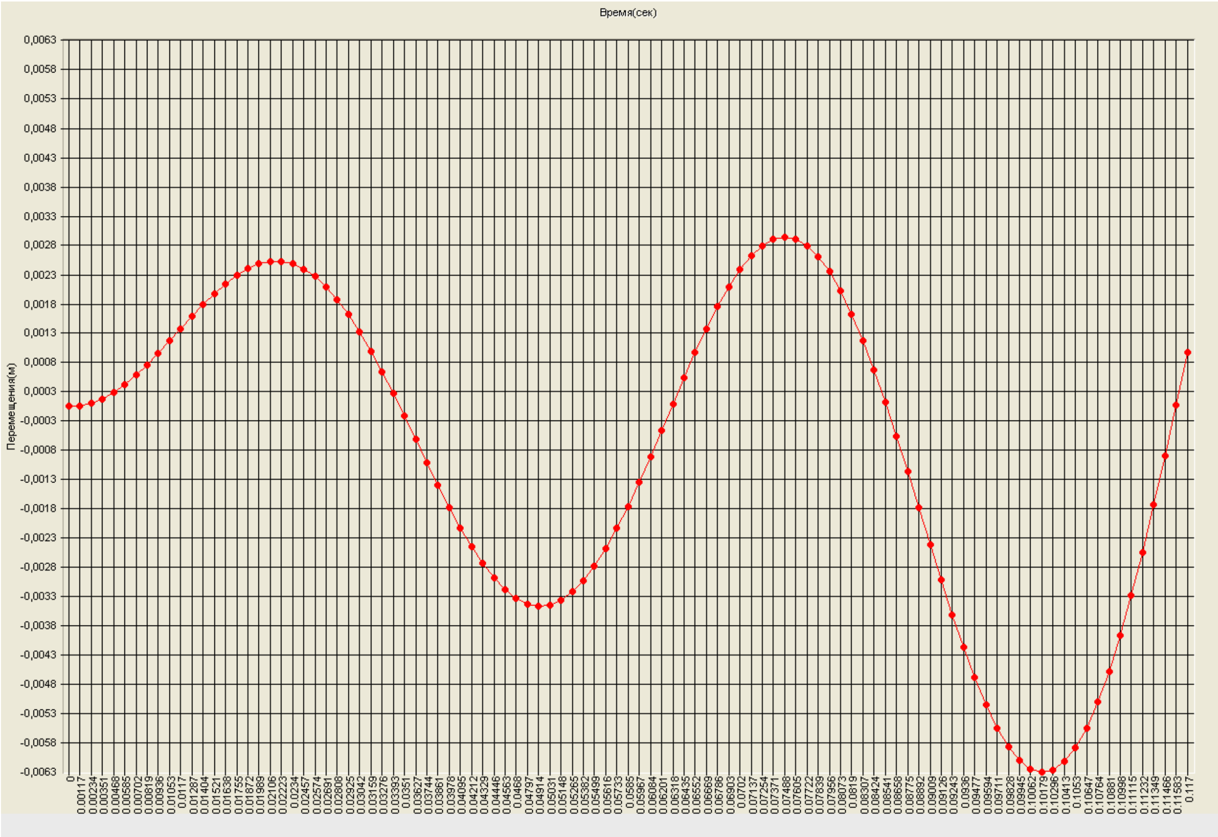
Graph of the variation of the deflection η_2 in the cross-section of the beam with the attached mass not subjected to the shear force, with time (m). Frequency of the harmonic exciting force $\omega_4 = 0.5\cdot(p_1 + p_2)$



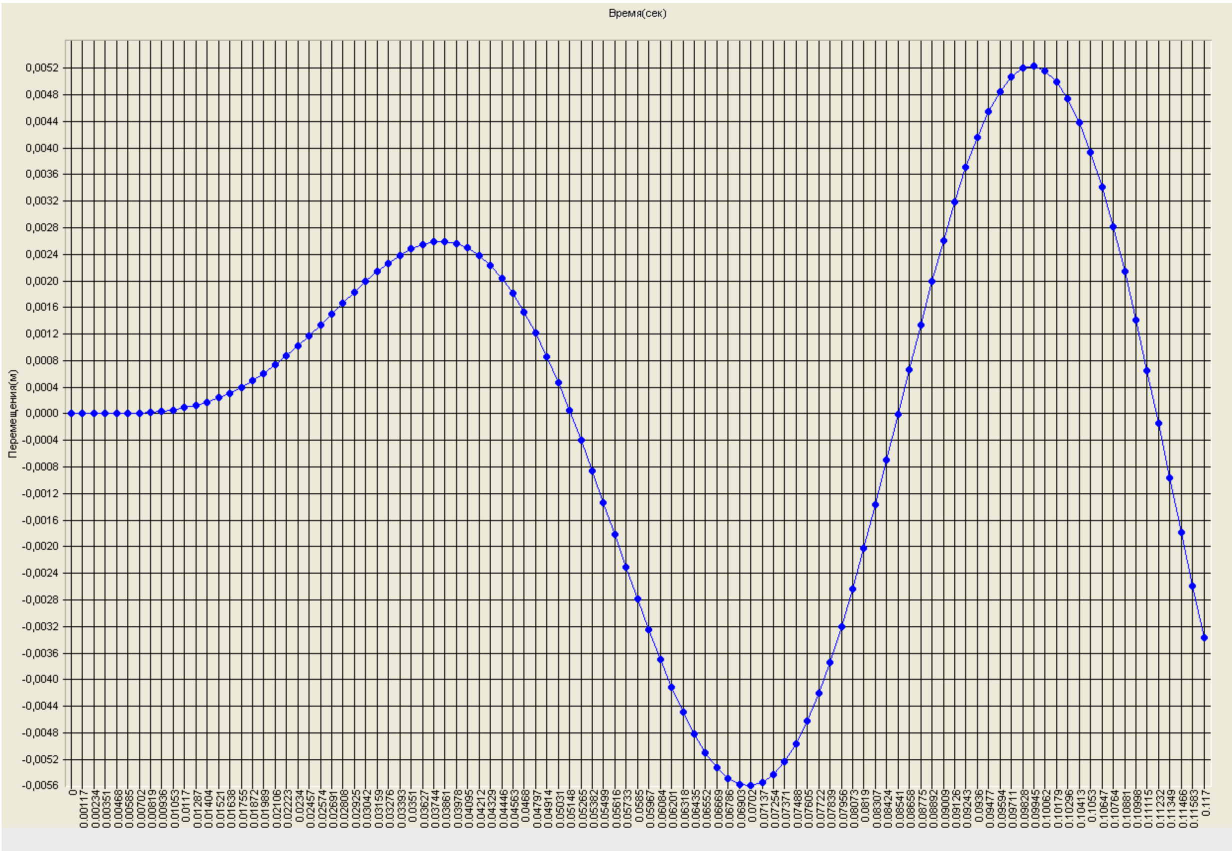
Amplitude values of the deflection η_1 in the cross-section of the beam with the attached mass subjected to the shear force and the deformed models at the respective time points (m). Frequency of the harmonic exciting force $\omega_4 = 0.5 \cdot (p_1 + p_2)$



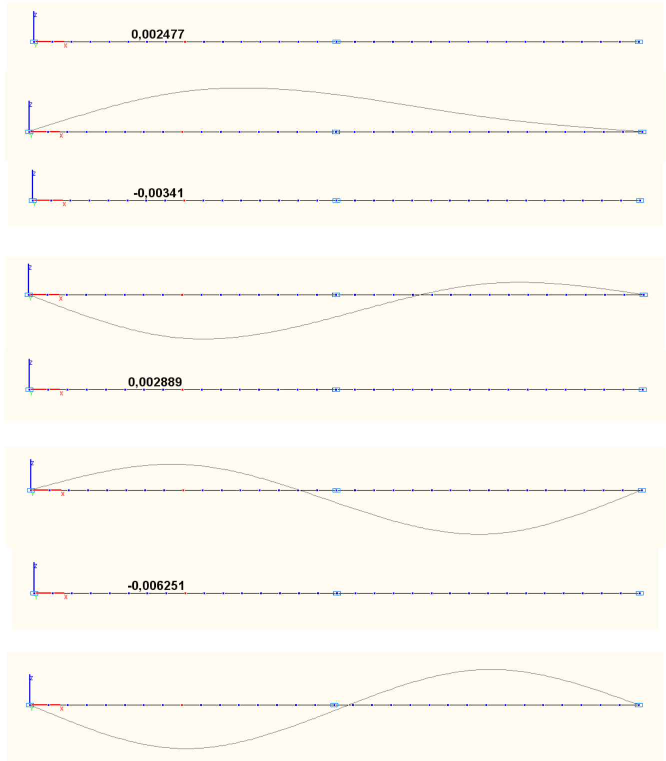
Amplitude values of the deflection η_2 in the cross-section of the beam with the attached mass not subjected to the shear force and the deformed models at the respective time points (m). Frequency of the harmonic exciting force $\omega_4 = 0.5 \cdot (p_1 + p_2)$



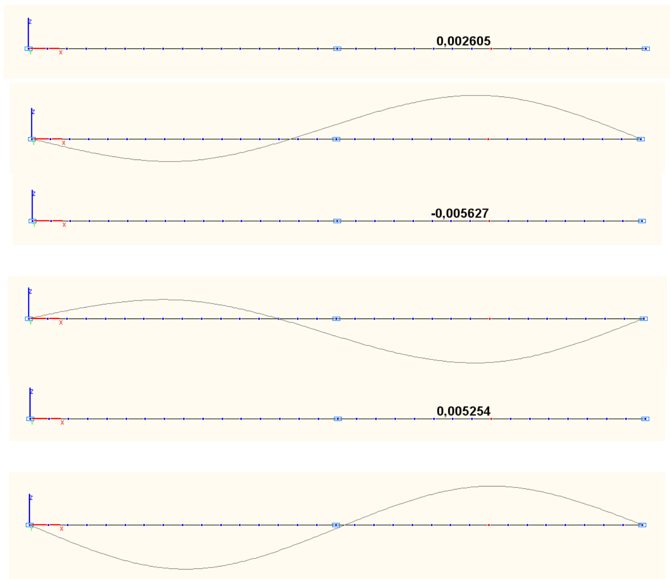
Amplitude value of the deflection η_1 in the cross-section of the beam with the attached mass subjected to the shear force and the deformed model at the respective time point (m). Frequency of the harmonic exciting force $\omega_5 = 0.95 \cdot p_2$



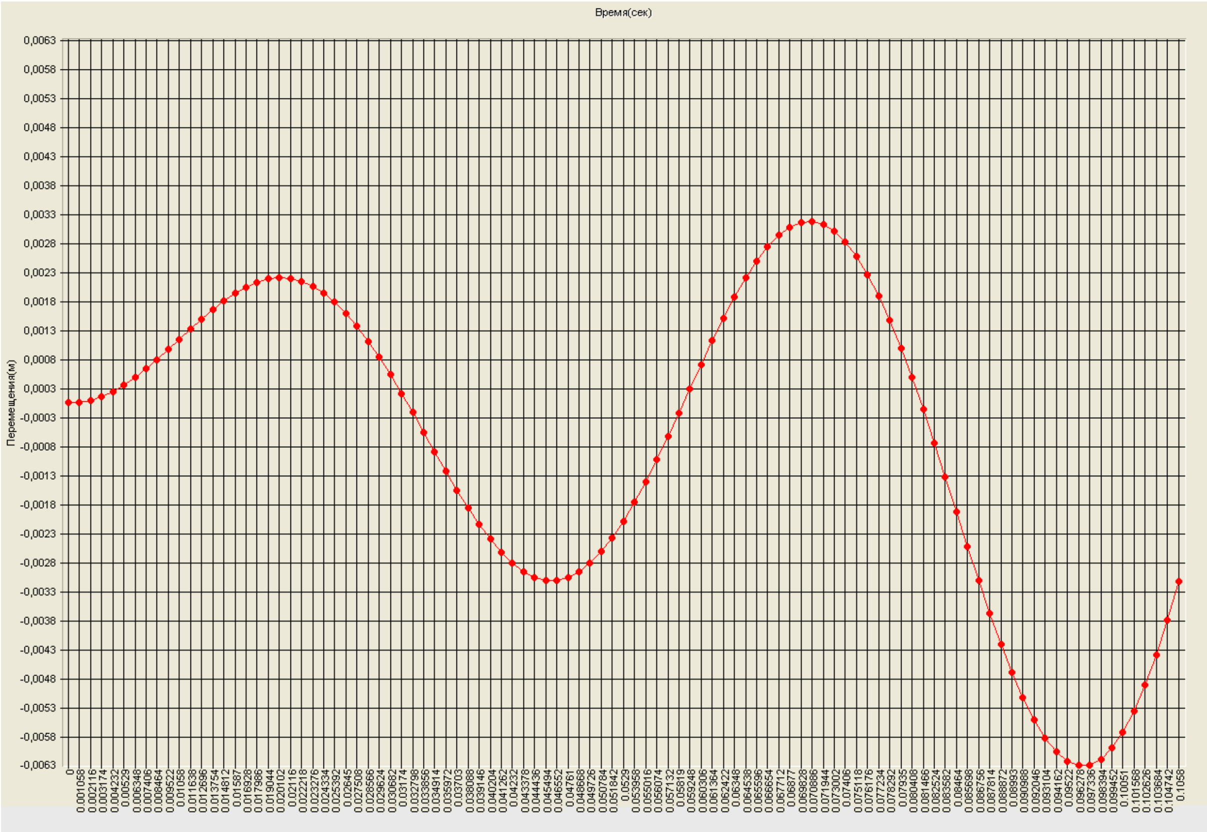
Graph of the variation of the deflection η_2 in the cross-section of the beam with the attached mass not subjected to the shear force, with time (m). Frequency of the harmonic exciting force $\omega_5 = 0.95 \cdot p_2$



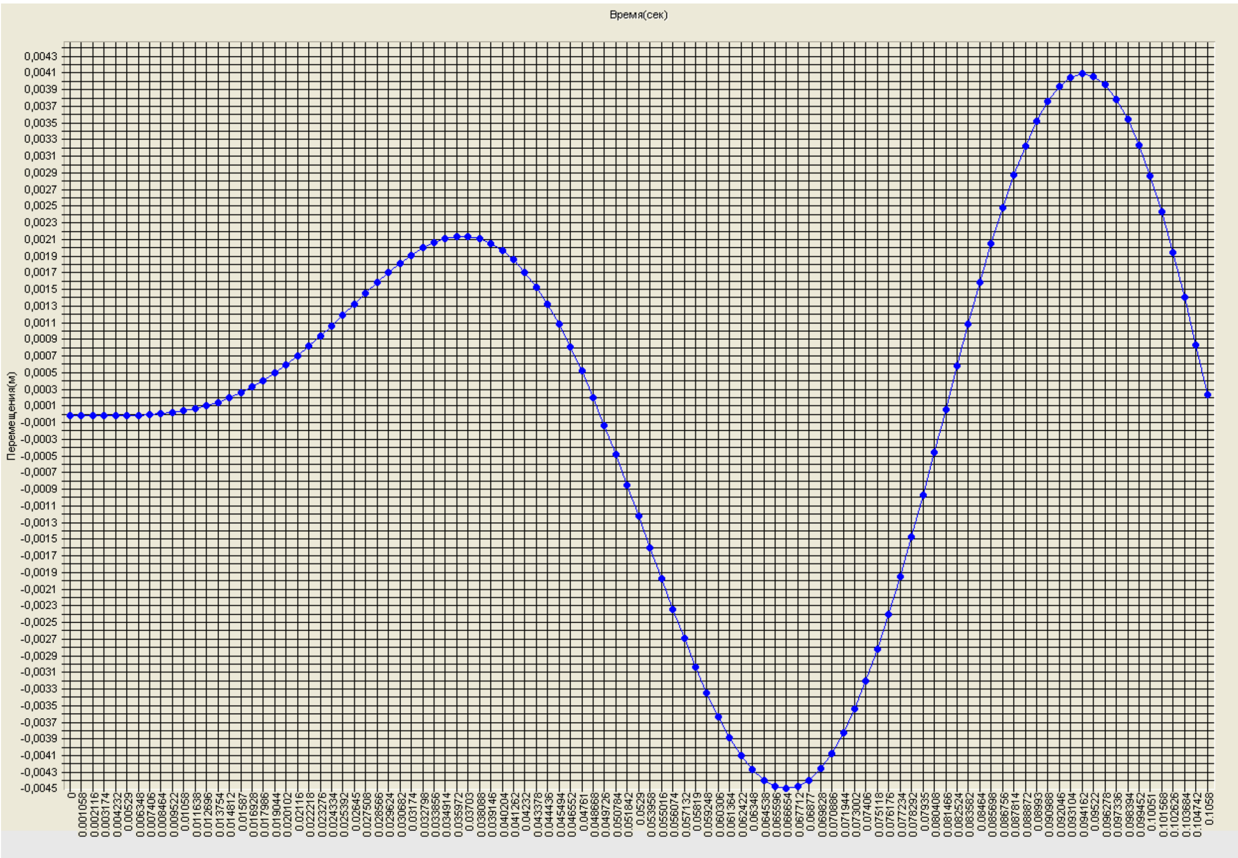
Amplitude values of the deflection η_1 in the cross-section of the beam with the attached mass subjected to the shear force and the deformed models at the respective time points (m). Frequency of the harmonic exciting force $\omega_5 = 0.95 \cdot p_2$



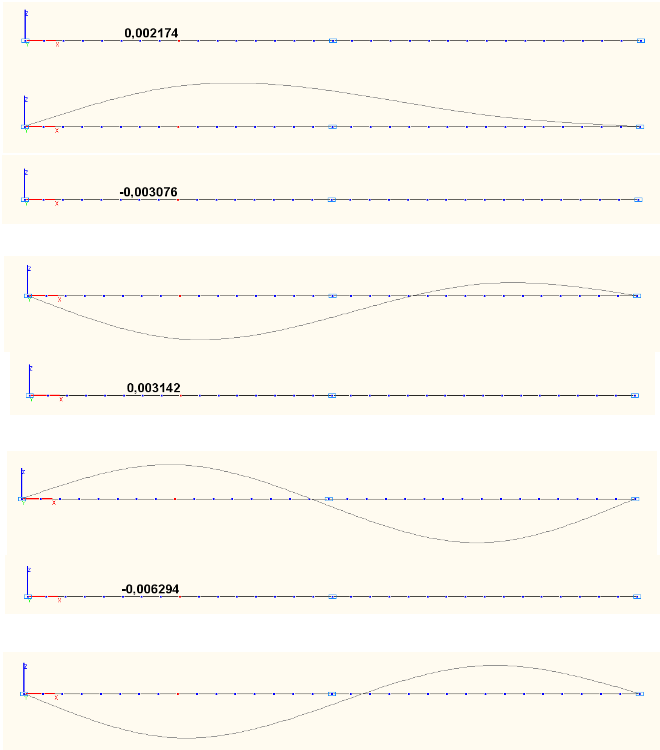
Amplitude values of the deflection η_2 in the cross-section of the beam with the attached mass not subjected to the shear force and the deformed models at the respective time points (m). Frequency of the harmonic exciting force $\omega_5 = 0.95 \cdot p_2$



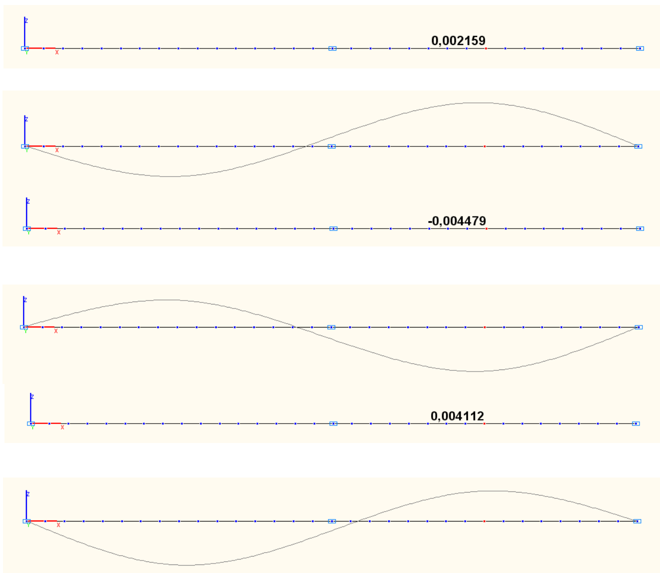
Amplitude value of the deflection η_1 in the cross-section of the beam with the attached mass subjected to the shear force and the deformed model at the respective time point (m). Frequency of the harmonic exciting force $\omega_6 = 1.05 \cdot p_2$



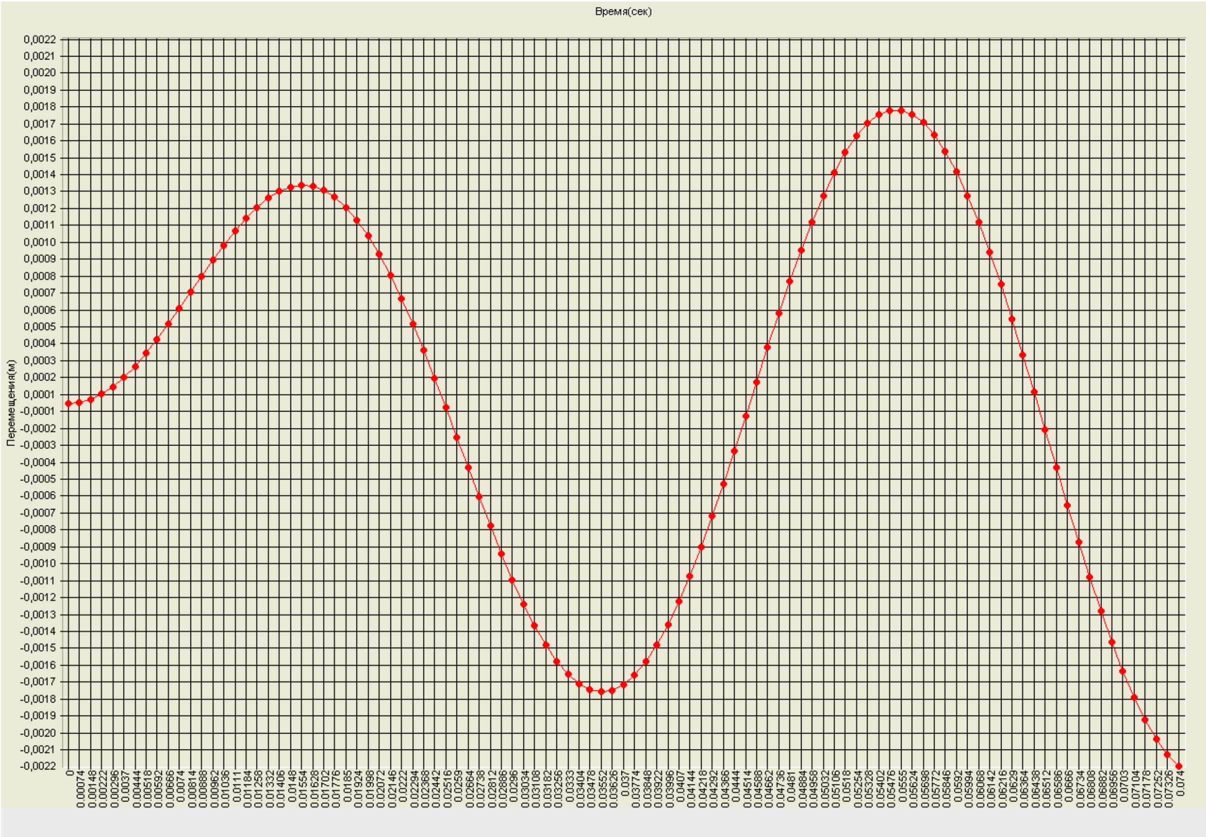
Graph of the variation of the deflection η_2 in the cross-section of the beam with the attached mass not subjected to the shear force, with time (m). Frequency of the harmonic exciting force $\omega_6 = 1.05 \cdot p_2$



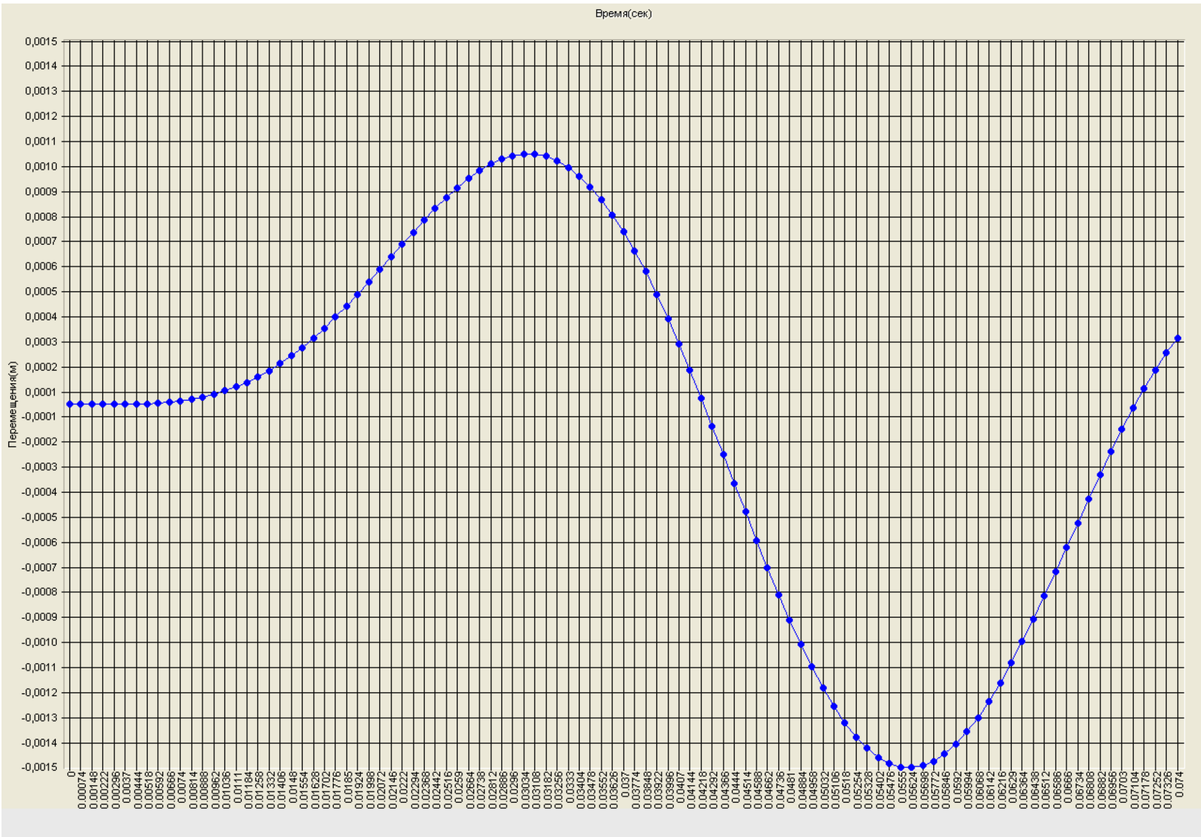
Amplitude values of the deflection η_1 in the cross-section of the beam with the attached mass subjected to the shear force and the deformed models at the respective time points (m). Frequency of the harmonic exciting force $\omega_6 = 1.05 \cdot p_2$



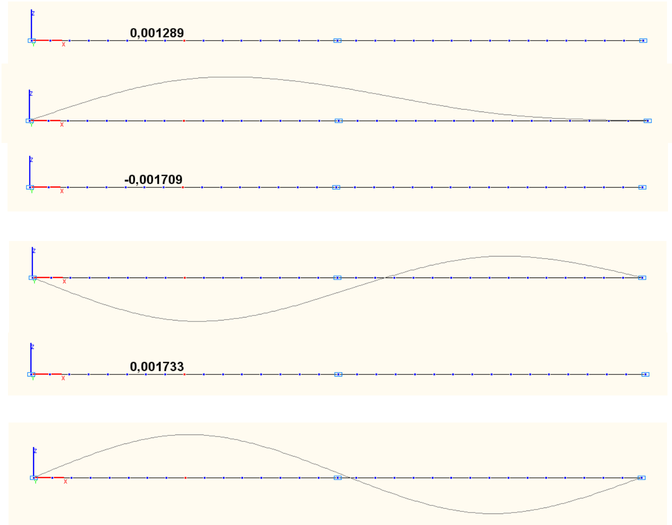
Amplitude values of the deflection η_2 in the cross-section of the beam with the attached mass not subjected to the shear force and the deformed models at the respective time points (m). Frequency of the harmonic exciting force $\omega_6 = 1.05 \cdot p_2$



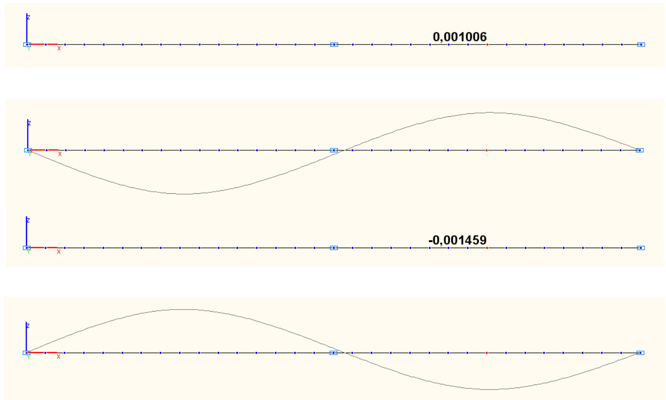
Amplitude value of the deflection η_1 in the cross-section of the beam with the attached mass subjected to the shear force and the deformed model at the respective time point (m). Frequency of the harmonic exciting force $\omega_7 = 1.5 \cdot p_2$



Graph of the variation of the deflection η_2 in the cross-section of the beam with the attached mass not subjected to the shear force with time (m). Frequency of the harmonic exciting force $\omega_7 = 1.5 \cdot p_2$

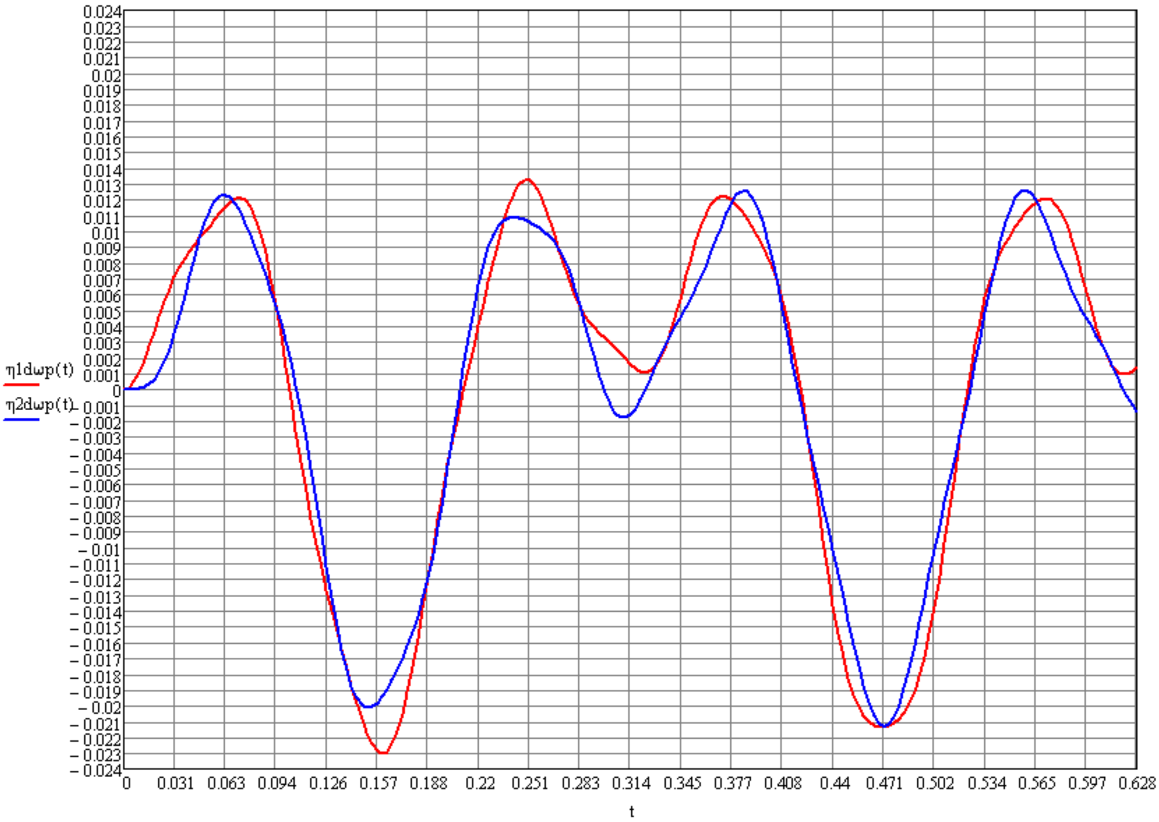


Amplitude values of the deflection η_1 in the cross-section of the beam with the attached mass subjected to the shear force and the deformed models at the respective time points (m). Frequency of the harmonic exciting force $\omega_7 = 1.5 \cdot p_2$

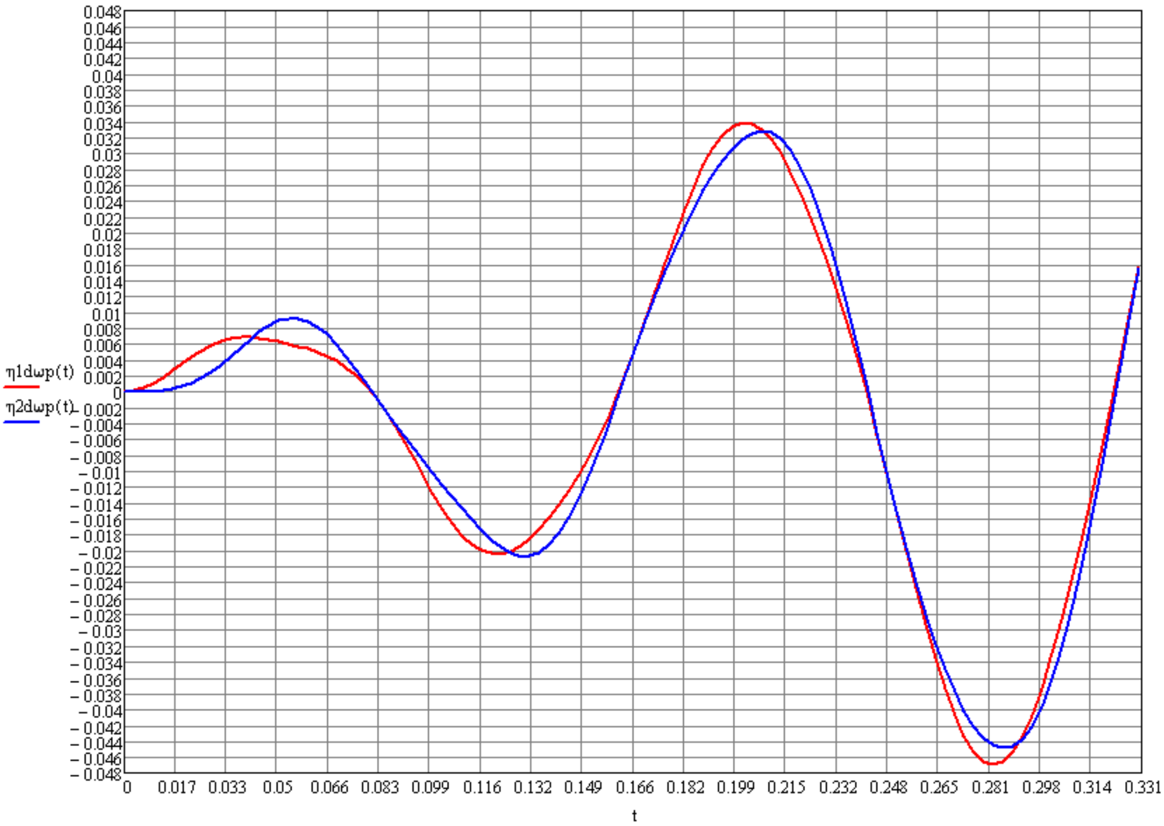


Amplitude values of the deflection η_2 in the cross-section of the beam with the attached mass not subjected to the shear force and the deformed models at the respective time points (m). Frequency of the harmonic exciting force $\omega_7 = 1.5 \cdot p_2$

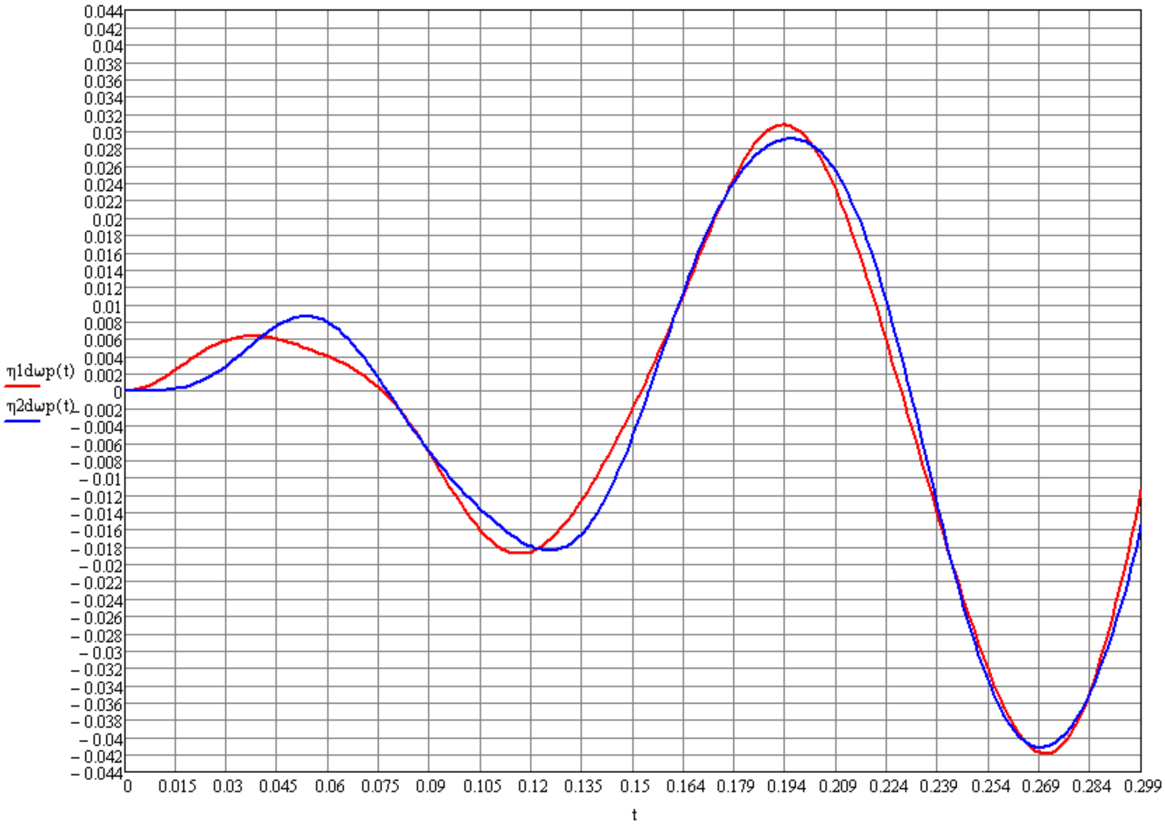
Comparison of solutions:



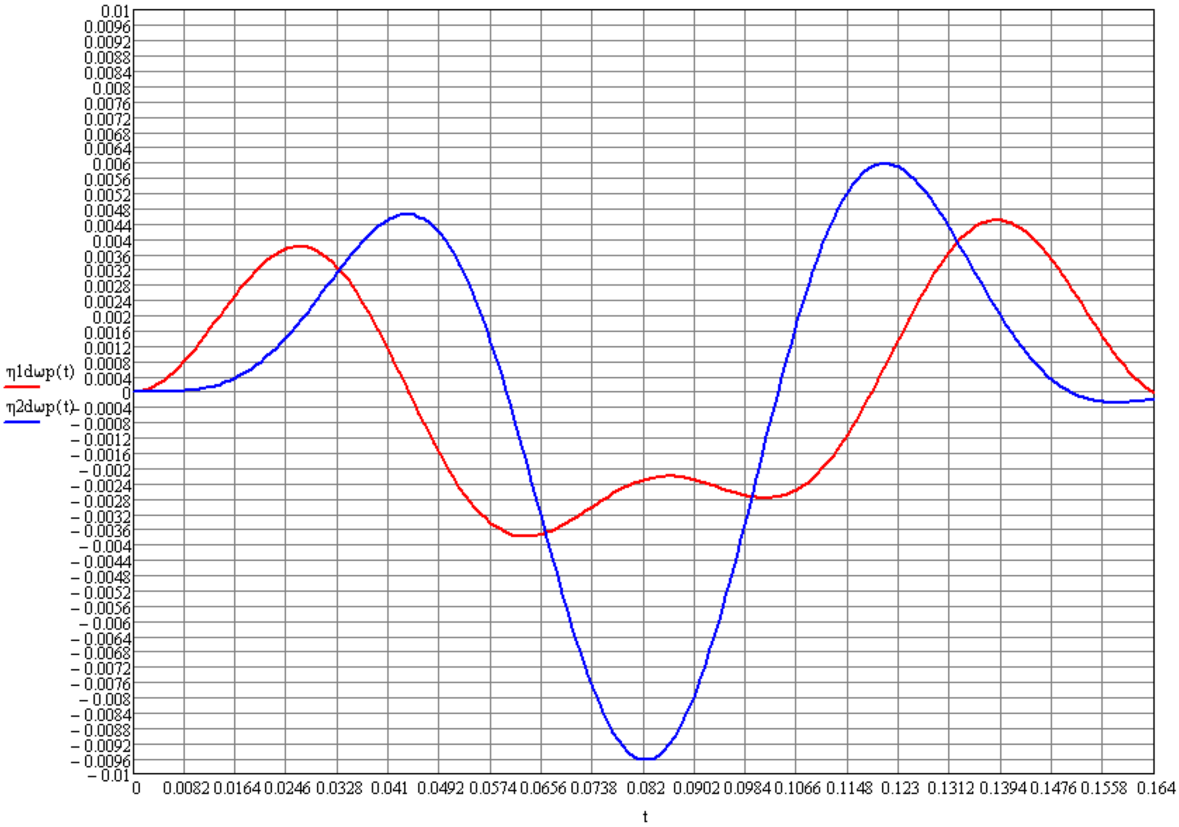
Graphs of the variation of the deflections η_1 and η_2 in the cross-sections of the beam with the attached masses with time according to the theoretical solution (m)
Frequency of the harmonic exciting force $\omega_1 = 0.5 \cdot p_1$



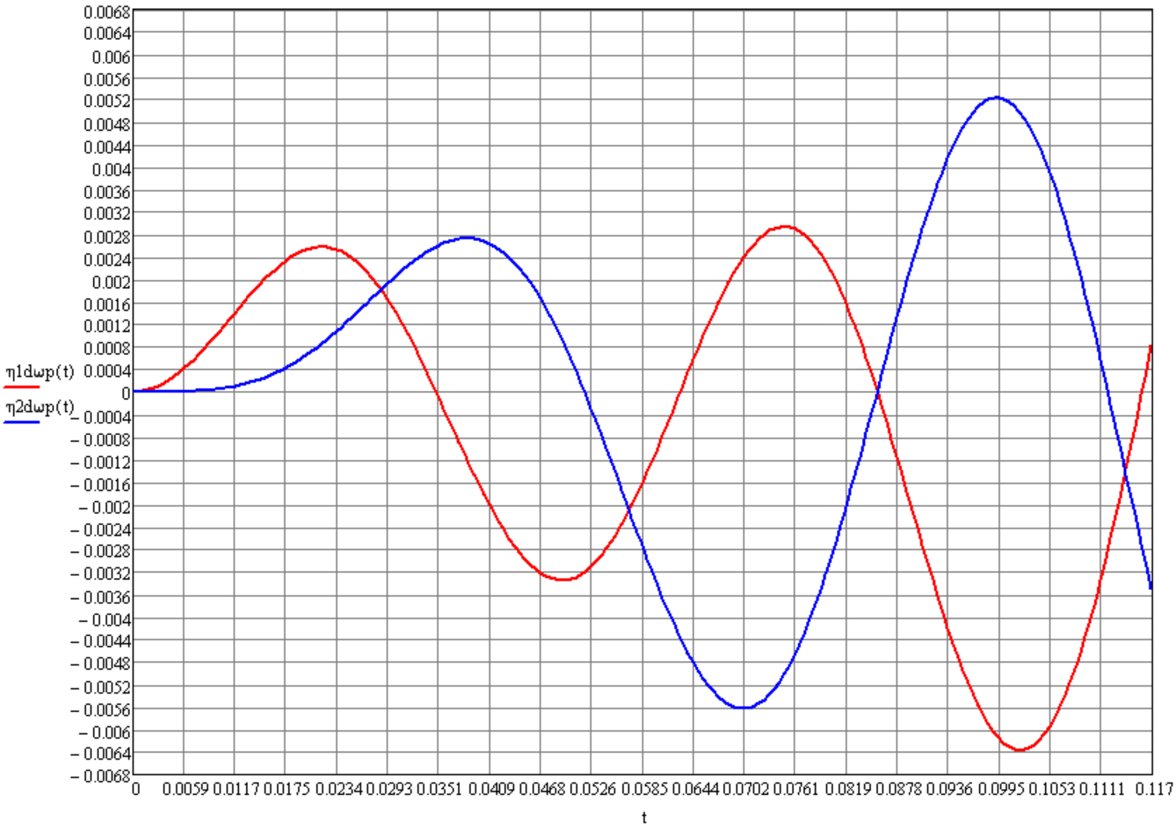
Graphs of the variation of the deflections η_1 and η_2 in the cross-sections of the beam with the attached masses with time according to the theoretical solution (m)
Frequency of the harmonic exciting force $\omega_2 = 0.95 \cdot p_1$



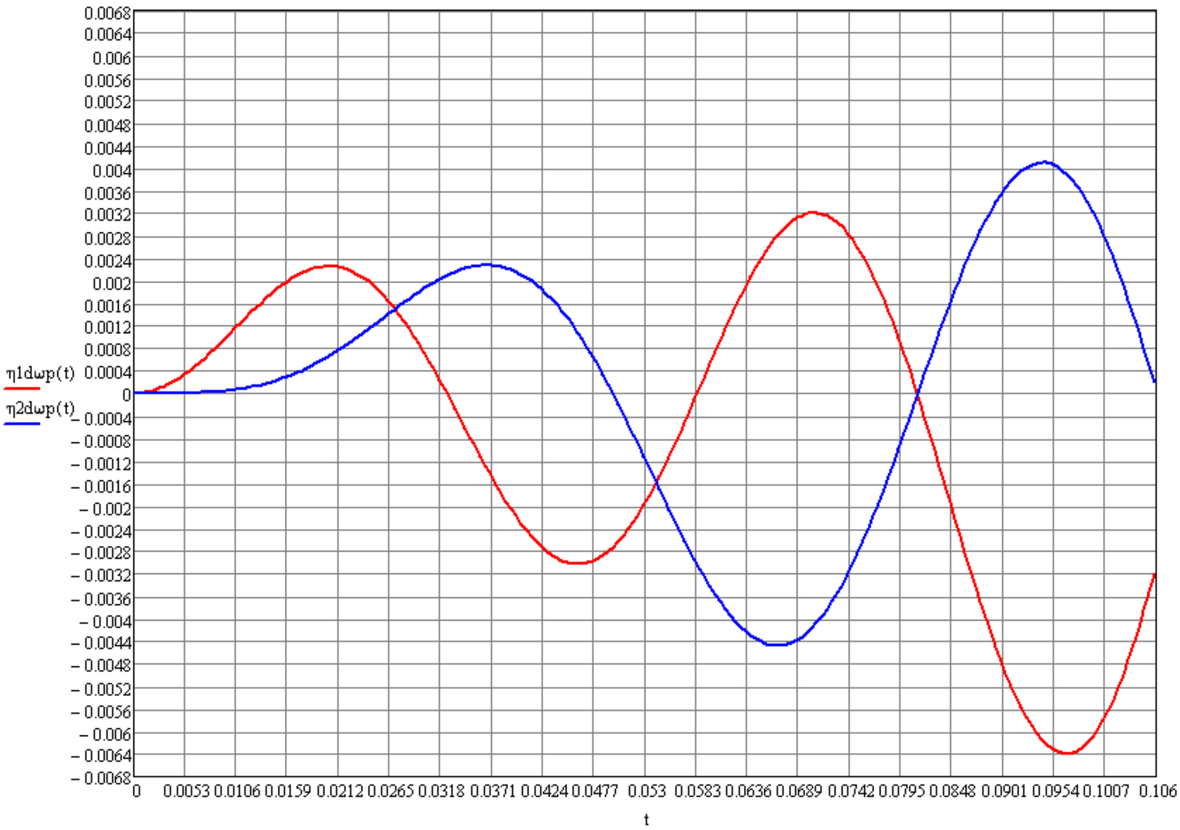
Graphs of the variation of the deflections η_1 and η_2 in the cross-sections of the beam with the attached masses with time according to the theoretical solution (m)
Frequency of the harmonic exciting force $\omega_3 = 1.05 \cdot p_1$



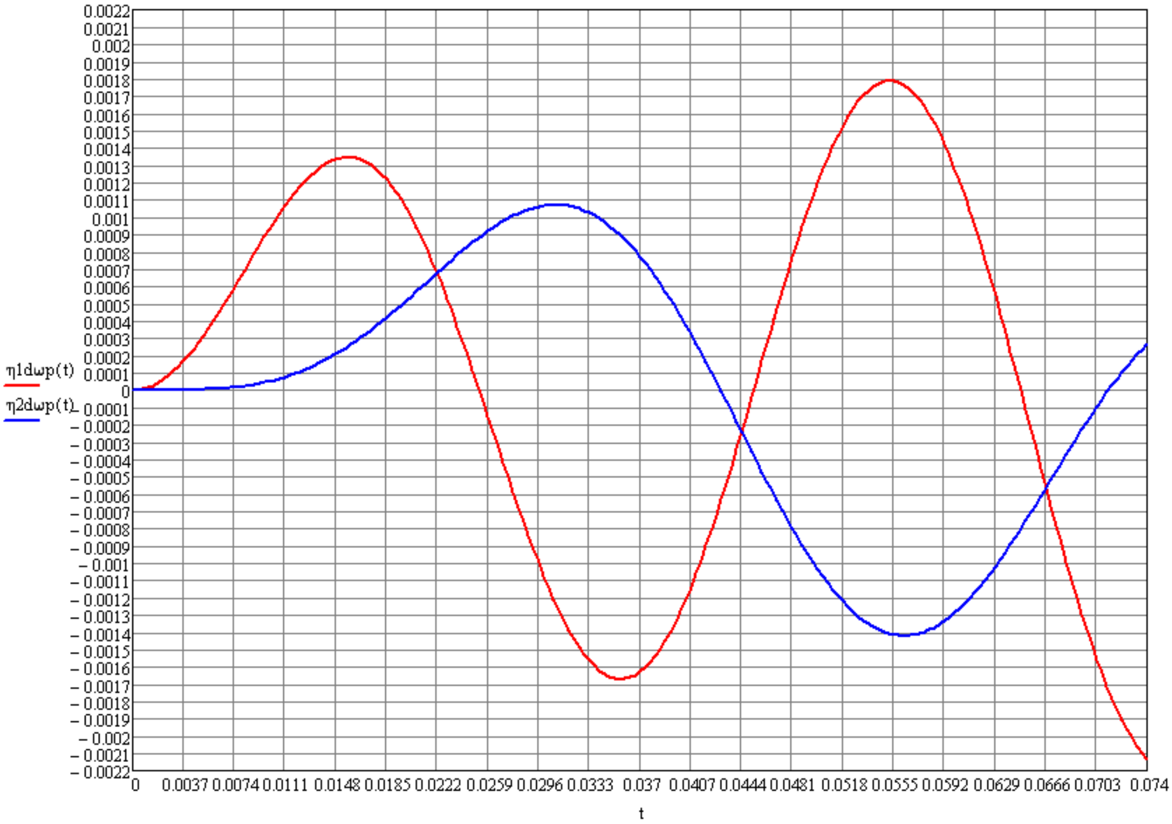
Graphs of the variation of the deflections η_1 and η_2 in the cross-sections of the beam with the attached masses with time according to the theoretical solution (m)
Frequency of the harmonic exciting force $\omega_4 = 0.5 \cdot (p_1 + p_2)$



Graphs of the variation of the deflections η_1 and η_2 in the cross-sections of the beam with the attached masses with time according to the theoretical solution (m)
Frequency of the harmonic exciting force $\omega_5 = 0.95 \cdot p_2$



Graphs of the variation of the deflections η_1 and η_2 in the cross-sections of the beam with the attached masses with time according to the theoretical solution (m)
Frequency of the harmonic exciting force $\omega_6 = 1.05 \cdot p_2$



Graphs of the variation of the deflections η_1 and η_2 in the cross-sections of the beam with the attached masses with time according to the theoretical solution (m)
Frequency of the harmonic exciting force $\omega_7 = 1.5 \cdot p_2$

Natural frequencies p, rad/s

Oscillation mode	**Theory**	**SCAD**	**Deviations, %**
1	40.000	40.000	0.00
2	113.137	113.137	0.00

Amplitude values of the deflections η in the cross-sections of the beam with the attached masses at the frequency of the harmonic exciting force $\omega_1 = 0.5 \cdot p_1$

Nodal mass	**Theory**		**SCAD**		
	Time, s	**Deflection, m**	**Time, s**	**Deflection, m**	**Deviations, %**
1	0.0715	0.012129	0.0754	0.011629	4.12
1	0.0161	-0.023034	0.1634	-0.022804	1.00
1	0.2497	0.013302	0.2514	0.013543	1.81
1	0.3230	0.001094	0.3268	0.001423	—
1	0.3714	0.012253	0.3770	0.012156	0.79
1	0.4700	-0.021344	0.4650	-0.021635	1.36
1	0.5714	0.012072	0.5718	0.011818	2.10
2	0.0621	0.012314	0.0628	0.012115	1.62
2	0.1515	-0.020119	0.1508	-0.020250	0.65
2	0.2412	0.010905	0.2451	0.011364	4.21
2	0.3099	-0.001817	0.3142	-0.001890	—
2	0.3845	0.012586	0.3896	0.011918	5.31
2	0.4716	-0.021331	0.4776	-0.021019	1.46
2	0.5586	0.012664	0.5656	0.012968	2.40

Amplitude values of the deflections η in the cross-sections of the beam
with the attached masses
at the frequency of the harmonic exciting force $\omega_2 = 0.95 \cdot p_1$

Nodal mass	**Theory**		**SCAD**		
	Time, s	**Deflection, m**	**Time, s**	**Deflection, m**	**Deviations, %**
1	0.0398	0.006829	0.0397	0.006657	2.52
1	0.1210	-0.020424	0.1223	-0.020227	0.96
1	0.2018	0.033848	0.2017	0.033642	0.61
1	0.2826	-0.046989	0.2843	-0.046665	0.69
2	0.0547	0.009260	0.0562	0.009000	2.81
2	0.1303	-0.020792	0.1289	-0.020352	2.12
2	0.2078	0.032790	0.2083	0.032331	1.40
2	0.2863	-0.044911	0.2876	-0.044388	1.16

Amplitude values of the deflections η in the cross-sections of the beam
with the attached masses
at the frequency of the harmonic exciting force $\omega_3 = 1.05 \cdot p_1$

Nodal mass	**Theory**		**SCAD**		
	Time, s	**Deflection, m**	**Time, s**	**Deflection, m**	**Deviations, %**
1	0.0374	0.006369	0.0389	0.006194	2.75
1	0.1161	-0.018850	0.1167	-0.018642	1.10
1	0.1938	0.030767	0.1945	0.030458	1.00
1	0.2710	-0.041915	0.2723	-0.041432	1.15
2	0.0534	0.008636	0.0539	0.008393	2.81
2	0.1249	-0.018428	0.1257	-0.018044	1.76
2	0.1959	0.029186	0.1945	0.028882	1.04
2	0.2695	-0.041172	0.2693	-0.041014	0.38

Amplitude values of the deflections η in the cross-sections of the beam
with the attached masses
at the frequency of the harmonic exciting force $\omega_4 = 0.5 \cdot (p_1 + p_2)$

Nodal mass	**Theory**		**SCAD**		
	Time, s	**Deflection, m**	**Time, s**	**Deflection, m**	**Deviations, %**
1	0.0267	0.003807	0.0263	0.003681	3.31
1	0.0631	-0.003795	0.0624	-0.003906	2.92
1	0.0859	-0.002215	0.0870	-0.002164	2.30
1	0.1016	-0.002783	0.1018	-0.002626	5.64
1	0.1387	0.004497	0.1396	0.004577	1.78
2	0.0440	0.004643	0.0443	0.004462	3.90
2	0.0823	-0.009649	0.0821	-0.009620	0.30
2	0.1207	0.005968	0.1251	0.006142	2.92
2	0.1579	-0.000288	0.1576	-0.000339	–

Amplitude values of the deflections η in the cross-sections of the beam
with the attached masses
at the frequency of the harmonic exciting force $\omega_5 = 0.95 \cdot p_2$

Nodal mass	**Theory**		**SCAD**		
	Time, s	**Deflection, m**	**Time, s**	**Deflection, m**	**Deviations, %**
1	0.0216	0.002575	0.0222	0.002477	3.81
1	0.0492	-0.003347	0.0491	-0.003410	1.88
1	0.0747	0.002938	0.0749	0.002889	1.67

Nodal mass	Theory		SCAD		
	Time, s	Deflection, m	Time, s	Deflection, m	Deviations, %
1	0.1018	-0.006366	0.1018	-0.006251	1.81
2	0.0383	0.002731	0.0386	0.002605	4.61
2	0.0700	-0.005629	0.0702	-0.005627	0.04
2	0.0991	0.005222	0.0995	0.005254	0.61

Amplitude values of the deflections η in the cross-sections of the beam
with the attached masses
at the frequency of the harmonic exciting force $\omega_6 = 1.05 \cdot p_2$

Nodal mass	Theory		SCAD		
	Time, s	Deflection, m	Time, s	Deflection, m	Deviations, %
1	0.0203	0.002260	0.0201	0.002174	3.81
1	0.0460	-0.003028	0.0466	-0.003076	1.59
1	0.0705	0.003206	0.0709	0.003142	2.00
1	0.0967	-0.006393	0.0963	-0.006294	1.55
2	0.0366	0.002273	0.0370	0.002159	5.02
2	0.0667	-0.004473	0.0667	-0.004479	0.13
2	0.0942	0.004097	0.0942	0.004112	0.37

Amplitude values of the deflections η in the cross-sections of the beam
with the attached masses
at the frequency of the harmonic exciting force $\omega_7 = 1.5 \cdot p_2$

Nodal mass	Theory		SCAD		
	Time, s	Deflection, m	Time, s	Deflection, m	Deviations, %
1	0.0157	0.001346	0.0155	0.001289	4.23
1	0.0356	-0.001671	0.0355	-0.001709	2.27
1	0.0552	0.001788	0.0555	0.001733	3.08
2	0.0308	0.001072	0.0303	0.001006	6.16
2	0.0563	-0.001420	0.0562	-0.001459	2.75

Notes: In the analytical solution the natural frequencies of oscillations *p* of the simply supported beam are determined according to the following formulas:

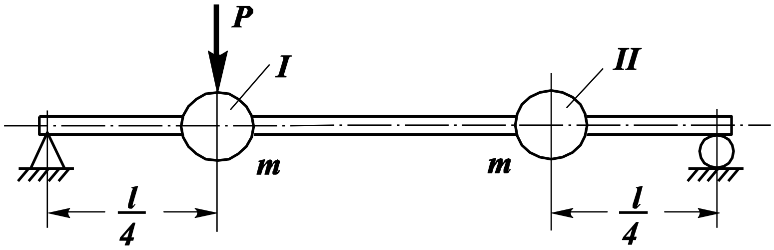
$$p_1 = \sqrt{\frac{48 \cdot E \cdot I}{m \cdot l^3}}; \qquad p_2 = \sqrt{\frac{384 \cdot E \cdot I}{m \cdot l^3}}.$$

In the analytical solution the deflections η in the cross-sections of the beam with the attached masses with time of the simply supported beam are determined according to the following formulas:

$$\eta_1(t) = \frac{P_0 \cdot l^3}{768 \cdot E \cdot I} \cdot \left[\left(\frac{8}{1 - \frac{\omega^2}{p_1^2}} + \frac{1}{1 - \frac{\omega^2}{p_2^2}} \right) \cdot cos(\omega \cdot t) - \left(\frac{8}{1 - \frac{\omega^2}{p_1^2}} \cdot cos(p_1 \cdot t) + \frac{1}{1 - \frac{\omega^2}{p_2^2}} \cdot cos(p_2 \cdot t) \right) \right];$$

$$\eta_2(t) = \frac{P_0 \cdot l^3}{768 \cdot E \cdot I} \cdot \left[\left(\frac{8}{1 - \frac{\omega^2}{p_1^2}} - \frac{1}{1 - \frac{\omega^2}{p_2^2}} \right) \cdot cos(\omega \cdot t) - \left(\frac{8}{1 - \frac{\omega^2}{p_1^2}} \cdot cos(p_1 \cdot t) - \frac{1}{1 - \frac{\omega^2}{p_2^2}} \cdot cos(p_2 \cdot t) \right) \right].$$

Simply Supported Weightless Beam with Two Concentrated Masses and Transverse Harmonic Exciting Force Applied to One of Them Taking into Account the Energy Dissipation due to Internal Friction



Objective: Determination of the strain state of a simply supported weightless beam with two concentrated masses subjected to a transverse harmonic exciting force applied to one of them taking into account the energy dissipation due to internal friction.

Initial data files: **5.12_Harm_L_Damp.SPR**
График_5.12_Harm_L_Forc_Freq_1.txt
График_5.12_Harm_L_Forc_Freq_2.txt
График_5.12_Harm_L_Forc_Freq_3.txt
График_5.12_Harm_L_Forc_Freq_4.txt
График_5.12_Harm_L_Forc_Freq_5.txt
График_5.12_Harm_L_Forc_Freq_6.txt
График_5.12_Harm_L_Forc_Freq_7.txt

Problem formulation: Two identical loads of mass *m* are attached to the simply supported beam of constant cross-section at a quarter span distance from each support. The mass of the beam is neglected in comparison with the masses of the loads. The force P_0 is applied to one of the masses at the initial time and varies harmonically with the frequency ω. Determine the natural oscillation modes and natural frequencies *p* of the simply supported beam, as well as the deflections η in the cross-sections of the beam with the attached masses with time taking into account the energy dissipation due to internal friction.

References: S.D. Ponomarev, V.L. Biederman, K.K. Likharev, V.M. Makushin, N.N. Malinin, V.I. Feodos'yev, Fundamentals of Modern Methods for Strength Analysis in Mechanical Engineering. Dynamic Analysis. Stability. Creep. Moscow, Mashgiz, 1952, p. 153.

Initial data:

$E = 3.0 \cdot 10^6$ tf/m^2	- elastic modulus;
$\nu = 0.2$	- Poisson's ratio;
b = 0.4 m	- width of the rectangular cross-section of the beam;
h = 0.8 m	- height of the rectangular cross-section of the beam;
l = 8.0 m	- beam span length;
m = 3.0 tf·s^2/m	- value of the concentrated masses attached to the beam;
P_0 = 76.8 tf	- amplitude value of the harmonic exciting force applied to one of the masses;
g = 10.00 m/s^2	- gravitational acceleration;
$I = b \cdot h^3/12 = 0.017067$	- cross-sectional moment of inertia of the beam.

The following values of frequencies of the harmonic exciting force ω_i depending on the values of natural frequencies of the beam p_i are considered:

$\omega_j = 0.5 \cdot p_1;\ 0.95 \cdot p_1;\ 1.05 \cdot p_1;\ 0.5 \cdot (p_1 + p_2);\ 0.95 \cdot p_2;\ 1.05 \cdot p_2;\ 1.5 \cdot p_2.$

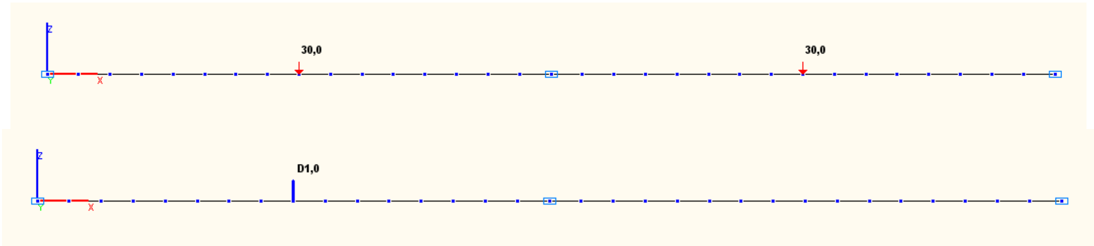
Critical damping ratios for the 1-st and 2-nd natural frequencies are taken with the maximum value:

$\xi_{1,2} = 0.9999.$

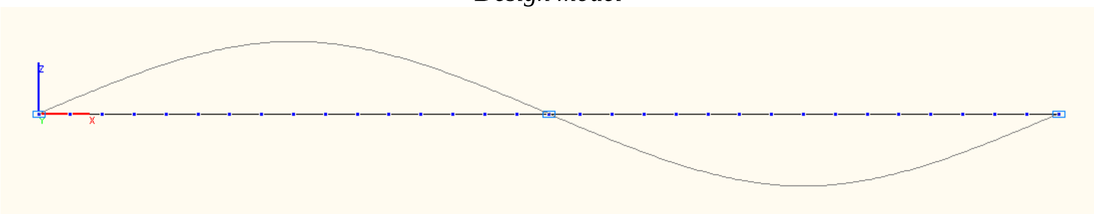
Finite element model: Design model – plane frame, 32 bar elements of type 2. Boundary conditions of the simply supported ends of the beam are provided by imposing constraints in the direction of the degree of freedom Z. The dimensional stability of the design model is provided by imposing a constraint in the node of the cross-section along the symmetry axis of the beam in the direction of the degree of freedom X. The concentrated masses are specified by transforming the static nodal loads $m \cdot g$.

The calculation is performed in two stages: first the natural oscillation modes and natural frequencies p are determined by the modal analysis, and then the deflections η in the cross-sections of the beam with the attached masses with time are determined by the direct integration of the equations of motion method. The action of the transverse harmonic exciting force is described by the graph of the load variation with time and is given in the form of a nodal force acting along the Z axis of the global coordinate system with the scale factor of 1.0 and the delay time 0.0 s. Intervals between the time points of the load variation graph are equal to $\Delta t_{int} = T_j/100$, where T_j – period of the harmonic exciting force, and correspond to the integration step. When plotting the graph, the action of the transverse harmonic exciting force is taken as $P_n = P_0 \cdot \cos(\omega_j \cdot n \cdot \Delta t_{int})$ at the time points $n \cdot \Delta t_{int}$. The duration of the process is equal to $t = 2 \cdot T_j$. The conversion factor for the added static loading is equal to k = 0.981 (mass generation). Number of nodes in the design model – 33. The modal integration method is used in the calculation. The determination of the natural oscillation modes and natural frequencies is performed by the method of subspace iteration. The matrix of concentrated masses is used in the calculation.

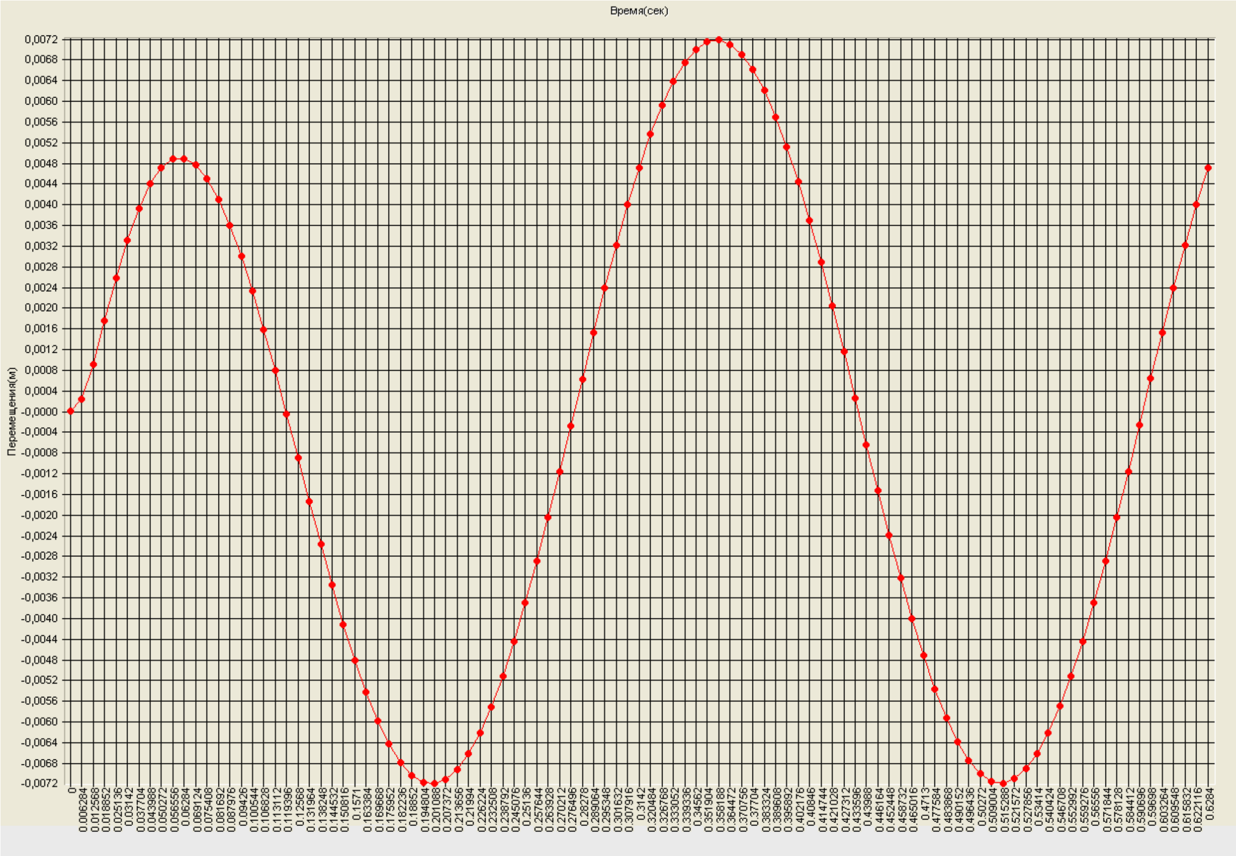
Results in SCAD



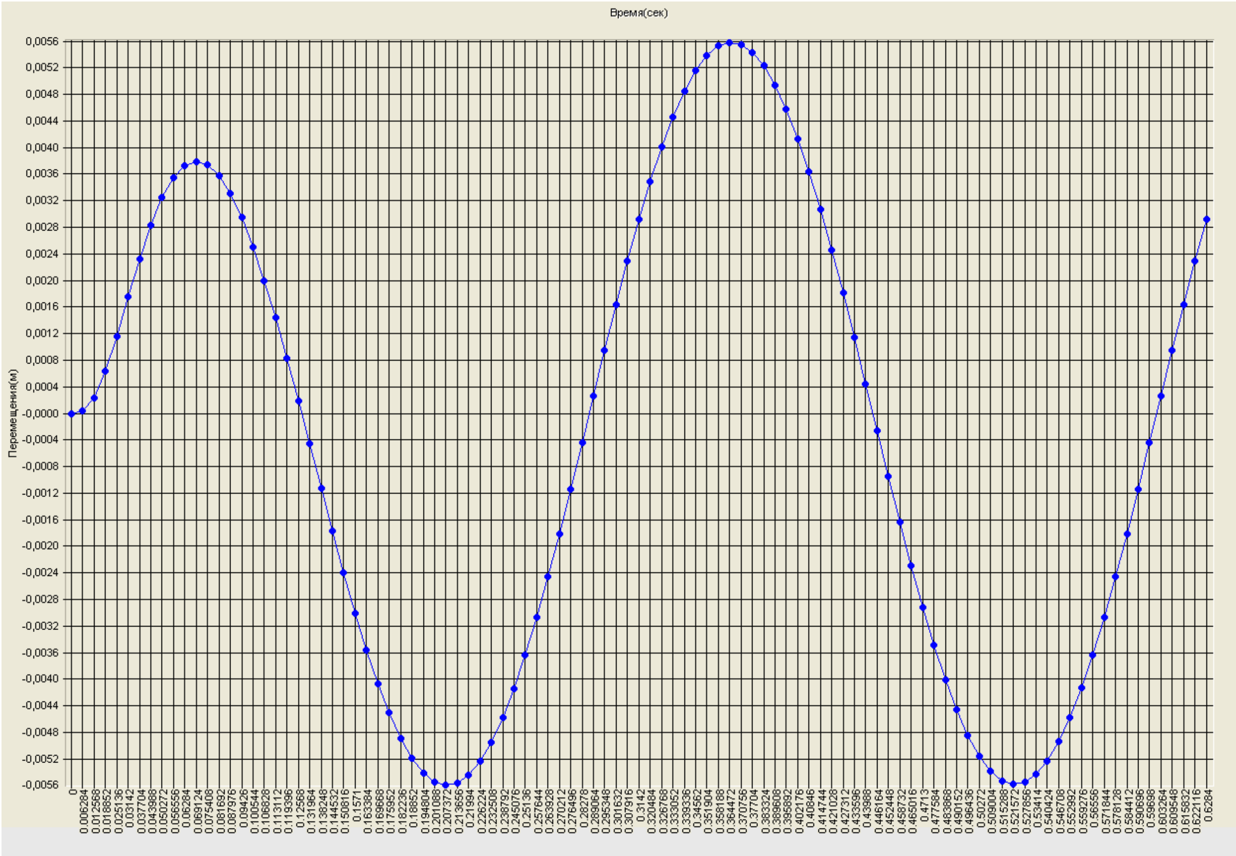
Design model



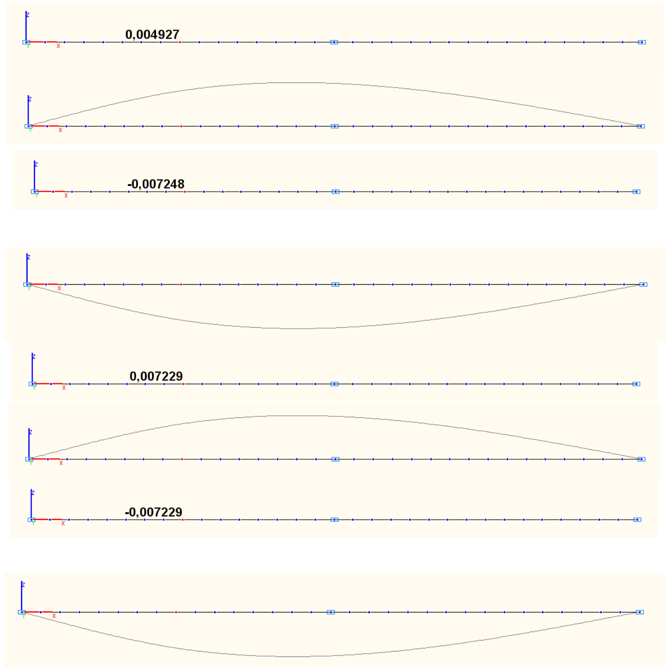
1-st and 2-nd natural oscillation modes



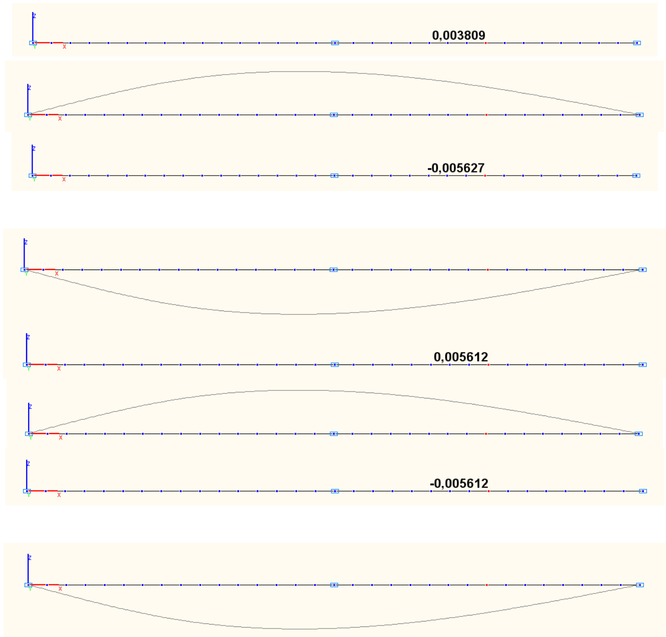
Graph of the variation of the deflection η_1 in the cross-section of the beam with the attached mass subjected to the shear force, with time (m).
Frequency of the harmonic exciting force $\omega_1 = 0.5 \cdot p_1$



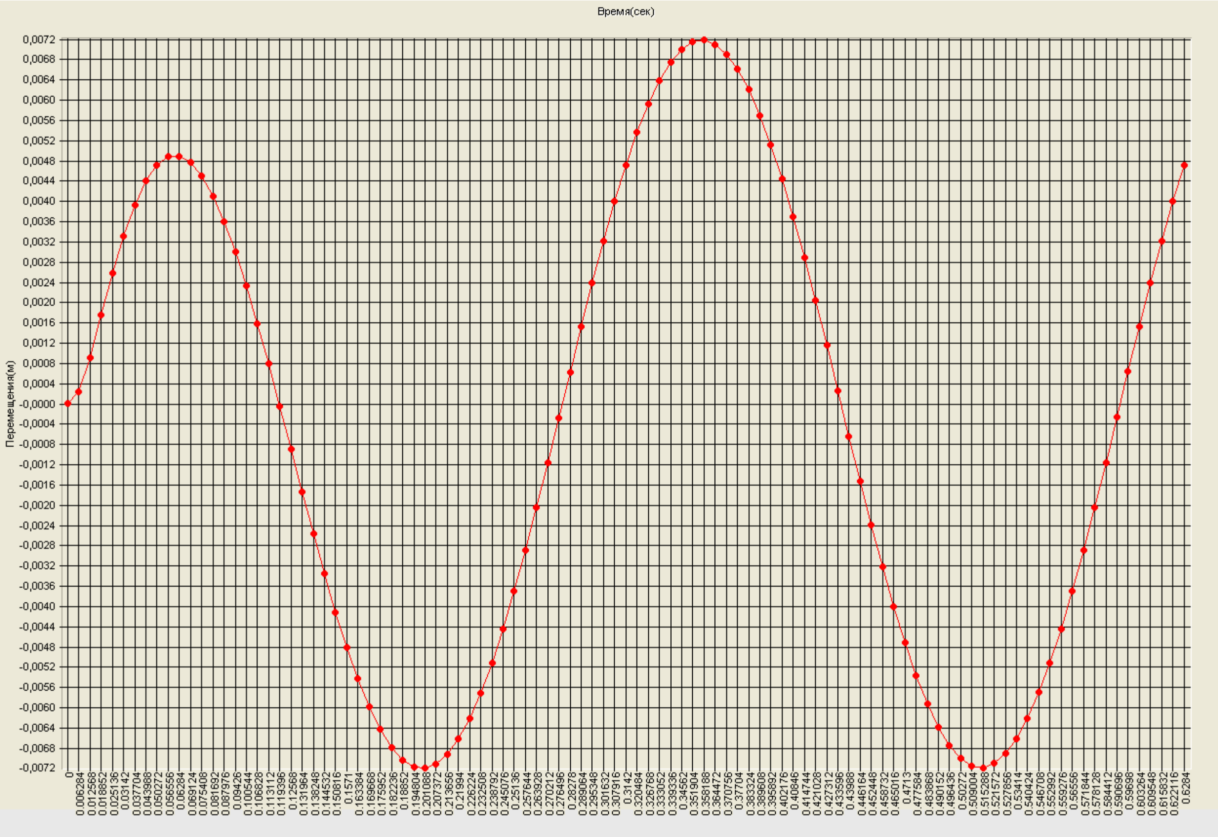
Graph of the variation of the deflection η_2 in the cross-section of the beam with the attached mass not subjected to the shear force, with time (m).
Frequency of the harmonic exciting force $\omega_1 = 0.5 \cdot p_1$



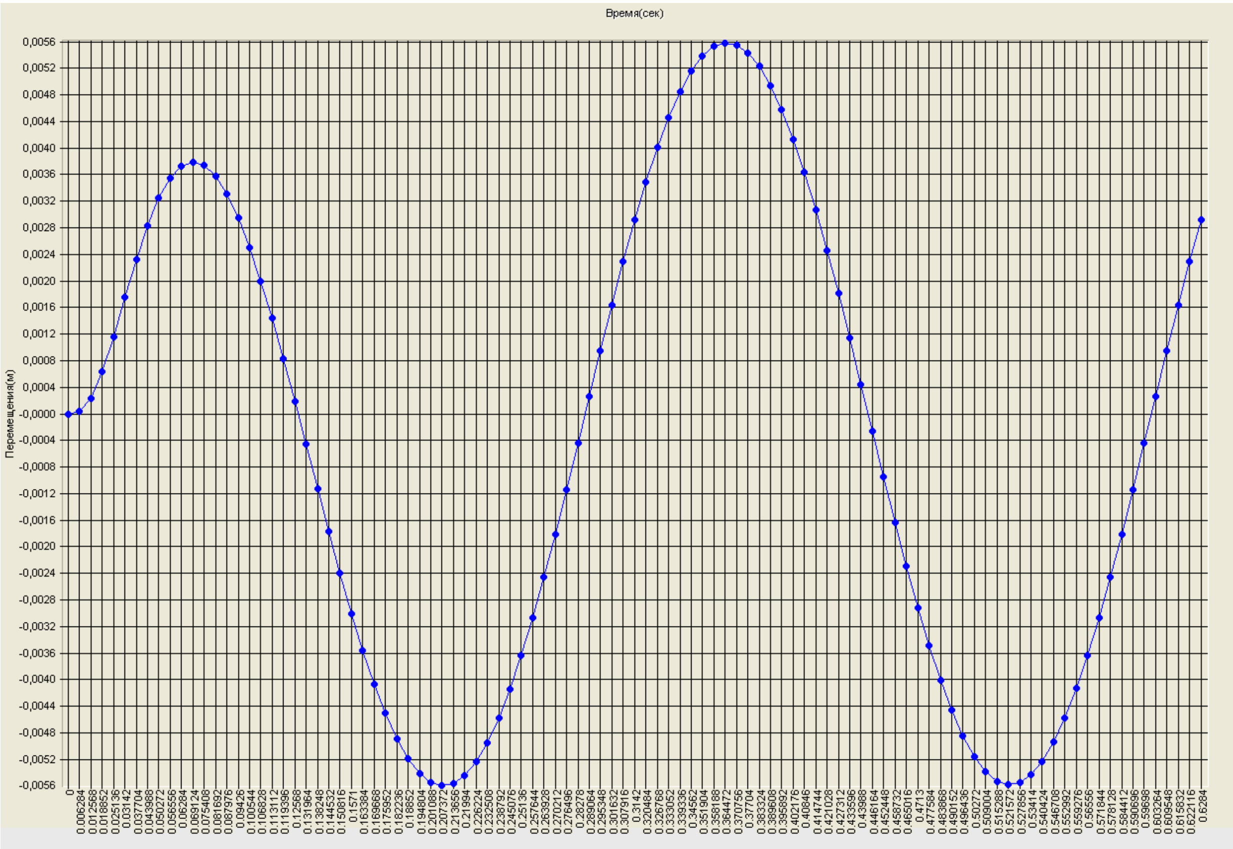
Amplitude values of the deflection η_1 in the cross-section of the beam
with the attached mass subjected to the shear force
and the deformed models at the respective time points (m).
Frequency of the harmonic exciting force $\omega_1 = 0.5 \cdot p_1$



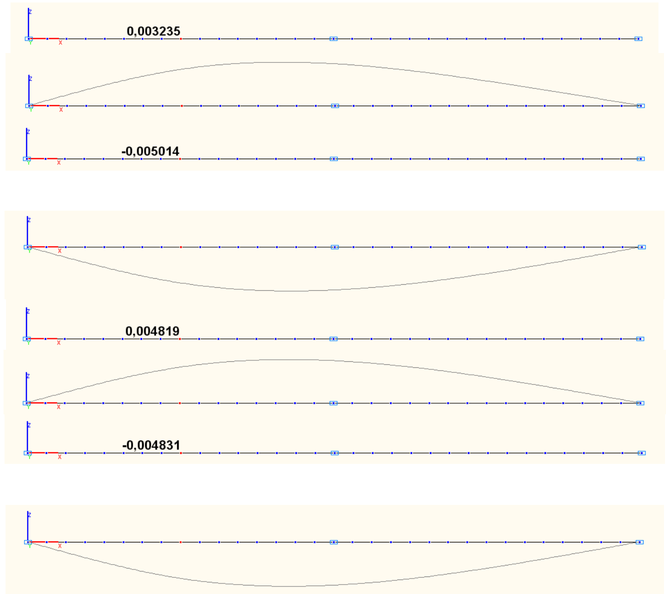
Amplitude values of the deflection η_2 in the cross-section of the beam
with the attached mass not subjected to the shear force
and the deformed models at the respective time points (m).
Frequency of the harmonic exciting force $\omega_1 = 0.5 \cdot p_1$



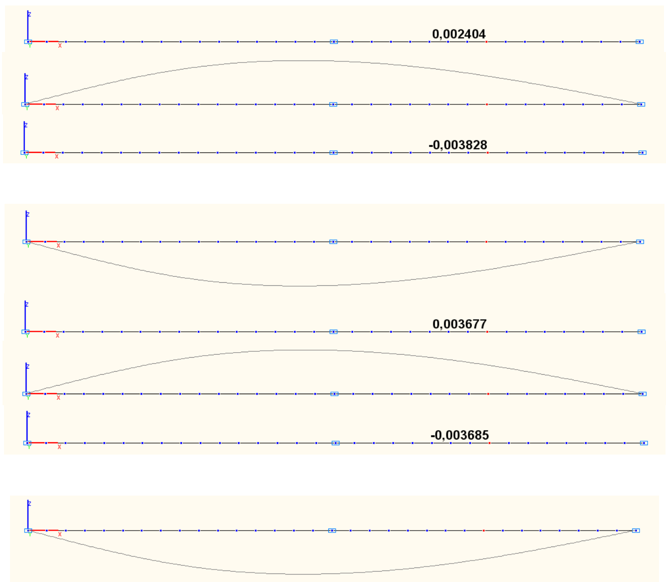
Amplitude value of the deflection η_1 in the cross-section of the beam with the attached mass subjected to the shear force and the deformed model at the respective time point (m). Frequency of the harmonic exciting force $\omega_2 = 0.95 \cdot p_1$



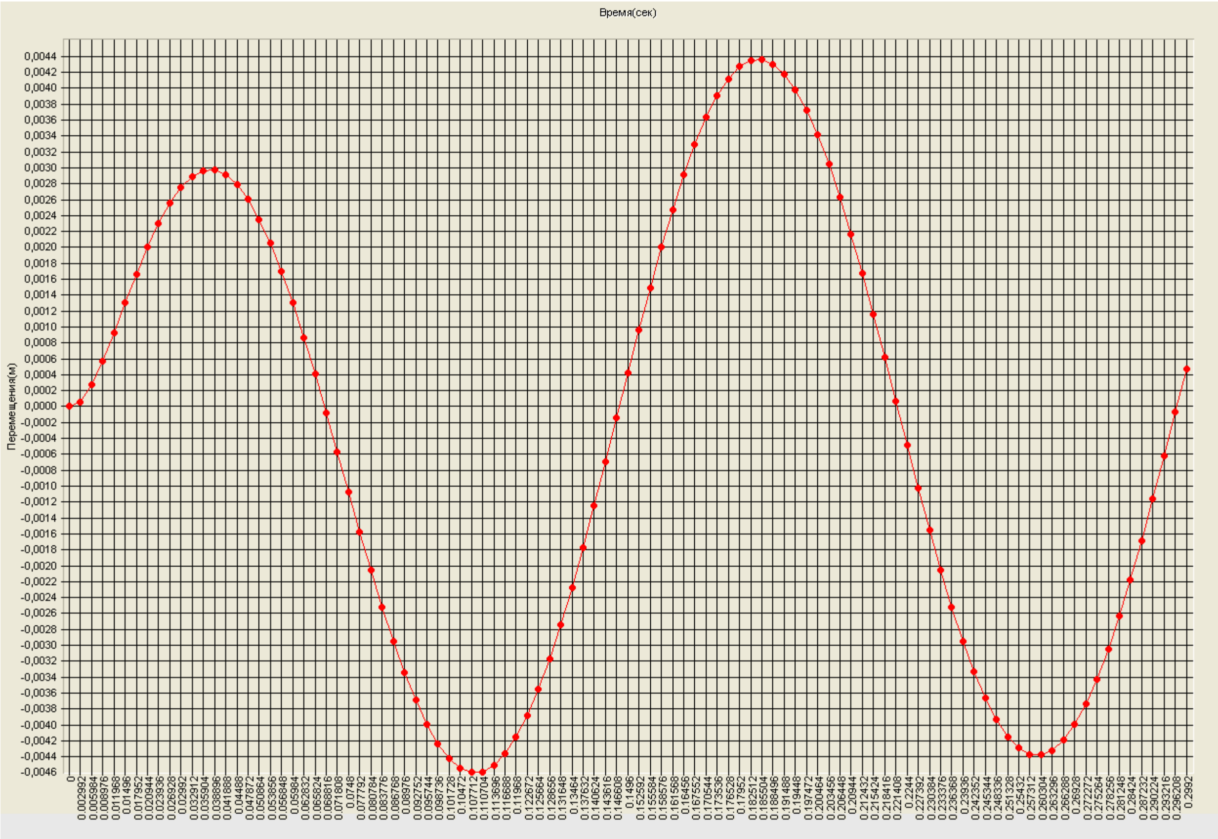
Graph of the variation of the deflection η_2 in the cross-section of the beam with the attached mass not subjected to the shear force, with time (m). Frequency of the harmonic exciting force $\omega_2 = 0.95 \cdot p_1$



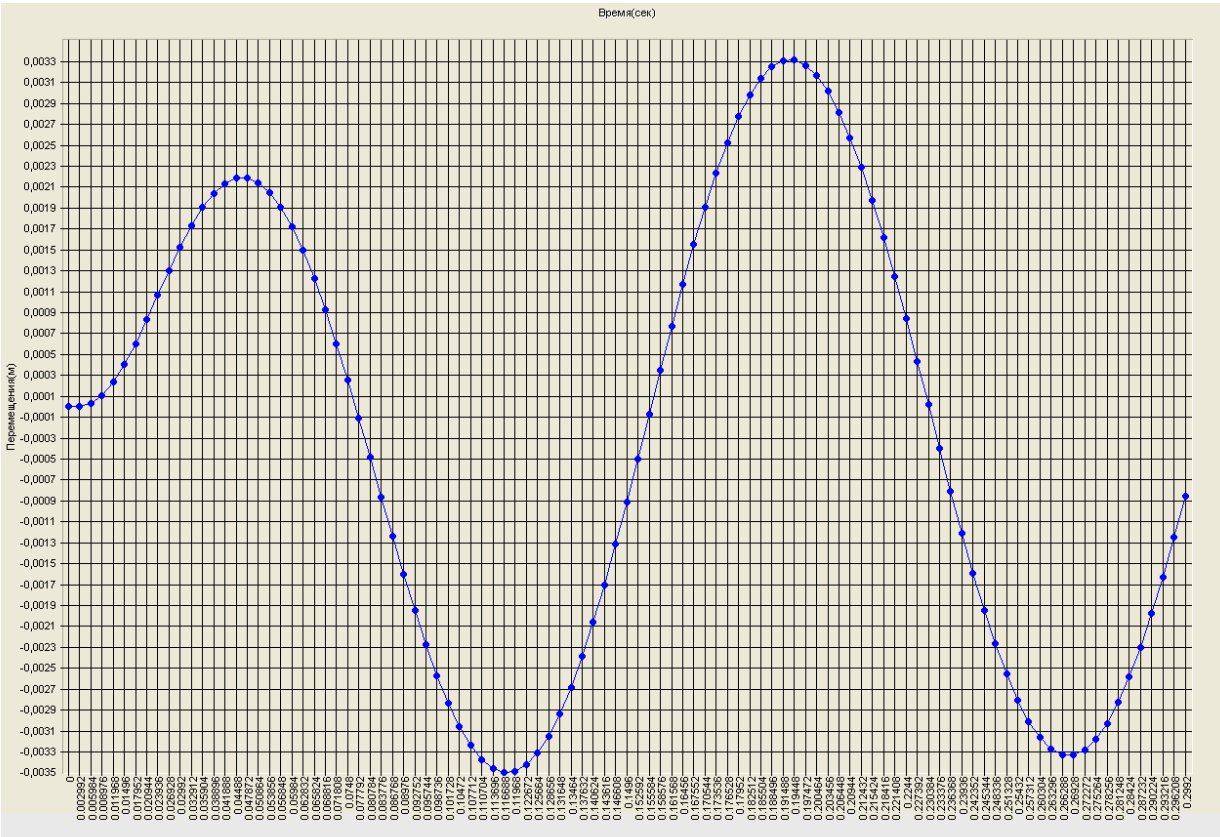
Amplitude values of the deflection η_1 in the cross-section of the beam with the attached mass subjected to the shear force and the deformed models at the respective time points (m). Frequency of the harmonic exciting force $\omega_2 = 0.95 \cdot p_1$



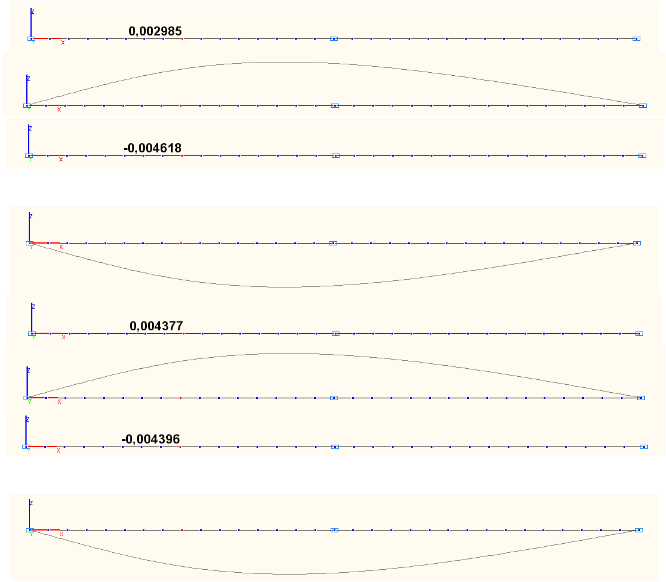
Amplitude values of the deflection η_2 in the cross-section of the beam with the attached mass not subjected to the shear force and the deformed models at the respective time points (m). Frequency of the harmonic exciting force $\omega_2 = 0.95 \cdot p_1$



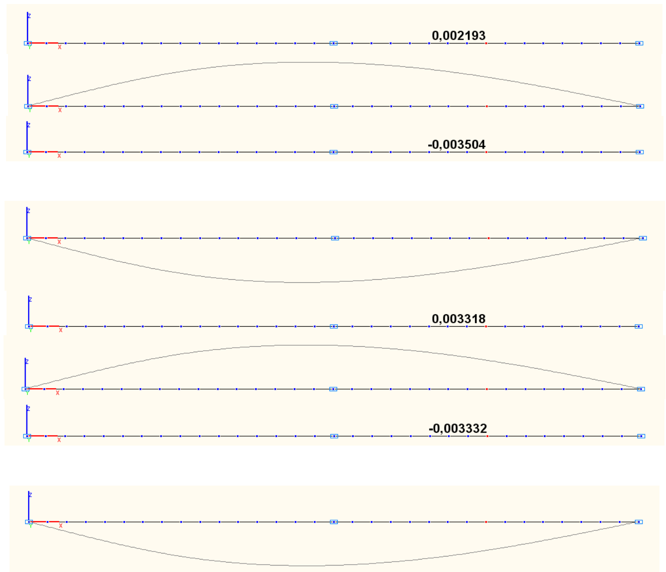
Amplitude value of the deflection η_1 in the cross-section of the beam with the attached mass subjected to the shear force and the deformed model at the respective time point (m). Frequency of the harmonic exciting force $\omega_3 = 1.05 \cdot p_1$



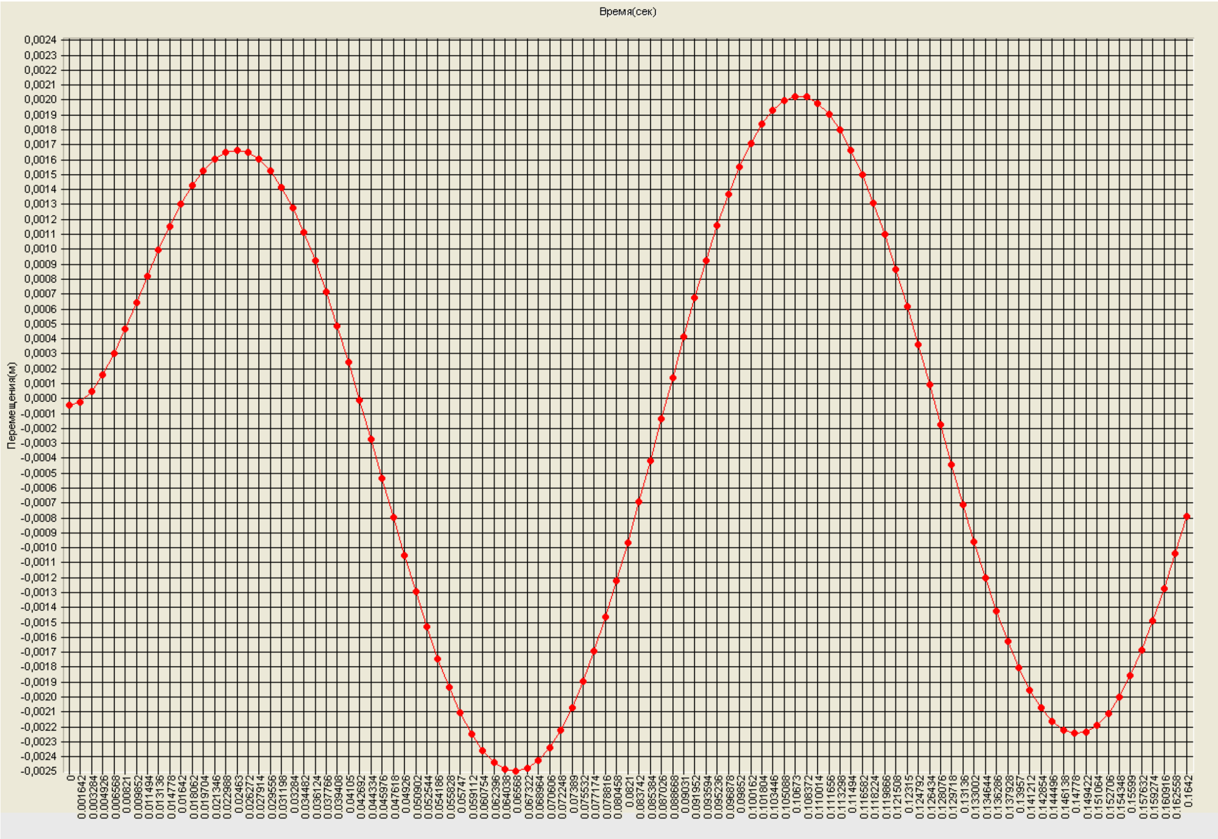
Graph of the variation of the deflection η_2 in the cross-section of the beam with the attached mass not subjected to the shear force, with time (m). Frequency of the harmonic exciting force $\omega_3 = 1.05 \cdot p_1$



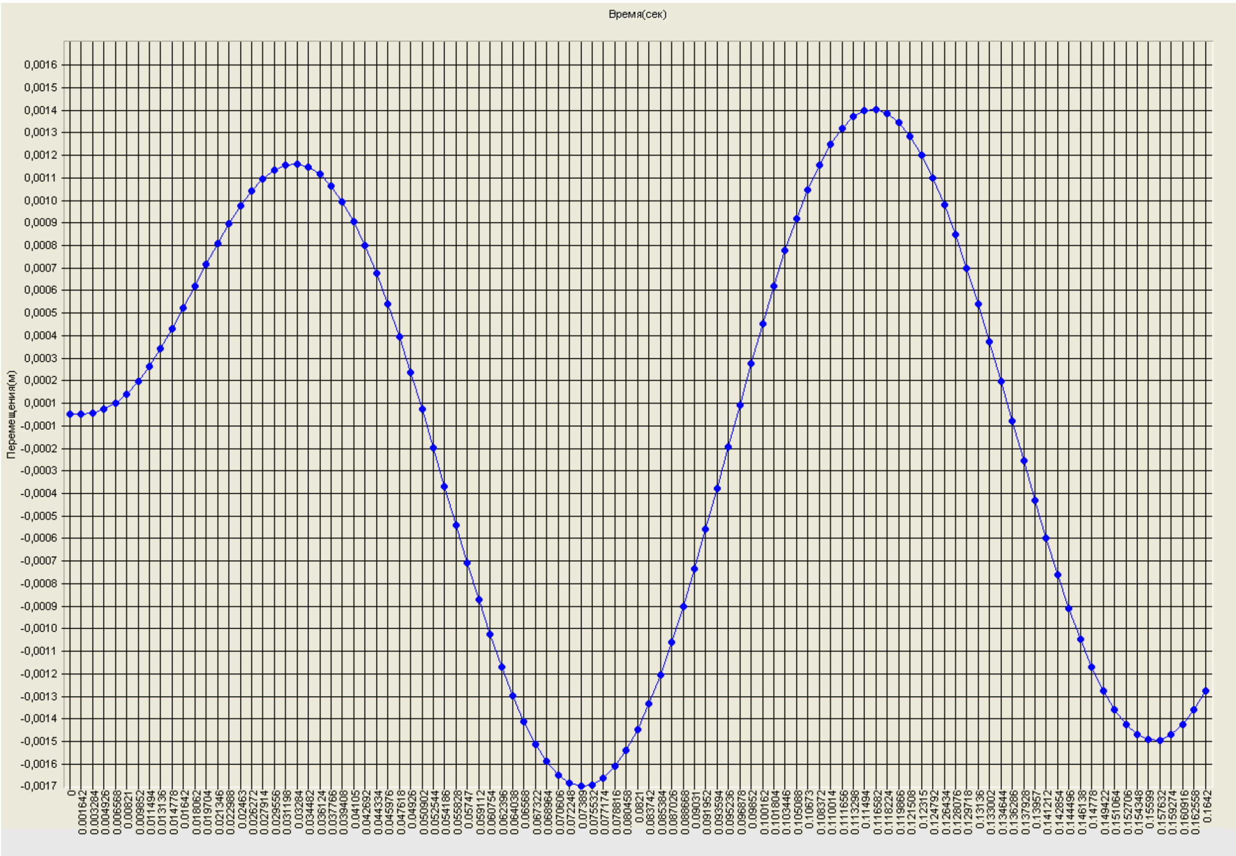
Amplitude values of the deflection η_1 in the cross-section of the beam with the attached mass subjected to the shear force and the deformed models at the respective time points (m). Frequency of the harmonic exciting force $\omega_3 = 1.05 \cdot p_1$



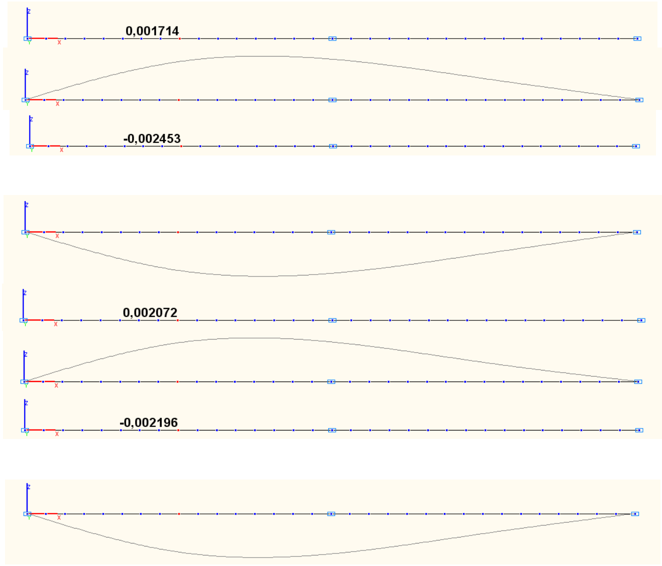
Amplitude values of the deflection η_2 in the cross-section of the beam with the attached mass not subjected to the shear force and the deformed models at the respective time points (m). Frequency of the harmonic exciting force $\omega_3 = 1.05 \cdot p_1$



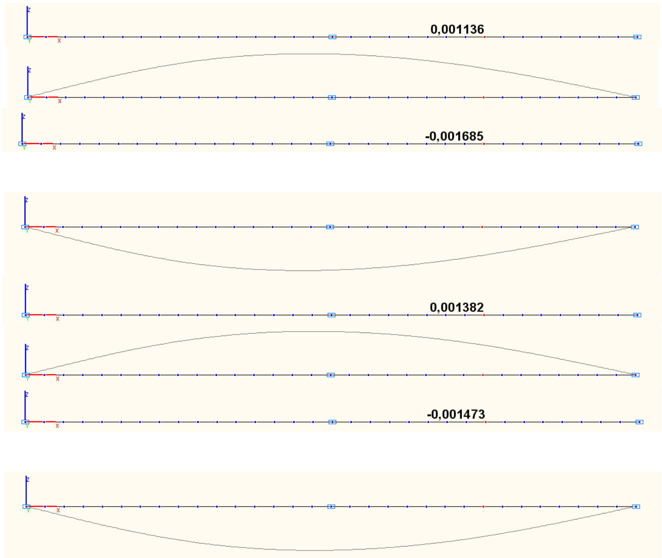
Amplitude value of the deflection η_1 in the cross-section of the beam with the attached mass subjected to the shear force and the deformed model at the respective time point (m). Frequency of the harmonic exciting force $\omega_4 = 0.5 \cdot (p_1 + p_2)$



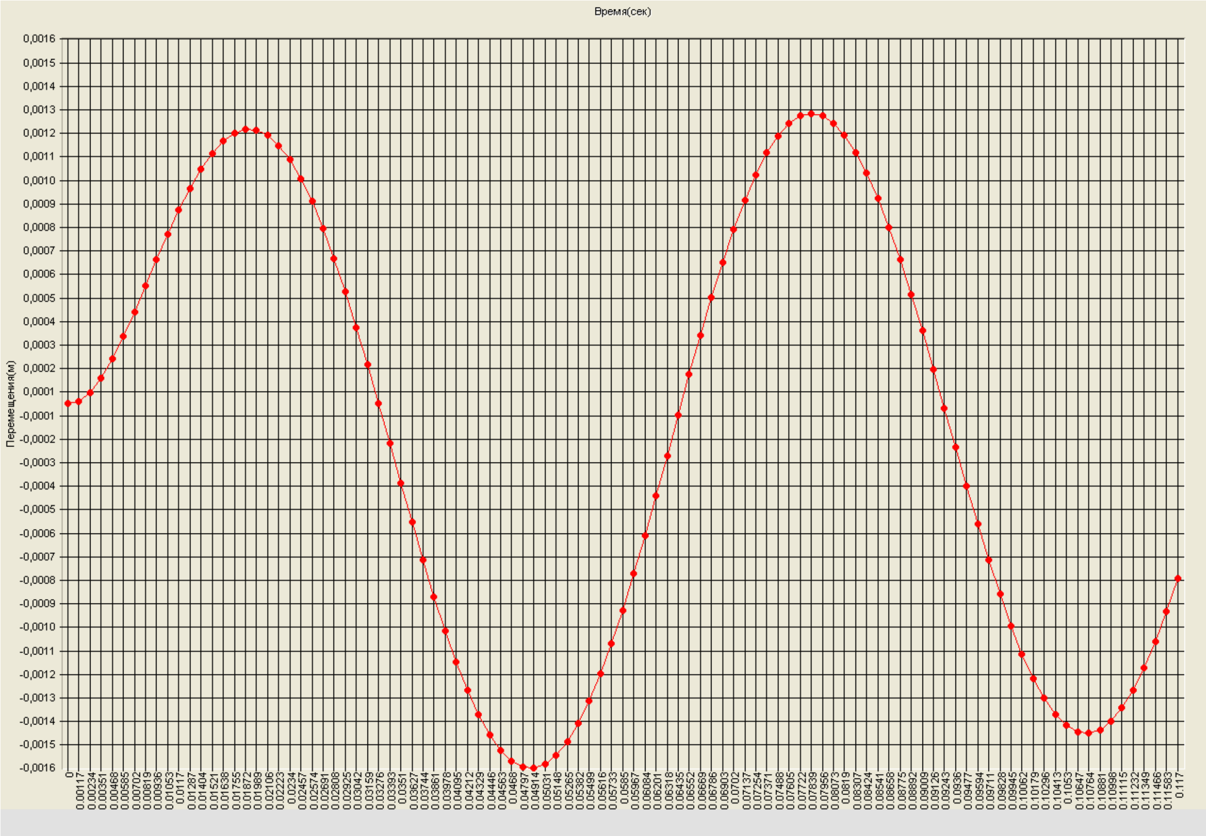
Graph of the variation of the deflection η_2 in the cross-section of the beam with the attached mass not subjected to the shear force, with time (m). Frequency of the harmonic exciting force $\omega_4 = 0.5 \cdot (p_1 + p_2)$



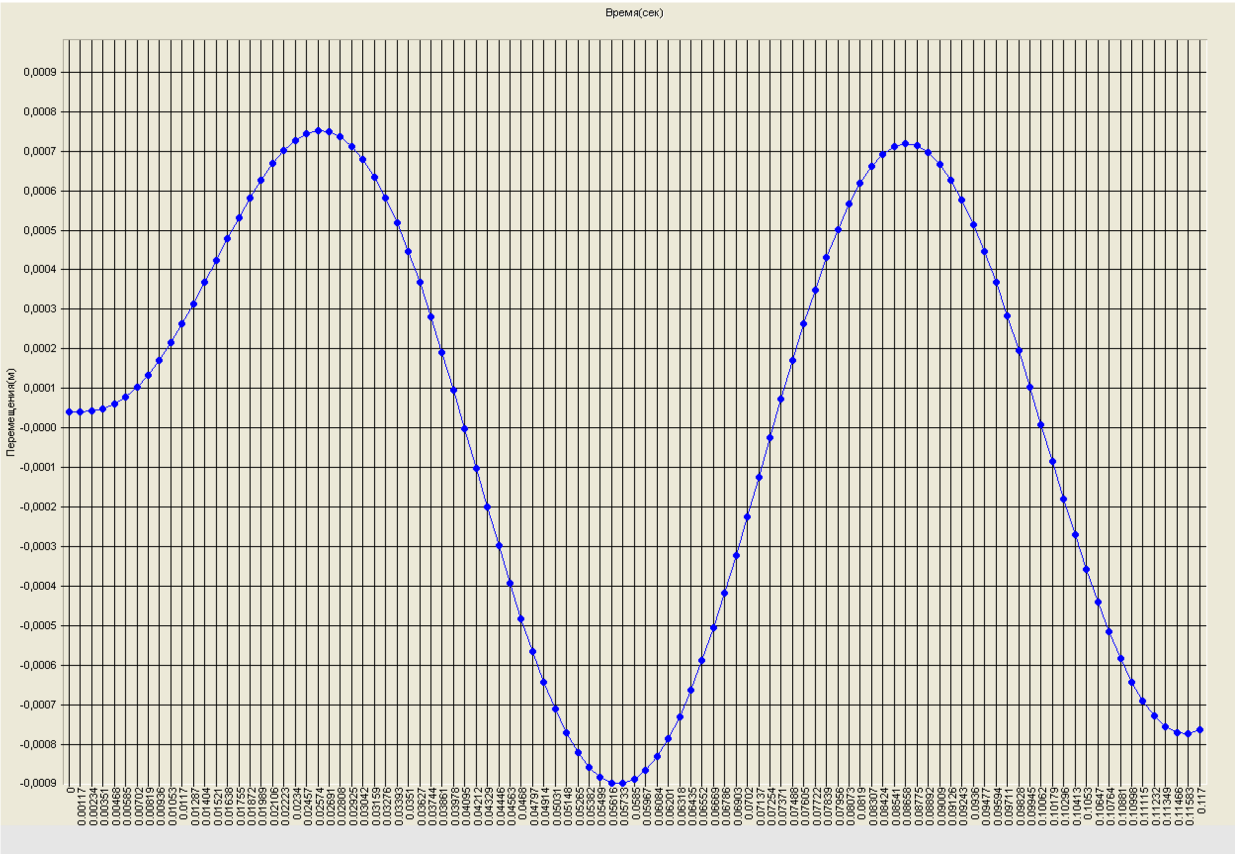
Amplitude values of the deflection η_1 in the cross-section of the beam with the attached mass subjected to the shear force and the deformed models at the respective time points (m). Frequency of the harmonic exciting force $\omega_4 = 0.5 \cdot (p_1 + p_2)$



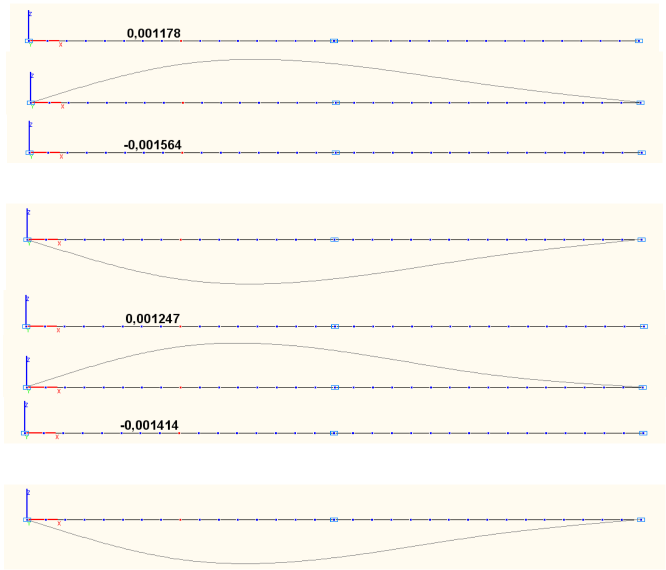
Amplitude values of the deflection η_2 in the cross-section of the beam with the attached mass not subjected to the shear force and the deformed models at the respective time points (m). Frequency of the harmonic exciting force $\omega_4 = 0.5 \cdot (p_1 + p_2)$



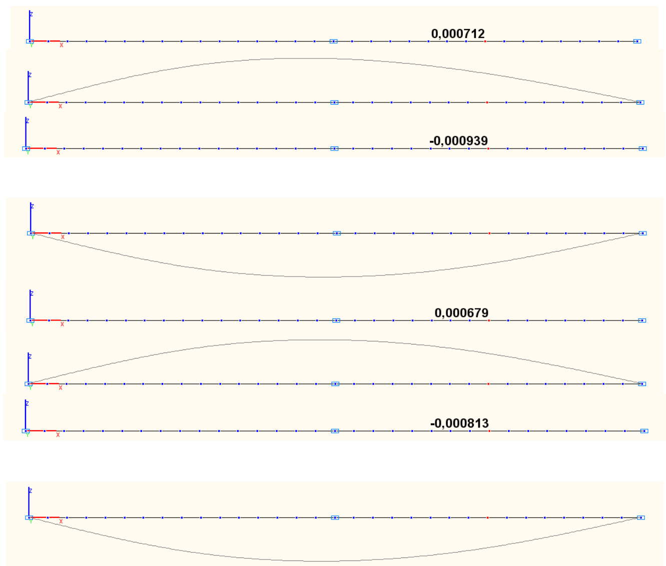
Amplitude value of the deflection η_1 in the cross-section of the beam with the attached mass subjected to the shear force and the deformed model at the respective time point (m). Frequency of the harmonic exciting force $\omega_5 = 0.95 \cdot p_2$



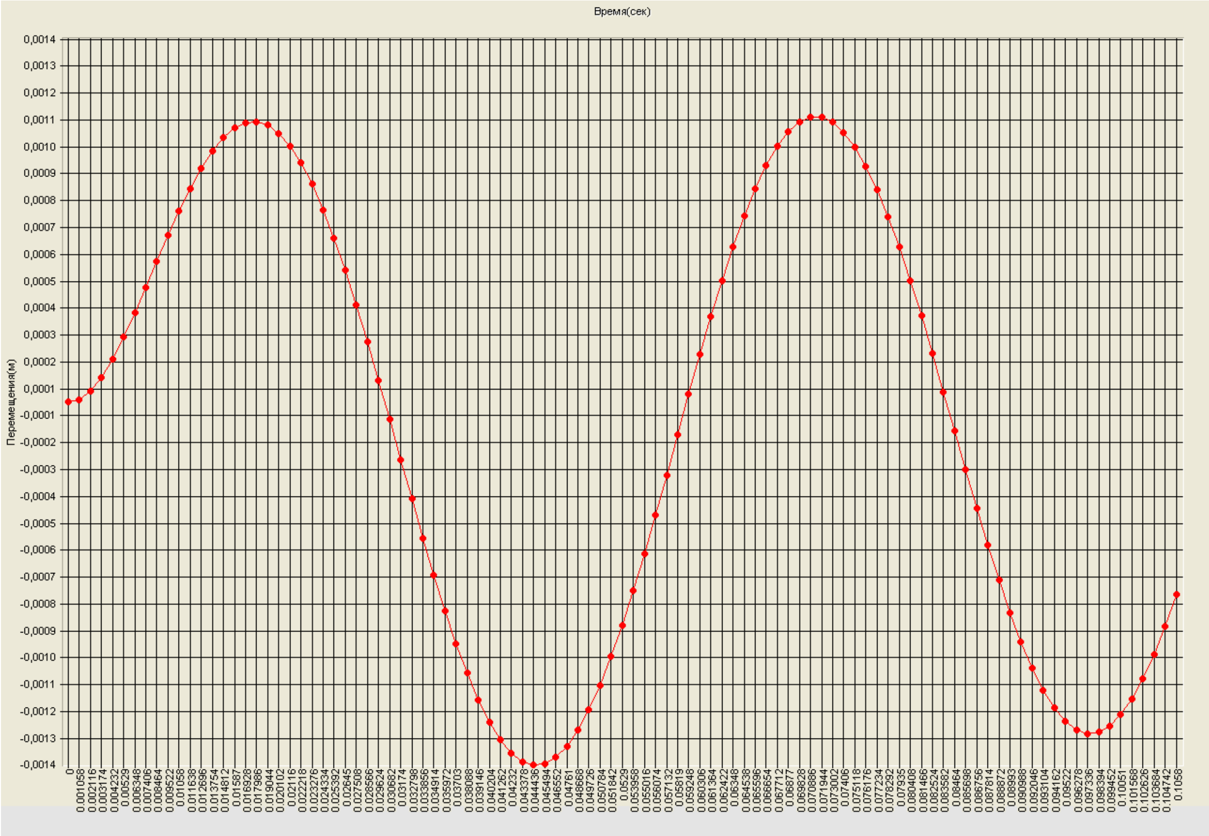
Graph of the variation of the deflection η_2 in the cross-section of the beam with the attached mass not subjected to the shear force, with time (m). Frequency of the harmonic exciting force $\omega_5 = 0.95 \cdot p_2$



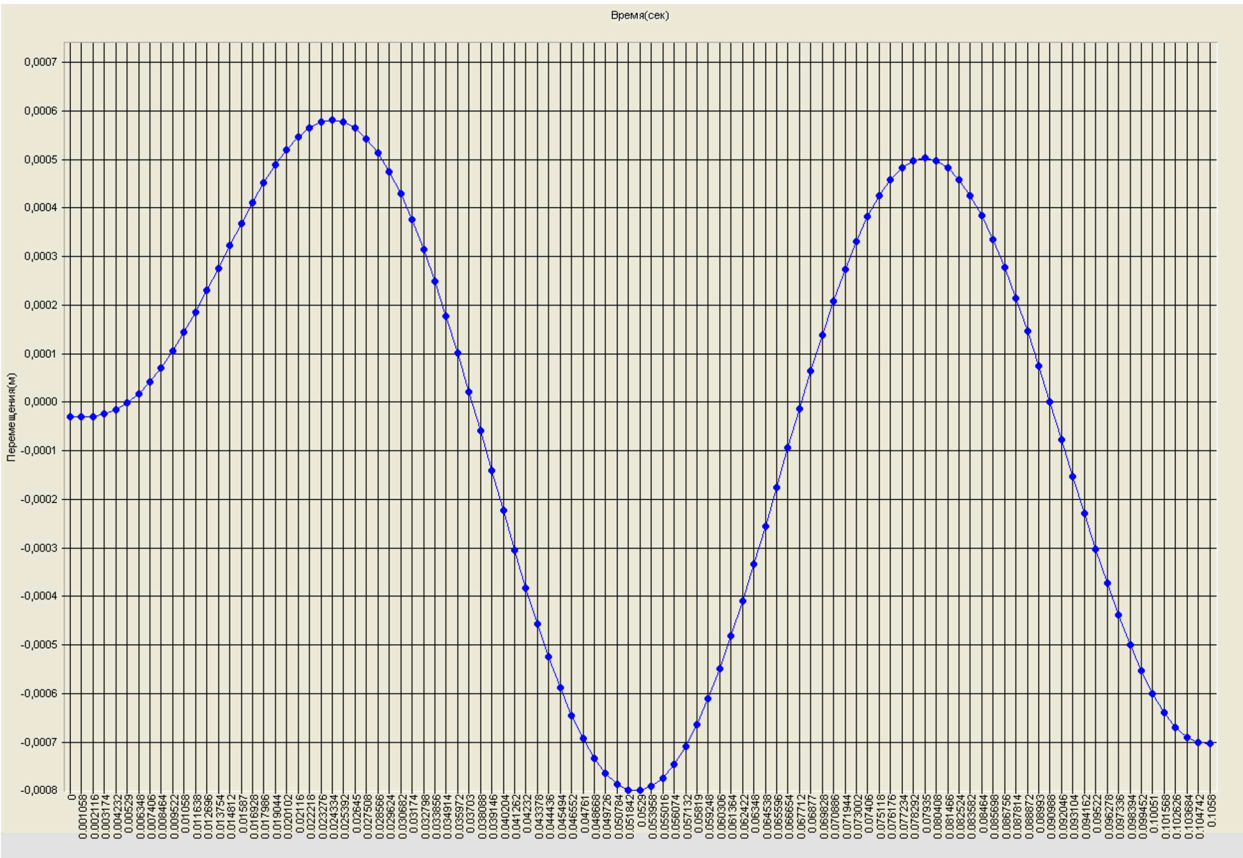
Amplitude values of the deflection η_1 in the cross-section of the beam with the attached mass subjected to the shear force and the deformed models at the respective time points (m). Frequency of the harmonic exciting force $\omega_5 = 0.95 \cdot p_2$



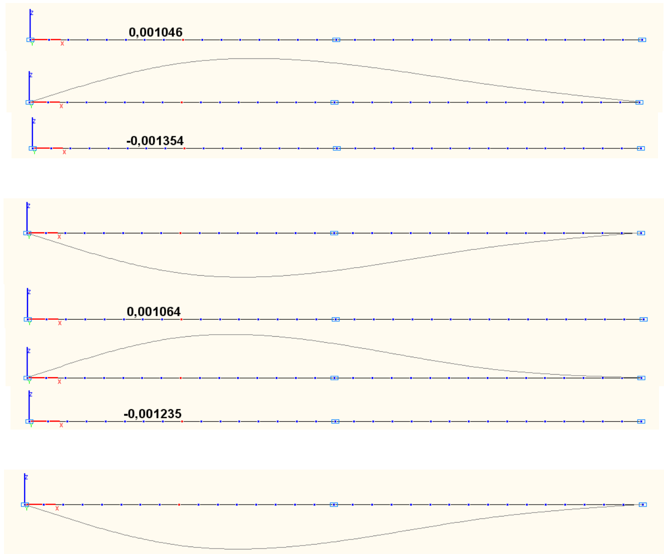
Amplitude values of the deflection η_2 in the cross-section of the beam with the attached mass not subjected to the shear force and the deformed models at the respective time points (m). Frequency of the harmonic exciting force $\omega_5 = 0.95 \cdot p_2$



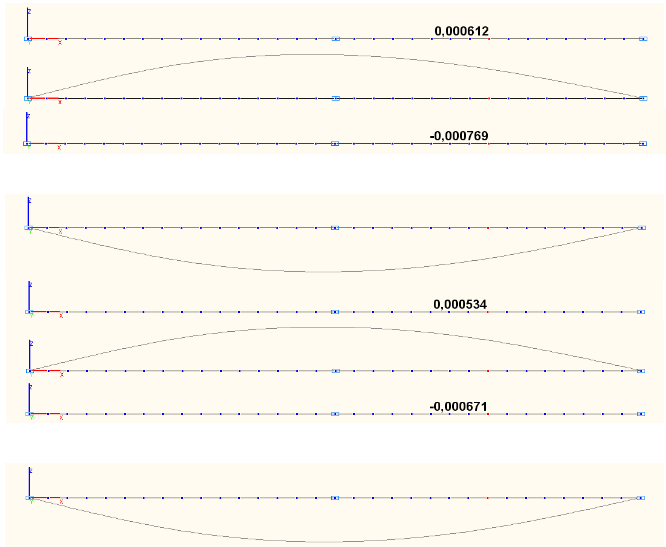
Amplitude value of the deflection η_1 in the cross-section of the beam with the attached mass subjected to the shear force and the deformed model at the respective time point (m). Frequency of the harmonic exciting force $\omega_6 = 1.05 \cdot p_2$



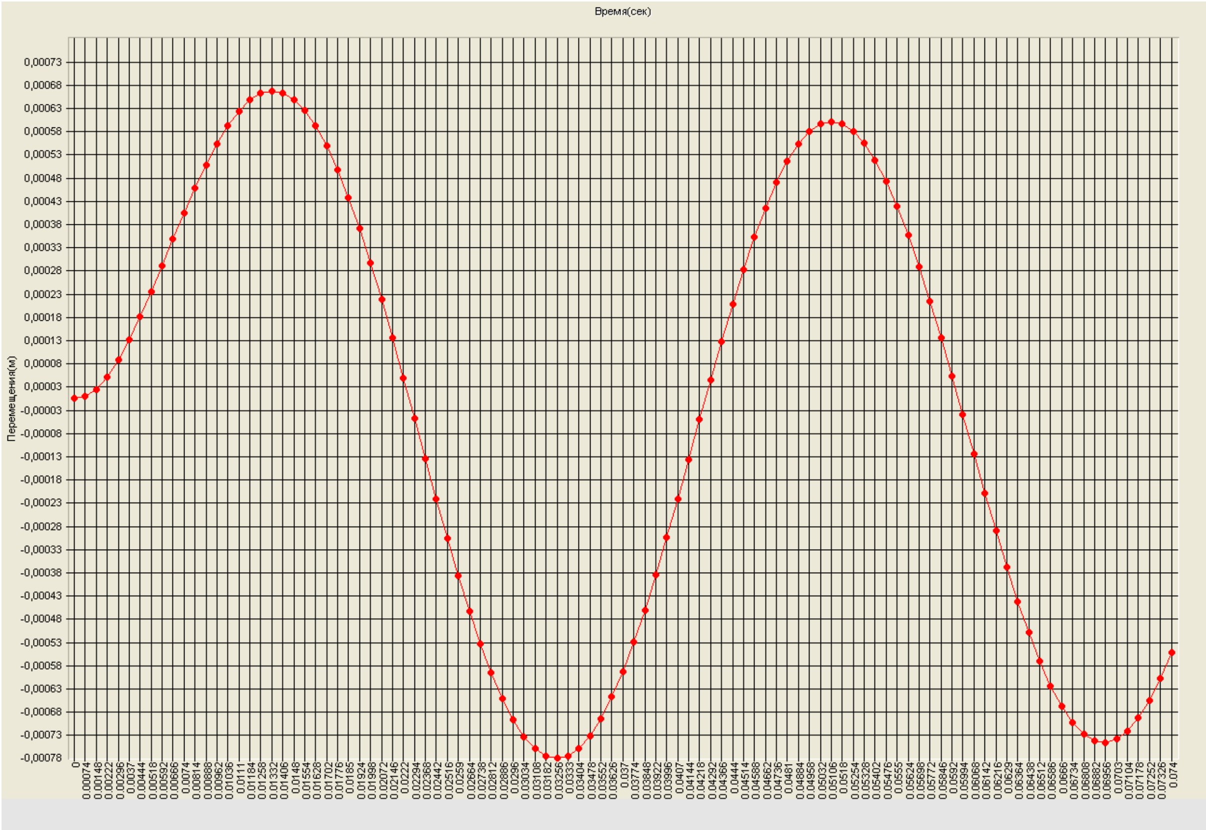
Graph of the variation of the deflection η_2 in the cross-section of the beam with the attached mass not subjected to the shear force, with time (m). Frequency of the harmonic exciting force $\omega_6 = 1.05 \cdot p_2$



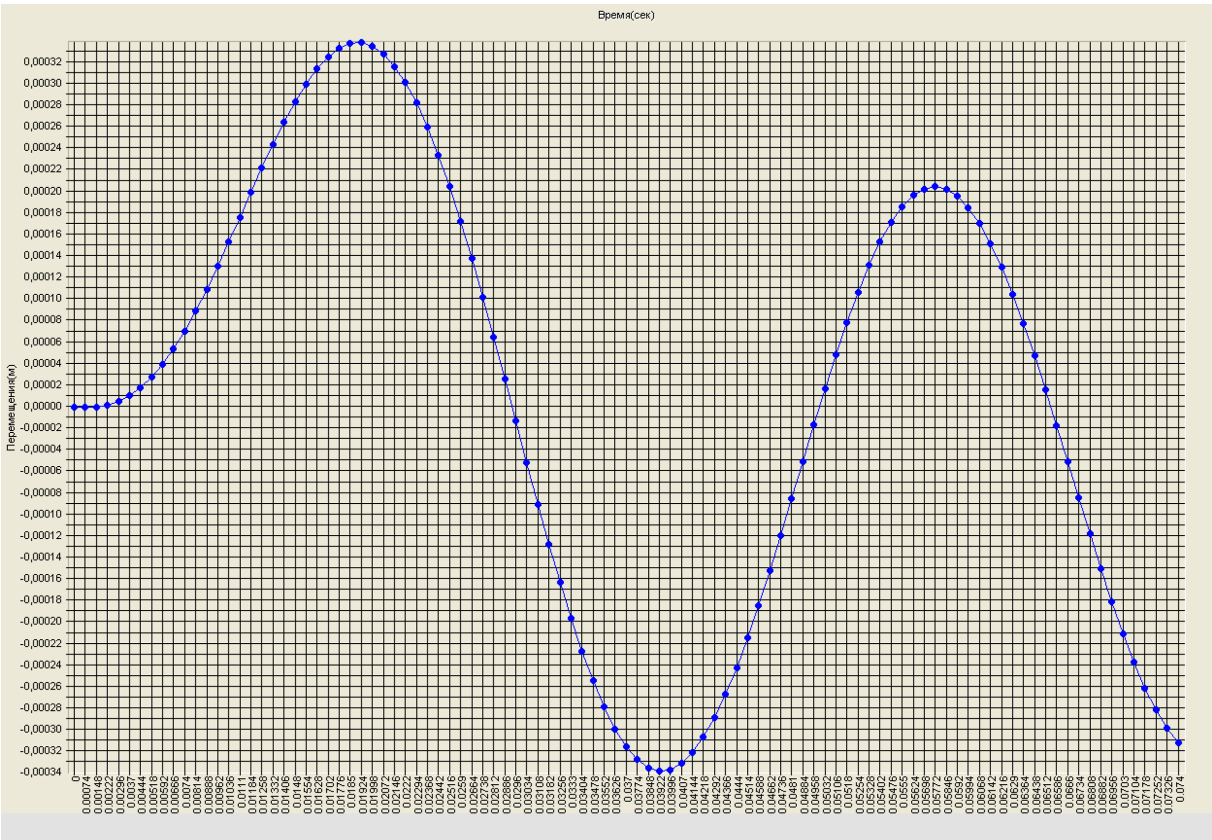
Amplitude values of the deflection η_1 in the cross-section of the beam with the attached mass subjected to the shear force and the deformed models at the respective time points (m). Frequency of the harmonic exciting force $\omega_6 = 1.05 \cdot p_2$



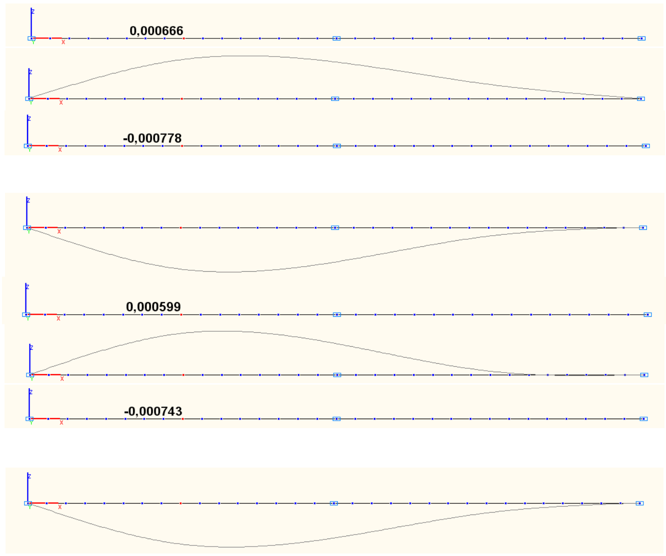
Amplitude values of the deflection η_2 in the cross-section of the beam with the attached mass not subjected to the shear force and the deformed models at the respective time points (m). Frequency of the harmonic exciting force $\omega_6 = 1.05 \cdot p_2$



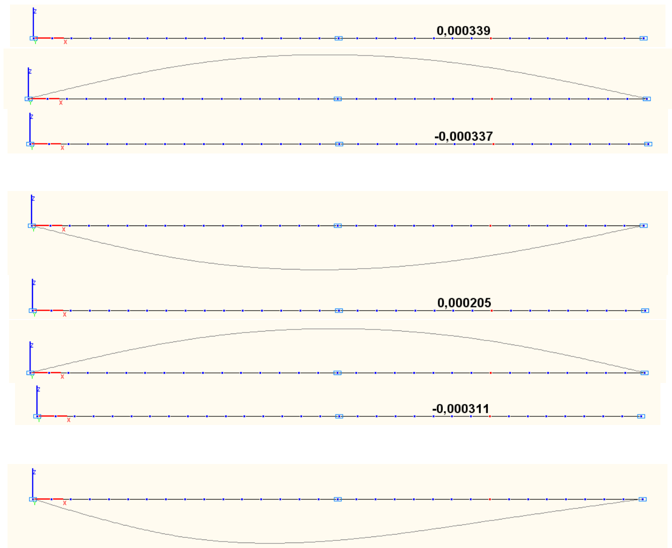
Amplitude value of the deflection η_1 in the cross-section of the beam with the attached mass subjected to the shear force and the deformed model at the respective time point (m). Frequency of the harmonic exciting force $\omega_7 = 1.5 \cdot p_2$



Graph of the variation of the deflection η_2 in the cross-section of the beam with the attached mass not subjected to the shear force, with time (m). Frequency of the harmonic exciting force $\omega_7 = 1.5 \cdot p_2$

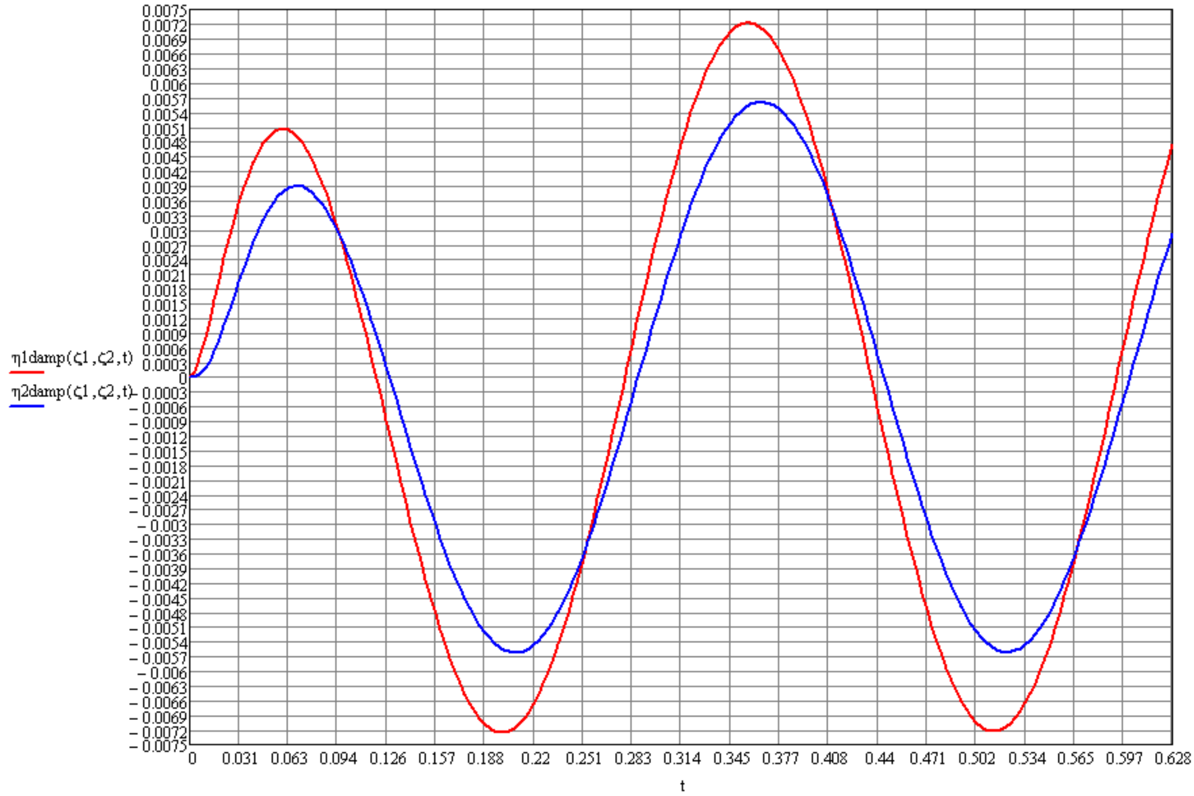


Amplitude values of the deflection η_1 in the cross-section of the beam with the attached mass subjected to the shear force and the deformed models at the respective time points (m). Frequency of the harmonic exciting force $\omega_7 = 1.5 \cdot p_2$

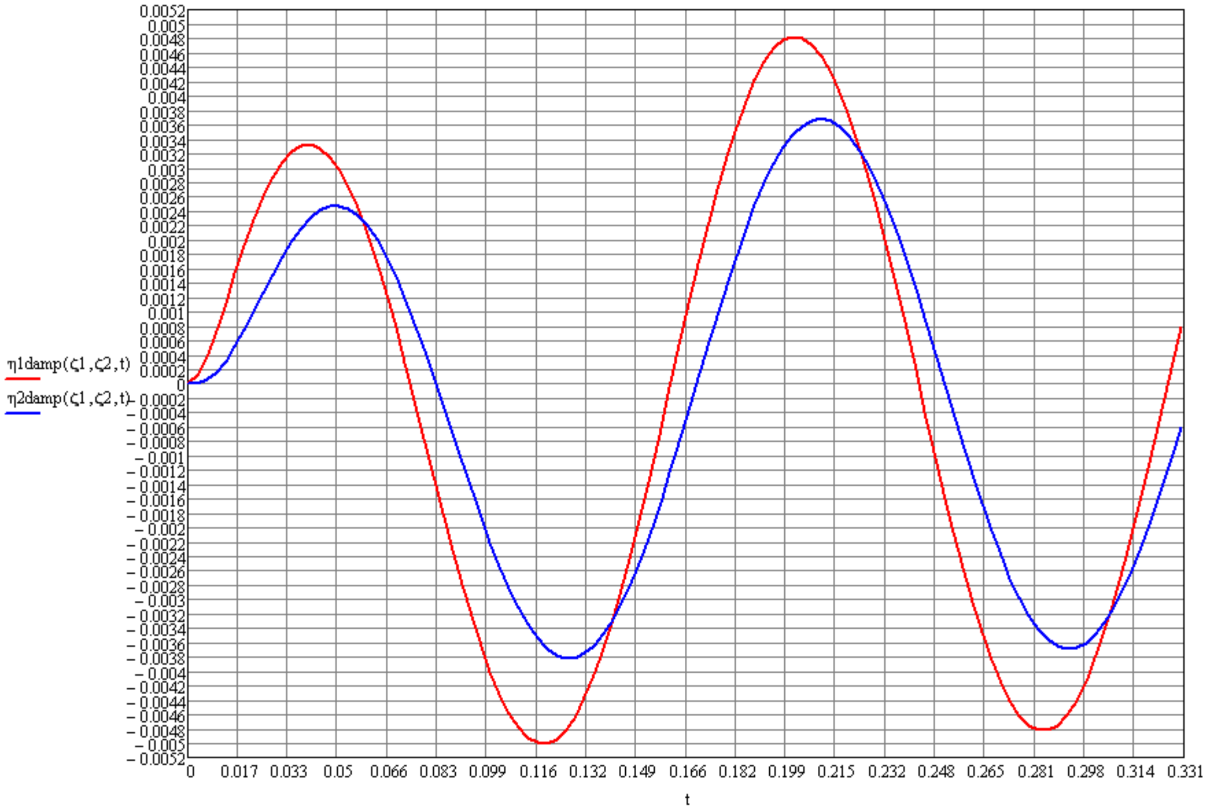


Amplitude values of the deflection η_2 in the cross-section of the beam with the attached mass not subjected to the shear force and the deformed models at the respective time points (m). Frequency of the harmonic exciting force $\omega_7 = 1.5 \cdot p_2$

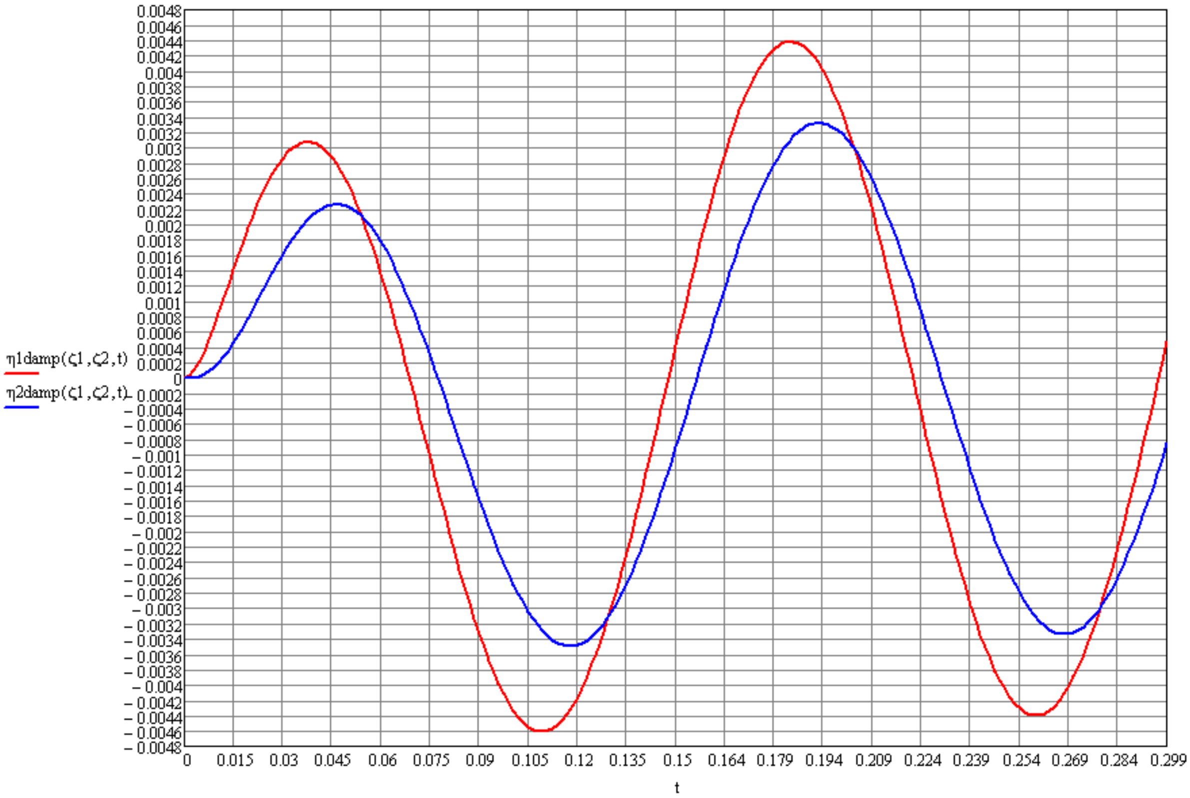
Comparison of solutions:



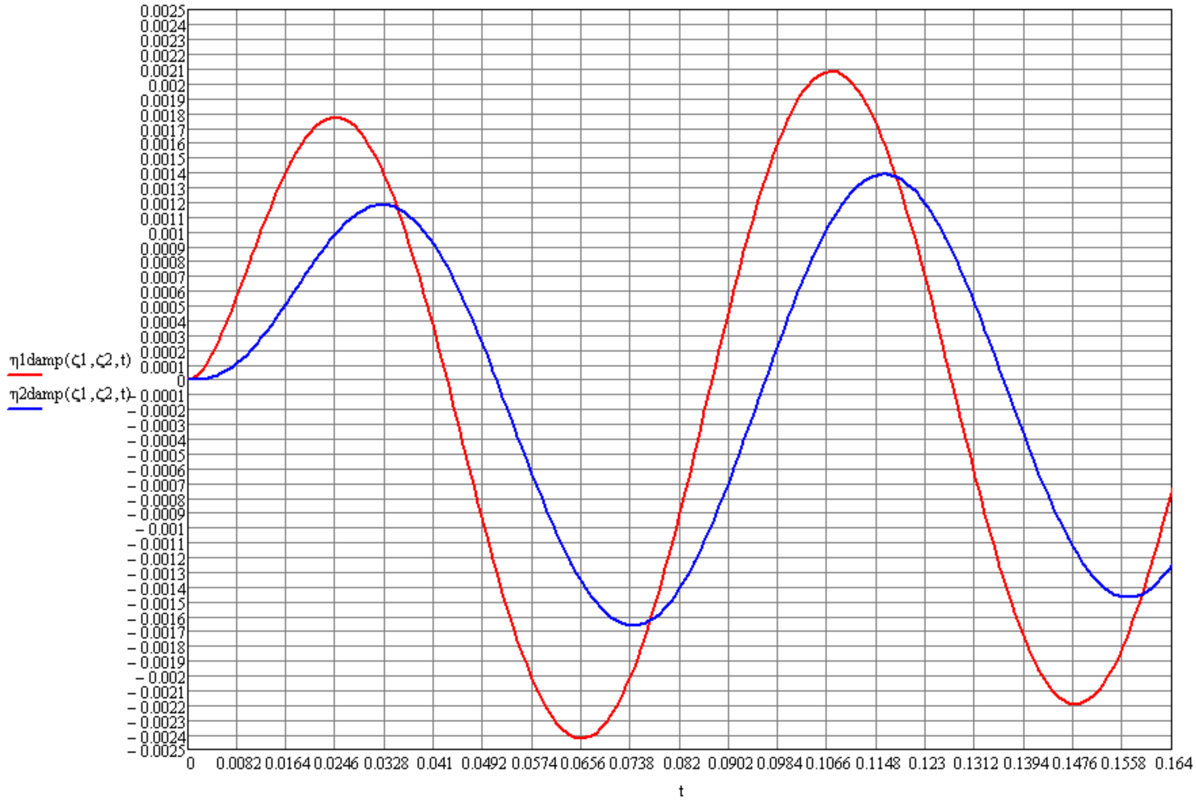
Graphs of the variation of the deflections η_1 and η_2 in the cross-sections of the beam with the attached masses with time according to the theoretical solution (m)
Frequency of the harmonic exciting force $\omega_1 = 0.5 \cdot p_1$



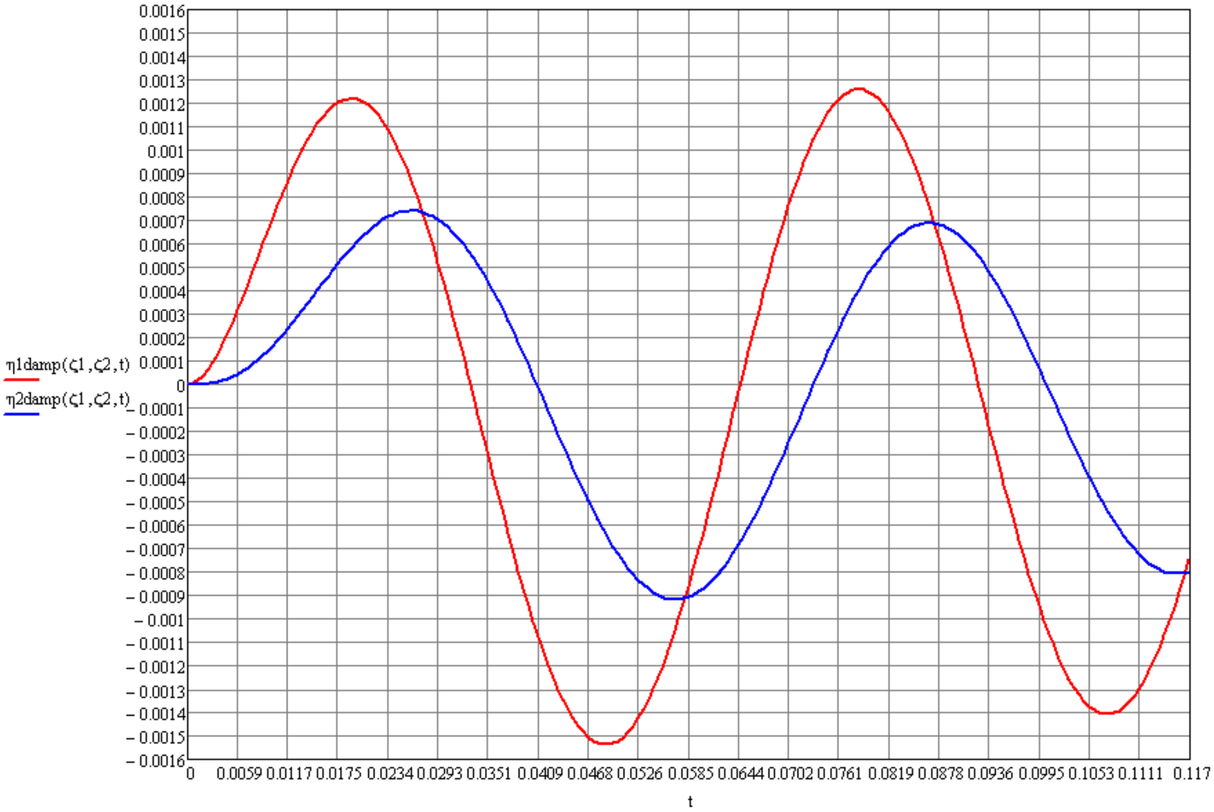
Graphs of the variation of the deflections η_1 and η_2 in the cross-sections of the beam with the attached masses with time according to the theoretical solution (m)
Frequency of the harmonic exciting force $\omega_2 = 0.95 \cdot p_1$



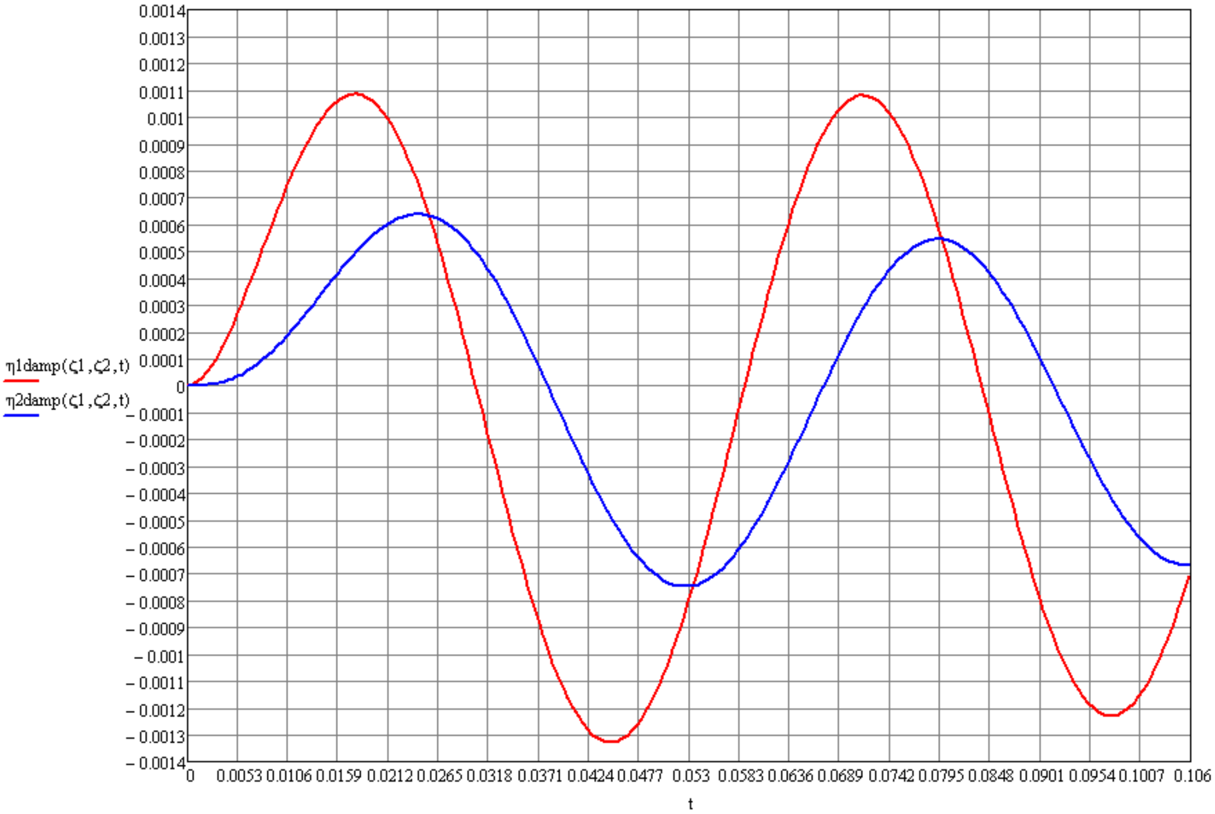
Graphs of the variation of the deflections η_1 and η_2 in the cross-sections of the beam with the attached masses with time according to the theoretical solution (m)
Frequency of the harmonic exciting force $\omega_3 = 1.05 \cdot p_1$



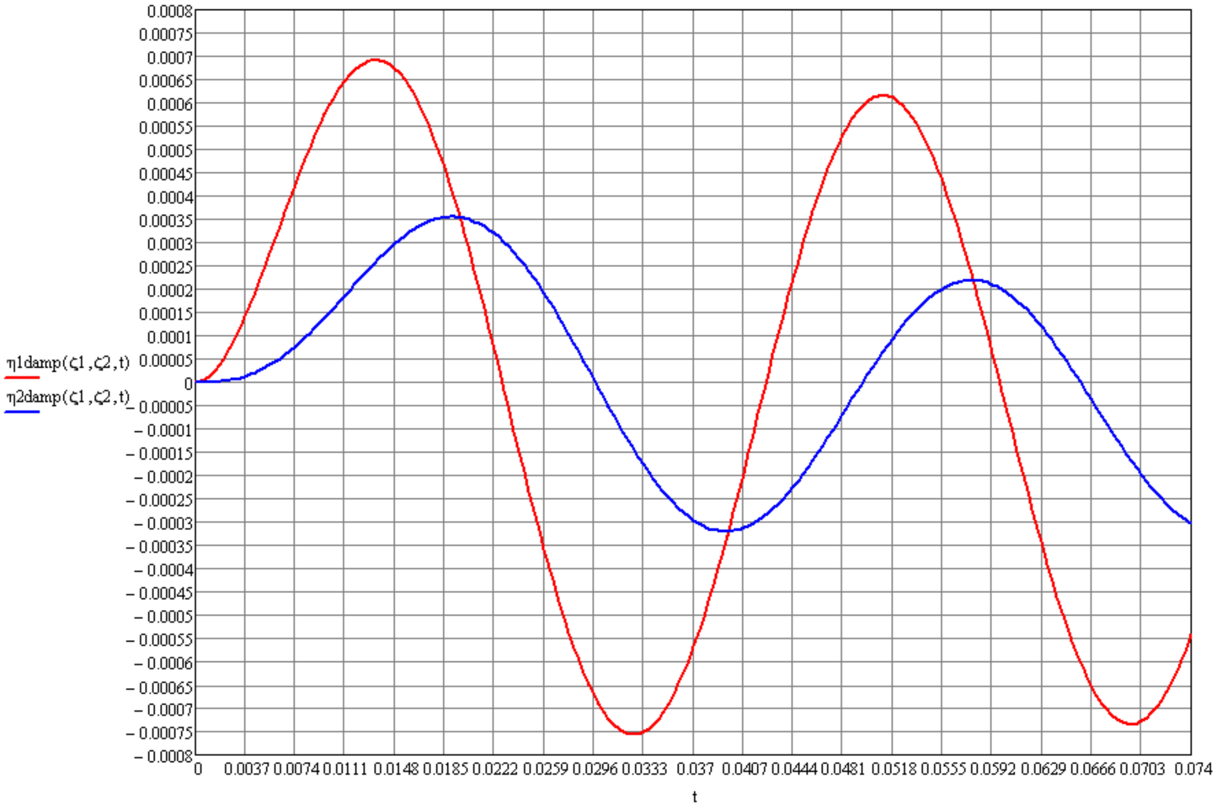
Graphs of the variation of the deflections η_1 and η_2 in the cross-sections of the beam with the attached masses with time according to the theoretical solution (m)
Frequency of the harmonic exciting force $\omega_4 = 0.5 \cdot (p_1 + p_2)$



Graphs of the variation of the deflections η_1 and η_2 in the cross-sections of the beam with the attached masses with time according to the theoretical solution (m)
Frequency of the harmonic exciting force $\omega_5 = 0.95 \cdot p_2$



Graphs of the variation of the deflections η_1 and η_2 in the cross-sections of the beam with the attached masses with time according to the theoretical solution (m)
Frequency of the harmonic exciting force $\omega_6 = 1.05 \cdot p_2$



Graphs of the variation of the deflections η_1 and η_2 in the cross-sections of the beam with the attached masses with time according to the theoretical solution (m)
Frequency of the harmonic exciting force $\omega_7 = 1.5 \cdot p_2$

Natural frequencies p, rad/s

Oscillation mode	Theory	SCAD	Deviations, %
1	40.000	40.000	0.00
2	113.137	113.137	0.00

Amplitude values of the deflections η in the cross-sections of the beam with the attached masses at the frequency of the harmonic exciting force $\omega_1 = 0.5 \cdot p_1$

Nodal mass	Theory		SCAD		
	Time, s	Deflection, m	Time, s	Deflection, m	Deviations, %
1	0.0595	0.005054	0.0628	0.004927	2.51
1	0.1996	-0.007251	0.2011	-0.007248	0.04
1	0.3569	0.007232	0.3582	0.007229	0.04
1	0.5139	-0.007232	0.5153	-0.007229	0.04
2	0.0685	0.003899	0.0691	0.003809	2.31
2	0.2079	-0.005627	0.2074	-0.005627	0.00
2	0.3652	0.005613	0.3645	0.005612	0.02
2	0.5223	-0.005613	0.5216	-0.005612	0.02

Amplitude values of the deflections η in the cross-sections of the beam
with the attached masses
at the frequency of the harmonic exciting force $\omega_2 = 0.95 \cdot p_1$

Nodal mass	Theory		SCAD		
	Time, s	**Deflection, m**	**Time, s**	**Deflection, m**	**Deviations, %**
1	0.0401	0.003330	0.0397	0.003235	2.85
1	0.1181	-0.005009	0.1190	-0.005014	0.10
1	0.2016	0.004822	0.2017	0.004819	0.06
1	0.2842	-0.004834	0.2843	-0.004831	0.06
2	0.0490	0.002478	0.0496	0.002404	2.99
2	0.1268	-0.003825	0.1256	-0.003828	0.08
2	0.2103	0.003684	0.2116	0.003677	0.19
2	0.2929	-0.003692	0.2942	-0.003685	0.19

Amplitude values of the deflections η in the cross-sections of the beam
with the attached masses
at the frequency of the harmonic exciting force $\omega_3 = 1.05 \cdot p_1$

Nodal mass	Theory		SCAD		
	Time, s	**Deflection, m**	**Time, s**	**Deflection, m**	**Deviations, %**
1	0.0375	0.003077	0.0389	0.002985	2.99
1	0.1088	-0.004609	0.1077	-0.004618	0.19
1	0.1845	0.004383	0.1855	0.004377	0.14
1	0.2592	-0.004402	0.2603	-0.004396	0.14
2	0.0464	0.002267	0.0479	0.002193	3.26
2	0.1175	-0.003497	0.1167	-0.003504	0.20
2	0.1932	0.003325	0.1945	0.003318	0.21
2	0.2679	-0.003339	0.2693	-0.003332	0.21

Amplitude values of the deflections η in the cross-sections of the beam
with the attached masses
at the frequency of the harmonic exciting force $\omega_4 = 0.5 \cdot (p_1 + p_2)$

Nodal mass	Theory		SCAD		
	Time, s	**Deflection, m**	**Time, s**	**Deflection, m**	**Deviations, %**
1	0.0246	0.001770	0.0246	0.001714	3.16
1	0.0656	-0.002427	0.0657	-0.002453	1.07
1	0.1072	0.002082	0.1067	0.002072	0.48
1	0.1480	-0.002194	0.1478	-0.002196	0.09
2	0.0324	0.001179	0.0328	0.001136	3.65
2	0.0742	-0.001664	0.0739	-0.001685	1.26
2	0.1160	0.001388	0.1166	0.001382	0.43
2	0.1568	-0.001474	0.1576	-0.001473	0.07

Amplitude values of the deflections η in the cross-sections of the beam
with the attached masses
at the frequency of the harmonic exciting force $\omega_5 = 0.95 \cdot p_2$

Nodal mass	Theory		SCAD		
	Time, s	**Deflection, m**	**Time, s**	**Deflection, m**	**Deviations, %**
1	0.0191	0.001221	0.0187	0.001178	3.52
1	0.0488	-0.001538	0.0491	-0.001564	1.69
1	0.0783	0.001259	0.0784	0.001247	0.95
1	0.1073	-0.001408	0.1076	-0.001414	0.43
2	0.0260	0.000741	0.0257	0.000712	3.91
2	0.0569	-0.000918	0.0573	-0.000939	2.29
2	0.0866	0.000689	0.0866	0.000679	1.45
2	0.1154	-0.000809	0.1158	-0.000813	0.49

Amplitude values of the deflections η in the cross-sections of the beam
with the attached masses
at the frequency of the harmonic exciting force $\omega_6 = 1.05 \cdot p_2$

Nodal mass	Theory		SCAD		
	Time, s	**Deflection, m**	**Time, s**	**Deflection, m**	**Deviations, %**
1	0.0177	0.001085	0.0180	0.001046	3.59
1	0.0447	-0.001329	0.0444	-0.001354	1.88
1	0.0714	0.001080	0.0709	0.001064	1.48
1	0.0976	-0.001229	0.0973	-0.001235	0.49
2	0.0244	0.000638	0.0243	0.000612	4.08
2	0.0526	-0.000748	0.0529	-0.000769	2.81
2	0.0793	0.000545	0.0794	0.000534	2.02
2	0.1054	-0.000667	0.1058	-0.000671	0.60

Amplitude values of the deflections η in the cross-sections of the beam
with the attached masses
at the frequency of the harmonic exciting force $\omega_7 = 1.5 \cdot p_2$

Nodal mass	Theory		SCAD		
	Time, s	**Deflection, m**	**Time, s**	**Deflection, m**	**Deviations, %**
1	0.0134	0.000692	0.0133	0.000666	3.76
1	0.0326	-0.000756	0.0326	-0.000778	2.91
1	0.0511	0.000616	0.0511	0.000599	2.76
1	0.0695	-0.000734	0.0696	-0.000743	1.23
2	0.0191	0.000355	0.0192	0.000339	4.51
2	0.0394	-0.000320	0.0392	-0.000337	5.31
2	0.0577	0.000219	0.0577	0.000205	6.39
2	0.0760	-0.000318	0.0740	-0.000311	2.20

Notes: In the analytical solution the natural frequencies of oscillations *p* of the simply supported beam are determined according to the following formulas:

$$p_1 = \sqrt{\frac{48 \cdot E \cdot I}{m \cdot l^3}}\ ; \qquad p_2 = \sqrt{\frac{384 \cdot E \cdot I}{m \cdot l^3}}\ .$$

In the analytical solution the deflections η in the cross-sections of the beam with the attached masses with time taking into account the energy dissipation into internal friction are determined according to the following formulas (the Voigt viscous friction hypothesis):

$$\eta_1(\zeta_1,\zeta_2,t)=\frac{P_0\cdot l^3}{768\cdot E\cdot I}\cdot\frac{8}{1-\frac{\omega^2}{p_1^{\,2}}+4\cdot\xi_1^{\,2}\cdot\frac{\frac{\omega^2}{p_1^{\,2}}}{1-\frac{\omega^2}{p_1^{\,2}}}}\cdot\left(\cos(\omega\cdot t)-e^{(-\xi_1\cdot p_1\cdot t)}\cdot\cos\left(p_1\cdot\sqrt{1-\xi_1^{\,2}}\cdot t\right)+\frac{2\cdot\xi_1\cdot\frac{\omega}{p_1}}{1-\frac{\omega^2}{p_1^{\,2}}}\cdot\sin(\omega\cdot t)-\right.$$

$$\left.\frac{\xi_1}{\sqrt{1-\xi_1^{\,2}}}\cdot\frac{1+\frac{\omega^2}{p_1^{\,2}}}{1-\frac{\omega^2}{p_1^{\,2}}}\cdot e^{(-\xi_1\cdot p_1\cdot t)}\cdot\sin\left(p_1\cdot\sqrt{1-\xi_1^{\,2}}\cdot t\right)\right)+\frac{P_0\cdot l^3}{768\cdot E\cdot I}\cdot\frac{1}{1-\frac{\omega^2}{p_2^{\,2}}+4\cdot\xi_2^{\,2}\cdot\frac{\frac{\omega^2}{p_2^{\,2}}}{1-\frac{\omega^2}{p_2^{\,2}}}}\cdot\quad;$$

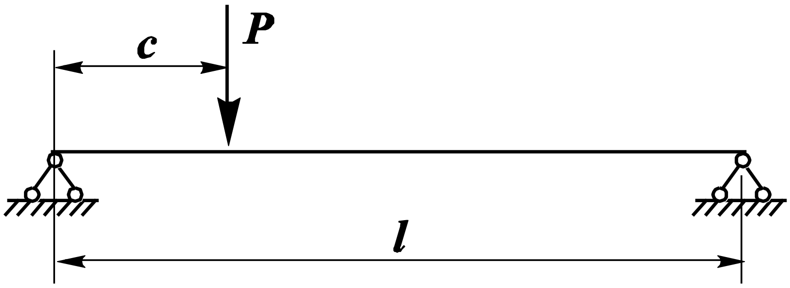
$$\left(\cos(\omega\cdot t)-e^{(-\xi_2\cdot p_2\cdot t)}\cdot\cos\left(p_2\cdot\sqrt{1-\xi_2^{\,2}}\cdot t\right)+\frac{2\cdot\xi_2\cdot\frac{\omega}{p_2}}{1-\frac{\omega^2}{p_2^{\,2}}}\cdot\sin(\omega\cdot t)-\frac{\xi_2}{\sqrt{1-\xi_2^{\,2}}}\cdot\frac{1+\frac{\omega^2}{p_2^{\,2}}}{1-\frac{\omega^2}{p_2^{\,2}}}\cdot e^{(-\xi_2\cdot p_2\cdot t)}\cdot\sin\left(p_2\cdot\sqrt{1-\xi_2^{\,2}}\cdot t\right)\right)$$

$$\eta_1(\zeta_1,\zeta_2,t)=\frac{P_0\cdot l^3}{768\cdot E\cdot I}\cdot\frac{8}{1-\frac{\omega^2}{p_1^{\,2}}+4\cdot\xi_1^{\,2}\cdot\frac{\frac{\omega^2}{p_1^{\,2}}}{1-\frac{\omega^2}{p_1^{\,2}}}}\cdot\left(\cos(\omega\cdot t)-e^{(-\xi_1\cdot p_1\cdot t)}\cdot\cos\left(p_1\cdot\sqrt{1-\xi_1^{\,2}}\cdot t\right)+\frac{2\cdot\xi_1\cdot\frac{\omega}{p_1}}{1-\frac{\omega^2}{p_1^{\,2}}}\cdot\sin(\omega\cdot t)-\right.$$

$$\left.\frac{\xi_1}{\sqrt{1-\xi_1^{\,2}}}\cdot\frac{1+\frac{\omega^2}{p_1^{\,2}}}{1-\frac{\omega^2}{p_1^{\,2}}}\cdot e^{(-\xi_1\cdot p_1\cdot t)}\cdot\sin\left(p_1\cdot\sqrt{1-\xi_1^{\,2}}\cdot t\right)\right)-\frac{P_0\cdot l^3}{768\cdot E\cdot I}\cdot\frac{1}{1-\frac{\omega^2}{p_2^{\,2}}+4\cdot\xi_2^{\,2}\cdot\frac{\frac{\omega^2}{p_2^{\,2}}}{1-\frac{\omega^2}{p_2^{\,2}}}}\cdot\quad.$$

$$\left(\cos(\omega\cdot t)-e^{(-\xi_2\cdot p_2\cdot t)}\cdot\cos\left(p_2\cdot\sqrt{1-\xi_2^{\,2}}\cdot t\right)+\frac{2\cdot\xi_2\cdot\frac{\omega}{p_2}}{1-\frac{\omega^2}{p_2^{\,2}}}\cdot\sin(\omega\cdot t)-\frac{\xi_2}{\sqrt{1-\xi_2^{\,2}}}\cdot\frac{1+\frac{\omega^2}{p_2^{\,2}}}{1-\frac{\omega^2}{p_2^{\,2}}}\cdot e^{(-\xi_2\cdot p_2\cdot t)}\cdot\sin\left(p_2\cdot\sqrt{1-\xi_2^{\,2}}\cdot t\right)\right)$$

Simply Supported Beam with a Distributed Mass Subjected to a Transverse Harmonic Exciting Force Applied in the Middle of the Span



Objective: Determination of the strain state of a simply supported beam with a distributed mass subjected to a transverse harmonic exciting force applied in the middle of the span.

Initial data file:**5.13.SPR**

Problem formulation: The force P_0 is applied in the middle of the span of the simply supported beam of constant cross-section with the uniformly distributed mass μ at the initial time and varies harmonically with the frequency ω. Determine the natural oscillation modes and natural frequencies *p* of the simply supported beam, as well as the deflection η in the cross-section in the middle of the beam span with time.

References: Timoshenko S.P., Course of the Theory of Elasticity, Kiev, Naukova Dumka, 1972, p. 343.

Initial data:

$E = 3.0 \cdot 10^6$ tf/m^2	- elastic modulus;
ν = 0.2	- Poisson's ratio;
b = 0.4 m	- width of the rectangular cross-section of the beam;
h = 0.8 m	- height of the rectangular cross-section of the beam;
l = 8.0 m	- beam span length;
γ = 2.5 tf/m^3	- specific weight of the beam material;
P_0 = 76.8 tf	- amplitude value of the harmonic exciting force applied in the middle of the span;
g = 10.00 m/s^2	- gravitational acceleration;

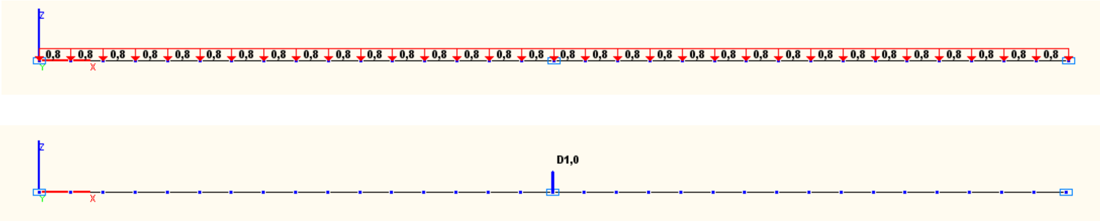
$\mu = 2.5 \cdot 0.4 \cdot 0.8/10.0 = 0.08$ tf·s^2/m^2	- value of the uniformly distributed mass of the beam;
$I = 0.4 \cdot (0.8)^3/12 = 0.017067$ m^4	- cross-sectional moment of inertia of the beam.

The frequency of the harmonic exciting force ω is taken depending on the value of the fundamental natural frequency of the beam p_1:

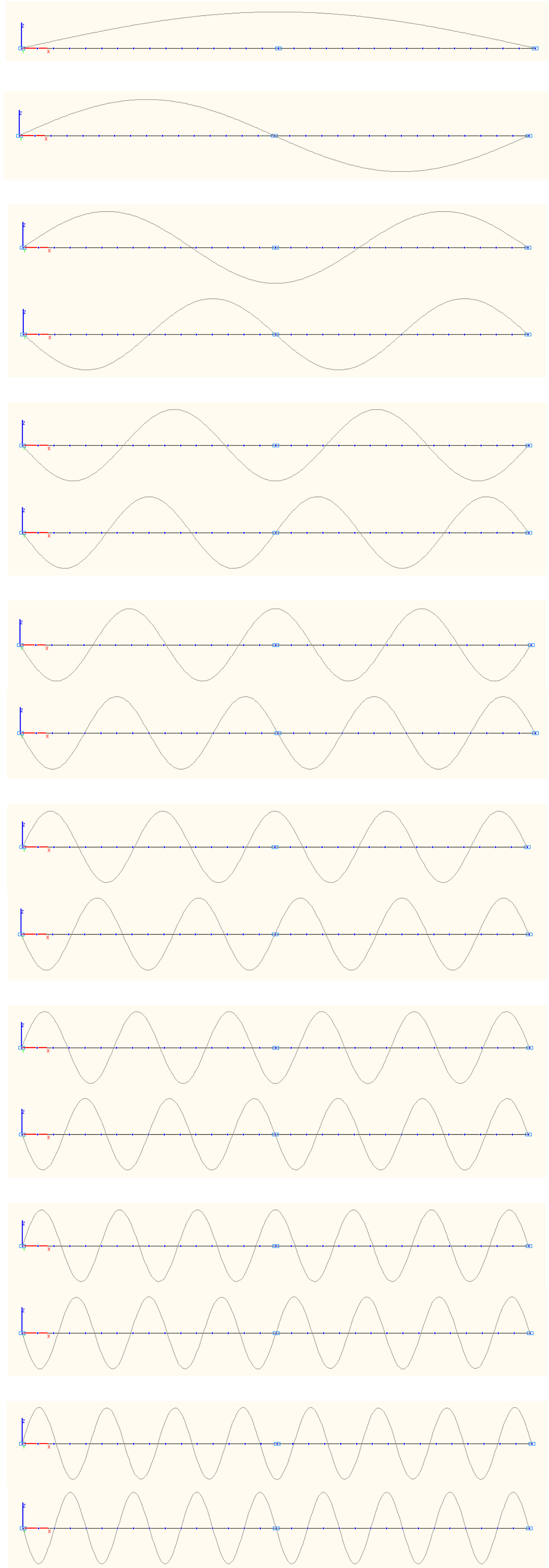
$\omega = 0.5 \cdot p_1$.

Finite element model: Design model – grade beam / plate, 32 bar elements of type 3. Boundary conditions of the simply supported ends of the beam are provided by imposing constraints in the direction of the degree of freedom Z. The dimensional stability of the design model is provided by imposing a constraint in the node of the cross-section along the symmetry axis of the beam in the direction of the degree of freedom UX. The distributed mass is specified by transforming the static load from the self-weight of the beam μ·g. The calculation is performed in two stages: first the natural oscillation modes and natural frequencies *p* are determined by the modal analysis, and then the deflections η in the cross-section in the middle of the beam span with time are determined by the direct integration of the equations of motion method. The action of the transverse harmonic exciting force is described by the graph of the load variation with time and is given in the form of a nodal force acting along the Z axis of the global coordinate system with the scale factor of 1.0 and the delay time 0.0 s. Intervals between the time points of the load variation graph are equal to Δt_{int} = T/100, where T – period of the harmonic exciting force, and correspond to the integration step. When plotting the graph, the action of the transverse harmonic exciting force is taken as $P_n = P_0 \cdot \cos(\omega \cdot n \cdot \Delta t_{int})$ at the time points $n \cdot \Delta t_{int}$. The duration of the process is equal to t = 2·T. Critical damping ratios for the 1-st and 2-nd natural frequencies are taken with the minimum value ξ = 0.0001. The conversion factor for the added static loading is equal to k = 0.981 (mass generation). Number of nodes in the design model – 33. The modal integration method is used in the calculation. The determination of the natural oscillation modes and natural frequencies is performed by the method of subspace iteration. The matrix of concentrated masses is used in the calculation.

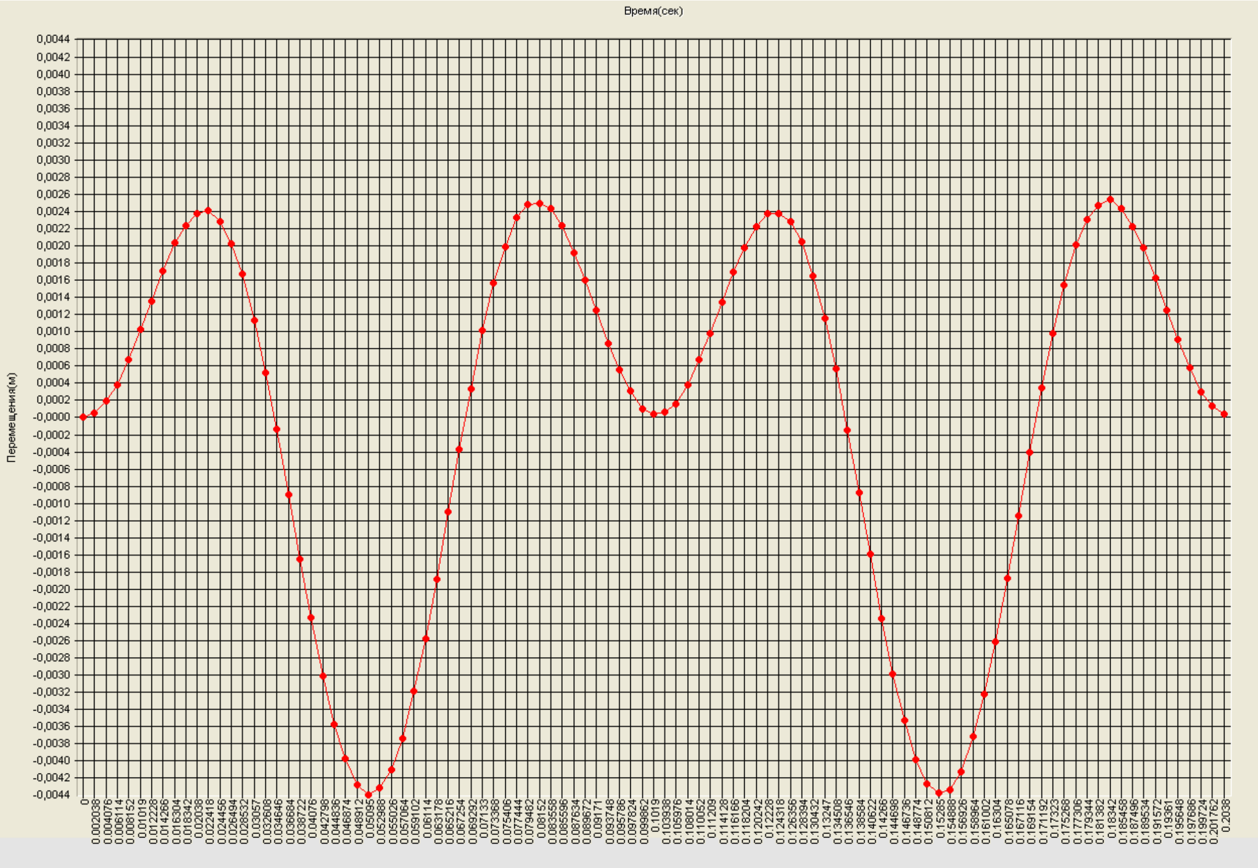
Results in SCAD



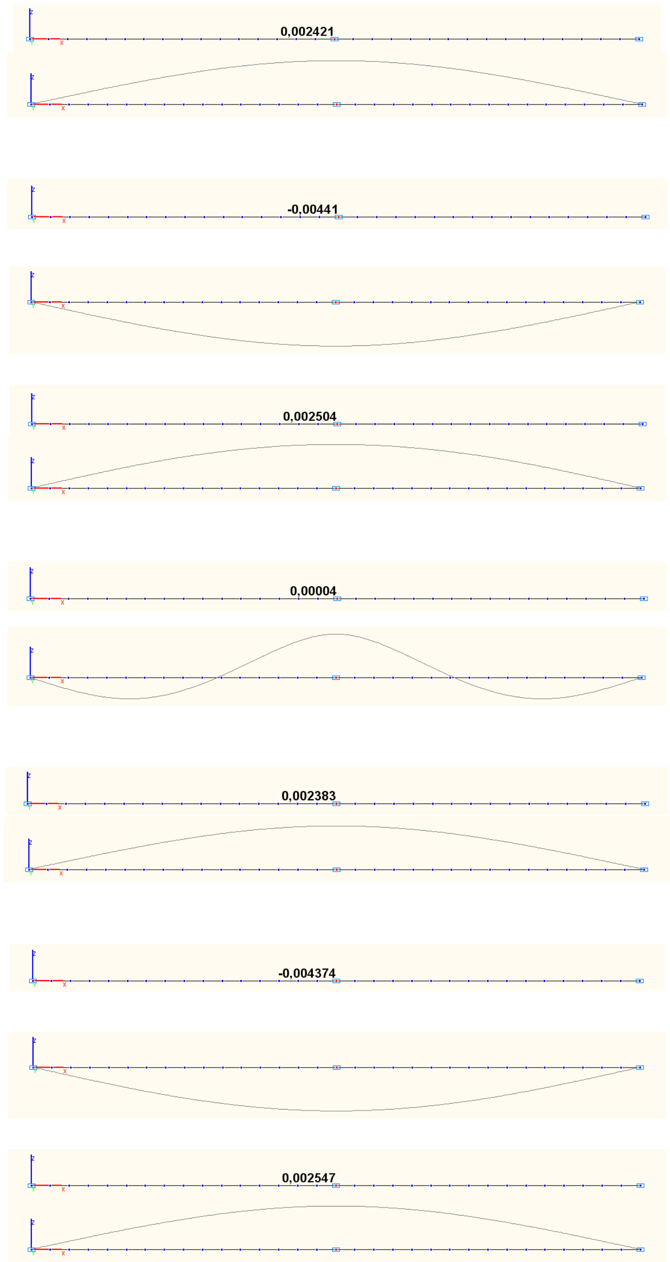
Design model



1-st -- 16-th natural oscillation modes

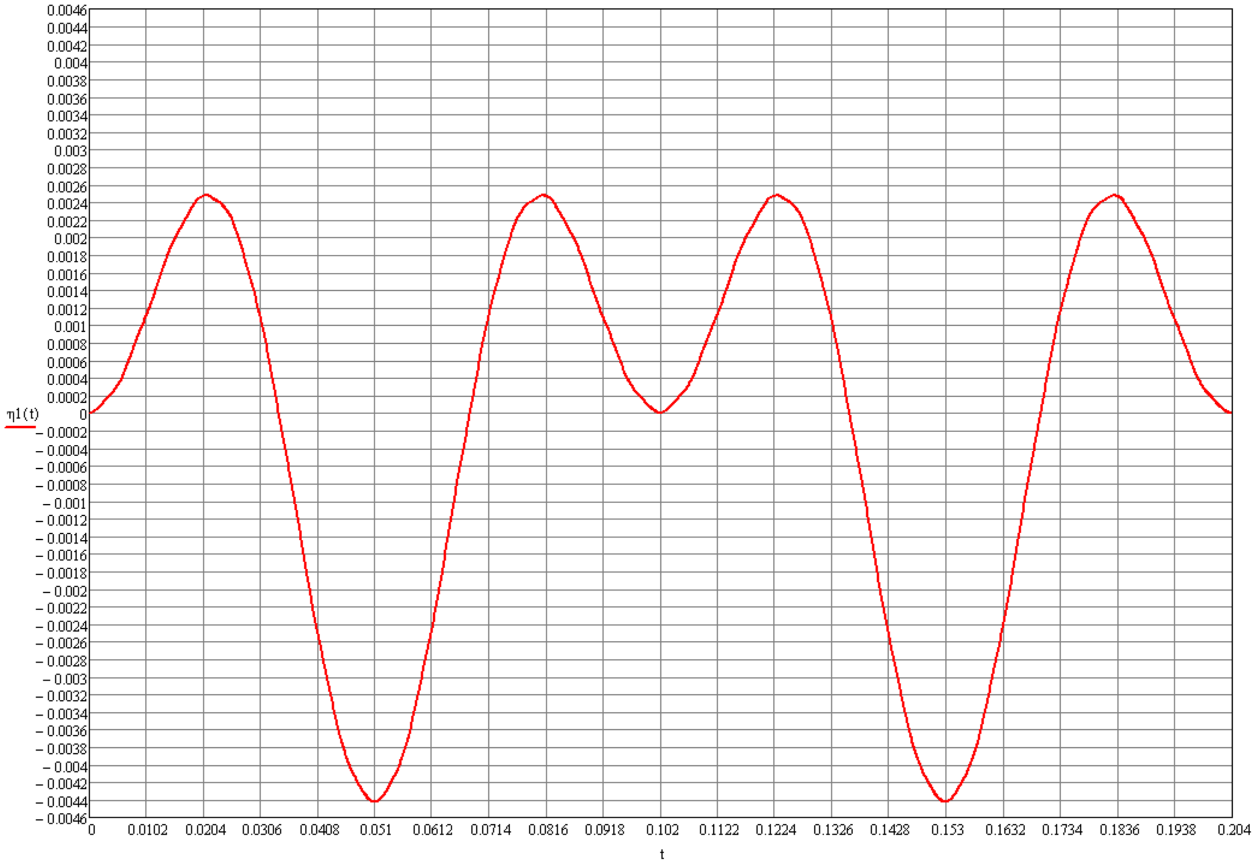


Graph of the variation of the deflection η in the cross-section in the middle of the beam span with time (m).



Amplitude values of the deflection η in the cross-section in the middle of the beam span and the deformed models at the respective time points (m).

Comparison of solutions:



Graph of the variation of the deflection η in the cross-section in the middle of the beam span with time according to the theoretical solution (m)

Natural frequencies *p*, rad/s

Oscillation mode	**Theory**	**SCAD**	**Deviations, %**
1	123.370	123.370	0.00
2	493.480	493.480	0.00
3	1110.330	1110.325	0.00
4	1973.921	1973.887	0.00
5	3084.251	3084.120	0.00
6	4441.322	4440.919	0.01
7	6045.133	6044.087	0.02
8	7895.684	7893.275	0.03
9	9992.974	9987.907	0.05
10	12337.005	12327.069	0.08
11	14927.777	14909.367	0.12
12	17765.288	17732.721	0.18
13	20849.539	20794.097	0.27
14	24180.531	24089.155	0.38
15	27758.262	27611.778	0.53
16	31582.734	31353.470	0.73

Amplitude values of the deflection η in the cross-section in the middle of the beam span at the frequency of the harmonic exciting force $\omega = 0.5 \cdot p_1$

Theory		SCAD		
Time, s	**Deflection, m**	**Time, s**	**Deflection, m**	**Deviations, %**
0.0210	0.002474	0.0224	0.002421	2.14
0.0510	-0.004428	0.0510	-0.004410	0.41
0.0809	0.002474	0.0815	0.002504	1.21
0.1017	0.000002	0.1019	0.000040	–
0.1228	0.002474	0.1223	0.002383	3.68
0.1528	-0.004428	0.1529	-0.004374	1.22
0.1828	0.002474	0.1834	0.002547	2.95

Notes: In the analytical solution the natural frequencies of oscillations *p* of the simply supported beam are determined according to the following formulas:

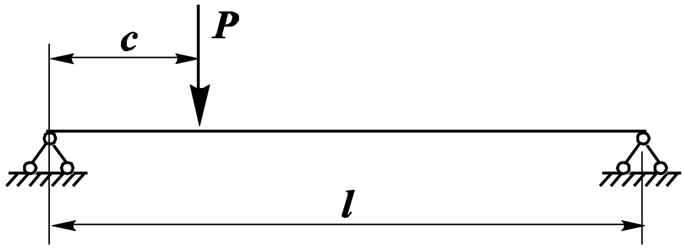
$$\frac{n^2 \cdot \pi^2}{l^2} \cdot \sqrt{\frac{E \cdot I}{\mu}} \, ,$$

where n = 1, 2, 3, 4, … – natural mode number.

In the analytical solution the deflections η in the cross-sections in the middle of the beam span with time are determined according to the following formula:

$$\eta(t) = \frac{2 \cdot P_0 \cdot l^3}{\pi^4 \cdot E \cdot I} \cdot \sum_{n=1} \left[\frac{\left(sin\left(\frac{n \cdot \pi}{2} \right) \right)^2}{n^4 \cdot \left(1 - \frac{\mu \cdot l^4 \cdot \omega^2}{n^4 \cdot \pi^4 \cdot E \cdot I} \right)} \cdot \left(cos(\omega \cdot t) - cos\left(\frac{n^2 \cdot \pi^2}{l^2} \cdot \sqrt{\frac{E \cdot I}{\mu}} \cdot t \right) \right) \right];$$

Simply Supported Beam with a Distributed Mass Subjected to a Constant Shear Force Moving along the Span of the Beam at a Constant Speed



Objective: Determination of the strain state of a simply supported beam with a distributed mass subjected to a constant shear force moving along the span of the beam at a constant speed.

Initial data files:

File name	Description
DIN_B_ML1.SPR **График_DIN_B_ML1.txt**	The action of the constant shear force moving along the beam span is specified in the form of forces applied in all nodes of the design model according to the following variant: The delay time for each nodal force is different. The graph describing the load variation with time is the same for all nodal forces.
DIN_B_ML2.SPR **График_DIN_B_ML2_2.txt** **График_DIN_B_ML2_3.txt** **График_DIN_B_ML2_4.txt** **График_DIN_B_ML2_5.txt** **График_DIN_B_ML2_6.txt** **График_DIN_B_ML2_7.txt** **График_DIN_B_ML2_8.txt** **График_DIN_B_ML2_9.txt** **График_DIN_B_ML2_10.txt** **График_DIN_B_ML2_11.txt** **График_DIN_B_ML2_12.txt** **График_DIN_B_ML2_13.txt** **График_DIN_B_ML2_14.txt** **График_DIN_B_ML2_15.txt** **График_DIN_B_ML2_16.txt** **График_DIN_B_ML2_17.txt** **График_DIN_B_ML2_18.txt** **График_DIN_B_ML2_19.txt** **График_DIN_B_ML2_20.txt** **График_DIN_B_ML2_21.txt** **График_DIN_B_ML2_22.txt** **График_DIN_B_ML2_23.txt** **График_DIN_B_ML2_24.txt** **График_DIN_B_ML2_25.txt** **График_DIN_B_ML2_26.txt** **График_DIN_B_ML2_27.txt** **График_DIN_B_ML2_28.txt** **График_DIN_B_ML2_29.txt** **График_DIN_B_ML2_30.txt** **График_DIN_B_ML2_31.txt** **График_DIN_B_ML2_32.txt**	The action of the constant shear force moving along the beam span is specified in the form of forces applied in all nodes of the design model according to the following variant: The delay time is the same for all nodal forces. Each nodal force has its corresponding graph describing the load variation with time.

Problem formulation: The constant shear force P moves at a constant speed v along the span of the simply supported beam with a uniformly distributed mass μ. Determine the natural oscillation modes and natural frequencies p of the simply supported beam, as well as the deflection η in the cross-section in the middle of the beam span with time.

References: Timoshenko S.P., Course of the Theory of Elasticity, Kiev, Naukova Dumka, 1972, p. 345.

Initial data:

$E = 3.0 \cdot 10^6$ tf/m^2	- elastic modulus;
$\nu = 0.2$	- Poisson's ratio;
b = 0.4 m	- width of the rectangular cross-section of the beam;
h = 0.8 m	- height of the rectangular cross-section of the beam;
l = 8.0 m	- beam span length;
$\gamma = 2.5$ tf/m^3	- specific weight of the beam material;
P = 76.8 tf	- value of the constant force moving along the beam span;
g = 10.00 m/s^2	- gravitational acceleration;

$\mu = 2.5 \cdot 0.4 \cdot 0.8/10.0 = 0.08$ tf·s^2/m^2	- value of the uniformly distributed mass of the beam;
$I = 0.4 \cdot (0.8)^3/12 = 0.017067$ m^4	- cross-sectional moment of inertia of the beam.

The speed of the constant force v is taken depending on the values of the beam span and the fundamental natural period of the beam T_1:

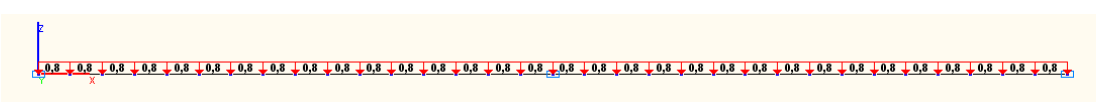
$v = l / T_1$.

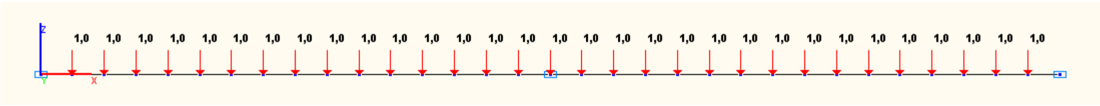
Finite element model: Design model – grade beam / plate, 32 bar elements of type 3. Boundary conditions of the simply supported ends of the beam are provided by imposing constraints in the direction of the degree of freedom Z. The dimensional stability of the design model is provided by imposing a constraint in the node of the cross-section along the symmetry axis of the beam in the direction of the degree of freedom UX. The distributed mass is specified by transforming the static load from the self-weight of the beam $\mu \cdot g$.
The calculation is performed in two stages: first the natural oscillation modes and natural frequencies p are determined by the modal analysis, and then the deflections η in the cross-section in the middle of the beam span are determined by the direct integration of the equations of motion method.
The action of the constant shear force moving along the beam span is specified in the form of forces applied in all nodes of the design model along the Z axis of the global coordinate system with the scale factor of 1.0 according to the following variants:

- The delay time for each nodal force is different and is determined as $t_0 = 2 \cdot (m-1) \cdot \Delta t_{int}$, where m is the number of finite elements counted from the support node of the beam to the node considered along the load path. The graph describing the load variation with time is the same for all nodal forces. When plotting the graph the nodal force is taken with consecutive values: 0; $0.5 \cdot P$; P; $0.5 \cdot P$; 0 at time points: 0; Δt_{int}; $2 \cdot \Delta t_{int}$; $3 \cdot \Delta t_{int}$; $4 \cdot \Delta t_{int}$; $5 \cdot \Delta t_{int}$, measured from the delay time t_0, at subsequent time points the nodal force is equal to 0.
- The delay time is the same for all nodal forces and is equal to $t_0 = 0$. Each nodal force has its corresponding graph describing the load variation with time. When plotting the graph the nodal force at the time points from 0 to $2 \cdot (m-1) \cdot \Delta t_{int}$ is equal to 0, at the time points from $2 \cdot (m-1) \cdot \Delta t_{int}$ to $2 \cdot (m+1) \cdot \Delta t_{int}$ inclusive is taken with consecutive values: 0; $0.5 \cdot P$; P; $0.5 \cdot P$; 0, at subsequent time points the nodal force is equal to 0, where m is the number of finite elements counted from the support node of the beam to the node considered along the load path.

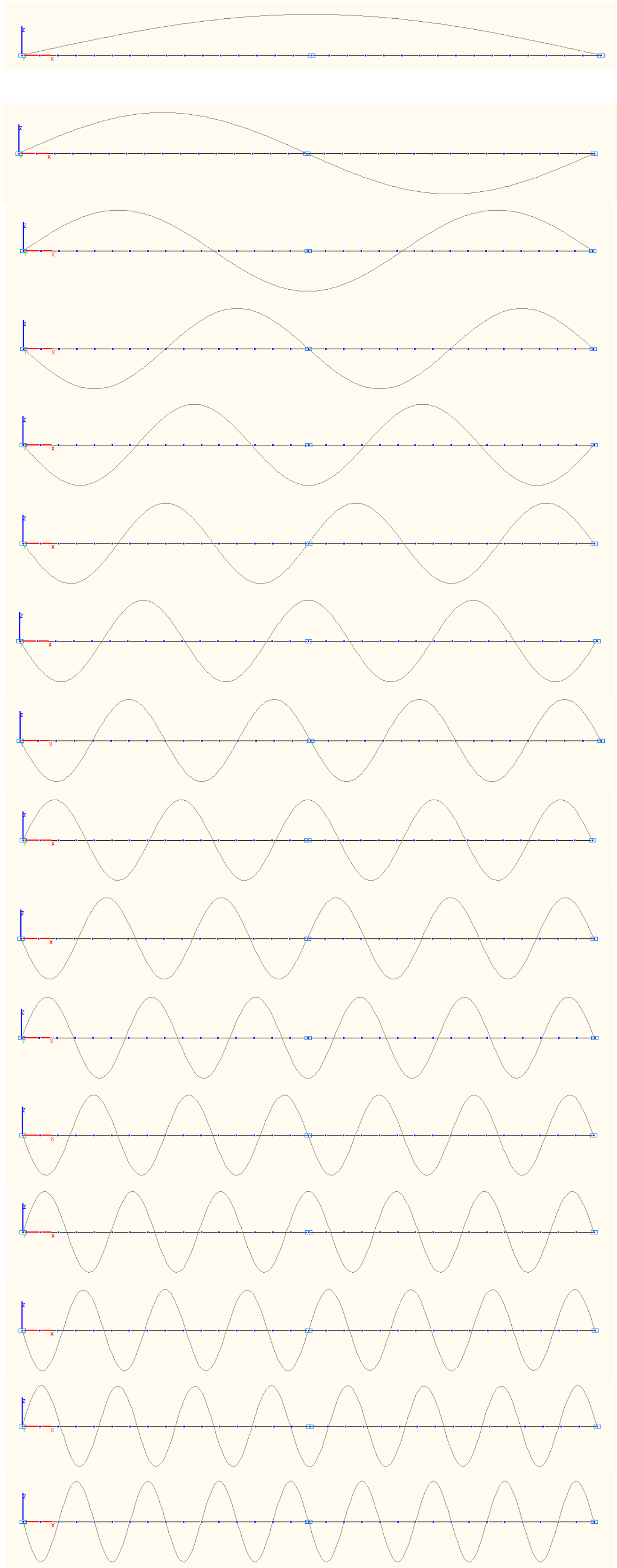
In both cases the intervals between the time points of the load variation graphs are equal to the time it takes to cover half the distance between the adjacent nodes of the design model at the speed v: $\Delta t_{int} = L / (2 \cdot n \cdot v) = T_1 / (2 \cdot n)$ and correspond to the integration step, where n is the number of finite elements in the design model. The duration of the process is equal to the time it takes the load moving at the speed v to cover the beam span l: $t = l/v = T_1$. Critical damping ratios for the 1-st and 2-nd natural frequencies are taken with the minimum value $\xi = 0.0001$. The conversion factor for the added static loading is equal to k = 0.981 (mass generation). Number of nodes in the design model – 33. The modal integration method is used in the calculation. The determination of the natural oscillation modes and natural frequencies is performed by the method of subspace iteration. The matrix of concentrated masses is used in the calculation.

Results in SCAD

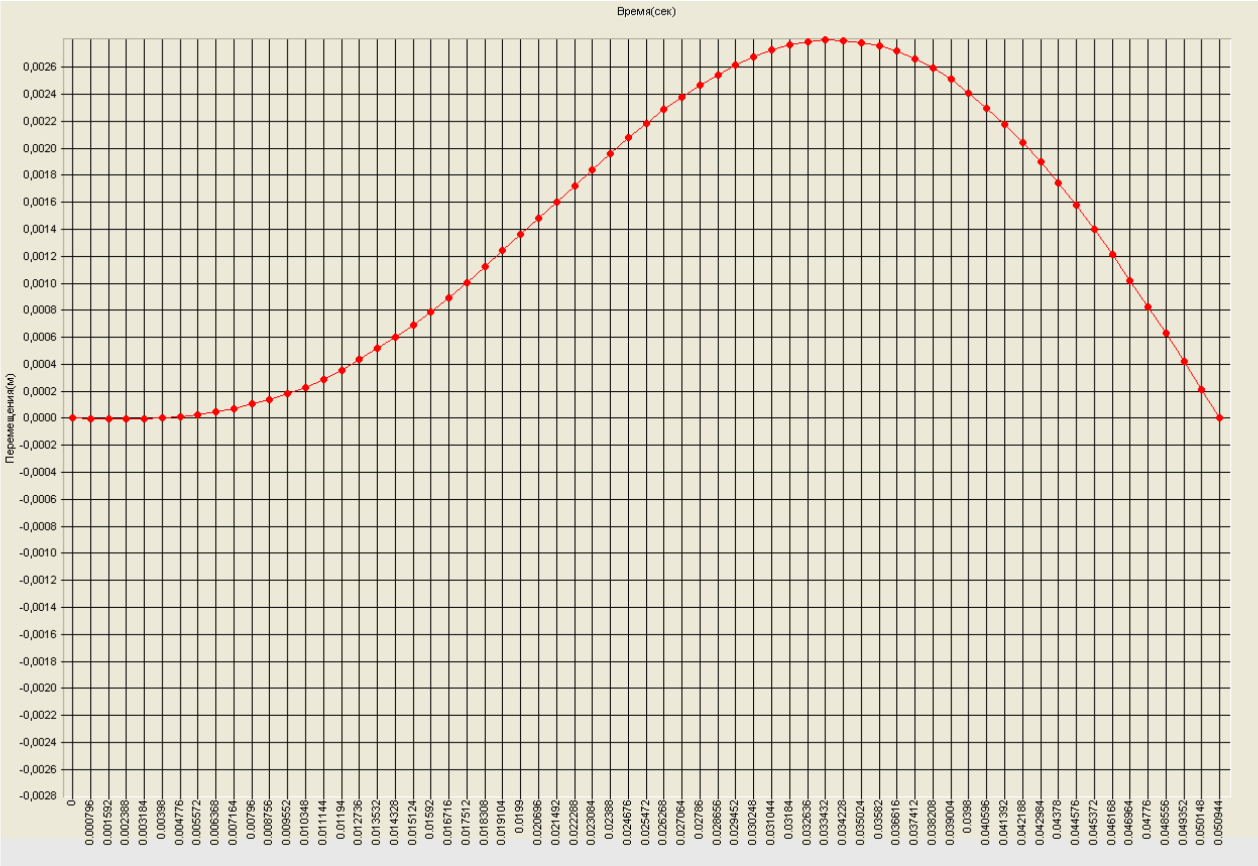




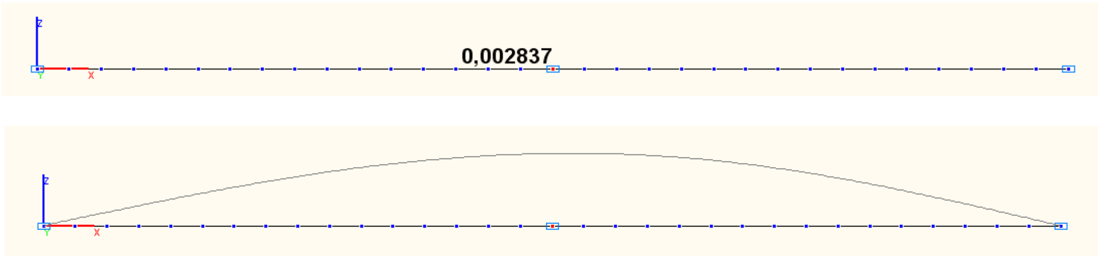
Design model



1-st -- 16-th natural oscillation modes



Graph of the variation of the deflection η in the cross-section in the middle of the beam span with time (m)



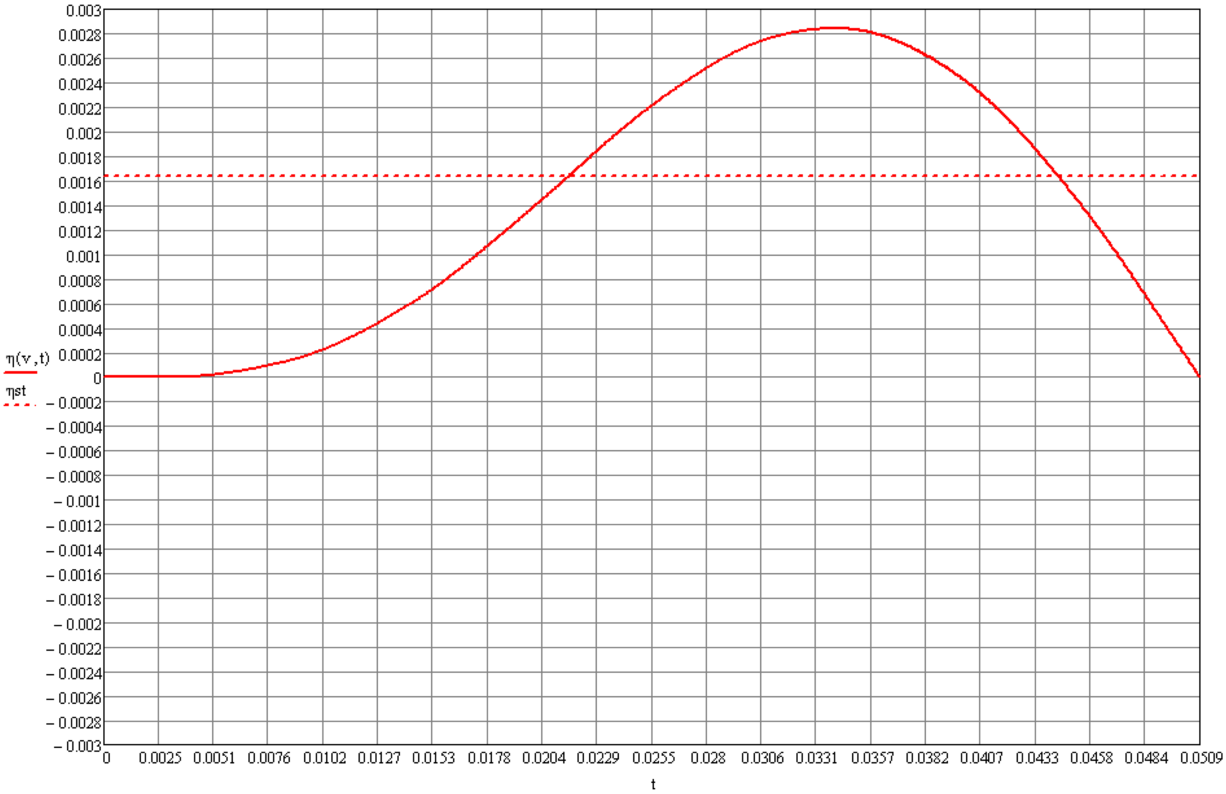
Amplitude value of the deflection η in the cross-section in the middle of the beam span and the deformed models at the respective time point (m)

Comparison of solutions:

Natural frequencies *p*, rad/s

Oscillation mode	Theory	SCAD	Deviations, %
1	123.370	123.370	0.00
2	493.480	493.480	0.00
3	1110.330	1110.325	0.00
4	1973.921	1973.887	0.00
5	3084.251	3084.120	0.00
6	4441.322	4440.919	0.01
7	6045.133	6044.087	0.02
8	7895.684	7893.275	0.03
9	9992.974	9987.907	0.05
10	12337.005	12327.069	0.08
11	14927.777	14909.367	0.12
12	17765.288	17732.721	0.18

Oscillation mode	Theory	SCAD	Deviations, %
13	20849.539	20794.097	0.27
14	24180.531	24089.155	0.38
15	27758.262	27611.778	0.53
16	31582.734	31353.470	0.73



The dashed line shows the value of the static deflection

Graph of the variation of the deflection η in the cross-section in the middle of the beam span with time according to the theoretical solution (m)

Amplitude value of the deflection η in the cross-section in the middle of the beam span, m

Theory		SCAD		
Time, s	Deflection, m	Time, s	Deflection, m	Deviation, %
0.0339	0.002842	0.0334	0.002837	0.18

Notes: In the analytical solution the natural frequencies of oscillations p of the simply supported beam are determined according to the following formula:

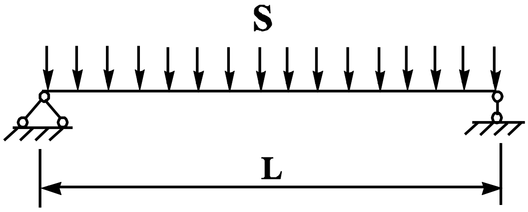
$$\frac{n^2 \cdot \pi^2}{l^2} \cdot \sqrt{\frac{E \cdot I}{\mu}},$$

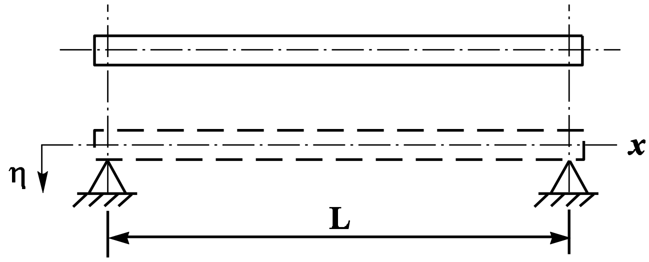
where n = 1, 2, 3, 4, … – natural mode number.

In the analytical solution the deflections η in the cross-section in the middle of the beam span with time are determined according to the following formula:

$$\eta(t)=\frac{2\cdot P\cdot l^3}{\pi^4\cdot E\cdot I}\cdot\sum_{n=1}\left[\frac{\sin\left(\frac{n\cdot\pi}{2}\right)}{n^4\cdot\left(1-\frac{\mu\cdot l^2\cdot v^2}{n^2\cdot\pi^2\cdot E\cdot I}\right)}\cdot\left(\sin\left(\frac{n\cdot\pi\cdot v}{l}\cdot t\right)-\frac{l\cdot v}{n\cdot\pi}\cdot\sqrt{\frac{\mu}{E\cdot I}}\cdot\sin\left(\frac{n^2\cdot\pi^2}{l^2}\cdot\sqrt{\frac{E\cdot I}{\mu}}\cdot t\right)\right)\right]$$

Simply Supported Beam with a Distributed Mass Subjected to a Uniformly Distributed Instantaneous Pulse (Impact of a Beam with Immovable Supports)





Objective: Determination of the stress-strain of a simply supported beam with a distributed mass subjected to a uniformly distributed instantaneous pulse.

Initial data file: **DIN_B_IL.SPR, График_DIN_B_IL.txt**

Problem formulation: The simply supported beam of constant cross-section with the uniformly distributed mass μ is subjected to the instantaneous transverse pulse S uniformly distributed over the entire span *L* (impacts the immovable supports at a speed $v_0 = S / \mu$). Determine the natural oscillation modes and natural frequencies *p* of the simply supported beam, as well as the deflection η and the bending moment *M* in the cross-section in the middle of the beam span with time.

References: Rabinovich I.M., Sinitsyn A.P., Luzhin O.V., Terenin V.M., Analysis of Structures Subject to Pulse Actions, Moscow, Stroyizdat, 1970, p. 83.
S.D. Ponomarev, V.L. Biederman, K.K. Likharev, V.M. Makushin, N.N. Malinin, V.I. Feodos'yev, Fundamentals of Modern Methods for Strength Analysis in Mechanical Engineering. Dynamic Analysis. Stability. Creep. Moscow, Mashgiz, 1952, p. 364.

Initial data:

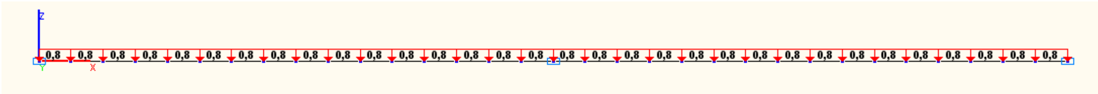
$E = 3.0 \cdot 10^6$ tf/m^2	- elastic modulus;
$\nu = 0.2$	- Poisson's ratio;
b = 0.4 m	- width of the rectangular cross-section of the beam;
h = 0.8 m	- height of the rectangular cross-section of the beam;
L = 8.0 m	- beam span length;
$\gamma = 2.5$ tf/m^3	- specific weight of the beam material;
S = 0.8· tf·s/m	- value of the uniformly distributed instantaneous pulse;
g = 10.00 m/s^2	- gravitational acceleration;

$\mu = 2.5 \cdot 0.4 \cdot 0.8/10.0 = 0.08$ tf·s^2/m^2	- value of the uniformly distributed mass of the beam;
$I = 0.4 \cdot (0.8)^3/12 = 0.017067$ m^4	- cross-sectional moment of inertia of the beam.

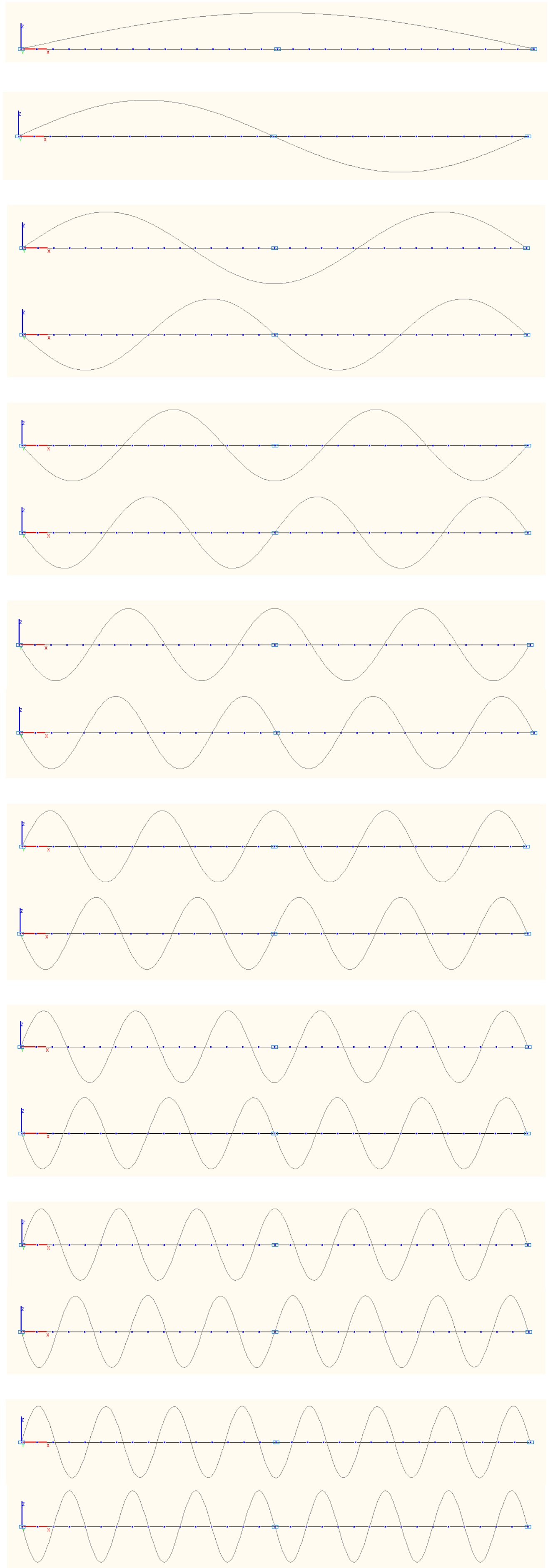
Finite element model: Design model – grade beam / plate, 32 bar elements of type 3. Boundary conditions of the simply supported ends of the beam are provided by imposing constraints in the direction of the degree of freedom Z. The dimensional stability of the design model is provided by imposing a constraint in the node of the cross-section along the symmetry axis of the beam in the direction of the degree of freedom UX. The distributed mass is specified by transforming the static load from the self-weight of the beam μ·g.
The calculation is performed in two stages: first the natural oscillation modes and natural frequencies *p* are determined by the modal analysis, and then the deflections η in the cross-section in the middle of the beam span with time are determined by the direct integration of the equations of motion method. The action of the uniformly distributed instantaneous transverse pulse is described by the graph of the load variation with time and is given in the form of nodal forces acting along the Z axis of the global coordinate system with the scale factor equal to the length of the bar finite element l / n = 0.25 m (n is the number of finite elements in the design model), and the delay time 0.0 s. Intervals between the time points of the load variation graph are equal to $\Delta t_{int} = 0.00001$ s and correspond to the integration step. When plotting the graph the pulse action is taken with a linear shape function, force value $P = S \cdot \Delta t_{int} = 80000$ tf and duration $\Delta t_{int} = 0.00001$

s. The duration of the process is equal to t = 0.05094 s, which corresponds to the value of the fundamental period T_1. Critical damping ratios for the 1-st and 2-nd natural frequencies are taken with the minimum value ξ = 0.0001. The conversion factor for the added static loading is equal to k = 0.981 (mass generation). Number of nodes in the design model – 33. The modal integration method is used in the calculation. The determination of the natural oscillation modes and natural frequencies is performed by the method of subspace iteration. The matrix of concentrated masses is used in the calculation.

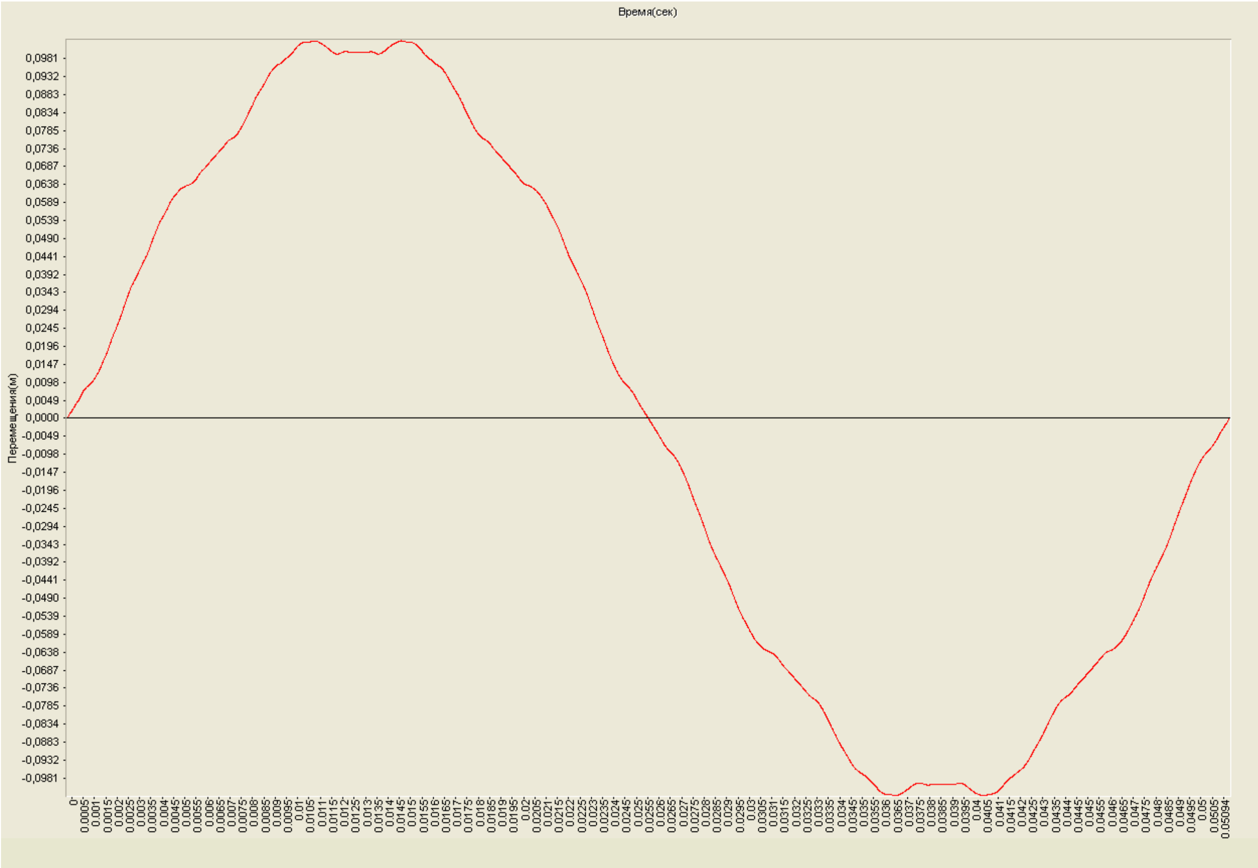
Results in SCAD



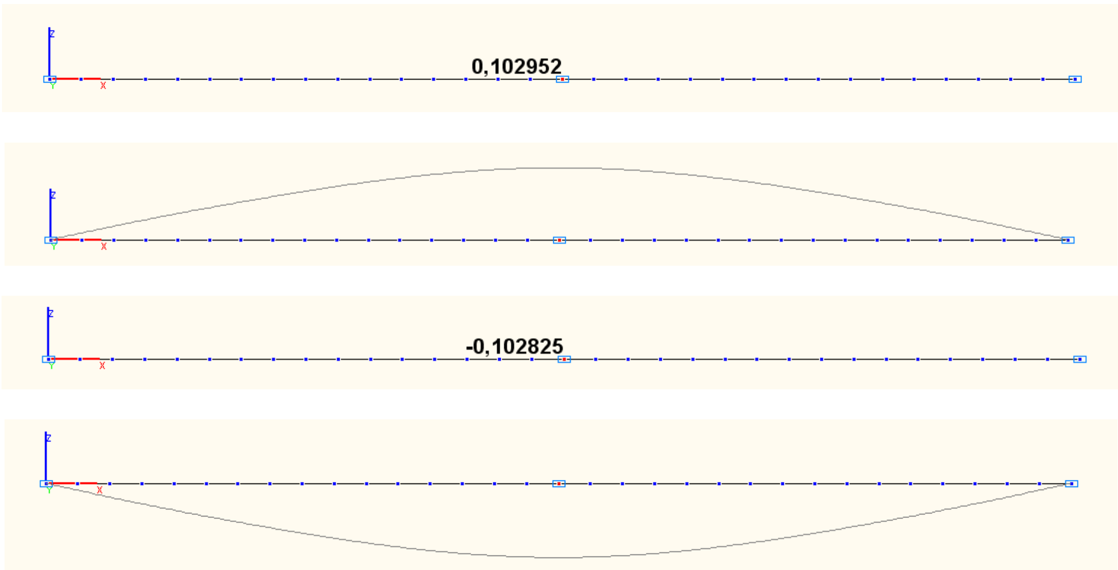
Design model



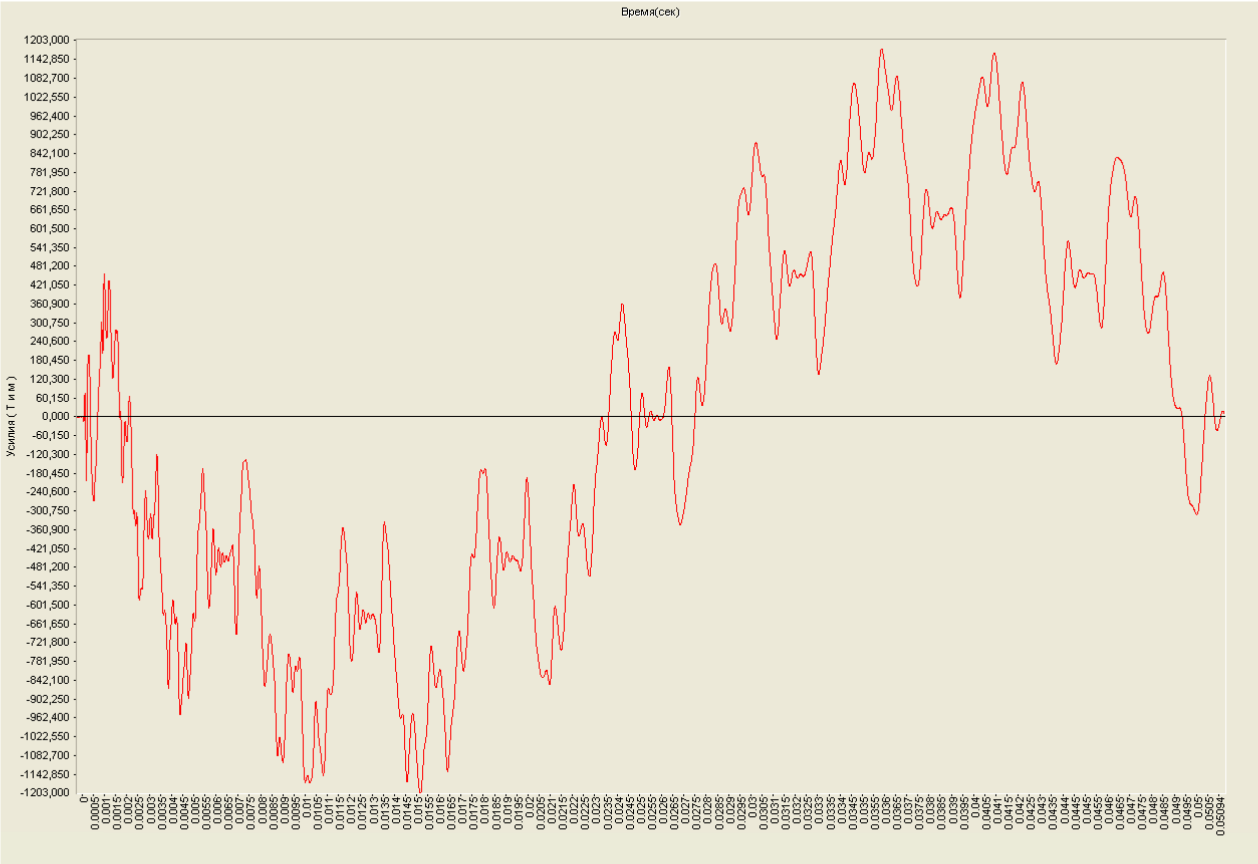
1-st — 16-th natural oscillation modes



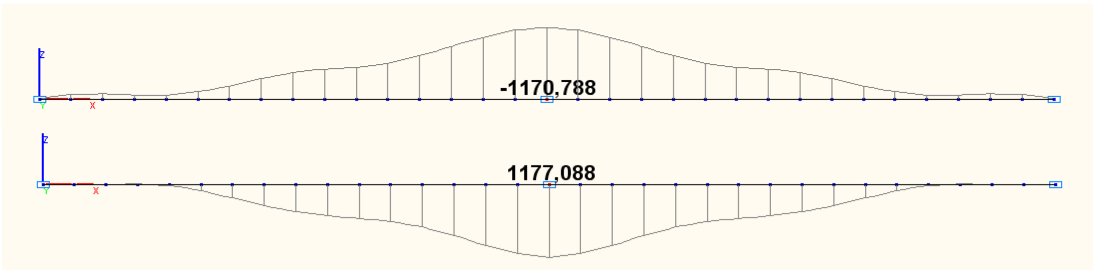
Graph of the variation of the deflection η in the cross-section in the middle of the beam span with time (m)



Amplitude values of the deflection η in the cross-section in the middle of the beam span and the deformed models at the respective time points (m)



Graph of the variation of the bending moment M
in the cross-section in the middle of the beam span with time (tm·m)



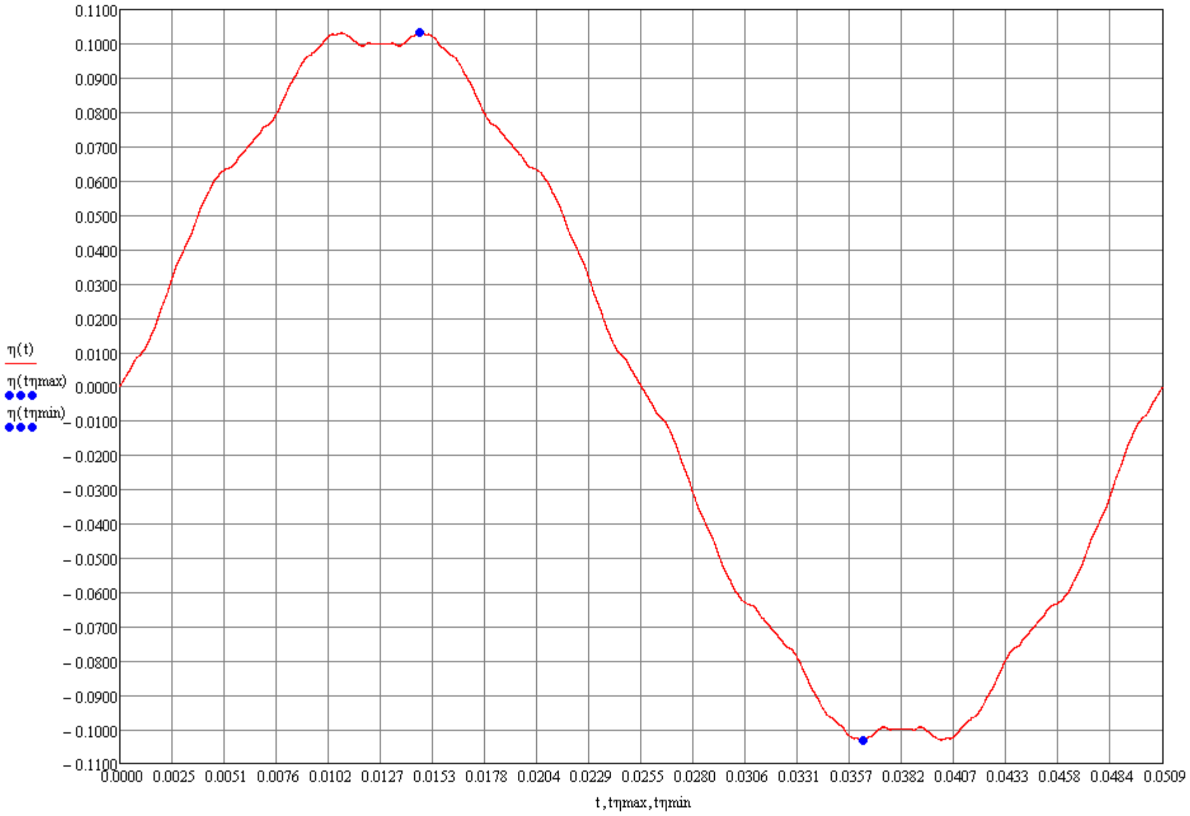
Amplitude values of the bending moment M
in the cross-section in the middle of the beam span (tm·m)

Comparison of solutions:

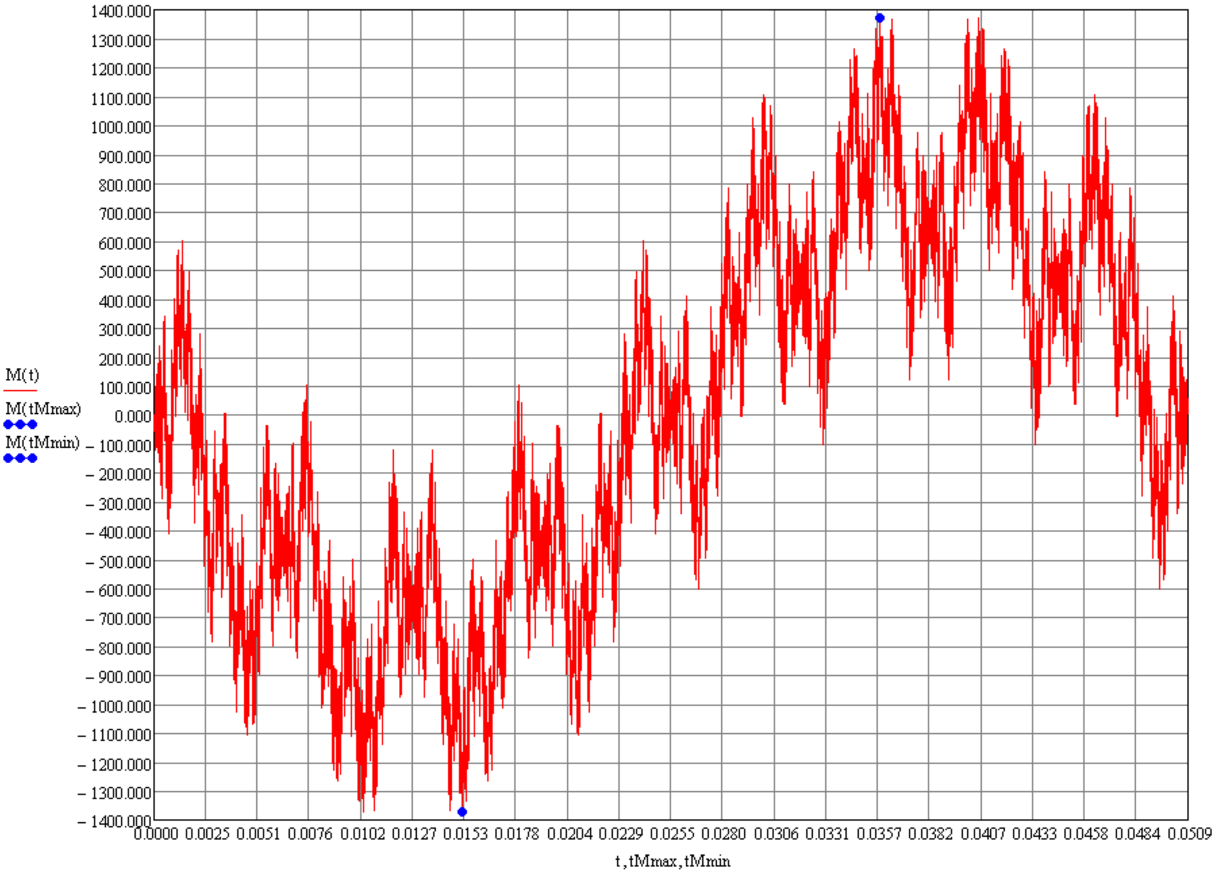
Natural frequencies *p*, rad/s

Oscillation mode	Theory	SCAD	Deviations, %
1	123.370	123.370	0.00
2	493.480	493.480	0.00
3	1110.330	1110.325	0.00
4	1973.921	1973.887	0.00
5	3084.251	3084.120	0.00
6	4441.322	4440.919	0.01
7	6045.133	6044.087	0.02
8	7895.684	7893.275	0.03
9	9992.974	9987.907	0.05
10	12337.005	12327.069	0.08
11	14927.777	14909.367	0.12
12	17765.288	17732.721	0.18
13	20849.539	20794.097	0.27

Oscillation mode	Theory	SCAD	Deviations, %
14	24180.531	24089.155	0.38
15	27758.262	27611.778	0.53
16	31582.734	31353.470	0.73



Graph of the variation of the deflection η in the cross-section in the middle of the beam span with time according to the theoretical solution (m)



Graph of the variation of the bending moment M
in the cross-section in the middle of the beam span with time
according to the theoretical solution (tm·m)

Amplitude values of the deflection η
in the cross-section in the middle of the beam span, m

Theory		**SCAD**		
Time, s	**Deflection, m**	**Time, s**	**Deflection, m**	**Deviations, %**
0.014617	0.103196	0.014660	0.102998	0.19
0.036313	-0.103196	0.036330	-0.102825	0.36

Amplitude value of the bending moment *M*
in the cross-section in the middle of the beam span, tf·m

Theory		**SCAD**		
Time, s	**Bending moment, tf·m**	**Time, s**	**Bending moment, tf·m**	**Deviations, %**
0.015162	-1369.739	0.015240	-1203.795	12.12
0.035768	1369.739	0.035710	1177.088	14.06

Notes: In the analytical solution the natural frequencies of oscillations p of the simply supported beam are determined according to the following formula:

$$\frac{n^2 \cdot \pi^2}{l^2} \cdot \sqrt{\frac{E \cdot I}{\mu}},$$

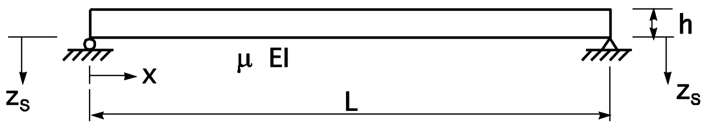
where $n = 1, 2, 3, 4, \ldots$ – natural mode number.

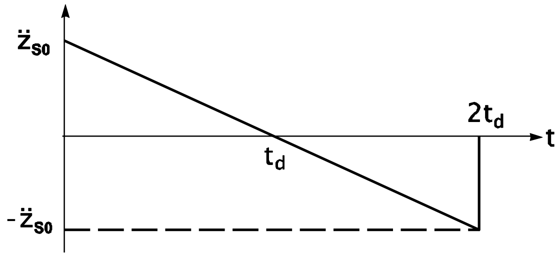
In the analytical solution the deflection η and the bending moment *M* in the cross-section in the middle of the beam span with time are determined according to the following formula:

$$\eta(t) = \frac{4 \cdot S \cdot l^2}{\pi^3 \cdot \sqrt{\mu \cdot E \cdot I}} \cdot \sum_{n=1} \left[\frac{\sin\left(\frac{n \cdot \pi}{2}\right) \cdot \sin\left(\frac{n^2 \cdot \pi^2}{l^2} \cdot \sqrt{\frac{E \cdot I}{\mu}} \cdot t\right)}{n^3} \right];$$

$$M(t) = \frac{4 \cdot S}{\pi} \cdot \sqrt{\frac{E \cdot I}{\mu}} \cdot \sum_{n=1} \left[\frac{\sin\left(\frac{n \cdot \pi}{2}\right) \cdot \sin\left(\frac{n^2 \cdot \pi^2}{l^2} \cdot \sqrt{\frac{E \cdot I}{\mu}} \cdot t\right)}{n} \right].$$

Simply Supported Beam with a Distributed Mass Subjected to a Kinematic Excitation of Supports (Seismic Action)





Objective: Determination of the stress-strain state of a simply supported beam with a distributed mass subjected to a kinematic excitation of supports.

Initial data file: **DIN_B_SL.SPR, DIN_B_SL.SPC**

Problem formulation: The simply supported beam of constant cross-section with the uniformly distributed mass μ is subjected to the kinematic excitation of supports according to the specified accelerogram:

$$\ddot{z}(t) = \ddot{z}_{s0} \cdot \left(1 - \frac{t}{t_d} \right).$$

Determine the natural oscillation mode and the fundamental natural frequency f of the simply supported beam, as well as the maximum amplitude values of the deflection z and the bending moment M in the cross-section in the middle of the beam span with time t.

References: John M. Biggs, Introduction to Structural Dynamics, McGraw-Hill Book Companies, New York, 1964, p.262.

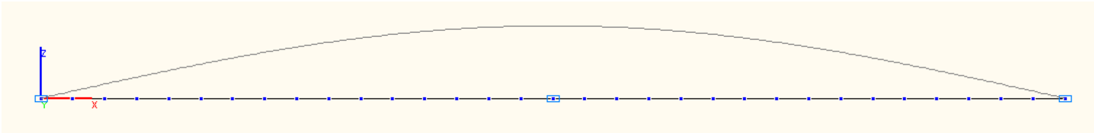
Initial data:

$E = 3.0 \cdot 10^7$ psi $= 2.1092 \cdot 10^7$ tf/m^2	- elastic modulus;
$I = 333.333$ in^4 $= 138.7448 \cdot 10^{-6}$ m^4	- cross-sectional moment of inertia of the beam.
h = 14 in = 0.3556 m	- height of the cross-section of the beam;
L = 240 in = 6.0960 m	- beam span length;
$\mu = 0.2$ lb·sec^2/in^2 $= 0.1406$ tf·s^2/m^2	- value of the uniformly distributed mass of the beam;
$\ddot{z}_{s0} = \pm 386.2200$ in/sec^2 $= \pm 9.81$ m/s^2	- amplitude values of the acceleration of the supports according to the accelerogram;
$t_d = 0.10$ sec = 0.10 s	- half-interval of the kinematic excitation of supports;
g = 386.2200 in/sec^2 =9.81 m/s^2	- gravitational acceleration.

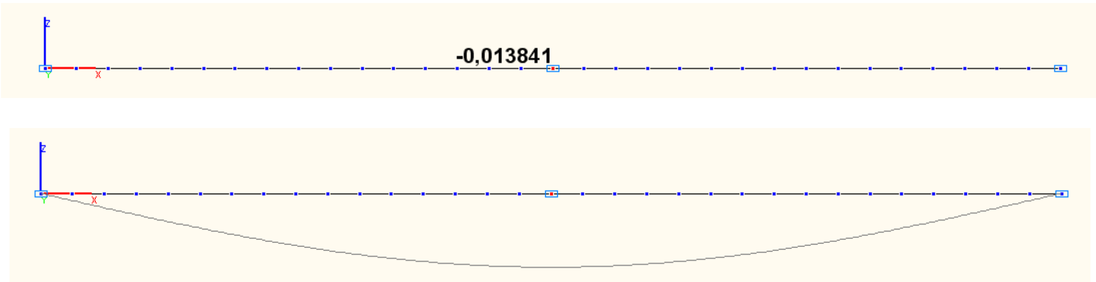
Finite element model: Design model – grade beam / plate, 32 bar elements of type 3. Boundary conditions of the simply supported ends of the beam are provided by imposing constraints in the direction of the degree of freedom Z. The dimensional stability of the design model is provided by imposing a constraint in the node of the cross-section along the symmetry axis of the beam in the direction of the degree of freedom UX. The distributed mass is specified by transforming the static load from the self-weight of the beam μ·g. The kinematic excitation of supports is described by the graph of the acceleration variation with time (accelerogram) and is given in the form of the action along the Z axis of the global coordinate system (direction cosines to the X, Y, Z axes: 0.00, 0.00, 1.00) with the scale factor to the values of the accelerogram equal to 1.00. The height of the beam structure in the model is directed along the Z axis of the global coordinate system. The dissipation factor (energy absorption factor) is taken with the minimum value of $\xi = 0.000001$. The intervals between the time points of the graph of the acceleration variation with time are equal to $\Delta t = 0.01$ s. When plotting the graph the acceleration is taken with the values $z(t) = \ddot{z}_{s0} \cdot \left(1 - n \cdot \Delta t / t_d\right)$ at the time points n·Δt. The conversion factor for the added static loading is equal to $k = 1.000$ (mass generation). Number of nodes in the design model – 33. The modal integration method is used in the calculation. The determination of the natural oscillation modes and natural frequencies is performed by the method of subspace iteration. The matrix of concentrated masses is used in the calculation.

Results in SCAD

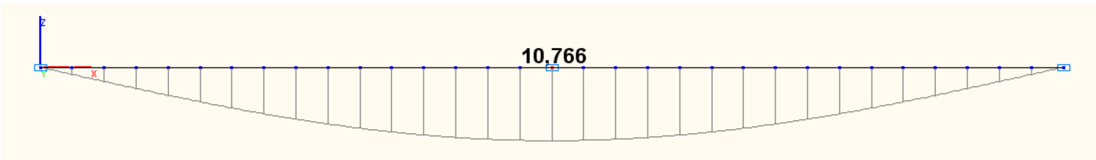
Design model and the specified accelerogram



1-st natural oscillation mode



Amplitude value of the deflection z in the cross-section in the middle of the beam span and the deformed model at the respective time point (m)



Amplitude value of the bending moment M
in the cross-section in the middle of the beam span (tm·m)

Comparison of solutions:

Fundamental natural frequency *f*, Hz

Oscillation mode	**Theory**	**SCAD**	**Deviations, %**
1	6.098	6.101	0.05

Maximum amplitude value of the deflection z
in the cross-section in the middle of the beam span, m

Theory		SCAD	
Time, s	**Deflection, m**	**Deflection, m**	**Deviations, %**
0.0163982	-0.013951	-0.013841	0.80

Maximum amplitude value of the bending moment M
in the cross-section in the middle of the beam span, tf·m

Theory		SCAD	
Time, s	**Bending moment, tf·m**	**Bending moment, tf·m**	**Deviations, %**
0.0163982	10.843	10.766	0.73

Notes: In the analytical solution the fundamental natural frequency f of the simply supported beam is determined according to the following formula:

$$f = \frac{\pi}{2 \cdot L^2} \cdot \sqrt{\frac{E \cdot I}{\mu}} .$$

In the analytical solution the deflection z and the bending moment M in the cross-section in the middle of the beam span with time are determined according to the following formula:

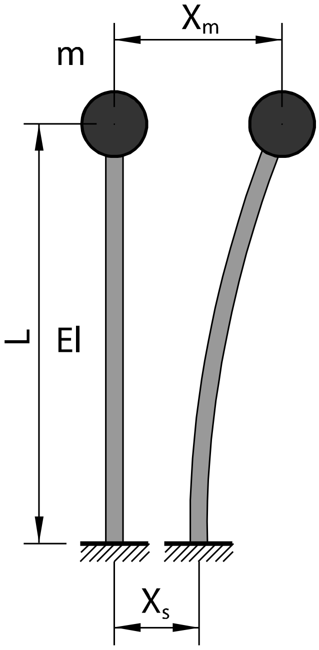
$$z(t) = \frac{4 \cdot z_{s0} \cdot L^4 \cdot \mu}{\pi^5 \cdot E \cdot I} \cdot \left[1 - \cos\left(\frac{\pi^2}{L^2} \cdot \sqrt{\frac{E \cdot I}{\mu}} \cdot t \right) + \frac{L^2}{\pi^2 \cdot t_d} \cdot \sqrt{\frac{\mu}{E \cdot I}} \cdot \sin\left(\frac{\pi^2}{L^2} \cdot \sqrt{\frac{E \cdot I}{\mu}} \cdot t \right) - \frac{1}{t_d} \cdot t \right] \quad \text{at}$$

$$t \le 2 \cdot t_d \, ;$$

$$M(t) = \frac{4 \cdot z_{s0} \cdot L^2 \cdot \mu}{\pi^3} \cdot \left[1 - \cos\left(\frac{\pi^2}{L^2} \cdot \sqrt{\frac{E \cdot I}{\mu}} \cdot t \right) + \frac{L^2}{\pi^2 \cdot t_d} \cdot \sqrt{\frac{\mu}{E \cdot I}} \cdot \sin\left(\frac{\pi^2}{L^2} \cdot \sqrt{\frac{E \cdot I}{\mu}} \cdot t \right) - \frac{1}{t_d} \cdot t \right] \quad \text{at}$$

$$t \le 2 \cdot t_d \, .$$

Cantilever Weightless Column with a Concentrated Mass at the Free End Subjected to a Horizontal Kinematic Displacement of a Support (Seismogram Based Analysis)



Objective: Determination of the strain state of a cantilever weightless column with a concentrated mass at the free end subjected to a horizontal kinematic displacement of a support.

Initial data file: 5.14.SPR
Seismogram file: 5.14_chart.txt

Problem formulation: The mass m is attached to the free end of the cantilever weightless column with a square cross-section. A horizontal kinematic action varying according to the harmonic law $X_s = \Delta \cdot \sin(\theta \cdot t)$ is applied to the support of the column at the initial time. Determine the natural oscillation mode and frequency ω of the cantilever column, as well as the deflection X_m of the free end of the column with the attached mass with time.

References: Kiselev V.A., Structural Mechanics. Special Course. Dynamics and Stability of Structures. Moscow, Stroyizdat, 1980, p. 65.

Initial data:

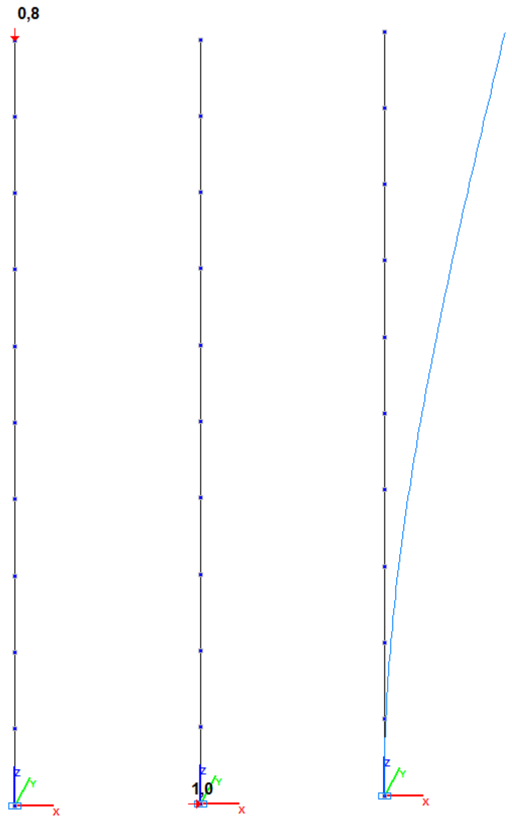
$E = 2.0 \cdot 10^8$ kN/m^2	- elastic modulus of the column material;
ν = 0.3	- Poisson's ratio;
b = 0.04 m	- width of the rectangular cross-section of the column;
h = 0.04 m	- height of the rectangular cross-section of the column;
L = 1.0 m	- length of the column;
m = 0.08 kN·s^2/m	- value of the concentrated mass attached to the free end of the column;
Δ = 0.1 m	- amplitude value of the horizontal kinematic harmonic excitation applied to the support of the column;
g = 10.00 m/s^2	- gravitational acceleration;
$I = b \cdot h^3/12 = 2.133333 \cdot 10^{-7}$ m^4	- cross-sectional moment of inertia of the column.

The following value of the frequency of the kinematic harmonic excitation θ depending on the value of the natural frequency of the column ω is considered: $\theta = 0.5 \cdot \omega$.

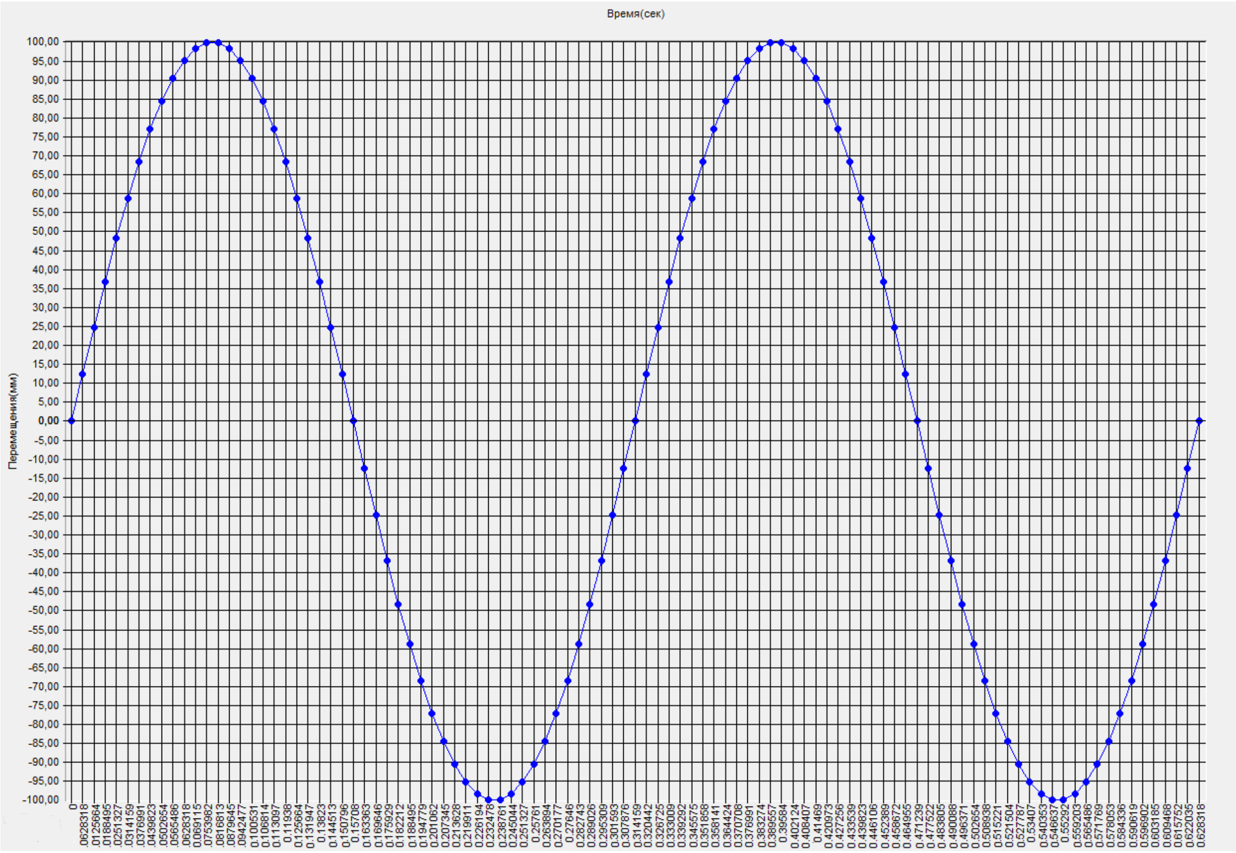
Finite element model: Design model – general type system, 10 bar elements of type 5. Boundary conditions are provided by imposing constraints in the node of the clamped end of the column in the directions of the degrees of freedom X, Y, Z, UX, UY, UZ. The concentrated mass is specified by transforming the static nodal load on the free end of the column *m·g*.

The calculation is performed in two stages: first the natural oscillation mode and natural frequency ω are determined by the modal analysis, and then the deflection X_m of the free end of the column with the attached mass with time is determined by the direct integration of the equations of motion method. The action of the kinematic harmonic excitation is described by the graph of the variation of the horizontal displacement of the support with time and is given in the form of the specified displacement of the constraint along the X axis of the global coordinate system with the scale factor of 1.0 and the delay time 0.0 s. Intervals between the time points of the displacement variation graph are equal to $\Delta t_{int} = T_\theta/100$, where T_θ – period of the kinematic harmonic excitation, and correspond to the integration step. When plotting the graph, the action of the specified displacement of the constraint is taken as $X_s = \Delta \cdot \sin(\theta \cdot n \cdot \Delta t_{int})$ at the time points $n \cdot \Delta t_{int}$. The duration of the process is equal to $t = 2 \cdot T_\theta$. Critical damping ratios for the 1-st and 2-nd natural frequencies are taken with the minimum value $\xi = 0.0001$. The conversion factor for the added static loading is equal to k = 0.981 (mass generation). Number of nodes in the design model – 11. The modal integration method is used in the calculation. The determination of the natural oscillation modes and natural frequencies is performed by the method of subspace iteration. The matrix of concentrated masses is used in the calculation.

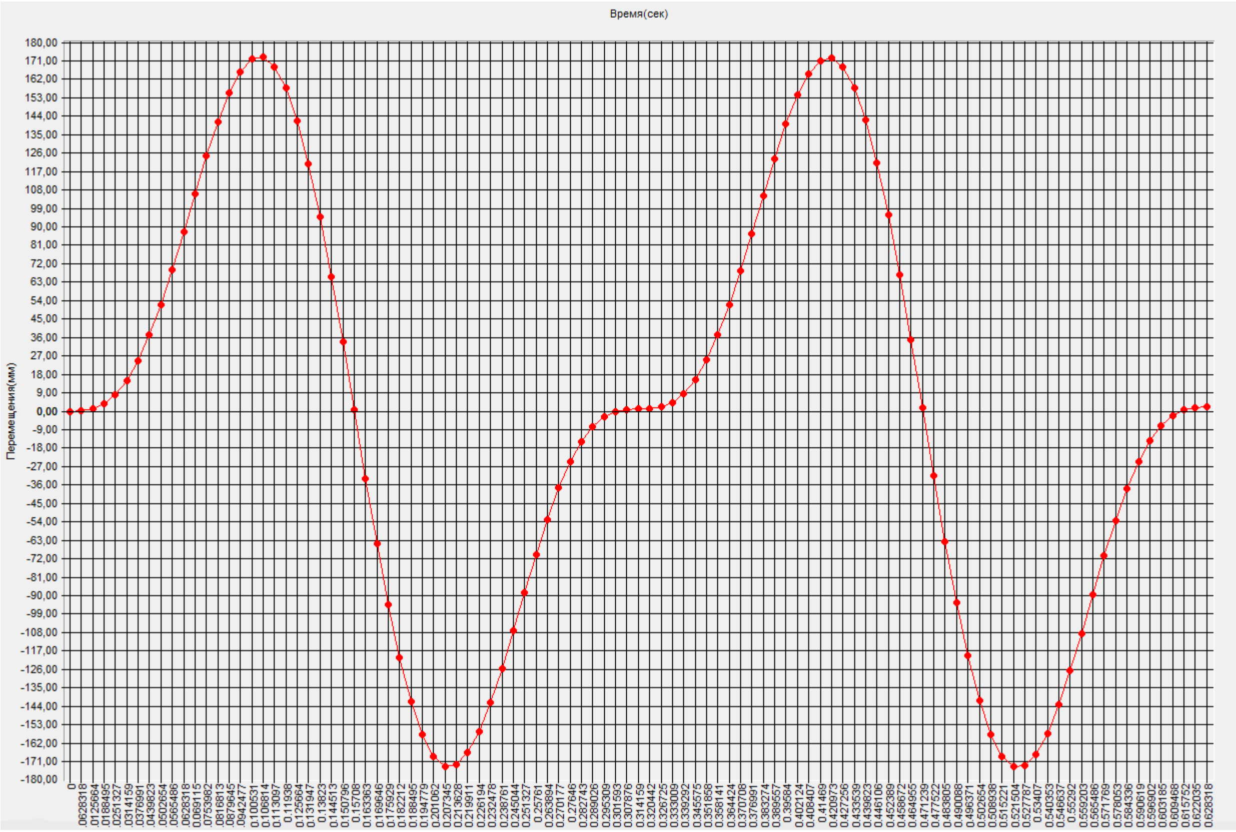
Results in SCAD



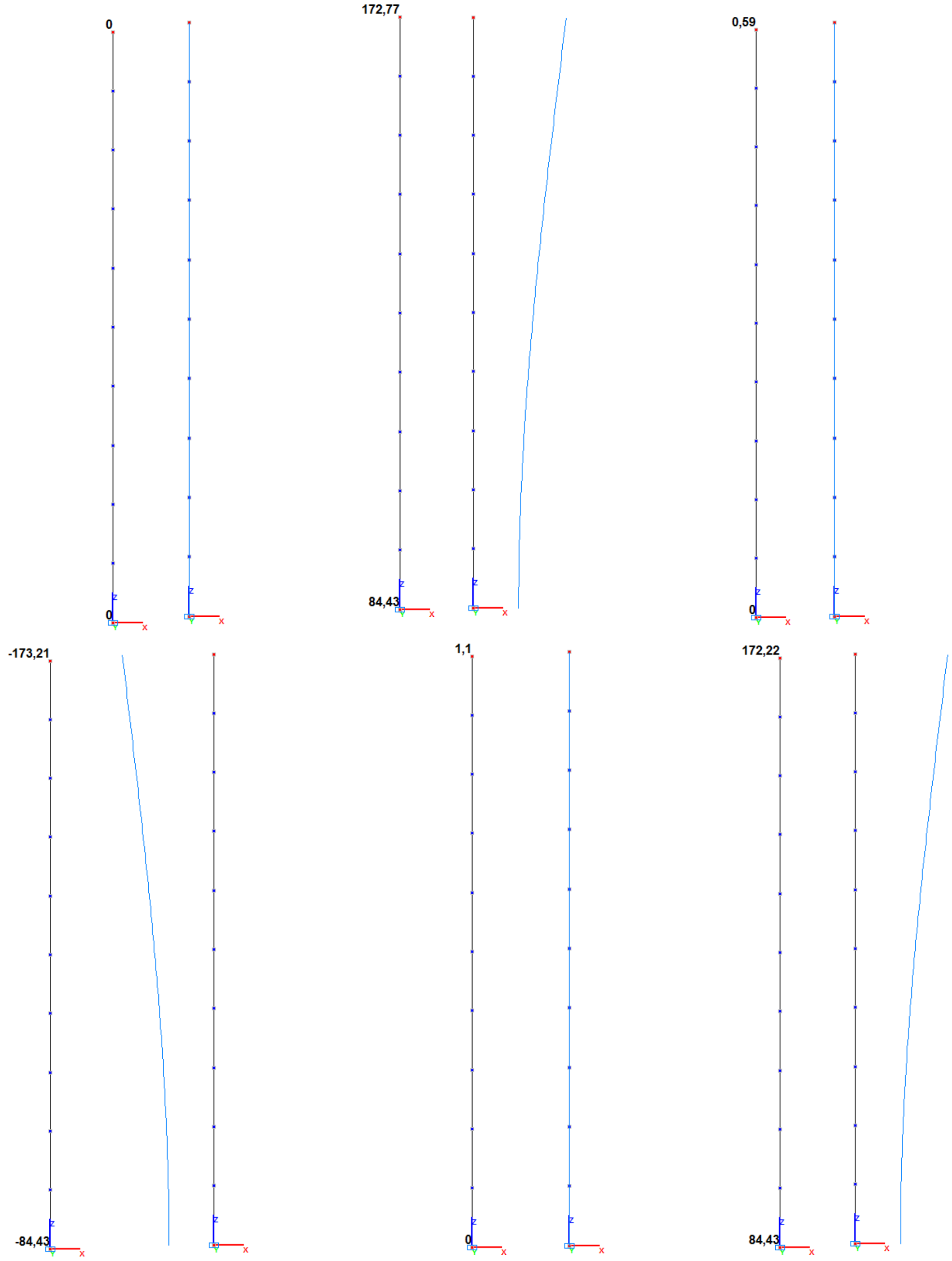
Design model and the 1-st oscillation mode

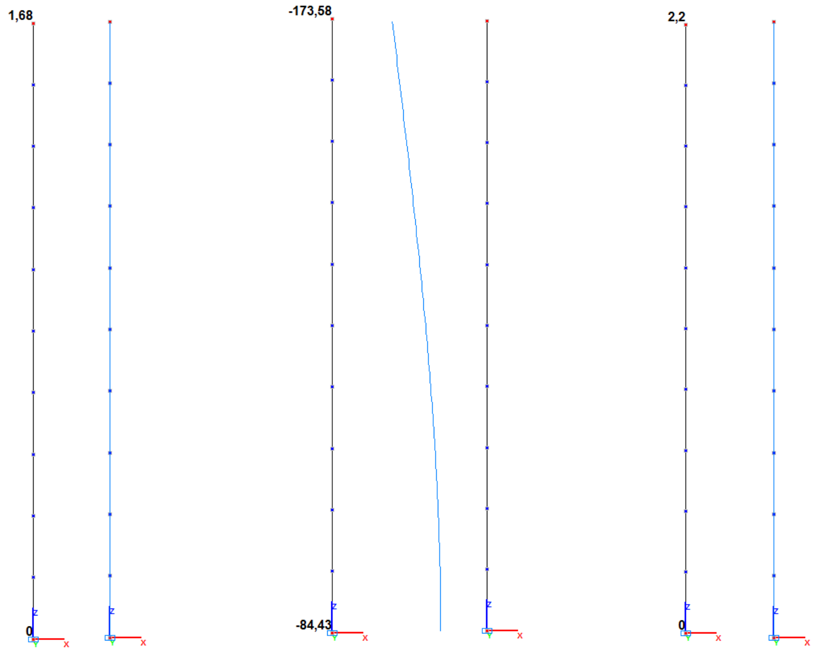


Graph of the variation of the horizontal displacement of the constraint X_s with time (mm).



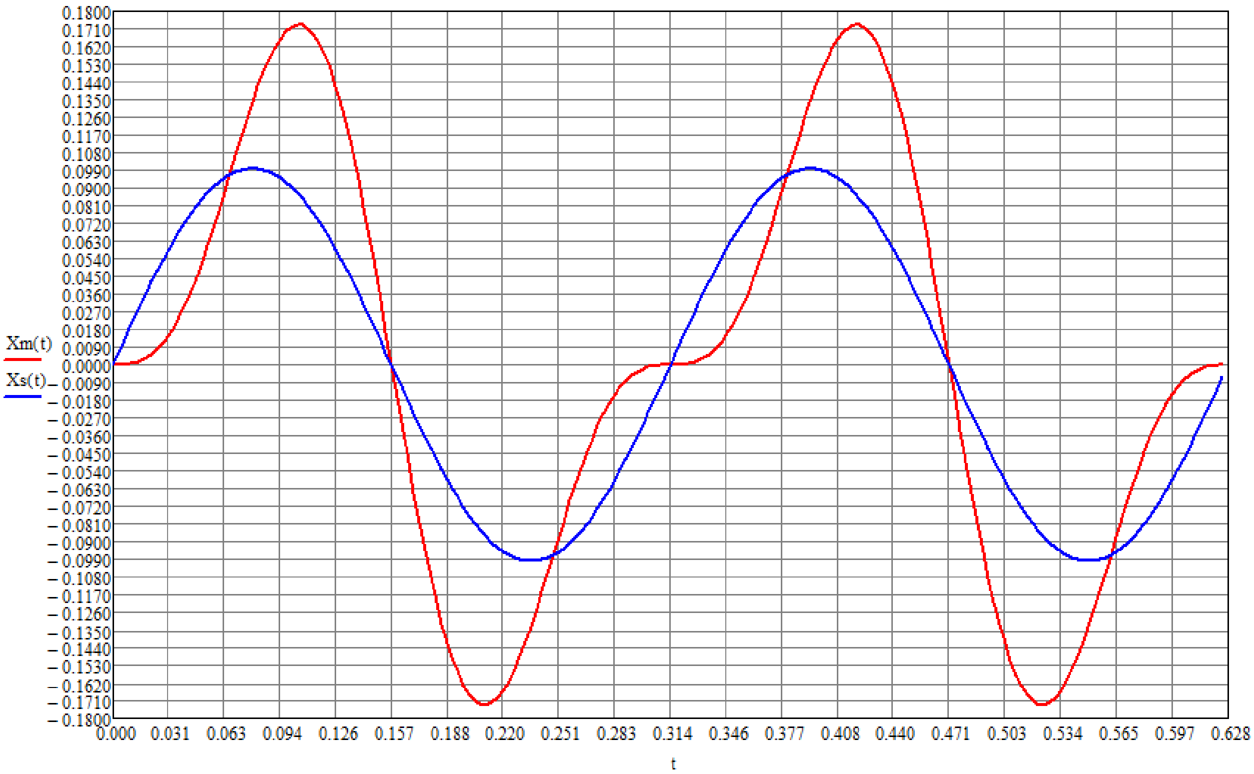
Graph of the variation of the deflection X_m of the free end of the column with the attached mass with time (mm)


0
0
172,77
84,43
0,59
0
-173,21
-84,43
1,1
0
172,22
84,43



Amplitude values of the deflection X_m of the free end of the column with the attached mass and the deformed models at the respective time points (mm).

Comparison of solutions:



Graphs of the variation of the horizontal displacement of the constraint X_s and the deflection X_m of the free end of the column with the attached mass with time (m)
Frequency of the kinematic harmonic excitation θ = 0.5·ω

Natural frequency ω, rad/s

Oscillation mode	Theory	SCAD	Deviations, %
1	40.000	40.000	0.00

Amplitude values of the deflection X_m of the free end of the column with the attached mass at the frequency of the kinematic harmonic excitation $\theta = 0.5 \cdot \omega$, mm

Theory		SCAD		
Time, s	Deflection, m	Time, s	Deflection, m	Deviation, %
0.000000	0.00	0.000000	0.00	—
0.106814	172.90	0.106814	172.77	0.08
0.157080	0.00	0.157080	0.59	—
0.207345	-172.90	0.207345	-173.21	0.18
0.314159	0.00	0.314159	1.10	—
0.420973	172.90	0.420973	172.22	0.39
0.471239	0.00	0.471239	1.68	—
0.521504	-172.90	0.521504	-173.58	0.39
0.628318	0.00	0.628318	2.20	—

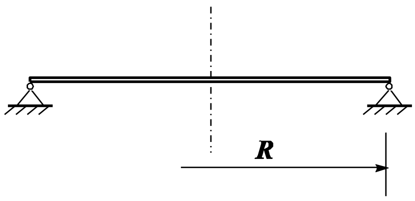
Notes: In the analytical solution the natural frequency ω of the cantilever column with the concentrated mass on the free end is determined according to the following formula:

$$\omega = \sqrt{\frac{3 \cdot E \cdot I}{m \cdot L^3}} .$$

In the analytical solution the deflection X_m of the free end of the column with the attached mass with time is determined according to the following formula:

$$X_m(t) = \frac{\Delta}{\left(1 - \frac{\theta^2}{\omega^2}\right)} \cdot \left(sin(\theta \cdot t) - \frac{\theta}{\omega} \cdot sin(\omega \cdot t) \right) .$$

Natural Oscillations of a Simply Supported Circular Plate



Objective: Modal analysis of a simply supported circular plate.

Initial data file: **5.7.SPR**

Problem formulation: Determine the natural oscillation modes and frequencies ω of the simply supported circular plate with the density of the material ρ.

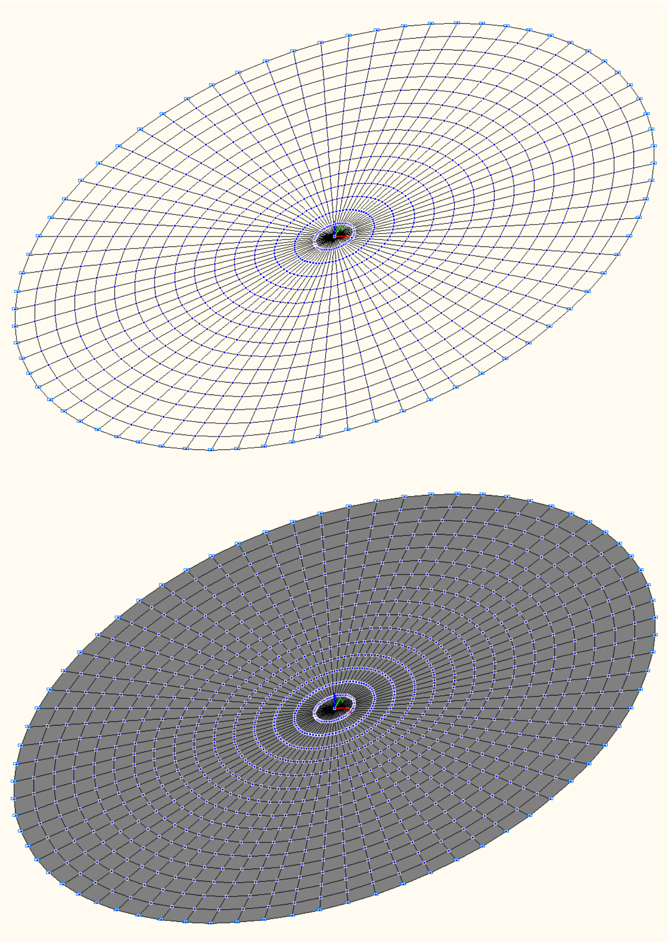
References: Chelomei V.N., Vibrations in Technology, Handbook in six volumes: Bolotin V.V., Volume 1, Vibrations of Linear Systems, Moscow, Mechanical engineering, 1978, p. 207.

Initial data:

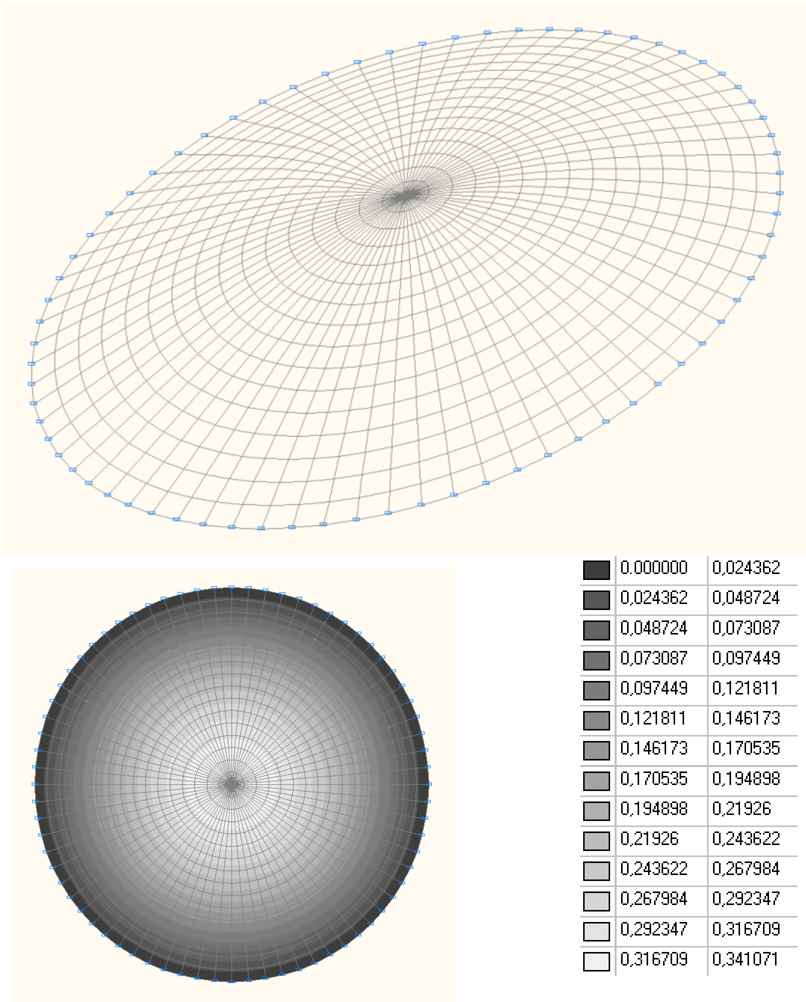
$E = 2.06 \cdot 10^8$ kPa	- elastic modulus;
$\nu = 0.3$	- Poisson's ratio;
$\rho = 7.85$ t/m^3	- density of the material;
h = 0.01 m	- thickness of the plate;
R = 0.5 m	- outer radius of the plate.

Finite element model: Design model – grade beam / plate, 1080 four-node plate elements of type 20 and 72 three-node plate elements of type 15. The spacing of the finite element mesh in the radial direction is 0.03125 m and in the tangential direction is 5.0º. Boundary conditions are provided by imposing constraints in the direction of the degree of freedom Z along the outer contour of the plate. The distributed mass is specified by transforming the static load from the self-weight of the plate ow = $\gamma \cdot h$, where $\gamma = \rho \cdot g = 77.01$ kN/m^3. Number of nodes in the design model – 1153. The determination of the natural oscillation modes and natural frequencies is performed by the method of subspace iteration. The matrix of concentrated masses is used in the calculation.

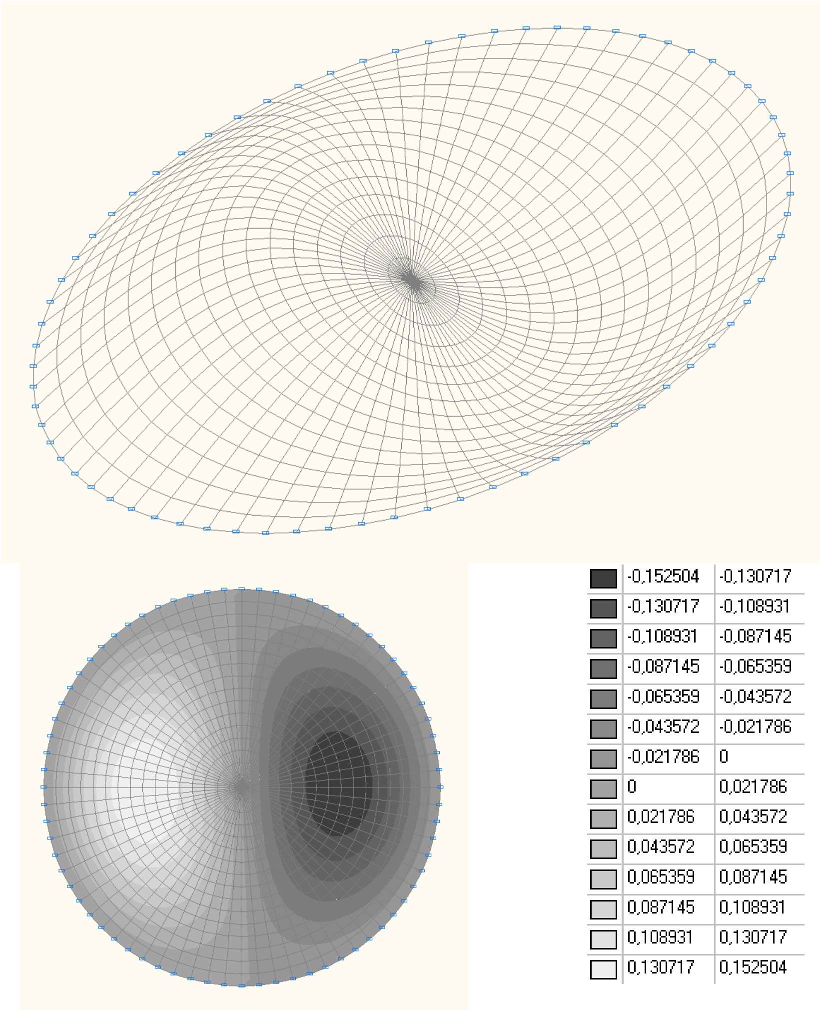
Results in SCAD



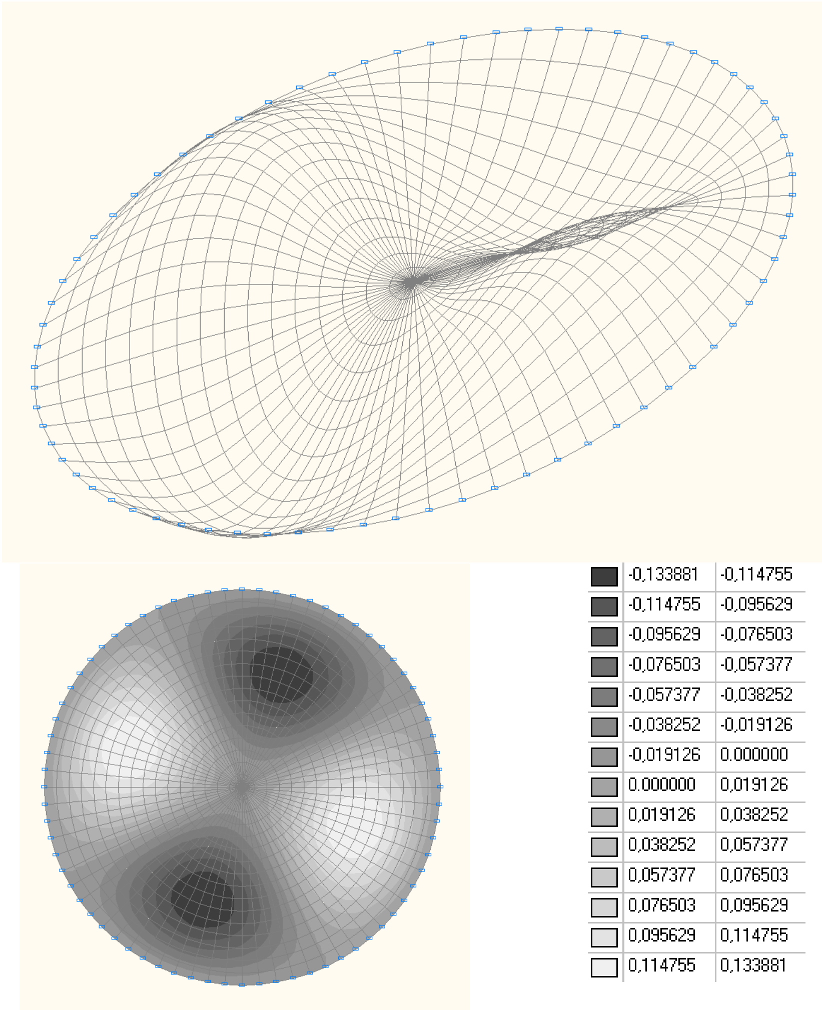
Design model



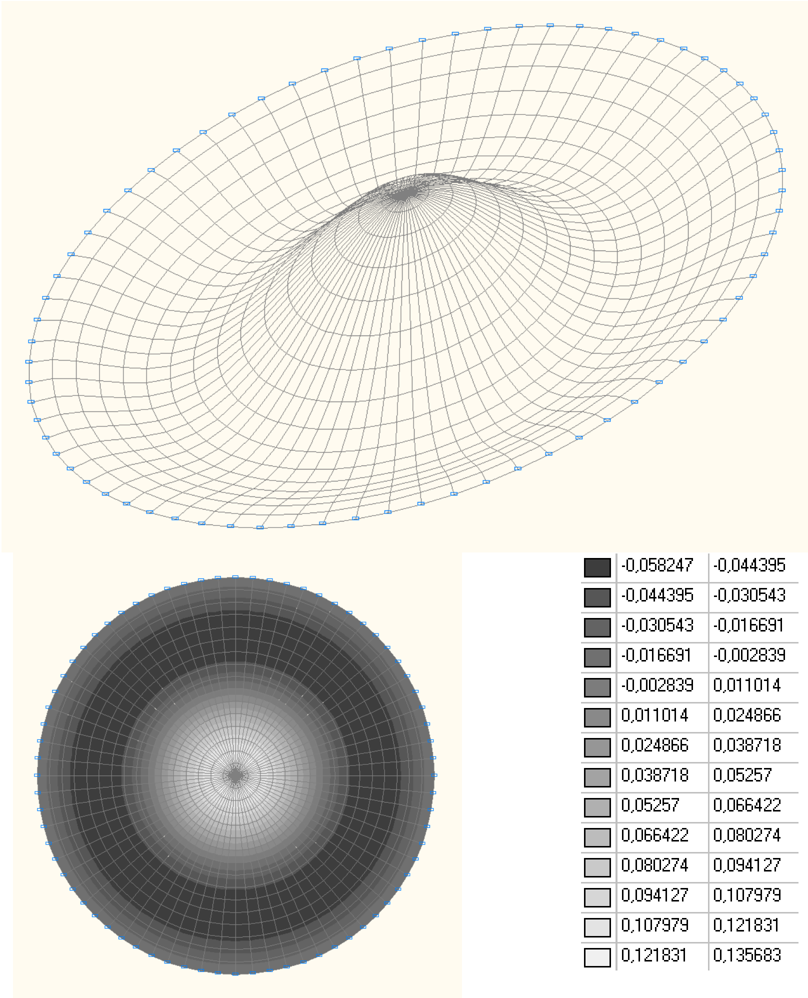
1-st natural oscillation mode



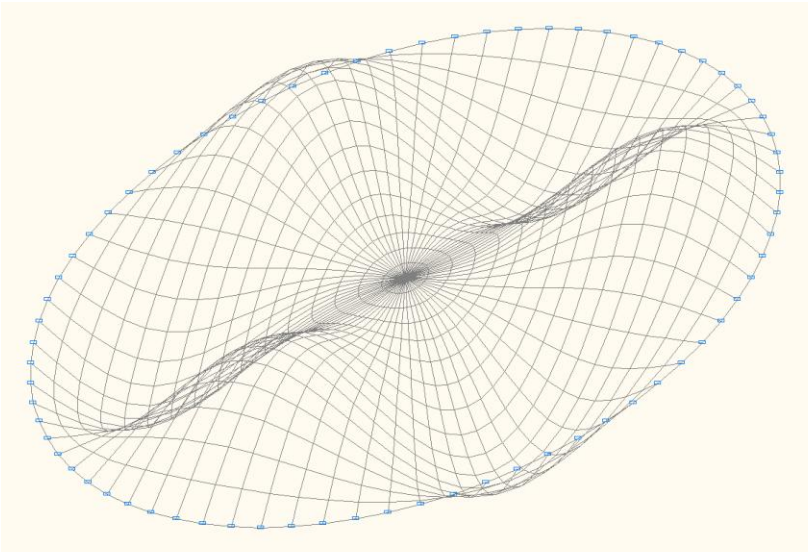
2-nd natural oscillation mode

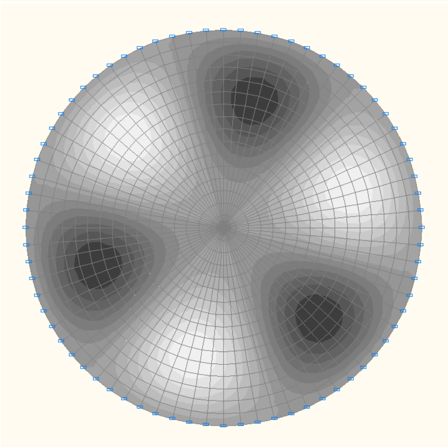


4-th natural oscillation mode



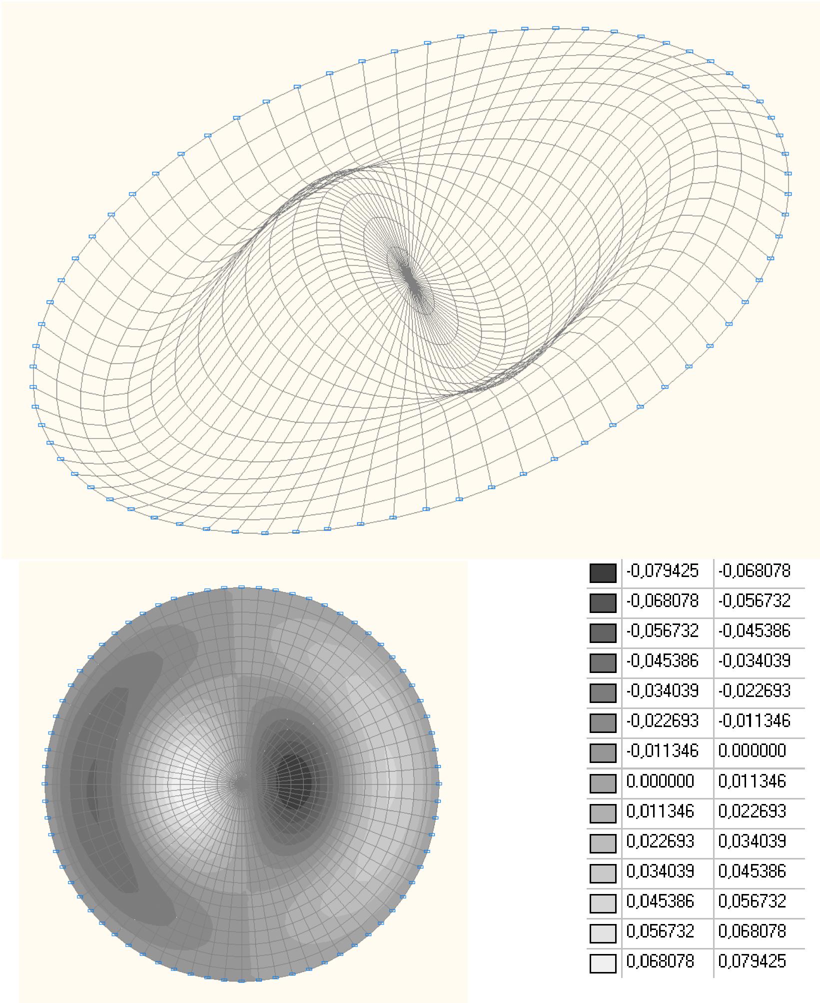
6-th natural oscillation mode



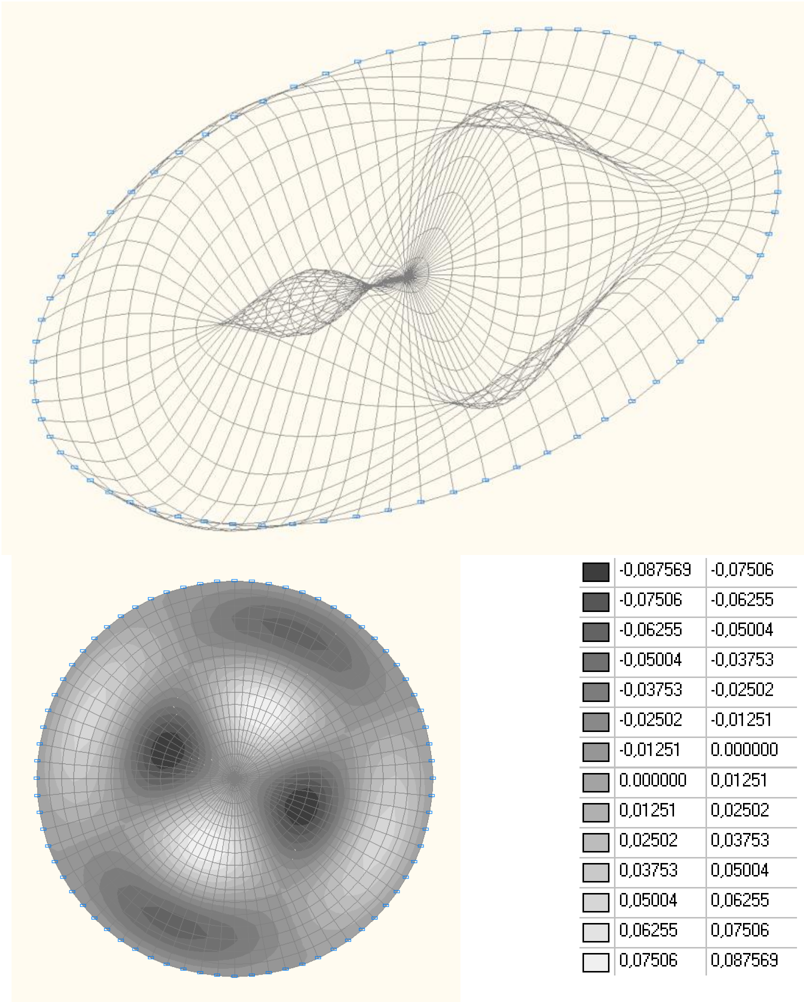


	-0,105057	-0,090049
	-0,090049	-0,075041
	-0,075041	-0,060033
	-0,060033	-0,045025
	-0,045025	-0,030016
	-0,030016	-0,015008
	-0,015008	0.000000
	0.000000	0,015008
	0,015008	0,030016
	0,030016	0,045025
	0,045025	0,060033
	0,060033	0,075041
	0,075041	0,090049
	0,090049	0,105057

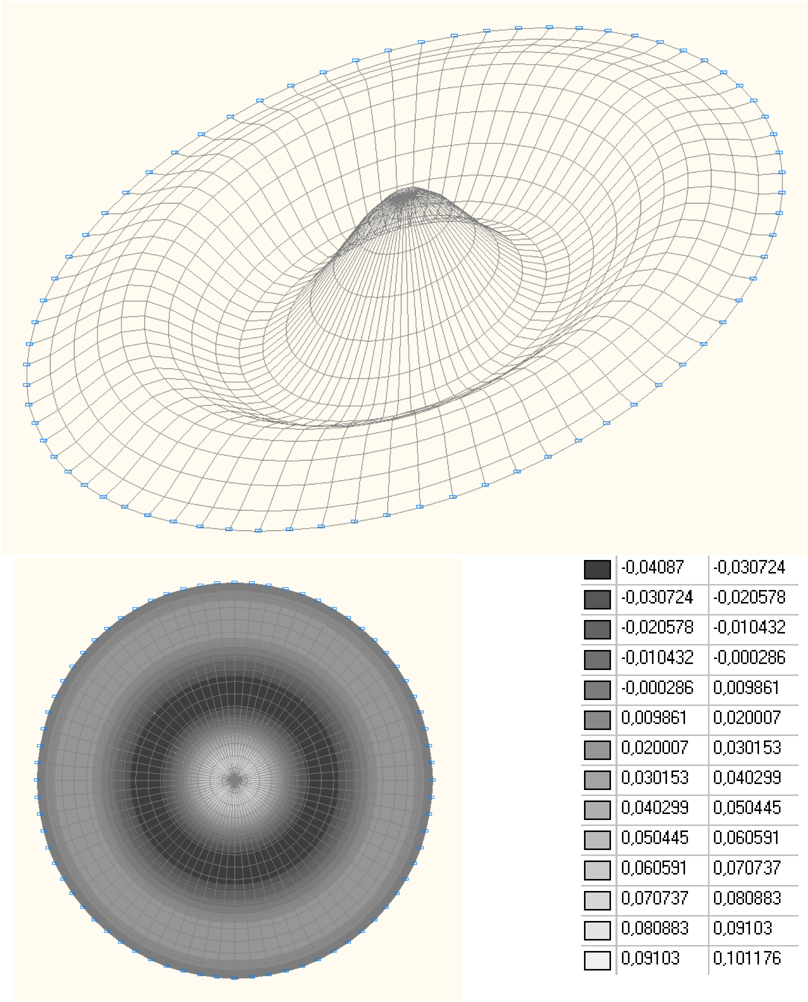
7-th natural oscillation mode



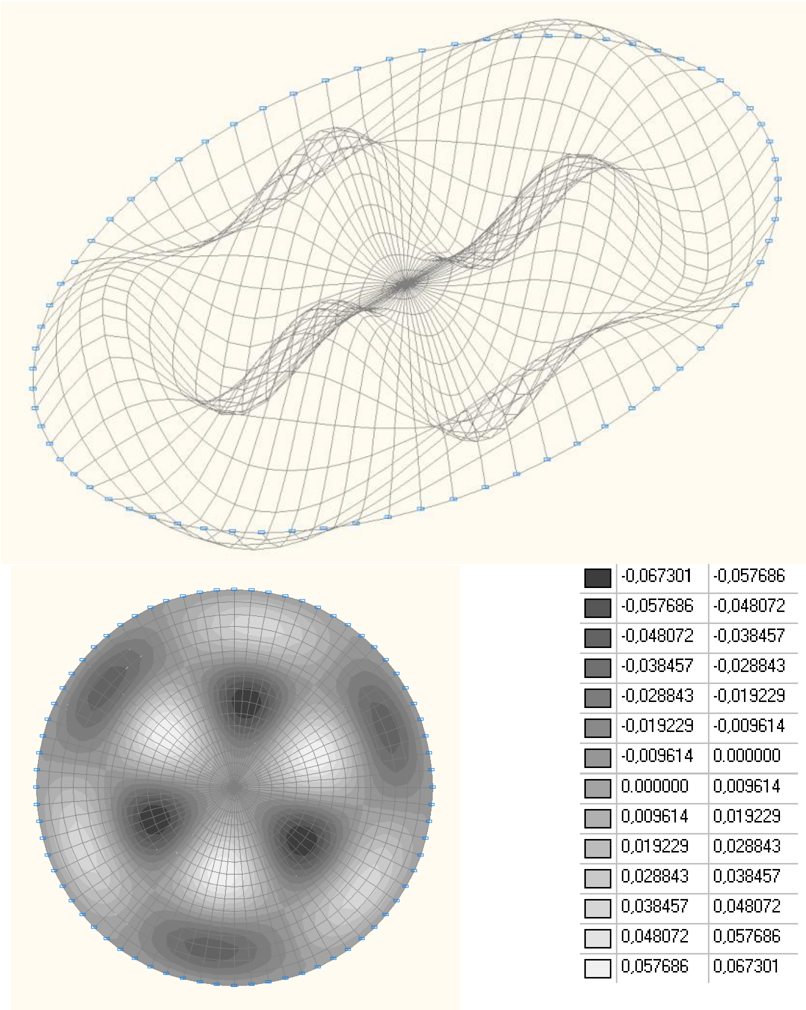
9-th natural oscillation mode



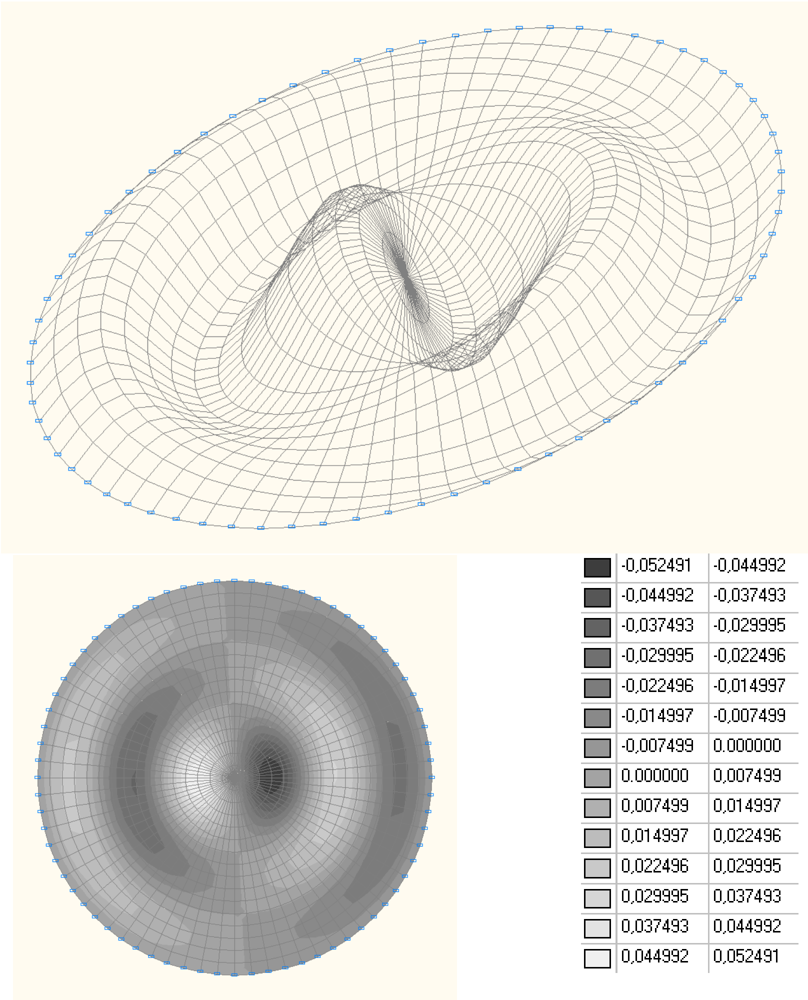
13-th natural oscillation mode



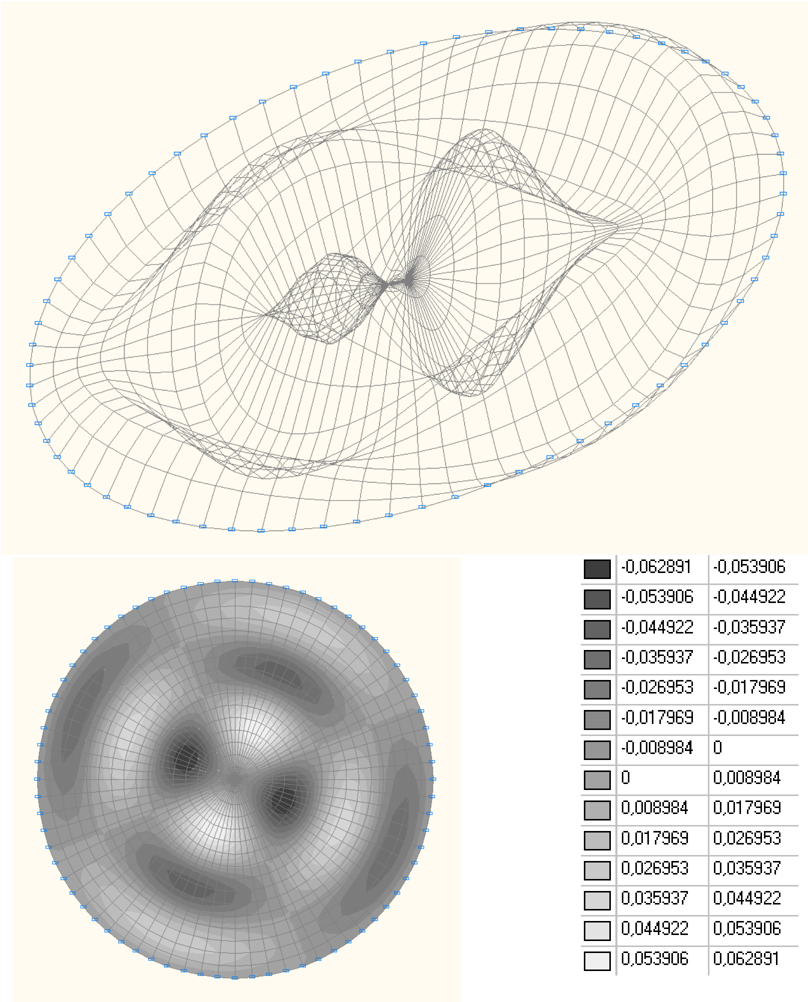
15-th natural oscillation mode



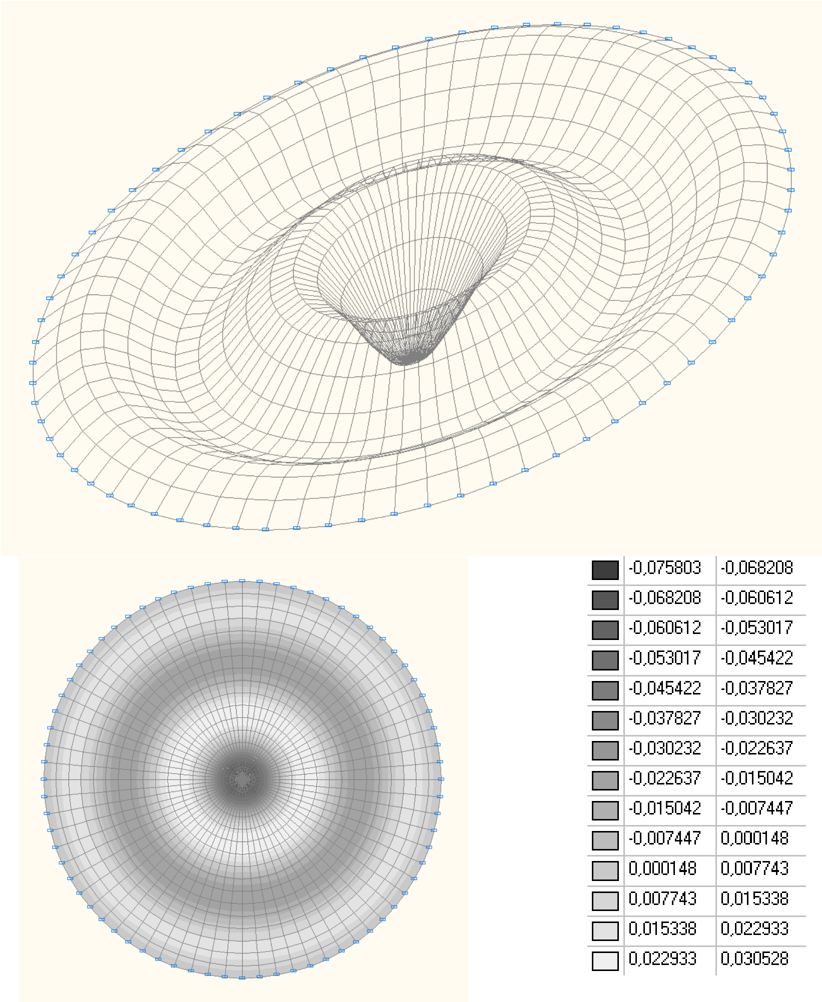
18-th natural oscillation mode



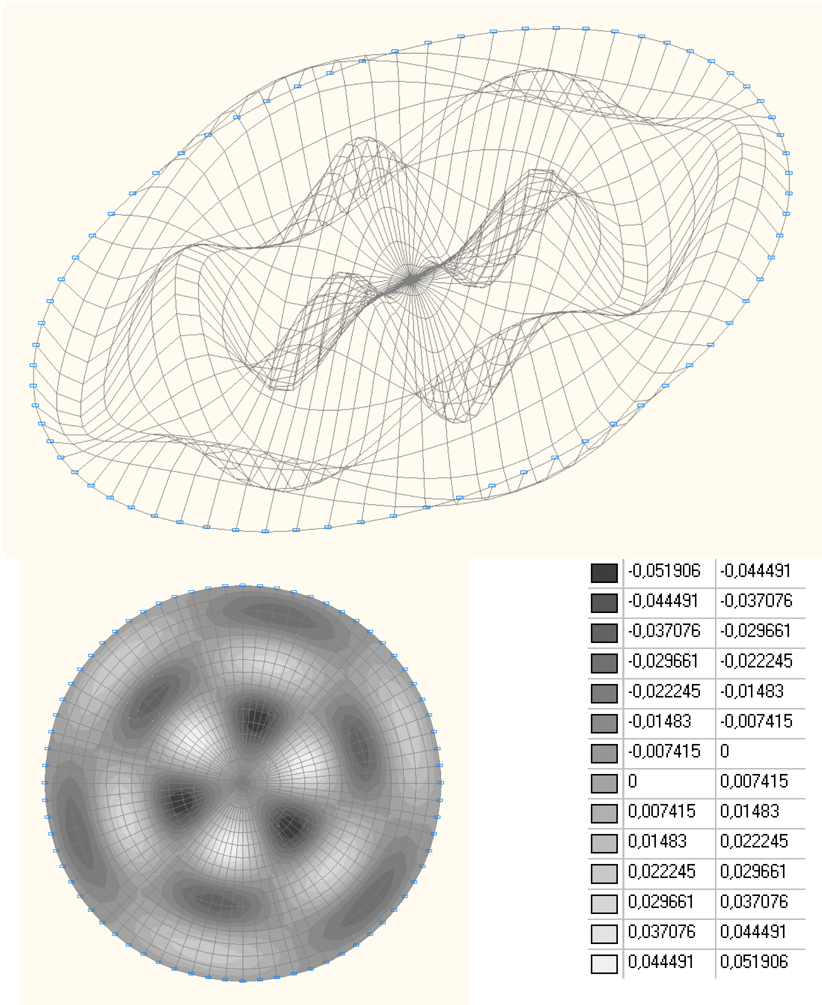
22-nd natural oscillation mode



28-th natural oscillation mode

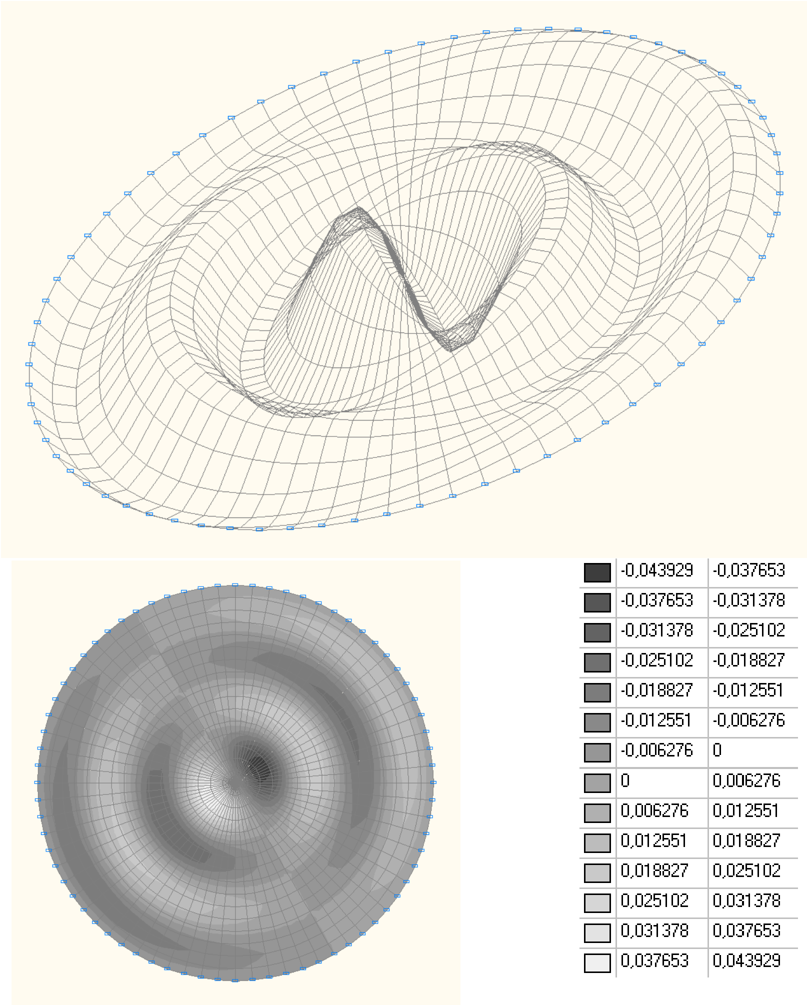


30-th natural oscillation mode

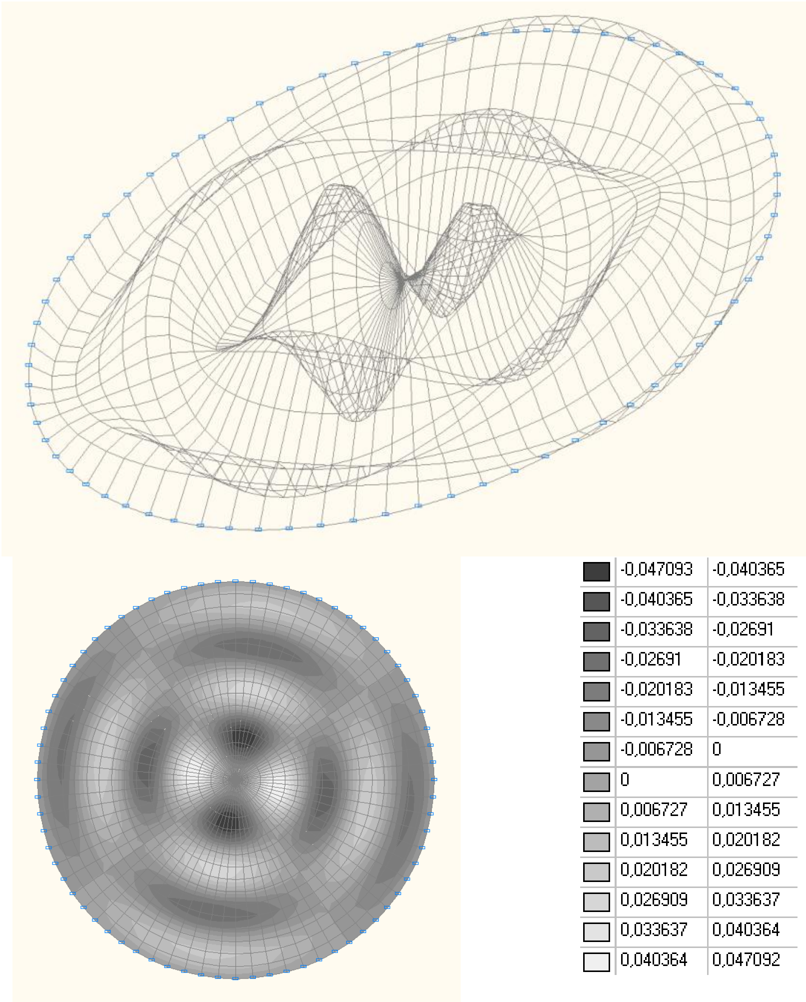


	-0,051906	-0,044491
	-0,044491	-0,037076
	-0,037076	-0,029661
	-0,029661	-0,022245
	-0,022245	-0,01483
	-0,01483	-0,007415
	-0,007415	0
	0	0,007415
	0,007415	0,01483
	0,01483	0,022245
	0,022245	0,029661
	0,029661	0,037076
	0,037076	0,044491
	0,044491	0,051906

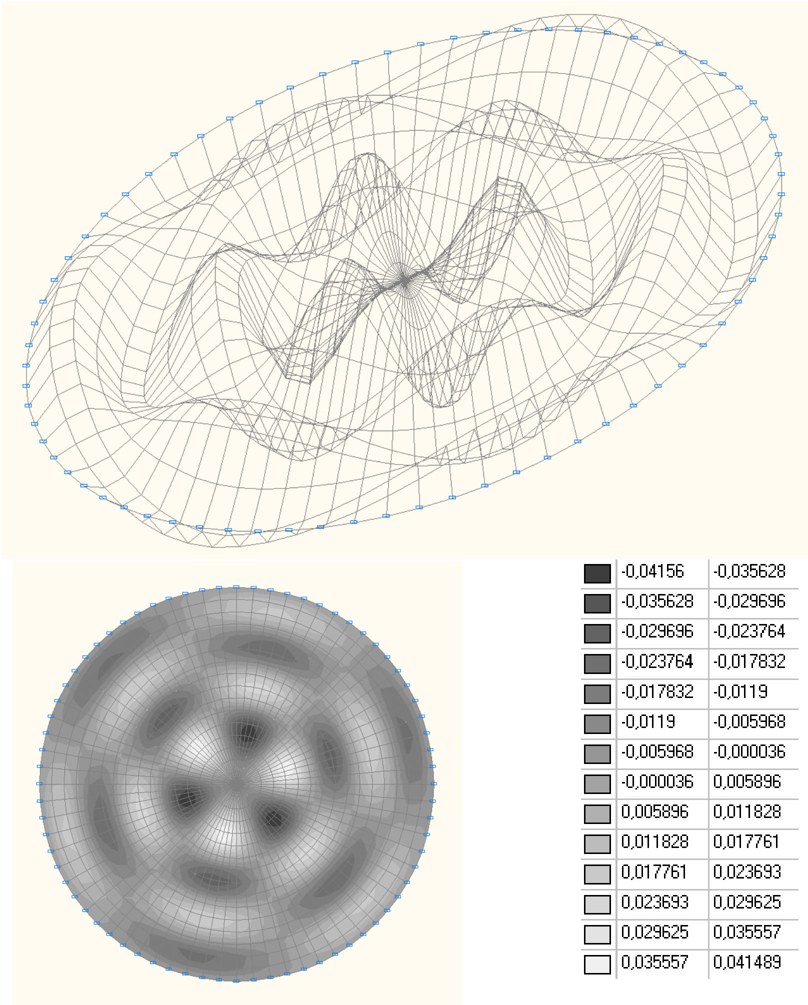
35-th natural oscillation mode



37-th natural oscillation mode



50-th (47-th theoretical) natural oscillation mode



58-th natural oscillation mode

Comparison of solutions:

Natural frequencies ω, rad / s

Oscillation mode	Number of nodal circles m and diameters n	Theory	SCAD	Deviations, %
1	0, 0	306.0	305.8	0.07
2, 3	0, 1	861.8	862.4	0.07
4, 5	0, 2	1588.2	1590.5	0.14
6	1, 0	1842.9	1839.3	0.20
7, 8	0, 3	2477.7	2483.2	0.22
9, 10	1, 1	3006.1	3011.2	0.17
11, 12	0, 4	3524.6	3532.7	0.23
13, 14	1, 2	4347.8	4366.4	0.43
15	2, 0	4598.3	4582.6	0.34
16, 17	0, 5	4725.2	4738.1	0.27
18, 19	1, 3	5862.8	5890.3	0.47
20, 21	0, 6	6076.4	6097.0	0.34
22, 23	2, 1	6372.8	6390.0	0.27
24, 25	0, 7	7546.5	7581.9	0.47
26, 27	1, 4	7576.1	7607.4	0.41
28, 29	2, 2	8327.5	8402.9	0.91
30	3, 0	8576.8	8534.9	0.49
31, 32	0, 8	9222.3	9267.5	0.49
33, 34	1, 5	9395.3	9441.9	0.50
35, 36	2, 3	10459.2	10539.6	0.77
37, 38	0, 9	10963.1	11004.7	0.38
39, 40	3, 1	11013.5	11076.0	0.57

Oscillation mode	Number of nodal circles m and diameters n	Theory	SCAD	Deviations, %
41, 42	1, 6	11406.2	11471.2	0.57
43, 44	2, 4	12764.4	12865.5	0.79
45, 46	0, 10	12948.4	13031.2	0.64
47, 48	3, 2	13530.3	13742.7	1.57
49, 50	1, 7	13576.7	13667.2	0.67
51	4, 0	13779.1	13690.3	0.64
52, 53	0, 11	15025.9	15131.7	0.70
54, 55	2, 5	15240.2	15359.6	0.78
56, 57	1, 8	15904.6	16028.2	0.78
58, 59	3, 3	16276.1	16457.3	1.11
60, 61	4, 1	16777.2	16859.0	0.49

Notes: In the analytical solution the natural frequencies ω of the simply supported circular plate with the density of the material ρ can be determined according to the following equation obtained on the basis of the factorization method:

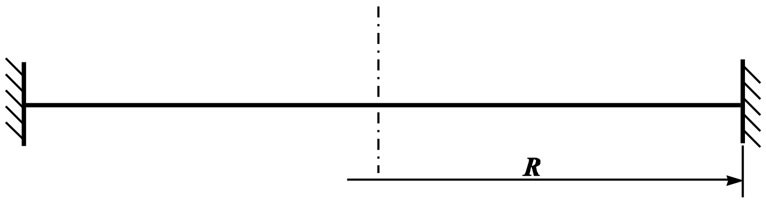
$$\frac{J_{n+1}(\beta \cdot R)}{J_n(\beta \cdot R)} + \frac{I_{n+1}(\beta \cdot R)}{I_n(\beta \cdot R)} = \frac{2 \cdot \beta \cdot R}{1-\nu}, \text{ where:}$$

$\beta = \left(\frac{\rho \cdot h \cdot \omega^2}{D} \right)^{\frac{1}{4}}$, $D = \frac{E \cdot h^3}{12 \cdot (1-\nu^2)}$, $n = 0,1,2,3\dots$ - number of nodal diameters,

$J_n(\beta \cdot R)$, $J_{n+1}(\beta \cdot R)$ - values of the Bessel function of the first kind of order *n*,

$I_n(\beta \cdot R)$, $I_{n+1}(\beta \cdot R)$ - values of the modified Bessel function of the first kind of order *n*.

Natural Oscillations of a Clamped Circular Plate



Objective: Modal analysis of a clamped circular plate.

Initial data file: **5.6.SPR**

Problem formulation: Determine the natural oscillation modes and frequencies ω of the clamped circular plate with the density of the material ρ.

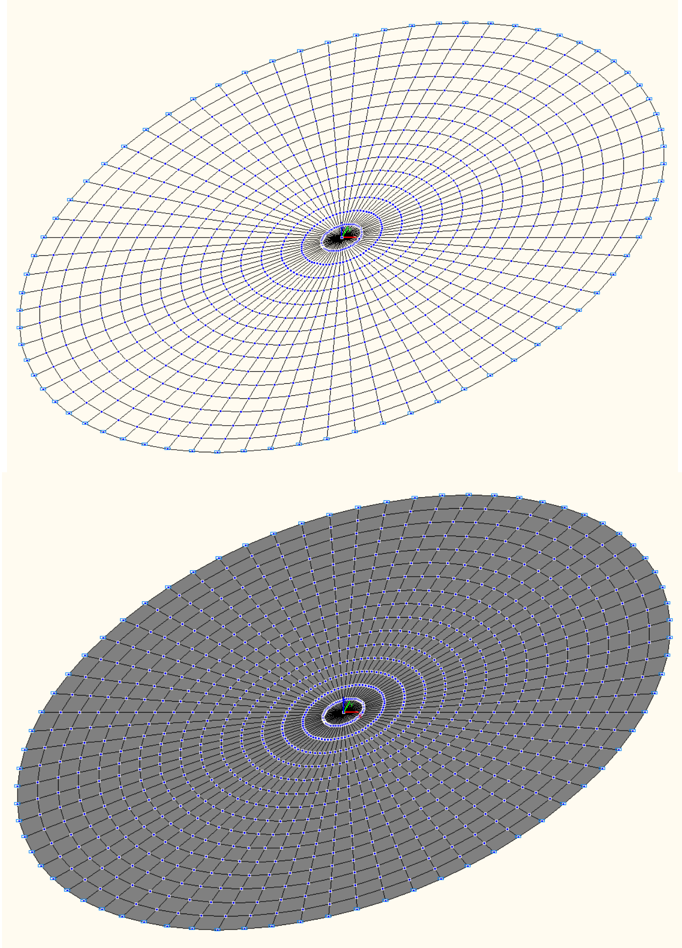
References: Chelomei V.N., Vibrations in Technology, Handbook in six volumes: Bolotin V.V., Volume 1, Vibrations of Linear Systems, Moscow, Mechanical engineering, 1978, p. 207.

Initial data:

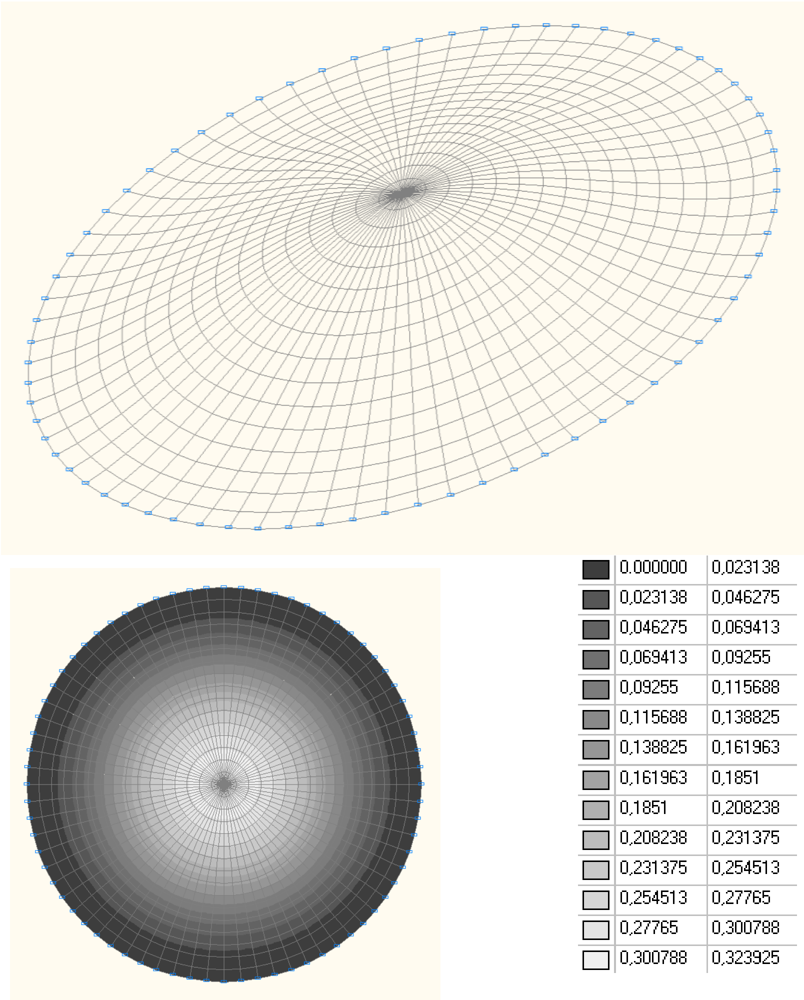
$E = 2.06 \cdot 10^8$ kPa	- elastic modulus;
ν = 0.3	- Poisson’s ratio;
$\rho = 7.85$ t/m^3	- density of the material;
h = 0.01 m	- thickness of the plate;
R = 0.5 m	- outer radius of the plate.

Finite element model: Design model – grade beam / plate, 1080 four-node plate elements of type 20 and 72 three-node plate elements of type 15. The spacing of the finite element mesh in the radial direction is 0.03125 m and in the tangential direction is 5.0º. Boundary conditions are provided by imposing constraints in the directions of the degrees of freedom Z, UX, UY along the outer contour of the plate. The distributed mass is specified by transforming the static load from the self-weight of the plate ow = γ·h, where γ = ρ·g = 77.01 kN/m^3. Number of nodes in the design model – 1153. The determination of the natural oscillation modes and natural frequencies is performed by the method of subspace iteration. The matrix of concentrated masses is used in the calculation.

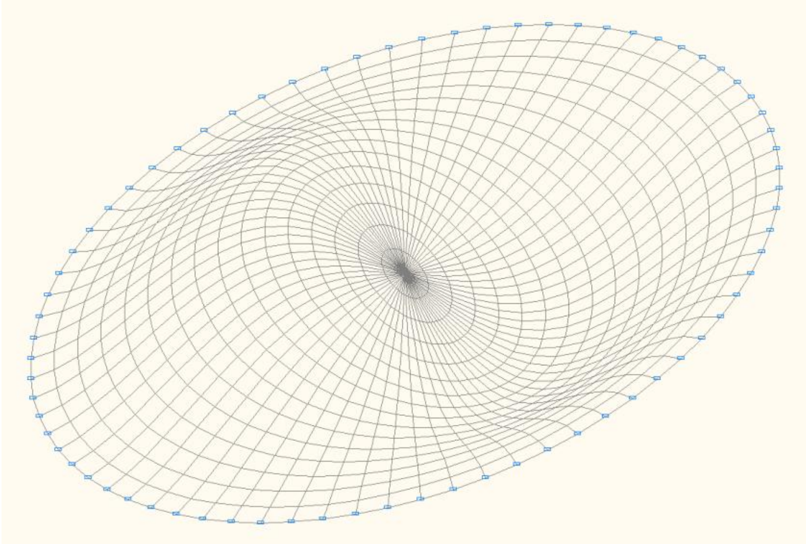
Results in SCAD

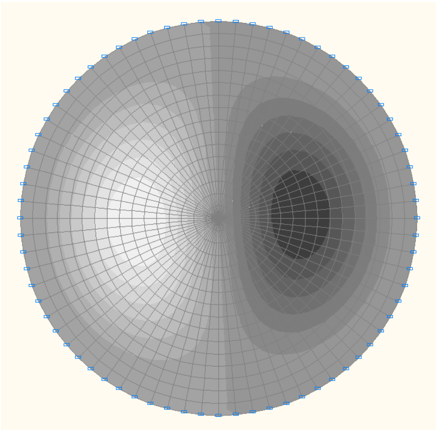


Design model



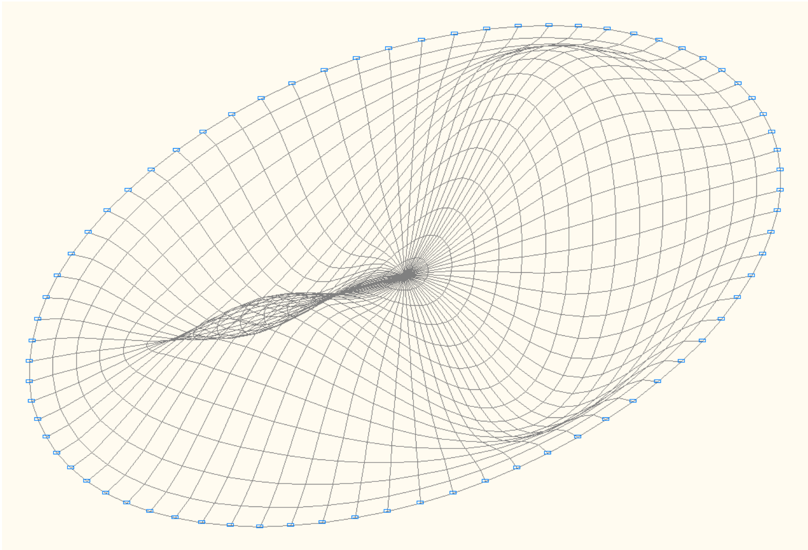
1-st natural oscillation mode

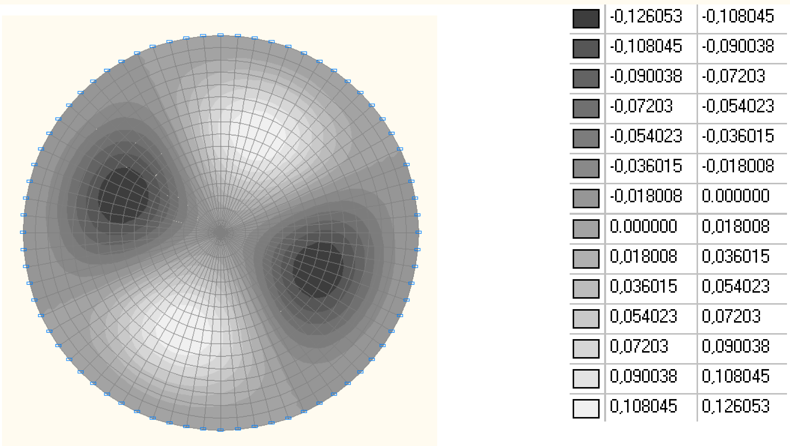




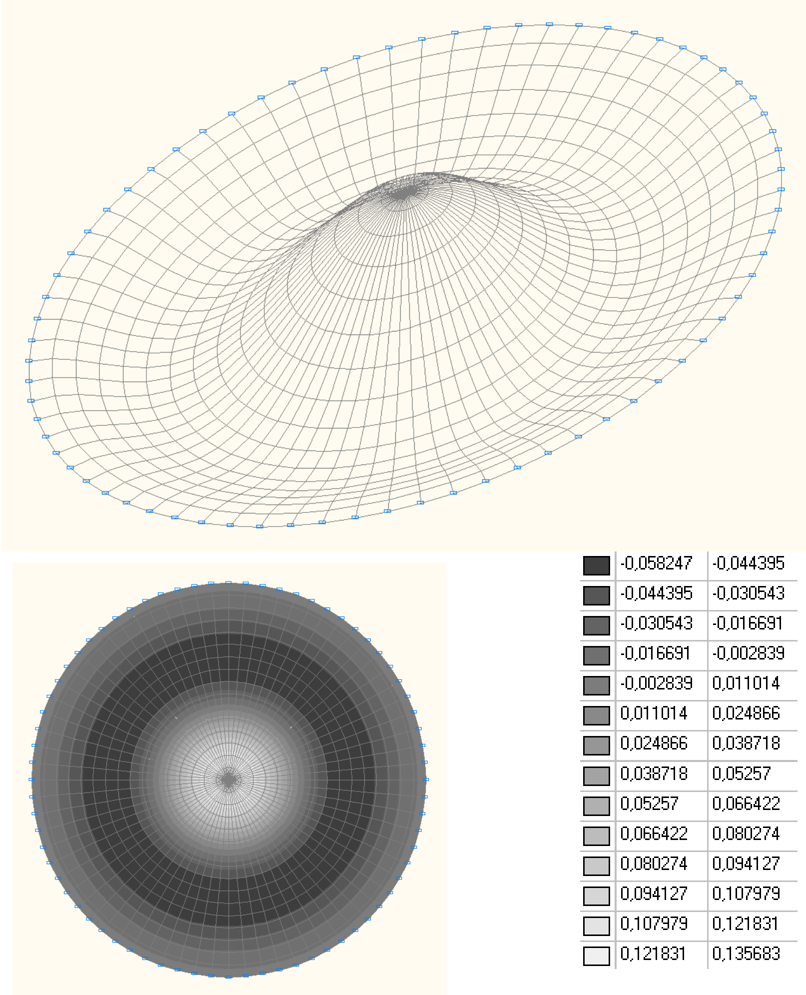
	-0,125385	-0,107473
	-0,107473	-0,089561
	-0,089561	-0,071649
	-0,071649	-0,053736
	-0,053736	-0,035824
	-0,035824	-0,017912
	-0,017912	0
	0	0,017912
	0,017912	0,035824
	0,035824	0,053736
	0,053736	0,071649
	0,071649	0,089561
	0,089561	0,107473
	0,107473	0,125385

2-nd natural oscillation mode

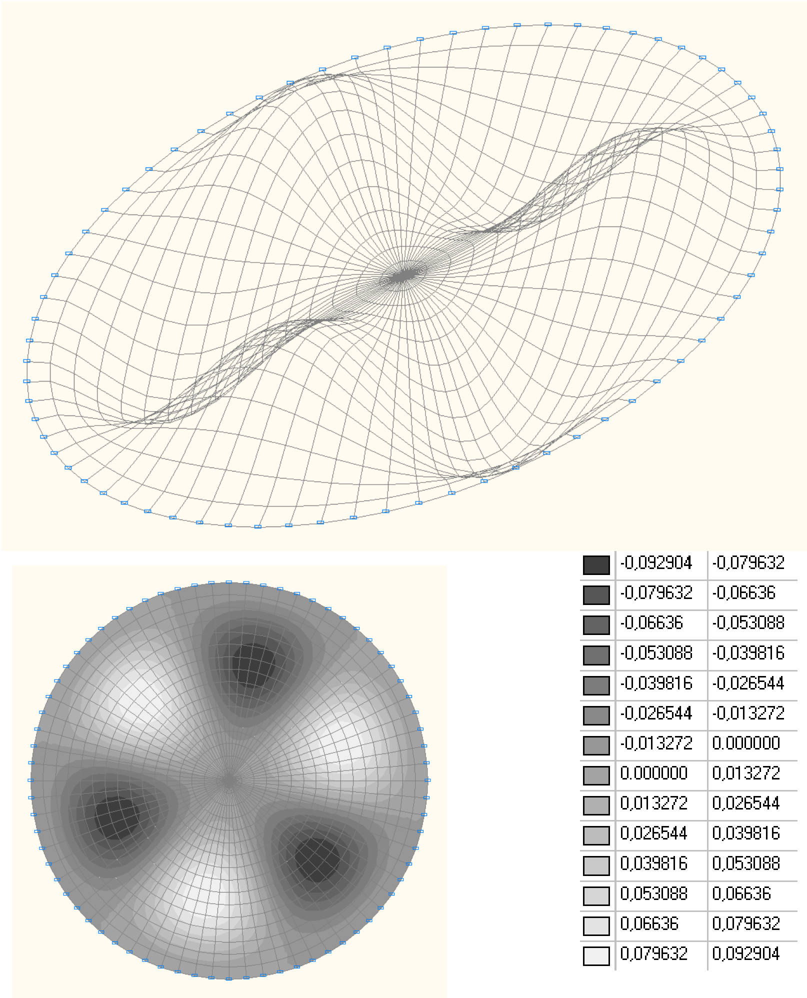




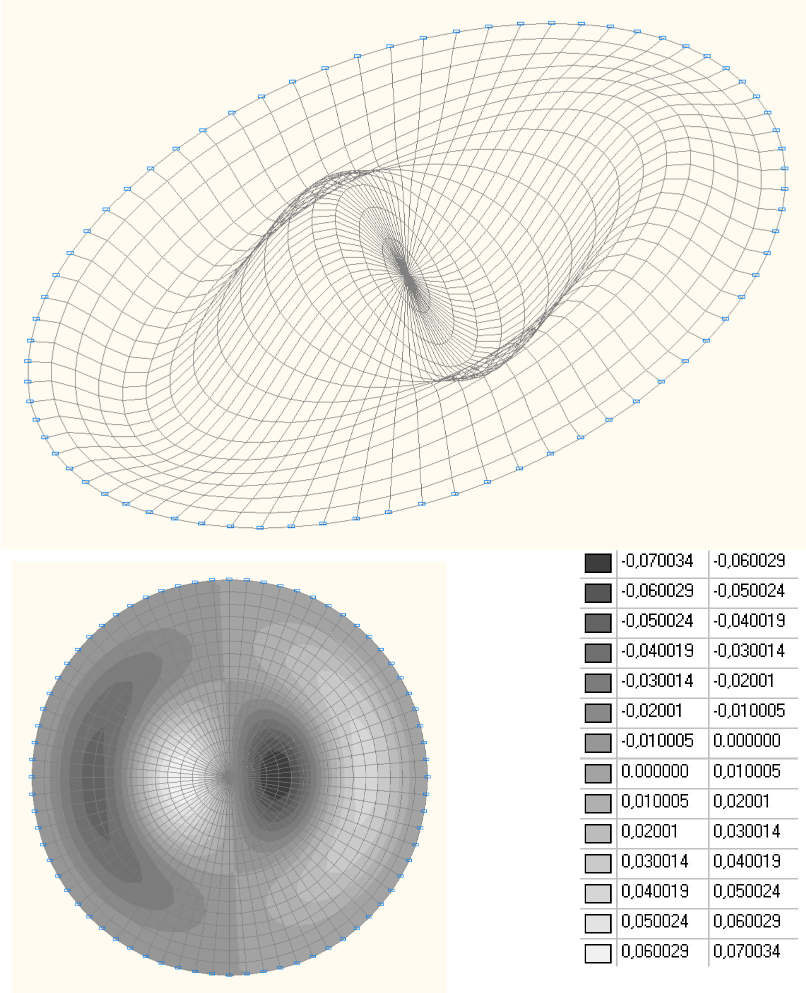
4-th natural oscillation mode



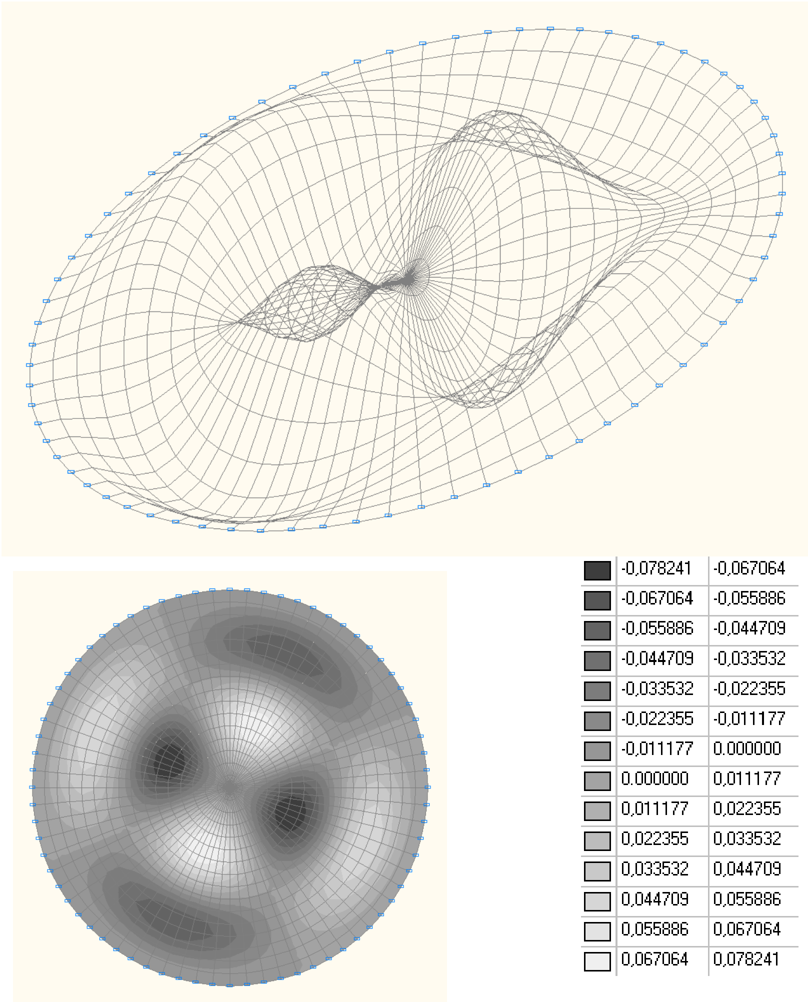
6-th natural oscillation mode



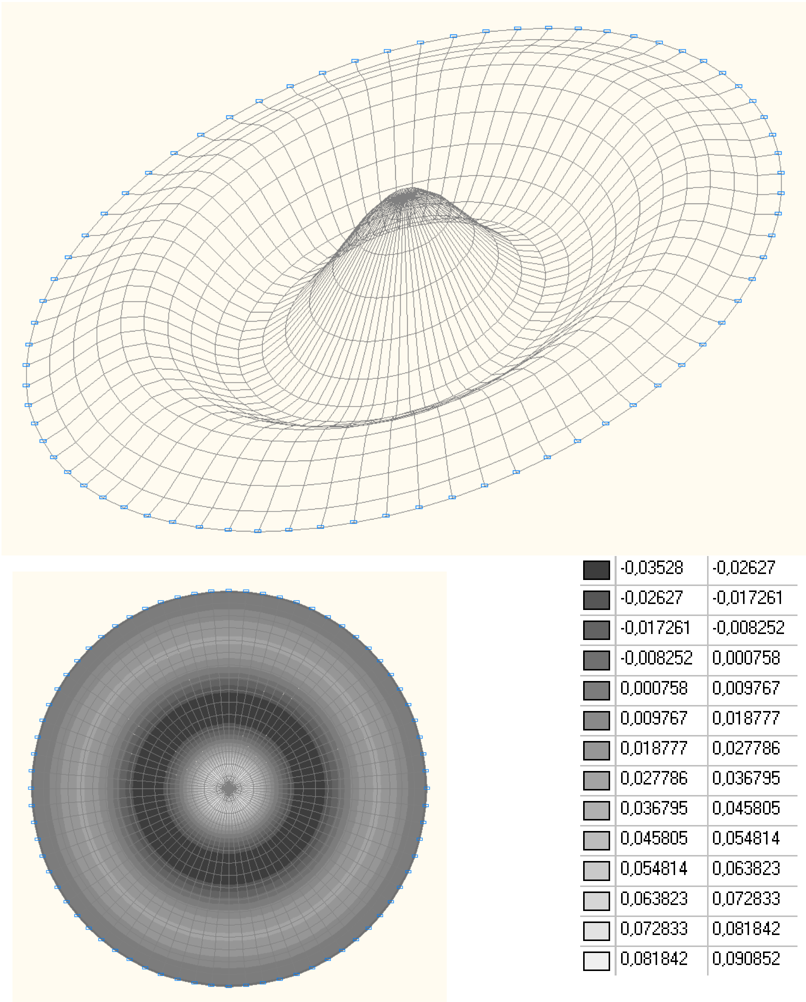
7-th natural oscillation mode



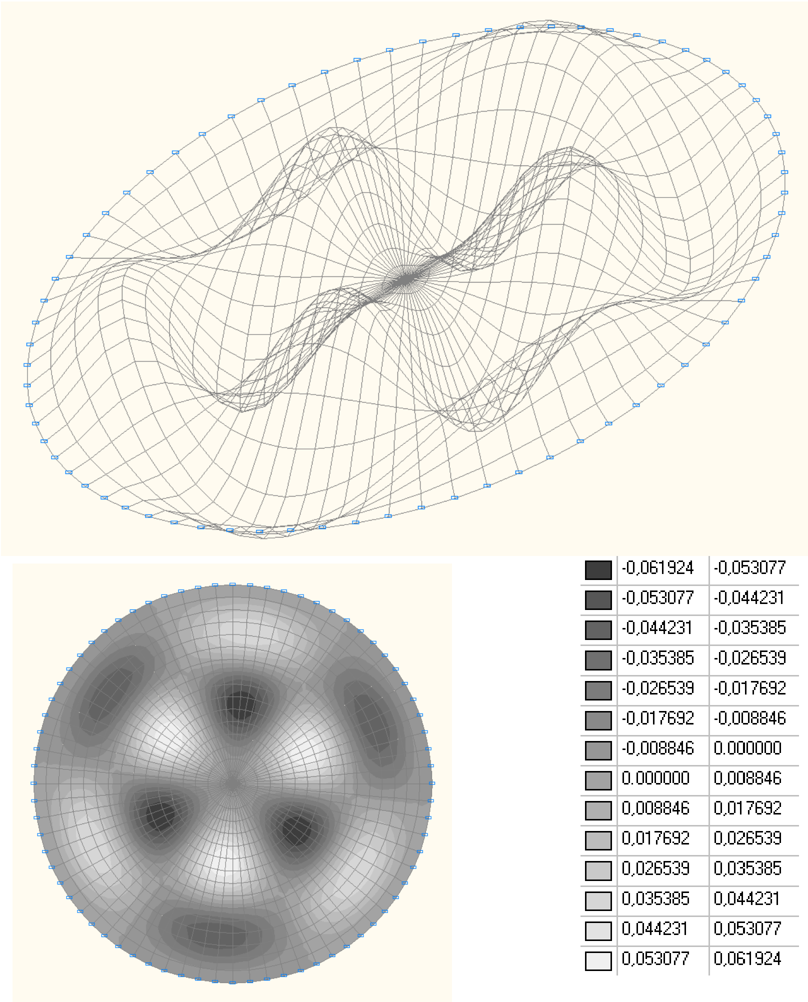
9-th natural oscillation mode



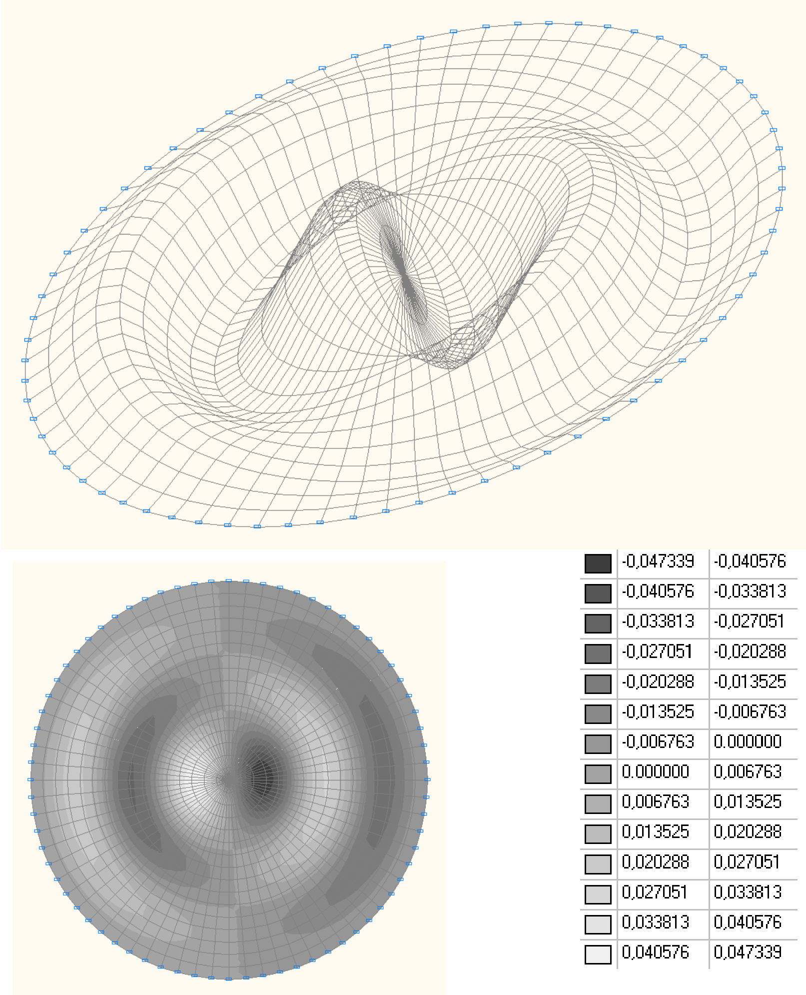
13-th natural oscillation mode



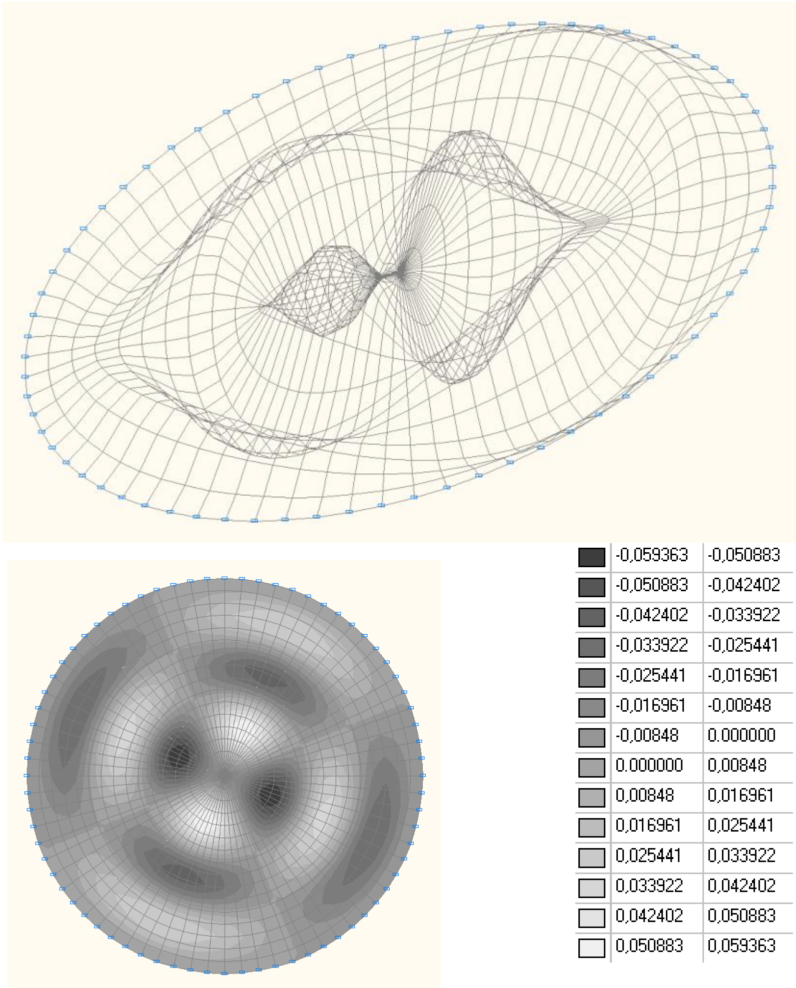
15-th natural oscillation mode



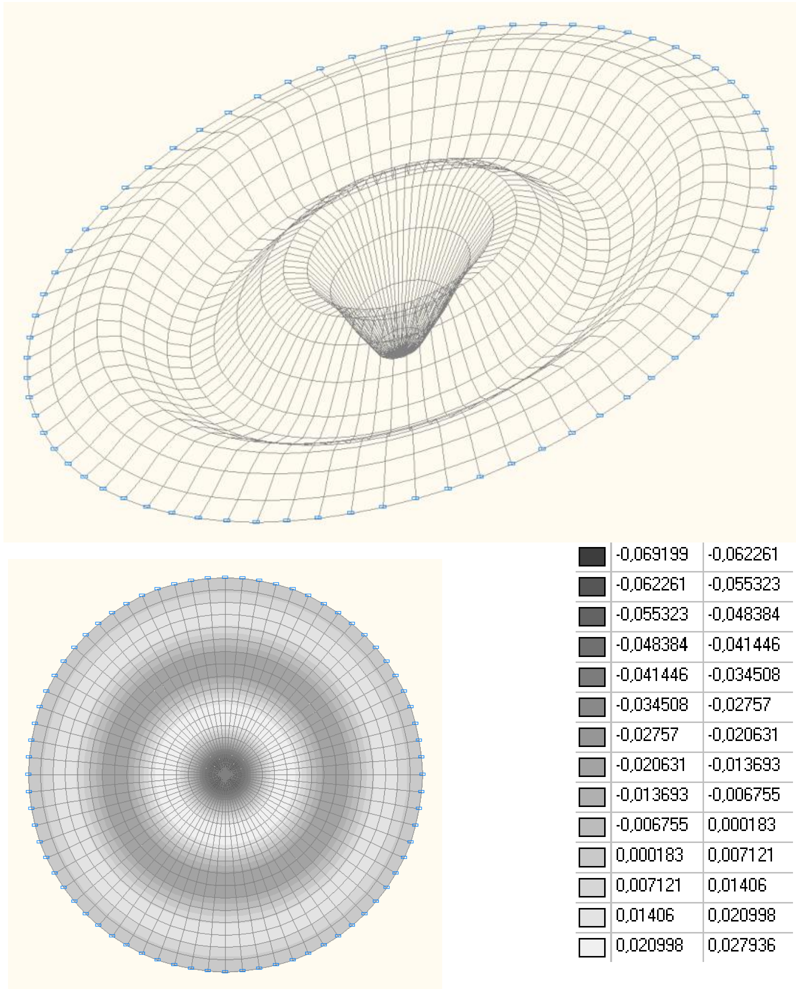
18-th natural oscillation mode



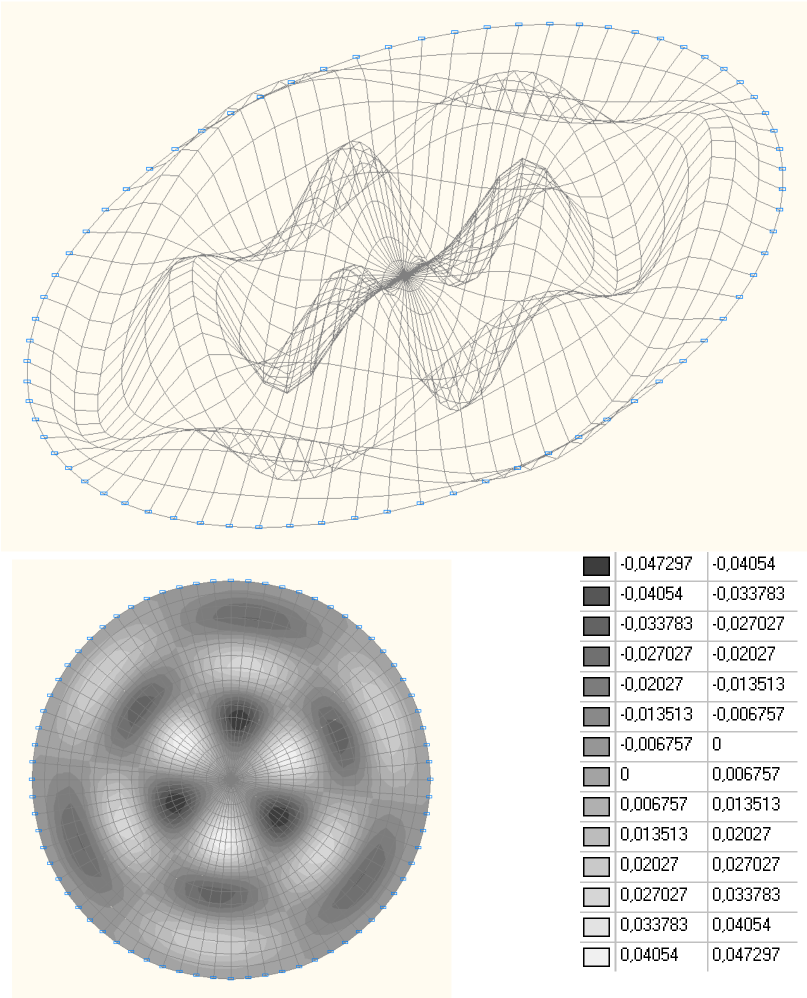
22-nd natural oscillation mode



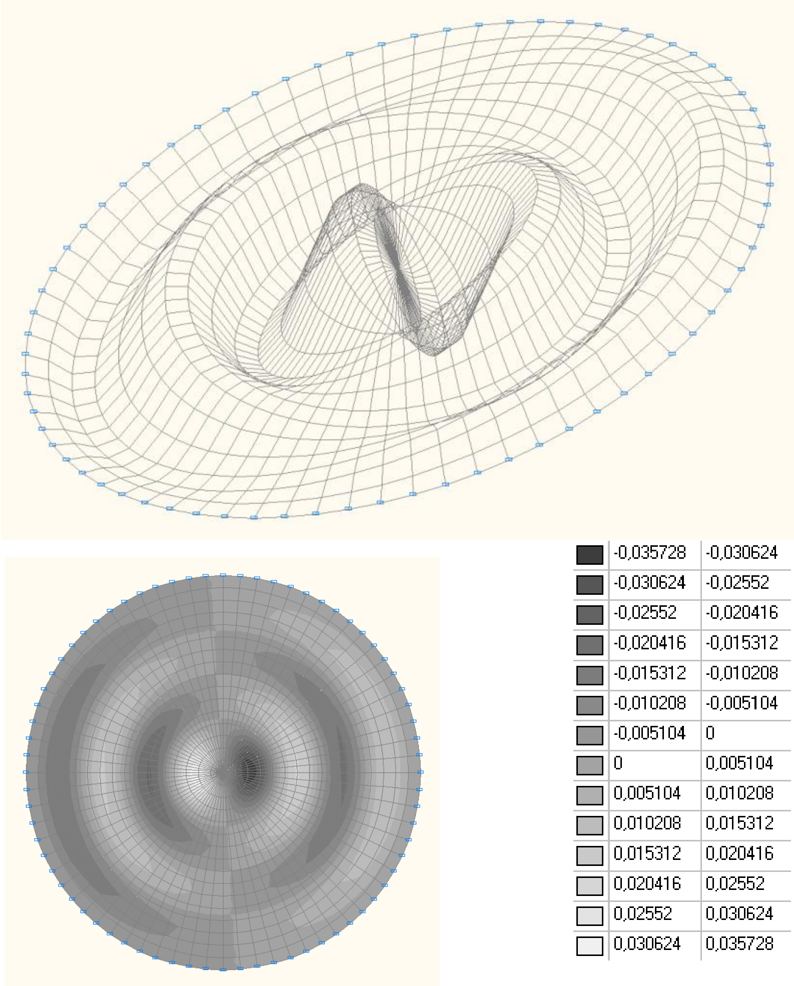
28-th natural oscillation mode



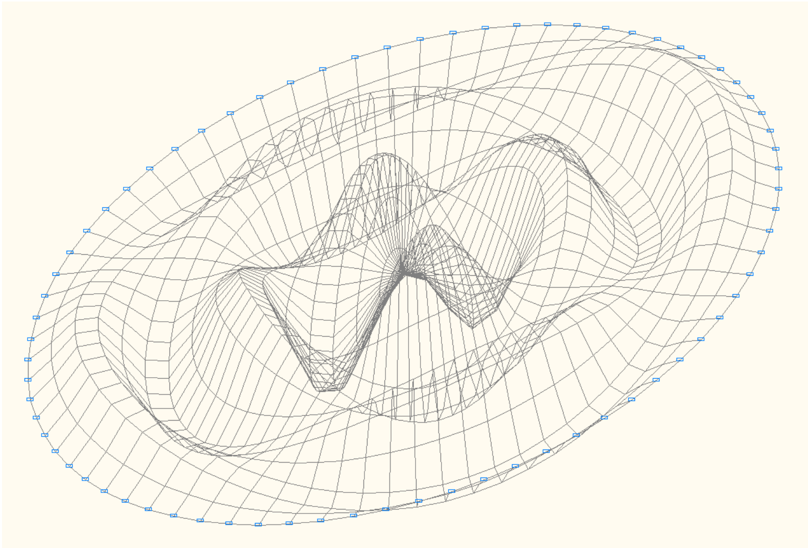
30-th natural oscillation mode

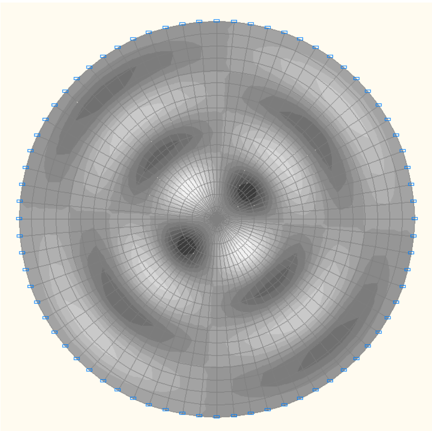


35-th natural oscillation mode



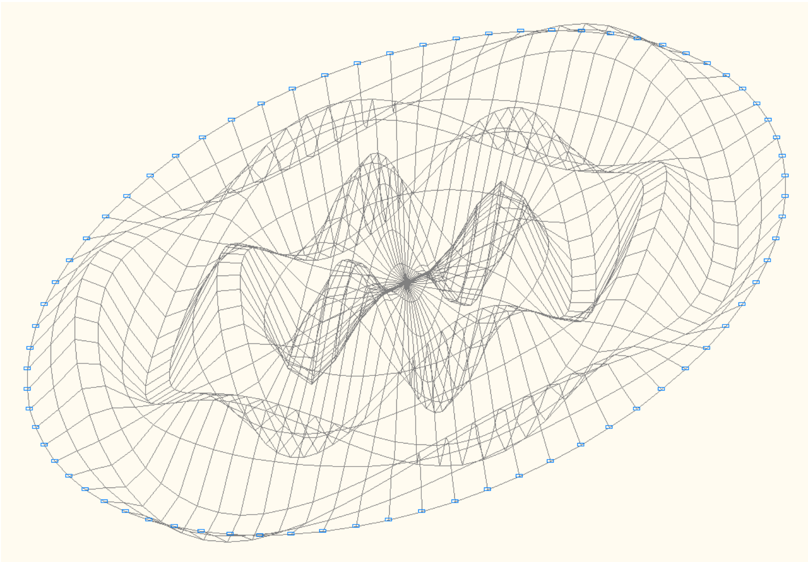
37-th natural oscillation mode

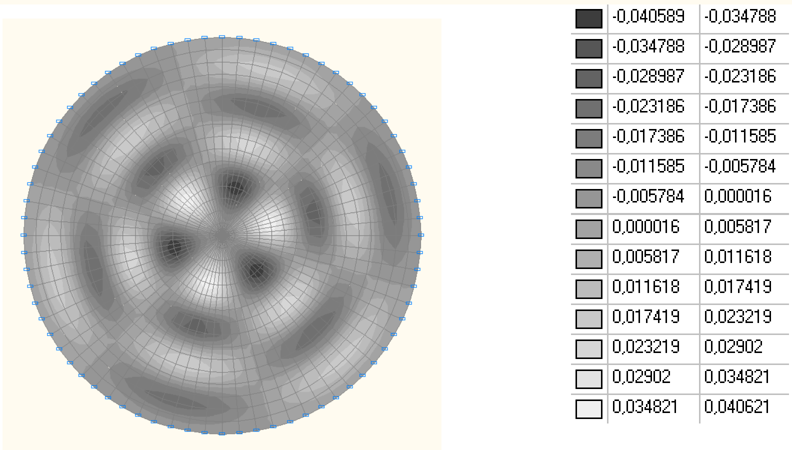




-0,045445	-0,038953
-0,038953	-0,032461
-0,032461	-0,025969
-0,025969	-0,019477
-0,019477	-0,012985
-0,012985	-0,006493
-0,006493	0
0	0,006492
0,006492	0,012984
0,012984	0,019476
0,019476	0,025968
0,025968	0,03246
0,03246	0,038952
0,038952	0,045444

50-th (47-th theoretical) natural oscillation mode





58-th natural oscillation mode

Comparison of solutions:

Natural frequencies ω, rad / s

Oscillation mode	Number of nodal circles m and diameters n	Theory	SCAD	Deviations, %
1	0, 0	633.5	633.8	0.05
2, 3	0, 1	1318.3	1321.7	0.26
4, 5	0, 2	2162.7	2170.6	0.37
6	1, 0	2466.1	2463.8	0.09
7, 8	0, 3	3164.3	3178.9	0.46
9, 10	1, 1	3771.9	3784.3	0.33
11, 12	0, 4	4319.8	4340.1	0.47
13, 14	1, 2	5244.8	5280.1	0.67
15	2, 0	5525.2	5511.5	0.25
16, 17	0, 5	5626.5	5655.5	0.52
18, 19	1, 3	6884.2	6931.9	0.69
20, 21	0, 6	7082.1	7123.6	0.59
22, 23	2, 1	7445.9	7477.9	0.43
24, 25	0, 7	8684.6	8742.7	0.67
26, 27	1, 4	8687.8	8748.6	0.70
28, 29	2, 2	9537.8	9652.0	1.20
30	3, 0	9808.7	9769.4	0.40
31, 32	0, 8	10432.5	10511.5	0.76
33, 34	1, 5	10653.2	10730.3	0.72
35, 36	2, 3	11800.3	11917.6	0.99
37, 38	0, 9	12324.5	12429.0	0.85
39, 40	3, 1	12342.9	12408.9	0.53

Oscillation mode	Number of nodal circles m and diameters n	Theory	SCAD	Deviations, %
41, 42	1, 6	12778.0	12880.4	0.80
43, 44	2, 4	14232.0	14378.8	1.03
45, 46	0, 10	14359.4	14494.1	0.94
47, 48	3, 2	15050.6	15335.5	1.89
49, 50	1, 7	15060.4	15196.7	0.91
51	4, 0	15316.4	15229.9	0.56
52, 53	0, 11	16536.2	16705.2	1.02
54, 55	2, 5	16830.7	17001.7	1.02
56, 57	1, 8	17498.5	17677.4	1.02
58, 59	3, 3	17931.5	18171.2	1.34
60, 61	4, 1	18463.5	18580.3	0.63

Notes: In the analytical solution the natural frequencies ω of the clamped circular plate with the density of the material ρ can be determined according to the following equation obtained on the basis of the factorization method:

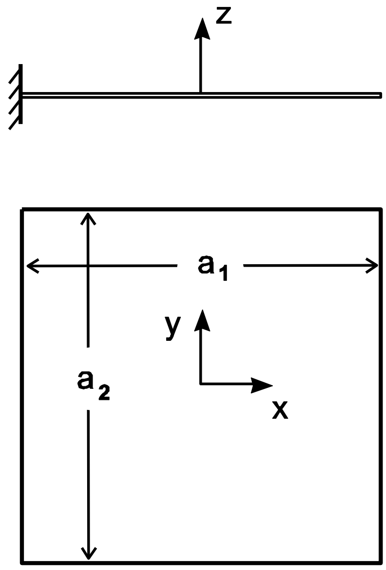
$$\frac{J_{n+1}(\beta \cdot R)}{J_n(\beta \cdot R)} + \frac{I_{n+1}(\beta \cdot R)}{I_n(\beta \cdot R)} = 0\text{, where:}$$

$$\beta = \left(\frac{\rho \cdot h \cdot \omega^2}{D} \right)^{\frac{1}{4}}, \qquad D = \frac{E \cdot h^3}{12 \cdot (1 - \nu^2)},\ n = 0,1,2,3\ldots$$ - number of nodal diameters,

$J_n(\beta \cdot R)$, $J_{n+1}(\beta \cdot R)$ - values of the Bessel function of the first kind of order *n*,

$I_n(\beta \cdot R)$, $I_{n+1}(\beta \cdot R)$ - values of the modified Bessel function of the first kind of order *n*.

Natural Oscillations of a Square Cantilever Plate



Objective: Modal analysis of a square cantilever plate.

Initial data file: **5.5.SPR**

Problem formulation: : Determine the natural oscillation modes and frequencies ω of the square cantilever plate with the density of the material ρ.

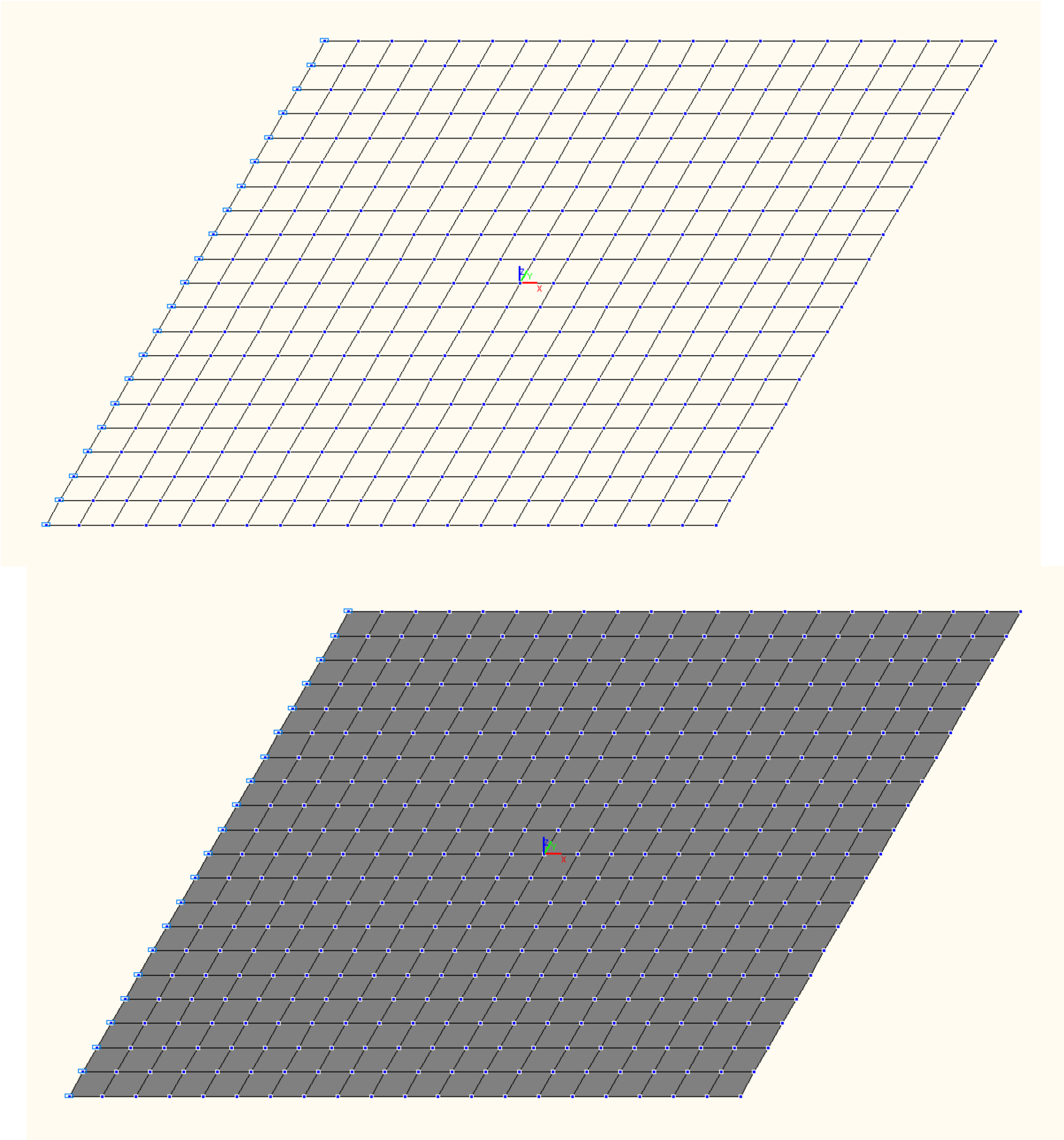
References: I.A. Birger, Ya.G. Panovko, Strength, Stability, Vibrations, Handbook in three volumes, Volume 3, Moscow, Mechanical engineering, 1968, p. 382.

Initial data:

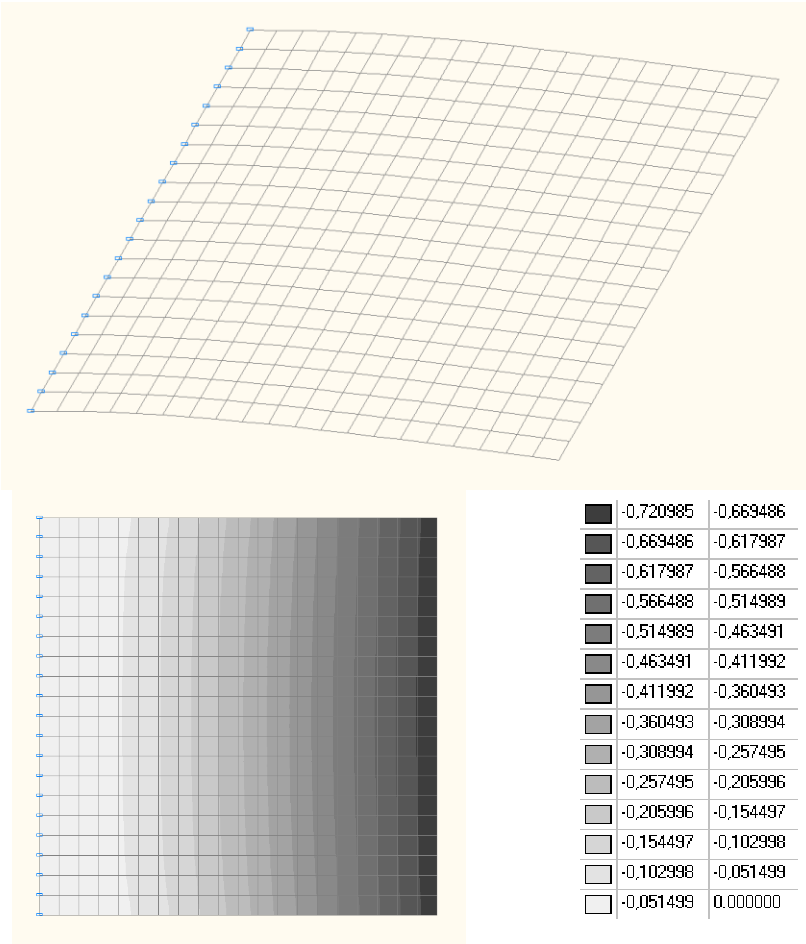
$E = 2.06 \cdot 10^8$ kPa	- elastic modulus;
$\nu = 0.3$	- Poisson's ratio;
$\rho = 7.85$ t/m^3	- density of the material;
$h = 0.01$ m	- thickness of the plate;
$a_1 = 1.0$ m	- long side of the plate (along the X axis of the global coordinate system);
$a_2 = 1.0$ m	- short side of the plate (along the Y axis of the global coordinate system).

Finite element model: Design model – grade beam / plate, 400 plate elements of type 20. The spacing of the finite element mesh along the sides of the plate (along the X, Y axes of the global coordinate system) is 0.05 m. Boundary conditions are provided by imposing constraints in the directions of the degrees of freedom Z, UX, UY for one of the edges parallel to the Y axis of the global coordinate system. The distributed mass is specified by transforming the static load from the self-weight of the plate ow = $\gamma \cdot h$, where $\gamma = \rho \cdot g = 77.01$ kN/m^3. Number of nodes in the design model – 441. The determination of the natural oscillation modes and natural frequencies is performed by the Lanczos method. A consistent mass matrix is used in the calculation.

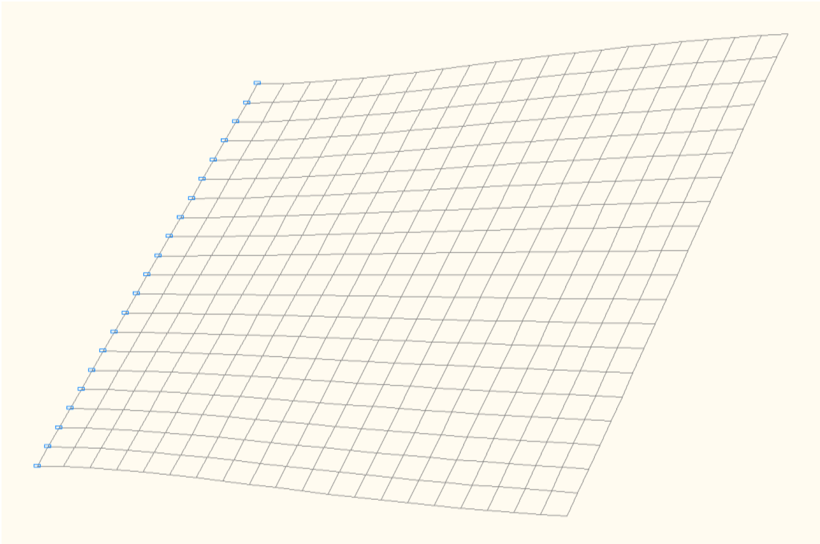
Results in SCAD

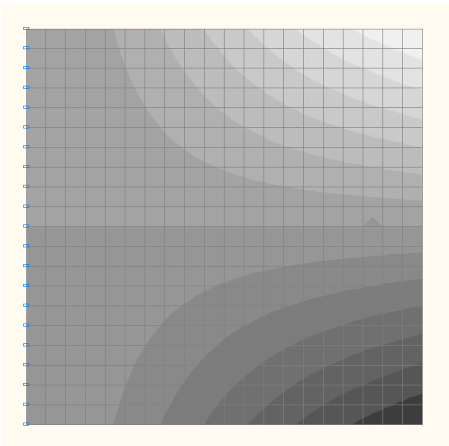


Design model



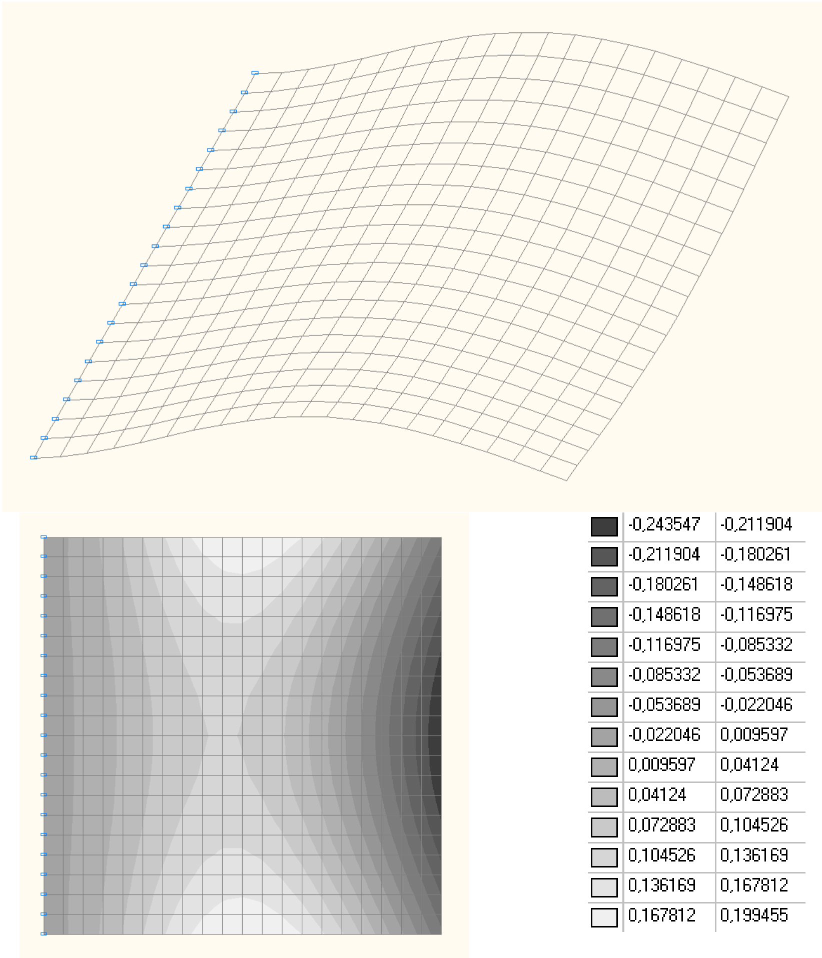
1-st natural oscillation mode



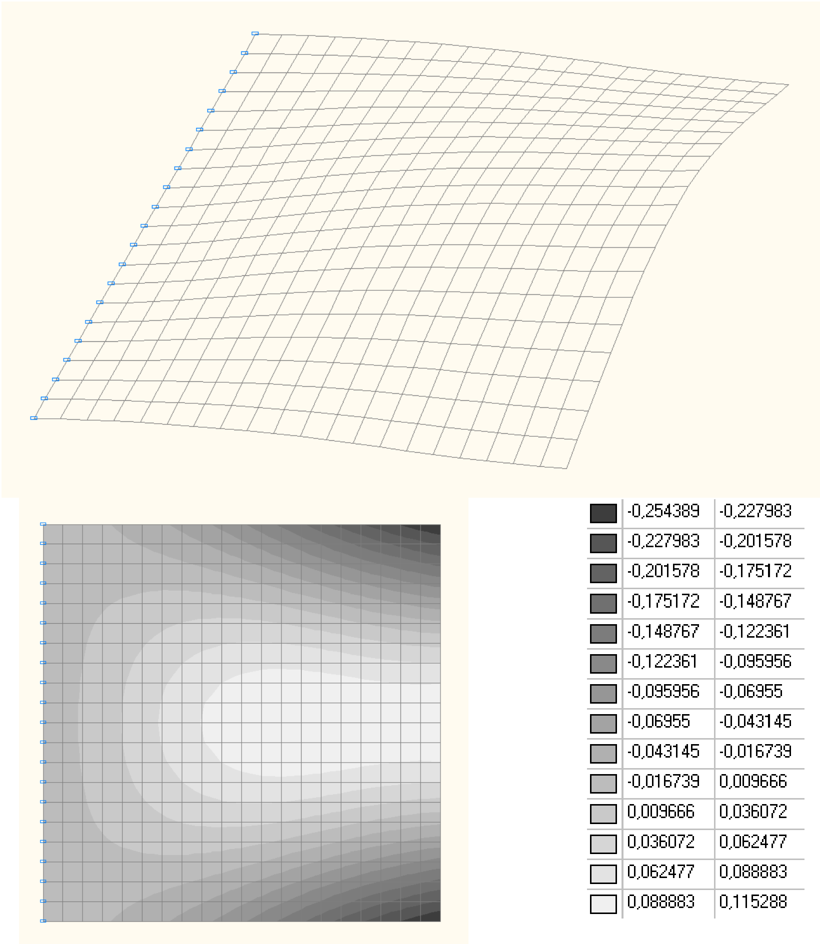


-0,45923	-0,393626
-0,393626	-0,328021
-0,328021	-0,262417
-0,262417	-0,196813
-0,196813	-0,131209
-0,131209	-0,065604
-0,065604	0
0	0,065604
0,065604	0,131209
0,131209	0,196813
0,196813	0,262417
0,262417	0,328021
0,328021	0,393626
0,393626	0,45923

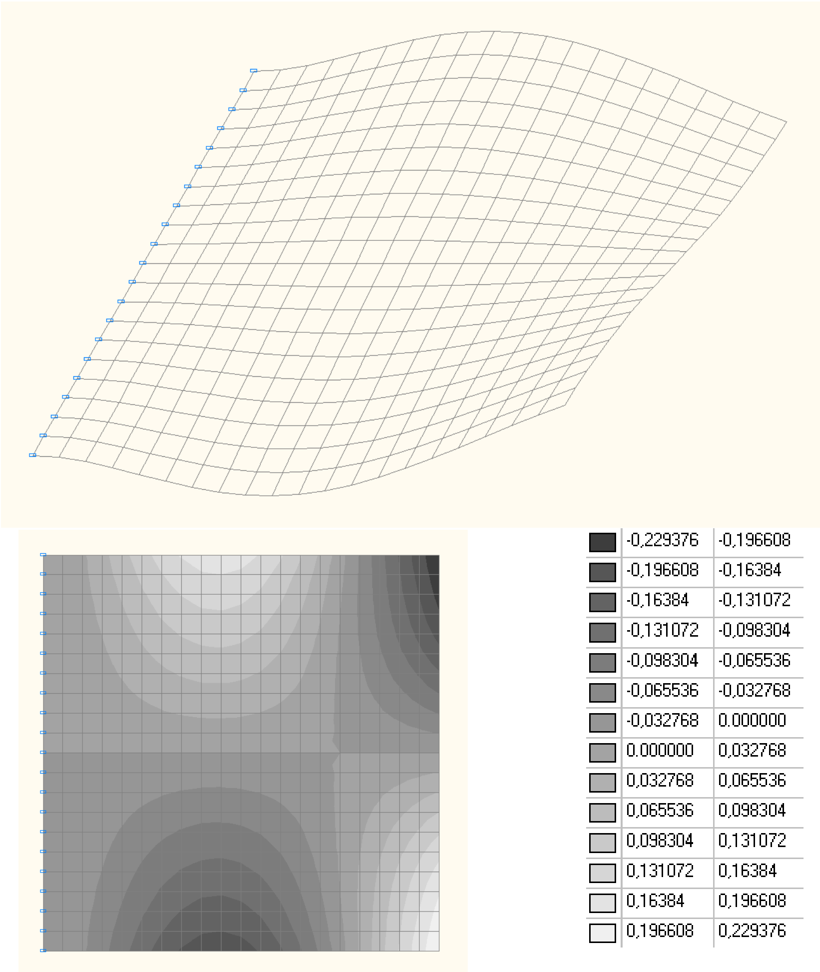
2-nd natural oscillation mode



3-rd natural oscillation mode



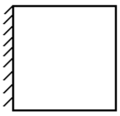
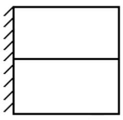
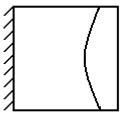
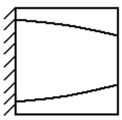
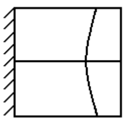
4-th natural oscillation mode



5-th natural oscillation mode

Comparison of solutions:

Natural frequencies ω, rad / s

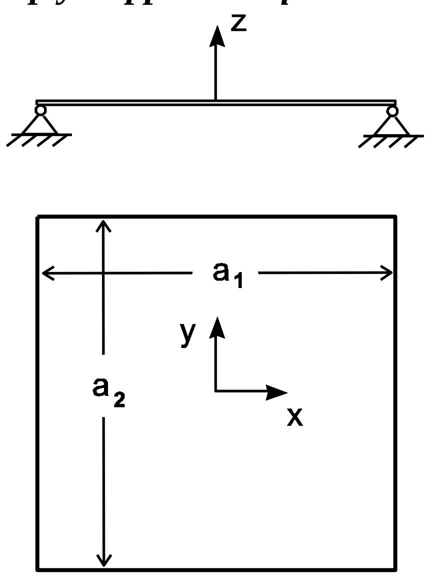
Oscillation mode	Nodal lines	Theory	SCAD	Deviations, %
1		54.2	53,8	0,71
2		132.5	131,9	0,43
3		332.4	330,0	0,72
4		425.7	421,8	0,91
5		483.2	480,3	0,61

Notes: In the analytical solution the natural frequencies ω of the square cantilever plate with the density of the material ρ can be determined according to the following formula obtained on the basis of the Rayleigh-Ritz method:

$$\omega = \frac{\omega_m^*}{a_1{}^2} \cdot \left(\frac{D}{\rho \cdot h} \right)^{\frac{1}{2}}, \text{ where at } \frac{a_2}{a_1} = 1:$$

$$\omega_1^* = 3.494, \quad \omega_2^* = 8.547, \quad \omega_3^* = 21.44, \quad \omega_2^* = 27.46, \quad \omega_2^* = 31.17, \quad D = \frac{E \cdot h^3}{12 \cdot \left(1 - \nu^2\right)}.$$

Natural Oscillations of a Simply Supported Square Plate



Objective: Modal analysis of a simply supported square plate.

***Initial data file*:** **5.2.SPR**

Problem formulation: Determine the natural oscillation modes and frequencies ω of the simply supported square plate with the density of the material ρ.

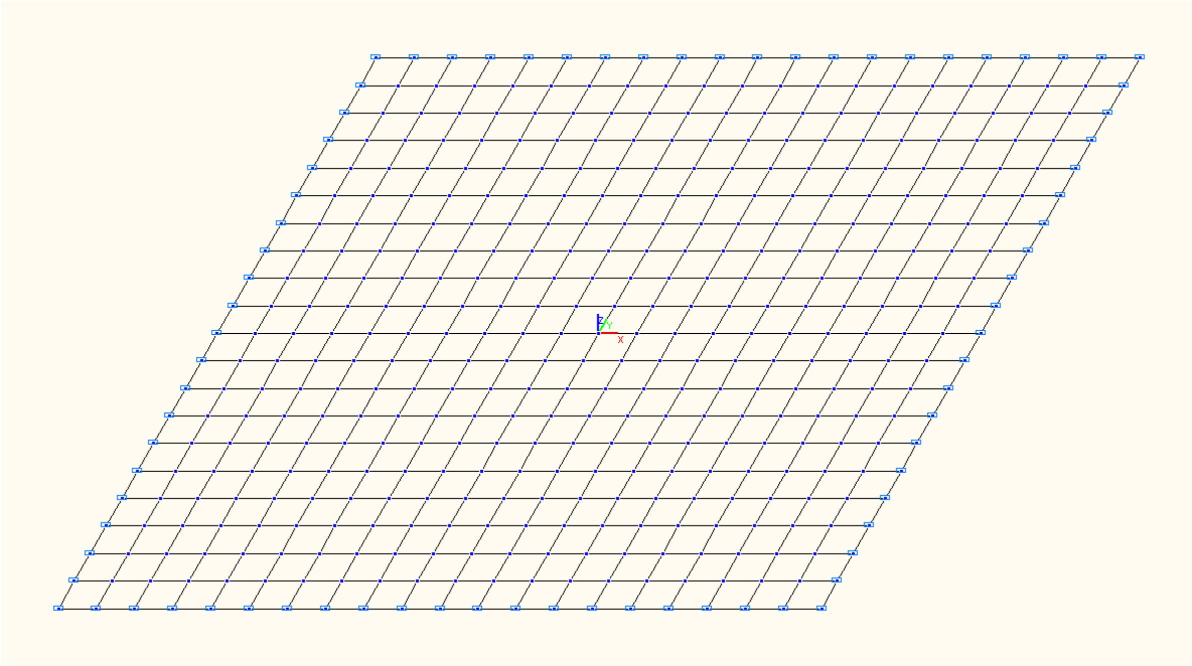
References: I.A. Birger, Ya.G. Panovko, Strength, Stability, Vibrations, Handbook in three volumes, Volume 3, Moscow, Mechanical engineering, 1968, p. 375.

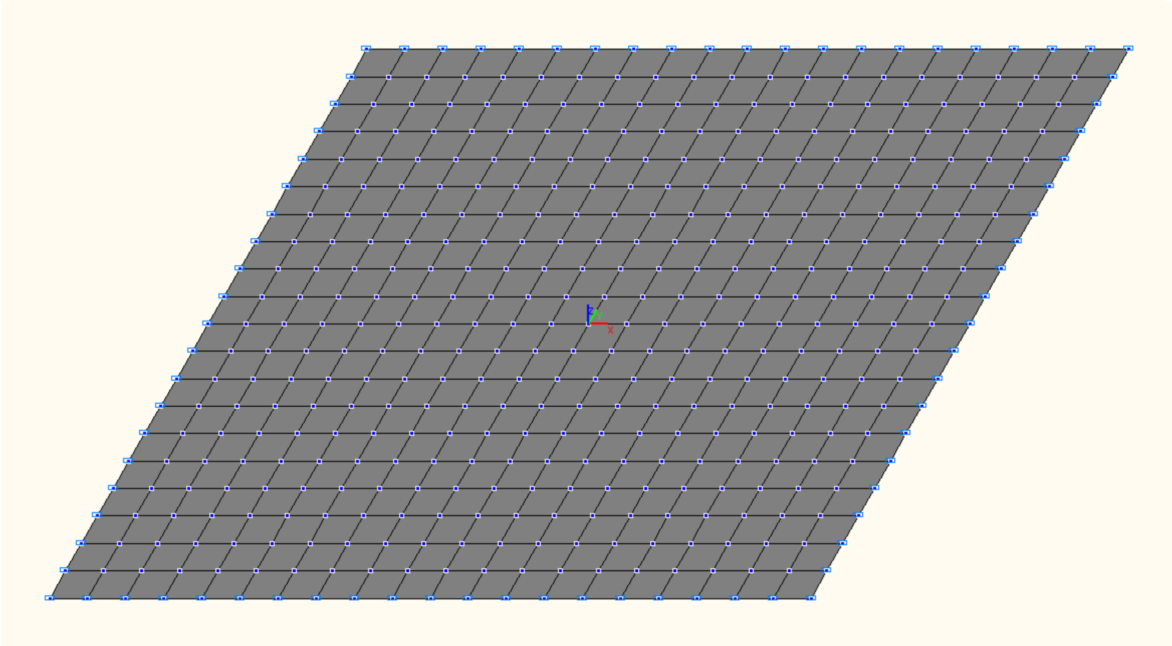
Initial data:

$E = 2.06 \cdot 10^8$ kPa	- elastic modulus;
$\nu = 0.3$	- Poisson's ratio;
$\rho = 7.85$ t/m^3	- density of the material;
$h = 0.01$ m	- thickness of the plate;
$a_1 = 1.0$ m	- long side of the plate (along the X axis of the global coordinate system);
$a_2 = 1.0$ m	- short side of the plate (along the Y axis of the global coordinate system).

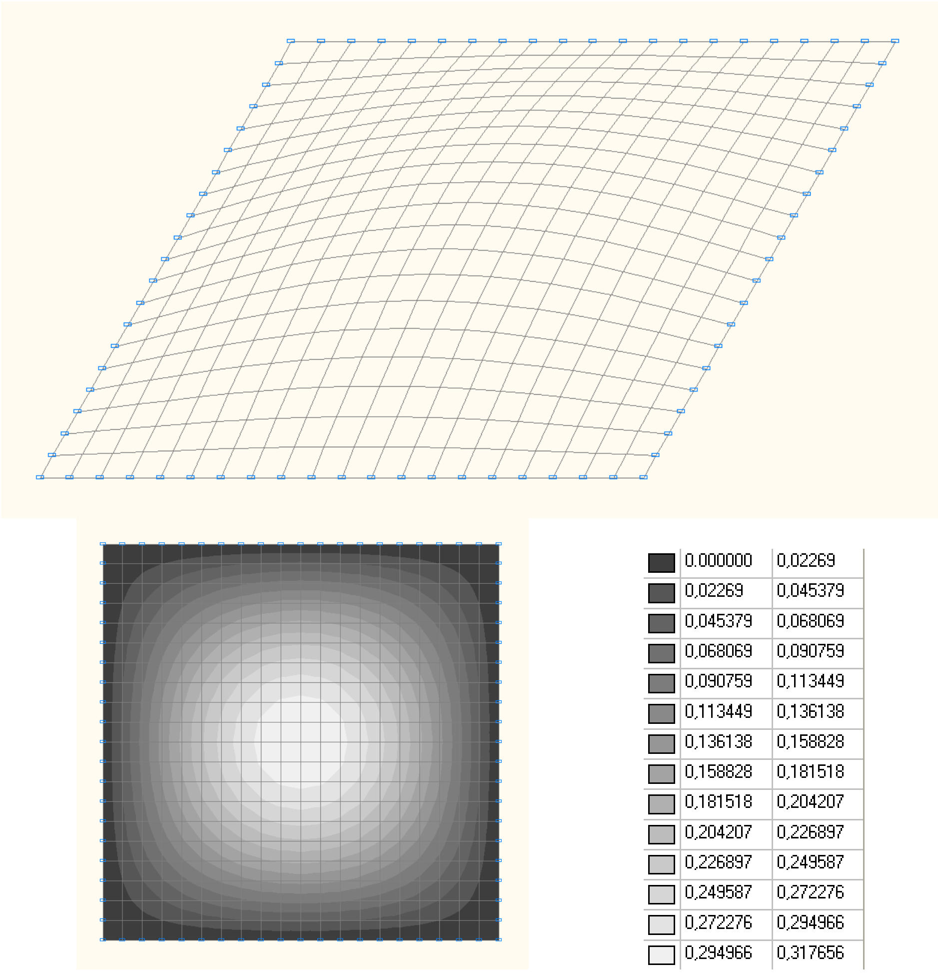
Finite element model: Design model – grade beam / plate, 400 plate elements of type 20. The spacing of the finite element mesh along the sides of the plate (along the X, Y axes of the global coordinate system) is 0.05 m. Boundary conditions are provided by imposing constraints in the direction of the degree of freedom Z for the edges parallel to the X and Y axes of the global coordinate system. The distributed mass is specified by transforming the static load from the self-weight of the plate ow = $\gamma \cdot h$, where $\gamma = \rho \cdot g = 77.01$ kN/m^3. Number of nodes in the design model – 441. The determination of the natural oscillation modes and natural frequencies is performed by the Lanczos method. A consistent mass matrix is used in the calculation.

Results in SCAD

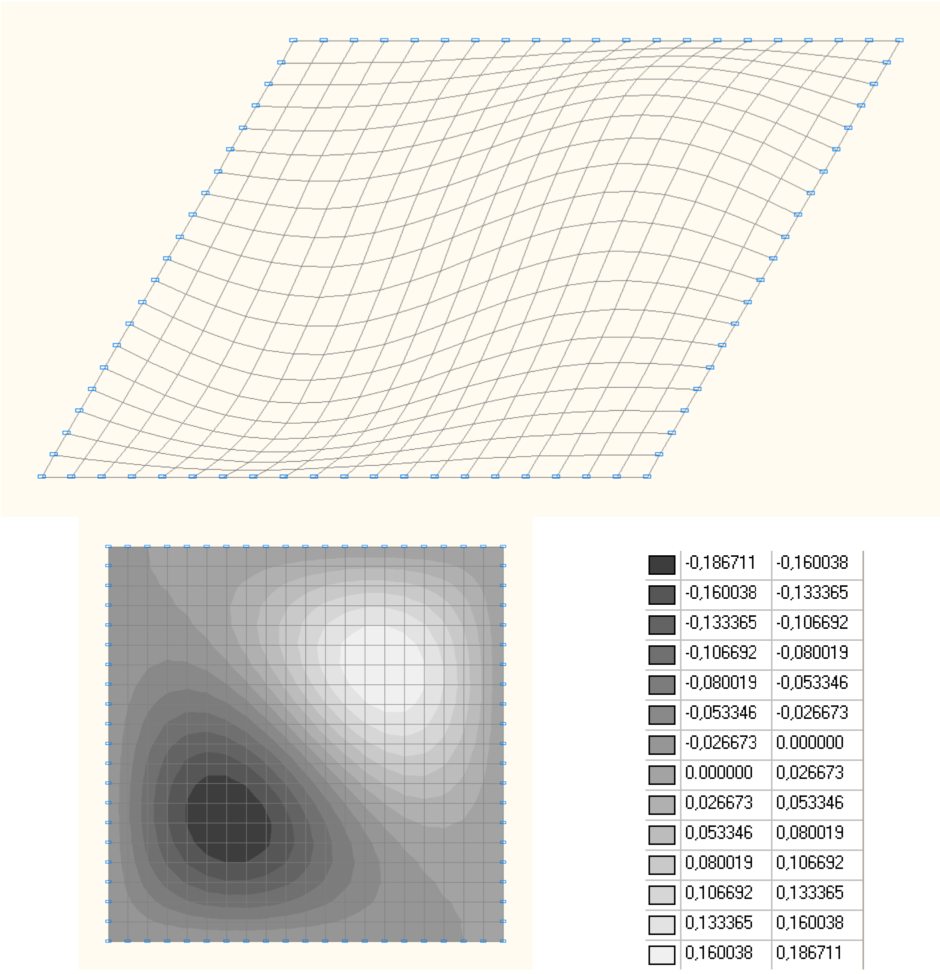




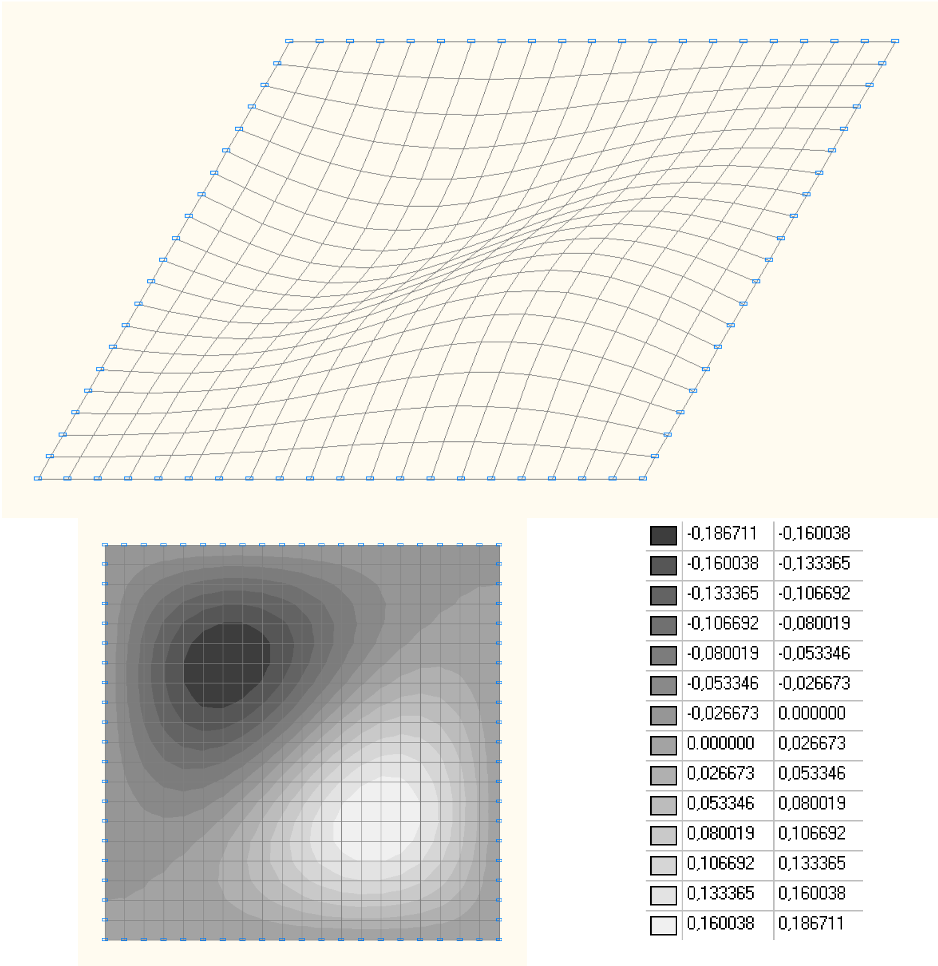
Design model



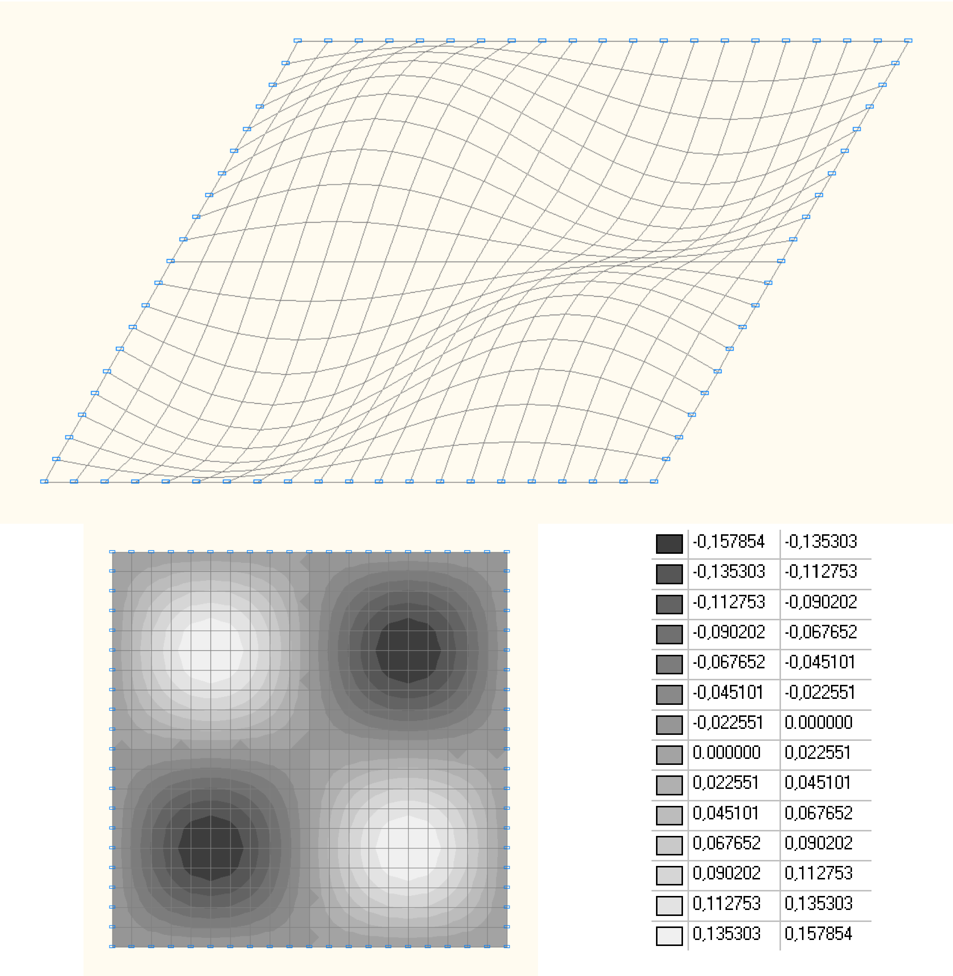
1-st natural oscillation mode



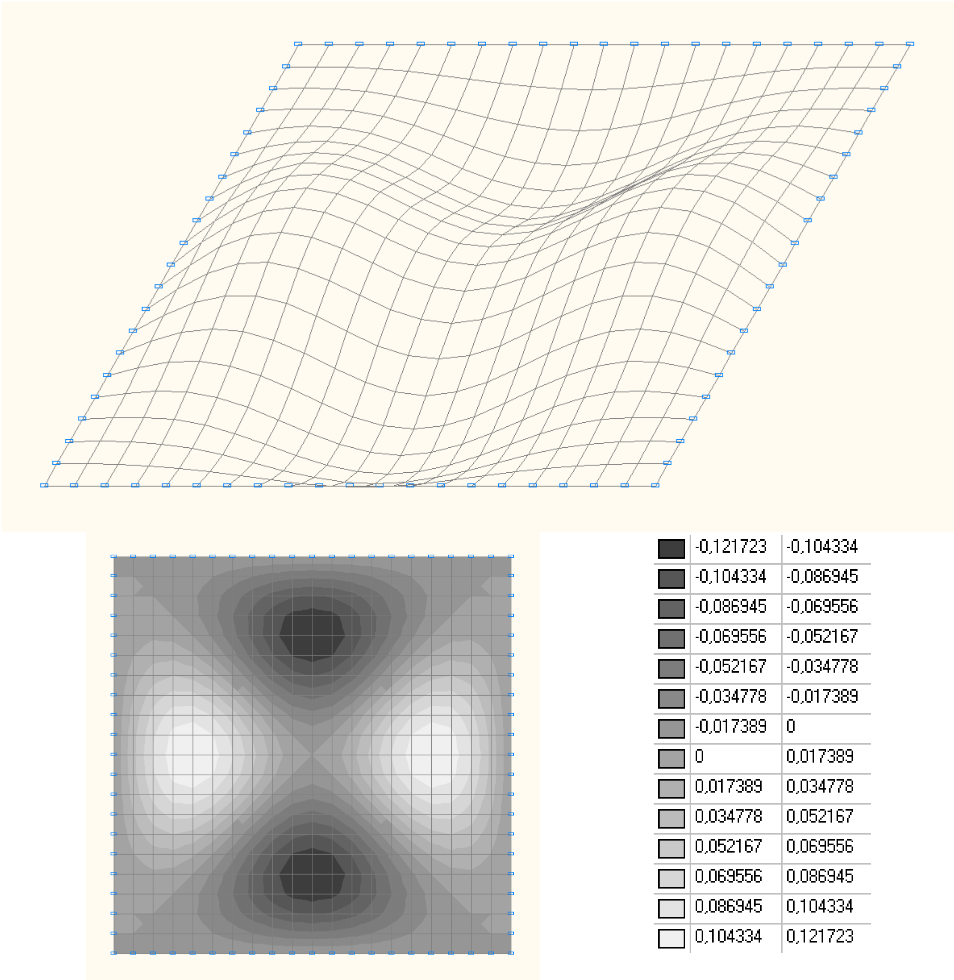
2-nd natural oscillation mode



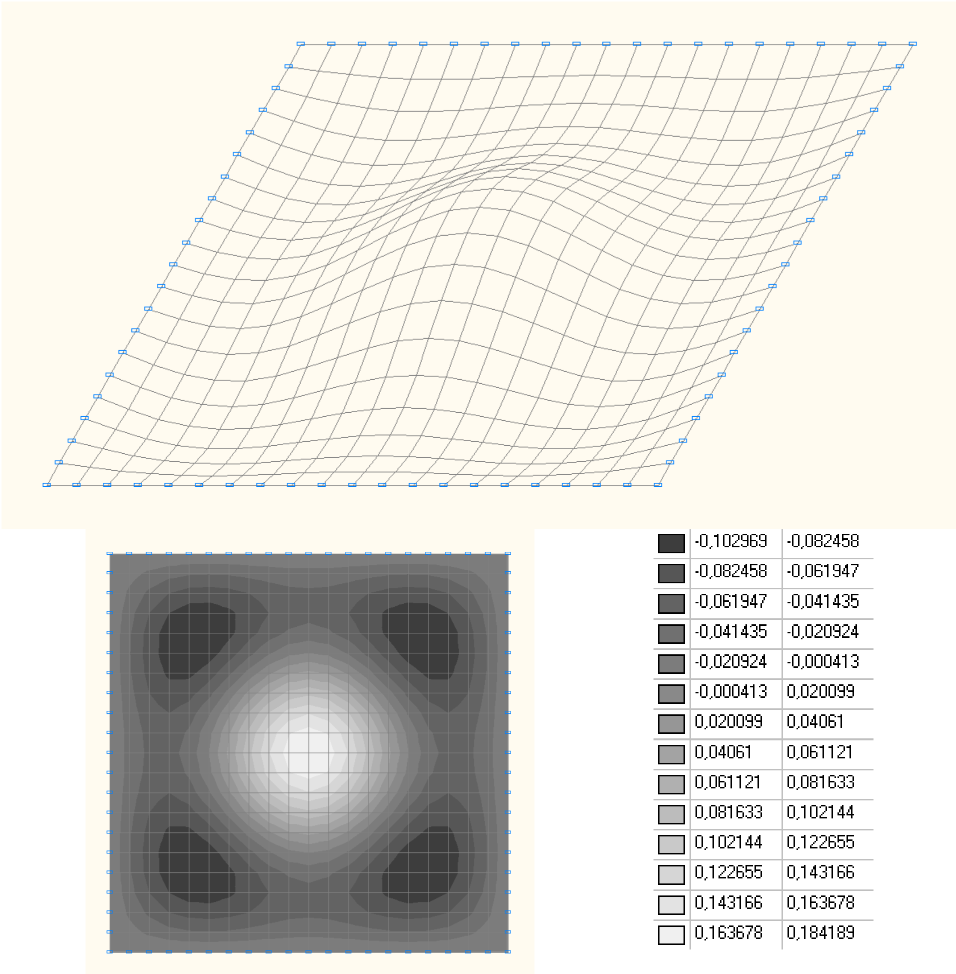
3-rd natural oscillation mode



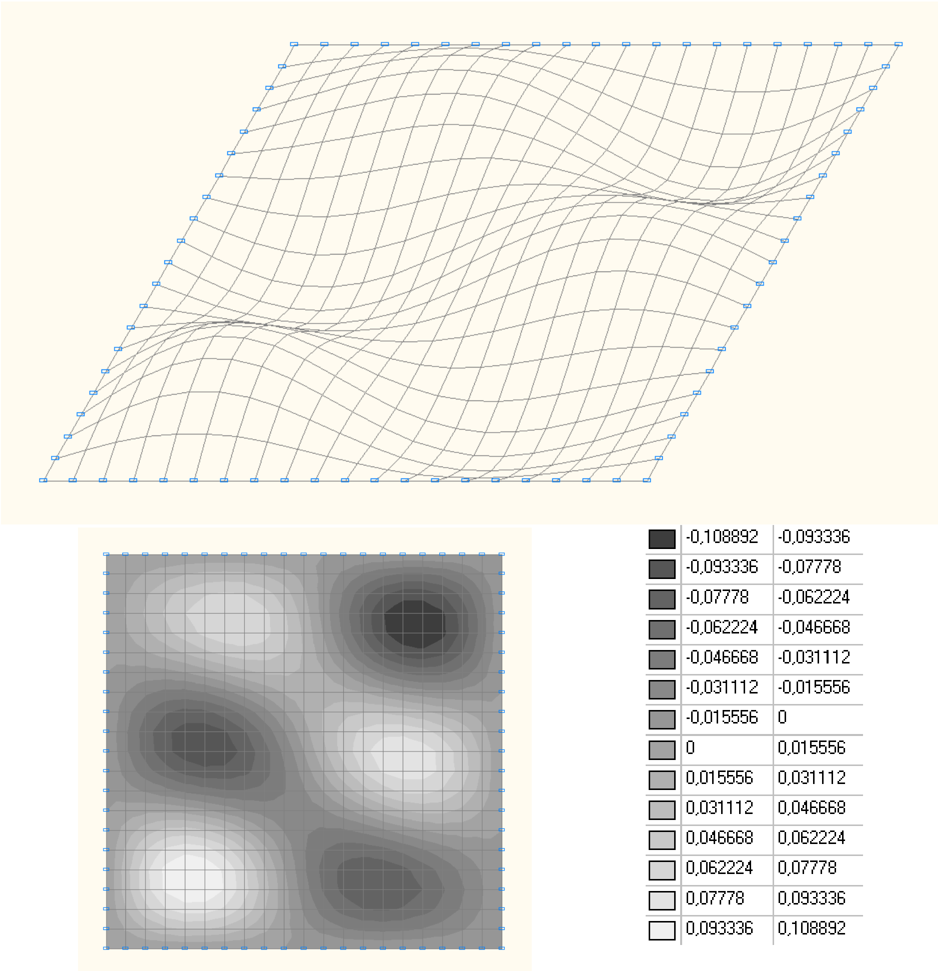
4-th natural oscillation mode



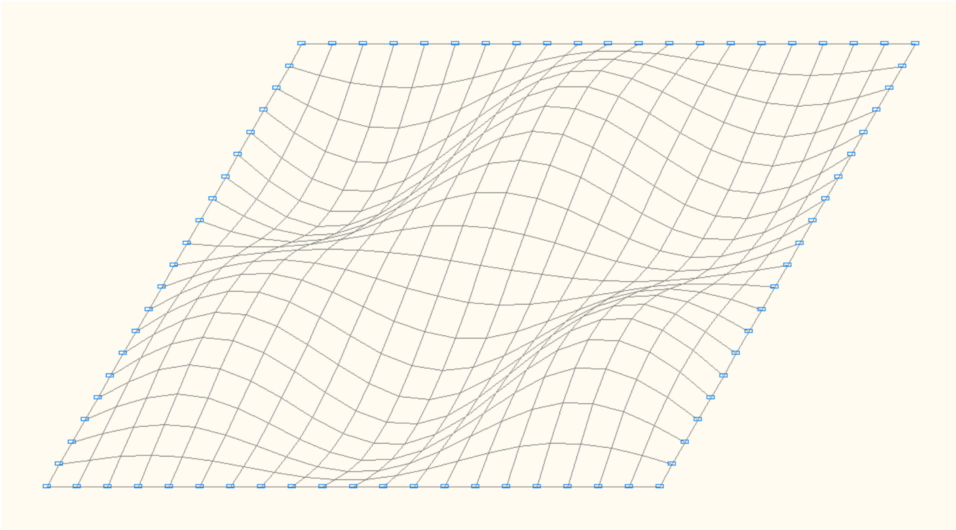
5-th natural oscillation mode

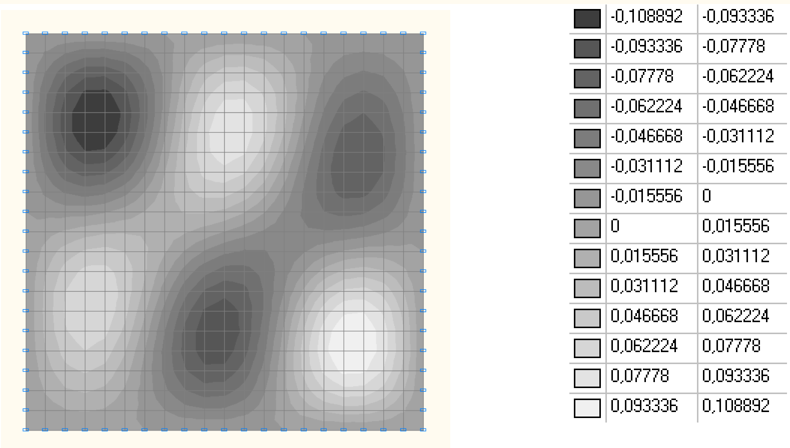


6-th natural oscillation mode

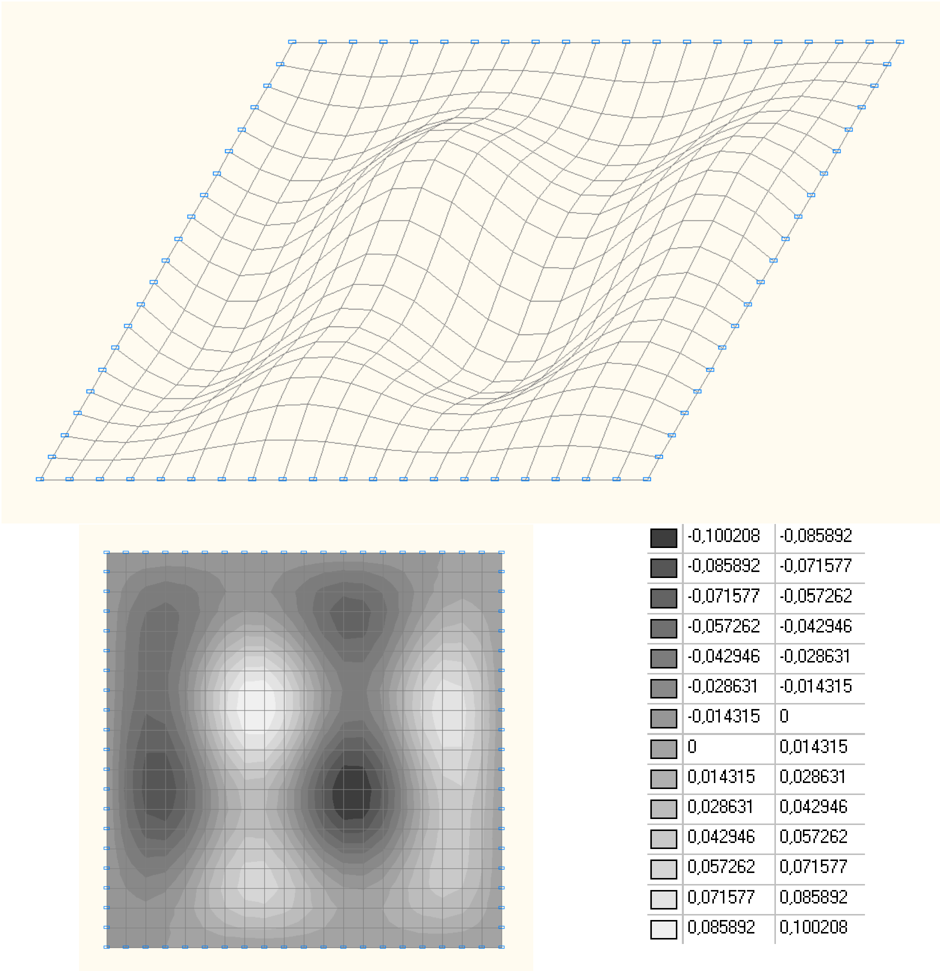


7-th natural oscillation mode

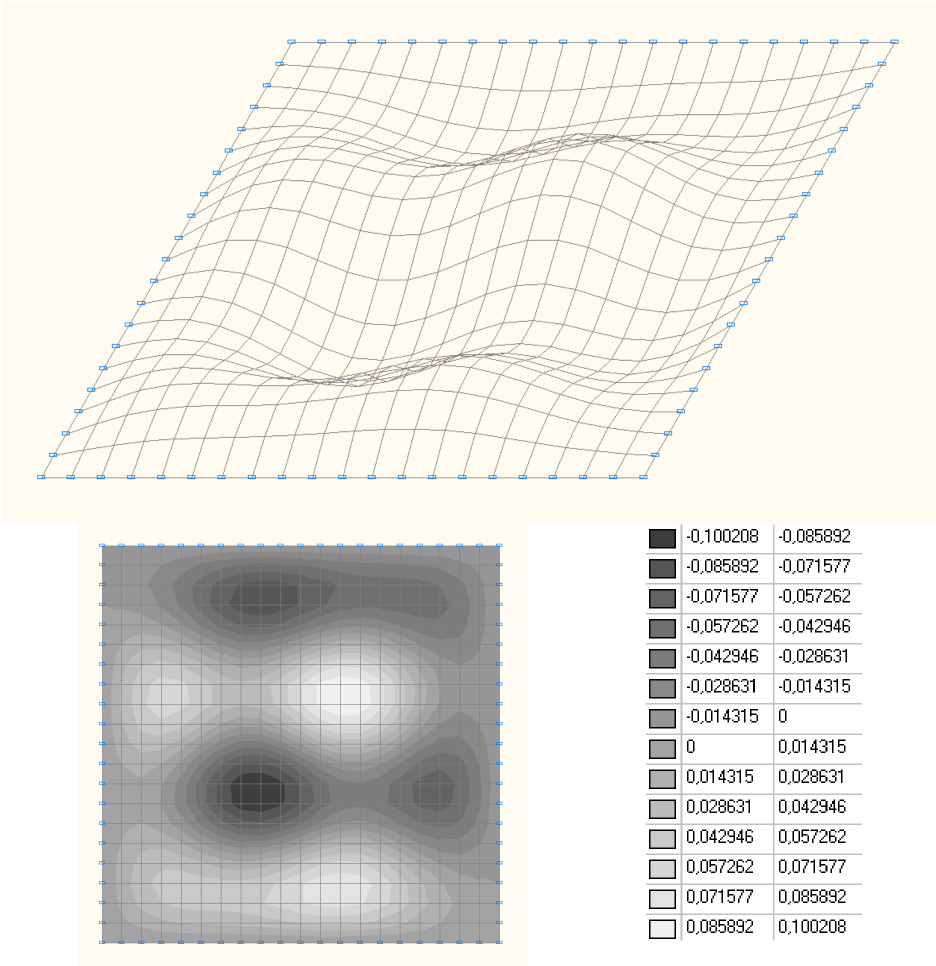




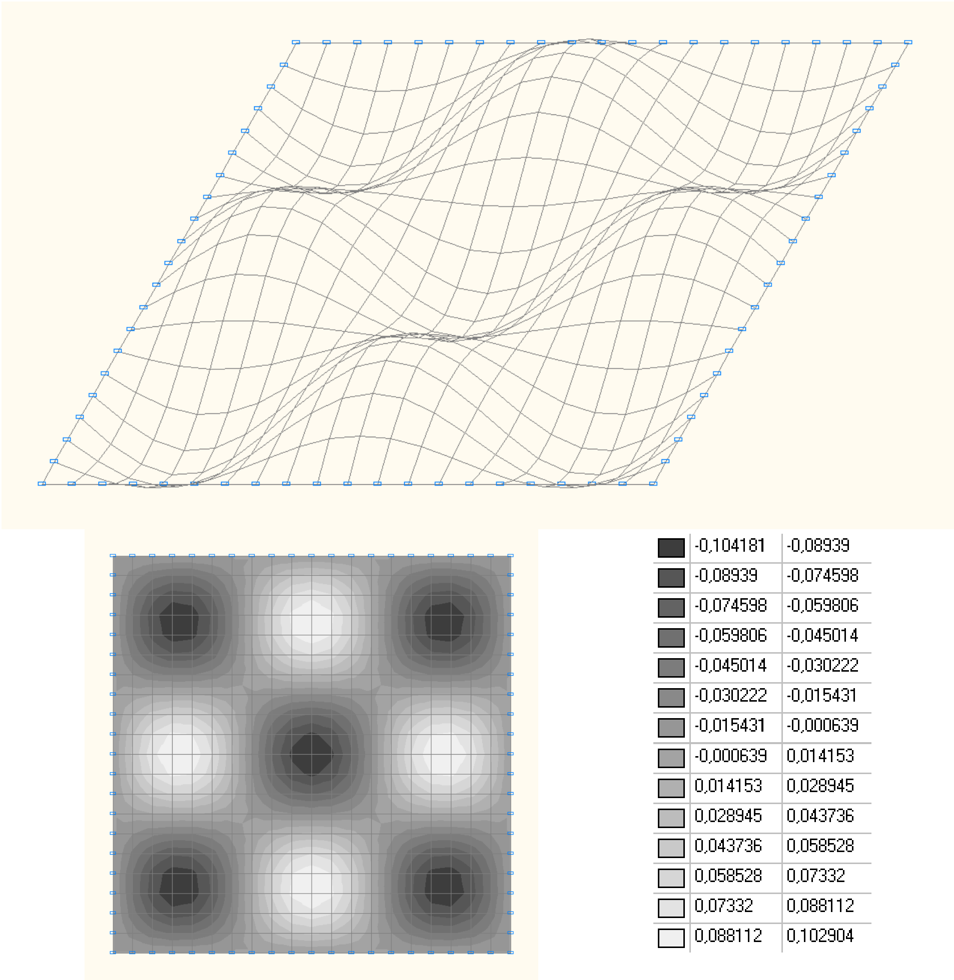
8-th natural oscillation mode



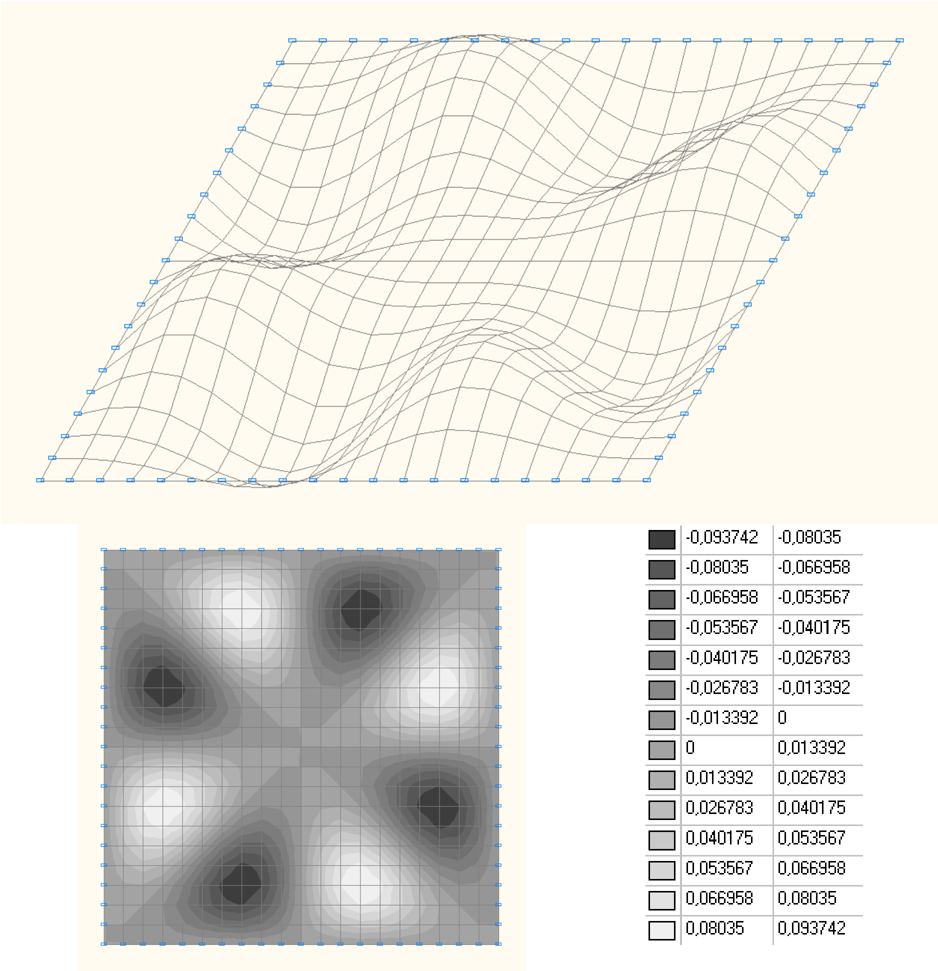
9-th natural oscillation mode



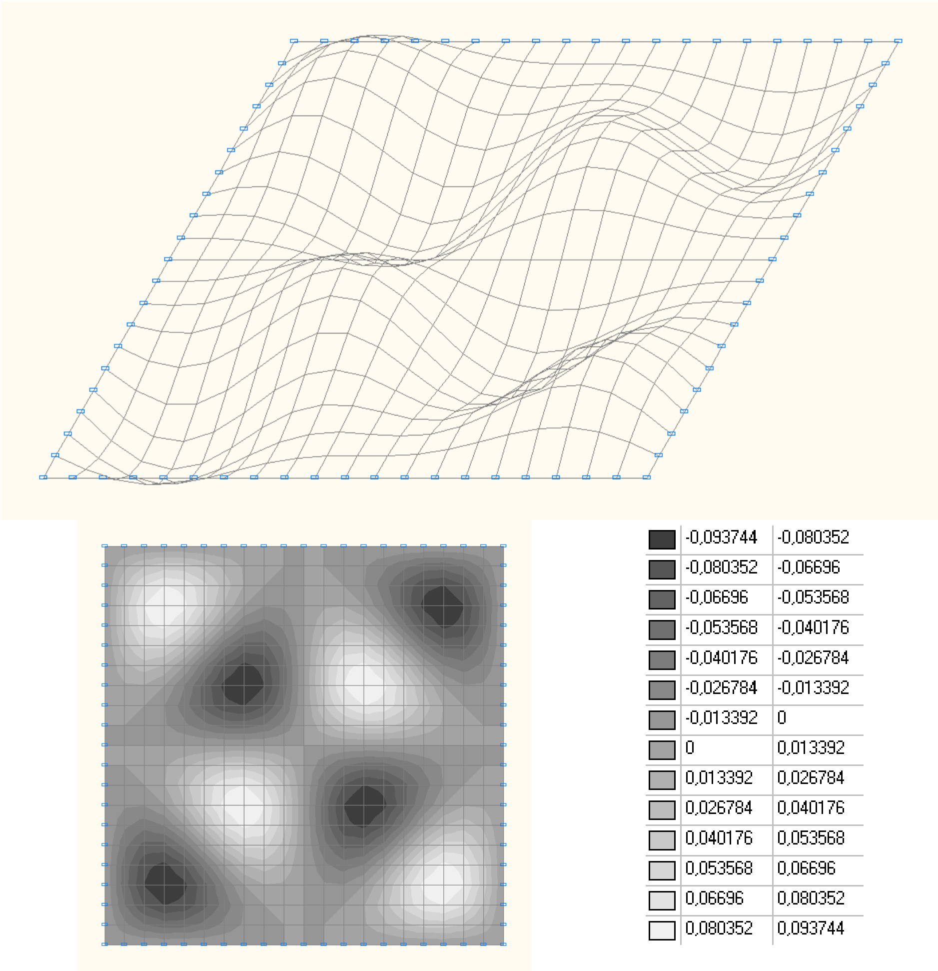
10-th natural oscillation mode



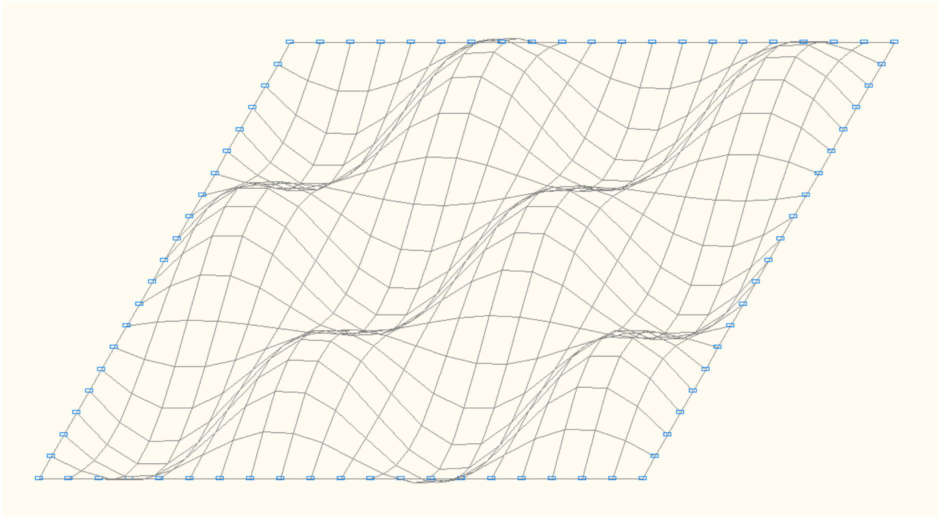
11-th natural oscillation mode

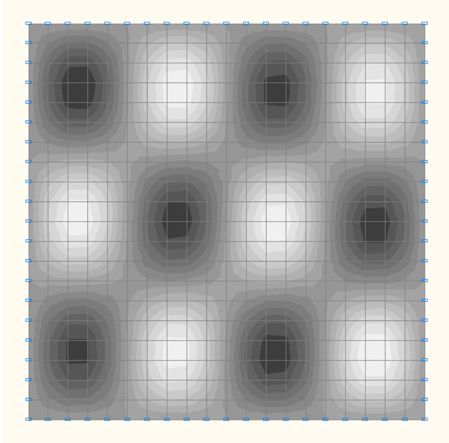


12-th natural oscillation mode



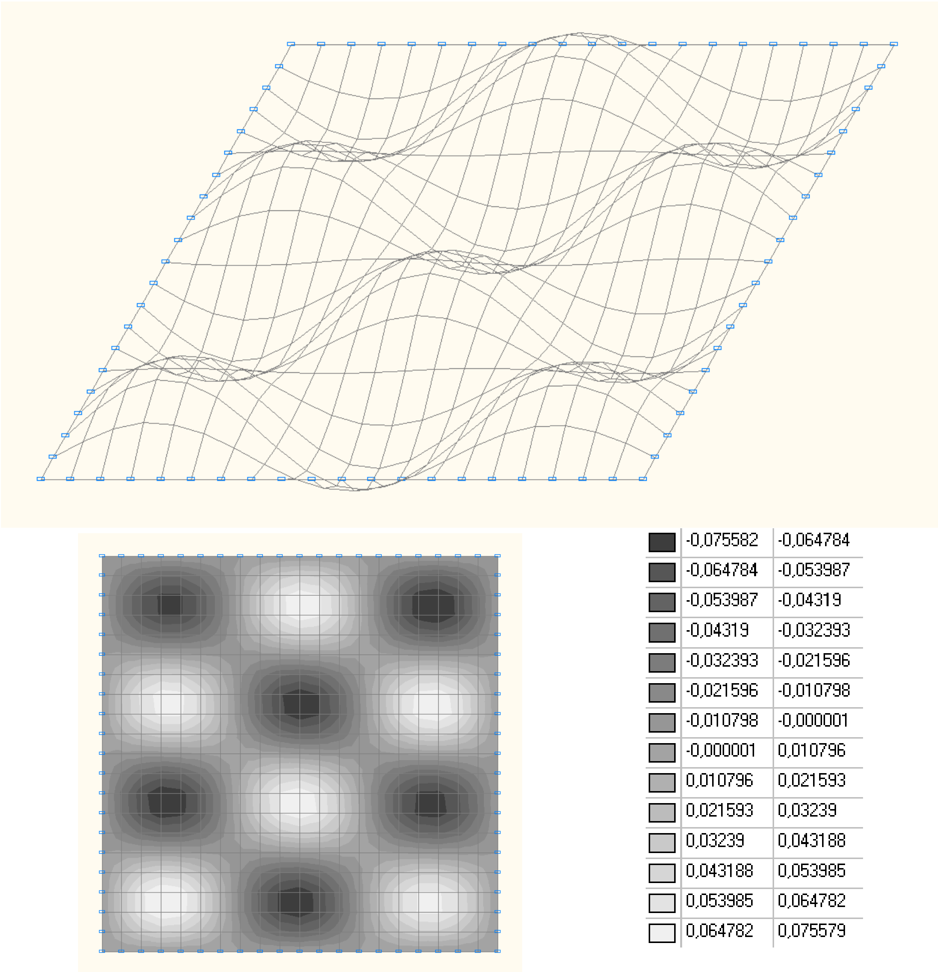
13-th natural oscillation mode



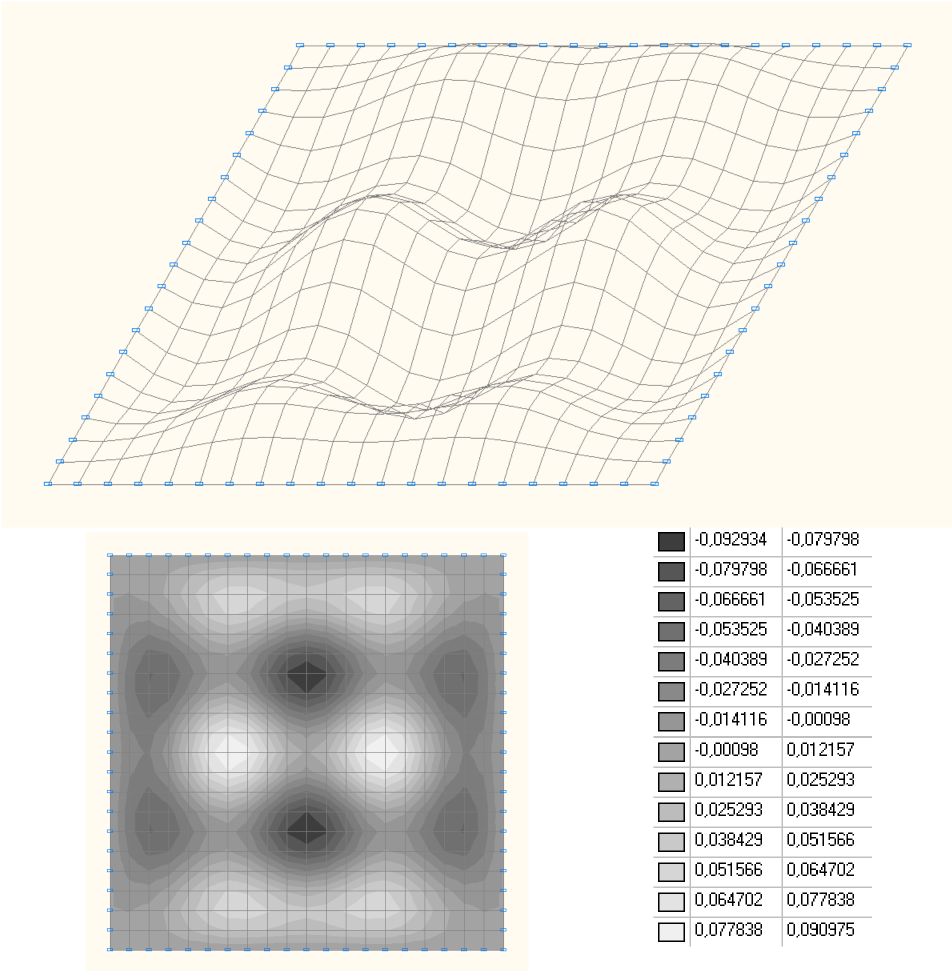


-0,075571	-0,064775
-0,064775	-0,053978
-0,053978	-0,043182
-0,043182	-0,032386
-0,032386	-0,02159
-0,02159	-0,010794
-0,010794	0,000003
0,000003	0,010799
0,010799	0,021595
0,021595	0,032391
0,032391	0,043188
0,043188	0,053984
0,053984	0,06478
0,06478	0,075576

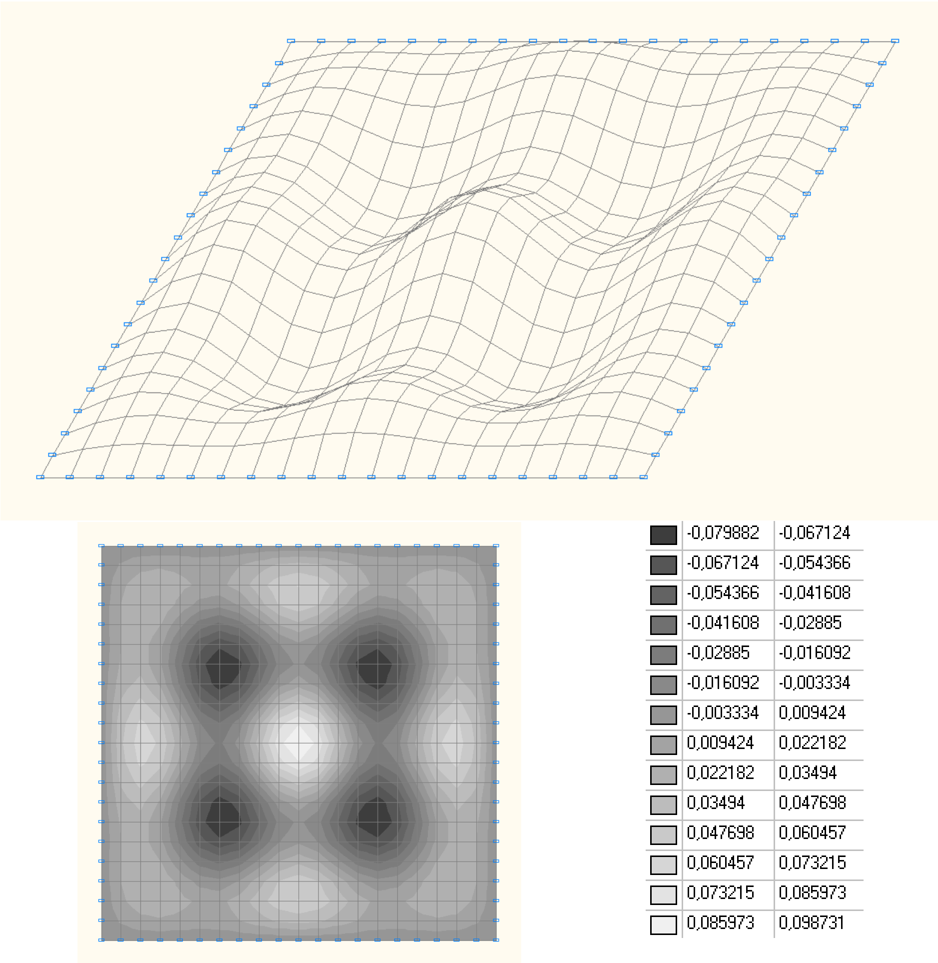
14-th natural oscillation mode



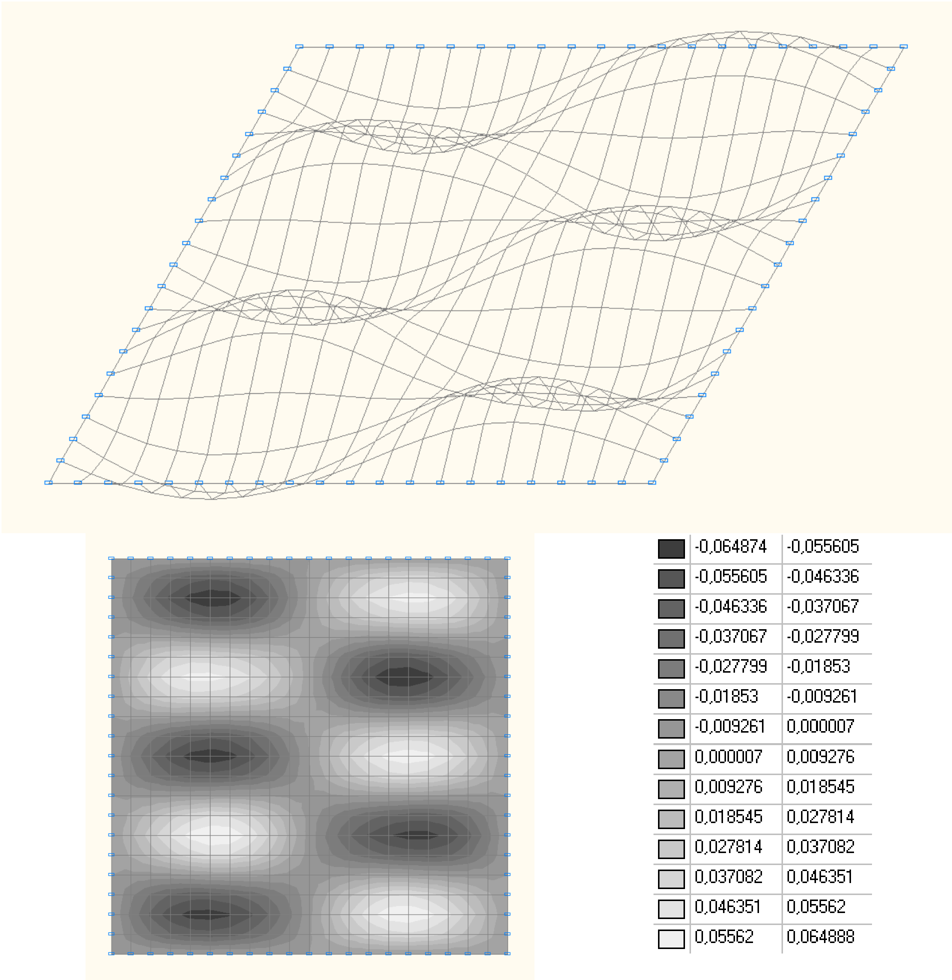
15-th natural oscillation mode



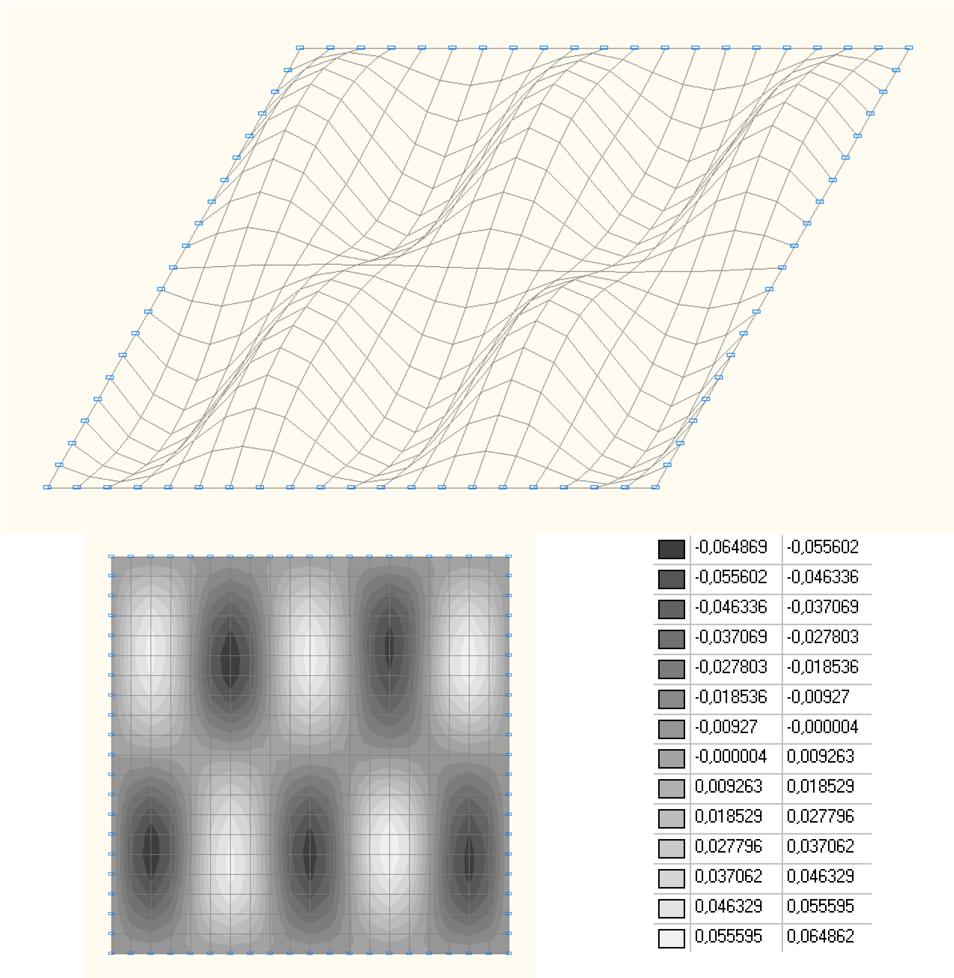
16-th natural oscillation mode



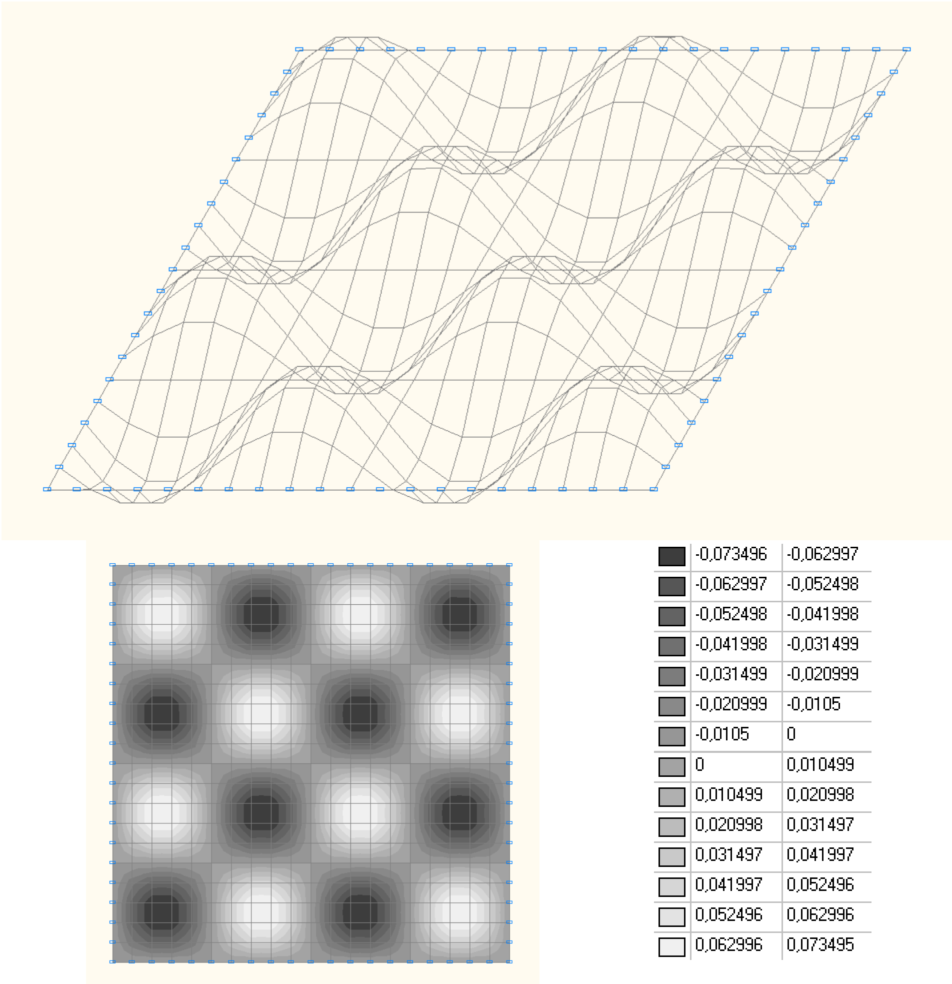
17-th natural oscillation mode



18-th natural oscillation mode



19-th natural oscillation mode



20-th natural oscillation mode

Comparison of solutions:

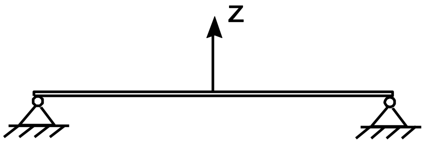
Natural frequencies ω, rad / s

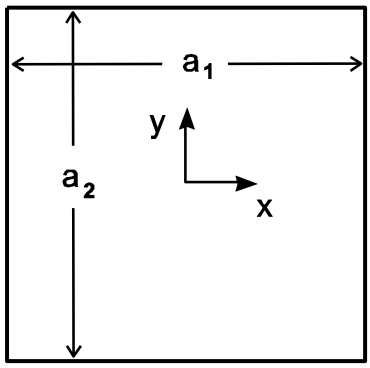
Oscillation mode	Number of half waves m1, m2	Theory	SCAD	Deviations, %
1	1, 1	306.0	306,1	0,05
2	1, 2	765. 0	765,6	0,08
3	2, 1	765.0	765,6	0,08
4	2, 2	1224.0	1226,4	0,19
5	1, 3	1530.0	1531,4	0,09
6	3, 1	1530.0	1531,4	0,09
7	2, 3	1989.0	1994,4	0,27
8	3, 2	1989.0	1994,4	0,27
9	1, 4	2601.0	2603,6	0,10
10	4, 1	2601.0	2603,6	0,10
11	3, 3	2754.0	2766,2	0,44
12	2, 4	3060.0	3069,7	0,32
13	4, 2	3060.0	3069,7	0,32
14	3, 4	3825.0	3846,8	0,57
15	4, 3	3825.0	3846,8	0,57
16	1, 5	3978.0	3982,6	0,12
17	5, 1	3978.0	3982,6	0,12
18	2, 5	4437.0	4452,7	0,35
19	5, 2	4437.0	4452,7	0,35
20	4, 4	4896.0	4934,7	0,79

Notes: In the analytical solution the natural frequencies ω of the simply supported square plate with the density of the material ρ can be determined according to the following formula:

$$\omega = \pi^2 \cdot \left(\frac{m_1^2}{a_2^2} + \frac{m_2^2}{a_2^2} \right) \cdot \left(\frac{D}{\rho \cdot h} \right)^{\frac{1}{2}}$$, where: $$D = \frac{E \cdot h^3}{12 \cdot \left(1 - \mu^2\right)},$$ $m_1, m_2 = 1,2,3 \ldots$

Natural Oscillations of a Simply Supported Rectangular Plate





Objective: Modal analysis of a simply supported rectangular plate.

Initial data file: **5.3.SPR**

Problem formulation: Determine the natural oscillation modes and frequencies ω of the simply supported rectangular plate with the density of the material ρ.

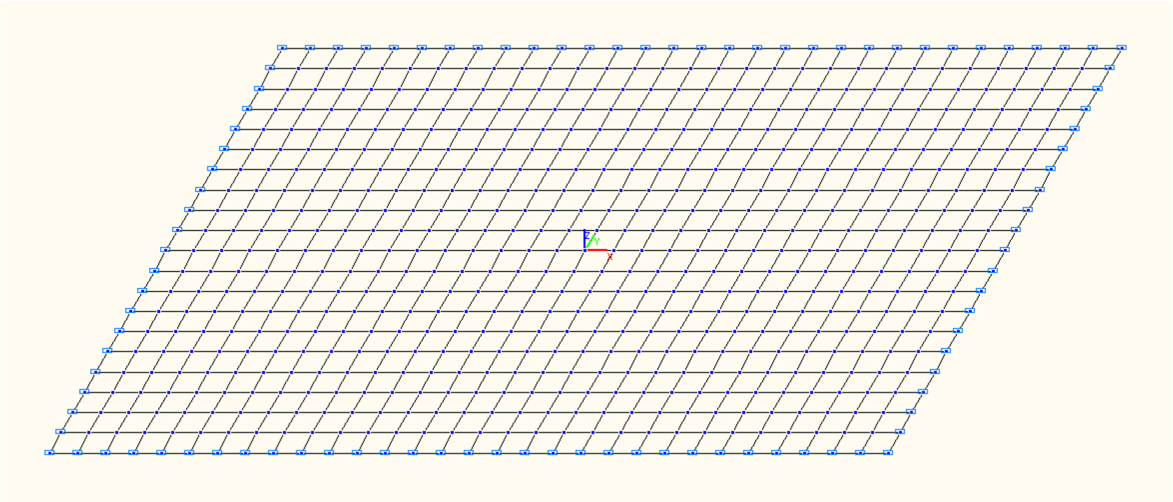
References: I.A. Birger, Ya.G. Panovko, Strength, Stability, Vibrations, Handbook in three volumes, Volume 3, Moscow, Mechanical engineering, 1968, p. 375.

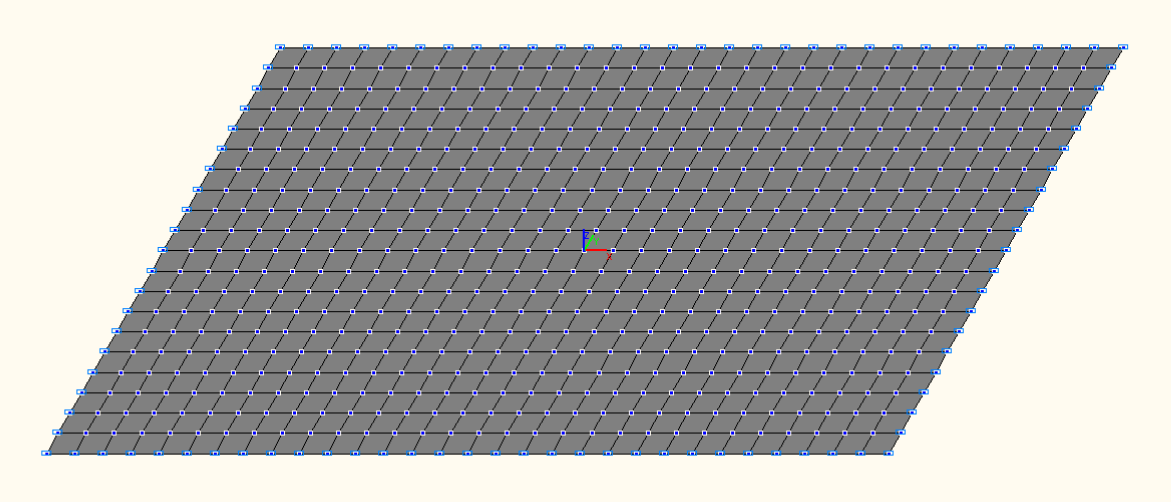
Initial data:

$E = 2.06 \cdot 10^8$ kPa	- elastic modulus;
$\nu = 0.3$	- Poisson's ratio;
$\rho = 7.85$ t/m^3	- density of the material;
$h = 0.01$ m	- thickness of the plate;
$a_1 = 1.5$ m	- long side of the plate (along the X axis of the global coordinate system);
$a_2 = 1.0$ m	- short side of the plate (along the Y axis of the global coordinate system).

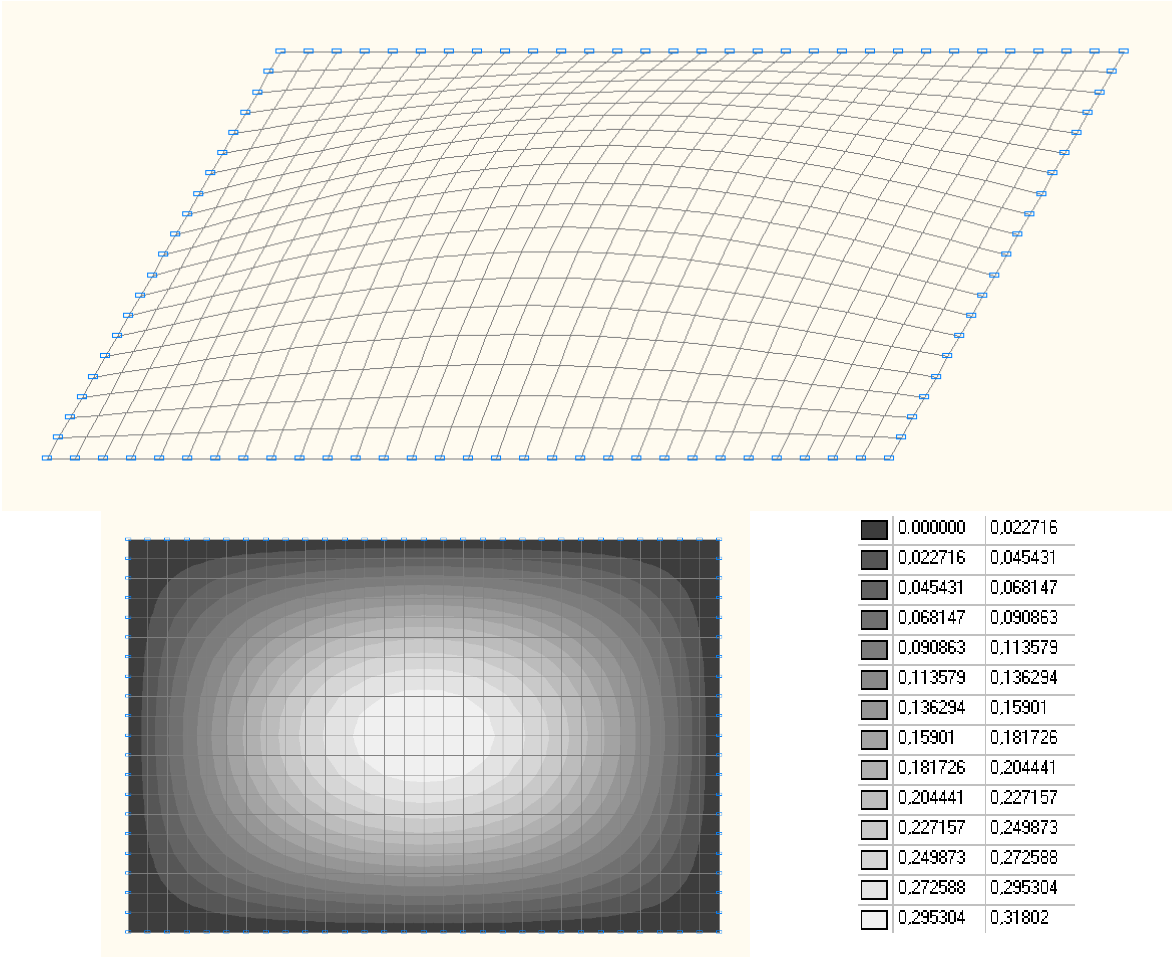
Finite element model: Design model – grade beam / plate, 600 plate elements of type 20. The spacing of the finite element mesh along the sides of the plate (along the X, Y axes of the global coordinate system) is 0.05 m. Boundary conditions are provided by imposing constraints in the direction of the degree of freedom Z for the edges parallel to the X and Y axes of the global coordinate system. The distributed mass is specified by transforming the static load from the self-weight of the plate ow = $\gamma \cdot h$, where $\gamma = \rho \cdot g = 77.01$ kN/m^3. Number of nodes in the design model – 651. The determination of the natural oscillation modes and natural frequencies is performed by the method of subspace iteration. The matrix of concentrated masses is used in the calculation.

Results in SCAD

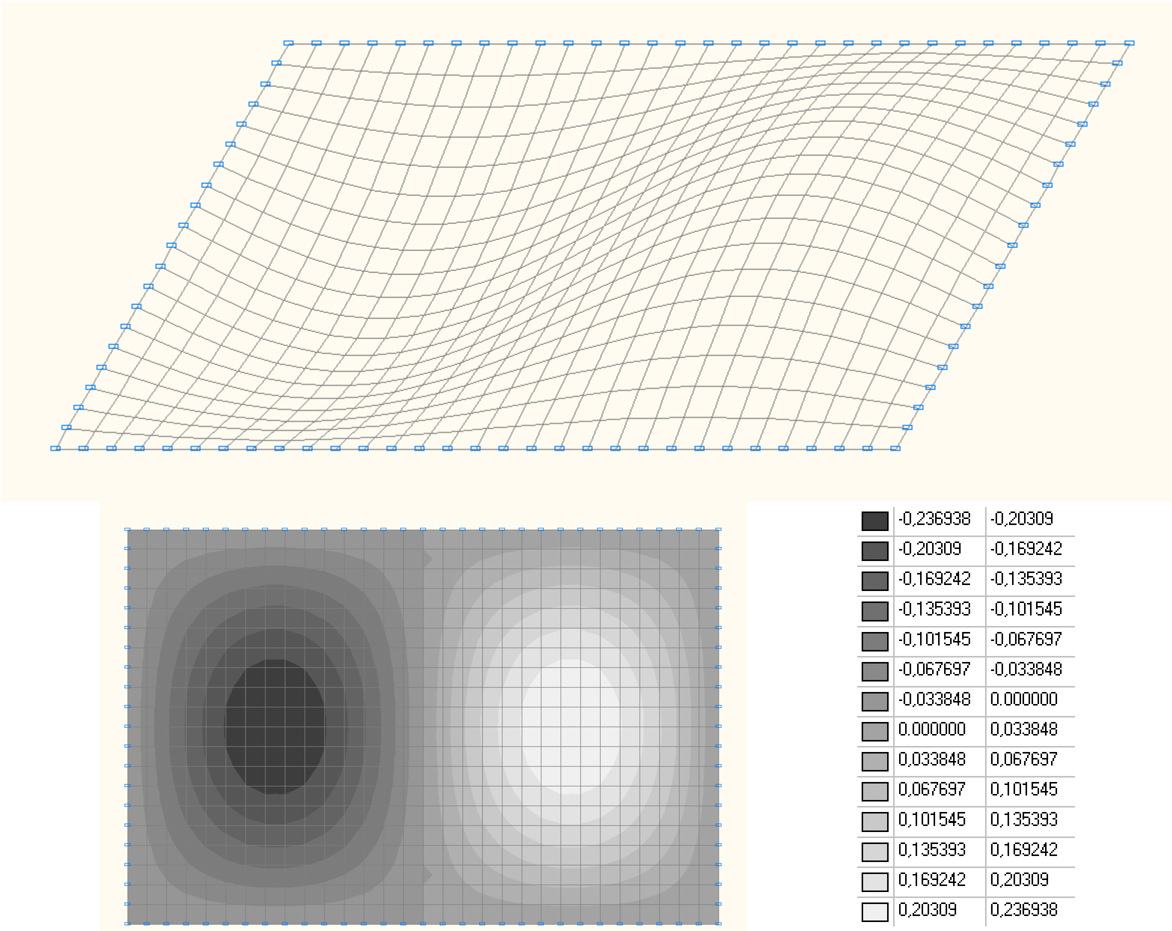




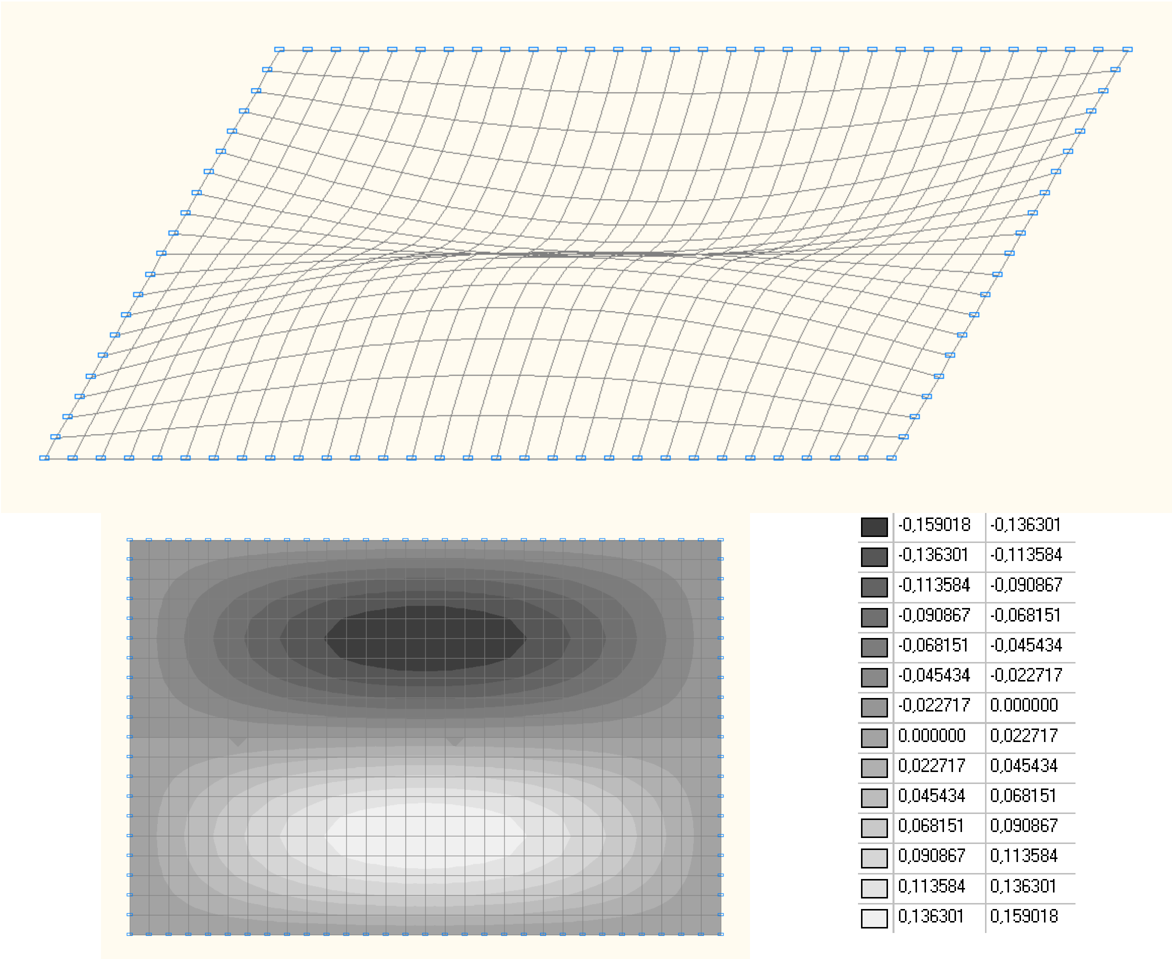
Design model



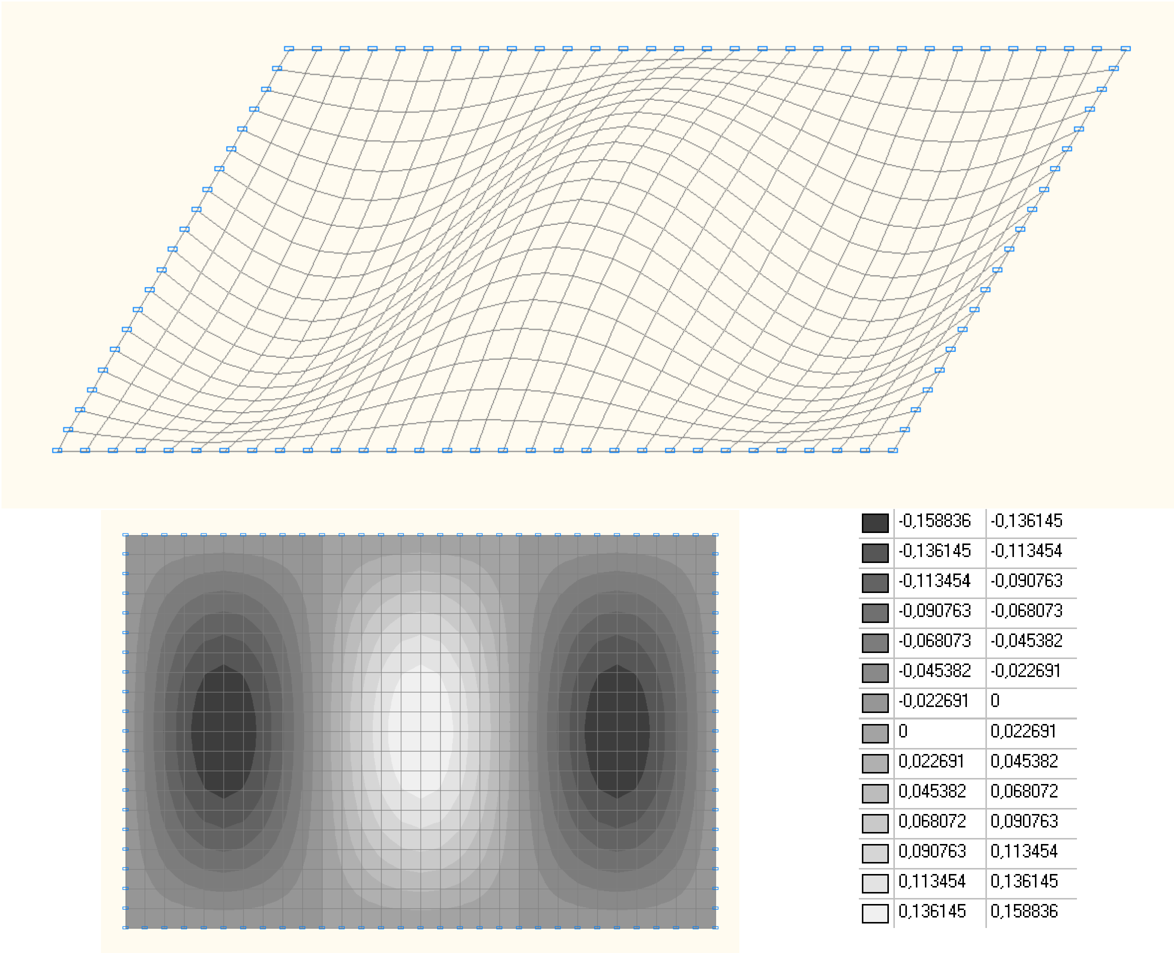
1-st natural oscillation mode



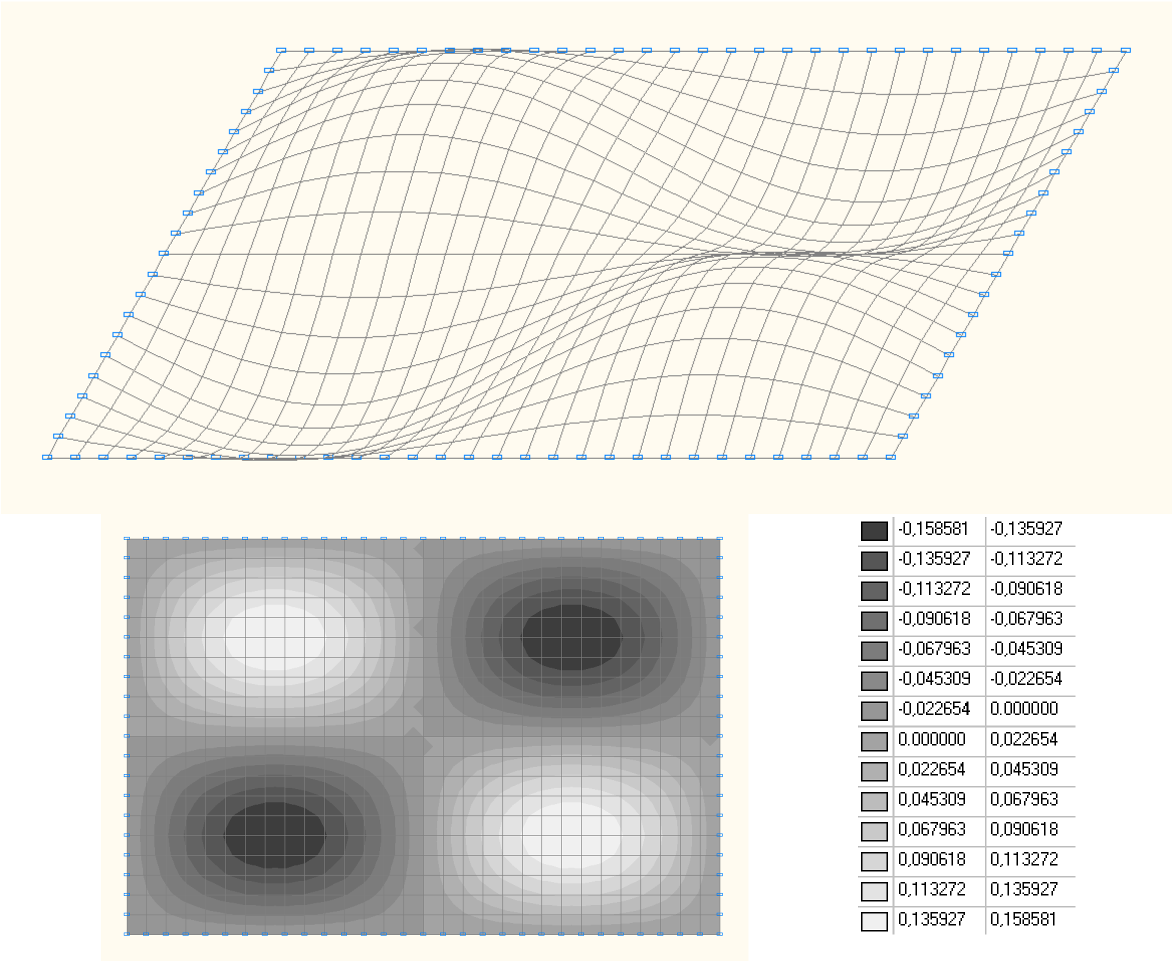
2-nd natural oscillation mode



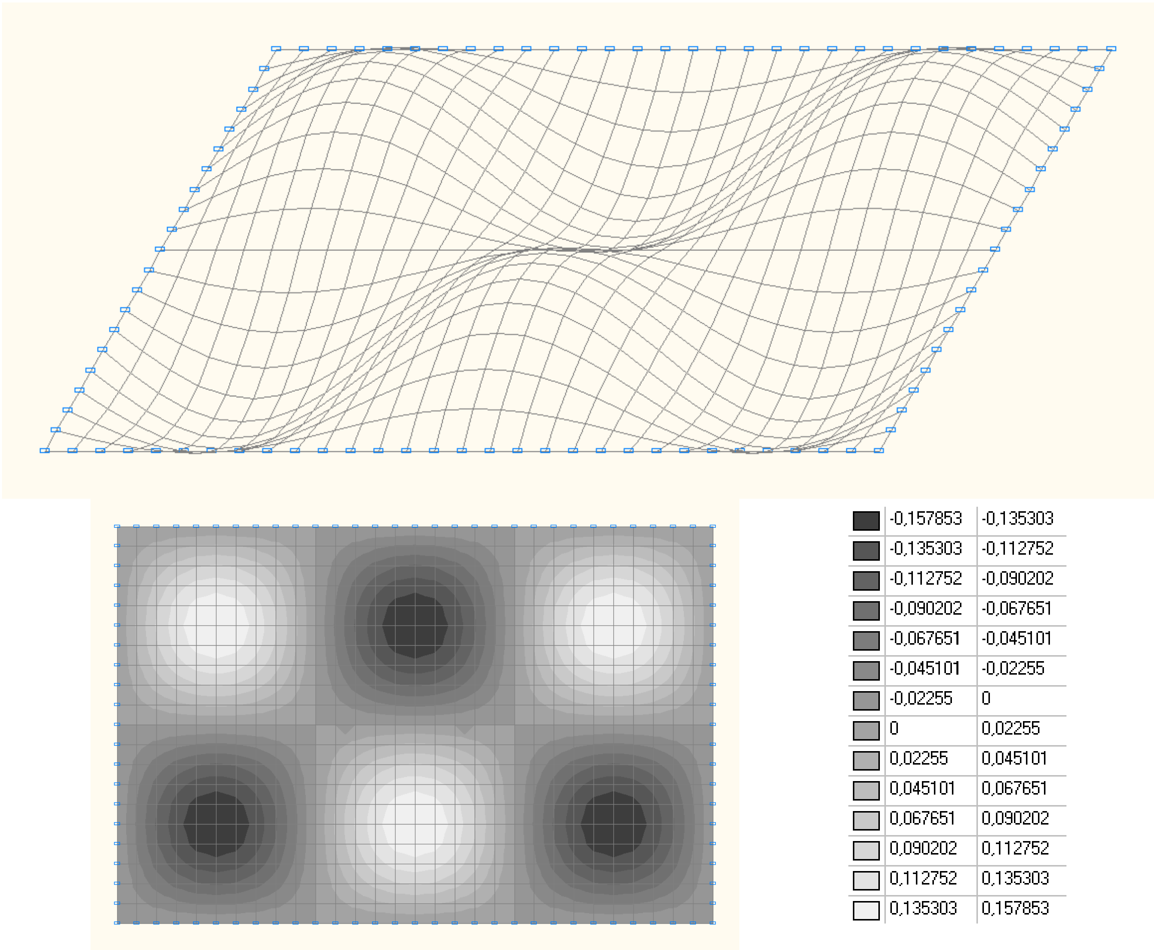
3-rd natural oscillation mode



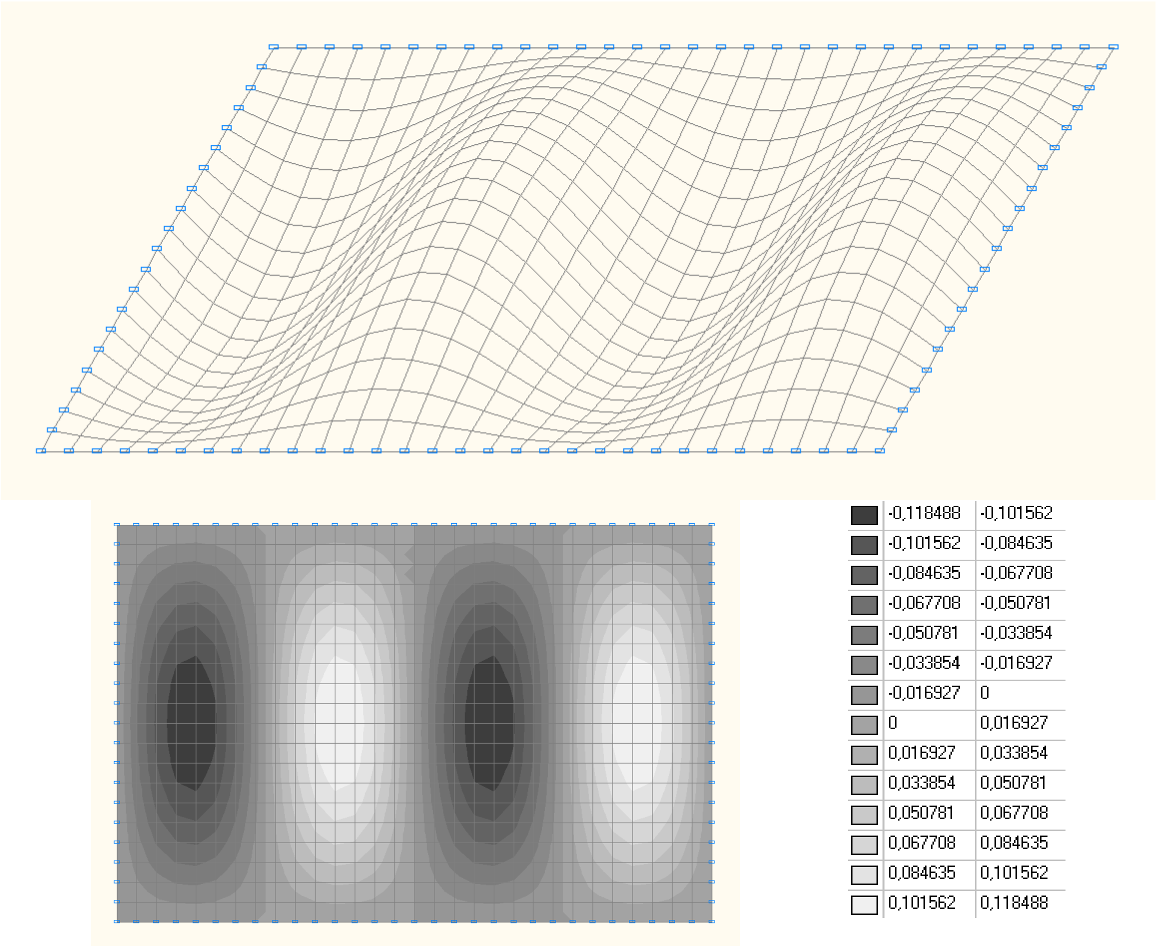
4-th natural oscillation mode



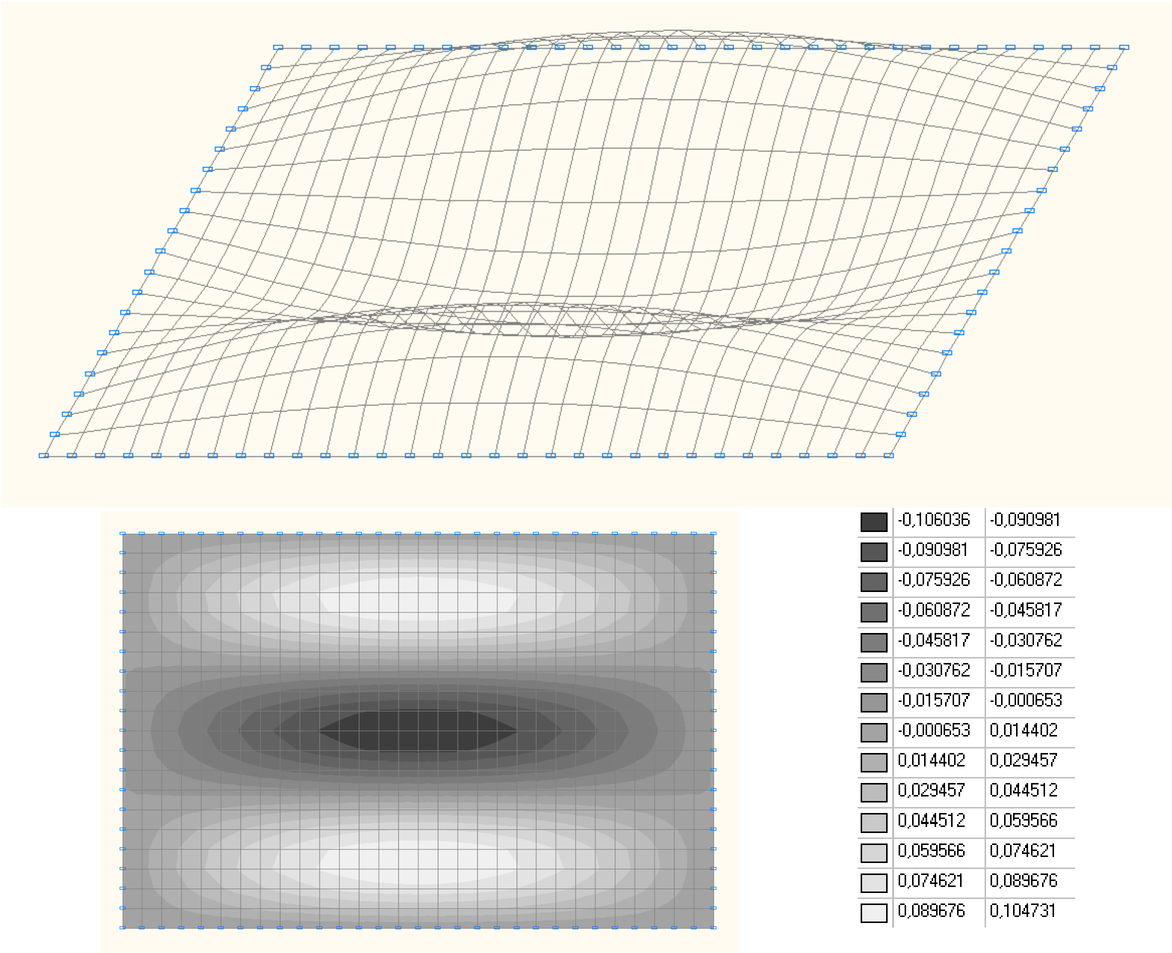
5-th natural oscillation mode



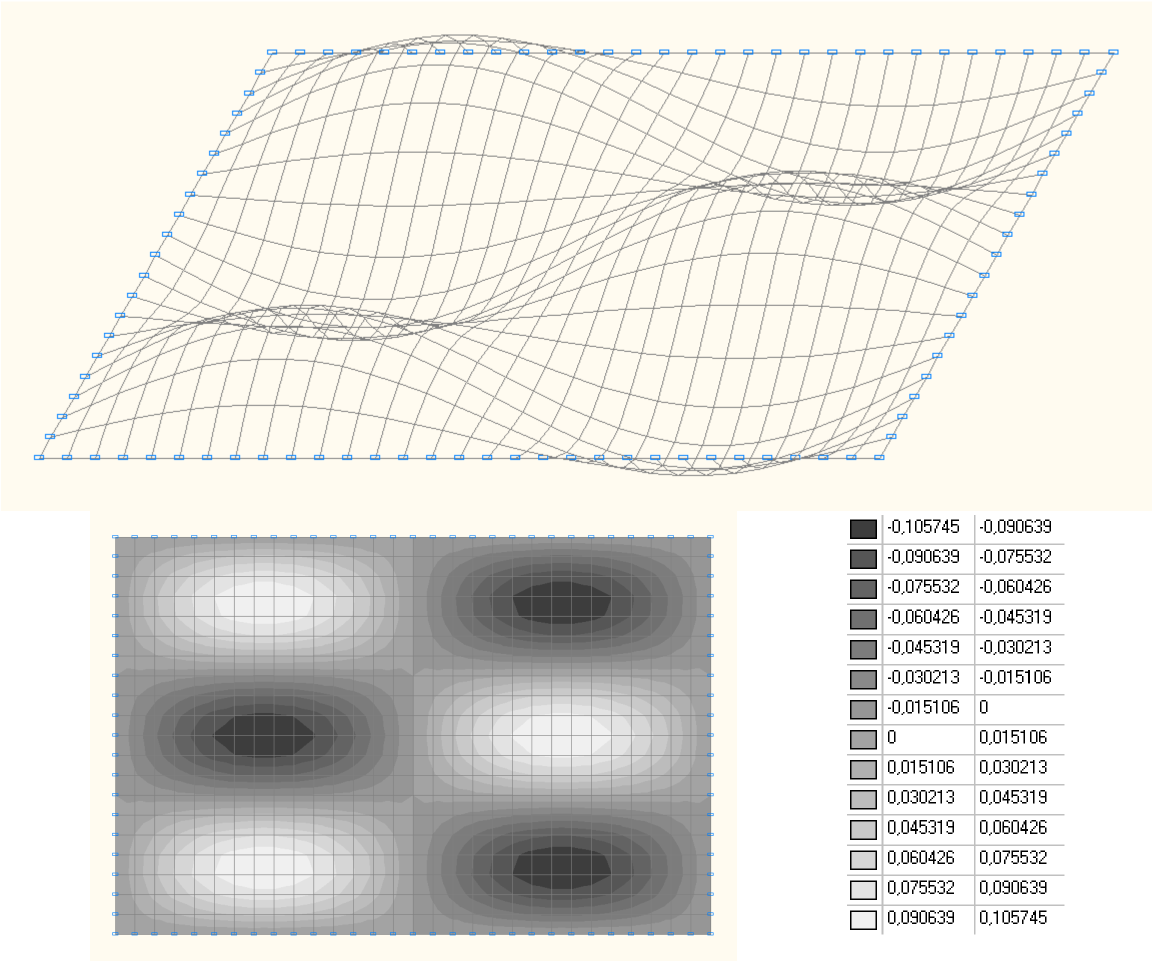
6-th natural oscillation mode



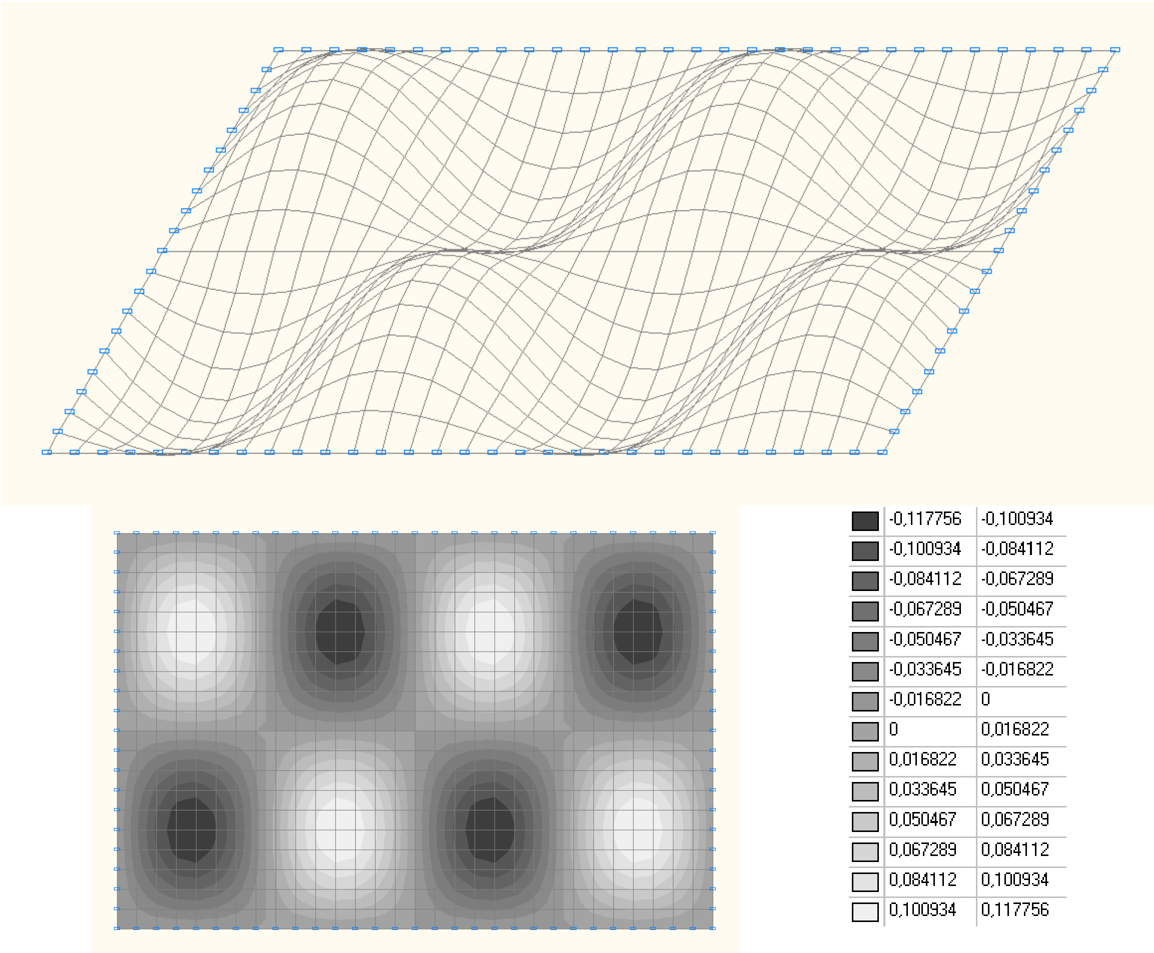
7-th natural oscillation mode



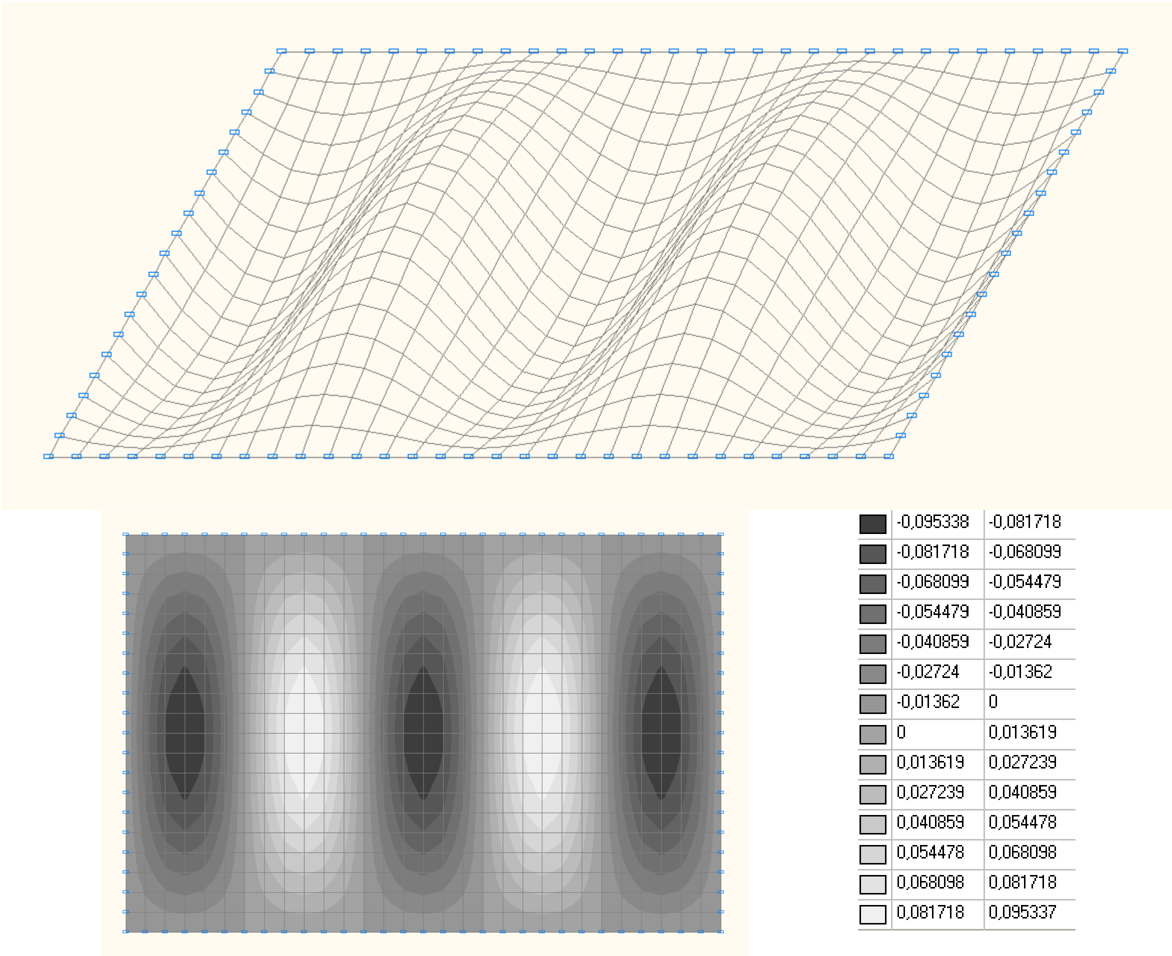
8-th natural oscillation mode



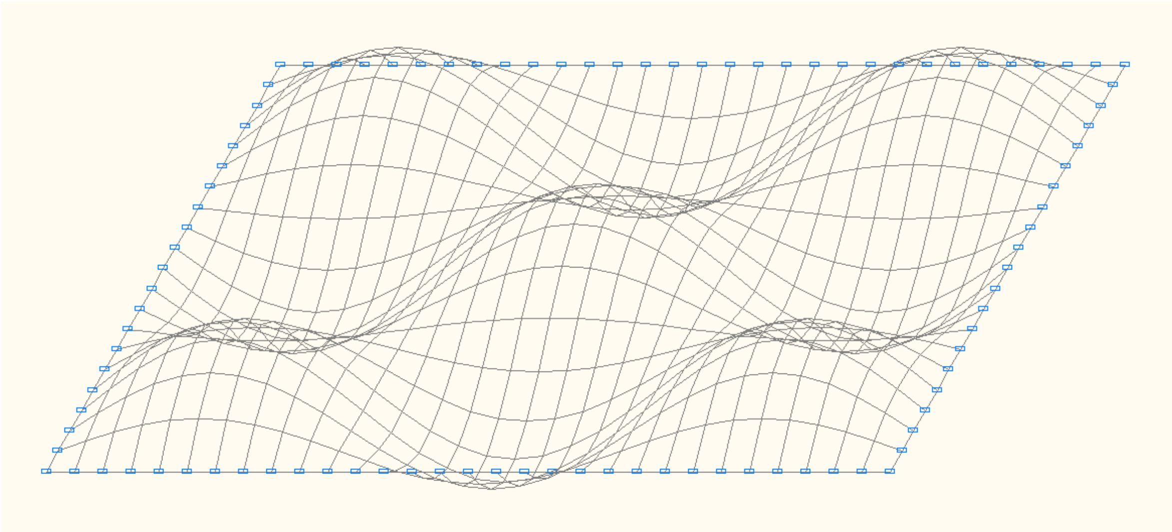
9-th natural oscillation mode

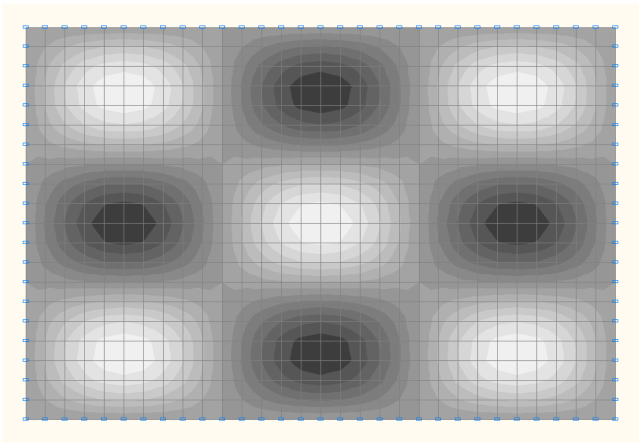


10-th natural oscillation mode



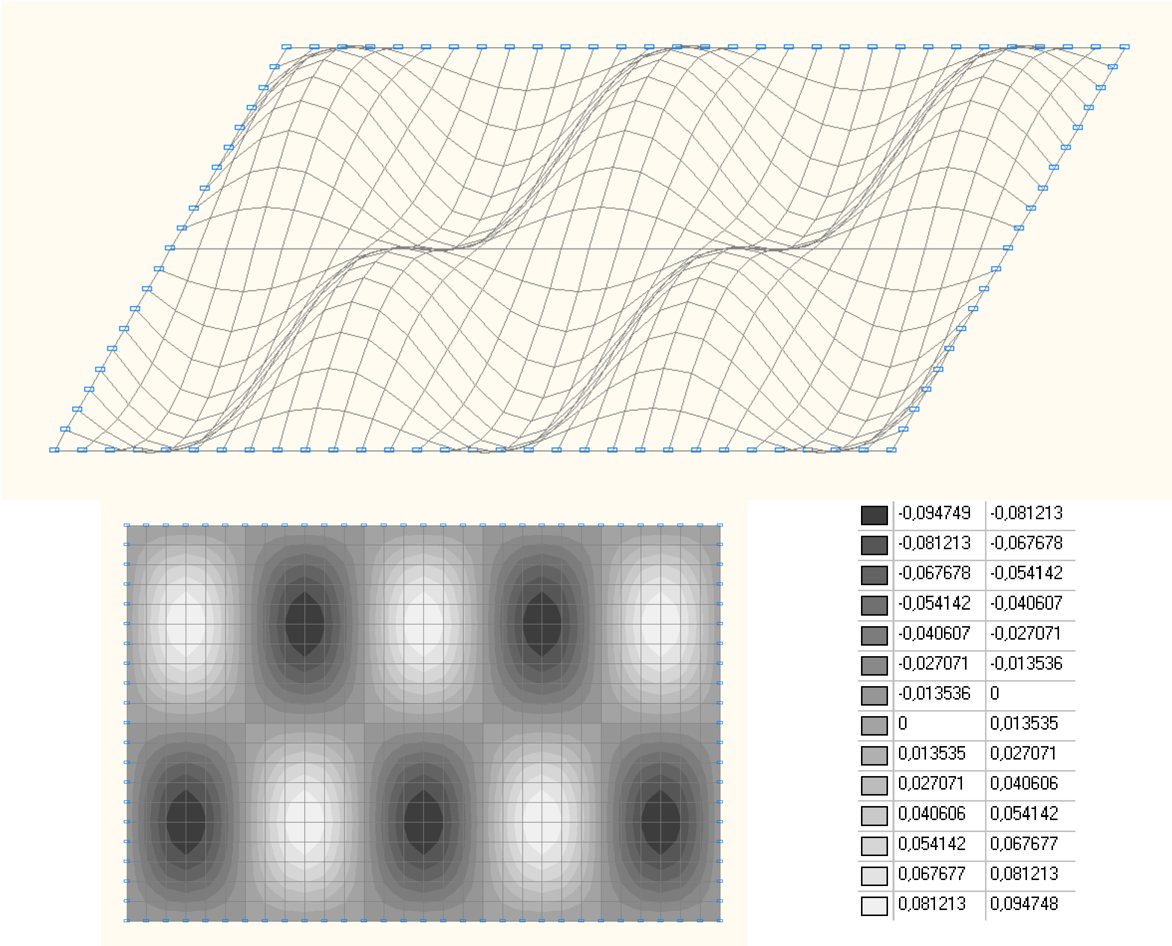
11-th natural oscillation mode



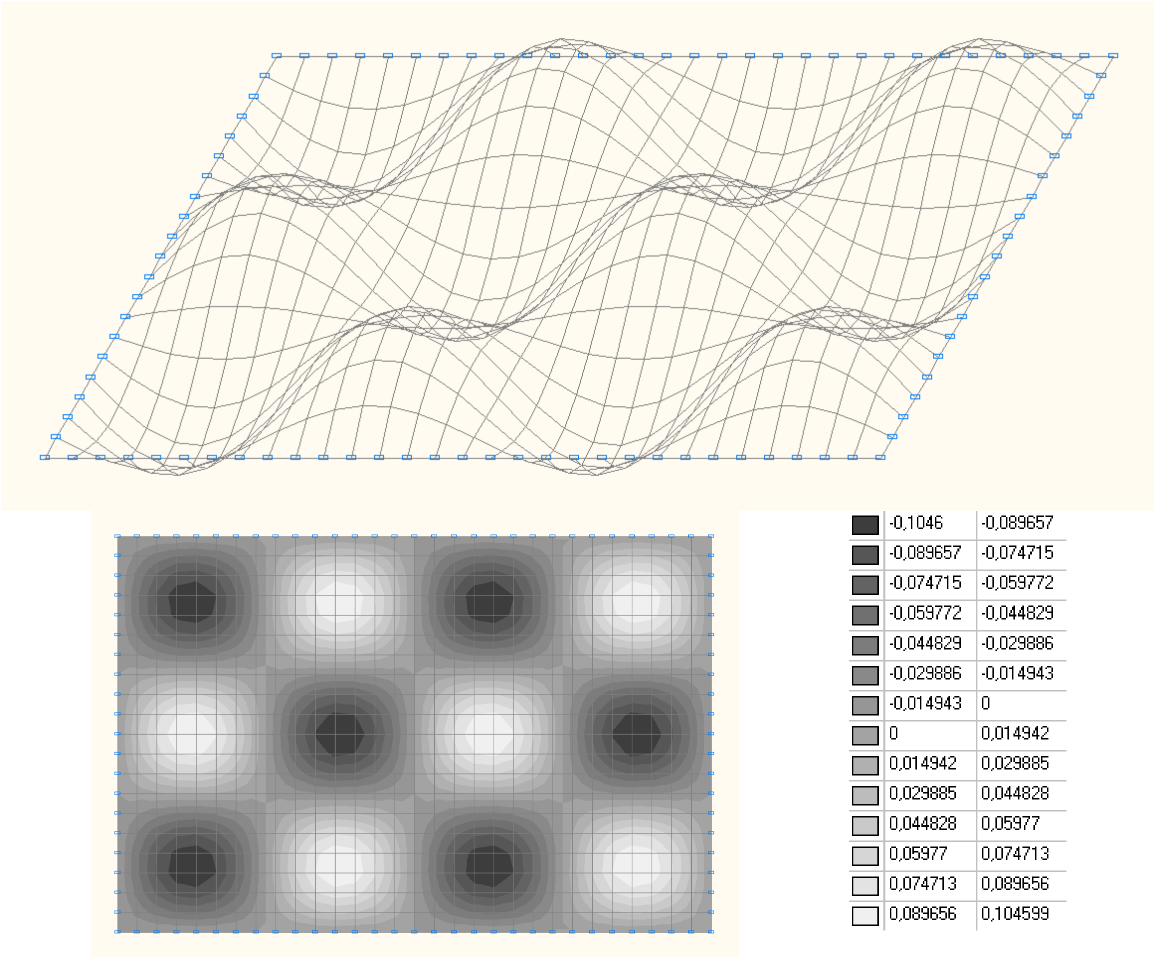


-0,105263	-0,090225
-0,090225	-0,075188
-0,075188	-0,06015
-0,06015	-0,045113
-0,045113	-0,030075
-0,030075	-0,015037
-0,015037	0
0	0,015038
0,015038	0,030075
0,030075	0,045113
0,045113	0,060151
0,060151	0,075188
0,075188	0,090226
0,090226	0,105263

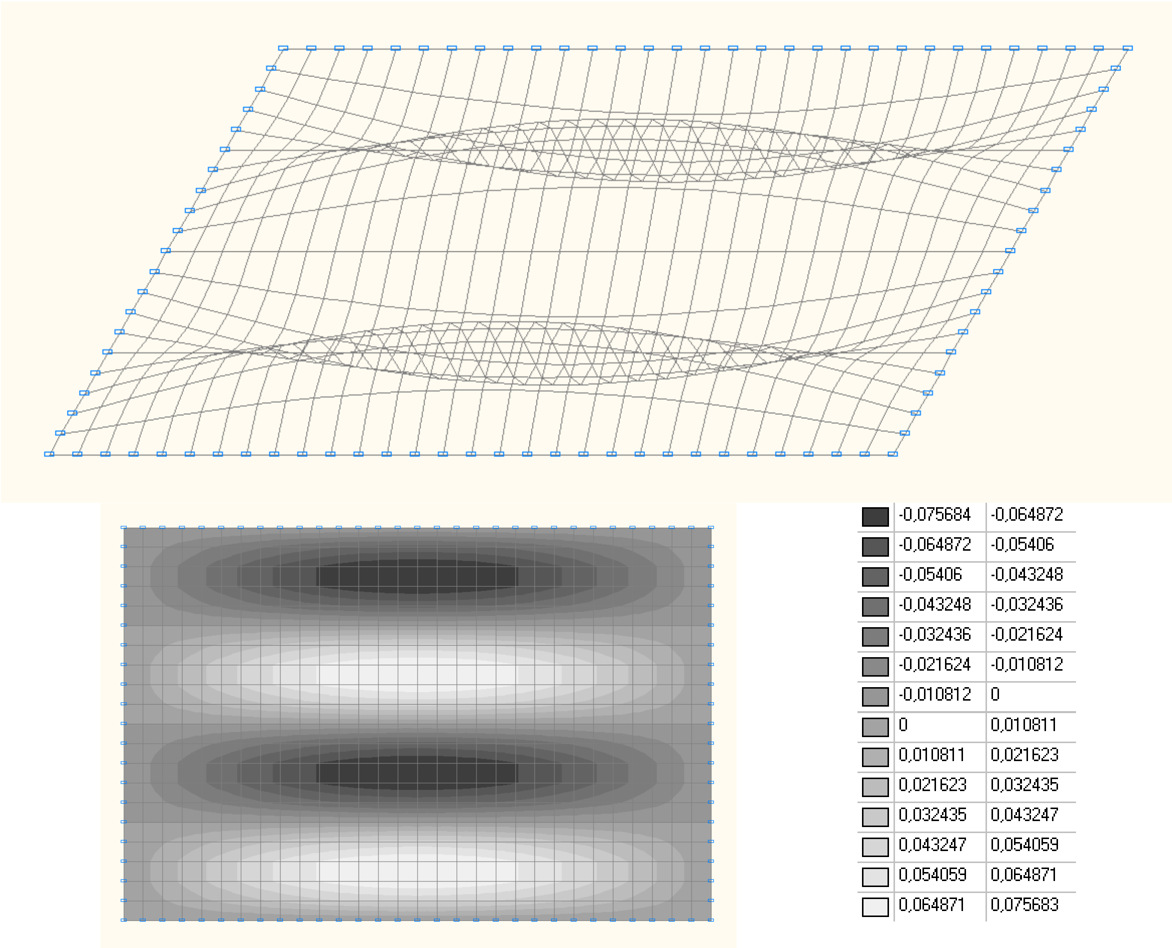
12-th natural oscillation mode



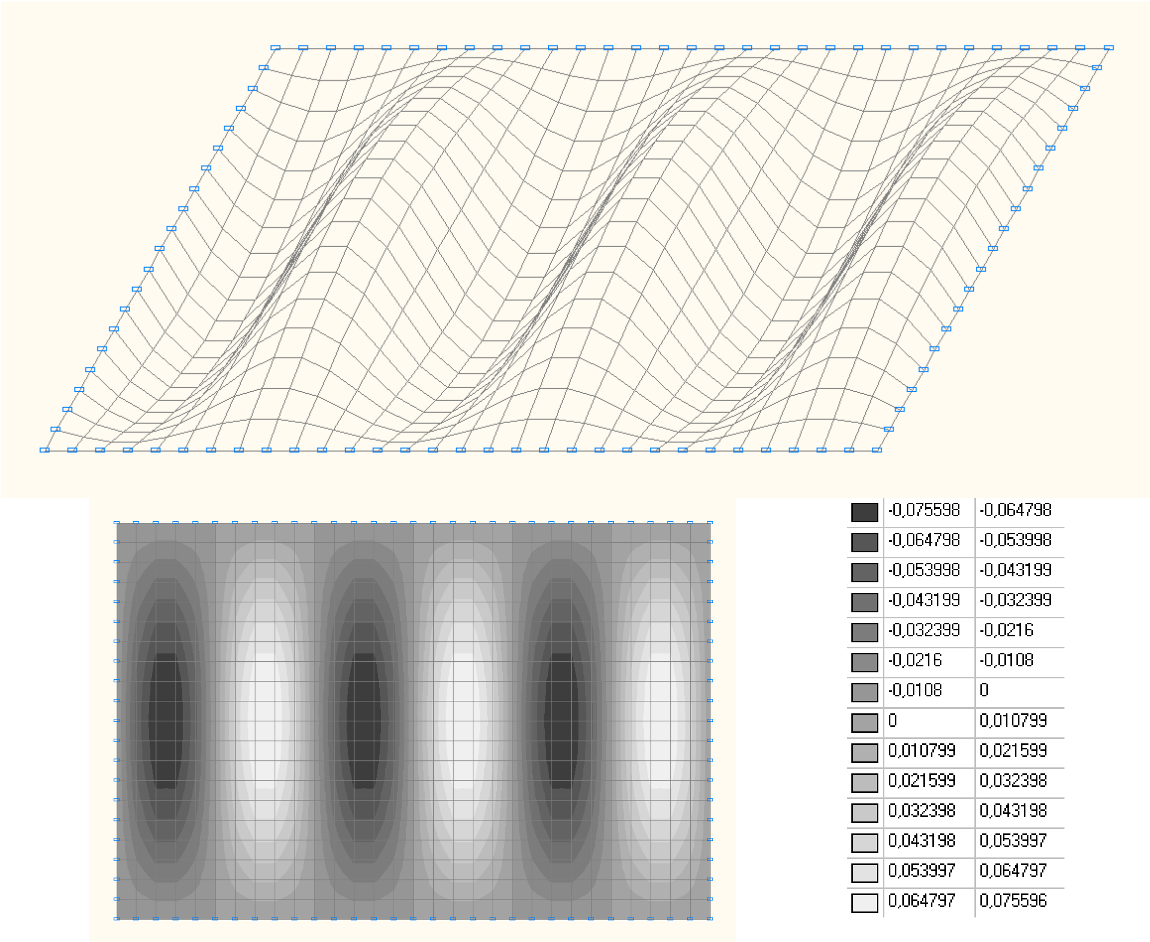
13-th natural oscillation mode



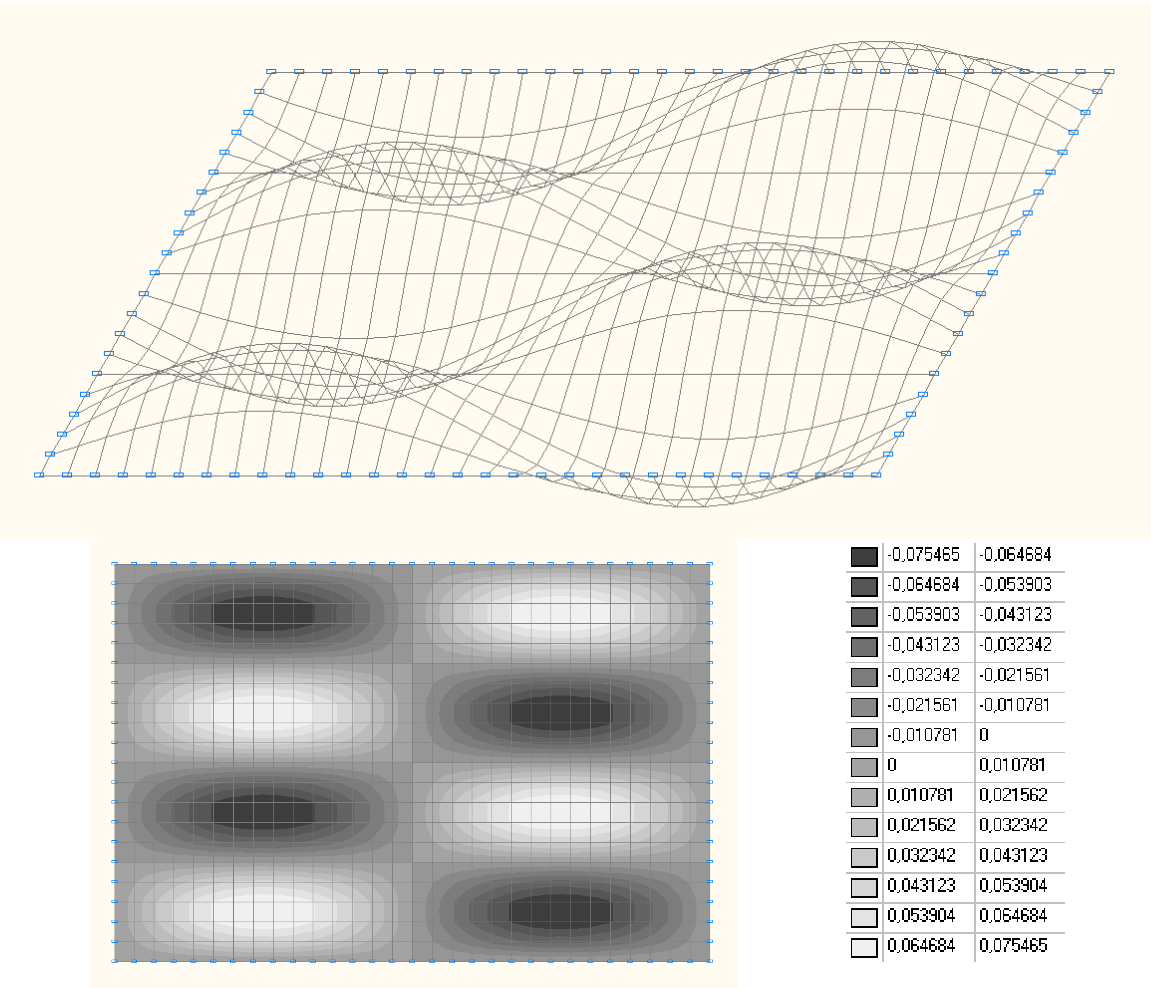
14-th natural oscillation mode



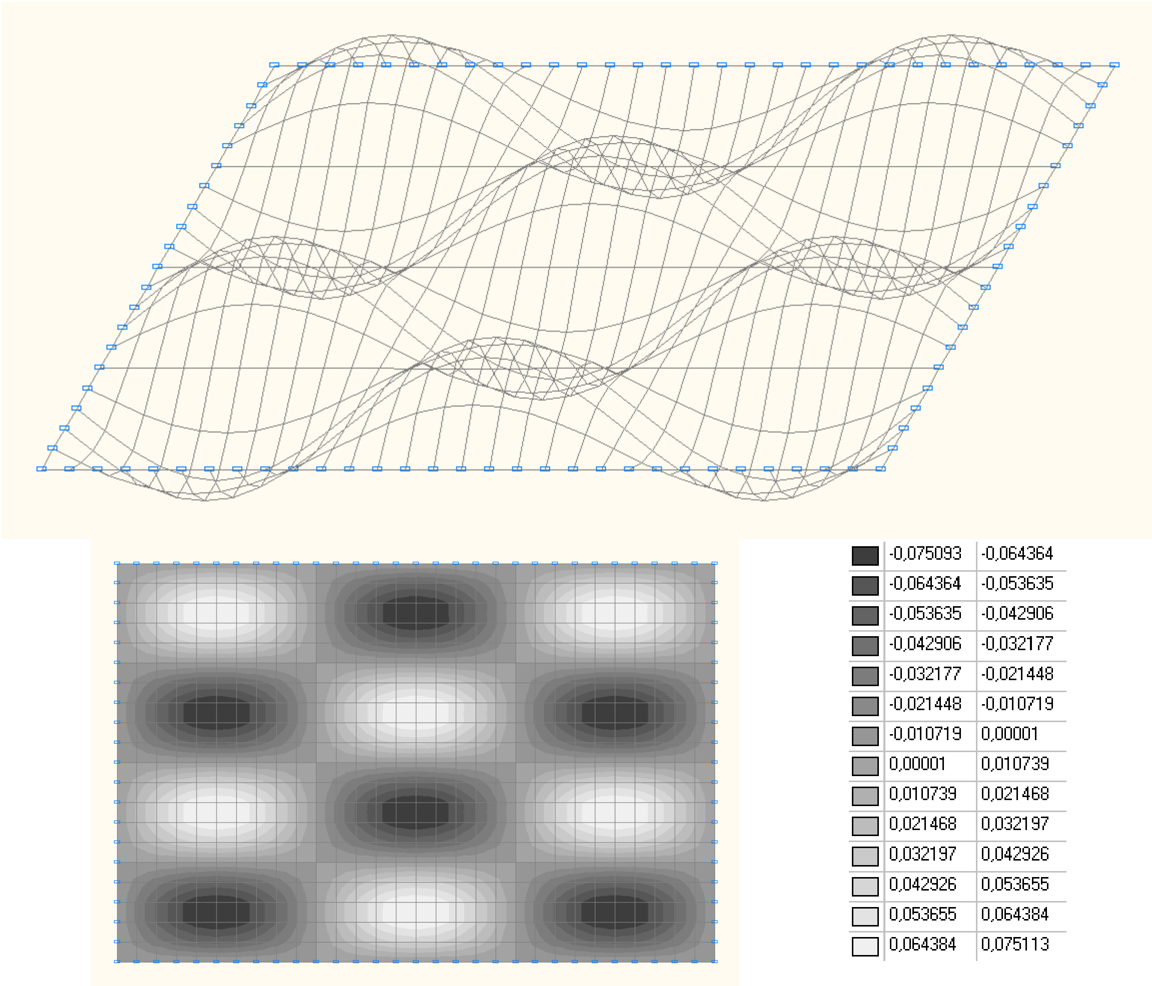
15-th natural oscillation mode



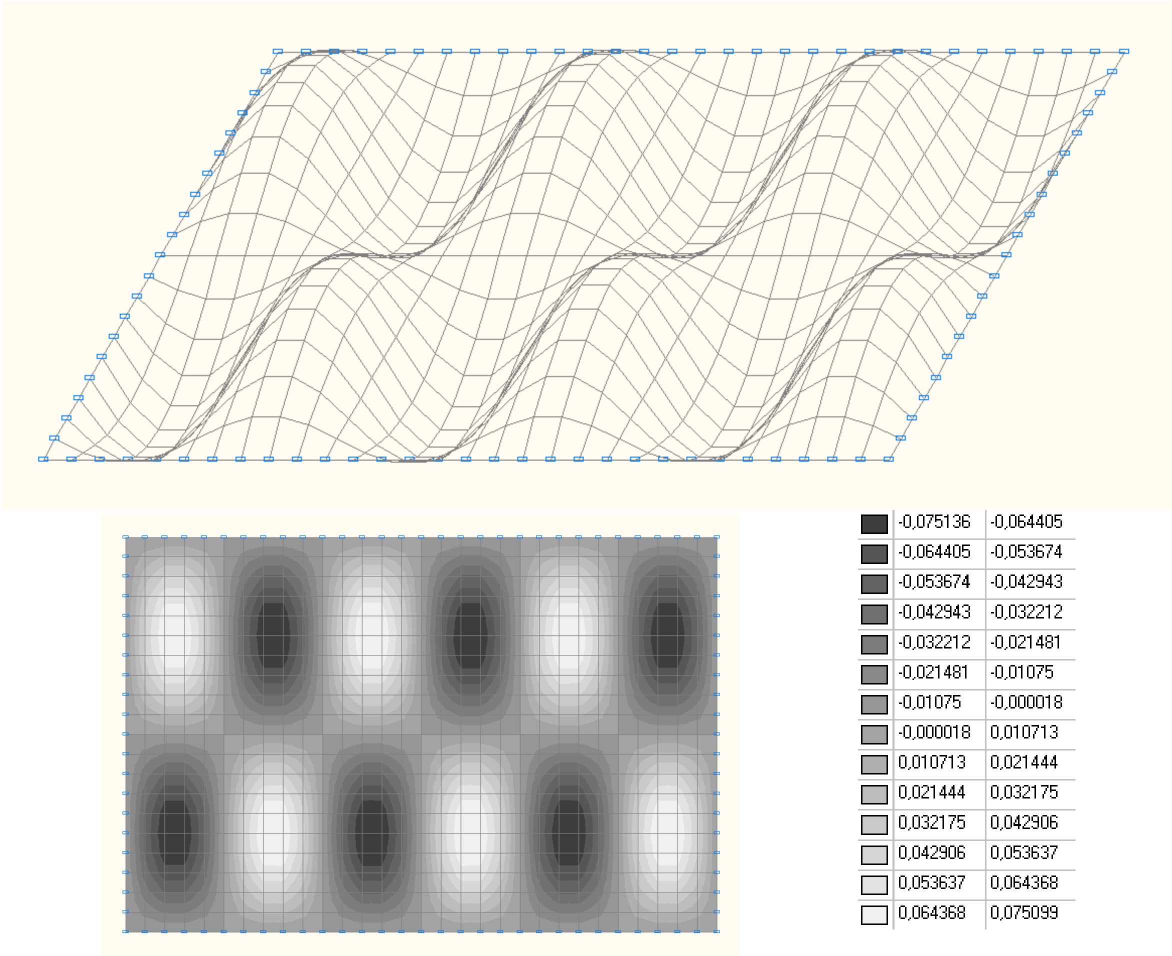
16-th natural oscillation mode



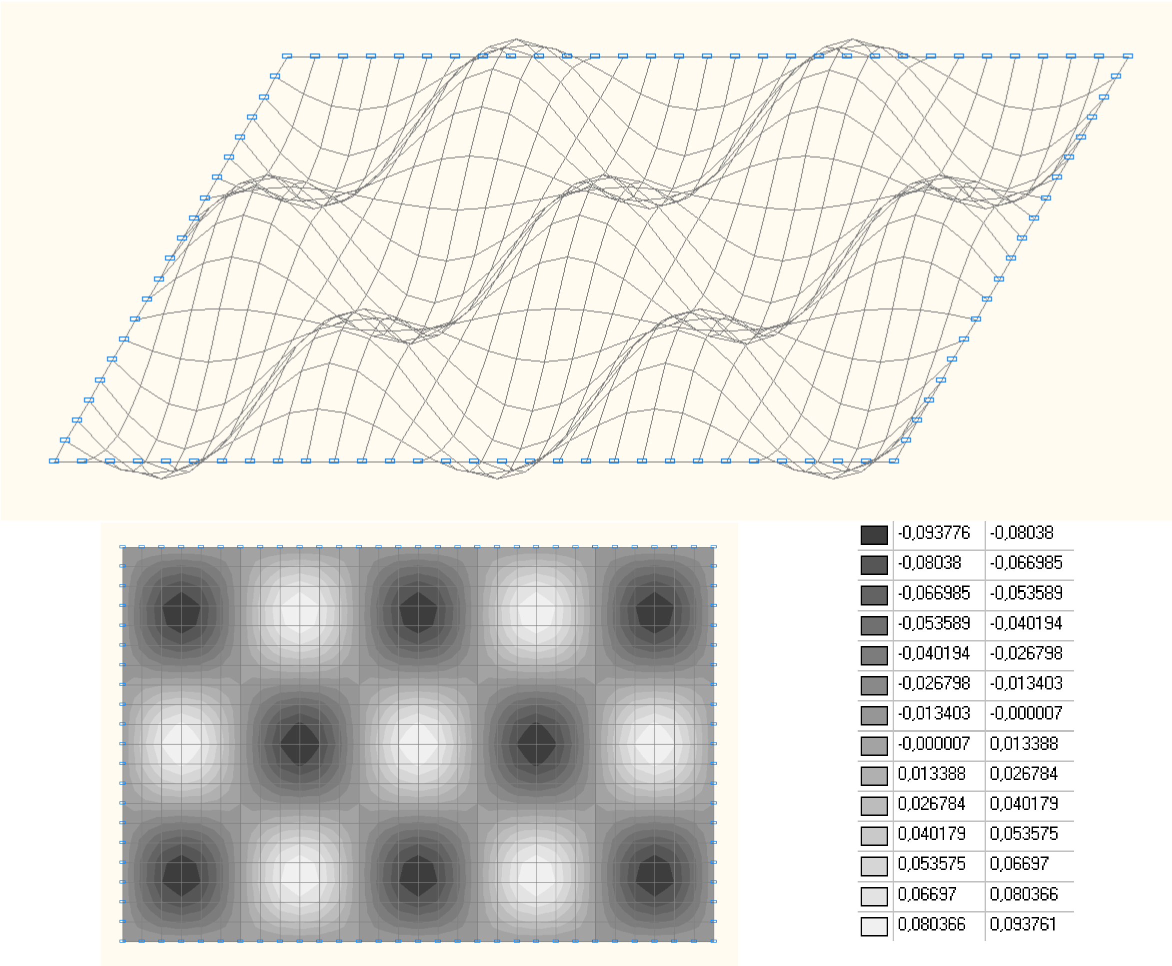
17-th natural oscillation mode



18-th natural oscillation mode



19-th natural oscillation mode



20-th natural oscillation mode

Comparison of solutions:

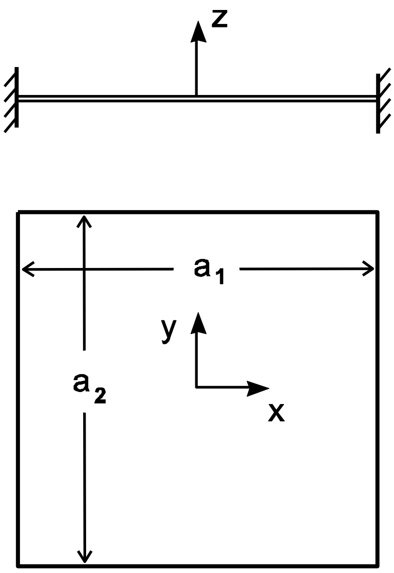
Natural frequencies ω, rad / s

Oscillation mode	Number of half waves m1, m2	Theory	SCAD	Deviations, %
1	1, 1	221.0	221.1	0.05
2	2, 1	425.0	425.3	0.07
3	1, 2	678.0	680.3	0.34
4	3, 1	765.0	765.6	0.08
5	2, 2	884.0	885.1	0.12
6	3, 2	1224.0	1226.4	0.20
7	4, 1	1241.0	1242.0	0.08
8	1, 3	1445.0	1445.5	0.03
9	2, 3	1649.0	1651.4	0.15
10	4, 2	1700.0	1704.3	0.25
11	5, 1	1853.0	1854.6	0.09
12	3, 3	1989.0	1994.5	0.28
13	5, 2	2312.0	2318.8	0.29
14	4, 3	2465.0	2474.9	0.40
15	1, 4	2516.0	2516.8	0.03
16	6, 1	2601.0	2603.1	0.08
17	2, 4	2720.0	2724.1	0.15
18	3, 4	3060.0	3069.7	0.32
19	6, 2	3060.0	3069.7	0.32
20	5, 3	3077.0	3092.5	0.50

Notes: In the analytical solution the natural frequencies ω of the simply supported rectangular plate with the density of the material ρ can be determined according to the following formula:

$$\omega = \pi^2 \cdot \left(\frac{m_1^{\ 2}}{a_2^{\ 2}} + \frac{m_2^{\ 2}}{a_2^{\ 2}} \right) \cdot \left(\frac{D}{\rho \cdot h} \right)^{\frac{1}{2}}$$, where: $D = \frac{E \cdot h^3}{12 \cdot (1 - \mu^2)}$, $m_1, m_2 = 1,2,3\ldots$

Natural Oscillations of a Clamped Square Plate



Objective: Modal analysis of a clamped square plate.

Initial data file: **5.4.SPR**

Problem formulation: Determine the natural oscillation modes and frequencies ω of the clamped square plate with the density of the material ρ.

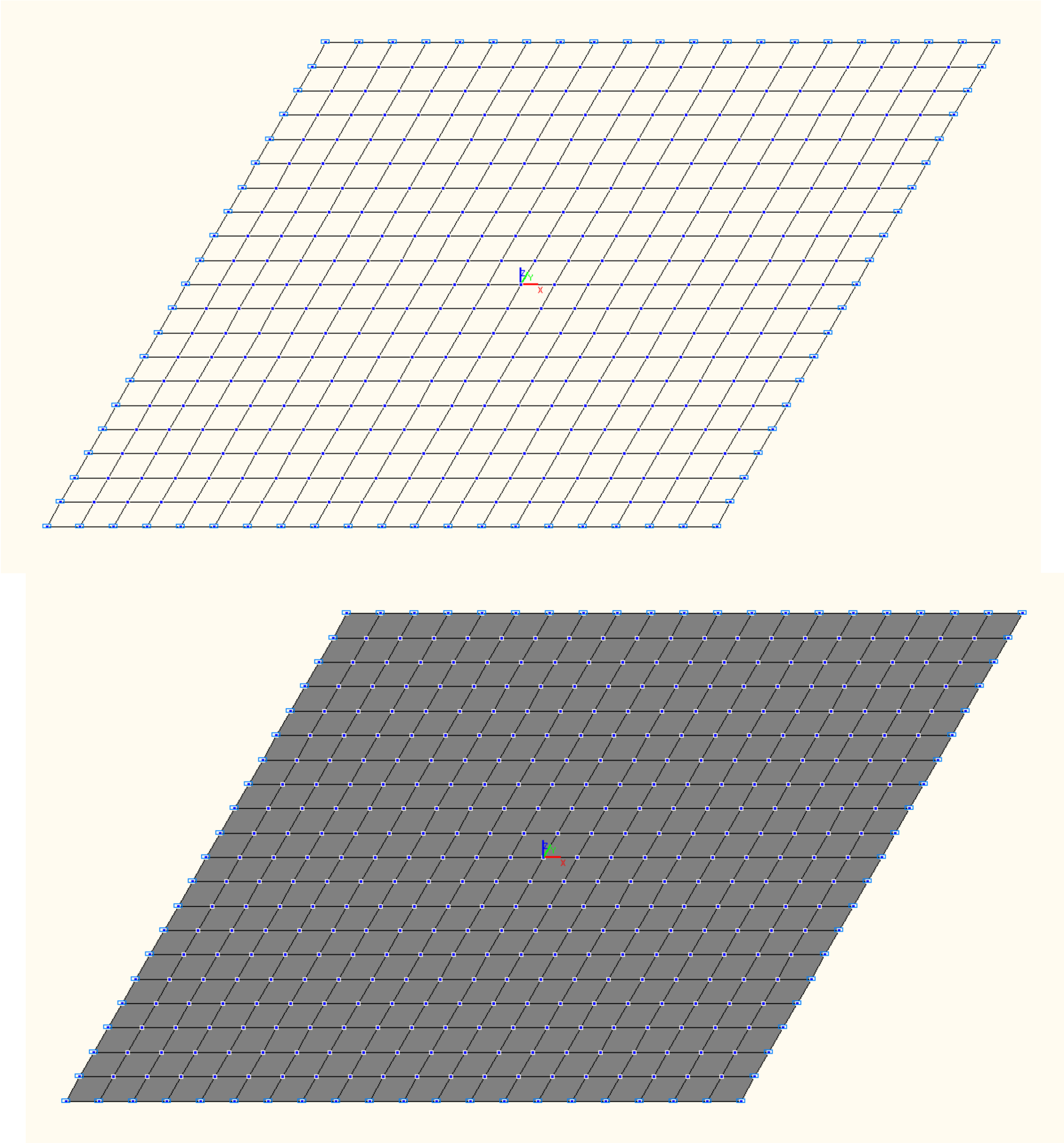
References: I.A. Birger, Ya.G. Panovko, Strength, Stability, Vibrations, Handbook in three volumes, Volume 3, Moscow, Mechanical engineering, 1968, p. 377.

Initial data:

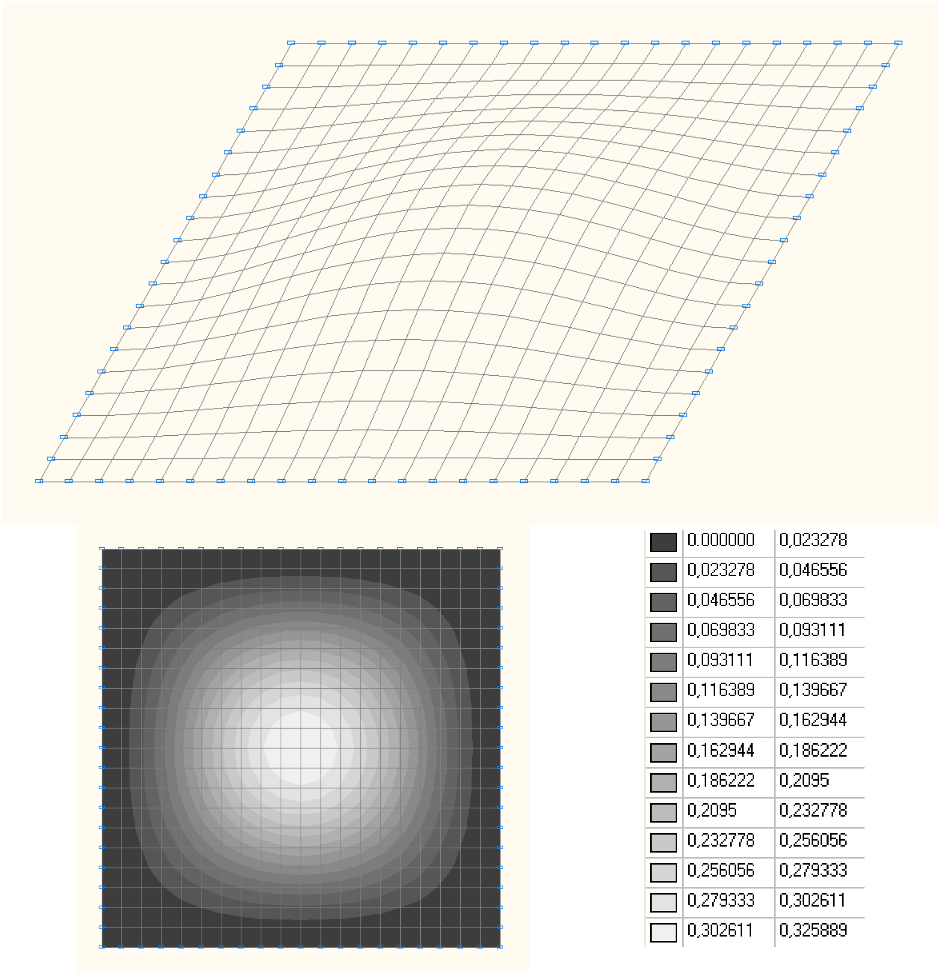
$E = 2.06 \cdot 10^8$ kPa	- elastic modulus;
$\nu = 0.3$	- Poisson's ratio;
$\rho = 7.85$ t/m^3	- density of the material;
$h = 0.01$ m	- thickness of the plate;
$a_1 = 1.0$ m	- long side of the plate (along the X axis of the global coordinate system);
$a_2 = 1.0$ m	- short side of the plate (along the Y axis of the global coordinate system).

Finite element model: Design model – grade beam / plate, 400 plate elements of type 20. The spacing of the finite element mesh along the sides of the plate (along the X, Y axes of the global coordinate system) is 0.05 m. Boundary conditions are provided by imposing constraints in the directions of the degrees of freedom Z, UX, UY for the edges parallel to the X and Y axes of the global coordinate system. The distributed mass is specified by transforming the static load from the self-weight of the plate $ow = \gamma \cdot h$, where $\gamma = \rho \cdot g = 77.01$ kN/m^3. Number of nodes in the design model – 441. The determination of the natural oscillation modes and natural frequencies is performed by the method of subspace iteration. The matrix of concentrated masses is used in the calculation.

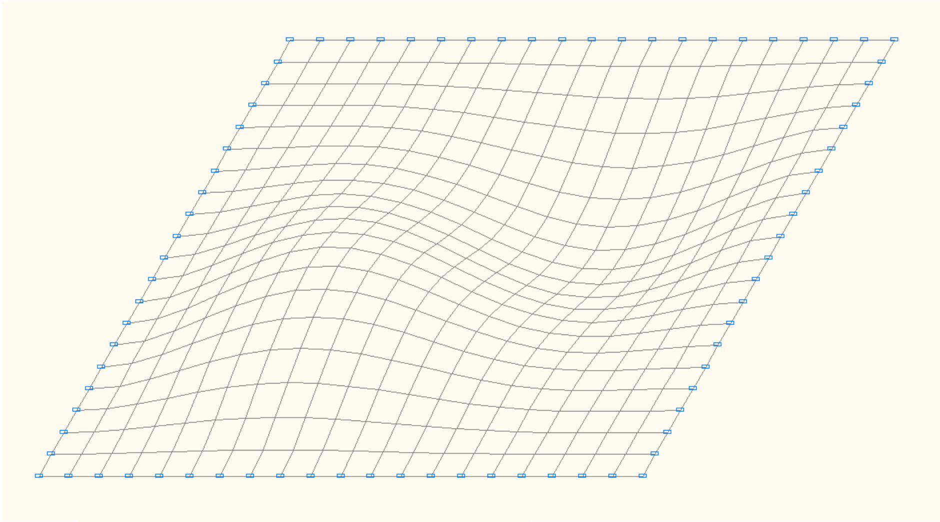
Results in SCAD

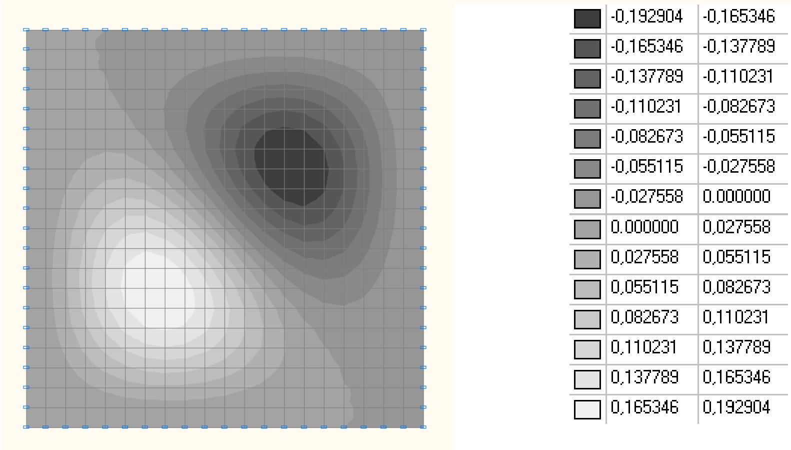


Design model

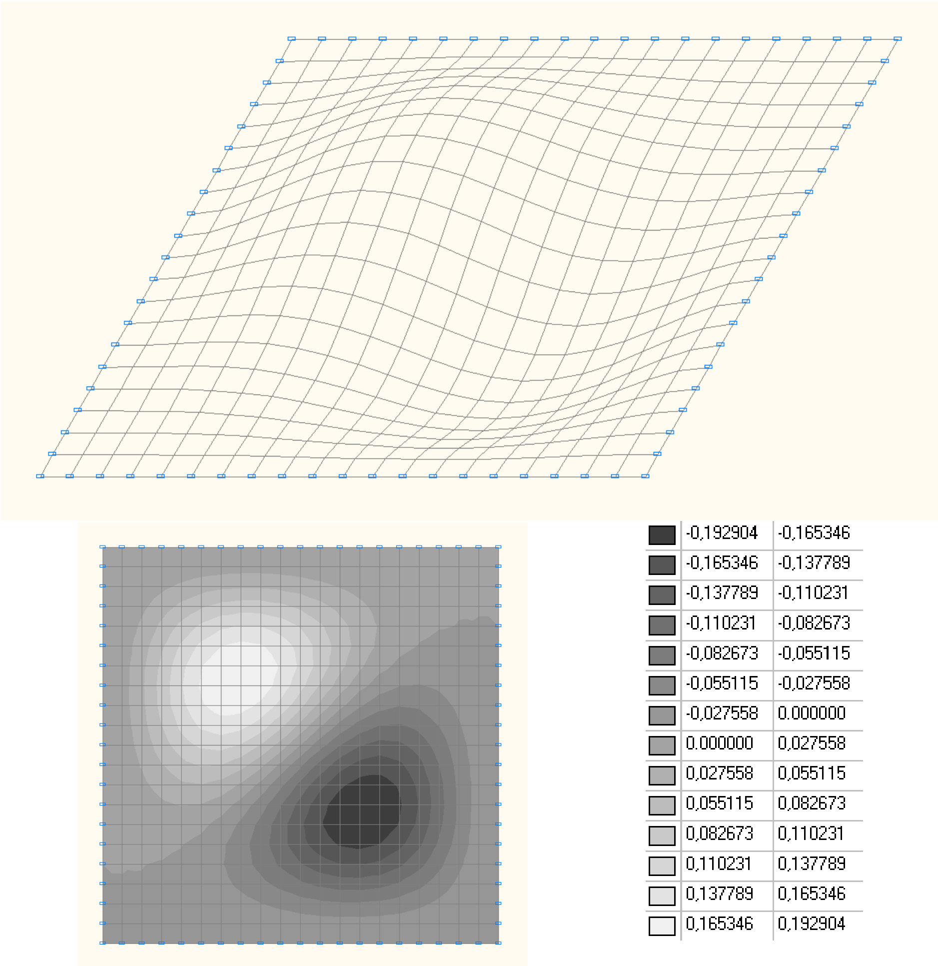


1-st natural oscillation mode

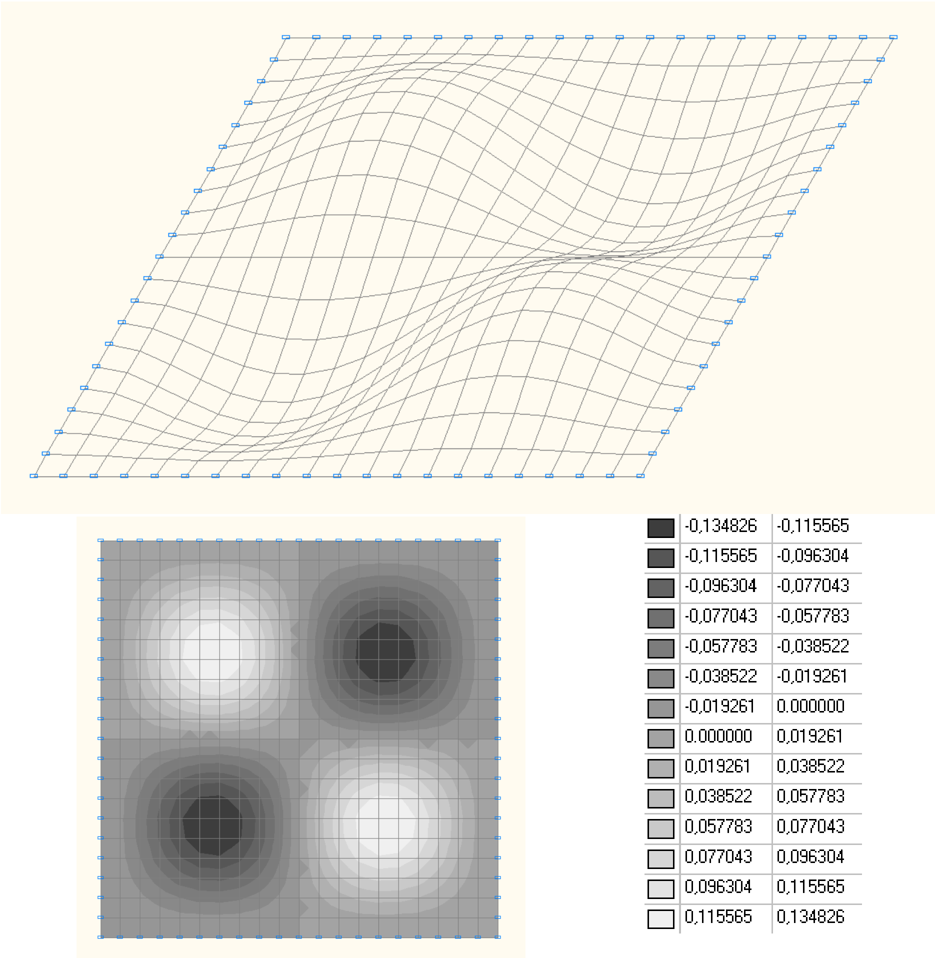




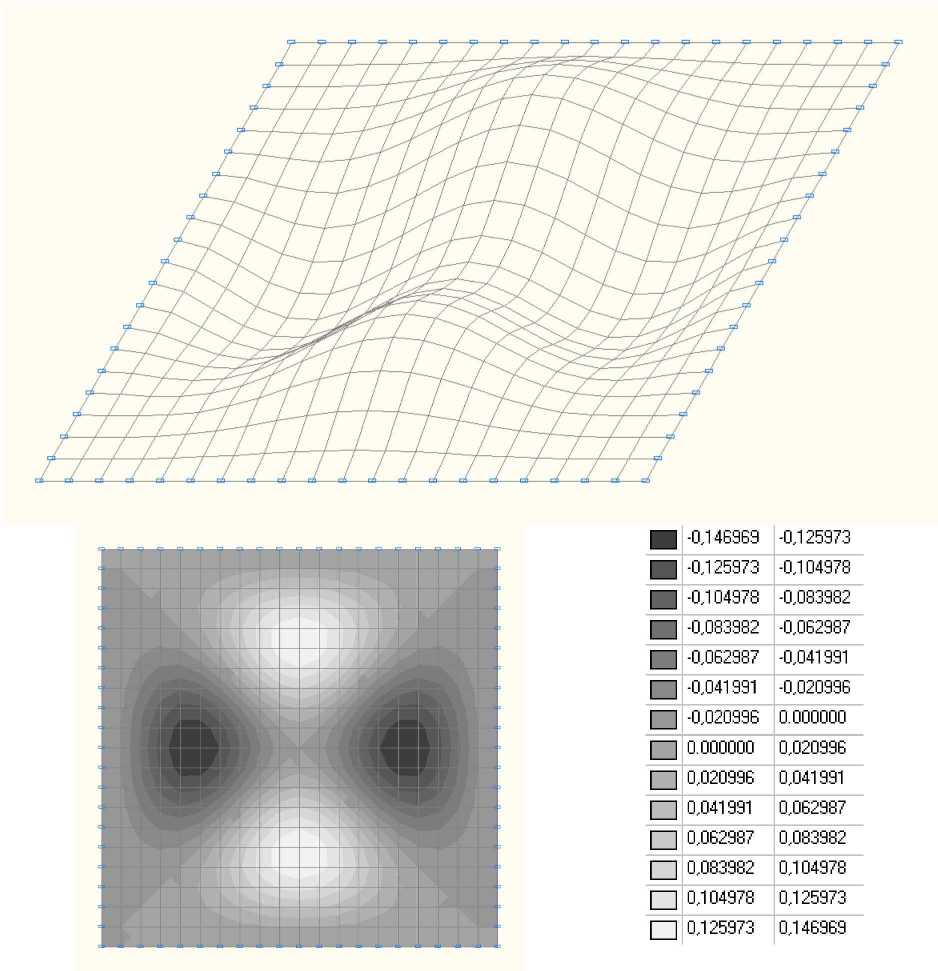
2-nd natural oscillation mode



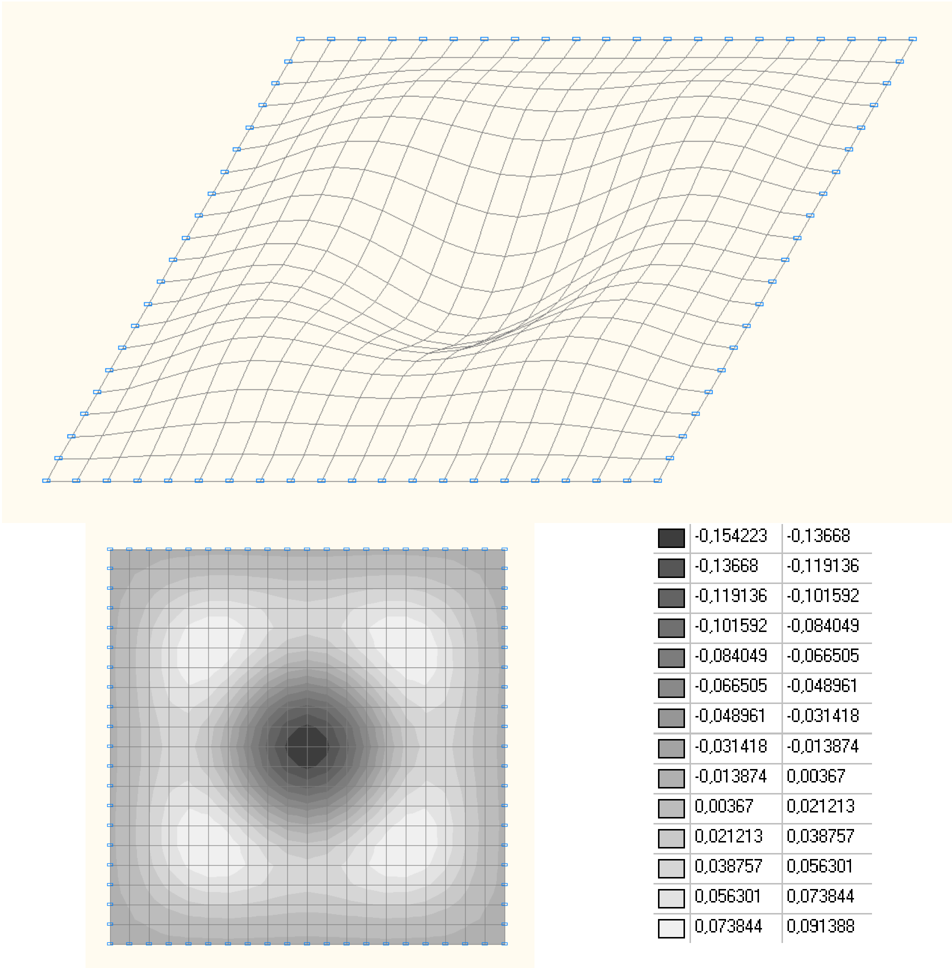
3-rd natural oscillation mode



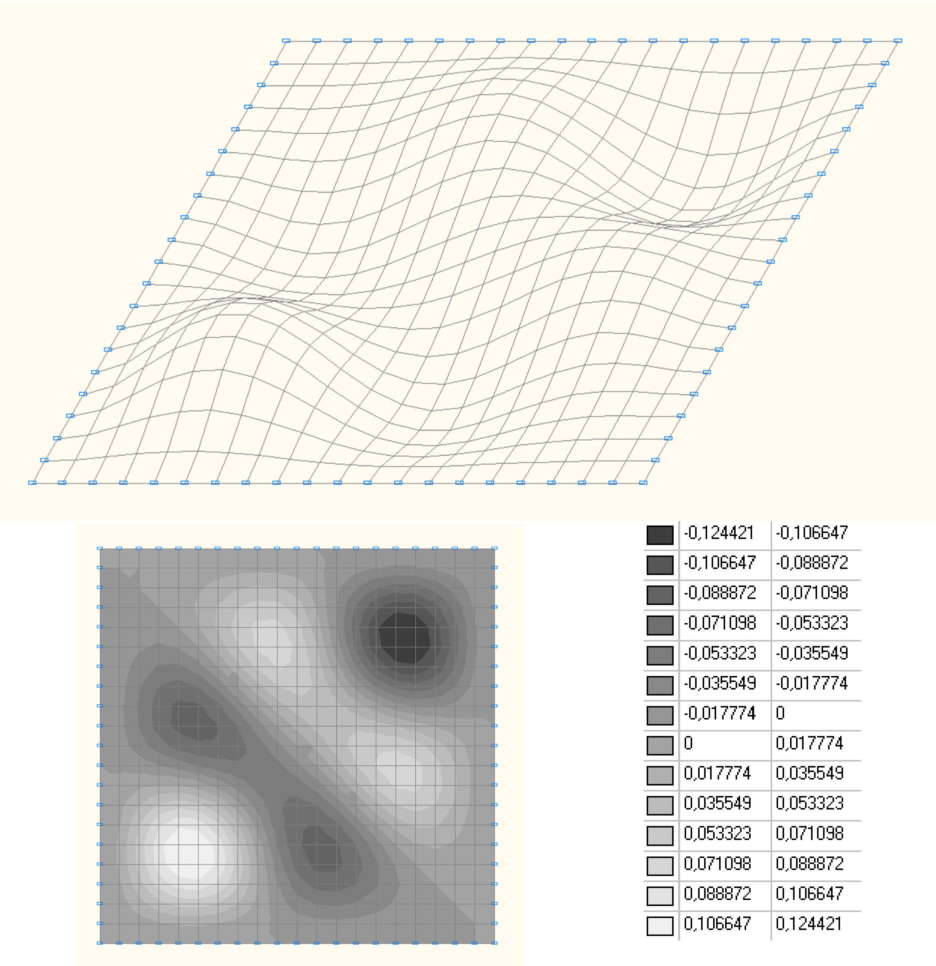
4-th natural oscillation mode



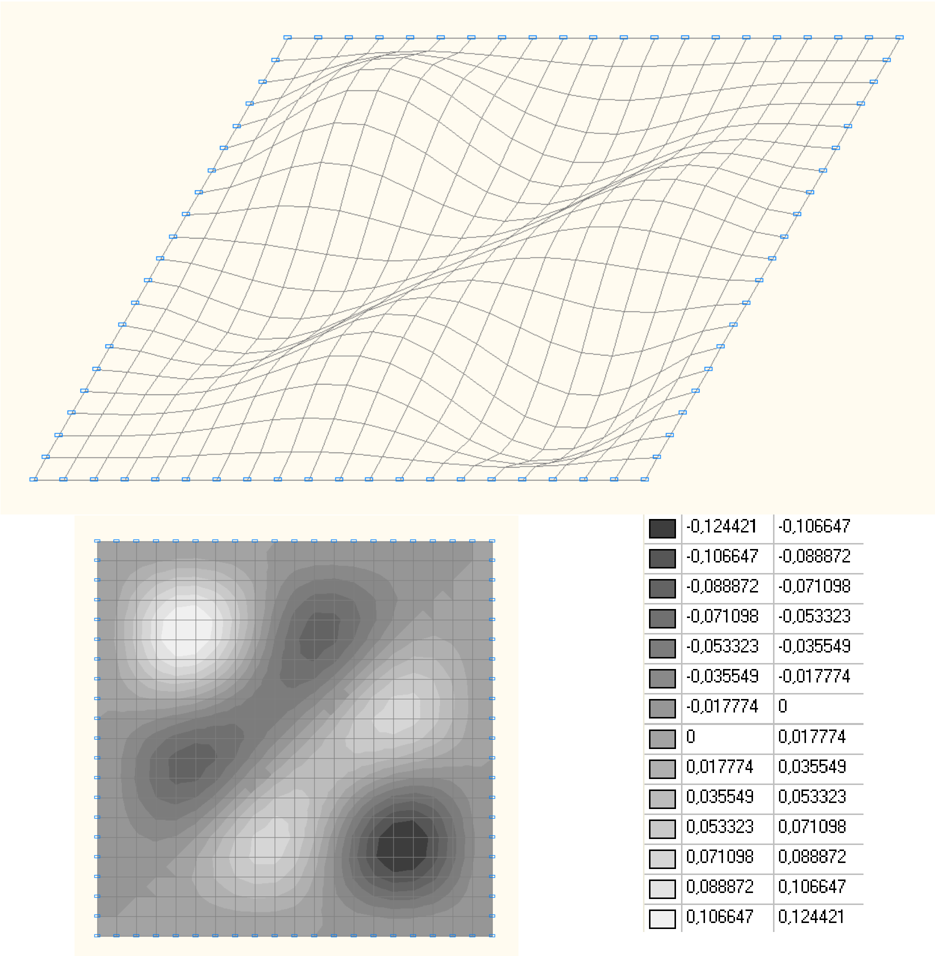
5-th natural oscillation mode



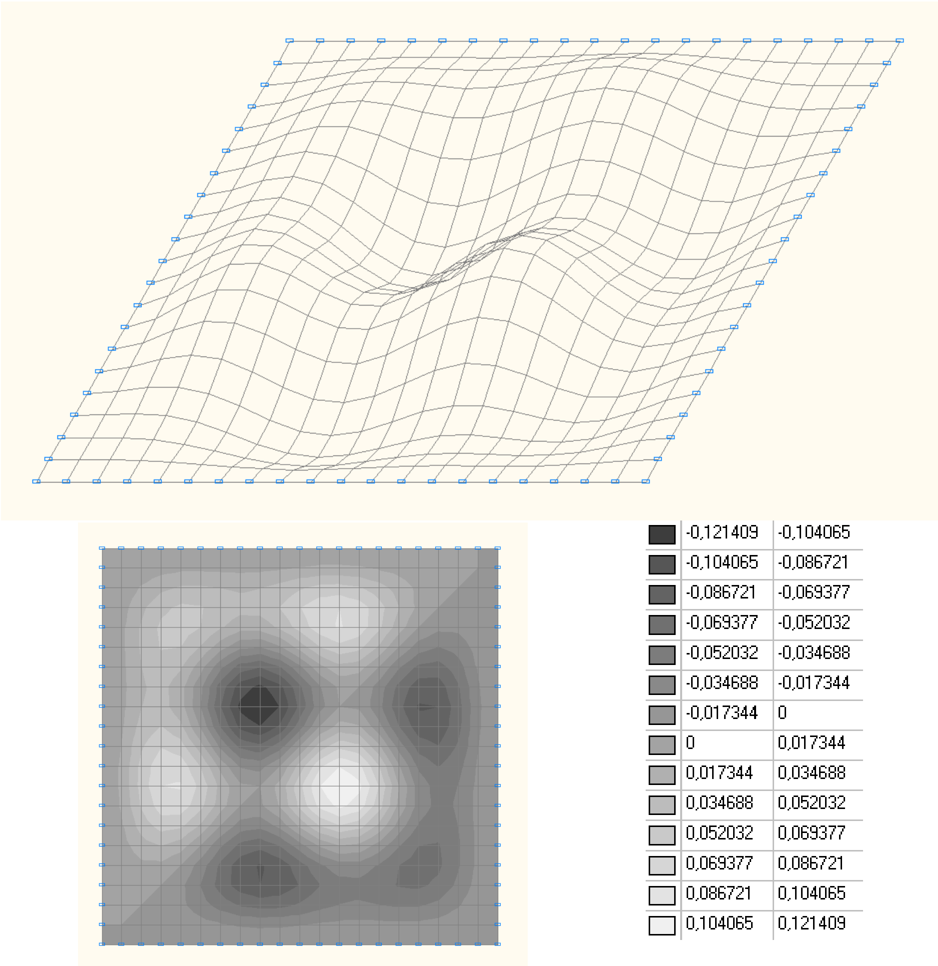
6-th natural oscillation mode



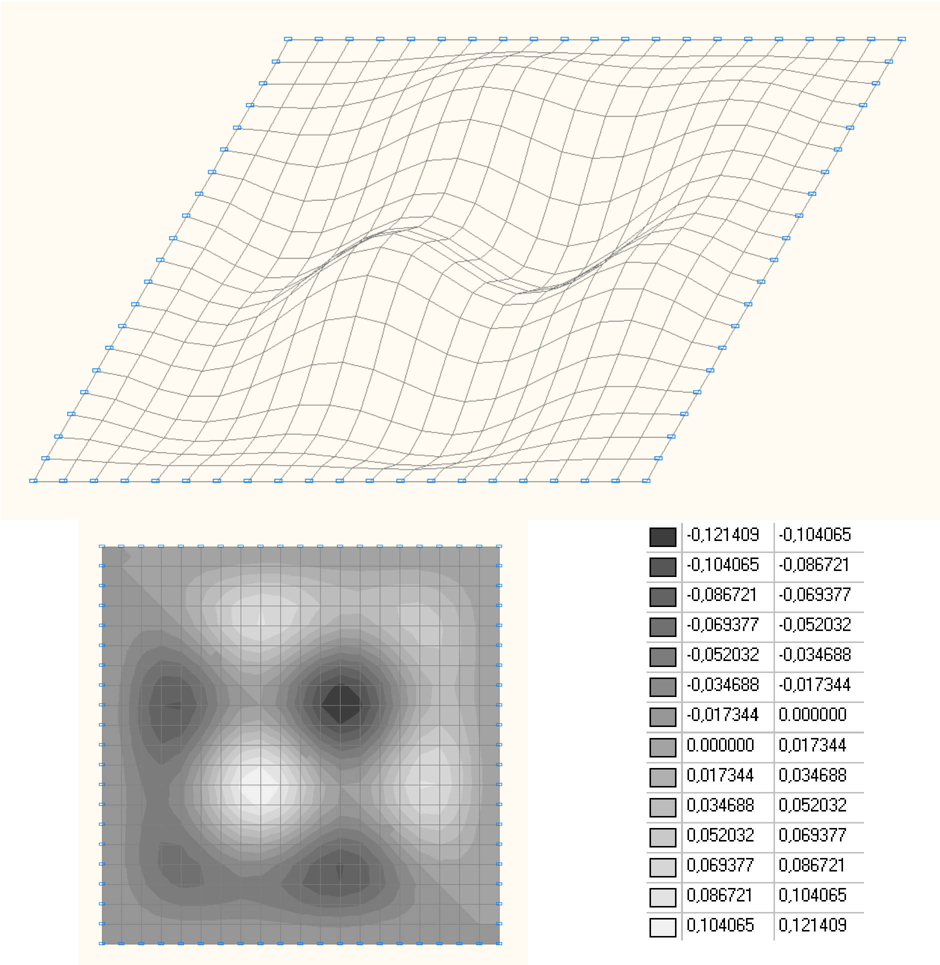
7-th natural oscillation mode



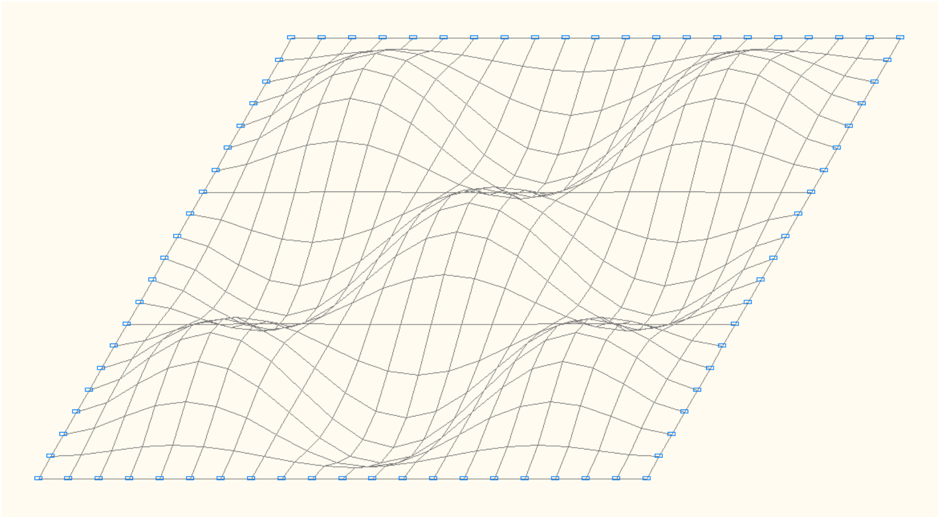
8-th natural oscillation mode

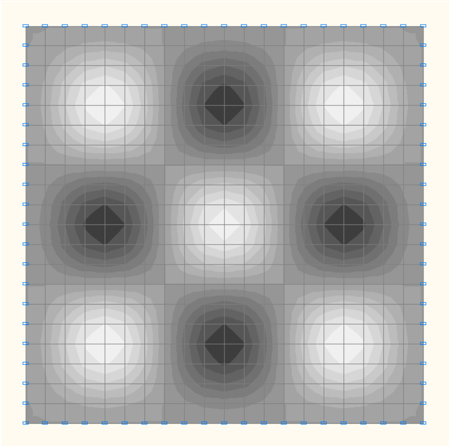


9-th natural oscillation mode



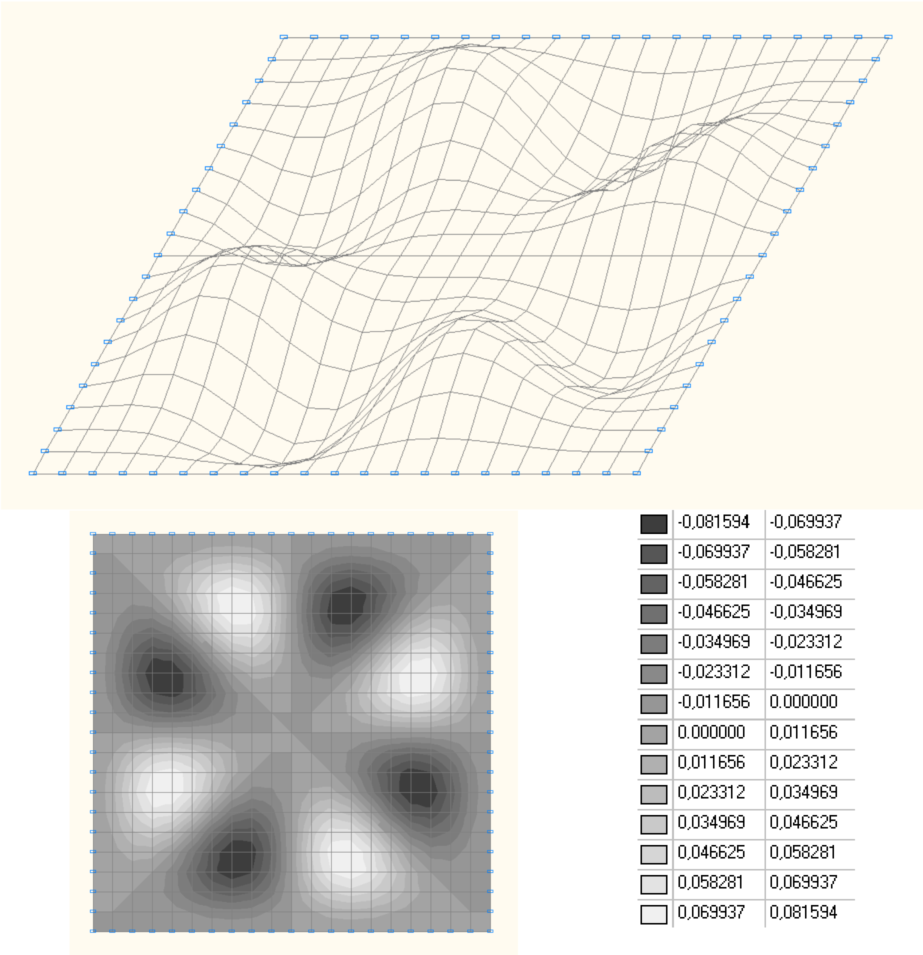
10-th natural oscillation mode



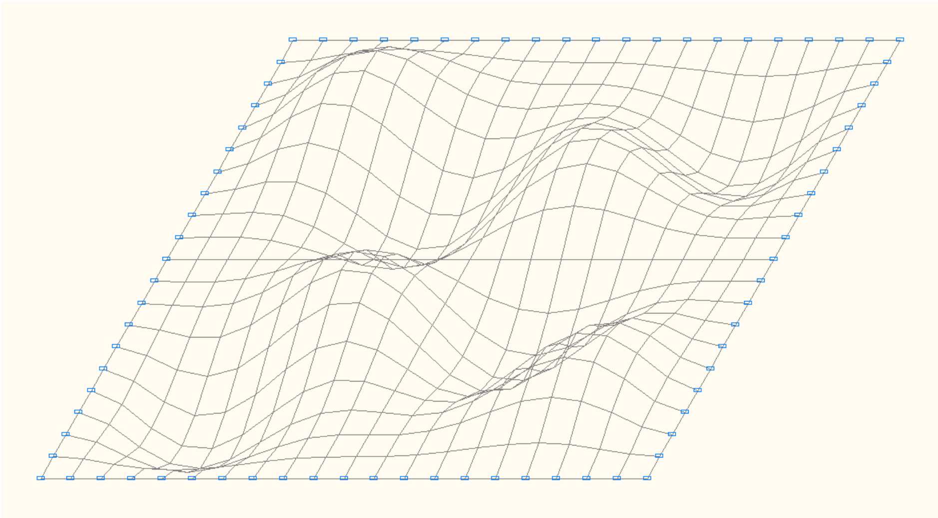


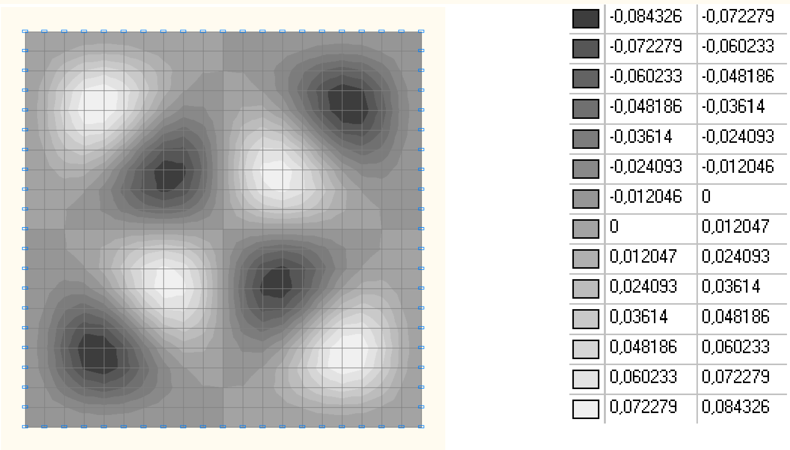
-0,093192	-0,079797
-0,079797	-0,066403
-0,066403	-0,053008
-0,053008	-0,039614
-0,039614	-0,02622
-0,02622	-0,012825
-0,012825	0,000569
0,000569	0,013964
0,013964	0,027358
0,027358	0,040752
0,040752	0,054147
0,054147	0,067541
0,067541	0,080936
0,080936	0,09433

11-th natural oscillation mode

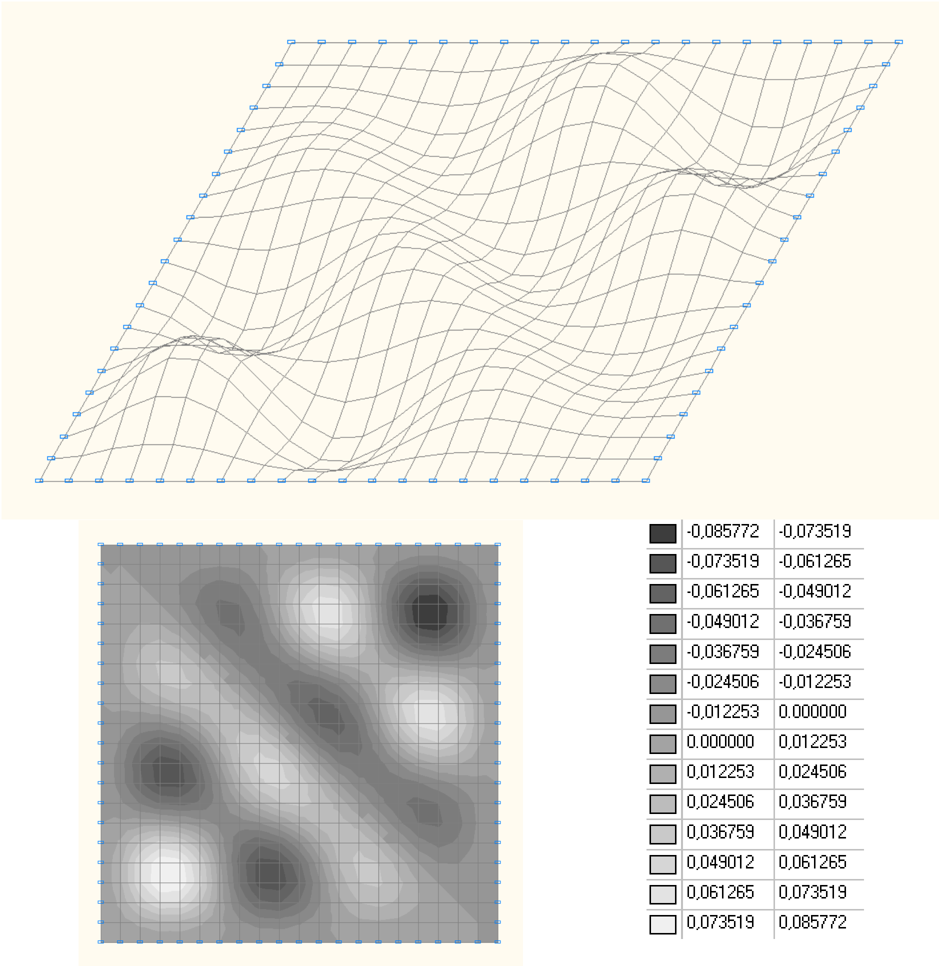


12-th natural oscillation mode

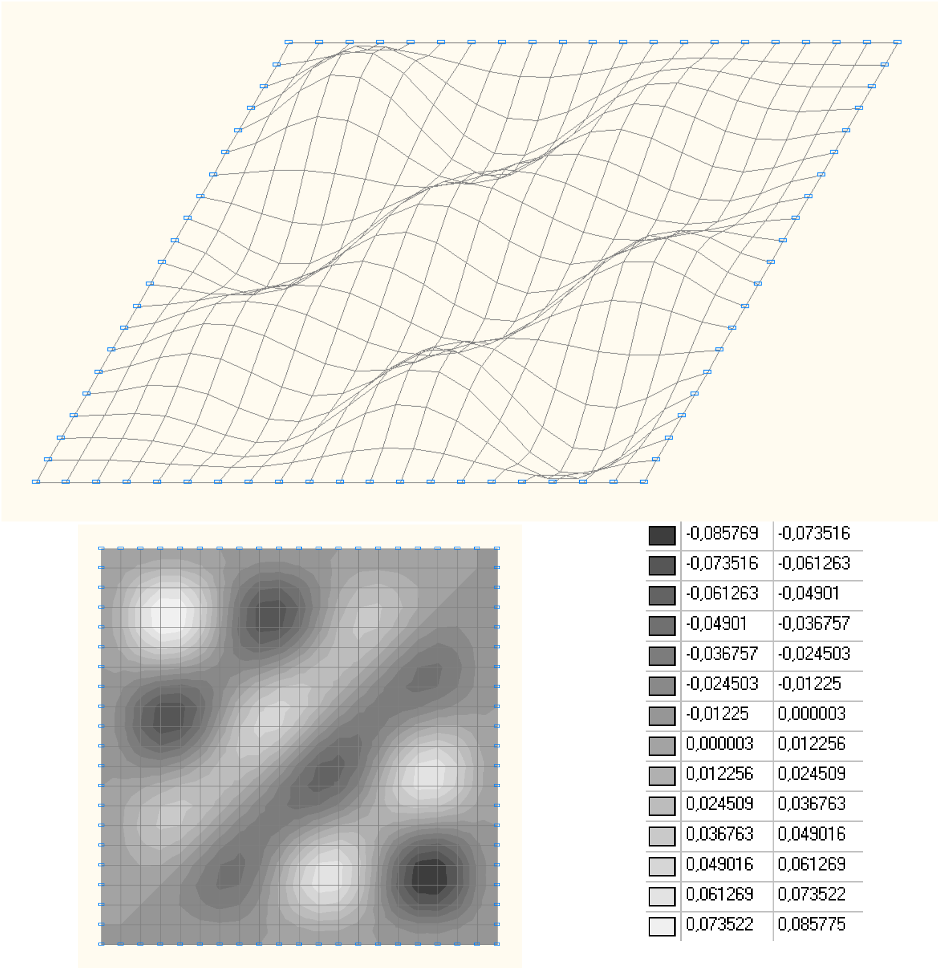




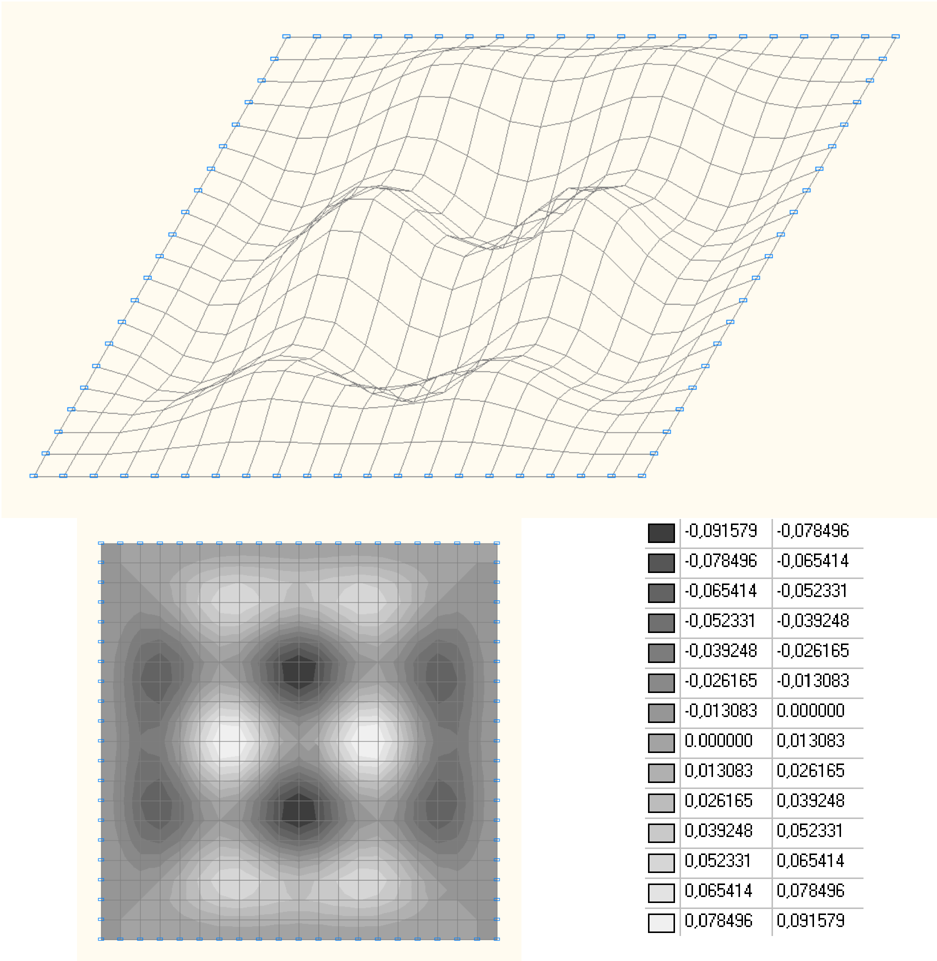
13-th natural oscillation mode



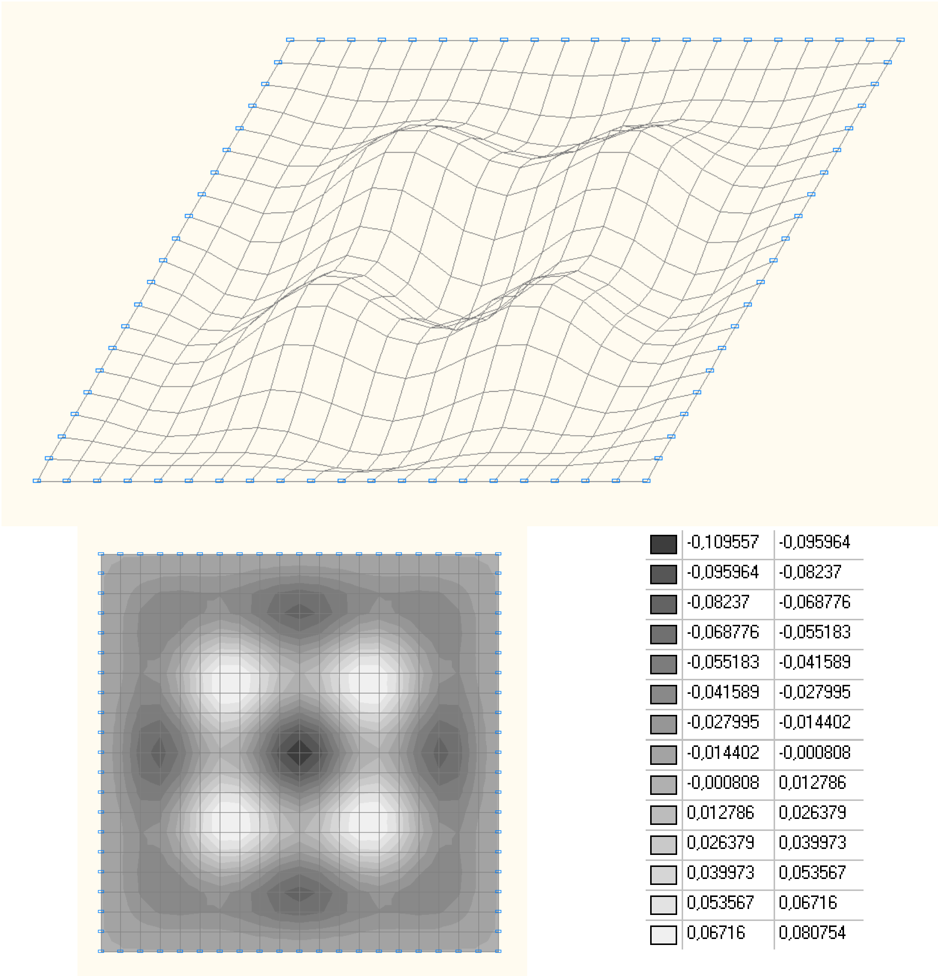
14-th natural oscillation mode



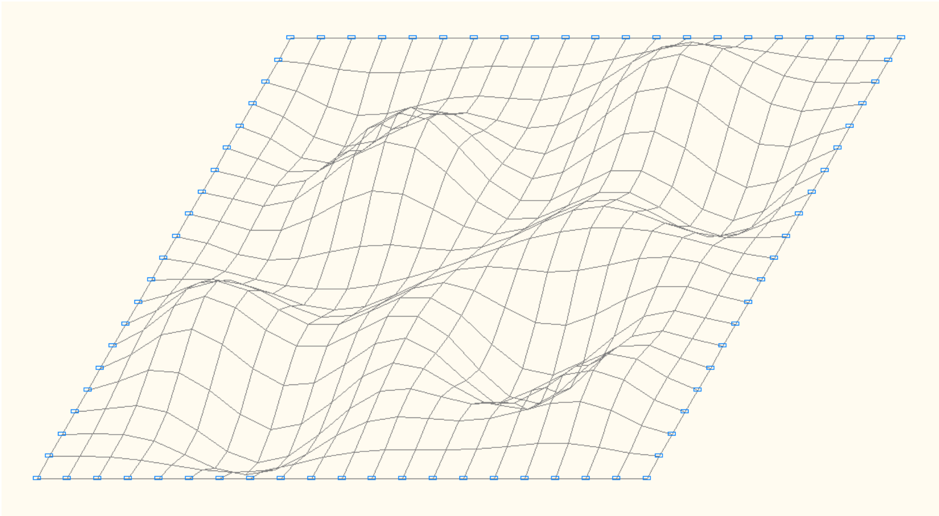
15-th natural oscillation mode

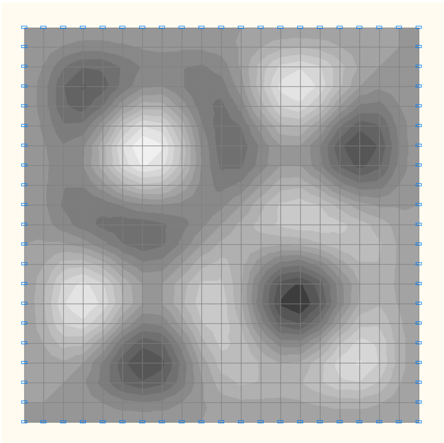


16-th natural oscillation mode



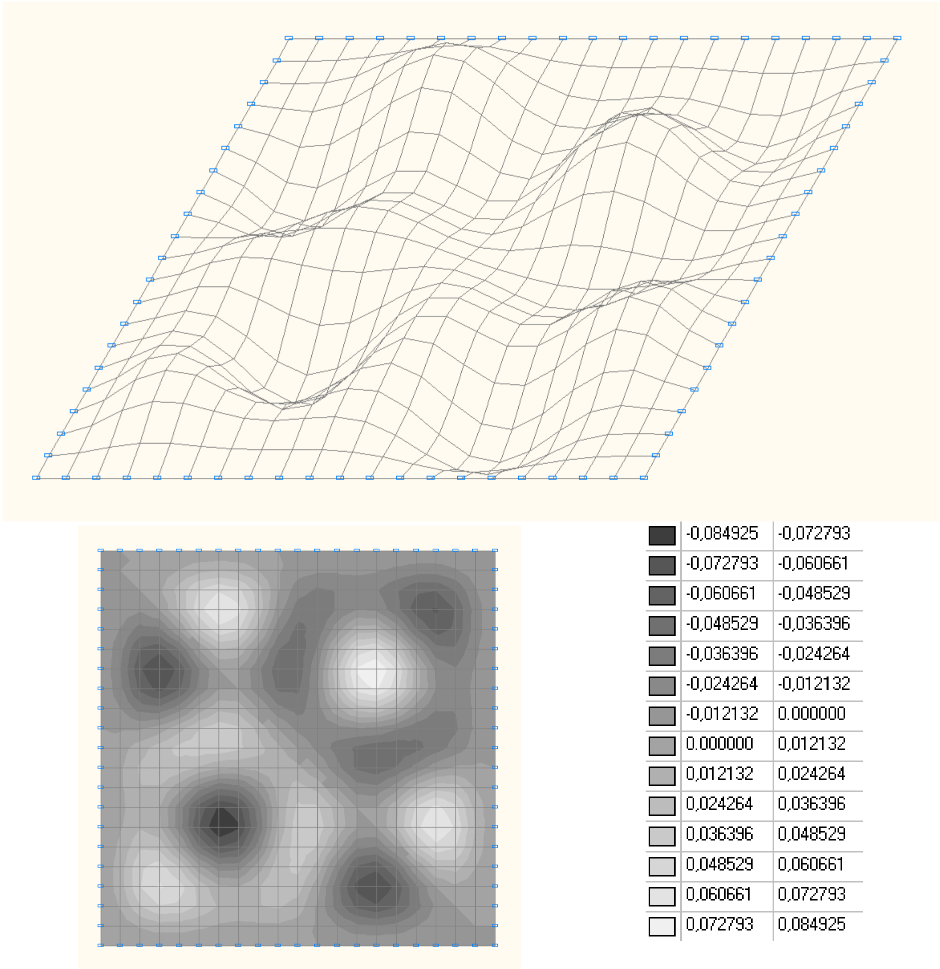
17-th natural oscillation mode



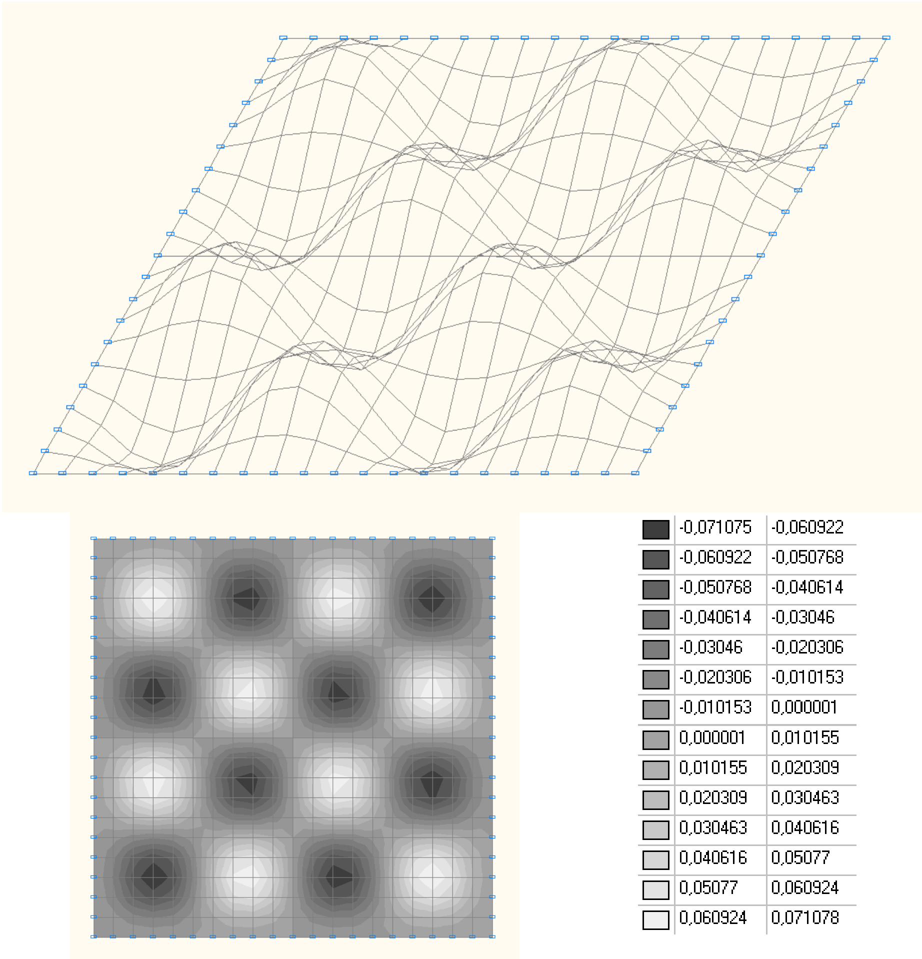


-0,084916	-0,072785
-0,072785	-0,060653
-0,060653	-0,048521
-0,048521	-0,03639
-0,03639	-0,024258
-0,024258	-0,012127
-0,012127	0,000005
0,000005	0,012137
0,012137	0,024268
0,024268	0,0364
0,0364	0,048532
0,048532	0,060663
0,060663	0,072795
0,072795	0,084927

18-th natural oscillation mode



19-th natural oscillation mode



20-th natural oscillation mode

Comparison of solutions:

Natural frequencies ω, rad / s

Oscillation mode	Number of half waves m1, m2	Theory	SCAD	Deviations, %
1	1, 1	560.1	558.5	0.29
2	1, 2	1143.2	1139.4	0.33
3	2, 1	1143.2	1139.4	0.33
4	2, 2	1686.6	1683.4	0.19
5	1, 3	2054.0	2042.8	0.55
6	3, 1	2054.0	2052.2	0.09
7	2, 3	2571.5	2569.1	0.09
8	3, 2	2571.5	2569.1	0.09
9	1, 4	3276.5	3267.5	0.27
10	4, 1	3276.5	3267.5	0.27
11	3, 3	3424.6	3434.5	0.29
12	2, 4	3782.2	3772.0	0.27
13	4, 2	3782.2	3786.2	0.11
14	3, 4	4611.8	4632.3	0.44
15	4, 3	4611.8	4632.3	0.44
16	1, 5	4806.6	4793.0	0.28
17	5, 1	4806.6	4796.7	0.21
18	2, 5	5307.4	5303.5	0.07
19	5, 2	5307.4	5303.5	0.07
20	4, 4	5774.8	5821.8	0.81

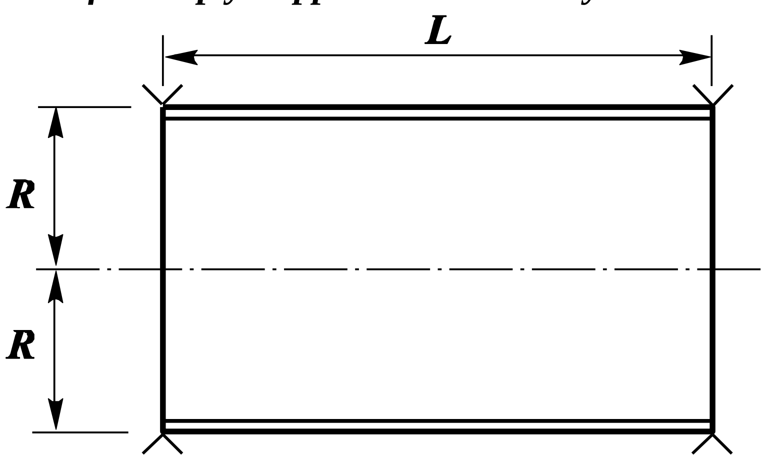
Notes: In the analytical solution the natural frequencies ω of the clamped square plate with the density of the material ρ can be determined according to the following formula obtained on the basis of the Rayleigh-Ritz method:

$$\omega = \pi^2 \cdot \left(\frac{D}{\rho \cdot h} \cdot \left(\frac{A_m{}^4}{a_1{}^4} + \frac{A_n{}^4}{a_2{}^4} + 2 \cdot \frac{B_m \cdot B_n}{a_1{}^2 \cdot a_2{}^2} \right) \right)^{\frac{1}{2}}$$, where:

$$A_m = \begin{cases} 1.506 & m = 1 \\ m + 0.5 & m \geq 2 \end{cases}, \quad A_n = \begin{cases} 1.506 & n = 1 \\ n + 0.5 & n \geq 2 \end{cases}, \quad B_m = \begin{cases} 1.248 & m = 1 \\ A_m \cdot \left(A_m - \frac{2}{\pi} \right) & m \geq 2 \end{cases}, \quad B_m = \begin{cases} 1.248 & n = 1 \\ A_n \cdot \left(A_n - \frac{2}{\pi} \right) & n \geq 2 \end{cases},$$

$$D = \frac{E \cdot h^3}{12 \cdot (1 - \nu^2)}, \qquad m_1, m_2 = 1, 2, 3 \ldots$$

Natural Oscillations of a Simply Supported Circular Cylindrical Shell



Objective: Modal analysis of a simply supported circular cylindrical shell.

Initial data file: **5.8_S.SPR**

Problem formulation: Determine the natural oscillation modes and frequencies ω of the simply supported circular cylindrical shell with the density of the material ρ.

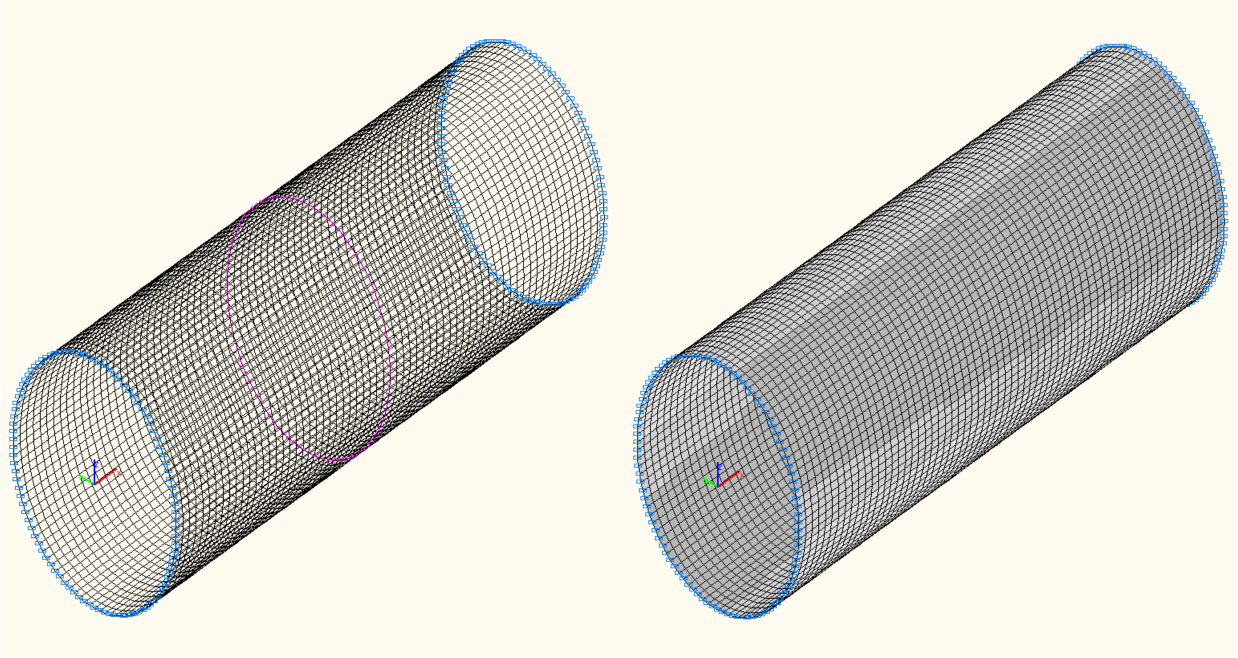
References: I.A. Birger, Ya.G. Panovko, Strength, Stability, Vibrations, Handbook in three volumes, Volume 3, Moscow, Mechanical engineering, 1968, p. 426.
V. L. Biderman, Theory of Mechanical Oscillations, Moscow, High School, 1980, p. 290.

Initial data:

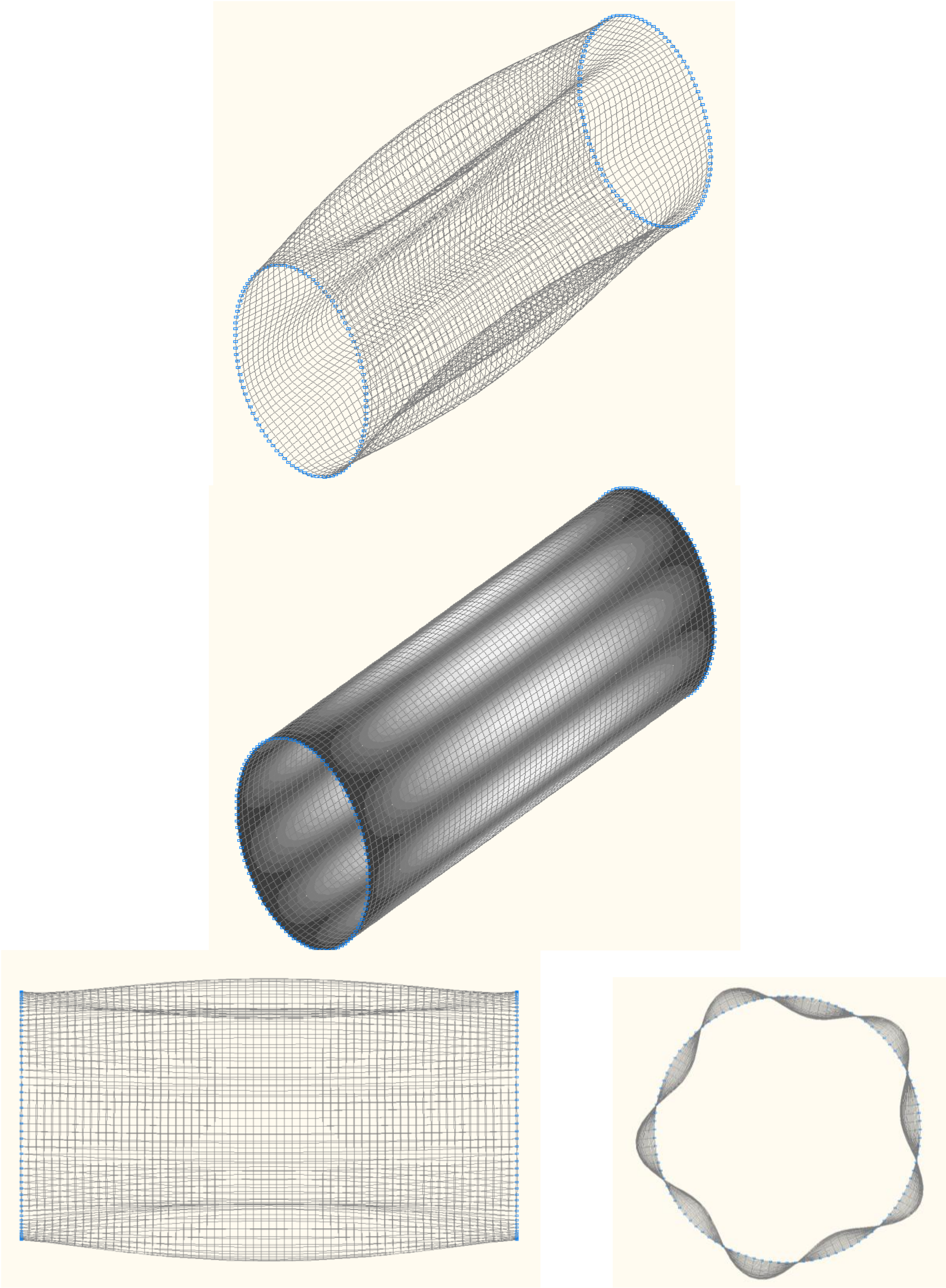
$E = 1.96 \cdot 10^8$ kPa	- elastic modulus;
$\nu = 0.3$	- Poisson's ratio;
$\rho = 7.70$ t/m^3	- density of the material;
$h = 0.25 \cdot 10^{-3}$ m	- thickness of the cylindrical shell;
R = 0.076 m	- radius of the midsurface of the cylindrical shell;
L = 0.305 m	- length of the cylindrical shell.

Finite element model: Design model – general type system, 6400 four-node shell elements of type 50. The spacing of the finite element mesh in the meridian direction is $4.765625 \cdot 10^{-3}$ m (64 elements) and in the circumferential is 3.6º (100 elements). Boundary conditions of the simply supported edges are provided by imposing constraints in the directions of the linear displacements in their plane (degrees of freedom Y, Z). The dimensional stability of the design model is provided by imposing constraints of finite rigidity (100 elements of type 51) in the nodes of the cross-section on the symmetry plane of the cylindrical shell in the meridian direction (k_x = 1.0 kN/m). The distributed mass is specified by transforming the static load from the self-weight of the cylindrical shell: ow = $\gamma \cdot h$, where $\gamma = \rho \cdot g = 75.537$ kN/m^3. Number of nodes in the design model – 6500. The determination of the natural oscillation modes and natural frequencies is performed by the method of subspace iteration. The matrix of concentrated masses is used in the calculation.

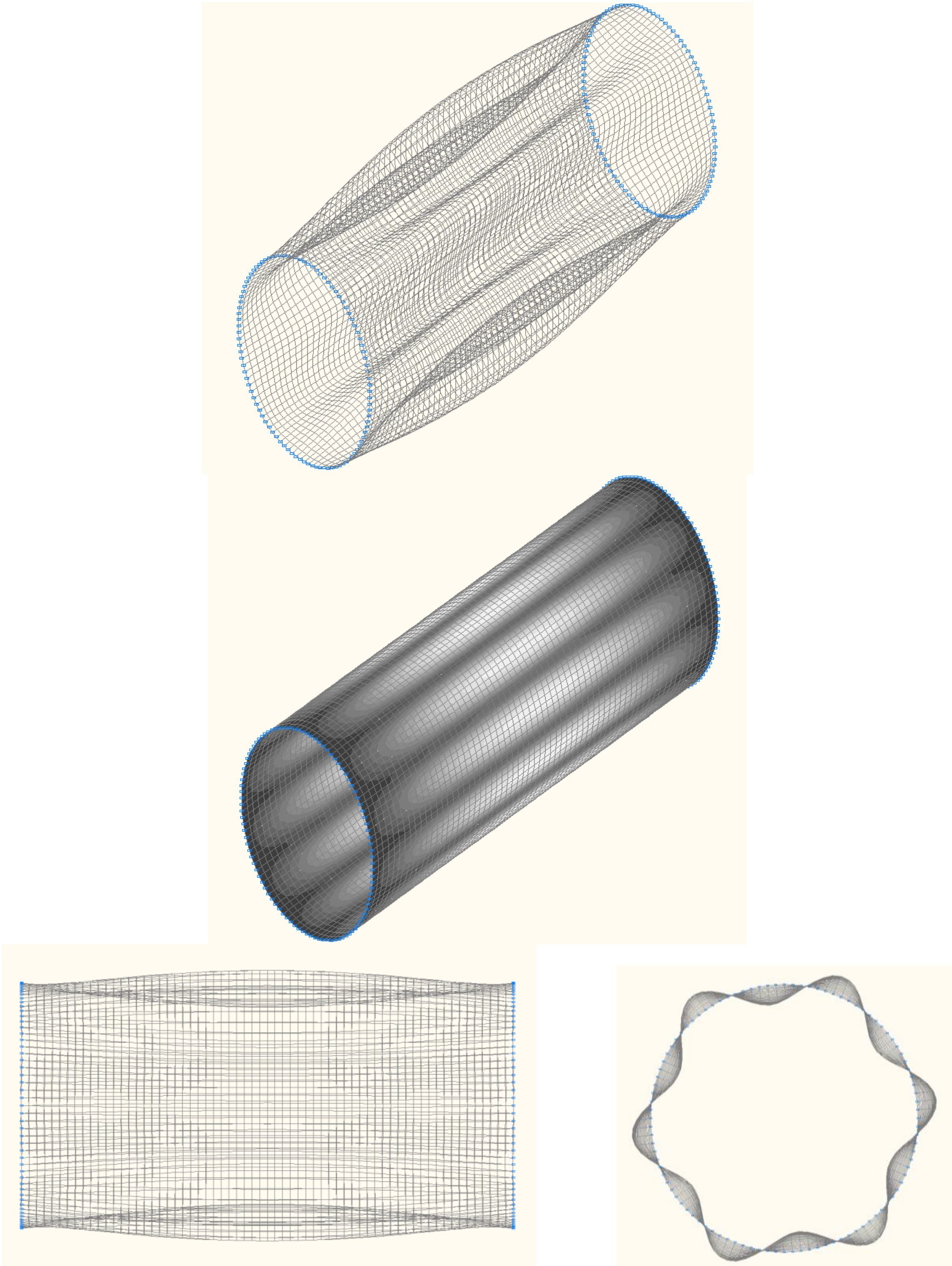
Results in SCAD



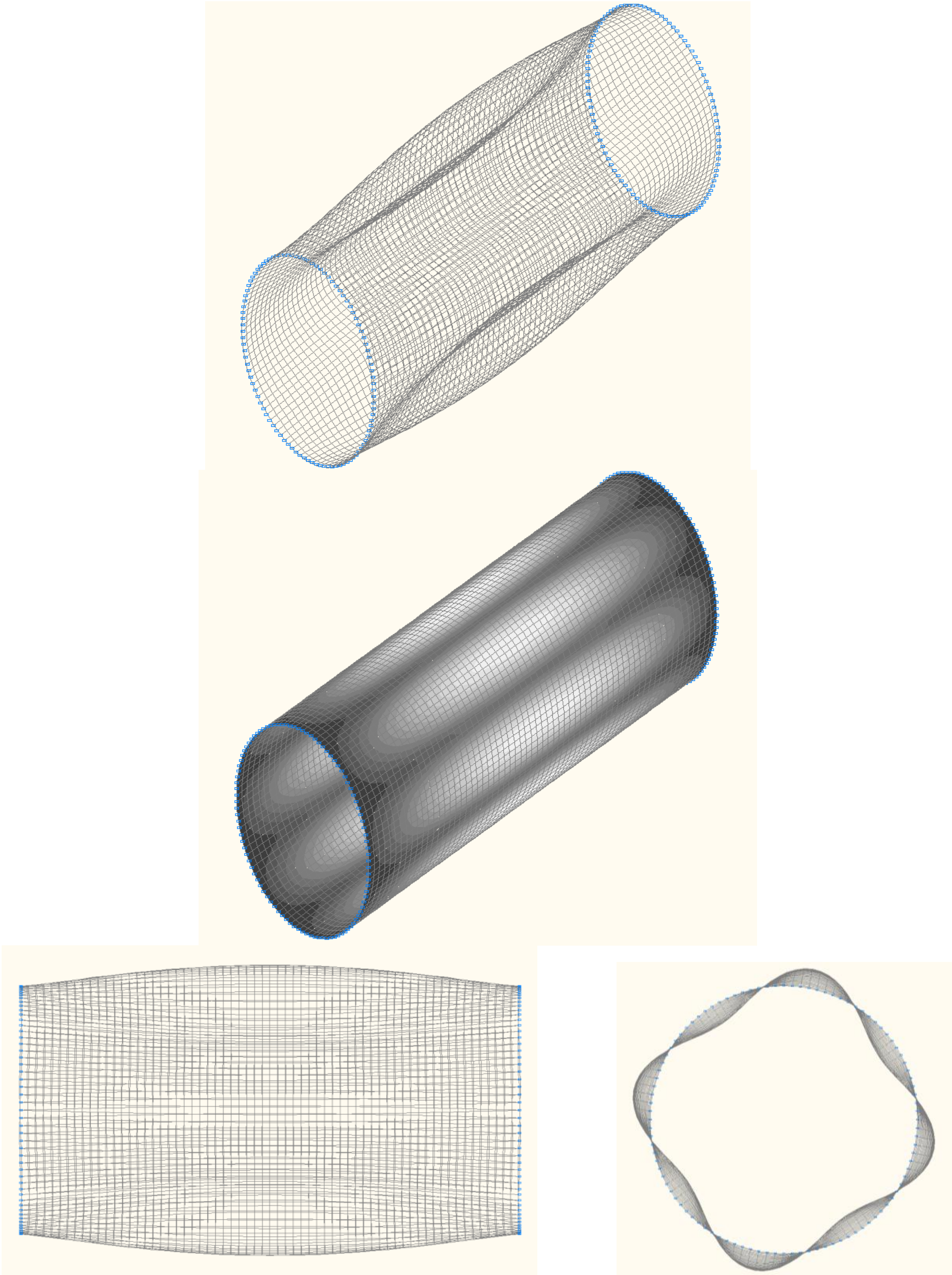
Design model



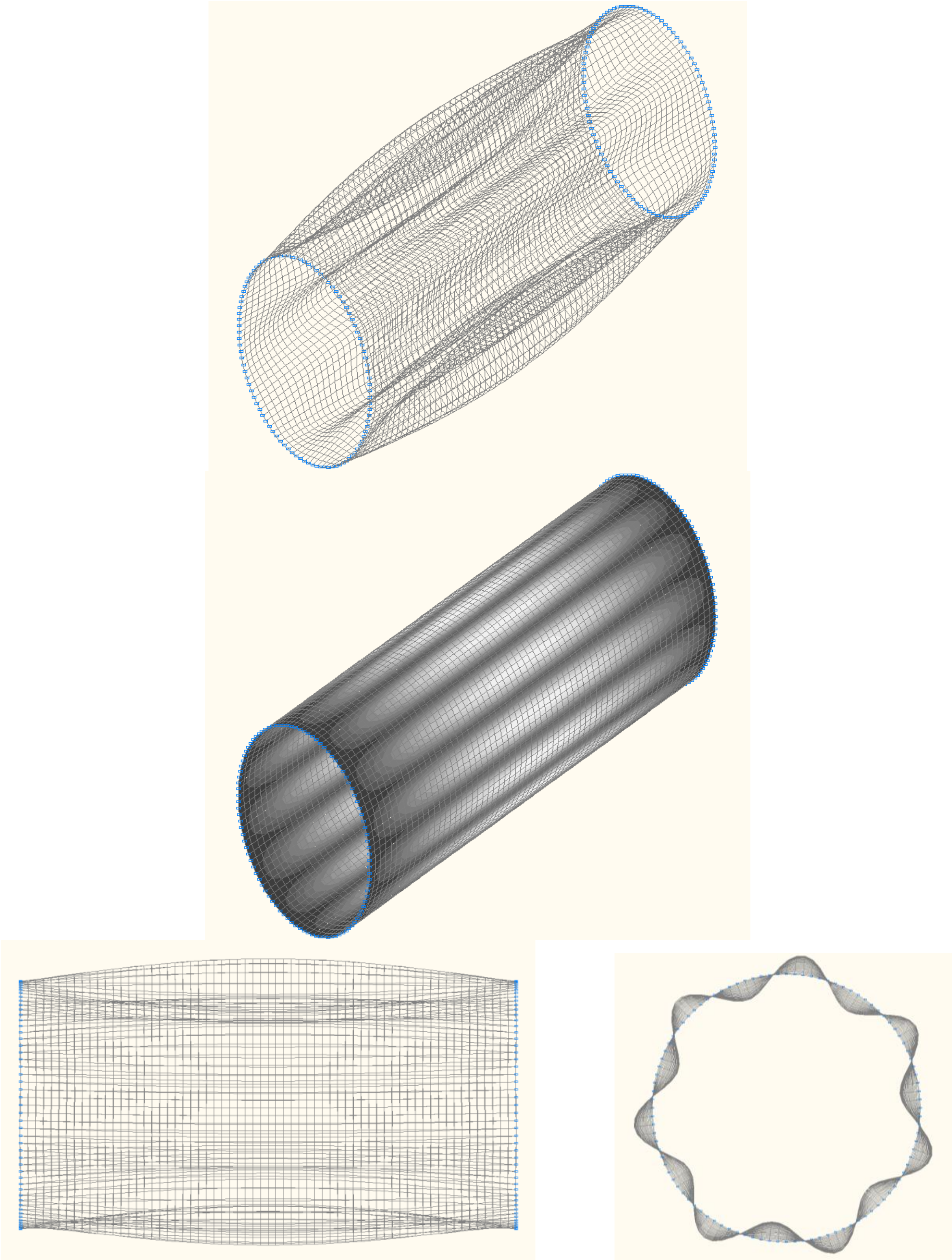
2-nd (1-st theoretical) natural oscillation mode



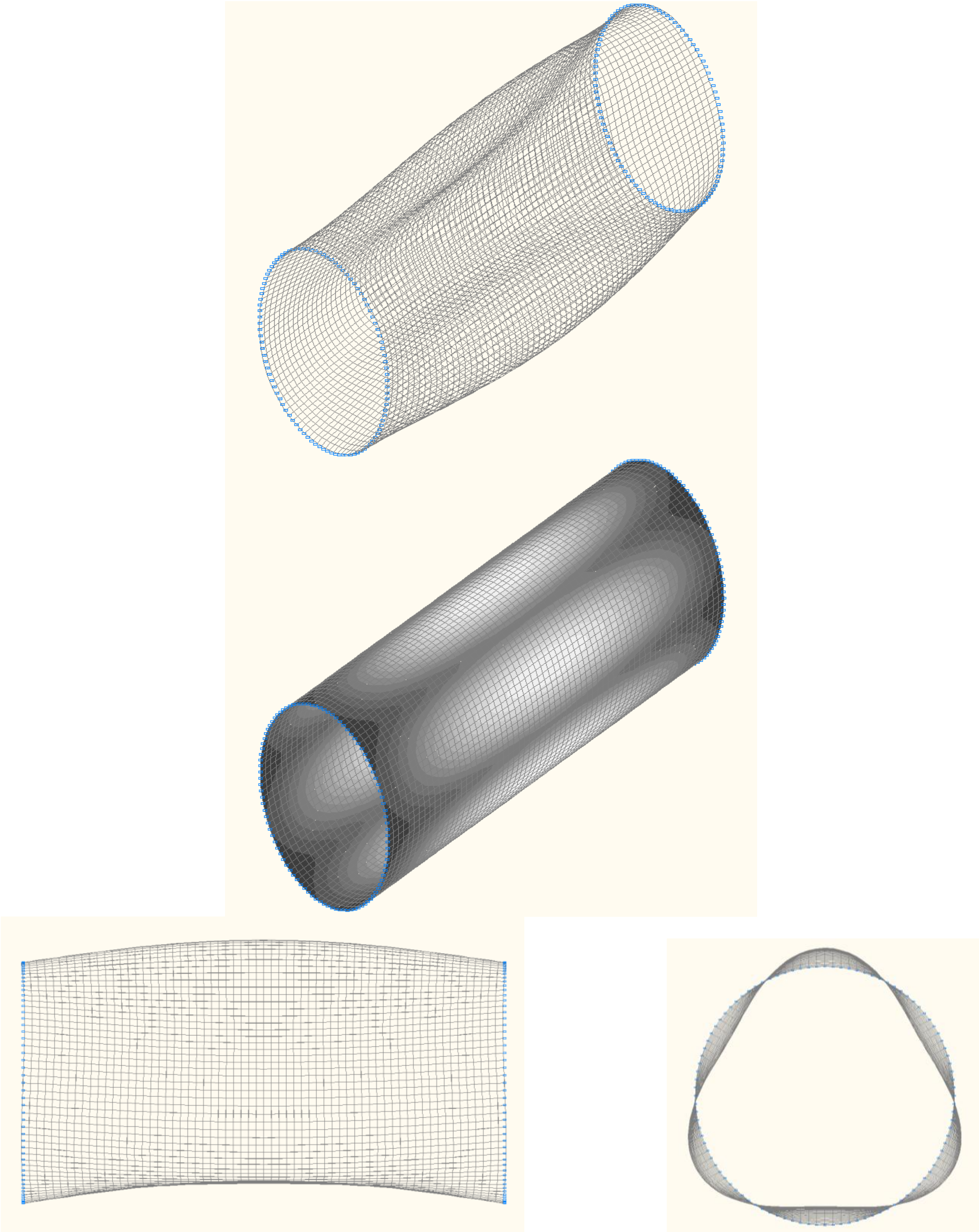
4-th (3-rd theoretical) natural oscillation mode



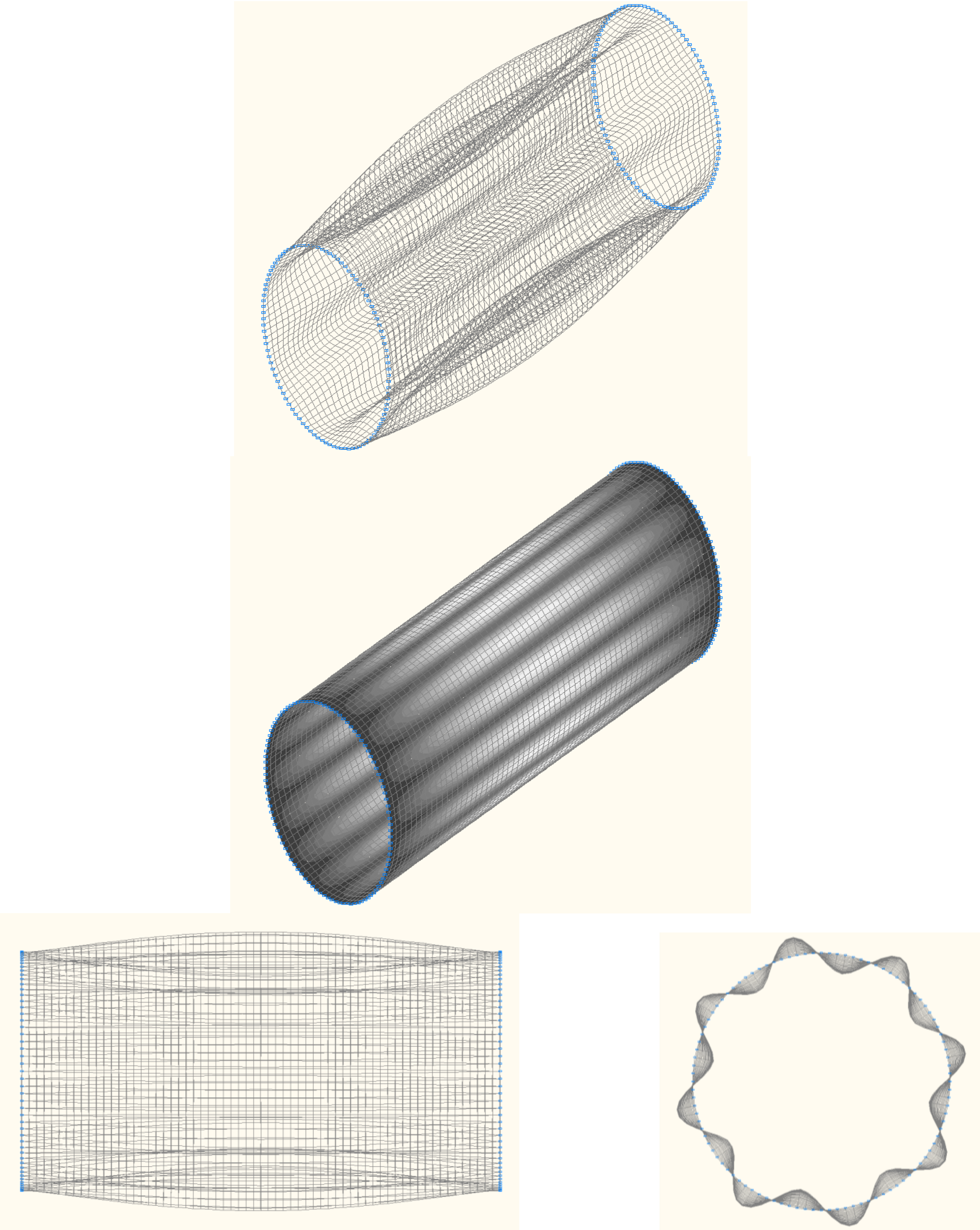
6-th (5-th theoretical) natural oscillation mode



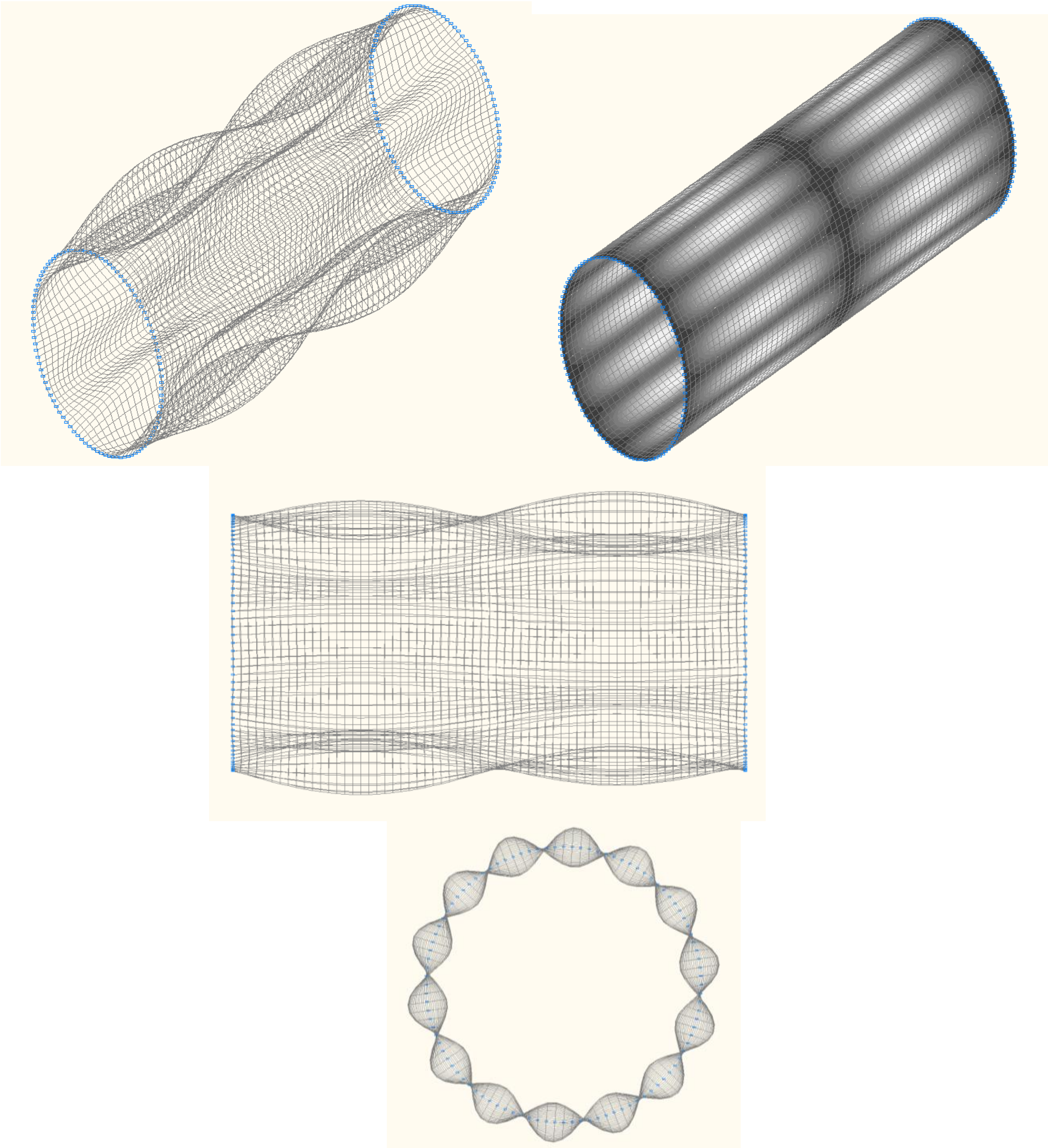
8-th (7-th theoretical) natural oscillation mode



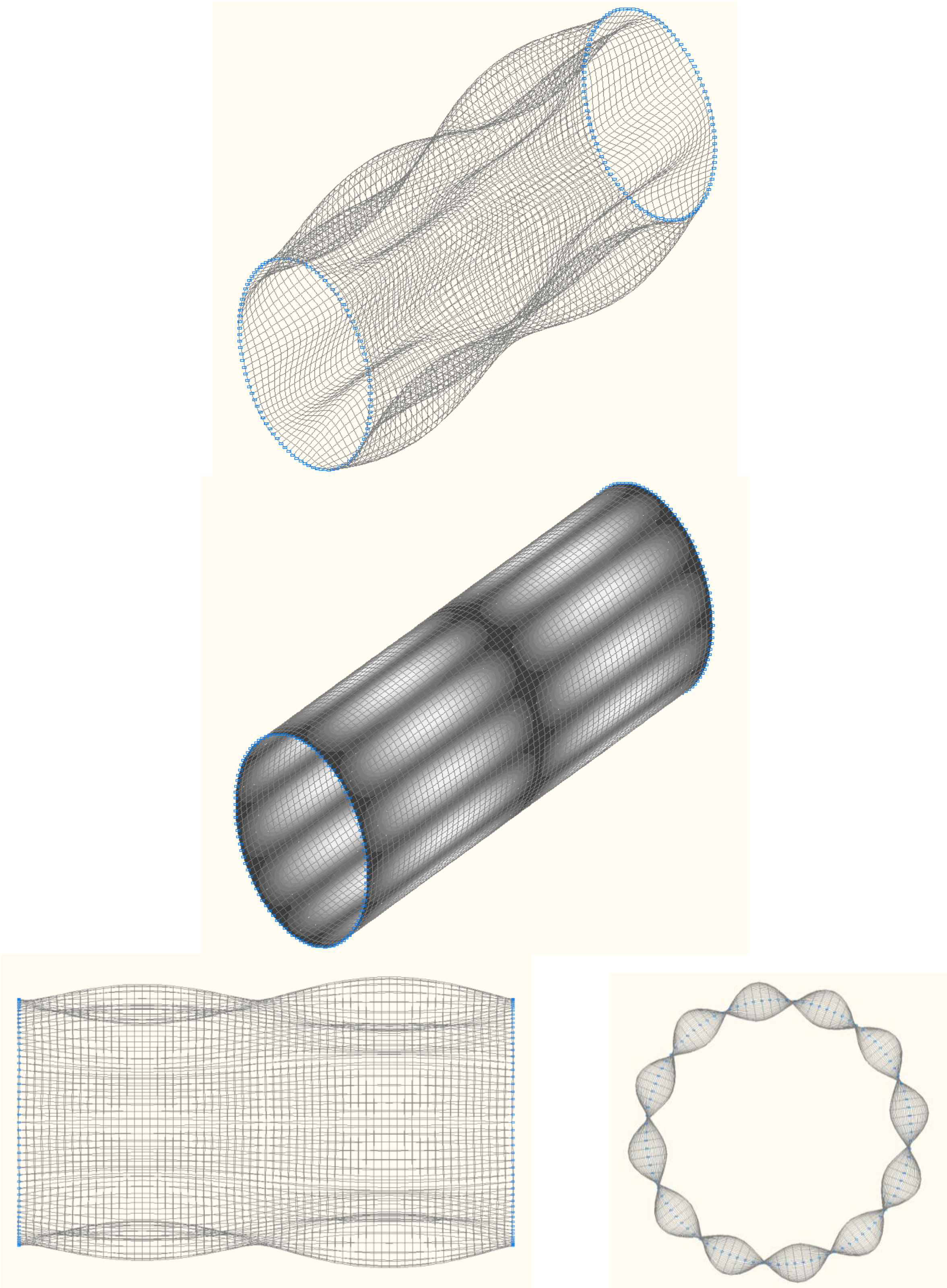
10-th (9-th theoretical) natural oscillation mode



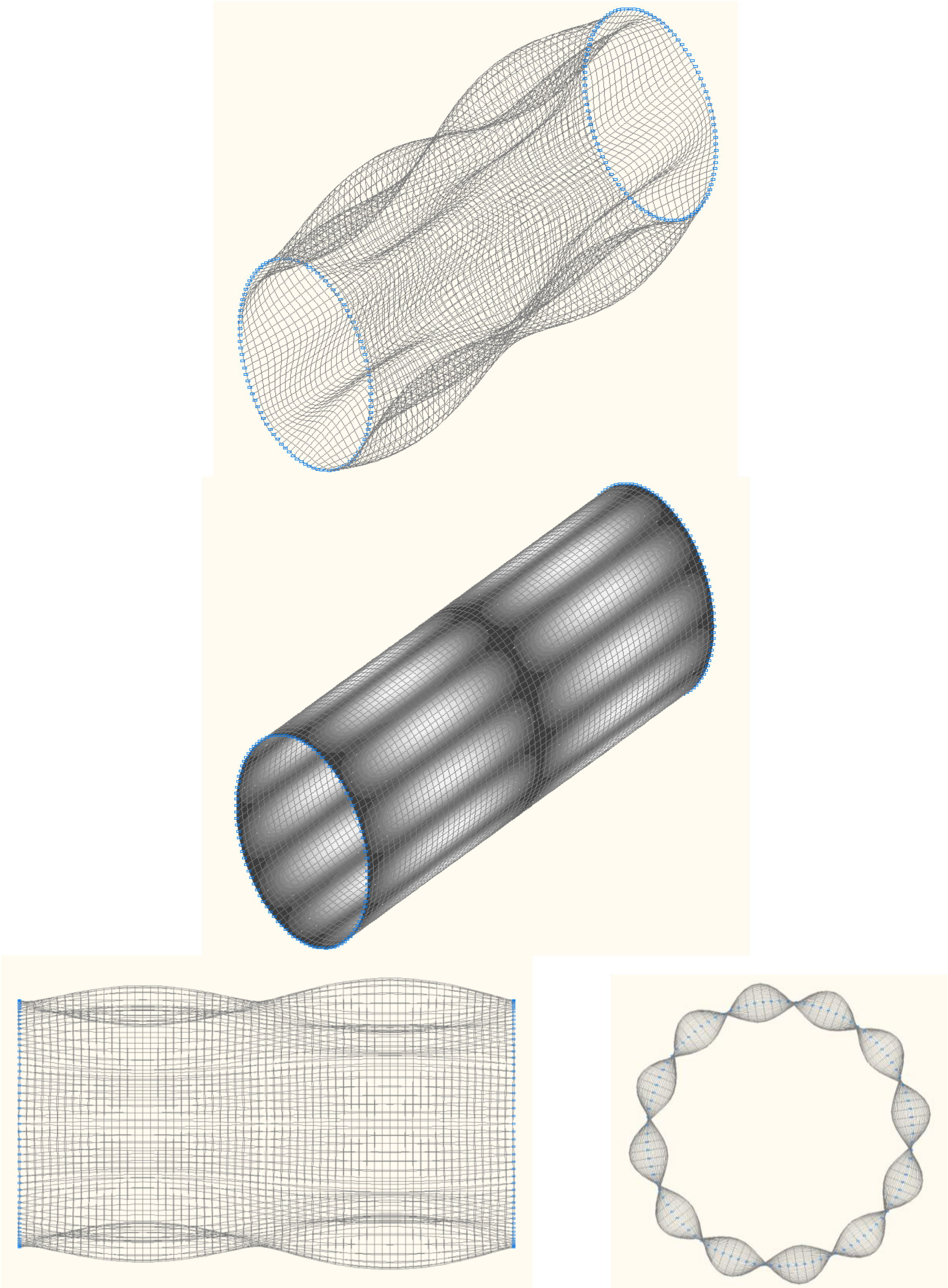
12-th (11-th theoretical) natural oscillation mode



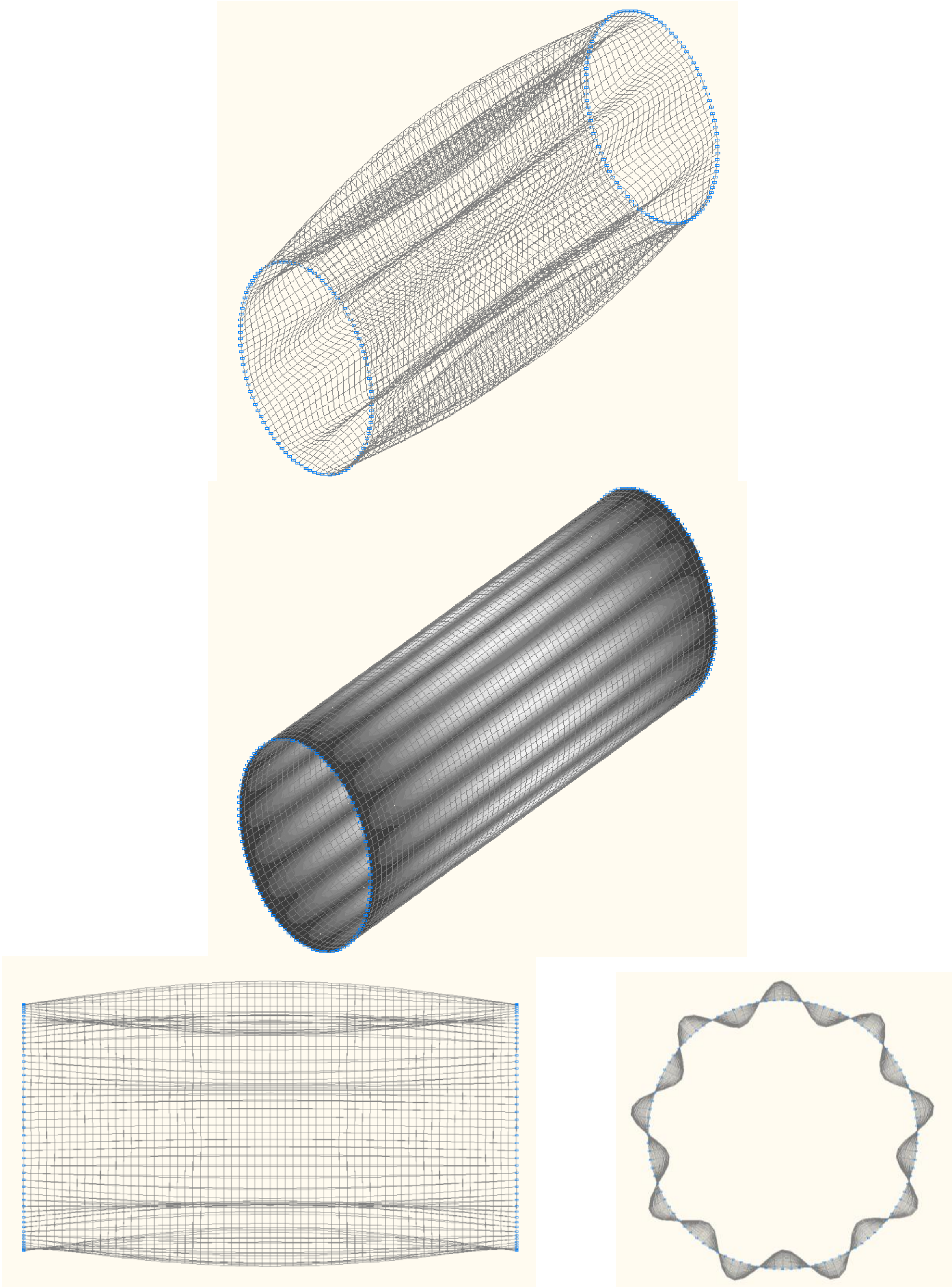
14-th (13-th theoretical) natural oscillation mode



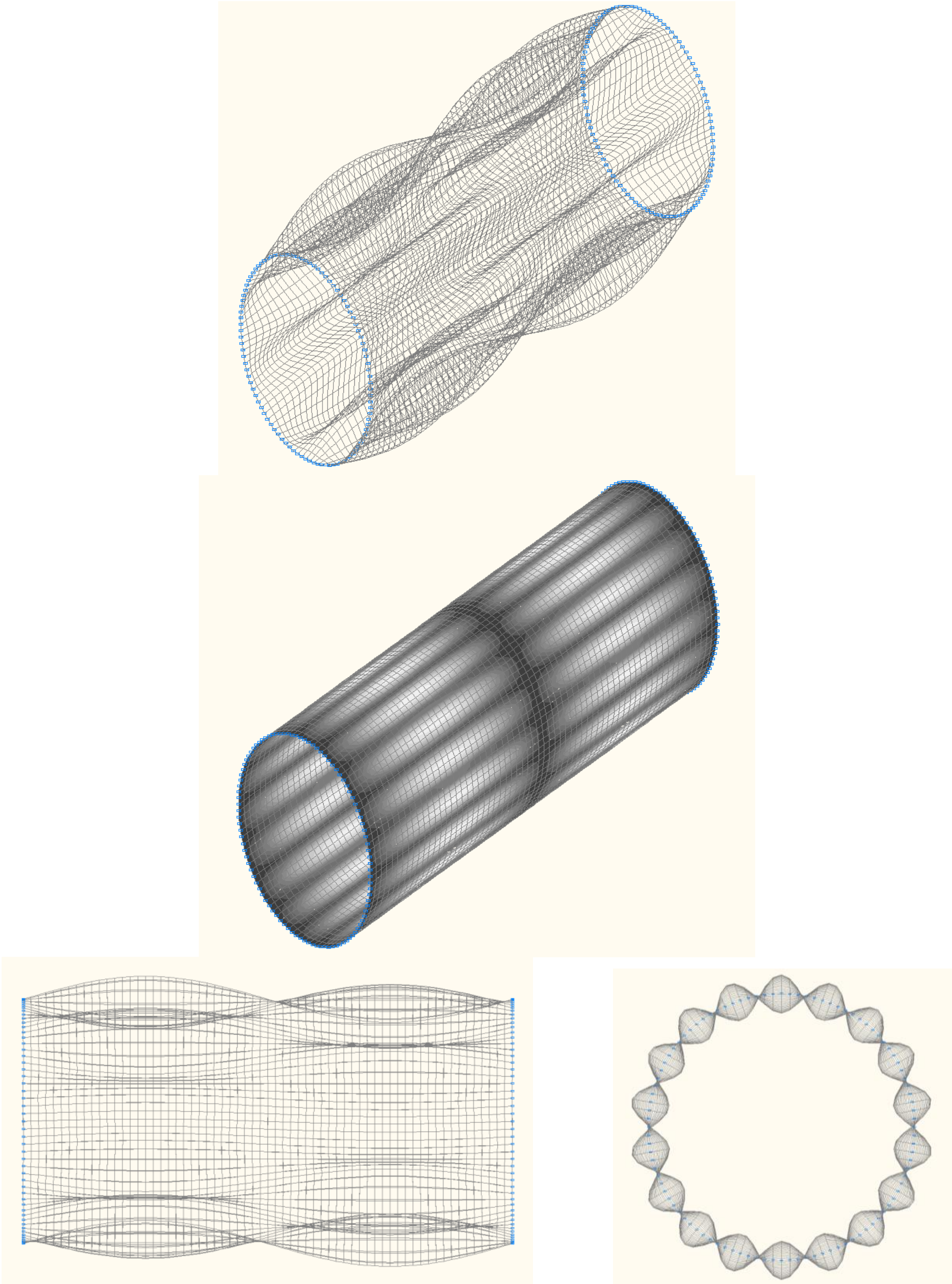
16-th (15-th theoretical) natural oscillation mode



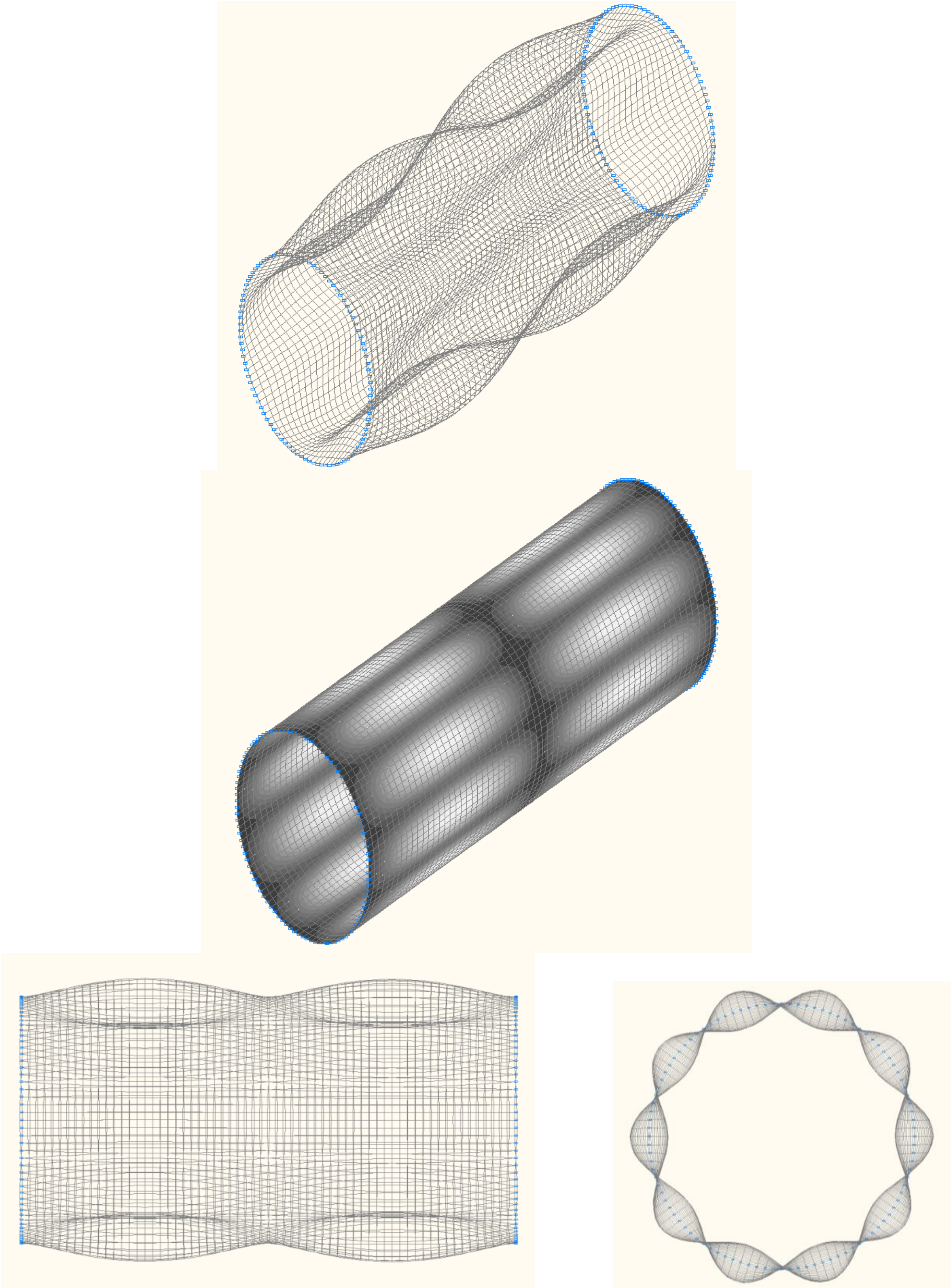
18-th (17-th theoretical) natural oscillation mode



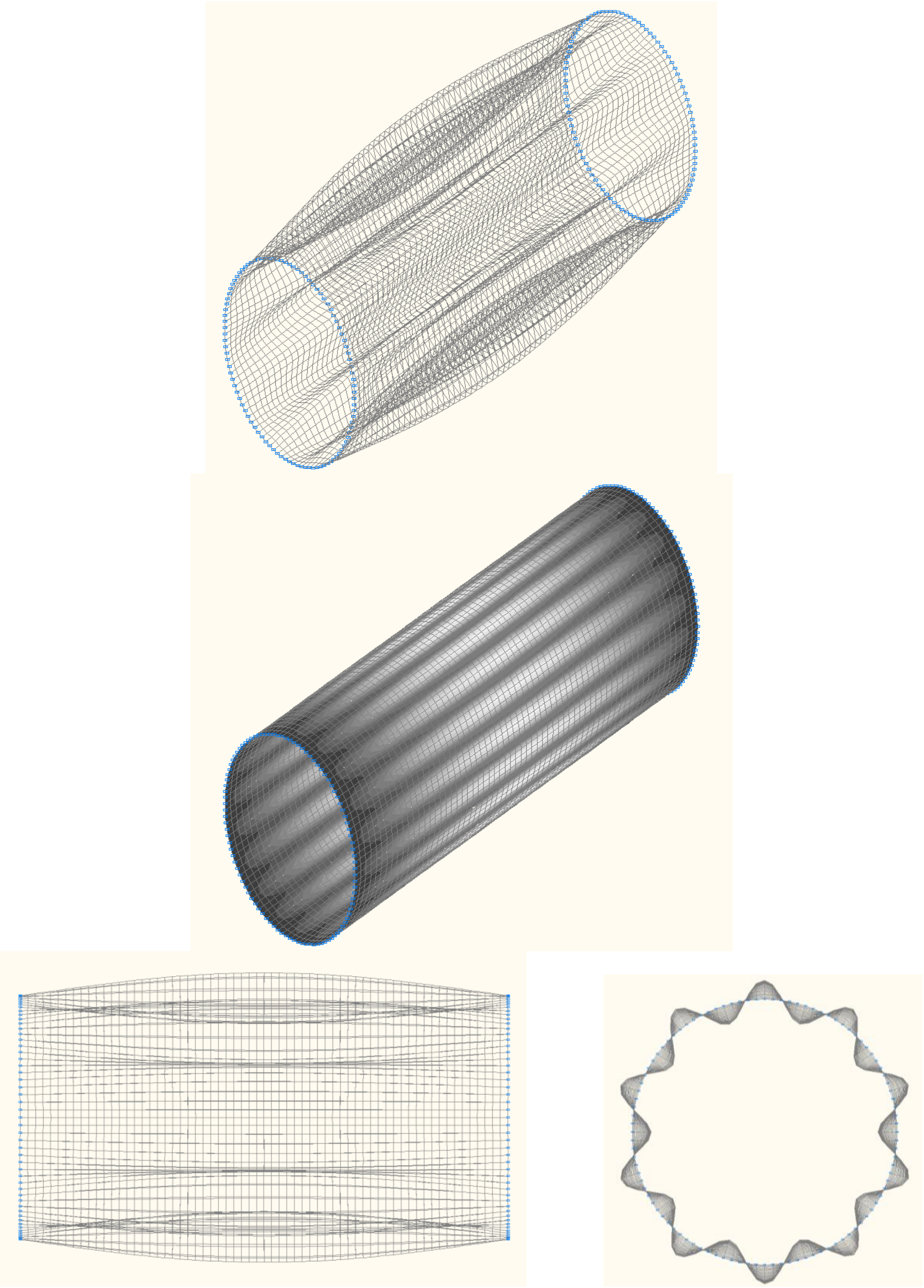
20-th (19-th theoretical) natural oscillation mode



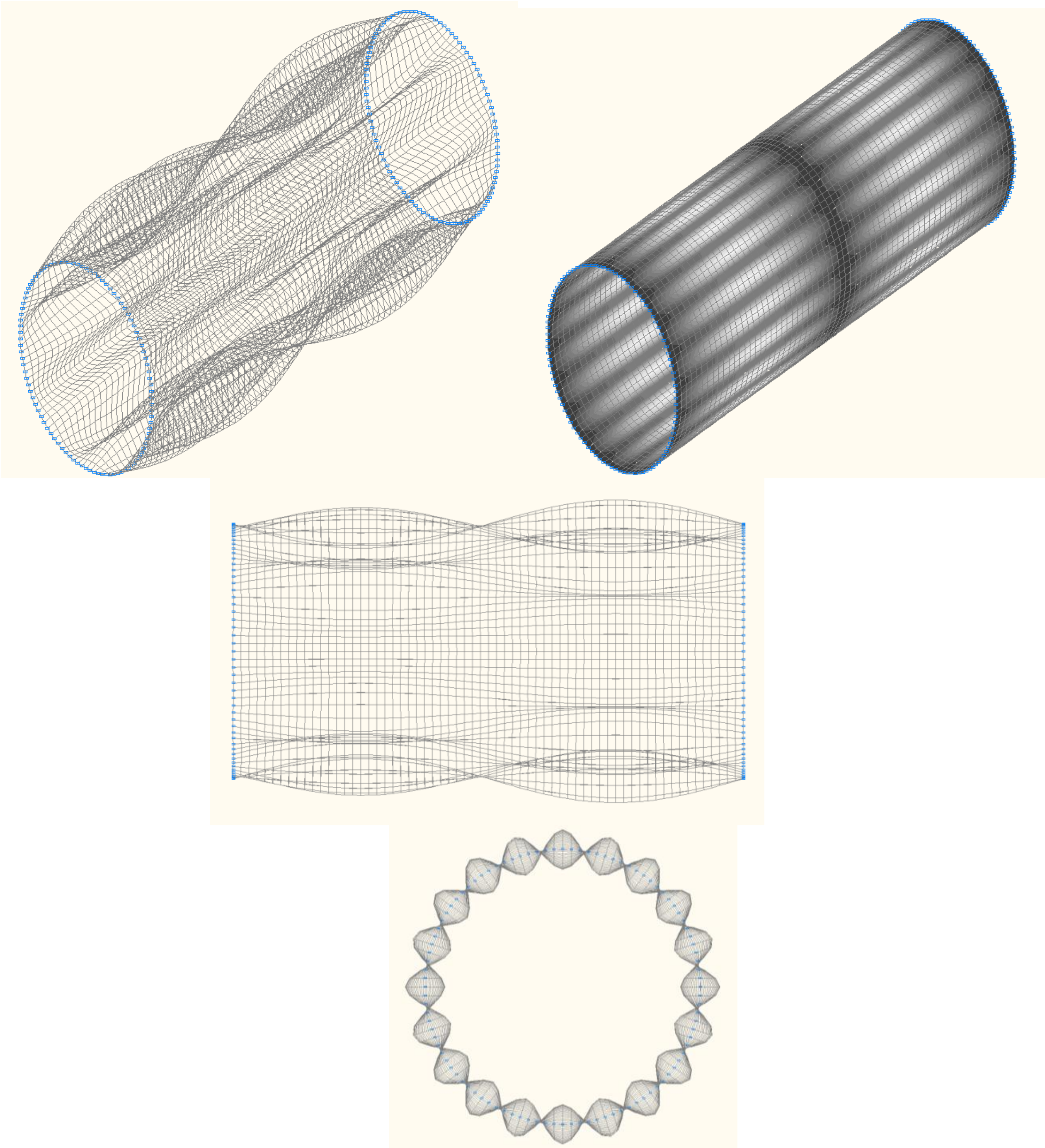
22-nd (21-st theoretical) natural oscillation mode



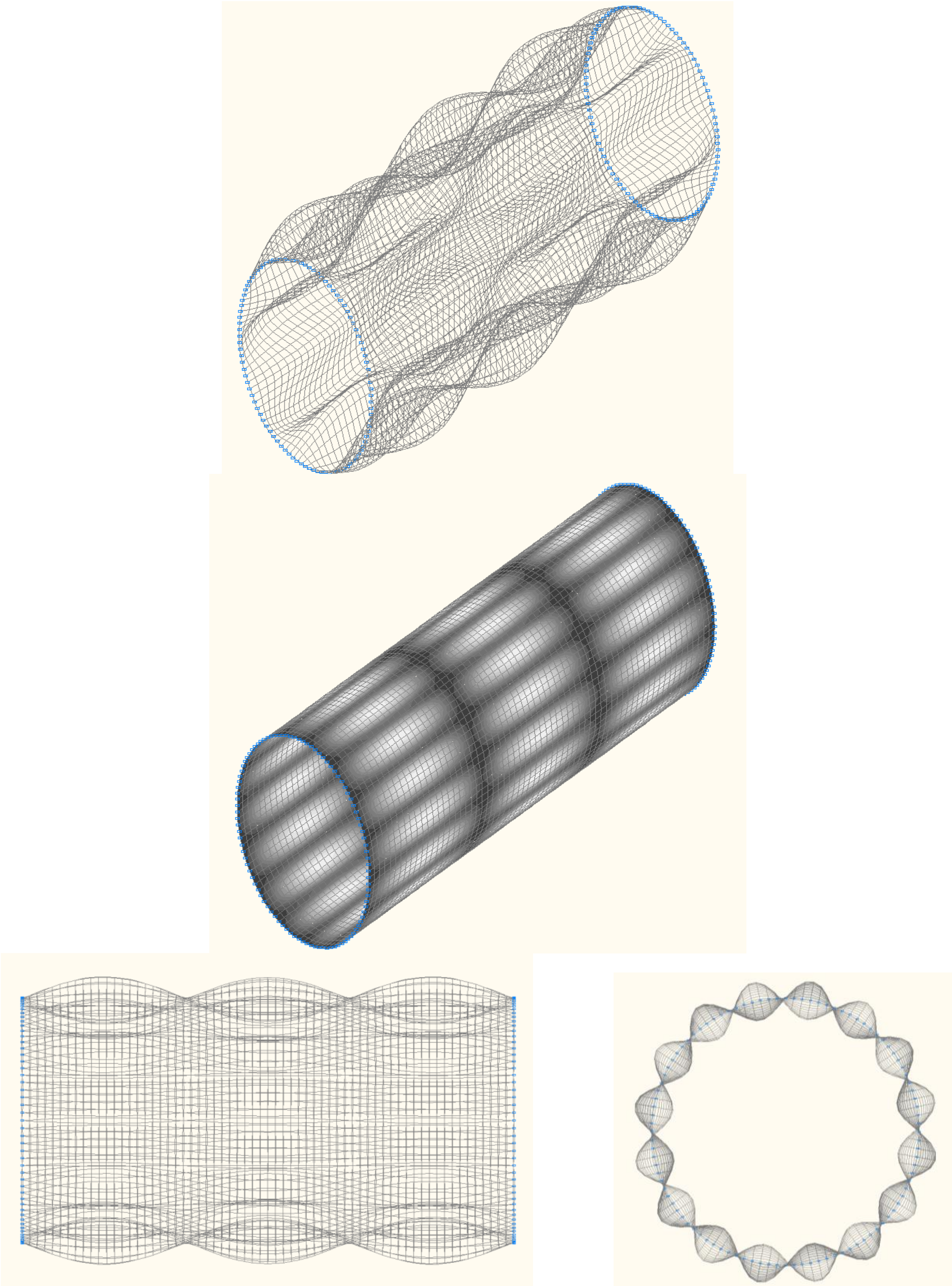
24-th (23-rd theoretical) natural oscillation mode



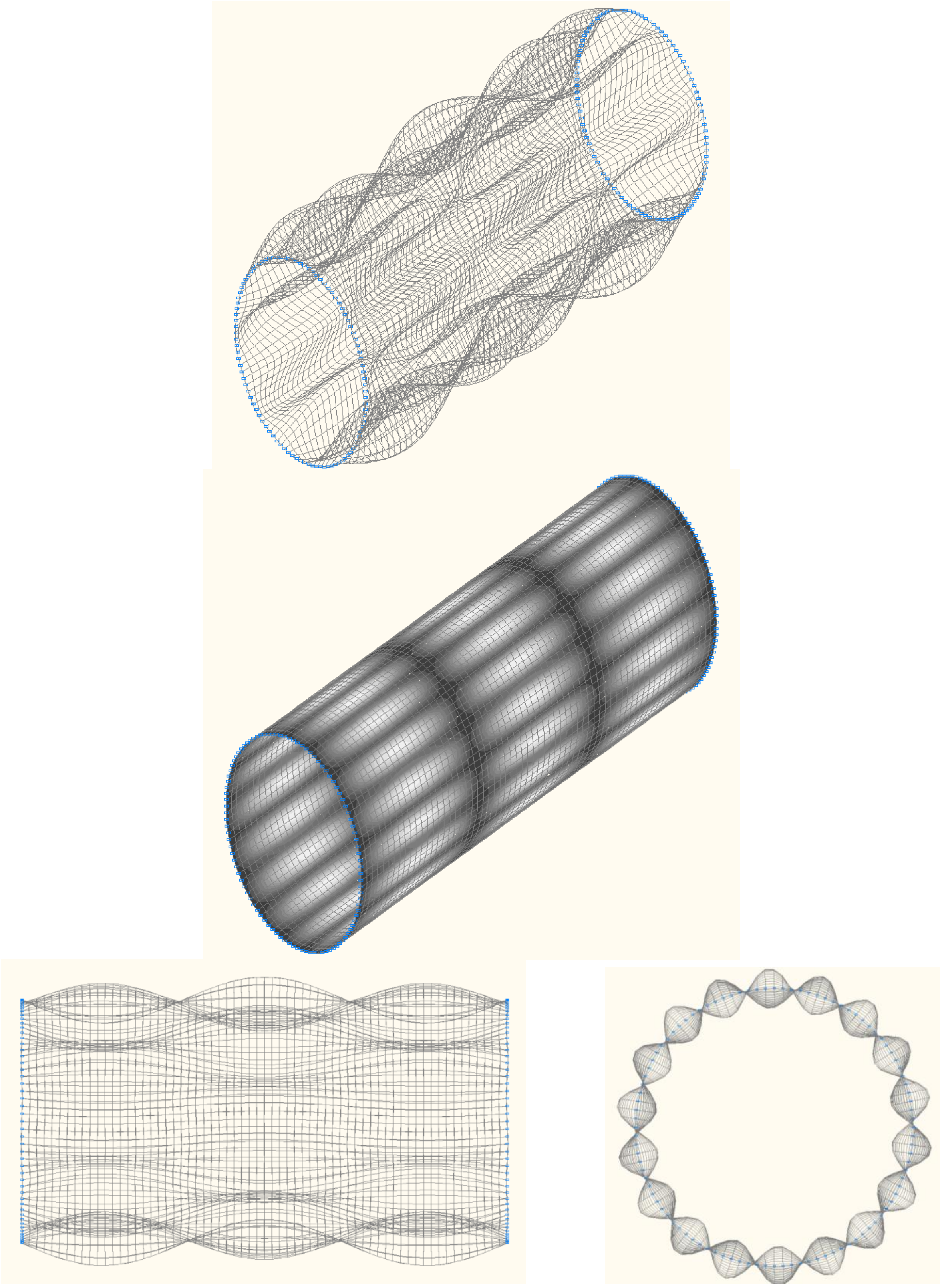
26-th (25-th theoretical) natural oscillation mode



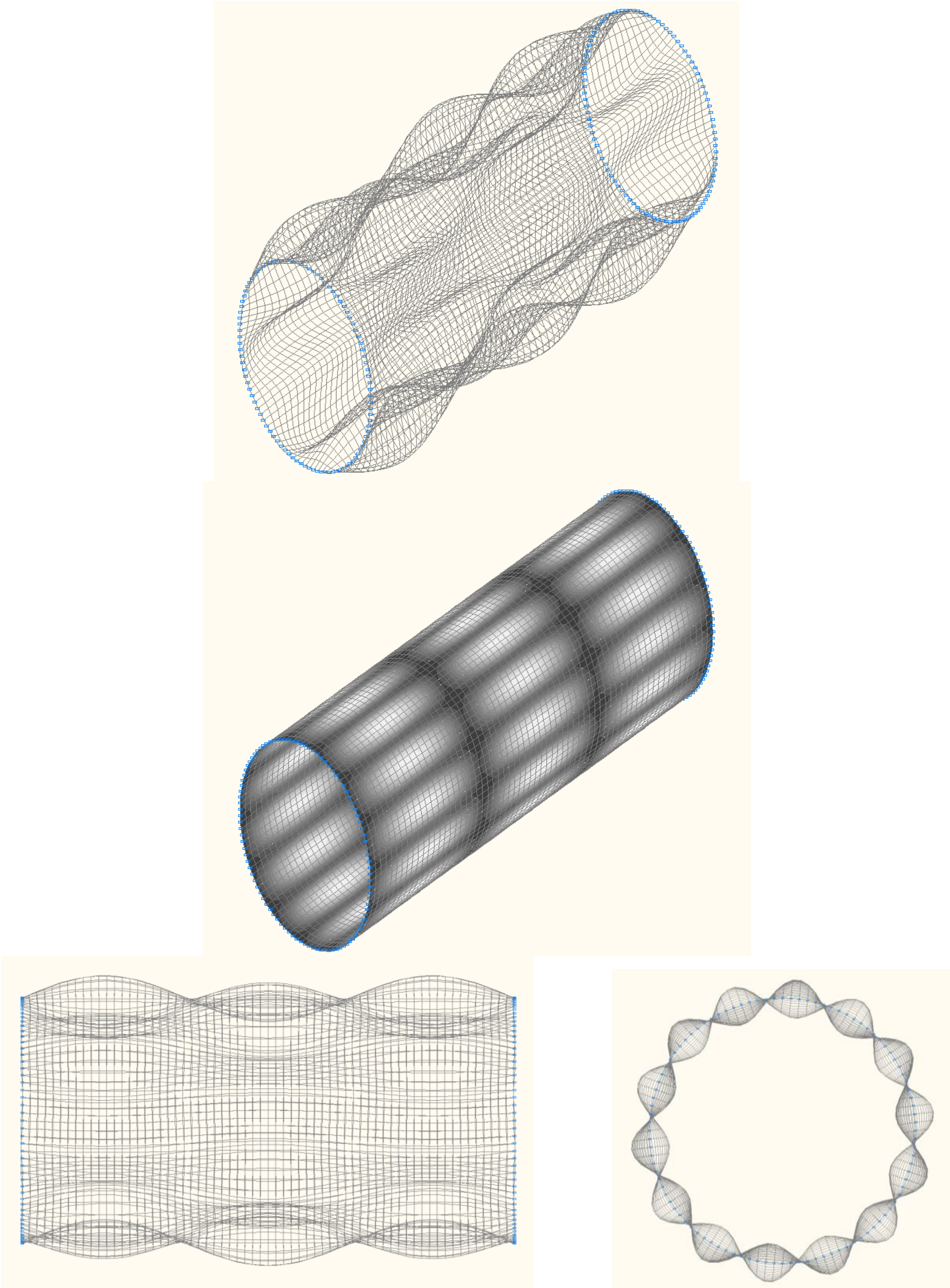
28-th (27-th theoretical) natural oscillation mode



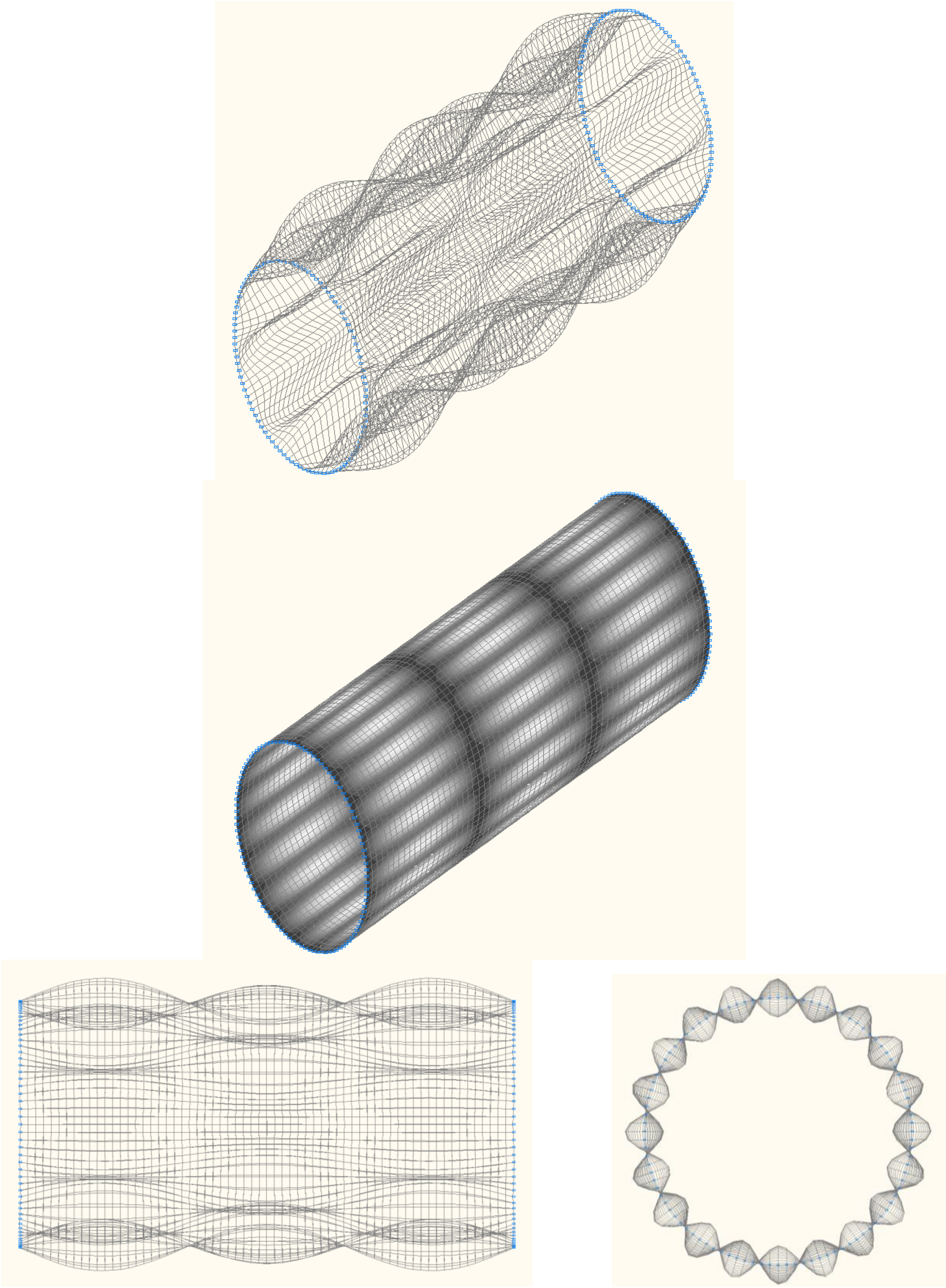
30-th (29-th theoretical) natural oscillation mode



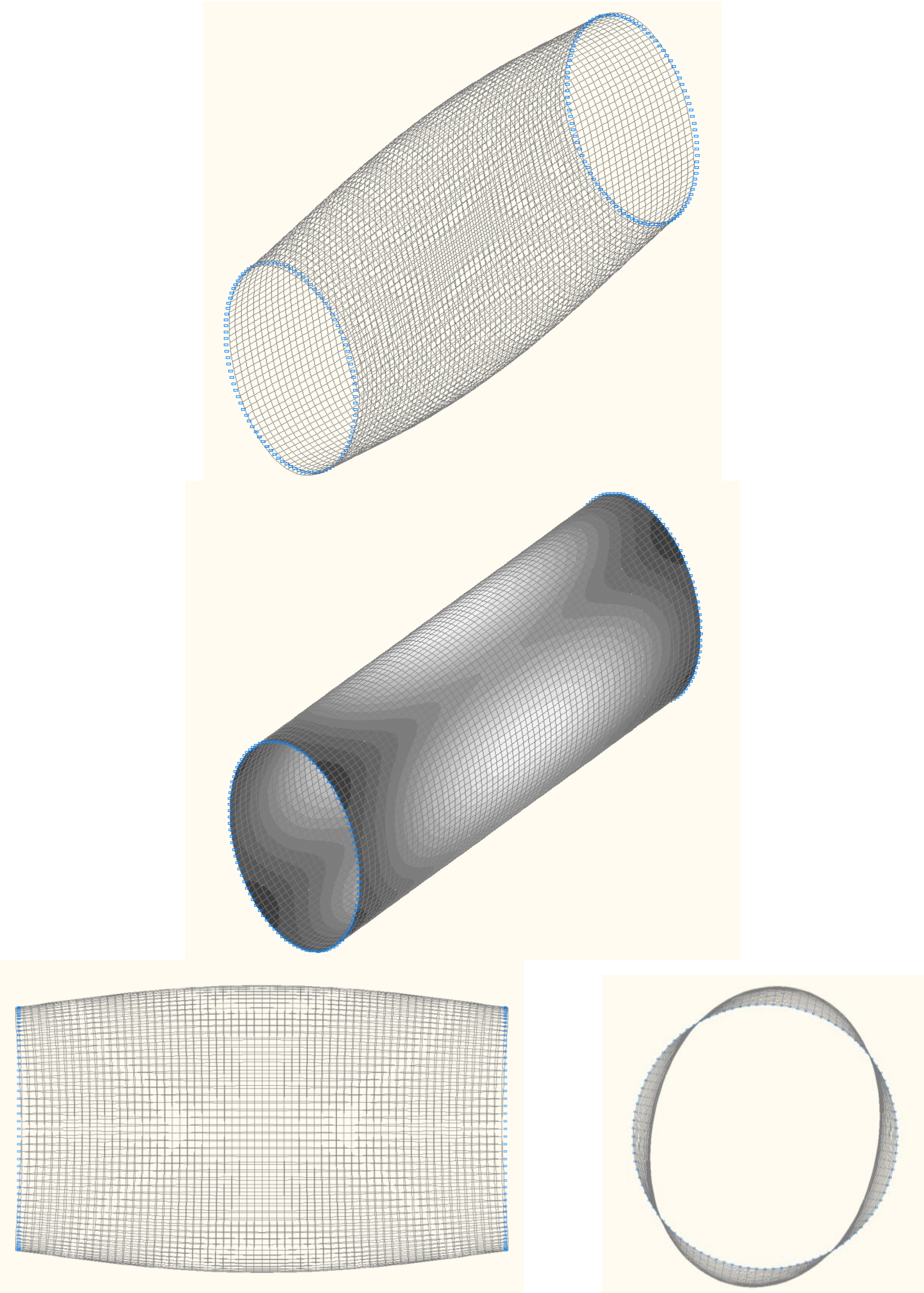
32-nd (31-st theoretical) natural oscillation mode



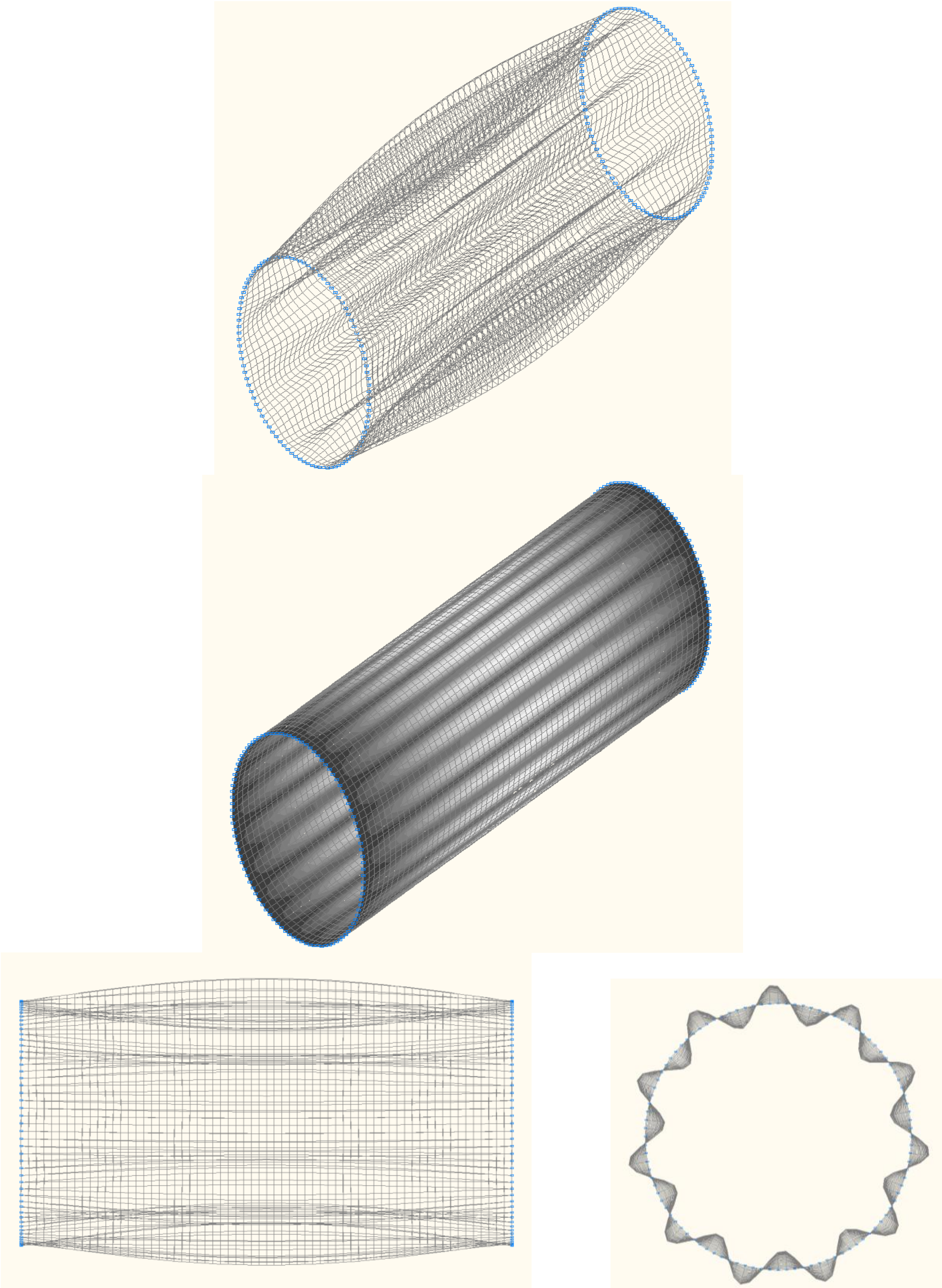
34-th (33-rd theoretical) natural oscillation mode



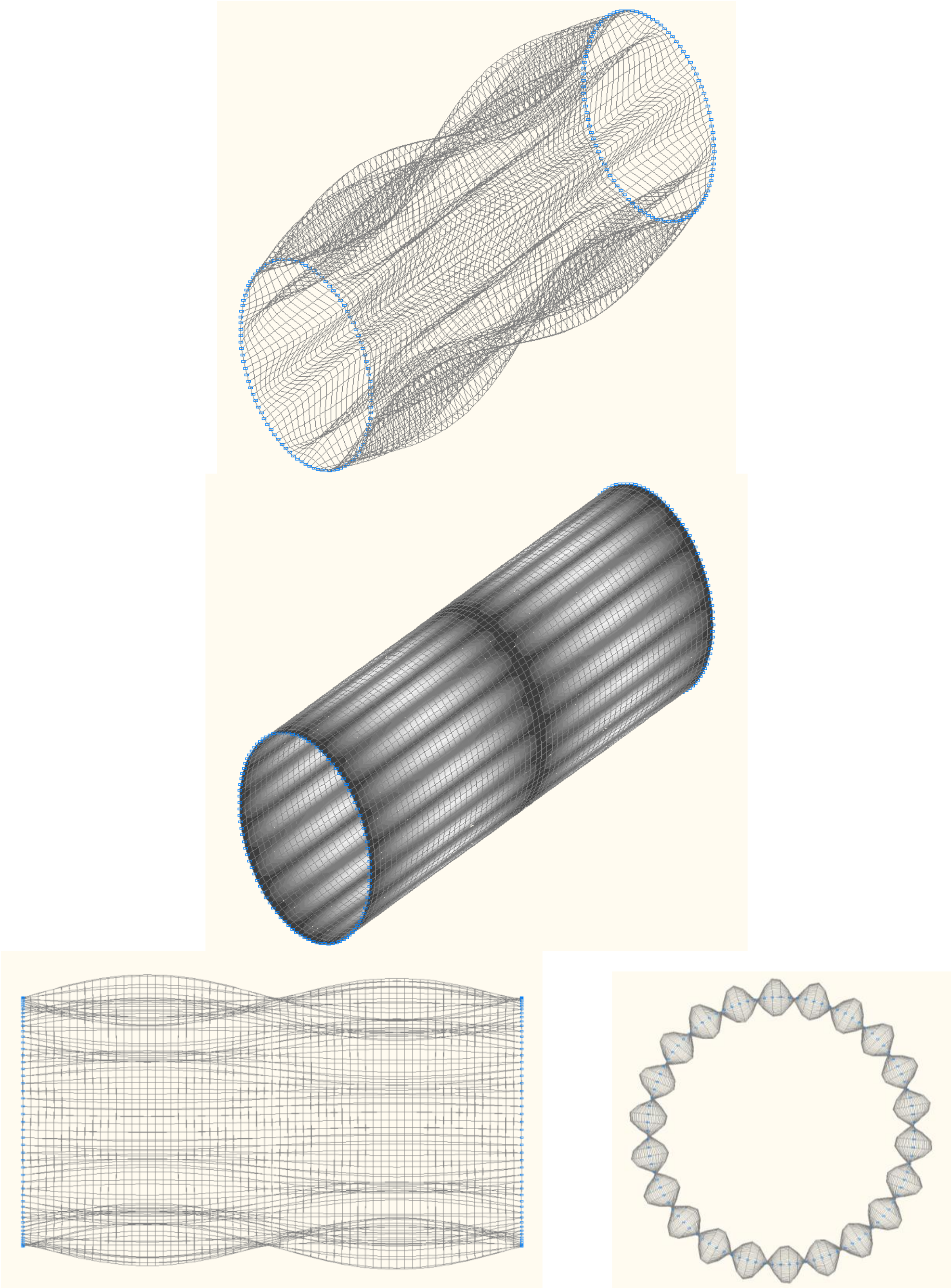
36-th (35-th theoretical) natural oscillation mode



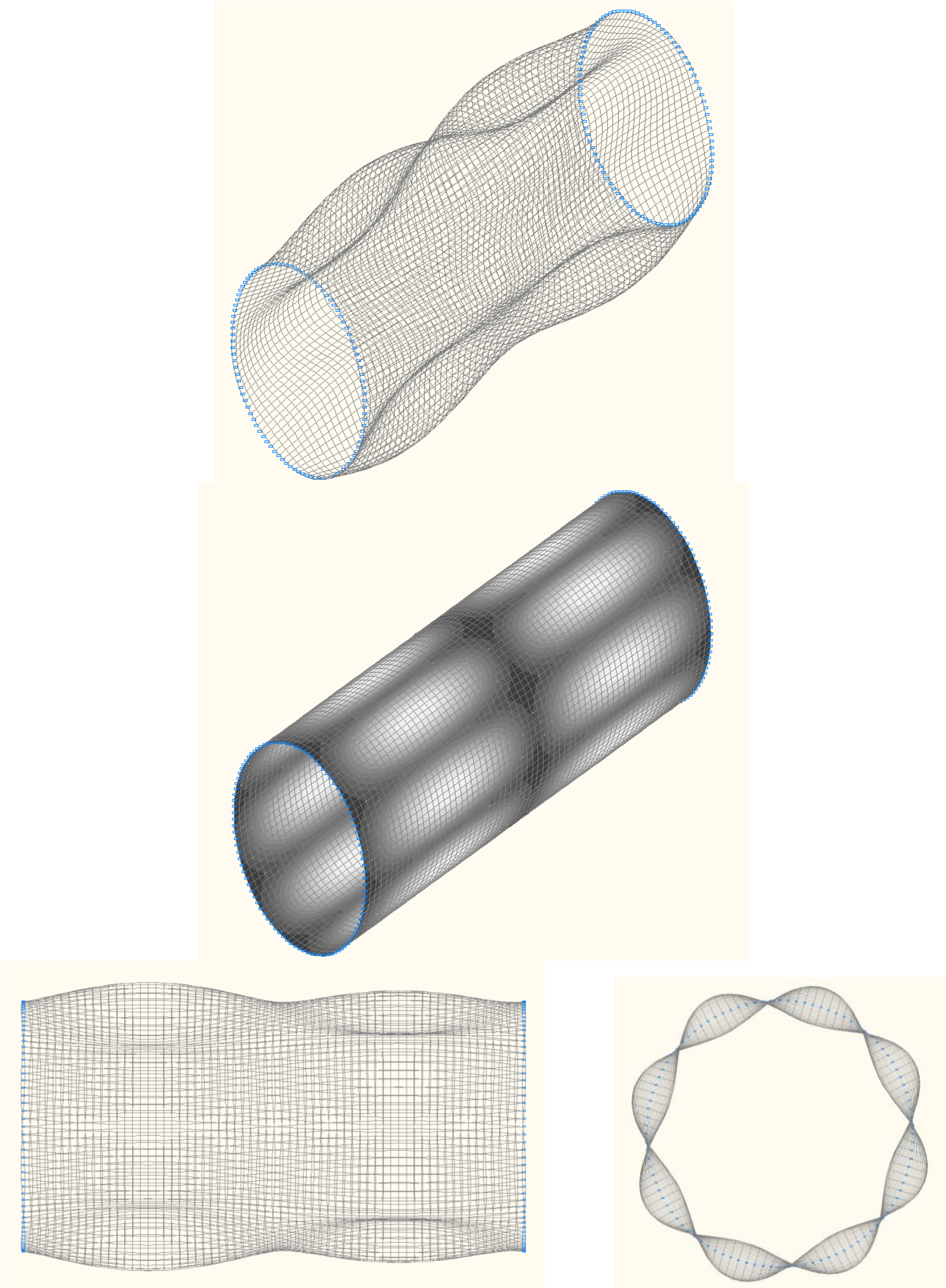
38-th (37-th theoretical) natural oscillation mode



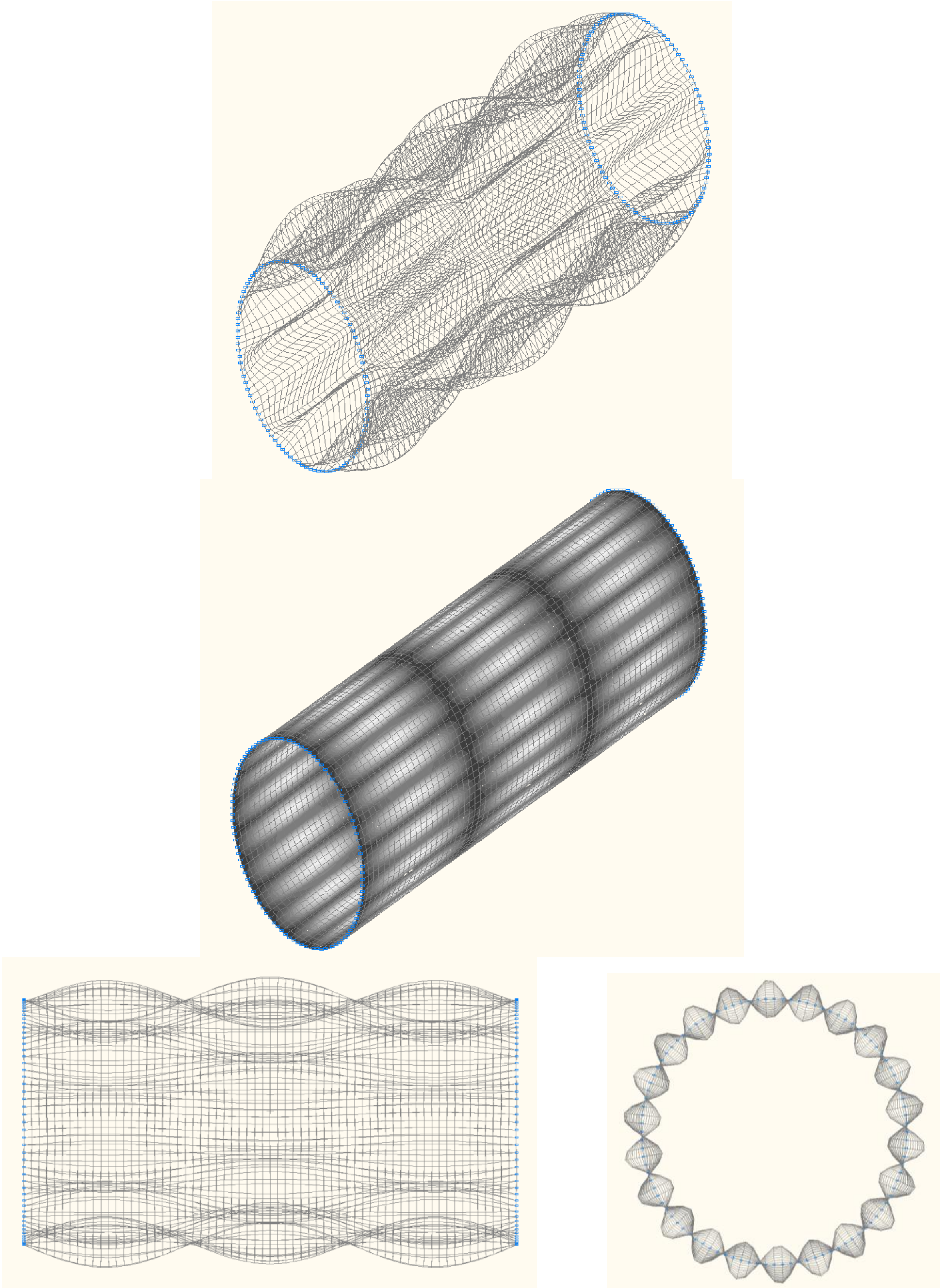
40-th (39-th theoretical) natural oscillation mode



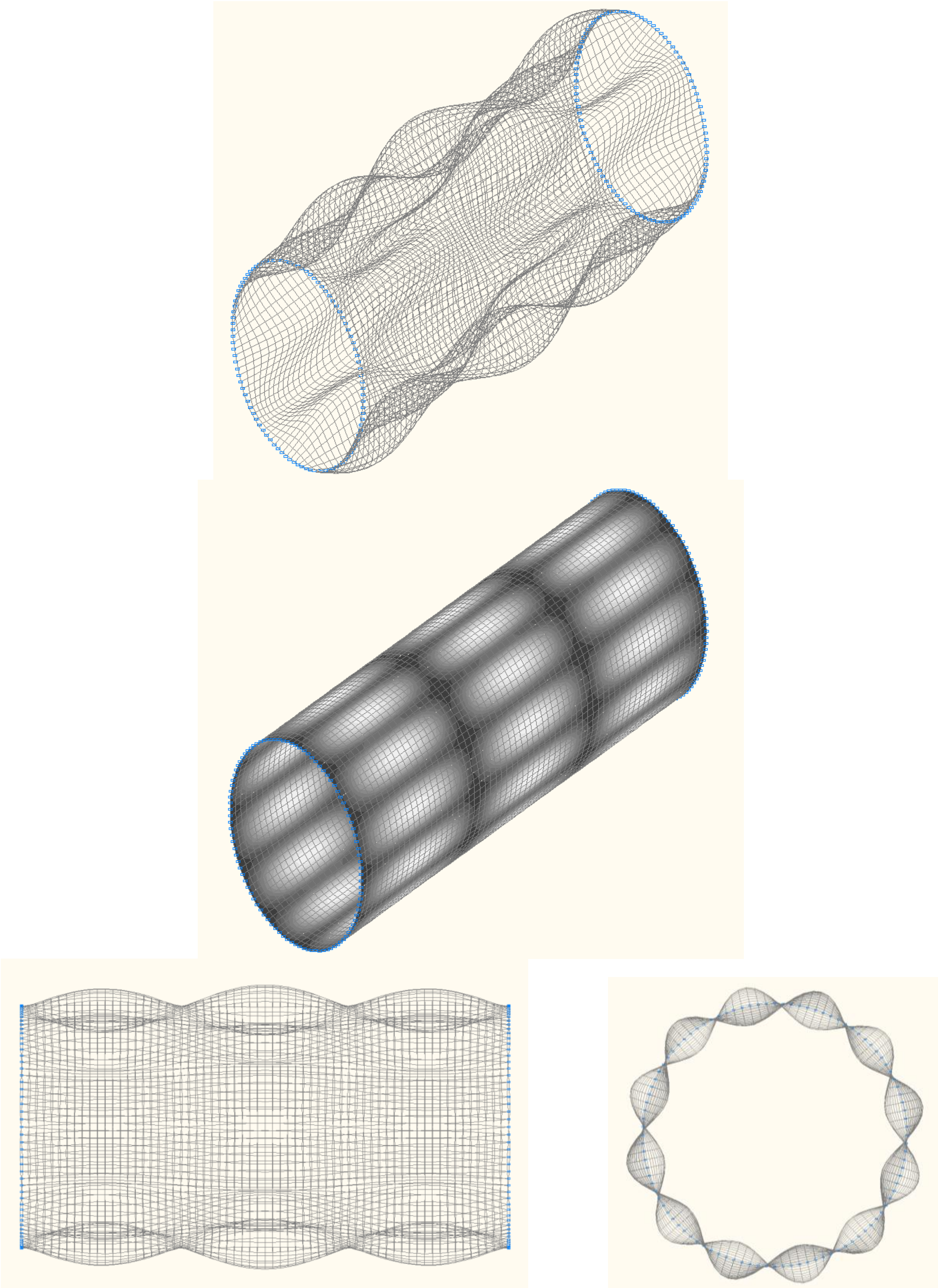
42-nd (41-st theoretical) natural oscillation mode



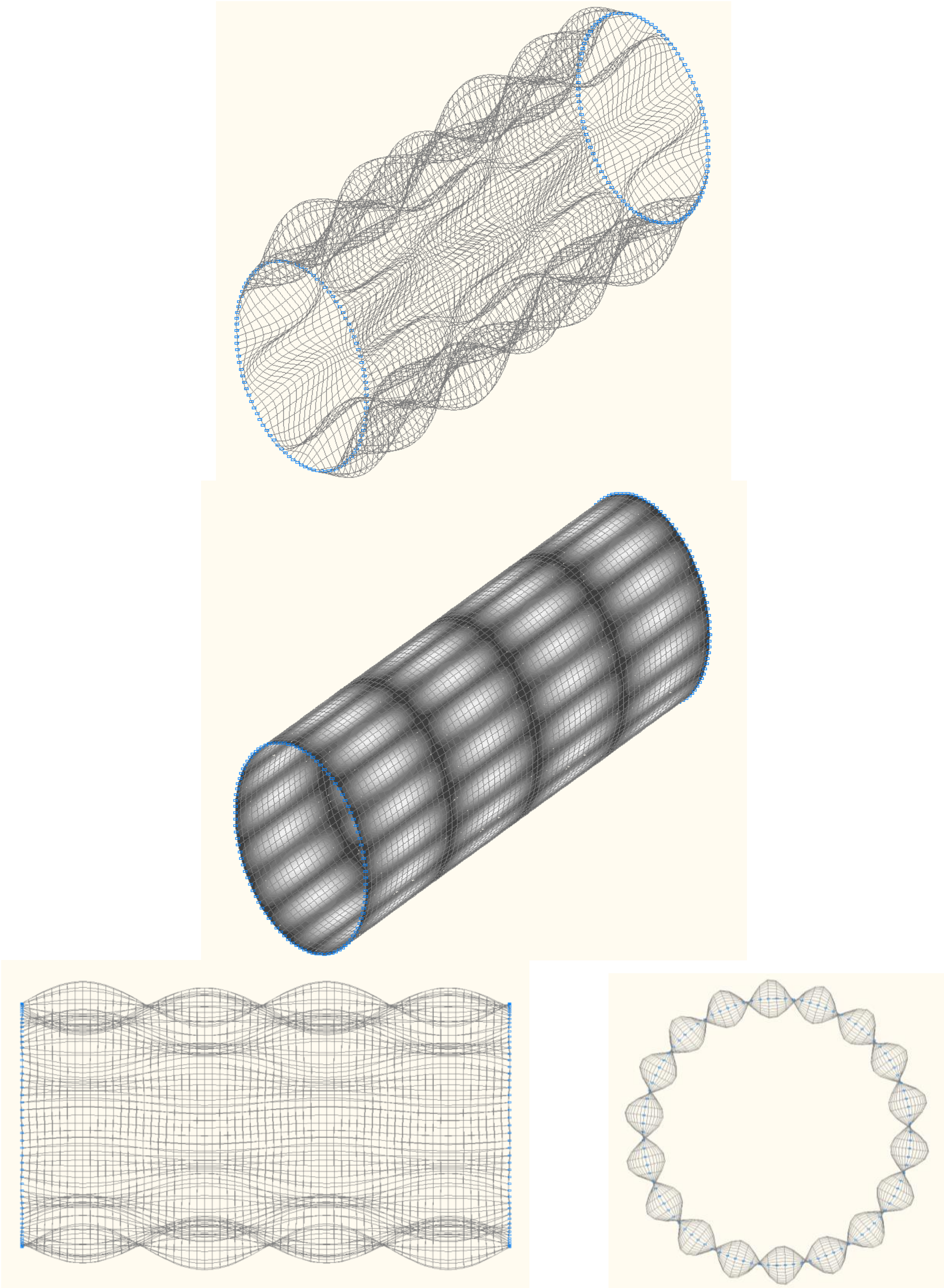
44-th (43-rd theoretical) natural oscillation mode



46-th (45-th theoretical) natural oscillation mode



48-th (47-th theoretical) natural oscillation mode



50-th (49-th theoretical) natural oscillation mode

Comparison of solutions:

Natural frequencies ω, Hz

Oscillation mode	Number of nodal circles m and meridians n	Theory	SCAD	Deviations, %
1, 2	2, 5	354.4	354.9	0.14
3, 4	2, 6	408.3	408.9	0.15
5, 6	2, 4	409.5	410.1	0.15
7, 8	2, 7	522.1	522.9	0.15

Oscillation mode	Number of nodal circles m and meridians n	Theory	SCAD	Deviations, %
9, 10	2, 3	642.1	642.8	0.11
11, 12	2, 8	671.1	672.0	0.13
13, 14	3, 7	723.2	724.9	0.24
15, 16	3, 6	768.5	770.3	0.23
17, 18	3, 8	784.3	785.9	0.20
19, 20	2, 9	846.2	847.3	0.13
21, 22	3, 9	914.9	916.6	0.19
23, 24	3, 5	962.3	964.5	0.23
25, 26	2, 10	1044.3	1045.7	0.13
27, 28	3, 10	1090.7	1092.5	0.17
29, 30	4, 8	1095.6	1099.3	0.34
31, 32	4, 9	1115.7	1119.2	0.31
33, 34	4, 7	1194.2	1198.2	0.33
35, 36	4, 10	1223.2	1226.5	0.27
37, 38	2, 2	1241.3	1242.5	0.10
39, 40	2, 11	1264.3	1265.9	0.13
41, 42	3, 11	1299.1	1301.2	0.16
43, 44	3, 4	1368.6	1370.9	0.17
45, 46	4, 11	1391.6	1395.0	0.24
47, 48	4, 6	1444.4	1448.8	0.30
49, 50	5, 9	1470.4	1477.2	0.46
51, 52	5, 10	1474.4	1480.6	0.42
53, 54	2, 12	1505.8	1507.5	0.11
55, 56	3, 12	1534.3	1536.6	0.15
57, 58	5, 11	1570.6	1576.5	0.38
59, 60	5, 8	1584.6	1591.9	0.46
61, 62	4, 12	1603.7	1607.1	0.21
63, 64	5, 12	1735.5	1741.2	0.33
65, 66	2, 13	1768.5	1770.3	0.10
67, 68	3, 13	1793.5	1795.9	0.13
69, 70	6, 10	1837.2	1848.0	0.59
71, 72	5, 7	1842.3	1850.1	0.42
73, 74	6, 11	1844.3	1854.3	0.54
75, 76	4, 13	1849.2	1852.7	0.19
77, 78	4, 5	1892.8	1897.7	0.26
79, 80	6, 12	1942.4	1951.9	0.49
81, 82	6, 9	1942.8	1954.3	0.59
83, 84	5, 13	1951.0	1956.7	0.29
85, 86	2, 14	2052.3	2054.1	0.09
87, 88	3, 14	2075.2	2077.7	0.12
89, 90	6, 13	2111.1	2120.1	0.43
91, 92	4, 14	2122.7	2126.3	0.17
93, 94	3, 3	2137.0	2140.0	0.14
95, 96	6, 8	2181.3	2193.4	0.55
97, 98	5, 14	2205.6	2211.2	0.25
99, 100	7, 11	2199.6	2215.4	0.72
101, 102	7, 12	2223.0	2237.8	0.67
103, 104	5, 6	2275.4	2283.7	0.36
105, 106	7, 10	2281.7	2298.3	0.73
107, 108	7, 13	2333.3	2347.3	0.60
109, 110	6, 14	2333.8	2342.5	0.37
111, 112	2, 15	2357.2	2358.9	0.07
113, 114	3, 15	2378.9	2381.2	0.10
115, 116	4, 15	2421.3	2424.8	0.14
117, 118	7, 9	2485.9	2503.2	0.70
119, 120	5, 15	2492.0	2497.5	0.22
121, 122	7, 14	2512.8	2526.3	0.54
123, 124	8, 12	2565.0	2586.6	0.84

Oscillation mode	Number of nodal circles m and meridians n	Theory	SCAD	Deviations, %
125, 126	6, 7	2574.4	2586.9	0.49
127, 128	6, 15	2598.7	2607.3	0.33
129,130	8, 13	2613.1	2633.7	0.79
131, 132	8, 11	2614.4	2637.0	0.86
133, 134	4, 4	2630.0	2635.4	0.21
135, 136	2, 16	2683.2	2684.5	0.05
137, 138	3, 16	2704.1	2706.1	0.07
139, 140	8, 14	2742.8	2762.4	0.71
141, 142	4, 16	2743.2	2746.5	0.12
143, 144	7, 15	2747.0	2759.9	0.47
145, 146	8, 10	2776.0	2799.3	0.84
147, 148	5, 16	2806.0	2811.3	0.19
149, 150	2, 1	2832.3	2835.3	0.11

Notes: In the analytical solution the natural frequencies ω of the simply supported circular cylindrical shell with the density of the material ρ can be determined from the characteristic equation:

$$\left(\frac{4\cdot\pi^2\cdot\rho\cdot R^2\cdot\left(1-\nu^2\right)}{E}\right)^3\cdot\omega^6+K2\cdot\left(\frac{4\cdot\pi^2\cdot\rho\cdot R^2\cdot\left(1-\nu^2\right)}{E}\right)^2\cdot\omega^4+K1\cdot\left(\frac{4\cdot\pi^2\cdot\rho\cdot R^2\cdot\left(1-\nu^2\right)}{E}\right)\cdot\omega^2+K0=0,$$

where:

$$K2=-1-\frac{1}{2}\cdot(3-\nu)\cdot\left[\left(\frac{(m-1)\cdot\pi\cdot R}{L}\right)^2+n^2\right]-\frac{h^2}{12\cdot R^2}\cdot\left\{\left[\left(\frac{(m-1)\cdot\pi\cdot R}{L}\right)^2+n^2\right]^2+2\cdot(1-\nu)\cdot\left(\frac{(m-1)\cdot\pi\cdot R}{L}\right)^2+n^2\right\}$$

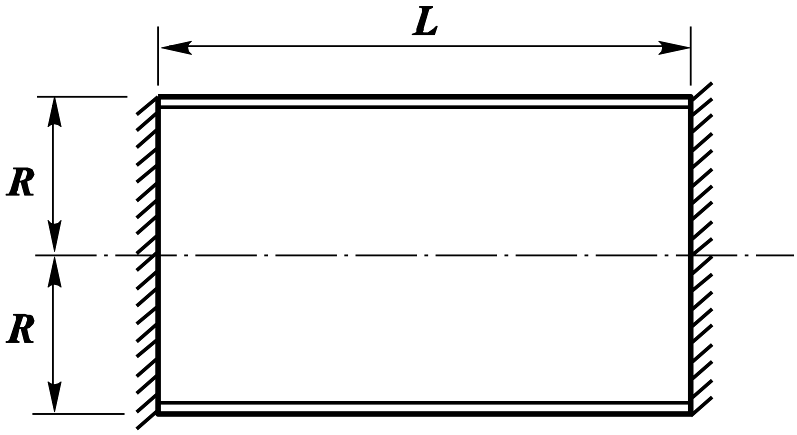
$$K1=\frac{1}{2}\cdot(1-\nu)\cdot\left[\left(\frac{(m-1)\cdot\pi\cdot R}{L}\right)^2+n^2\right]^2+\frac{1}{2}\cdot\left(3-\nu-2\cdot\nu^2\right)\cdot\left(\frac{(m-1)\cdot\pi\cdot R}{L}\right)^2+\frac{1}{2}\cdot(1-\nu)\cdot n^2+$$
$$\frac{h^2}{12\cdot R^2}\cdot\left\{\frac{1}{2}\cdot(3-\nu)\cdot\left[\left(\frac{(m-1)\cdot\pi\cdot R}{L}\right)^2+n^2\right]^3+2\cdot(1-\nu)\cdot\left(\frac{(m-1)\cdot\pi\cdot R}{L}\right)^4-\left(2-\nu^2\right)\cdot\left(\frac{(m-1)\cdot\pi\cdot R}{L}\right)^2\cdot n^2-\frac{1}{2}(3+\nu)\cdot n^4+\right.$$
$$\left.2\cdot(1-\nu)\cdot\left(\frac{(m-1)\cdot\pi\cdot R}{L}\right)^2+n^2\right\}+\frac{h^4}{144\cdot R^4}\cdot\left\{2\cdot(1-\nu)\cdot\left(\frac{(m-1)\cdot\pi\cdot R}{L}\right)^6+\left(1-\nu^2\right)\cdot\left(\frac{(m-1)\cdot\pi\cdot R}{L}\right)^4\cdot n^2\right\}$$

$$K0=-\frac{1}{2}\cdot(1-\nu)\cdot\left(1-\nu^2\right)\cdot\left(\frac{(m-1)\cdot\pi\cdot R}{L}\right)^4-\frac{1}{2}\cdot(1-\nu)\cdot\frac{h^2}{12\cdot R^2}\cdot\left\{\left[\left(\frac{(m-1)\cdot\pi\cdot R}{L}\right)^2+n^2\right]^4-2\cdot\left(4-\nu^2\right)\cdot\left(\frac{(m-1)\cdot\pi\cdot R}{L}\right)^4\cdot n^2-\right.$$
$$\left.8\cdot\left(\frac{(m-1)\cdot\pi\cdot R}{L}\right)^2\cdot n^4-2\cdot n^6+4\cdot\left(1-\nu^2\right)\cdot\left(\frac{(m-1)\cdot\pi\cdot R}{L}\right)^4+4\cdot\left(\frac{(m-1)\cdot\pi\cdot R}{L}\right)^2\cdot n^2+n^4\right\}-$$
$$\frac{1}{2}\cdot(1-\nu)\cdot\frac{h^4}{144\cdot R^4}\cdot\left\{4\cdot\left(\frac{(m-1)\cdot\pi\cdot R}{L}\right)^8-4\cdot\left(\frac{(m-1)\cdot\pi\cdot R}{L}\right)^6\cdot n^2+\left(1-\nu^2\right)\cdot\left(\frac{(m-1)\cdot\pi\cdot R}{L}\right)^4\cdot n^4\right\}$$

$m=2,3,4\ldots$ - number of nodal lines in the circumferential direction, taking into account the lines along the end support contours,

$n=0,1,2,\ldots$ - number of pairs of nodal lines in the meridian direction when each pair is located on one diameter.

Natural Oscillations of a Clamped Circular Cylindrical Shell



Objective: Modal analysis of a clamped circular cylindrical shell.

Initial data file: **5.8_C.SPR**

Problem formulation: Determine the natural oscillation modes and frequencies ω of the clamped circular cylindrical shell with the density of the material ρ.

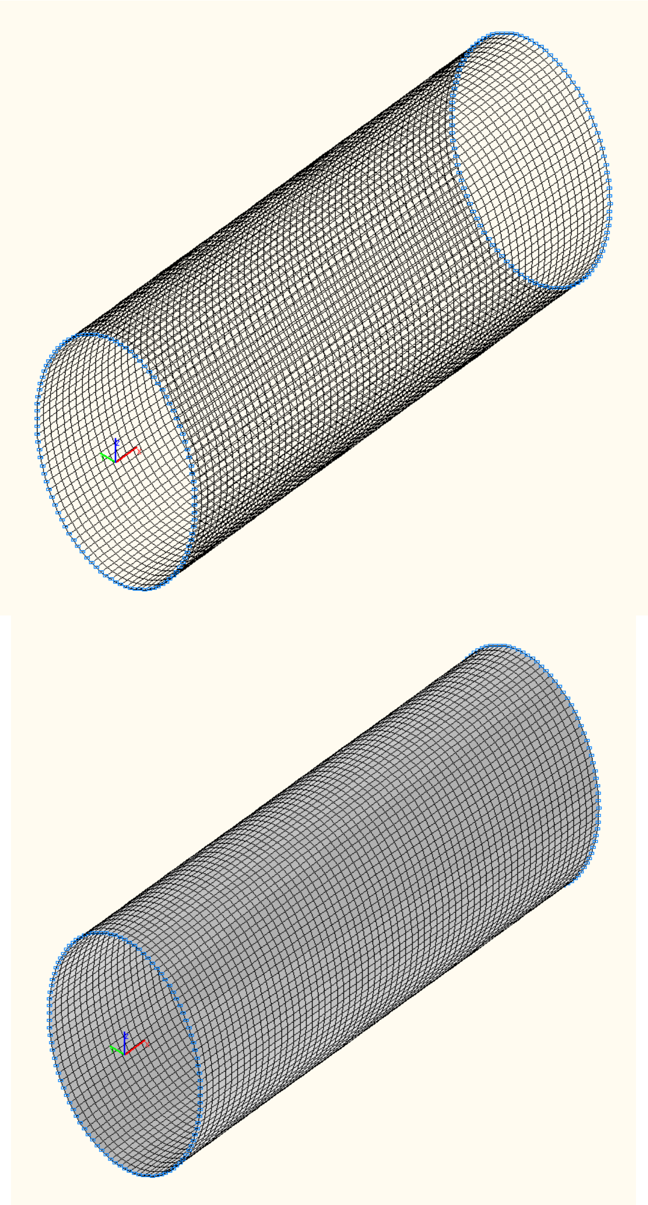
References: I.A. Birger, Ya.G. Panovko, Strength, Stability, Vibrations, Handbook in three volumes, Volume 3, Moscow, Mechanical engineering, 1968, p. 437.
V. S. Gontkevich, Natural Vibrations of Orthotropic Cylindrical Shells, Proceedings of the Conference on the Theory of Shells and Plates, Kazan, KFAN, 1961.

Initial data:

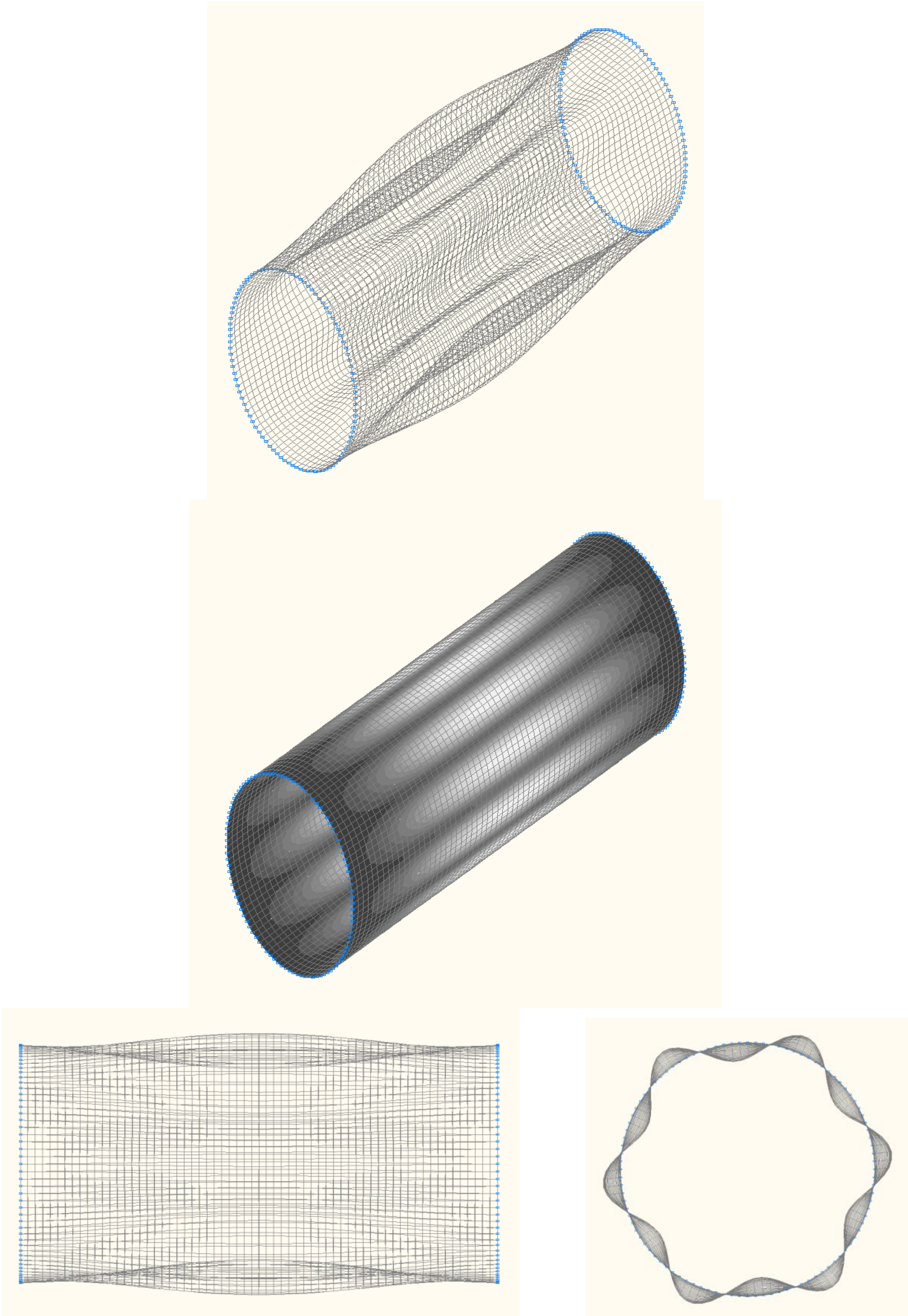
$E = 1.96 \cdot 10^8$ kPa	- elastic modulus;
$\nu = 0.3$	- Poisson's ratio;
$\rho = 7.70$ t/m^3	- density of the material;
$h = 0.25 \cdot 10^{-3}$ m	- thickness of the cylindrical shell;
$R = 0.076$ m	- radius of the midsurface of the cylindrical shell;
$L = 0.305$ m	- length of the cylindrical shell.

Finite element model: Design model – general type system, 6400 four-node shell elements of type 50. The spacing of the finite element mesh in the meridian direction is $4.765625 \cdot 10^{-3}$ m (64 elements) and in the circumferential is 3.6º (100 elements). Boundary conditions of the simply supported edges are provided by imposing constraints in the directions of all linear and angular displacements (degrees of freedom X, Y, Z, UX, UY, UZ). The distributed mass is specified by transforming the static load from the self-weight of the cylindrical shell: ow = $\gamma \cdot h$, where $\gamma = \rho \cdot g = 75.537$ kN/m^3. Number of nodes in the design model – 6500. The determination of the natural oscillation modes and natural frequencies is performed by the method of subspace iteration. The matrix of concentrated masses is used in the calculation.

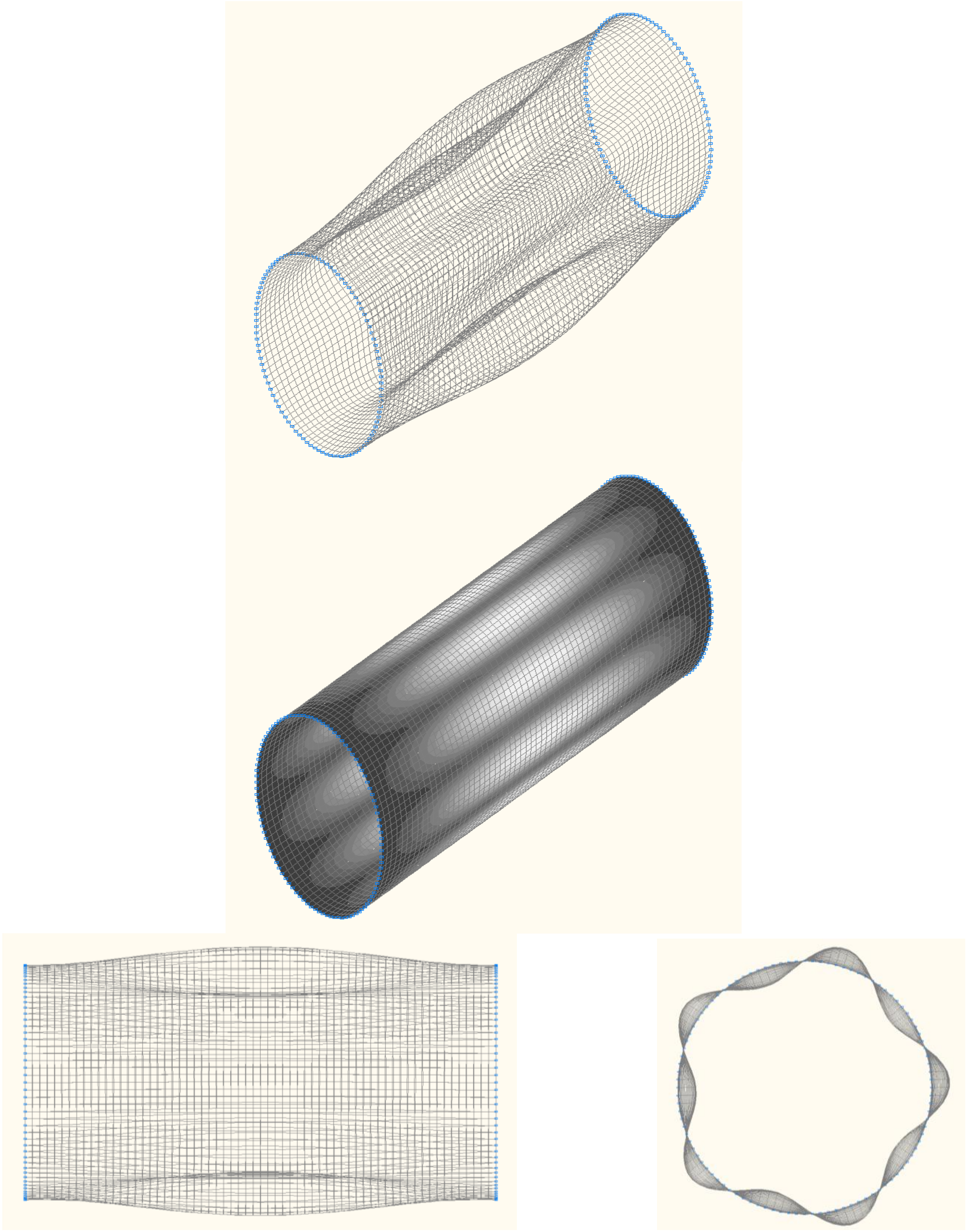
Results in SCAD



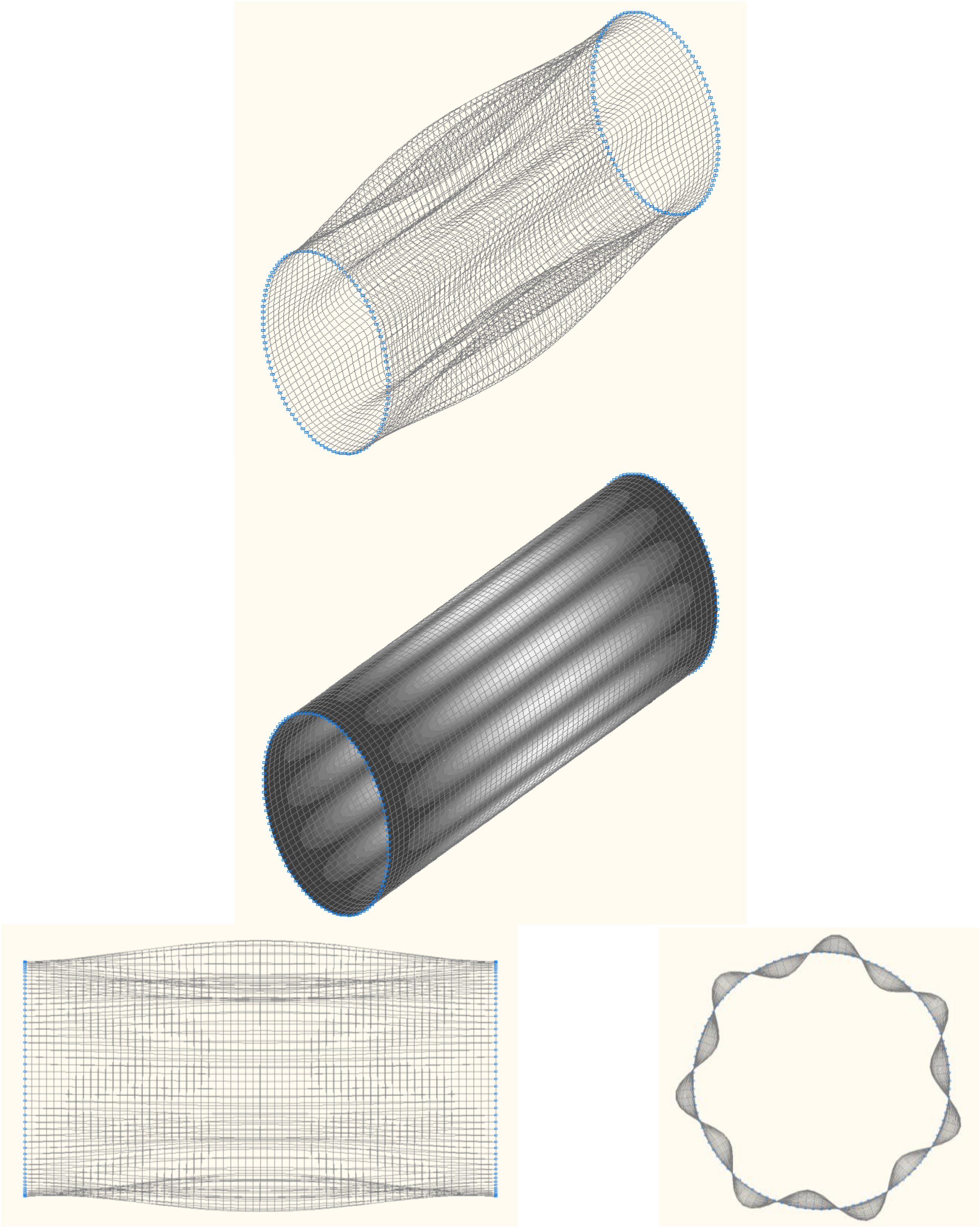
Design model



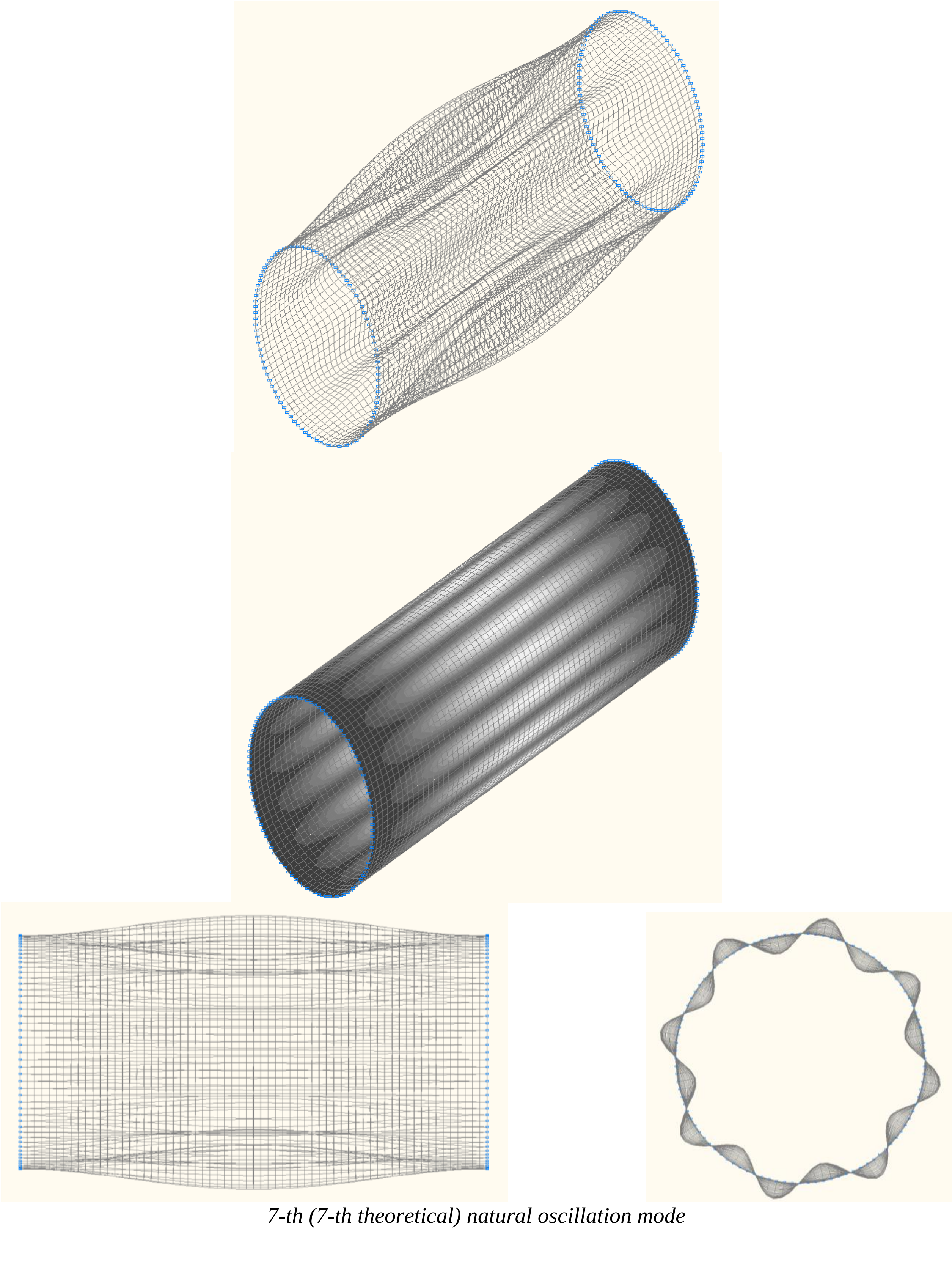
1-st (1-st theoretical) natural oscillation mode



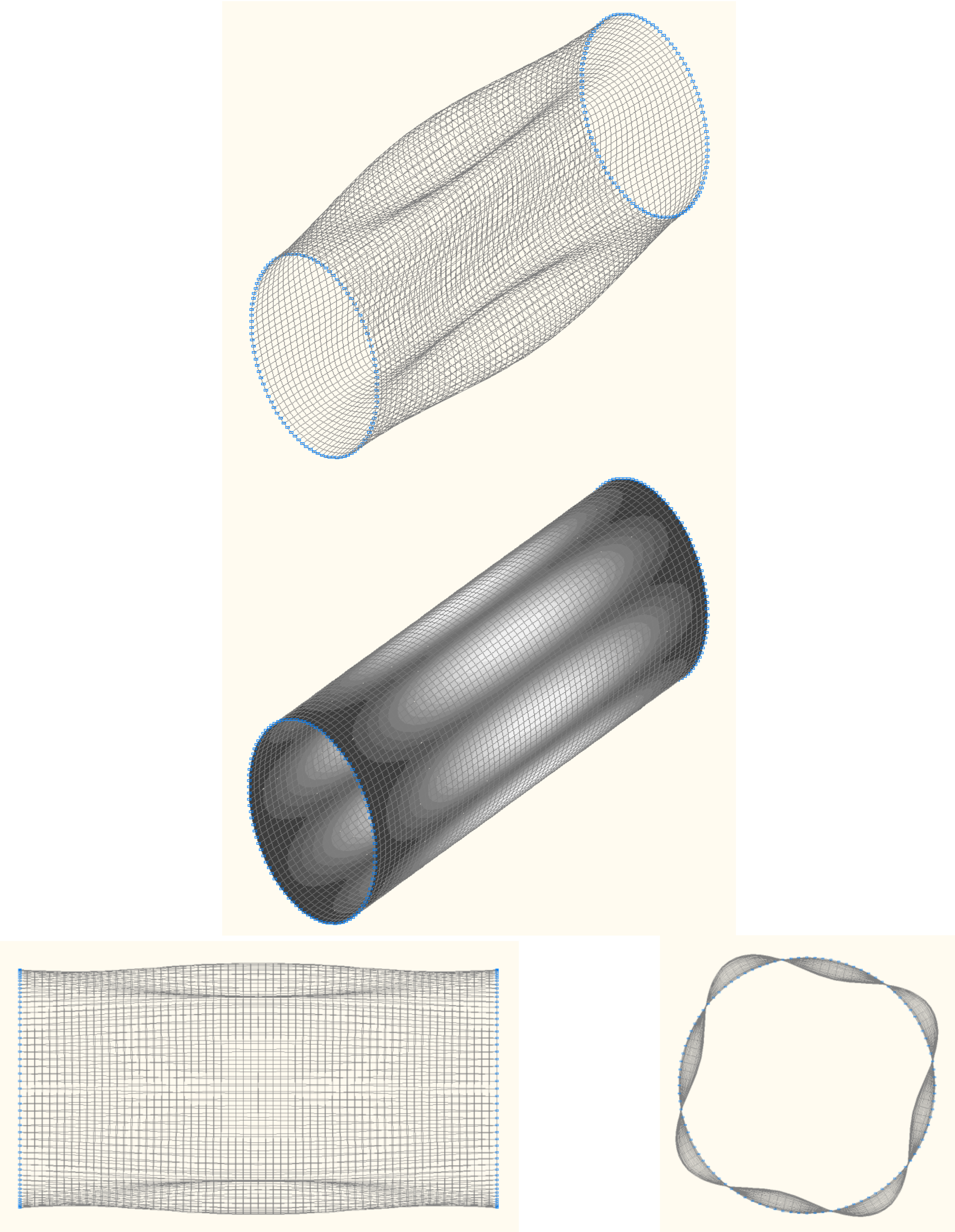
3-rd (3-rd theoretical) natural oscillation mode



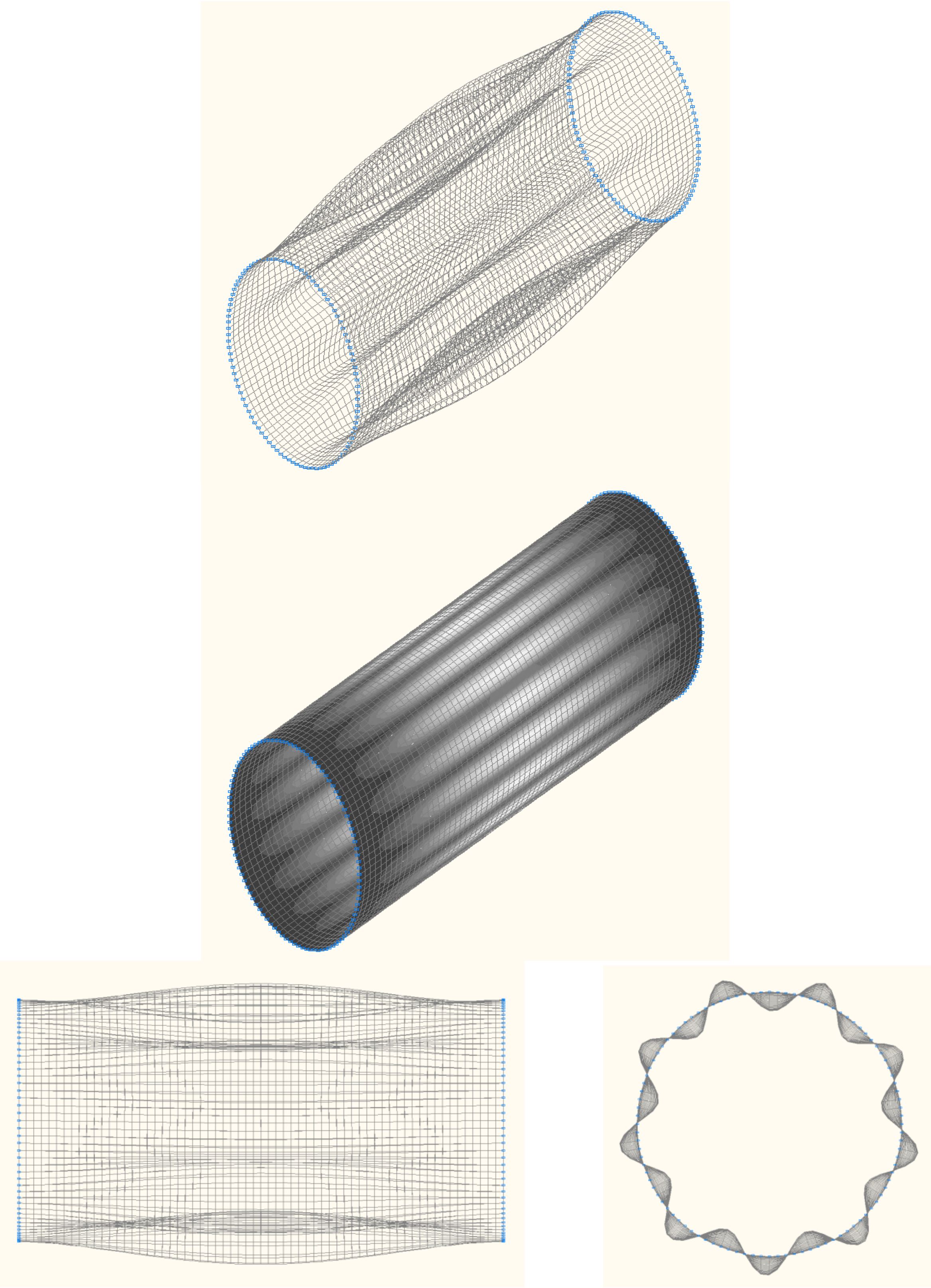
5-th (5-th theoretical) natural oscillation mode



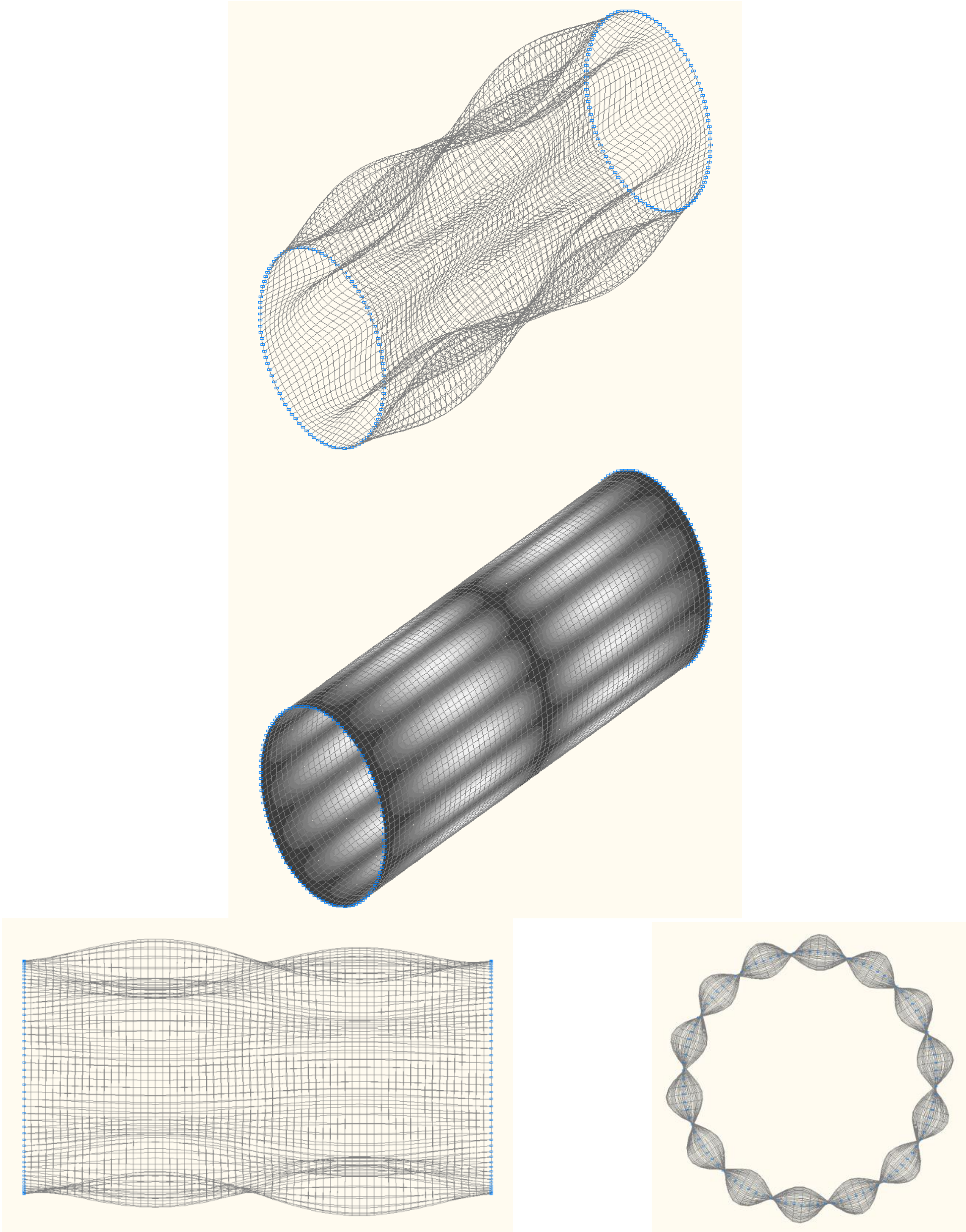
7-th (7-th theoretical) natural oscillation mode



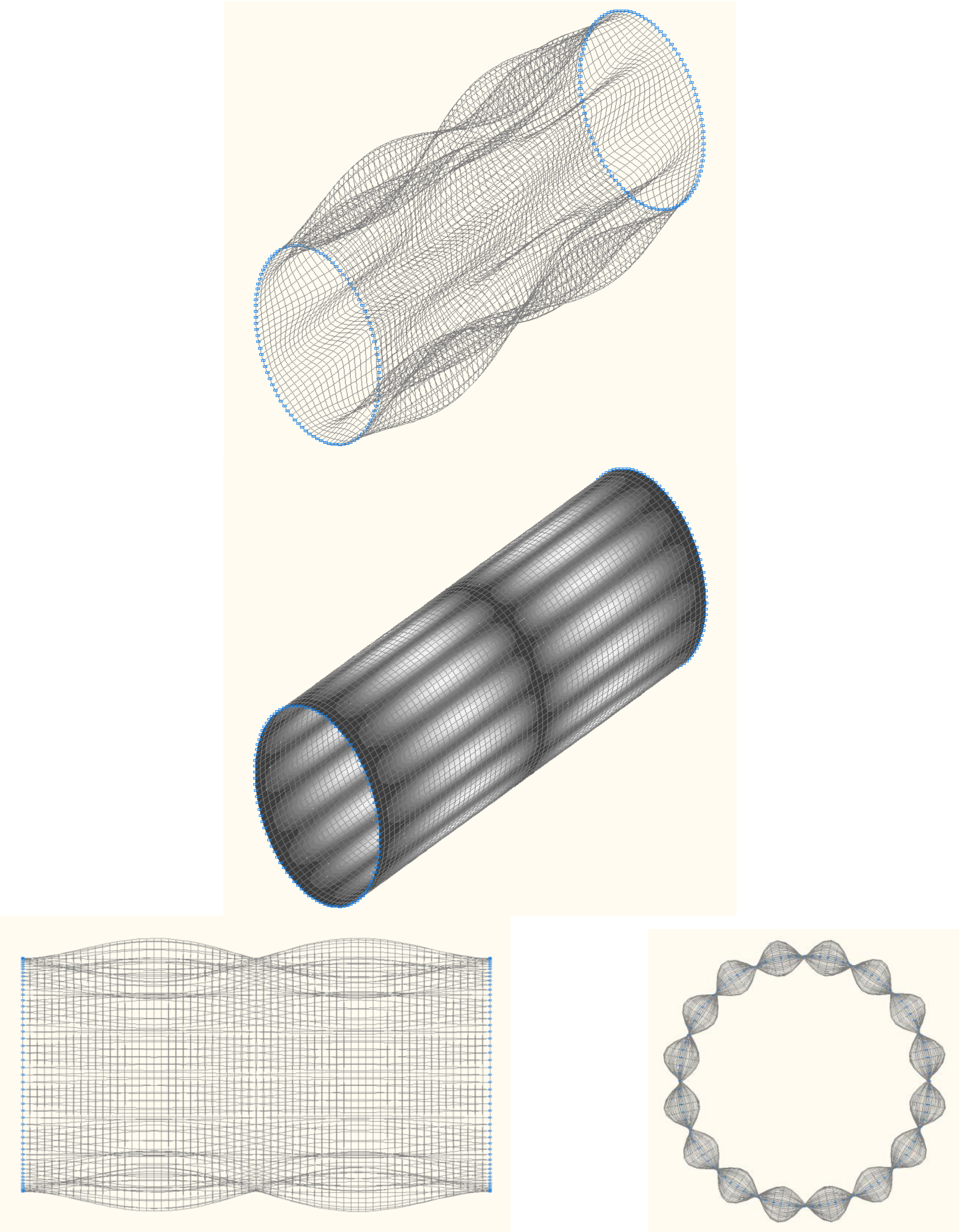
9-th (9-th theoretical) natural oscillation mode



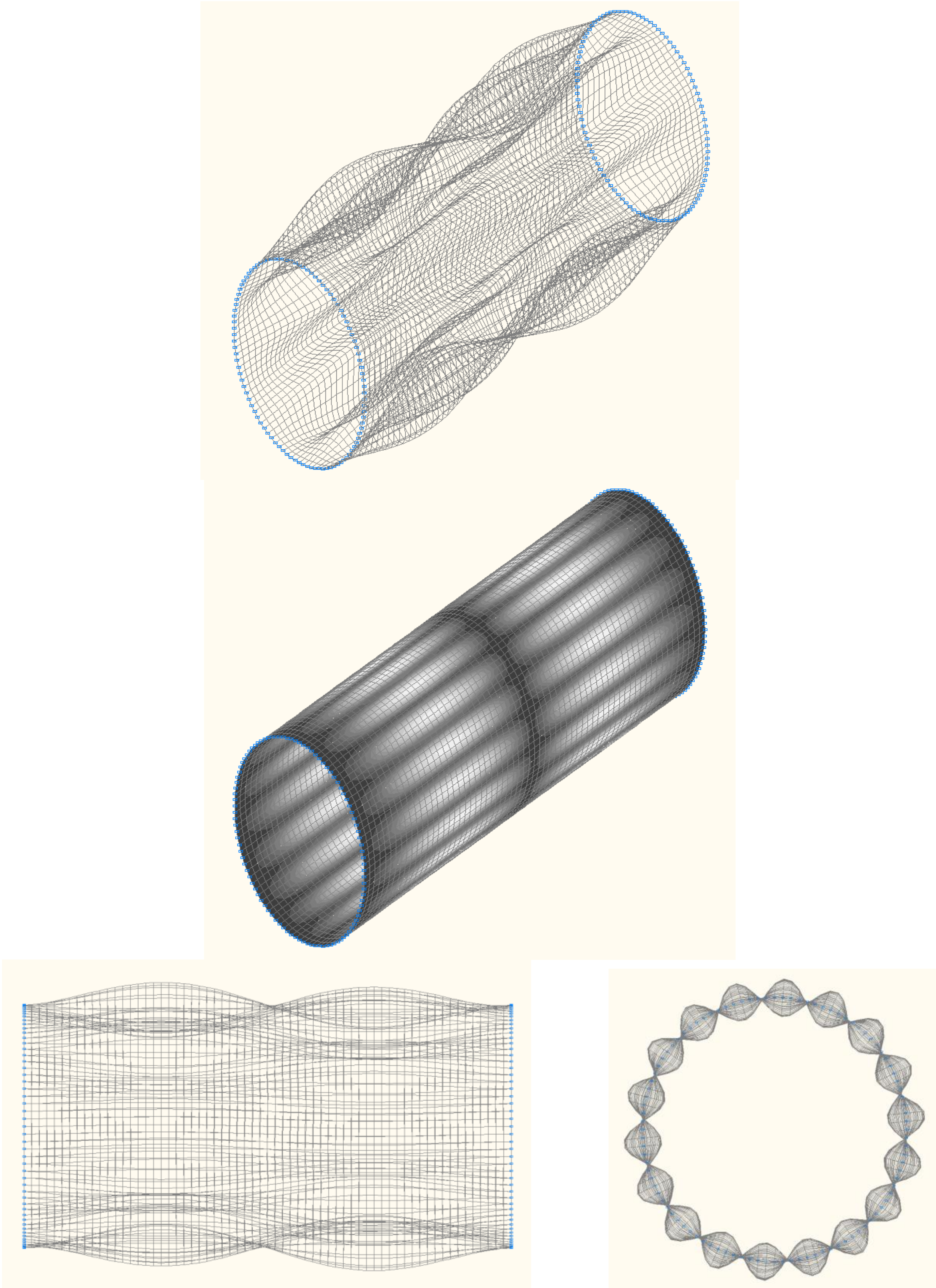
11-th (11-th theoretical) natural oscillation mode



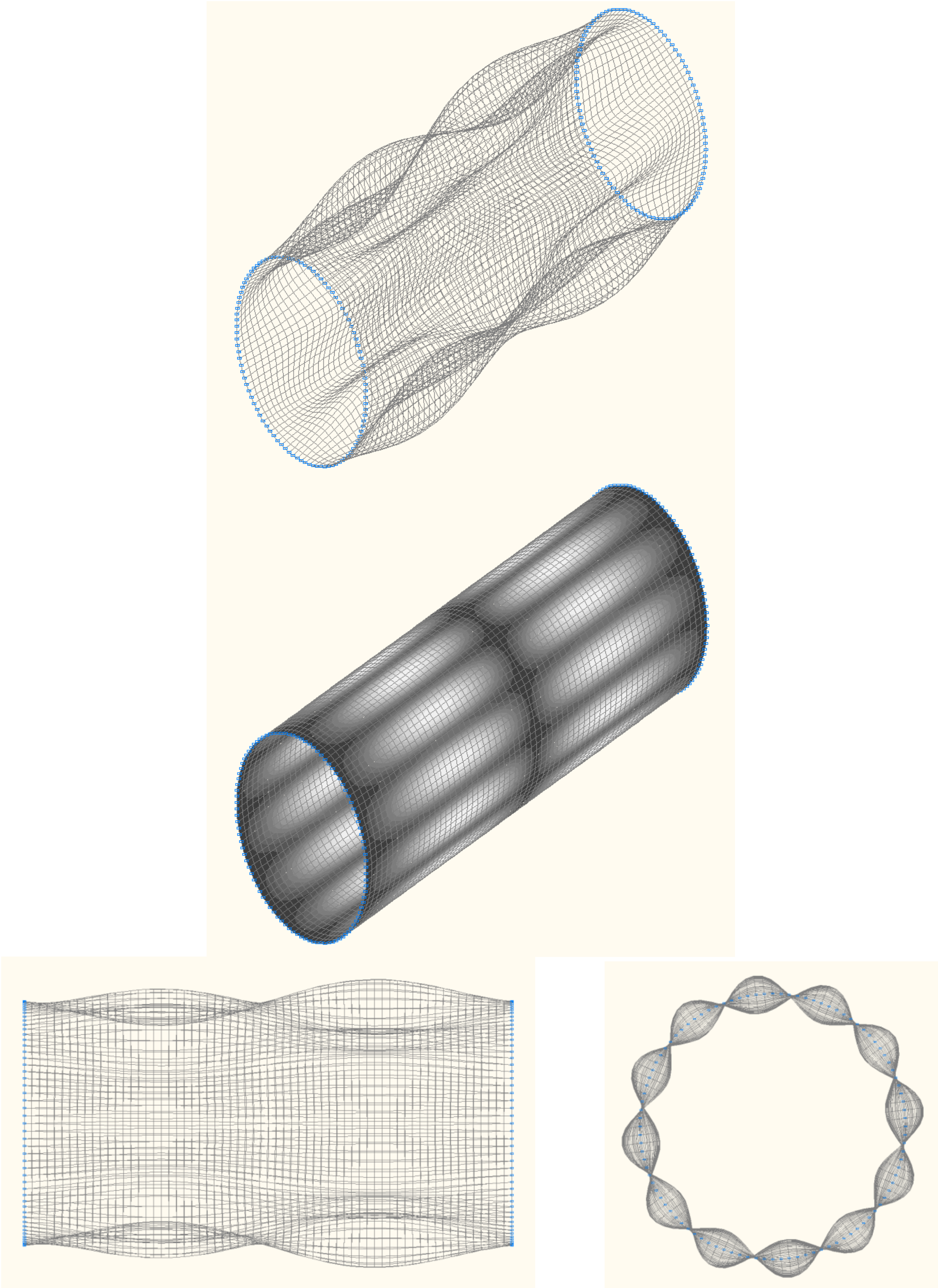
13-th (13-th theoretical) natural oscillation mode



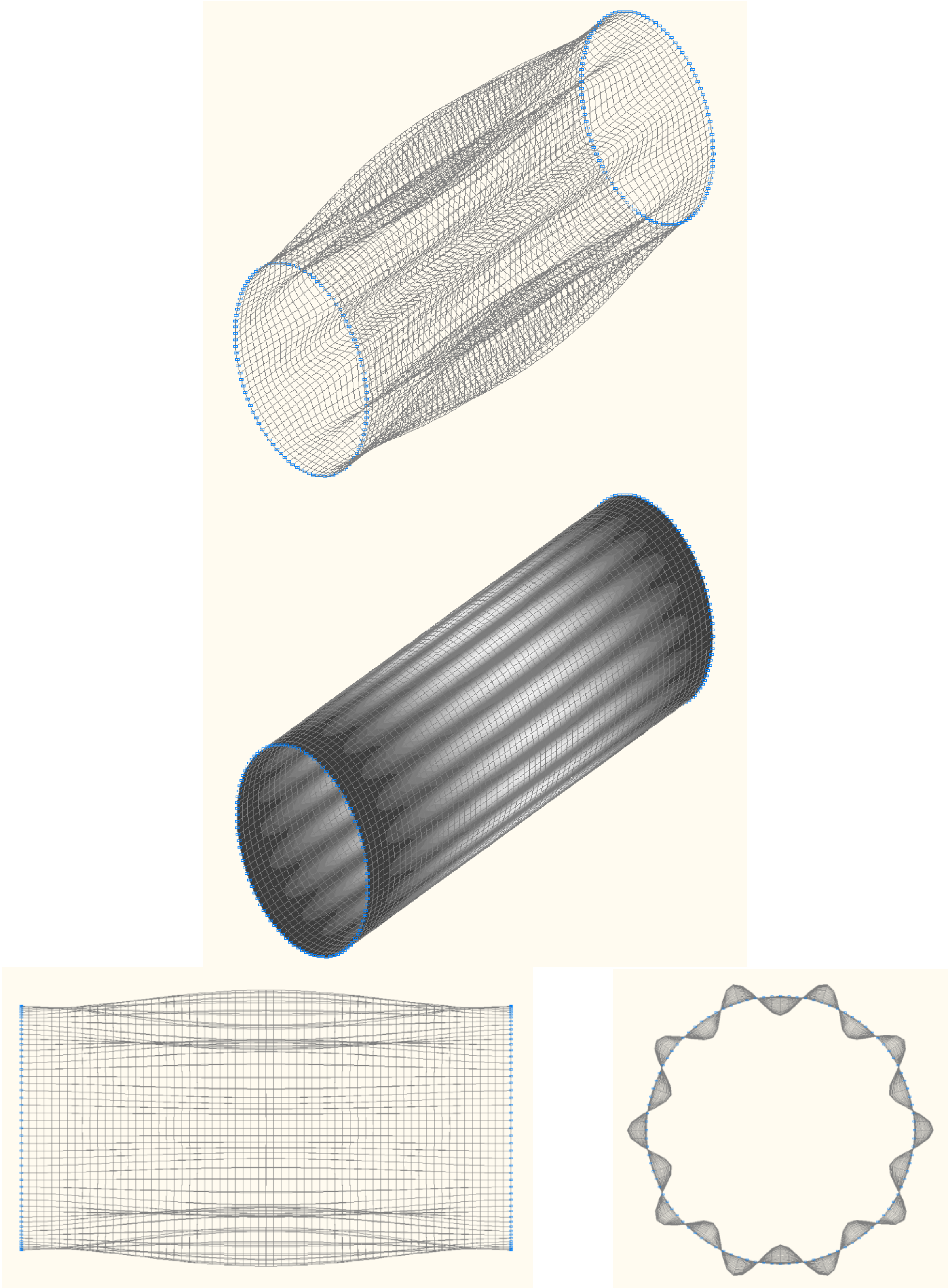
15-th (15-th theoretical) natural oscillation mode



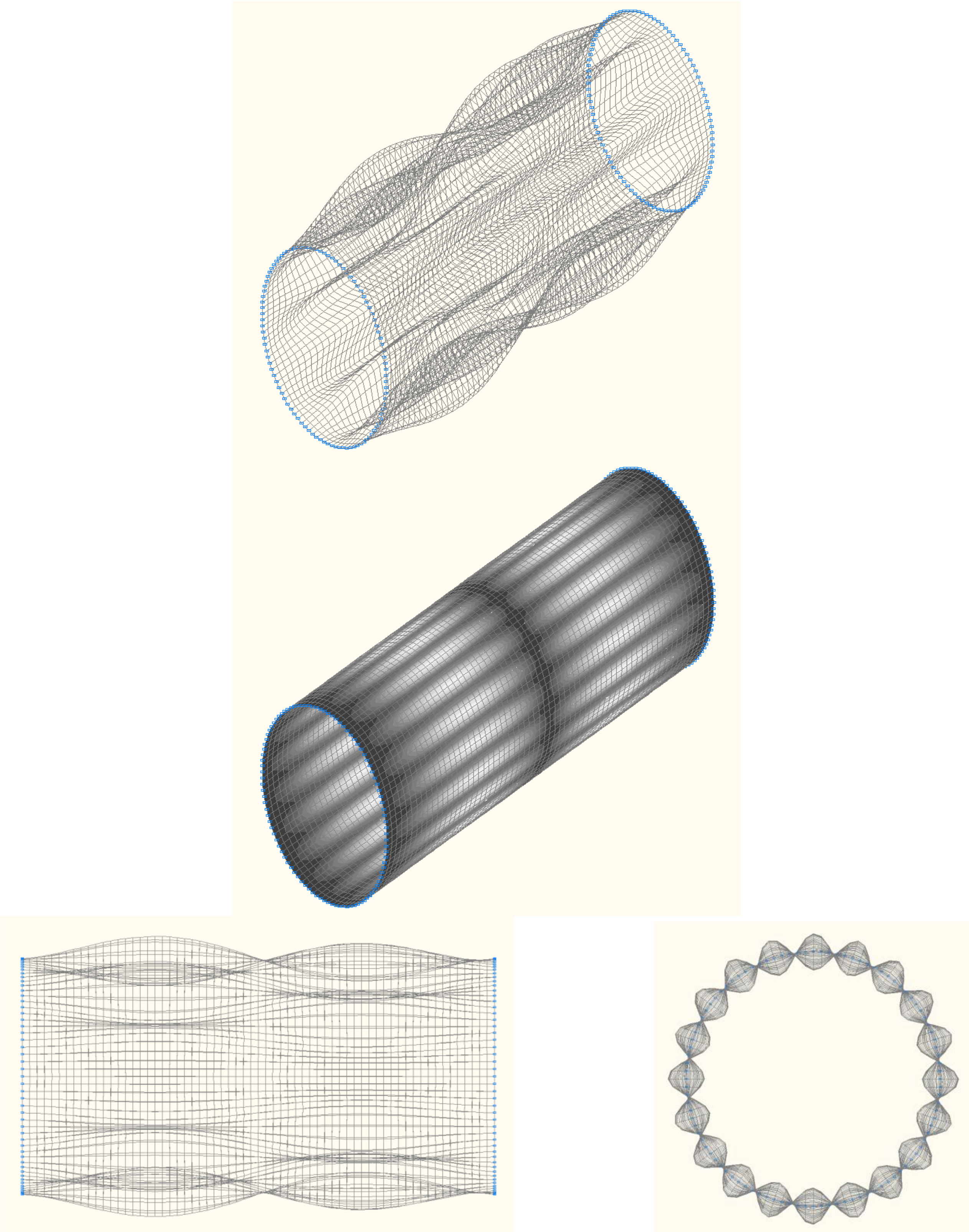
17-th (17-th theoretical) natural oscillation mode



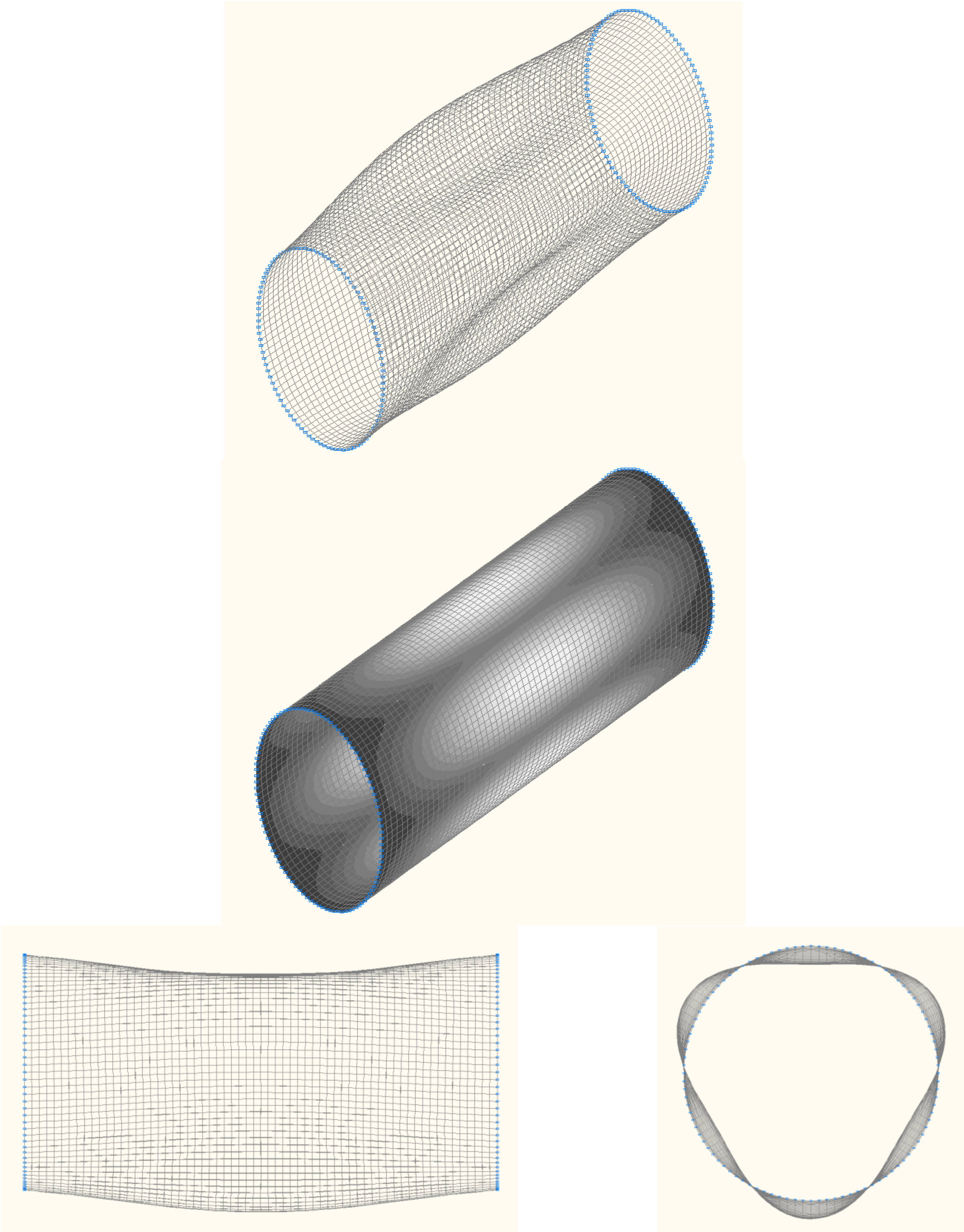
19-th (19-th theoretical) natural oscillation mode



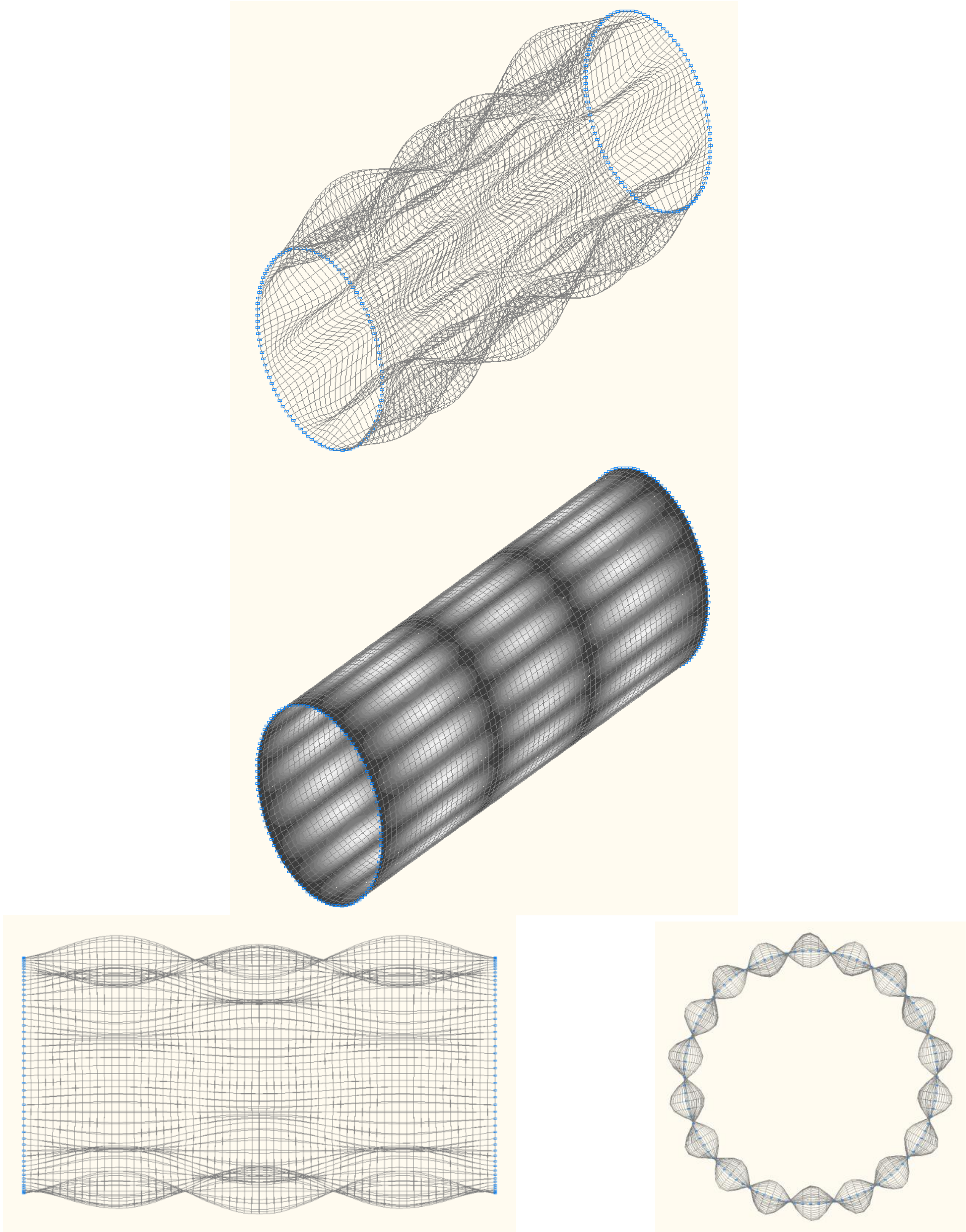
21-st (21-st theoretical) natural oscillation mode



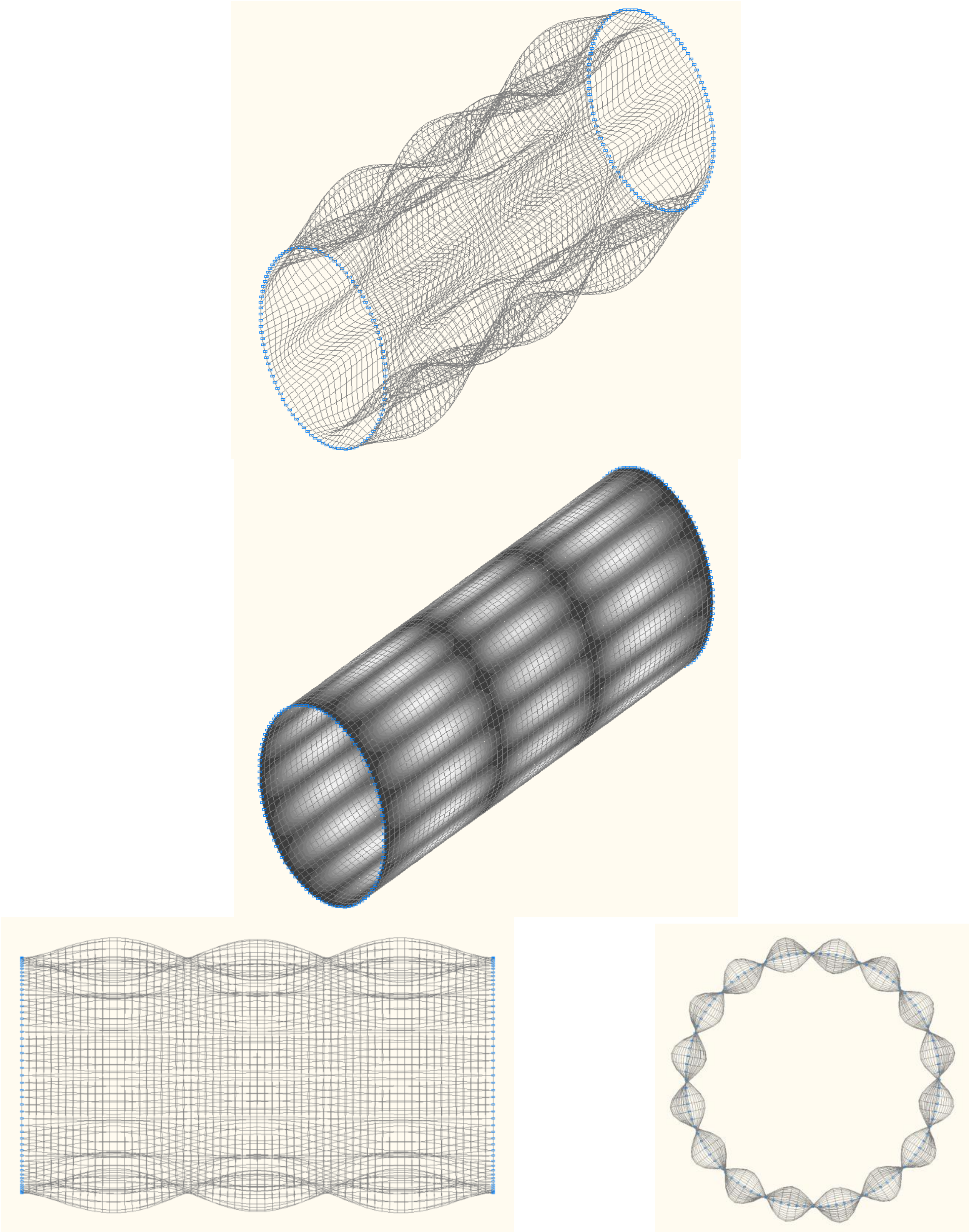
23-rd (25-th theoretical) natural oscillation mode



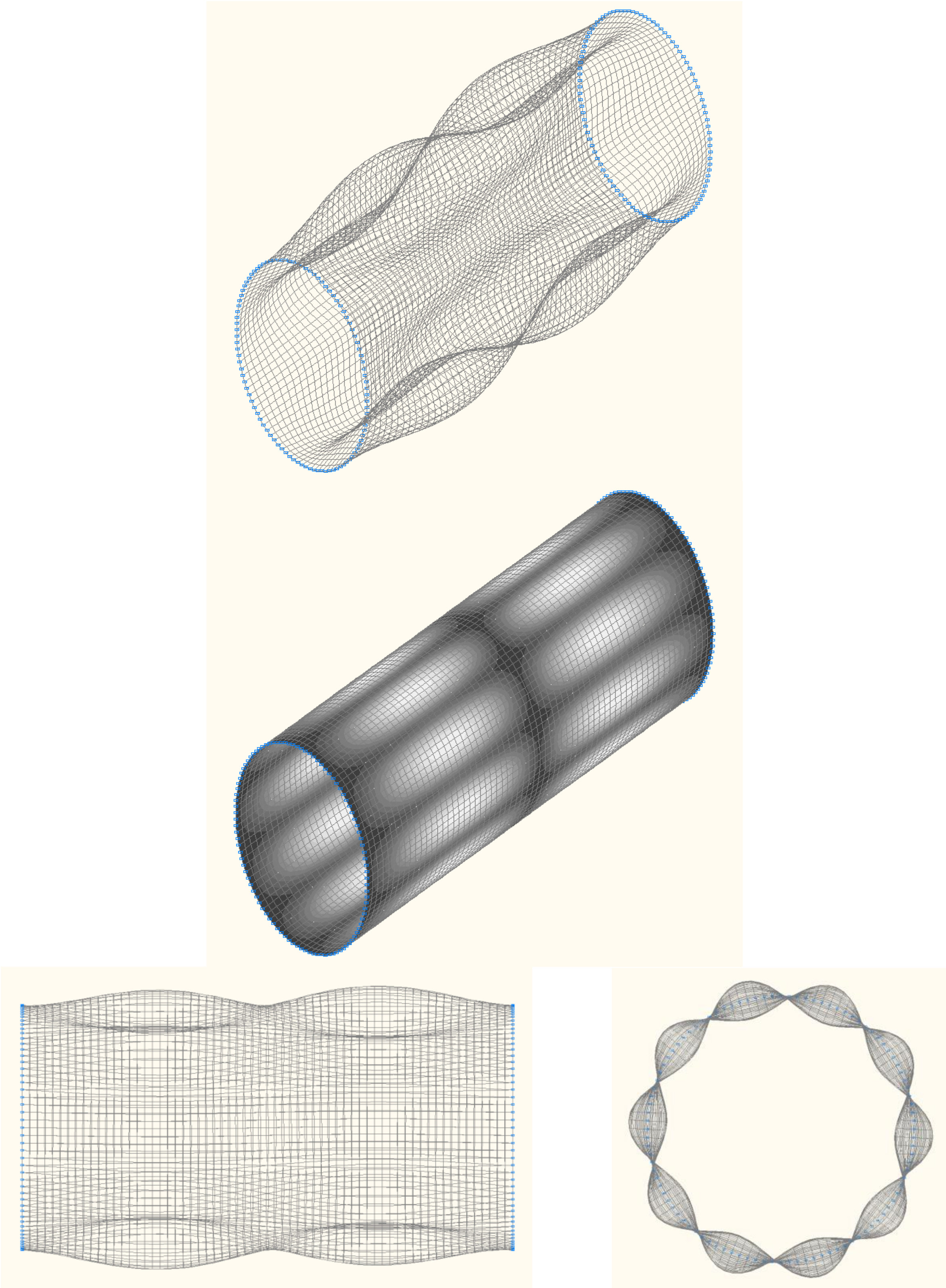
25-th (23-rd theoretical) natural oscillation mode



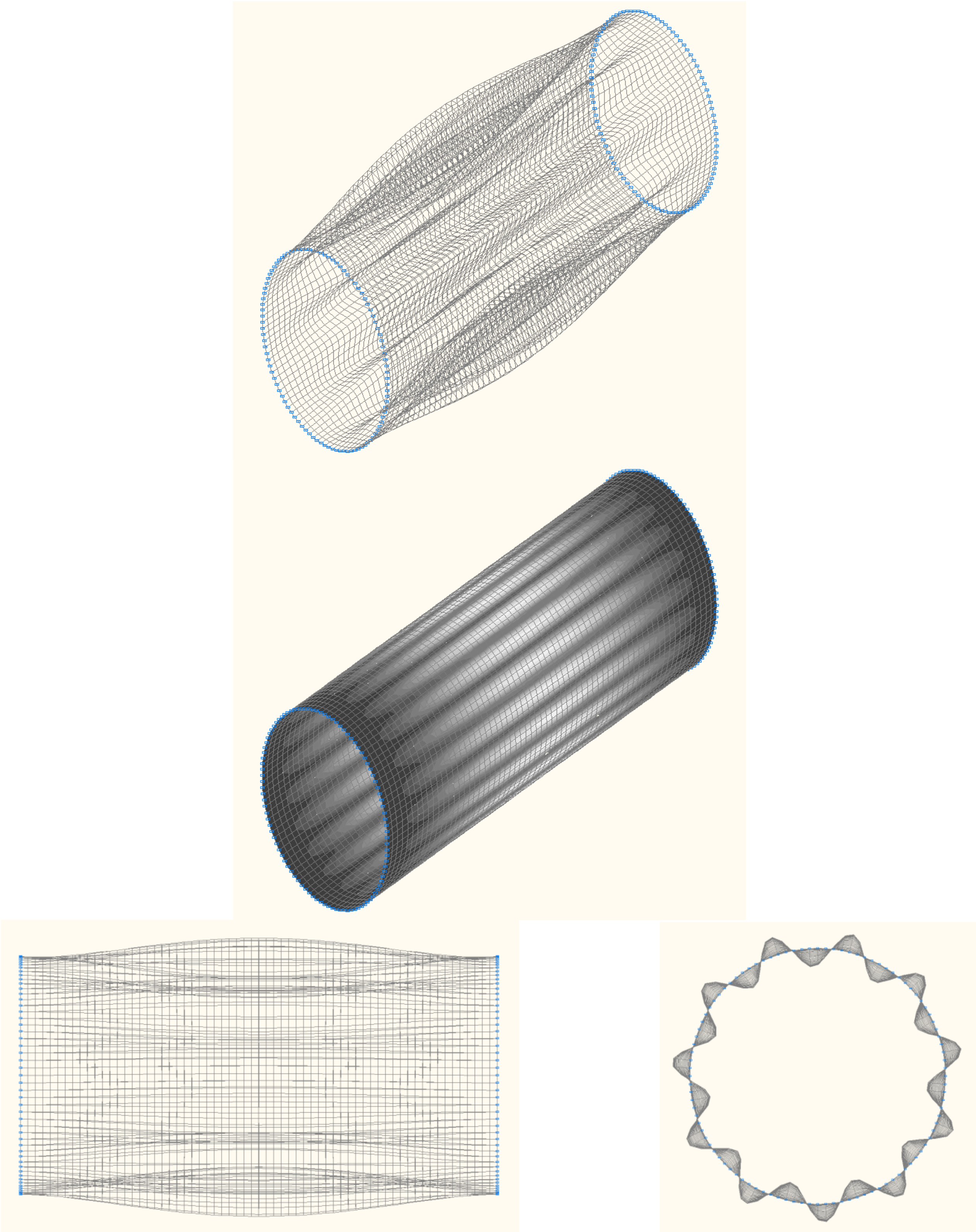
27-th (27-th theoretical) natural oscillation mode



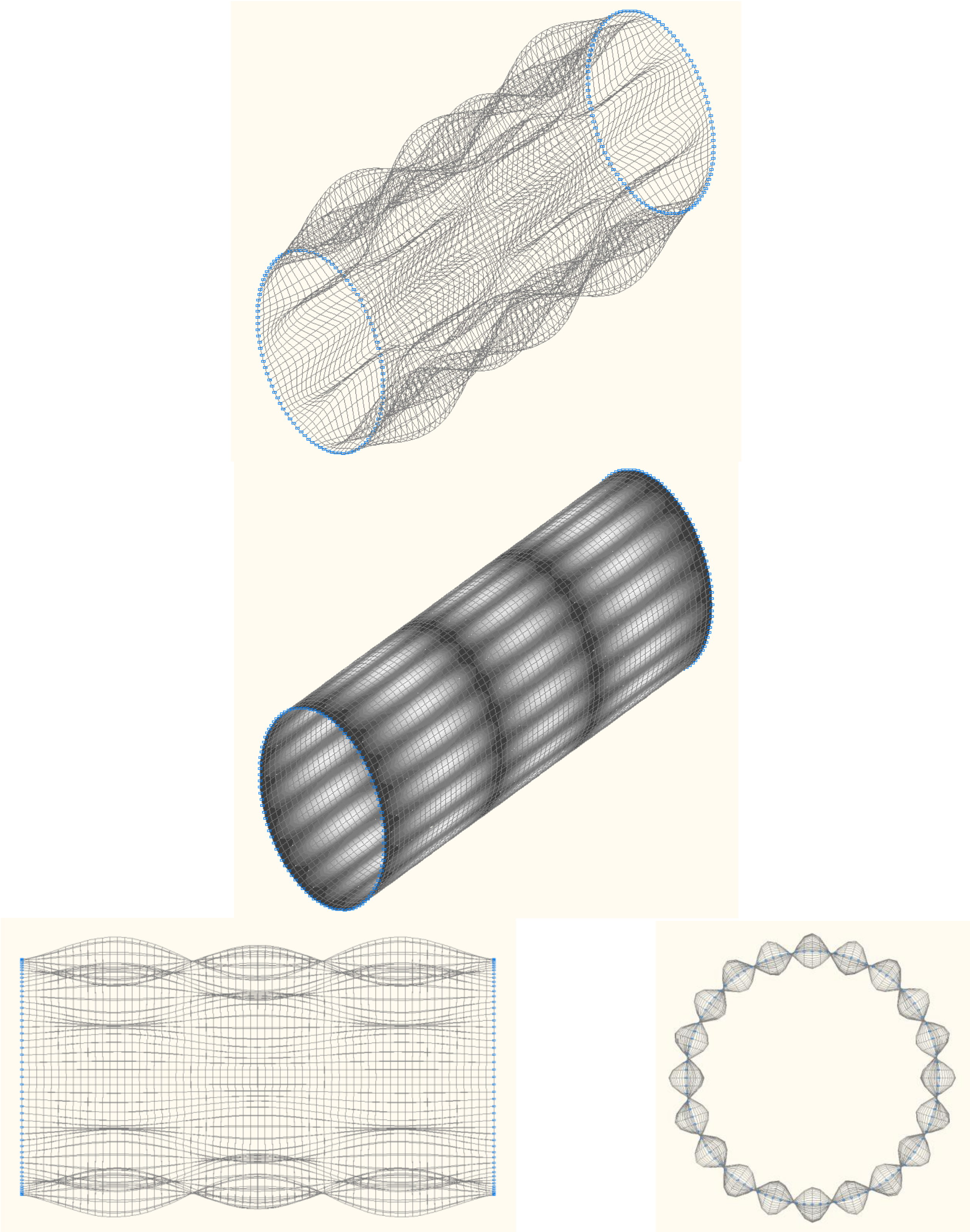
29-th (31-st theoretical) natural oscillation mode



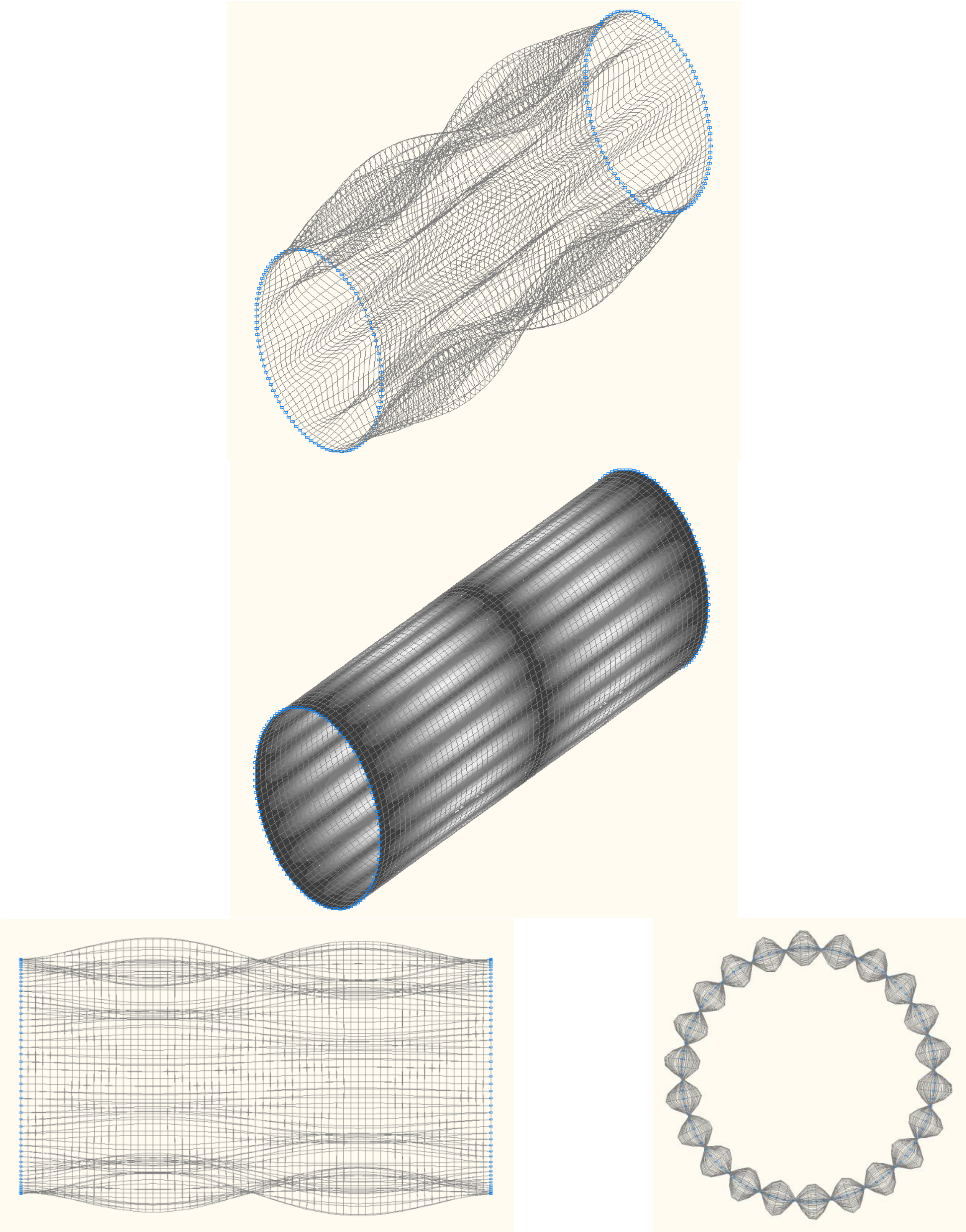
31-st (29-th theoretical) natural oscillation mode



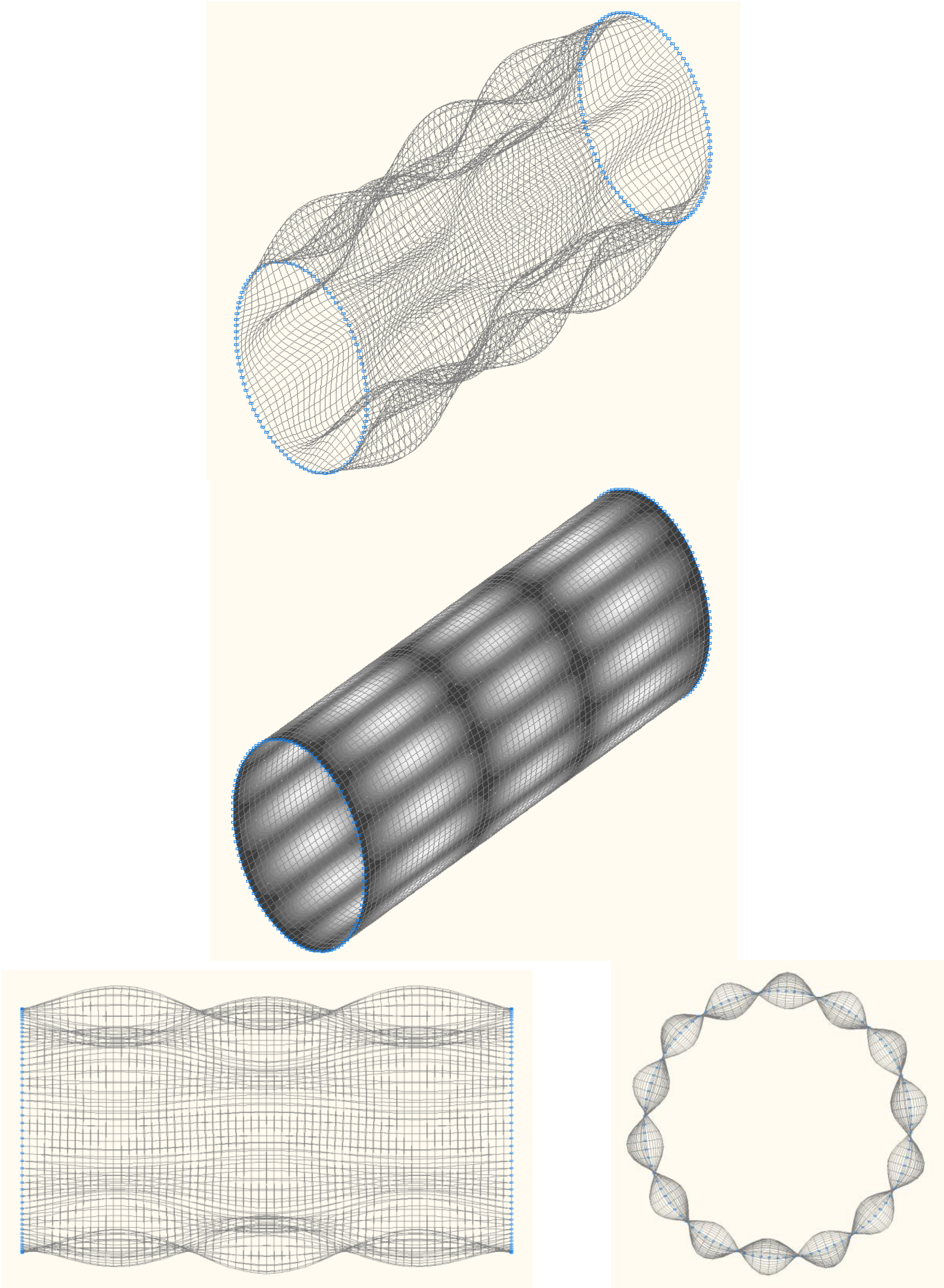
33-rd (33-rd theoretical) natural oscillation mode



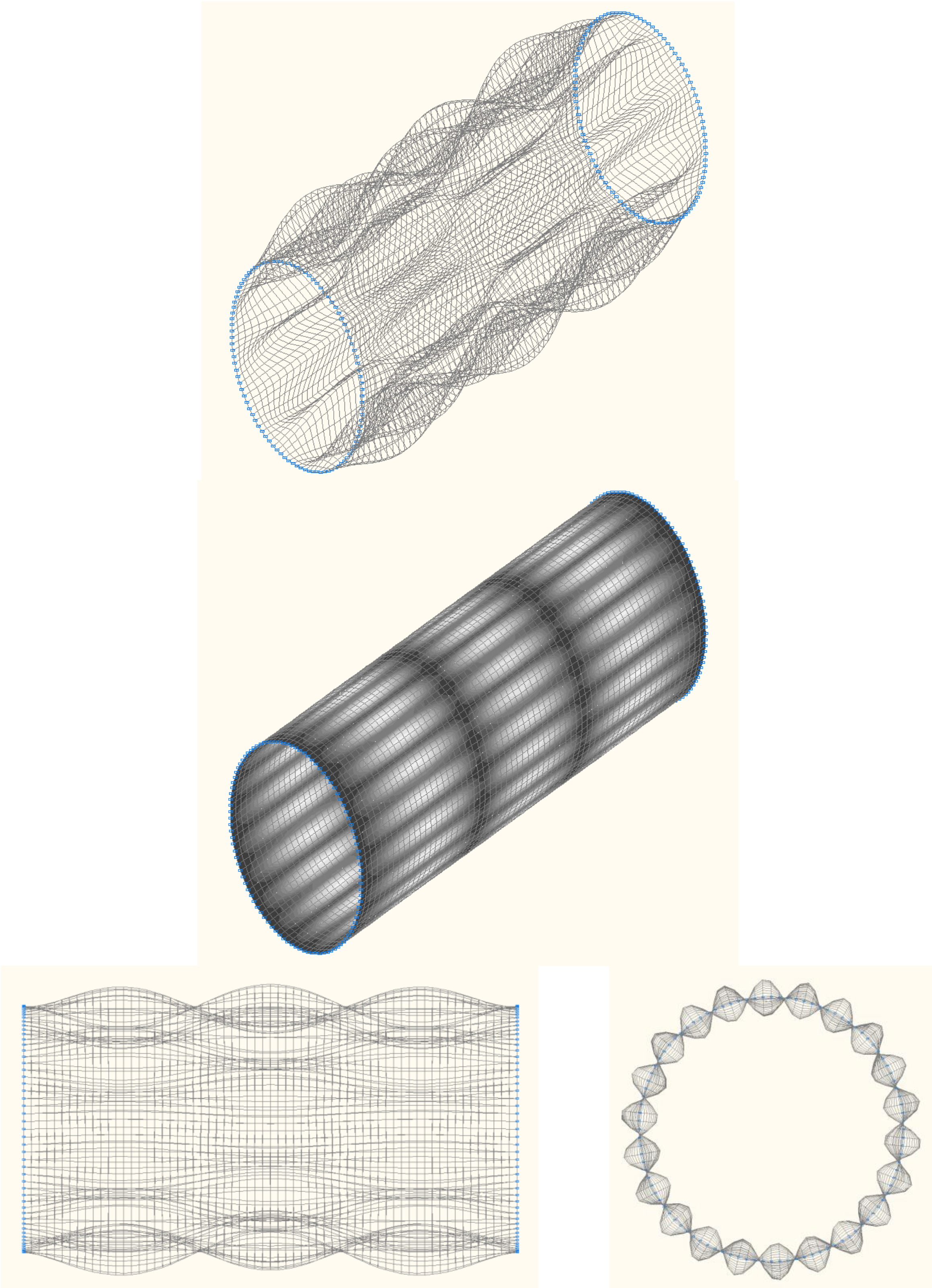
35-th (35-th theoretical) natural oscillation mode



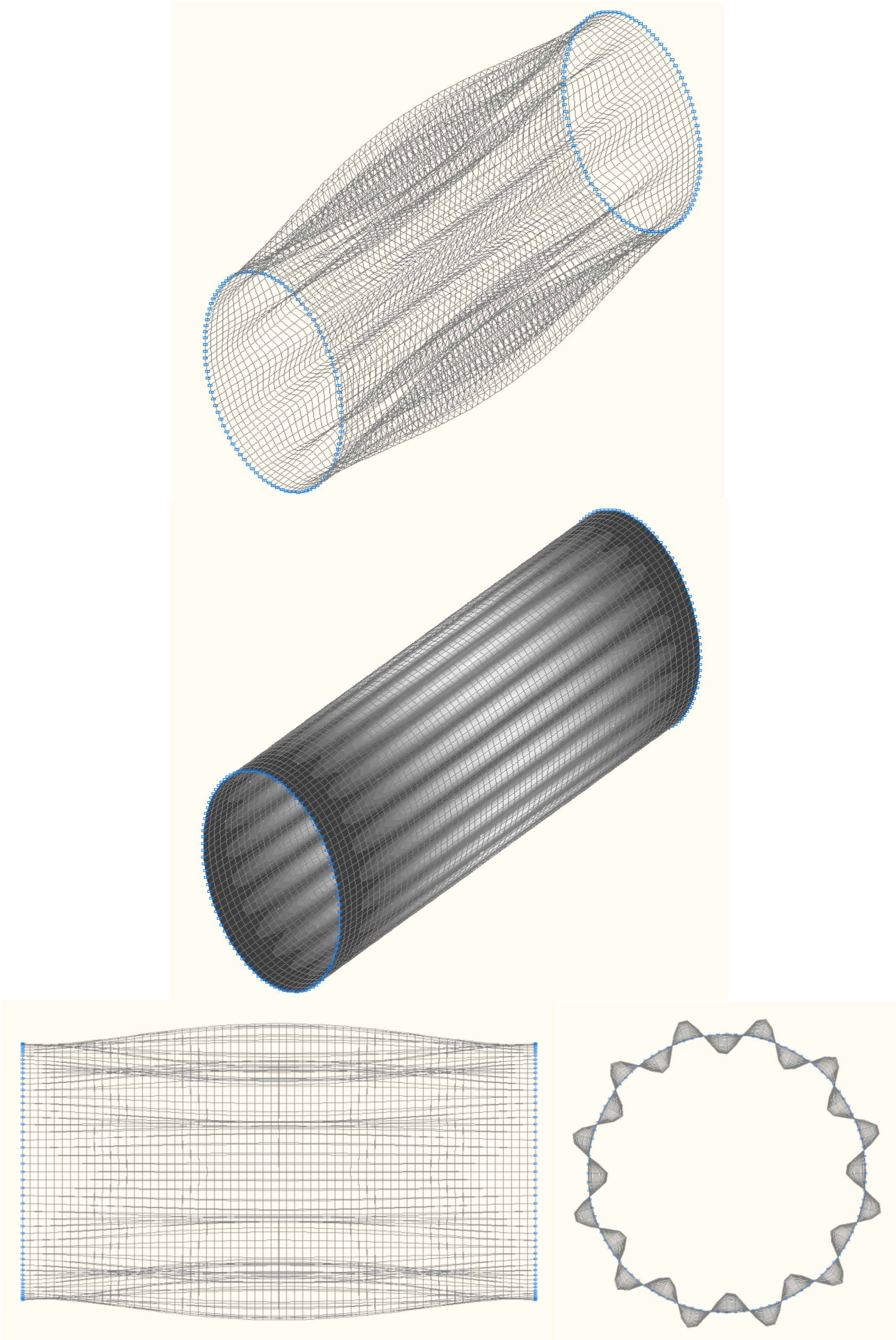
37-th (37-th theoretical) natural oscillation mode



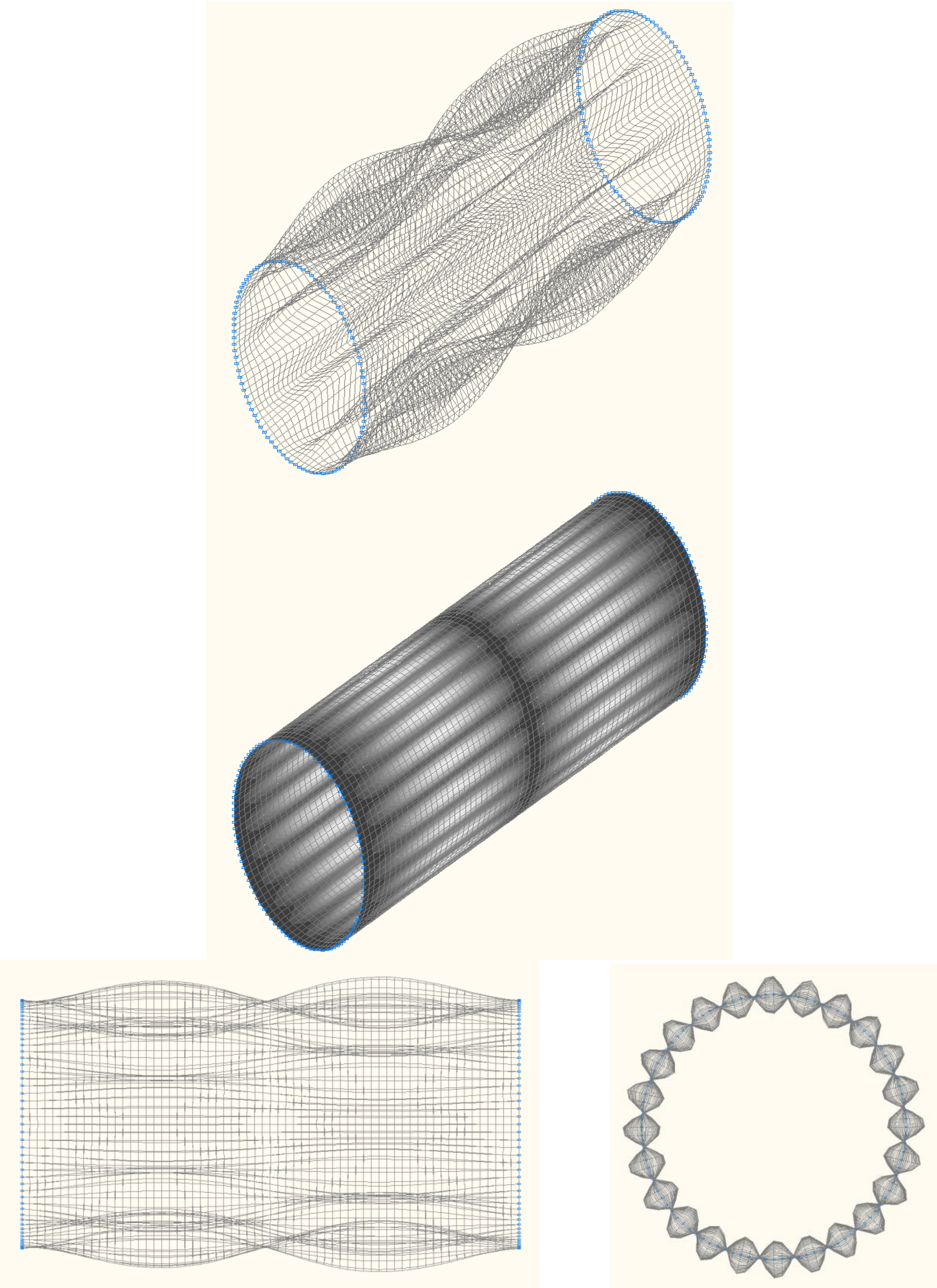
39-th (39-th theoretical) natural oscillation mode



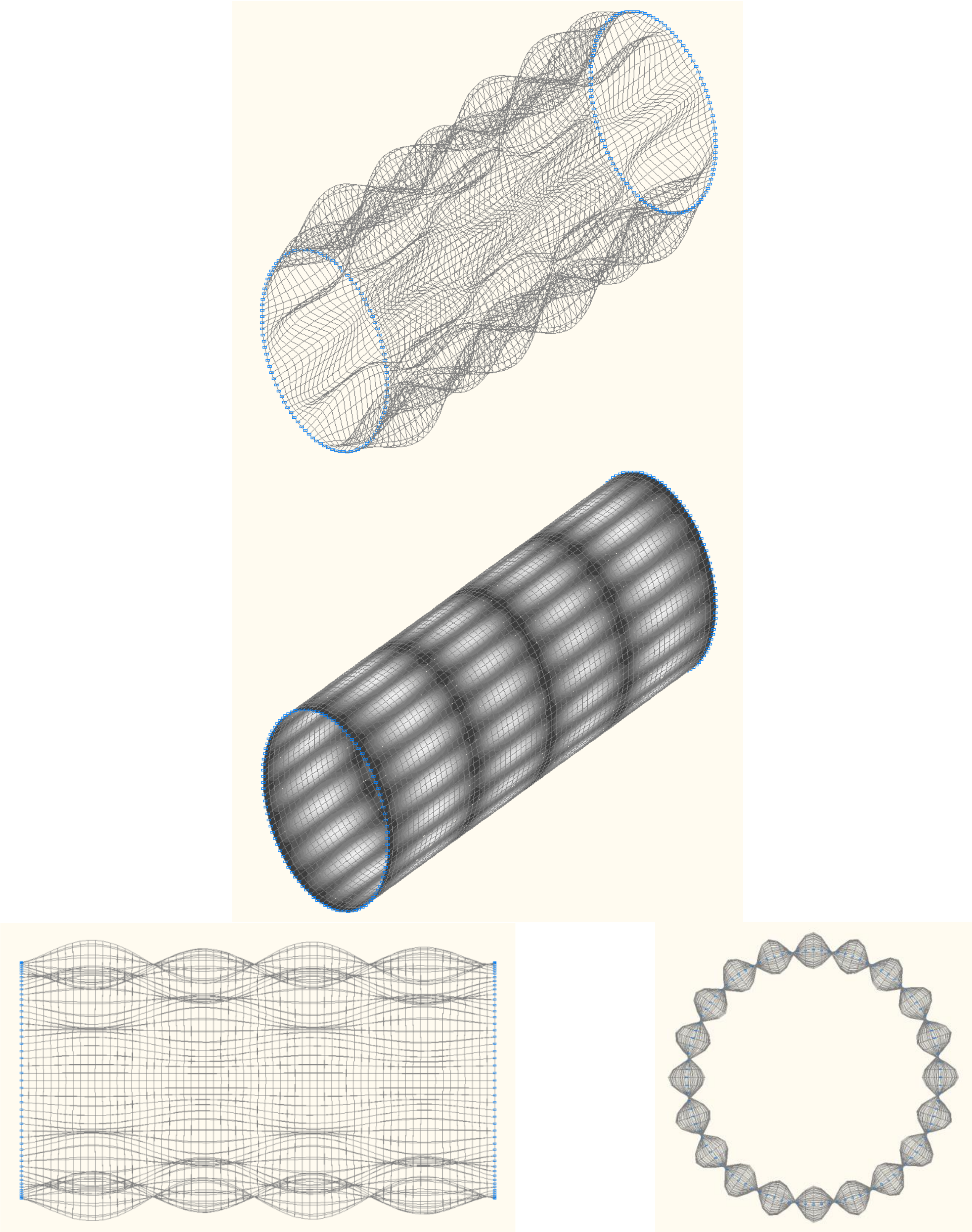
41-st (41-st theoretical) natural oscillation mode



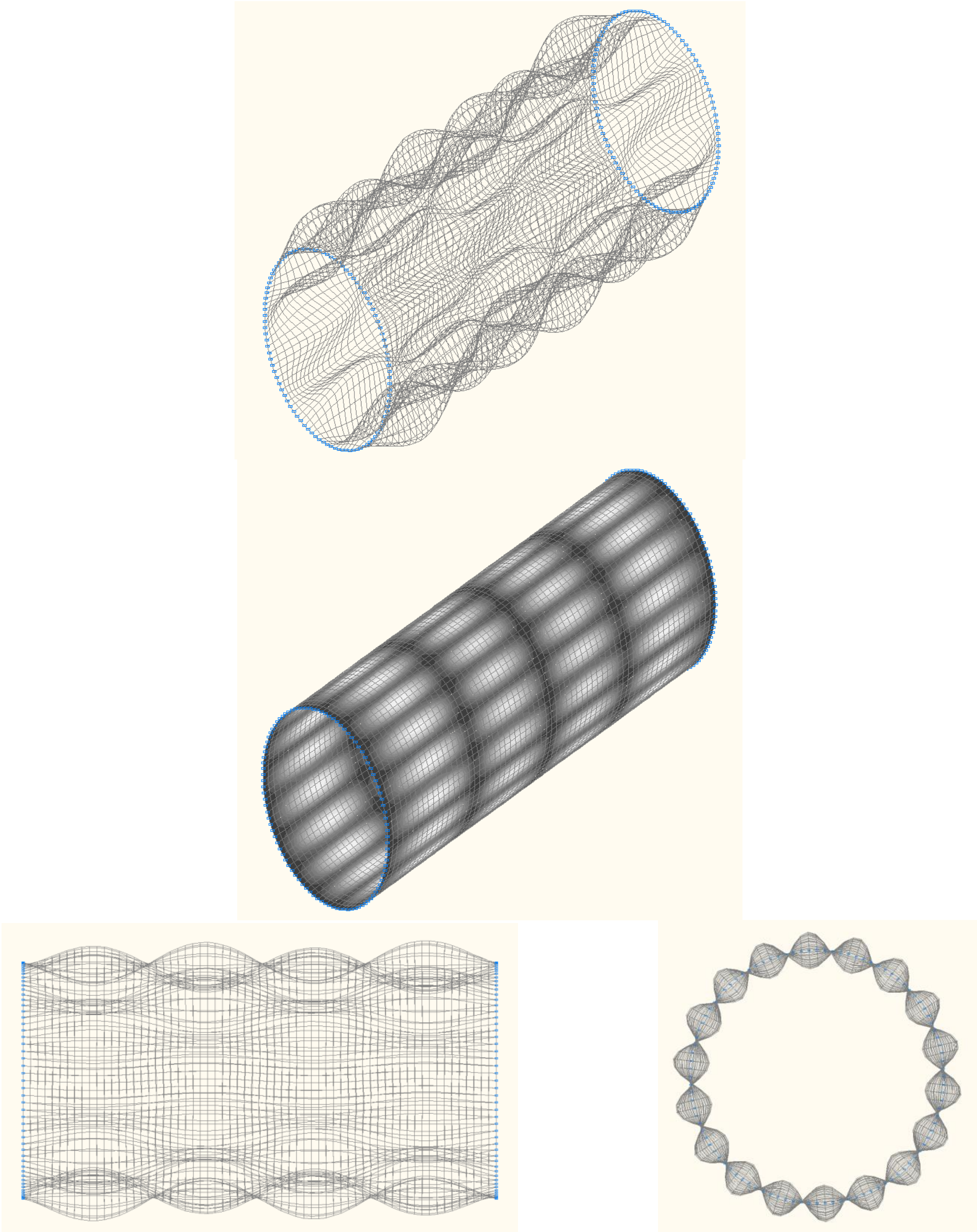
43-rd (43-rd theoretical) natural oscillation mode



45-th (45-th theoretical) natural oscillation mode



47-th (47-th theoretical) natural oscillation mode



49-th (49-th theoretical) natural oscillation mode

Comparison of solutions:

Natural frequencies ω, Hz

Oscillation mode	Number of nodal circles m and meridians n	Theory	SCAD	Deviations, %
1, 2	2, 6	533 (529.2)	522.2	2.03
3, 4	2, 5	574 (585.3)	567.0	1.22
5, 6	2, 7	593 (579.2)	578.9	2.38

Oscillation mode	Number of nodal circles m and meridians n	Theory	SCAD	Deviations, %
7, 8	2, 8	717 (697.2)	700.3	2.33
9, 10	2, 4	755 (787.9)	751.1	0.52
11, 12	2, 9	881 (857.8)	862.6	2.09
13, 14	3, 7	898 (910.0)	888.2	1.09
15, 16	3, 8	903 (897.8)	889.5	1.50
17, 18	3, 9	996 (979.9)	979.5	1.66
19, 20	3, 6	1011 (1047.7)	1004.6	0.63
21, 22	2, 10	1075 (1048.9)	1054.6	1.90
23, 24	2, 3	1140 (1209.6)	1136.7	0.29
25, 26	3, 10	1151 (1127.1)	1131.1	1.73
27, 28	4, 9	1251 (1251.3)	1238.2	1.02
29, 30	3, 5	1272 (1344.8)	1267.7	0.34
31, 32	4, 8	1273 (1293.0)	1264.2	0.69
33, 34	2, 11	1295 (1265.4)	1271.5	1.81
35, 36	4, 10	1325 (1310.9)	1308.2	1.27
37, 38	3, 11	1348 (1319.3)	1325.7	1.65
39, 40	4, 7	1415 (1460.8)	1409.3	0.40
41, 42	4, 11	1471 (1446.7)	1450.2	1.41
43, 44	2, 12	—— (1504.9)	1511.3	—
45, 46	3, 12	—— (1545.3)	1552.9	—
47, 48	5, 10	—— (1611.9)	1597.9	—
49, 50	5, 9	—— (1657.6)	1627.9	—
51, 52	3, 12	—— (1637.7)	1644.9	—
53, 54	5, 11	—— (1666.7)	1663.6	—
55, 56	4, 6	1700 (1781.0)	1696.6	0.20
57, 58	3, 4	1731 (1863.8)	1728.3	0.16
59, 60	5, 8	—— (1824.3)	1772.9	—
61, 62	2, 13	—— (1766.4)	1773.0	—
63, 64	5, 12	—— (1800.5)	1804.6	—
65, 66	3, 13	—— (1799.0)	1807.3	—
67, 68	4, 13	—— (1869.9)	1879.4	—
69, 70	2, 2	—— (2045.1)	1889.1	—
71, 72	6, 11	—— (1975.1)	1963.8	—
73, 74	6, 10	—— (2007.8)	1982.0	—
75, 76	5, 13	—— (1994.2)	2002.9	—
77, 78	6, 12	—— (2038.4)	2037.7	—
79, 80	5, 7	—— (2131.6)	2051.4	—
81, 82	2, 14	—— (2049.6)	2056.1	—
83, 84	3, 14	—— (2077.5)	2086.0	—
85, 86	6, 9	—— (2154.2)	2109.1	—
87, 88	4, 14	—— (2135.1)	2145.7	—
89, 90	4, 5	2165 (2295.4)	2163.0	0.09
91, 92	6, 13	—— (2179.6)	2186.2	—
93, 94	5, 14	—— (2233.9)	2245.5	—
95, 96	7, 11	—— (2352.7)	2334.1	—
97, 98	7, 12	—— (2343.7)	2338.1	—
99, 100	6, 8	—— (2429.8)	2360.2	—
101, 102	2, 15	—— (2354.1)	2360.4	—
103, 104	3, 15	—— (2379.1)	2387.6	—
105, 106	6, 14	—— (2381.8)	2393.2	—
107, 108	7, 13	—— (2425.1)	2429.2	—
109, 110	7, 10	—— (2467.4)	2432.1	—
111, 112	4, 15	—— (2428.2)	2439.5	—
113, 114	5, 6	—— (2606.7)	2488.7	—
115, 116	3, 3	2505 (2740.1)	2502.6	0.10
117, 118	5, 15	—— (2510.3)	2523.6	—
119, 120	7, 14	—— (2581.1)	2592.0	—
121, 122	7, 14	—— (2700.8)	2644.8	—

Oscillation mode	Number of nodal circles m and meridians n	Theory	SCAD	Deviations, %
123, 124	7, 9	—— (2632.0)	2646.5	—
125, 126	6, 15	—— (2679.8)	2685.7	—
127, 128	2, 16	—— (2701.6)	2692.7	—
129,130	8, 12	—— (2702.9)	2711.1	—
131, 132	3, 16	—— (2723.2)	2725.6	—
133, 134	8, 13	—— (2852.2)	2752.7	—
135, 136	6, 7	—— (2777.7)	2754.5	—
137, 138	8, 11	—— (2746.6)	2757.9	—
139, 140	4, 16	—— (2796.9)	2812.5	—
141, 142	5, 16	—— (2817.6)	2831.6	—
143, 144	7, 15	—— (2829.0)	2839.8	—
145, 146	4, 4	2884 (3082.7)	2883.1	0.03
147, 148	8, 10	—— (2963.2)	2922.5	—
149, 150	6, 16	—— (2921.1)	2937.5	—

The values of the exact solution are given before brackets in the “Theory” column, and the values of the approximate solution by the Rayleigh-Ritz method with the displacement components expressed by beam functions are given in brackets.

Notes: In the analytical solution by the Rayleigh-Ritz method with the displacement components expressed by beam functions the natural frequencies ω of the clamped circular cylindrical shell with the density of the material ρ can be determined from the characteristic equation:

$$\left(\frac{4\cdot\pi^2\cdot\rho\cdot R^2\cdot\left(1-\nu^2\right)}{E}\right)^3\cdot\omega^6+K2\cdot\left(\frac{4\cdot\pi^2\cdot\rho\cdot R^2\cdot\left(1-\nu^2\right)}{E}\right)^2\cdot\omega^4+K1\cdot\left(\frac{4\cdot\pi^2\cdot\rho\cdot R^2\cdot\left(1-\nu^2\right)}{E}\right)\cdot\omega^2+K0=0\,,$$

where:

$$K2=-1-\frac{1}{2}\cdot\left[\left(\frac{2}{\delta_m}+\delta_m-\nu\cdot\delta_m\right)\cdot\left(\frac{\lambda_m\cdot R}{L}\right)^2+\left(3-\nu\right)\cdot n^2\right]-$$

$$\frac{h^2}{12\cdot R^2}\cdot\left\{\left[\left(\frac{\lambda_m\cdot R}{L}\right)^4+2\cdot\delta_m\cdot\left(\frac{\lambda_m\cdot R}{L}\right)^2\cdot n^2+n^4\right]+2\cdot\left(1-\nu\right)\cdot\delta_m\cdot\left(\frac{\lambda_m\cdot R}{L}\right)^2+n^2\right\}$$

$$K1=\frac{1}{2}\cdot\left(1-\nu\right)\cdot\left[\left(\frac{\lambda_m\cdot R}{L}\right)^4+2\cdot\left(\frac{1-\nu\cdot\delta_m{}^2}{\left(1-\nu\right)\cdot\delta_m}\right)\cdot\left(\frac{\lambda_m\cdot R}{L}\right)^2\cdot n^2+n^4\right]+$$

$$\frac{1}{2}\cdot\left(\frac{2}{\delta_m}+\delta_m-\nu\cdot\delta_m-2\cdot\nu^2\cdot\delta_m\right)\cdot\left(\frac{\lambda_m\cdot R}{L}\right)^2+\frac{1}{2}\cdot\left(1-\nu\right)\cdot n^2+$$

$$\frac{h^2}{12\cdot R^2}\cdot\left\{\frac{1}{2}\cdot\left[\left(\frac{2}{\delta_m}+\delta_m-\nu\cdot\delta_m\right)\cdot\left(\frac{\lambda_m\cdot R}{L}\right)^6+\left(7+2\cdot\delta_m{}^2-\left(1+2\cdot\delta_m{}^2\right)\cdot\nu\right)\cdot\left(\frac{\lambda_m\cdot R}{L}\right)^4\cdot n^2+\right.\right.$$

$$\left.\left(\frac{2}{\delta_m}+7\cdot\delta_m-3\cdot\nu\cdot\delta_m\right)\cdot\left(\frac{\lambda_m\cdot R}{L}\right)^2\cdot n^4+\left(3-\nu\right)\cdot n^6\right]+2\cdot\left(1-\nu\right)\cdot\left(\frac{\lambda_m\cdot R}{L}\right)^4-$$

$$\left.\left(3\cdot\delta_m-\frac{1}{\delta_m}-\nu^2\cdot\delta_m\right)\cdot\left(\frac{\lambda_m\cdot R}{L}\right)^2\cdot n^2-\frac{1}{2}\cdot\left(3+\nu\right)\cdot n^4+2\cdot\left(1-\nu\right)\cdot\delta_m\cdot\left(\frac{\lambda_m\cdot R}{L}\right)^2+n^2\right\}$$

$$K0=-\frac{1}{2}\cdot(1-\nu)\cdot\left(1-\nu^2\cdot\delta_m{}^2\right)\cdot\left(\frac{\lambda_m\cdot R}{L}\right)^4-\frac{1}{2}\cdot(1-\nu)\cdot\frac{h^2}{12\cdot R^2}\cdot\left\{\left[\left(\frac{\lambda_m\cdot R}{L}\right)^8+2\cdot\left(\frac{1+\delta_m{}^2-2\cdot\nu\cdot\delta_m}{(1-\nu)\cdot\delta_m}\right)\cdot\left(\frac{\lambda_m\cdot R}{L}\right)^6\cdot n^2+\right.\right.$$

$$\left.\left(\frac{6-2\cdot\nu\cdot\left(1+2\cdot\delta_m{}^2\right)}{1-\nu}\right)\cdot\left(\frac{\lambda_m\cdot R}{L}\right)^4\cdot n^4+2\cdot\left(\frac{1+\delta_m{}^2-2\cdot\nu\cdot\delta_m}{(1-\nu)\cdot\delta_m}\right)\cdot\left(\frac{\lambda_m\cdot R}{L}\right)^2\cdot n^6+n^8\right]-$$

$$2\cdot\left(\frac{4-2\cdot\nu\cdot\left(1+\delta_m{}^2\right)-\nu^2\cdot\delta_m{}^2\cdot(1-\nu)}{1-\nu}\right)\cdot\left(\frac{\lambda_m\cdot R}{L}\right)^4\cdot n^4-\left(\frac{4\cdot\left(1+\delta_m{}^2\right)-8\cdot\nu\cdot\delta_m{}^2}{(1-\nu)\cdot\delta_m}\right)\cdot\left(\frac{\lambda_m\cdot R}{L}\right)^2\cdot n^4-2\cdot n^6+$$

$$\left.4\cdot\left(1-\nu^2\cdot\delta_m{}^2\right)\cdot\left(\frac{\lambda_m\cdot R}{L}\right)^4+\left(\frac{2\cdot\left(1+\delta_m{}^2\right)-4\cdot\nu\cdot\delta_m{}^2}{1-\nu}\right)\cdot\left(\frac{\lambda_m\cdot R}{L}\right)^2\cdot n^2+n^4\right\}$$

$$\delta_m=1-\frac{2}{\lambda_m}\cdot\left(\frac{sh(\lambda_m)\cdot ch(\lambda_m)-\lambda_m\cdot sh(\lambda_m)\cdot sin(\lambda_m)-sh(\lambda_m)\cdot cos(\lambda_m)-ch(\lambda_m)\cdot sin(\lambda_m)+sin(\lambda_m)\cdot cos(\lambda_m)}{(sh(\lambda_m)-sin(\lambda_m))^2}\right)$$

Eigenvalues of the m-th beam function are determined from the following equation:

$$ch(\lambda_m)\cdot cos(\lambda_m)=1$$

$m=2,3,4\ldots$ - number of nodal lines in the circumferential direction, taking into account the lines along the end support contours,

$n=0,1,2,\ldots$ - number of pairs of nodal lines in the meridian direction when each pair is located on one diameter.

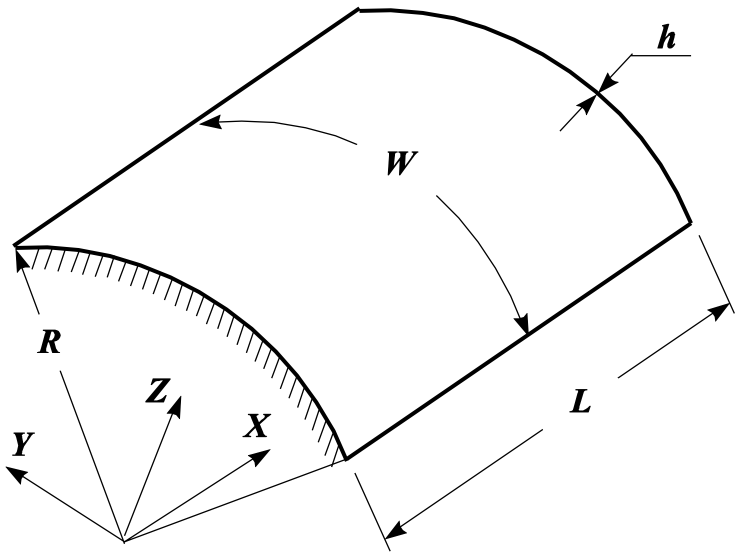
The deviations from the theory for the initial natural frequencies are due to the fact that the natural modes and frequencies are determined by the program for a design model with all degrees of freedom of nodal displacements, i.e. tangential inertia forces were taken into account as well. These forces are especially noticeable in natural modes with a small number of half waves *m* in the circumferential direction.

The exact solution from the source does not take into account the tangential inertia forces. However, the page 440 of this handbook provides a formula for the estimation of the error introduced by this assumption. It gives a value of the correction to the square of the natural frequency:

k=1/(1+z).

For the first modes when m=2 the calculations gave the value z=0,042. Therefore, we can expect a 2% correction to the theoretical value.

Natural Oscillations of a Cantilever Open Cylindrical Shell



Objective: Modal analysis of a cantilever open cylindrical shell.

Initial data file: **5.9.SPR**

Problem formulation: Determine the natural oscillation modes and frequencies ω of the cantilever open cylindrical shell with the density of the material ρ.

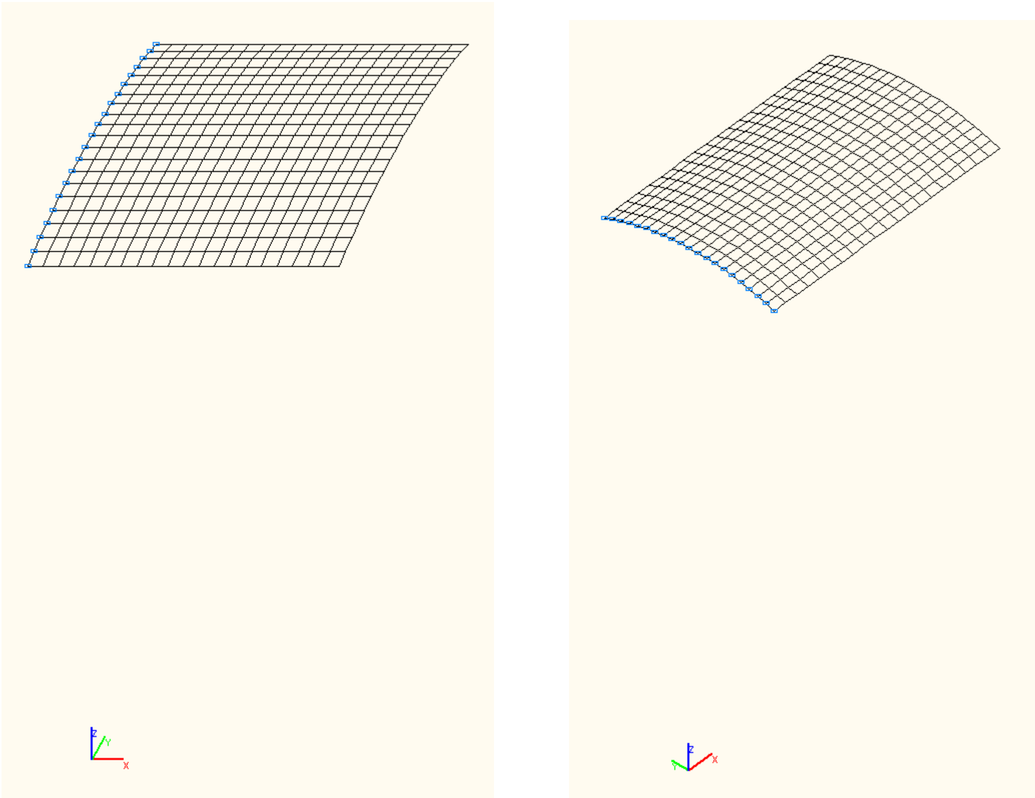
References: Olson M. D., Lindberg G. M., Vibration analysis of cantilevered curved plates using a new cylindrical shell finite element, Second conference on matrix methods in structural mechanics at Wright – Patterson Air Force Base in Ohio, AFFDL-TR-68-155, 1969, p. 247-269.

Initial data:

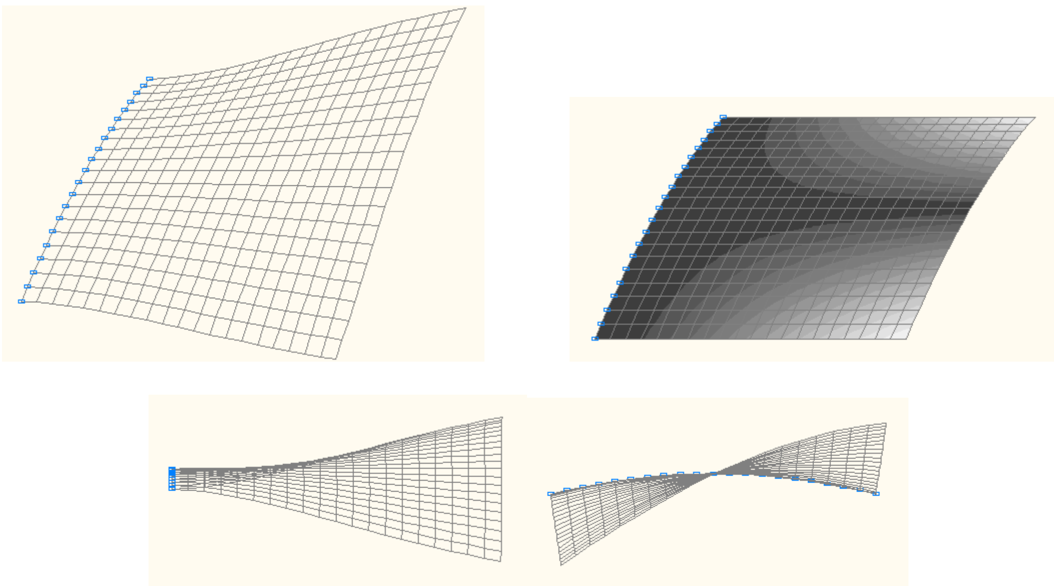
$E = 30.0 \cdot 10^6$ PSI = $2.0685 \cdot 10^8$ kPa	- elastic modulus;
ν = 0.3	- Poisson’s ratio;
ρ = 0.28386 lb/in^3 = 7.8572 t/m^3	- density of the material;
h = 0.12 in = $3.048 \cdot 10^{-3}$ m	- thickness of the cylindrical shell;
R = 24 in = 0.6096 m	- radius of the midsurface of the cylindrical shell;
L = 12 in = 0.3048 m	- length of the generatrix of the cylindrical shell;
W =12 in = 0.3048 m	- length of the arc of the director of the cylindrical shell.

Finite element model: Design model – general type system, 400 four-node shell elements of type 50. The spacing of the finite element mesh in the meridian and in the circumferential directions is 0.01524 m (20 elements). Boundary conditions of the clamped curvilinear edge are provided by imposing constraints in the directions of all linear and angular displacements (degrees of freedom X, Y, Z, UX, UY, UZ). The distributed mass is specified by transforming the static load from the self-weight of the cylindrical shell: ow = γ·h, where γ = ρ·g = 77.0791 kN/m^3. Number of nodes in the design model – 441. The determination of the natural oscillation modes and natural frequencies is performed by the Lanczos method. A consistent mass matrix is used in the calculation.

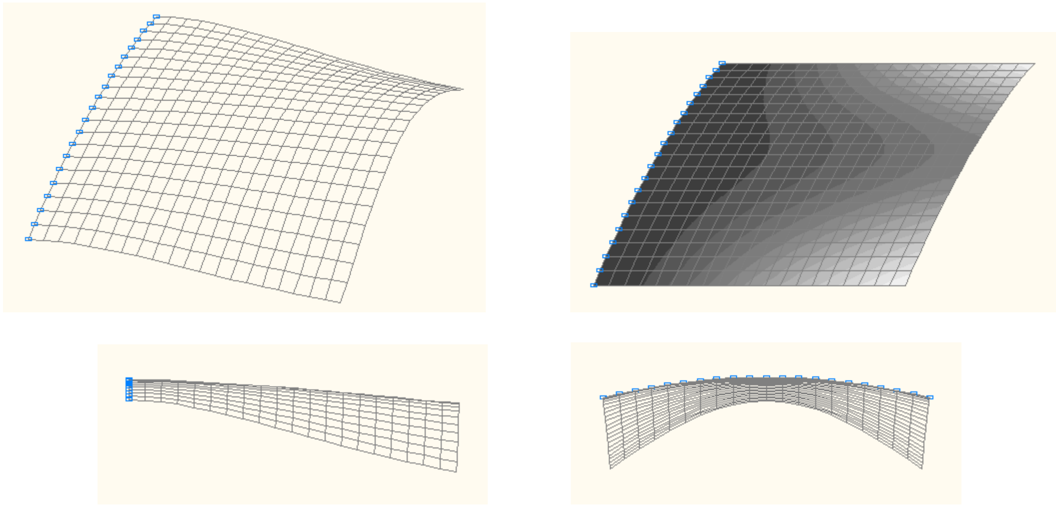
Results in SCAD



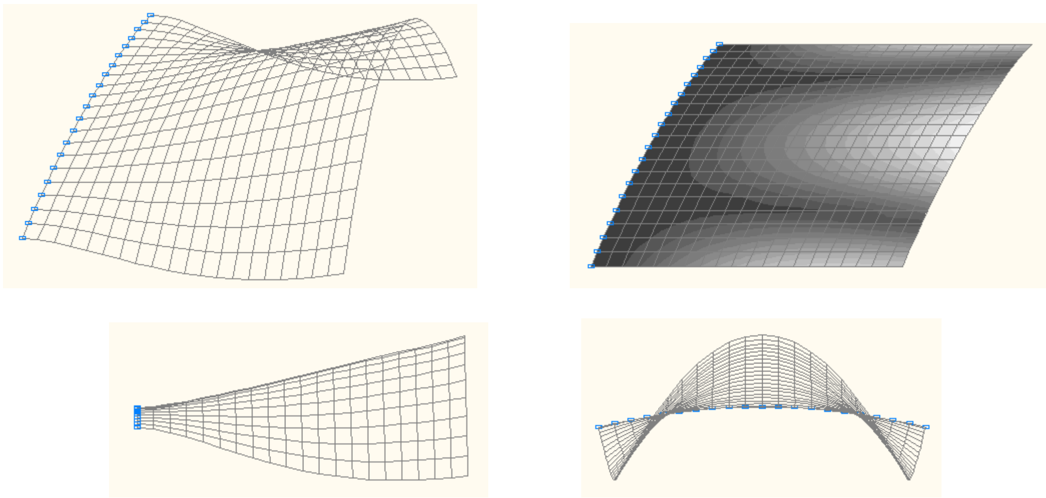
Design model



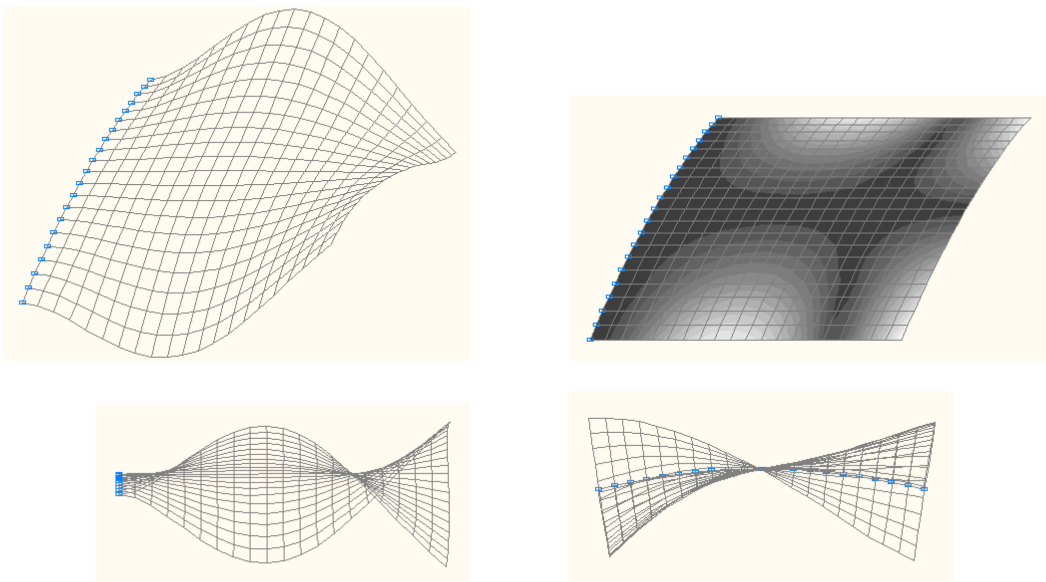
1-st natural oscillation mode



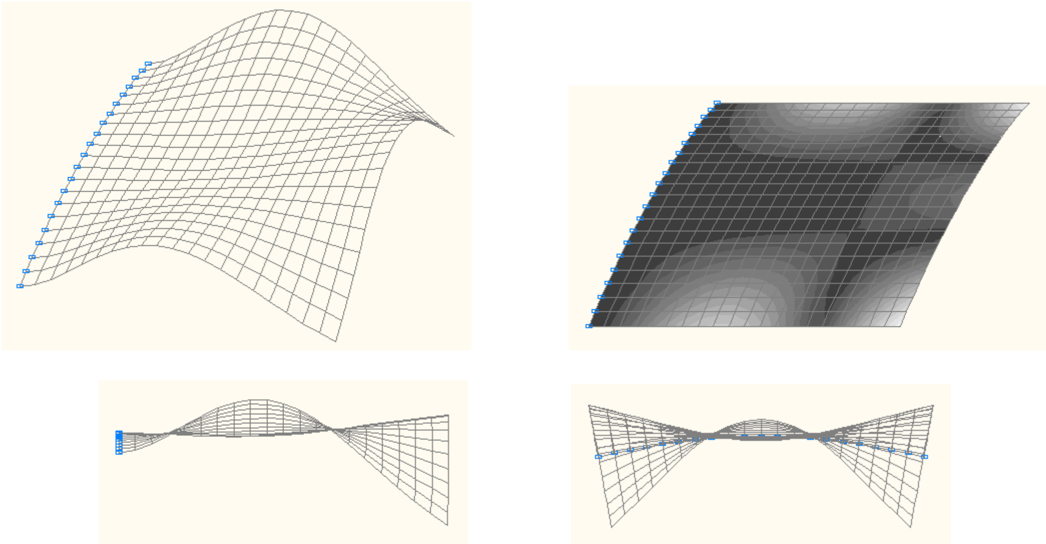
2-nd natural oscillation mode



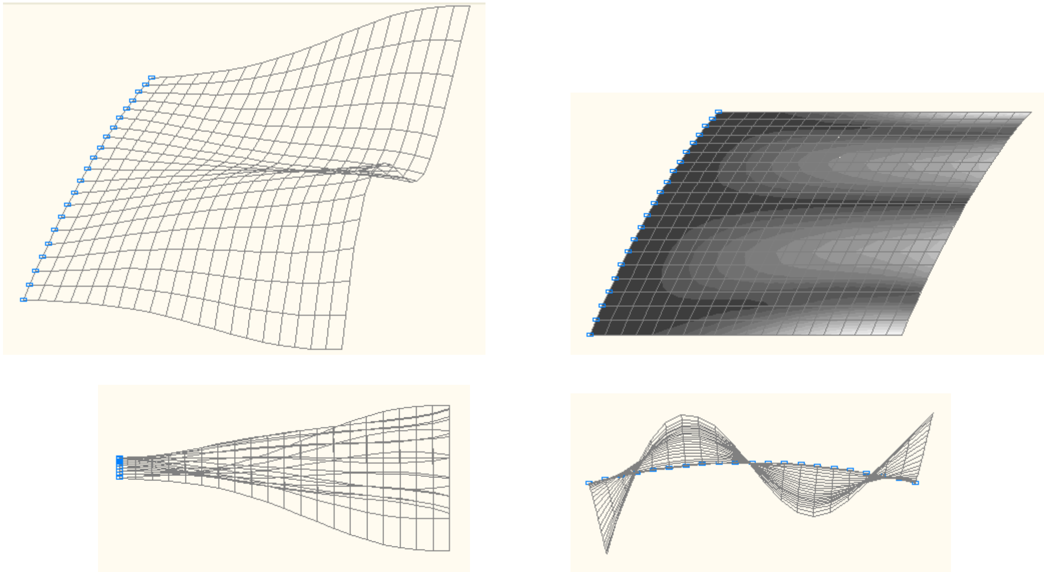
3-rd natural oscillation mode



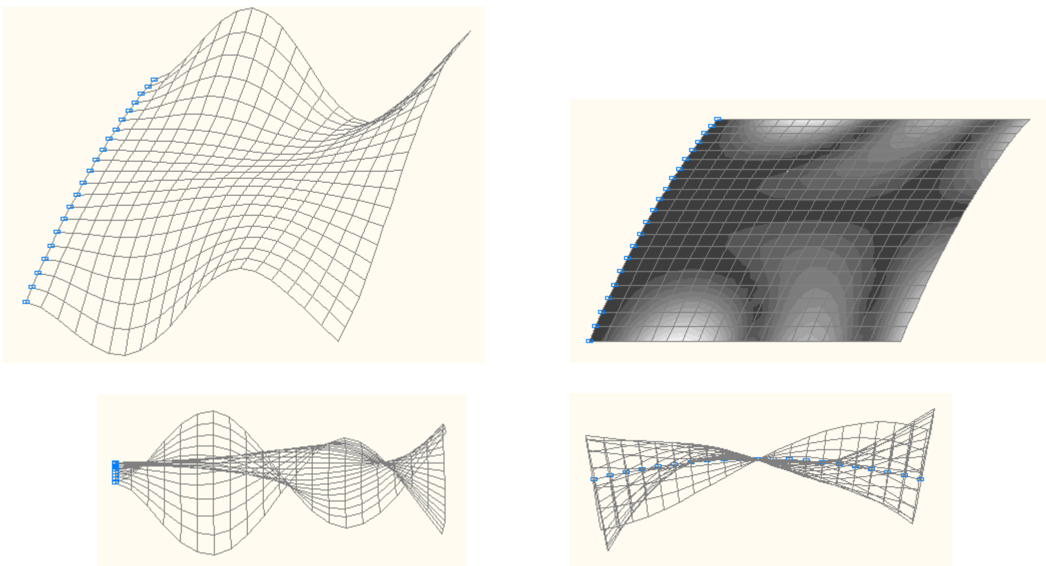
4-th natural oscillation mode



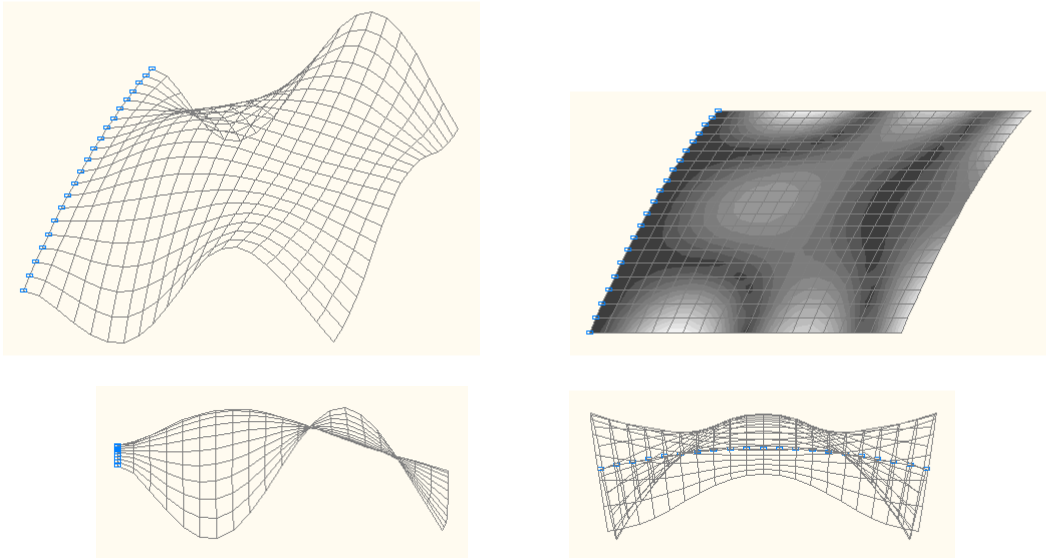
5-th natural oscillation mode



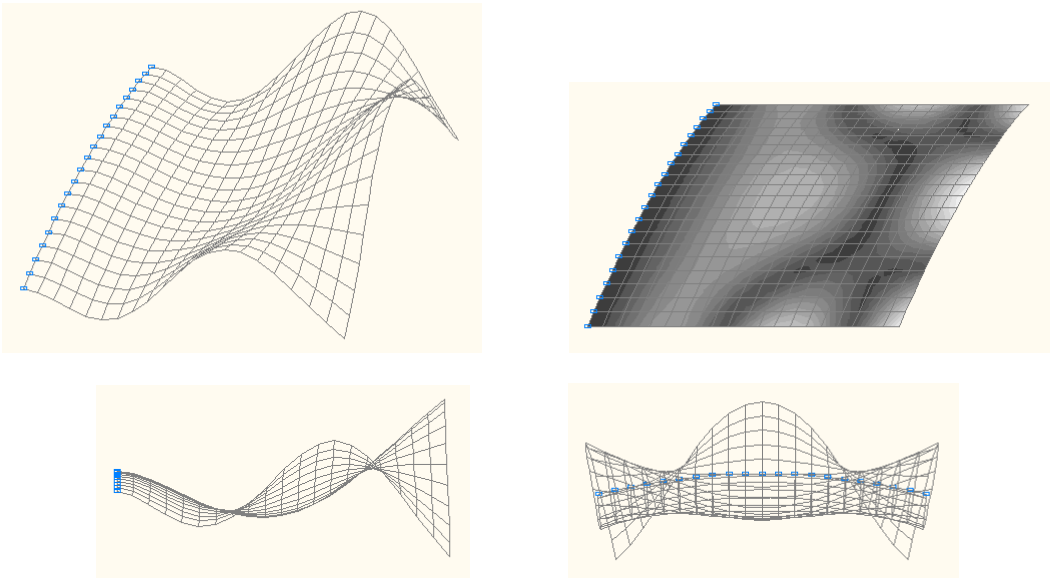
6-th natural oscillation mode



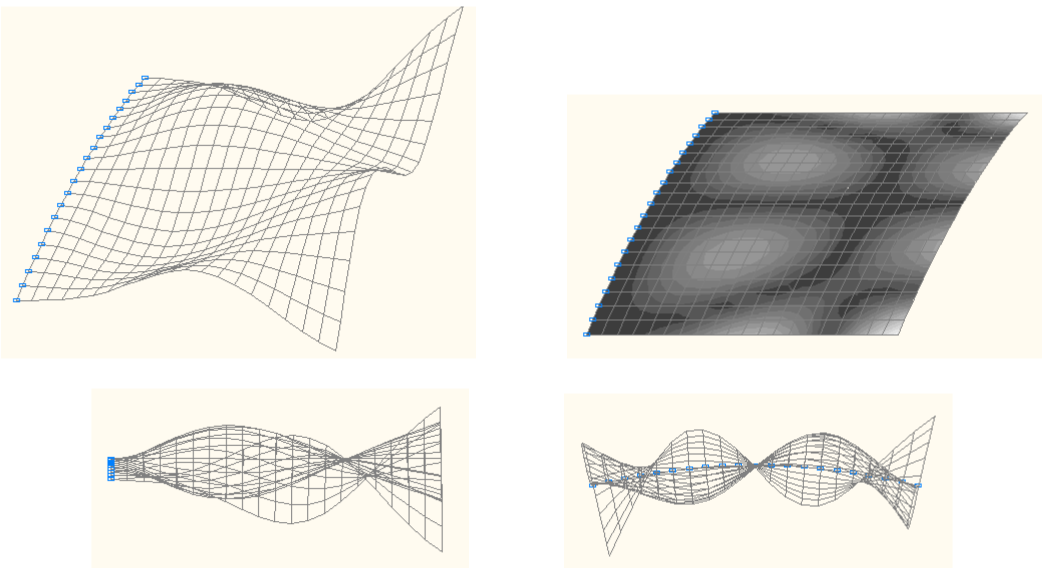
7-th natural oscillation mode



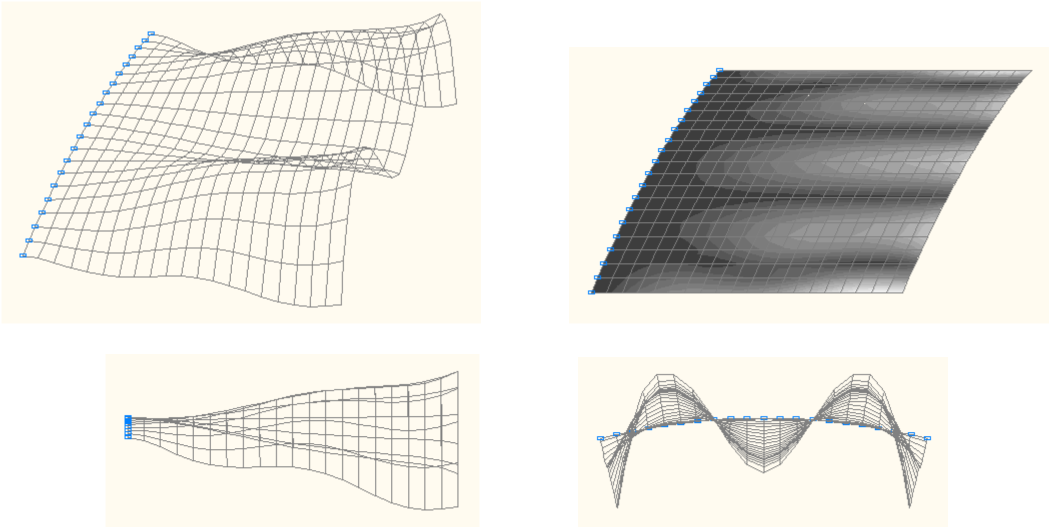
8-th natural oscillation mode



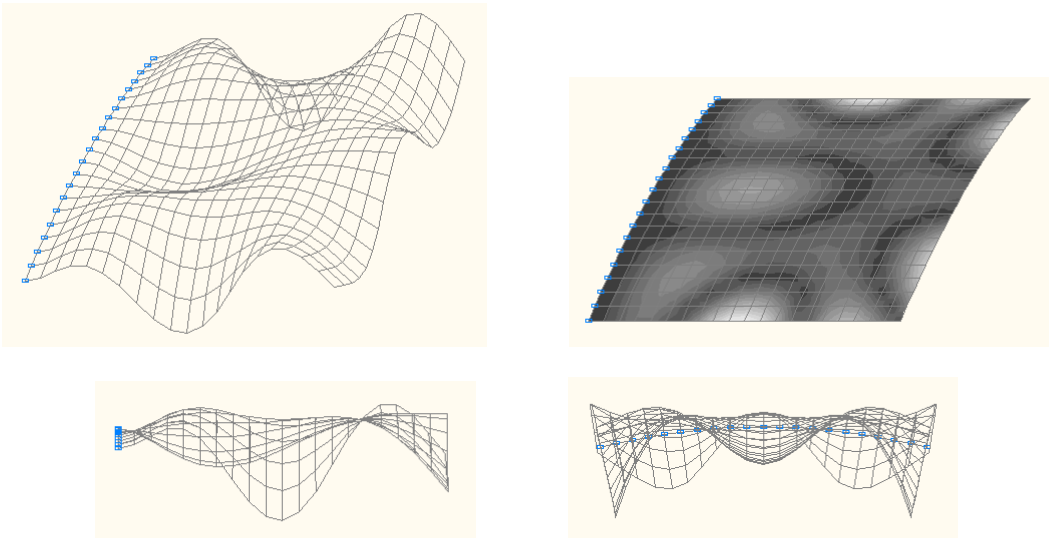
9-th natural oscillation mode



10-th natural oscillation mode



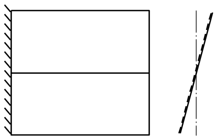
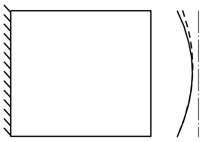
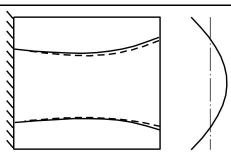
11-th natural oscillation mode

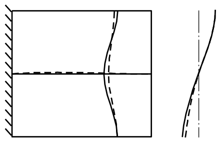
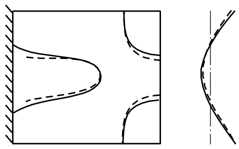
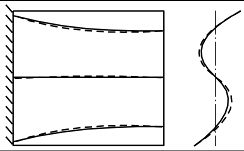
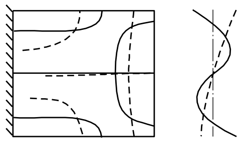
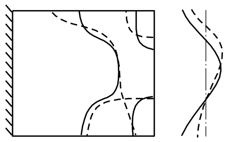
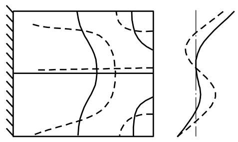
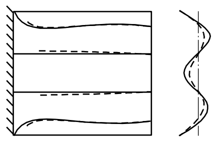
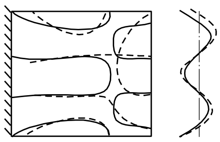


12-th natural oscillation mode

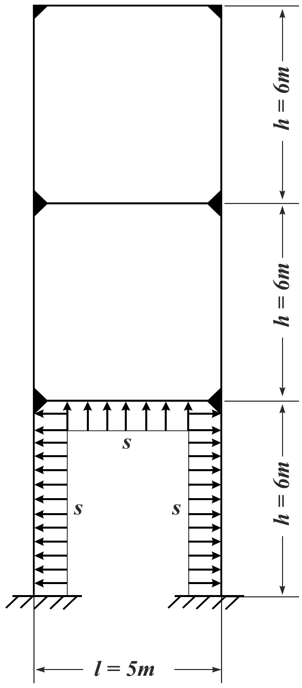
Comparison of solutions:

Natural frequencies ω, Hz

Oscillation mode	Nodal lines	Experiment	SCAD	Deviations, %
1		85.6	86,2	0,35
2		135.5	139,2	0,57
3		258.9	248,2	0,95

Oscillation mode	Nodal lines	Experiment	SCAD	Deviations, %
4		350.6	344,2	0,75
5		395.2	388,2	0,89
6		531.1	529,9	1,39
7		743.2	730,9	1,33
8		751.2	732,9	1,22
9		792.1	776,5	0,87
10		809.2	805,4	1,21
11		996.8	999,1	1,97
12		1215.0	1210,5	1,85

Plane Frame Subjected to a Uniformly Distributed Instantaneous Pulse



Objective: Determination of the stress-strain state of a plane frame subjected to a uniformly distributed instantaneous pulse.

Initial data file: DI_F.SPR

Problem formulation: The three-storey single-span plane frame with clamped columns and mass uniformly distributed over the columns m_1 and girders m_2 is subjected to an instantaneous pulse s uniformly distributed along the contour of the first storey. Determine the amplitude values of the bending moment M in the girder of the first storey in the section of its connection with the left column taking into account the following assumption made when deriving the analytical solution: it is assumed that there are no linear displacements of the beam-to-column joints when the symmetric design model is subjected to a symmetric loading and the longitudinal deformations of the frame structural members are neglected.

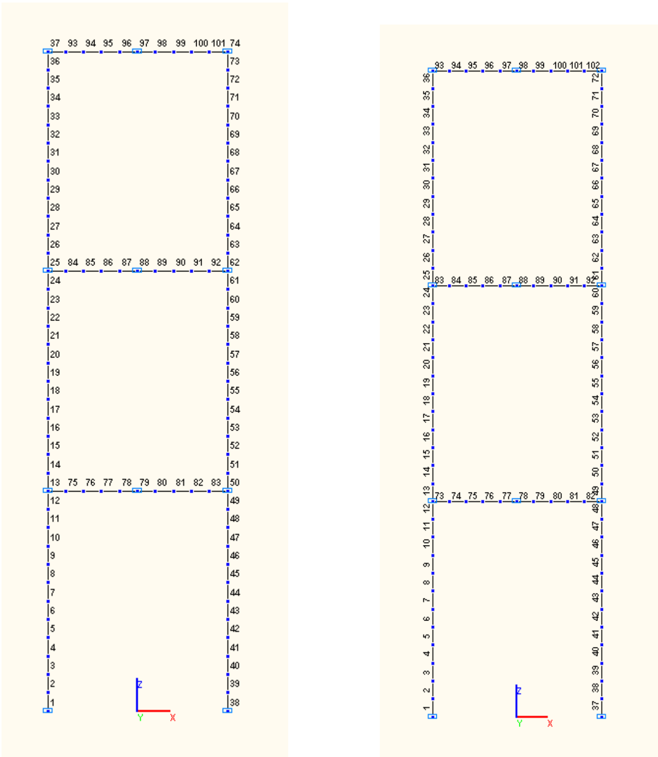
References: Rabinovich I.M., Sinitsyn A.P., Luzhin O.V., Terenin V.M., Analysis of Structures Subject to Pulse Actions, Moscow, Stroyizdat, 1970, p. 91;
Korenev B.G., Rabinovich I.M., Dynamic Analysis of Buildings and Structures (Designer's handbook), Moscow, Stroyizdat, 1984, p. 79.

Initial data:

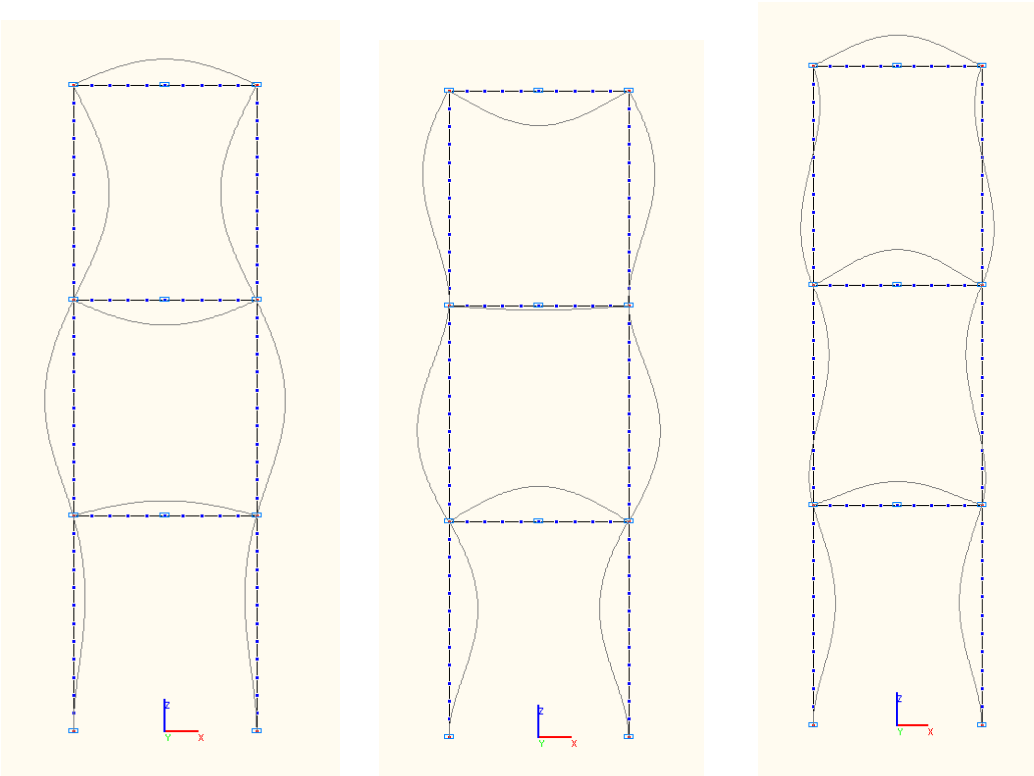
$E = 2.1 \cdot 10^7$ tf/m^2	- elastic modulus;
h = 6.0 m	- height of the frame columns;
$I_1 = 1 \cdot 10^{-4}$ m^4	- cross-sectional moment of inertia of the frame columns;
$F_1 = 2 \cdot 10^{-1}$ m^2	- cross-sectional area of the frame columns;
$m_1 = 0.0204$ tf·s^2/m^2	- value of the mass uniformly distributed over the frame columns;
l = 5.0 m	- length of the span of the frame girders;
$I_2 = 2 \cdot 10^{-4}$ m^4	- cross-sectional moment of inertia of the frame girders;
$F_2 = 4 \cdot 10^{-1}$ m^2	- cross-sectional area of the frame girders;
$m_2 = 0.0510$ tf·s^2/m^2	- value of the mass uniformly distributed over the frame girders;
s = 0.3· tf·s/m	- value of the uniformly distributed instantaneous pulse;
g = 9.81 m/s^2	- gravitational acceleration.

Finite element model: Design model – plane frame, 102 bar elements of type 2.
The spacing of the finite element mesh along the longitudinal axes of the columns and girders of the frame is 0.5 m. Boundary conditions of the support nodes of the columns of the first storey are provided by imposing constraints in the directions of the following degrees of freedom: X, Z, UY. Boundary conditions of the beam-to-column joints according to the assumption made when deriving the analytical solution are provided by imposing constraints in the directions of the following degrees of freedom: X, Z. Boundary conditions of the nodes in the center of the girder spans according to the assumption made when deriving the analytical solution are provided by imposing constraints in the directions of the following degrees of freedom: X, UY. The distributed mass is specified by transforming the static load on the columns $m_1 \cdot g$ and on the girders $m_2 \cdot g$ of the frame. The action of the distributed instantaneous pulse is reduced to a number of nodal actions with the values 0.5·s. Number of nodes in the design model – 101. The determination of the natural oscillation modes and natural frequencies is performed by the method of subspace iteration. The matrix of concentrated masses is used in the calculation.

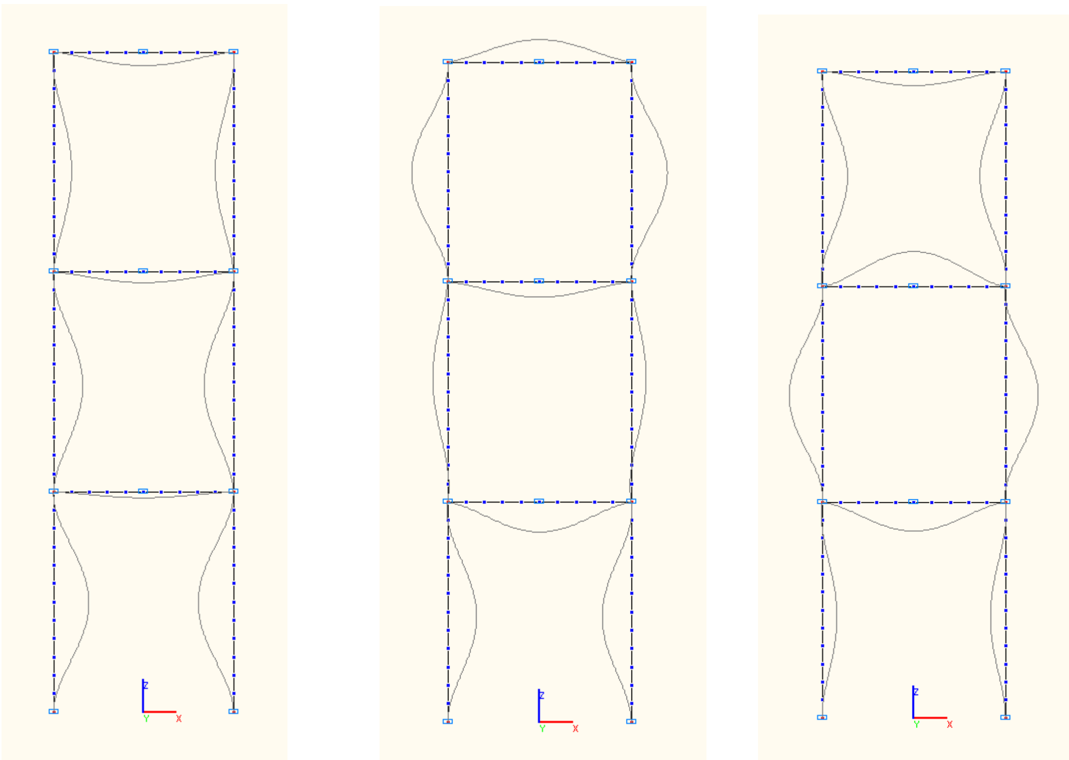
Results in SCAD:



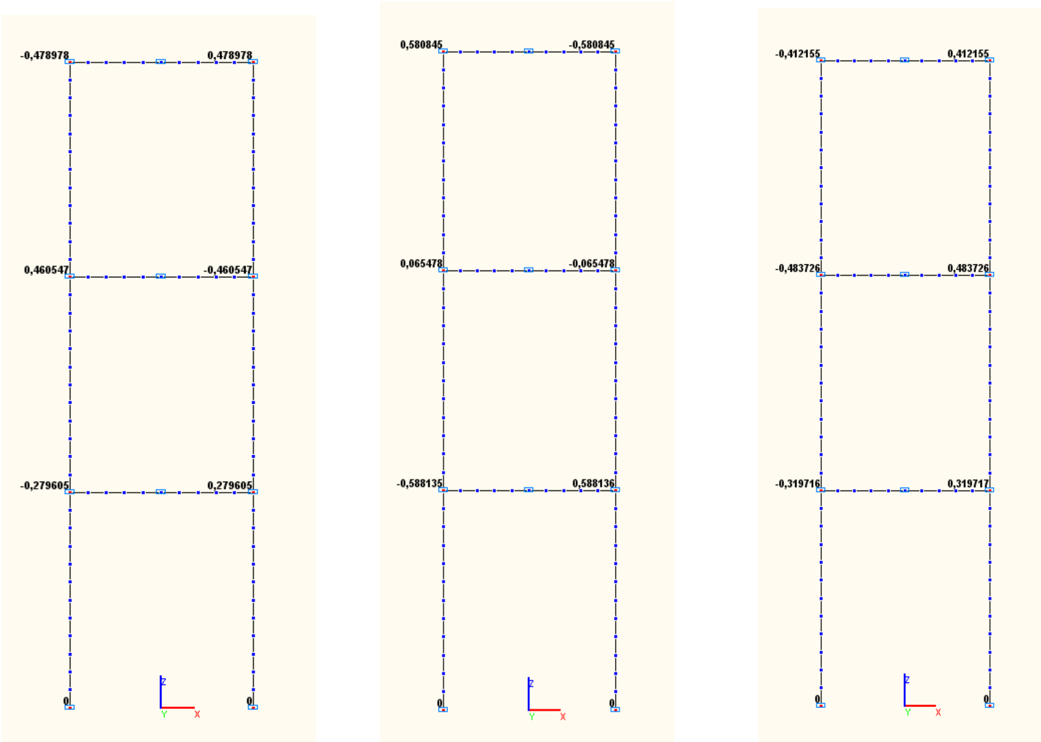
Design model



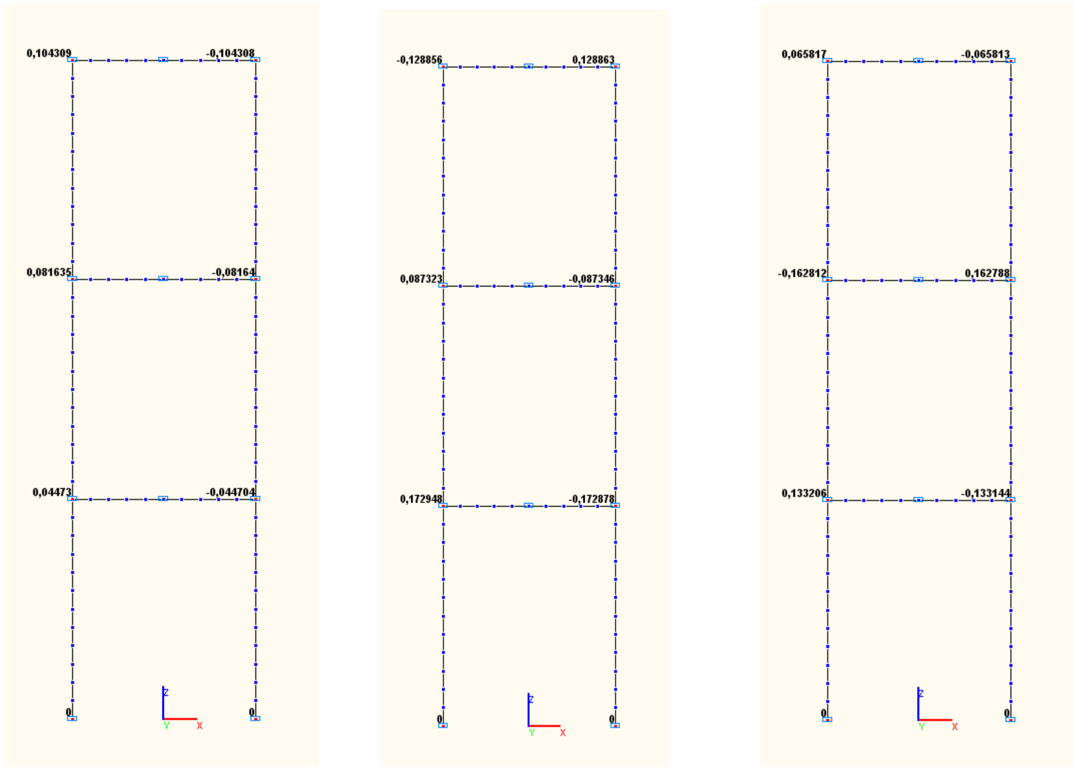
1-st, 2-nd, 3-rd natural oscillation modes



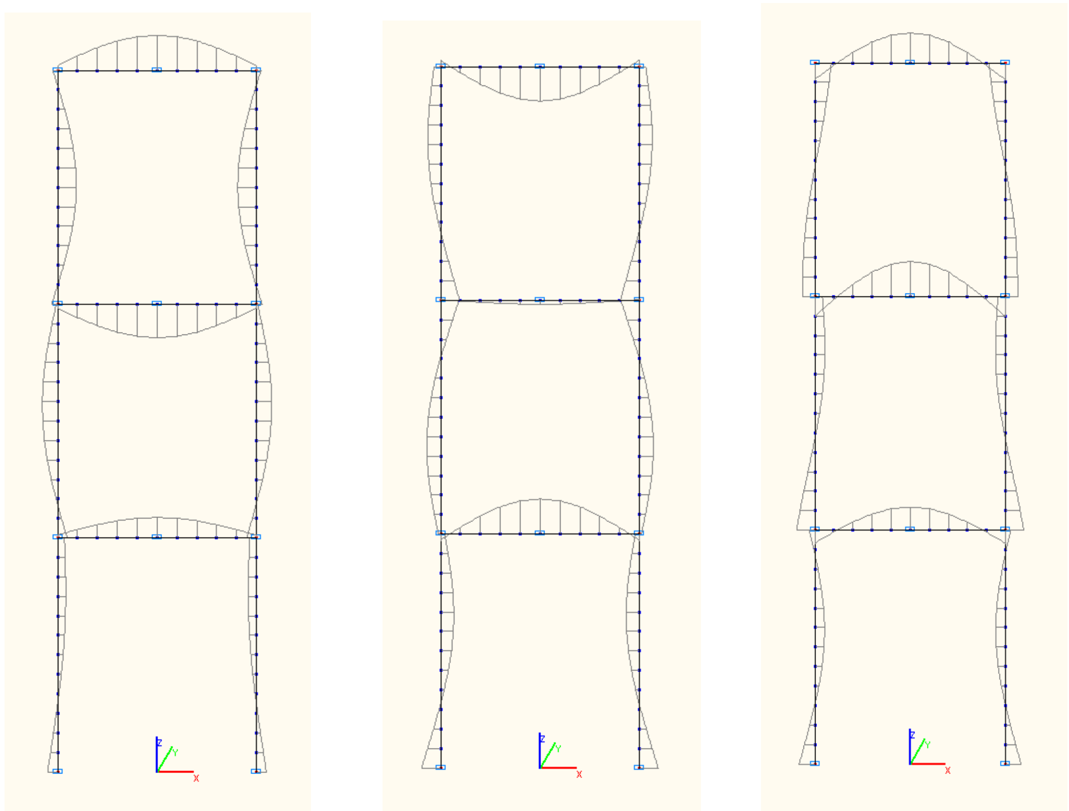
7-th, 8-th, 9-th natural oscillation modes



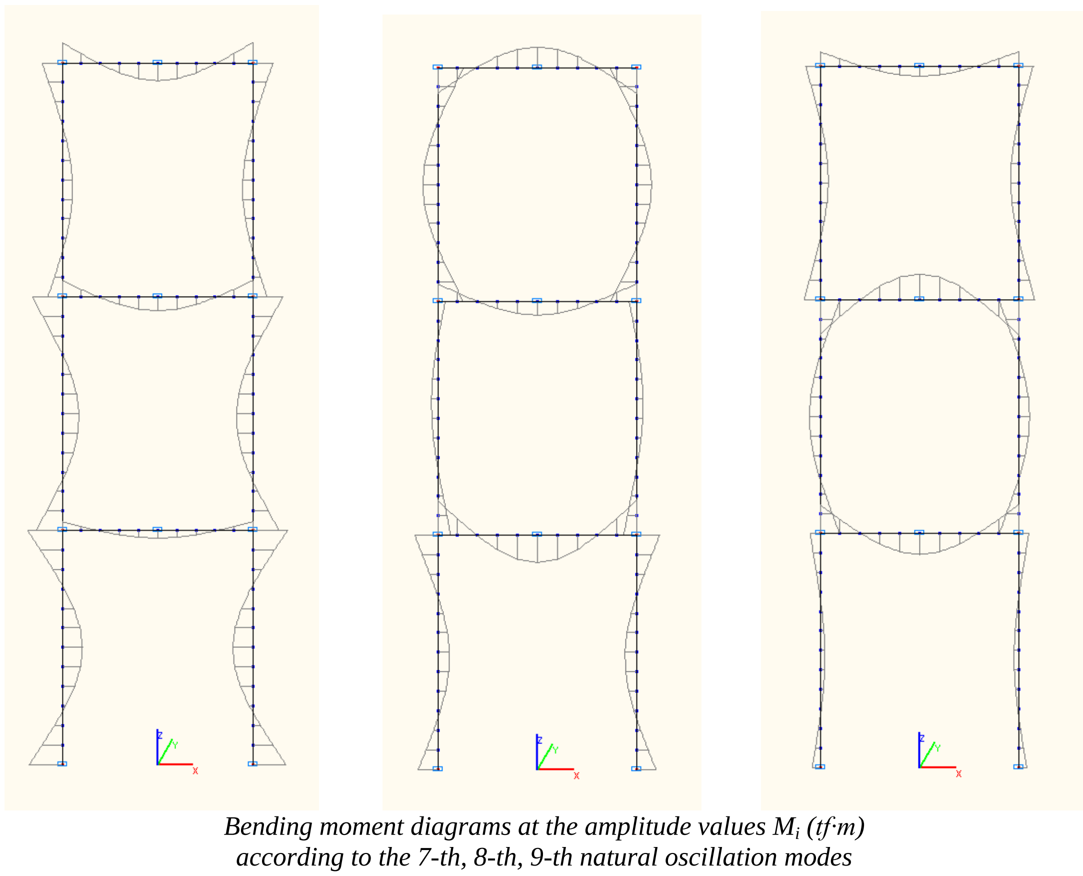
Amplitude values of the angular displacements UY_{ij} (rad)
in the beam-to-column joints
according to the 1-st, 2-nd, 3-rd natural oscillation modes
(modal analysis)



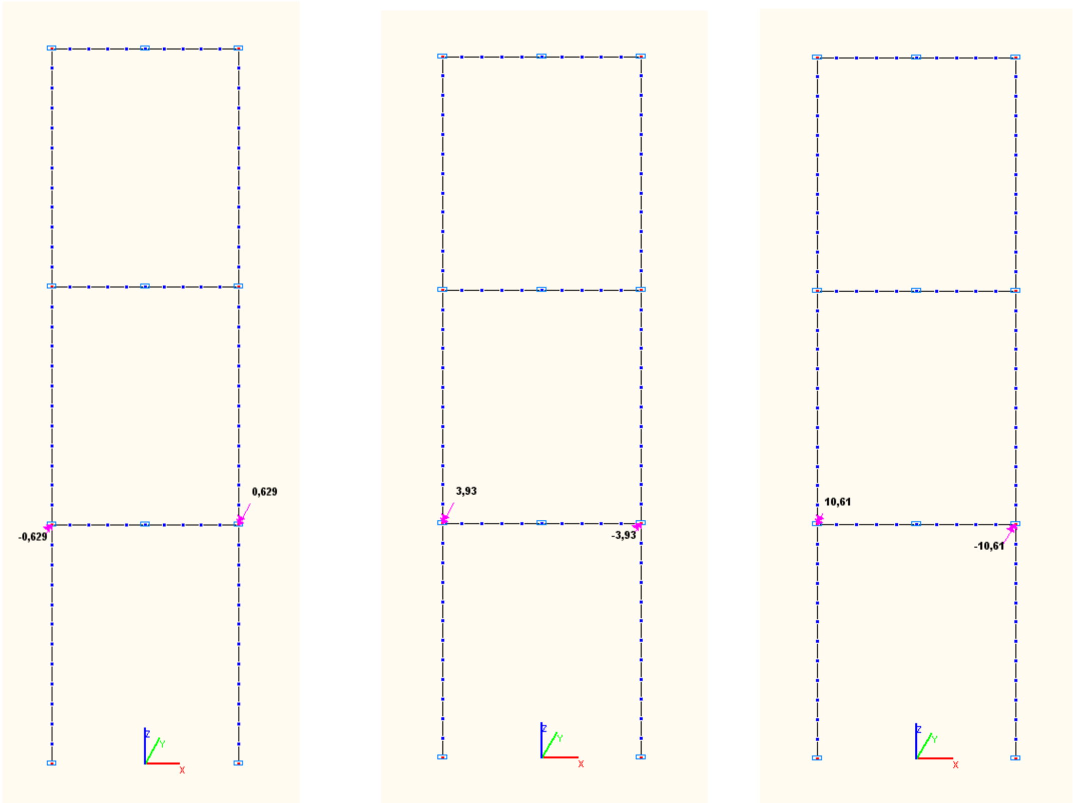
Amplitude values of the angular displacements UY_{ij} (rad)
in the beam-to-column joints
according to the 7-th, 8-th, 9-th natural oscillation modes
(modal analysis)



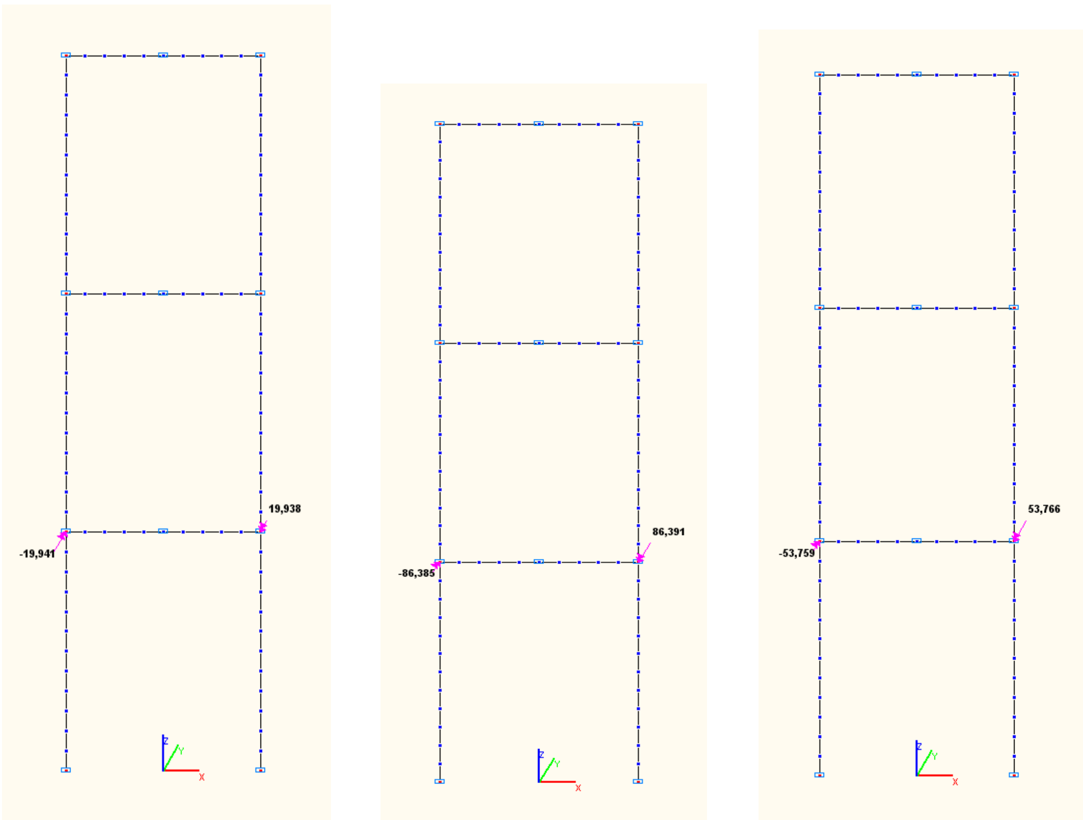
Bending moment diagrams at the amplitude values M_i (tf·m)
according to the 1-st, 2-nd, 3-rd natural oscillation modes



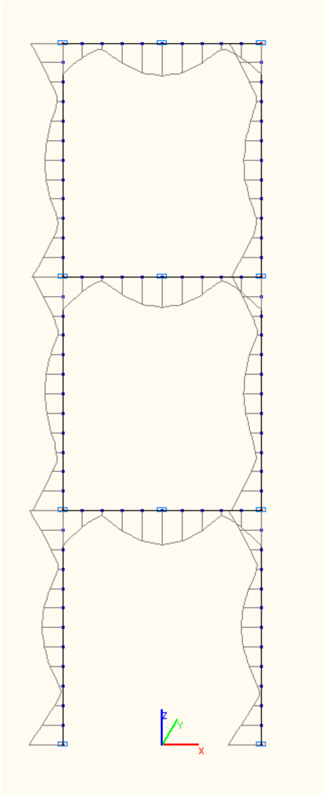
Bending moment diagrams at the amplitude values M_i (tf·m)
according to the 7-th, 8-th, 9-th natural oscillation modes

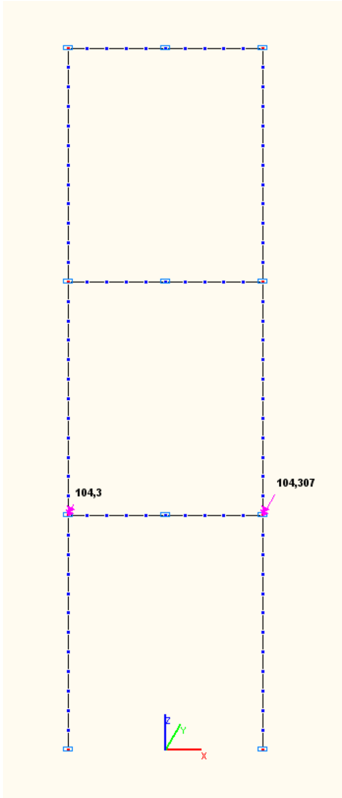


Amplitude values of the bending moments M_i (tf·m)
in the girder of the first floor in the sections of its connections with the columns
according to the 1-st, 2-nd, 3-rd natural oscillation modes



Amplitude values of the bending moments M_i (tf·m)
in the girder of the first floor in the sections of its connections with the columns
according to the 7-th, 8-th, 9-th natural oscillation modes





Bending moment diagram at the amplitude values M (tf·m)
from the total pulse load

Amplitude values of the bending moments M (tf·m)
in the girder of the first floor in the sections of its connections with the columns
from the total pulse load

Comparison of solutions:

Natural periods T, s

Oscillation mode	**Theory**	**SCAD**	**Deviations, %**
1	0.060607	0.060607	0.00
2	0.049785	0.049785	0.00
3	0.040435	0.040436	0.00
7	0.030924	0.030925	0.00
8	0.028903	0.028904	0.00
9	0.027787	0.027788	0.00

Amplitude values of the angular displacements UY_{ij} (rad)
in the beam-to-column joints
according to the 1-st, 2-nd, 3-rd, 7-th, 8-th, 9-th natural oscillation modes
(modal analysis)

Oscillation mode	**Storey**	**Theory**	**SCAD**	**Deviation, %**
1	1	+0.583753	-0.279605 / -0.478978 = = +0.583753	0.00
1	2	-0.961520	+0.460547 / -0.478978 = = -0.961520	0.00
1	3	+1.000000	-0.478978 / -0.478978 = = +1.000000	0.00

2	1	-1.012550	-0.588135 / +0.580845 = = -1.012551	0.00
2	2	+0.112727	+0.065478 / +0.580845 = = +0.112729	0.00
2	3	+1.000000	+0.580845 / +0.580845 = = +1.000000	0.00
3	1	+0.775708	-0.319716 / -0.412155 = = +0.775718	0.00
3	2	+1.173640	-0.483726 / -0.412155 = = +1.173651	0.00
3	3	+1.000000	-0.412155 / -0.412155 = = +1.000000	0.00
7	1	+0.428722	+0.044730 / +0.104309 = = +0.428822	0.00
7	2	+0.782640	+0.081635 / +0.104309 = = +0.782627	0.00
7	3	+1.000000	+0.044730 / +0.104309 = = +1.000000	0.00
8	1	-1.342142	+0.172948 / -0.128856 = = -1.342180	0.00
8	2	-0,677645	+0.087323 / -0.128856 = = -0,677679	0.00
8	3	+1.000000	-0.128856 / -0.128856 = = +1.000000	0.00
9	1	+2.023786	+0.133206 / +0.065817 = = +2.023884	0.00
9	2	-2.473762	-0.162812 / +0.065817 = = -2.473707	0.00
9	3	+1.000000	+0.065817 / +0.065817 = = +1.000000	0.00

Amplitude values of the bending moments M_i (tf·m)
in the girder of the first floor in the section of its connection with the left column
according to the 1-st, 2-nd, 3-rd, 7-th, 8-th, 9-th natural oscillation modes

Oscillation mode	**Theory**	**SCAD**	**Deviations, %**
1	+0.629	-0.629	0.00
2	-3.931	+3.930	0.03
3	-10.611	+10.610	0.01
7	+19.939	-19.941	0.01
8	+86.385	-86.385	0.00
9	+53.755	-53.759	0.01

Parameter	**Theory**	**SCAD**	**Deviations, %**
Amplitude values of the bending moment M in the girder of the first floor in the section of its connection with the left column from the total pulse load, tf·m	+165.576 (+173.075)	+175.241	5.84 (1.25)

The theoretical value of the bending moment in the girder corresponds to the time point t = 0.036 s from the start of the action of the pulse load;
The theoretical value of the bending moment in the girder given in the brackets was determined taking into account the phase shift of the harmonics.

Notes: In the analytical solution the natural periods T, amplitude values of the angular displacements UY_{ij} in the beam-to-column joints at the modal analysis, amplitude values of the bending moments in the girder of the first storey in the section of its connection with the left column according to the natural oscillation modes M_i and from the total pulse load M are determined according to the following formulas:

$$T_i = \frac{2 \cdot \pi}{\omega_i}\ ; \qquad \omega_i = \frac{\lambda_i^{\,2}}{h^2} \cdot \sqrt{\frac{E \cdot I_1}{m_1}}\ ;$$

λ_i – are determined from the following expression:

$$\Big\{\ 4 \cdot [ch(\lambda) \cdot \sin(\lambda) - sh(\lambda) \cdot \cos(\lambda)]^3 - 3 \cdot [sh(\lambda) - \sin(\lambda)]^2 \cdot [ch(\lambda) \cdot \sin(\lambda) - sh(\lambda) \cdot \cos(\lambda)]\Big\} \Big/ [1 - ch(\lambda) \cdot \cos(\lambda)]^3 +$$

$$+ \beta^2 \cdot \alpha^2 \cdot \Big\{\ 5 \cdot [ch(\lambda) \cdot \sin(\lambda) - sh(\lambda) \cdot \cos(\lambda)] \cdot [ch(\alpha \cdot \lambda) \cdot \sin(\alpha \cdot \lambda) - sh(\alpha \cdot \lambda) \cdot \cos(\alpha \cdot \lambda)]^2 +$$

$$+ 5 \cdot [sh(\alpha \cdot \lambda) - \sin(\alpha \cdot \lambda)]^2 \cdot [ch(\lambda) \cdot \sin(\lambda) - sh(\lambda) \cdot \cos(\lambda)] -$$

$$-10 \cdot [sh(\alpha \cdot \lambda) - \sin(\alpha \cdot \lambda)] \cdot [ch(\lambda) \cdot \sin(\lambda) - sh(\lambda) \cdot \cos(\lambda)] \cdot [ch(\alpha \cdot \lambda) \cdot \sin(\alpha \cdot \lambda) - sh(\alpha \cdot \lambda) \cdot \cos(\alpha \cdot \lambda)]\Big\} /$$

$$\Big/ \Big\{\ [1 - ch(\lambda) \cdot \cos(\lambda)] \cdot [1 - ch(\alpha \cdot \lambda) \cdot \cos(\alpha \cdot \lambda)]^2 \Big\} +$$

$$+ \beta \cdot \alpha \cdot \Big\{\ 8 \cdot [ch(\lambda) \cdot \sin(\lambda) - sh(\lambda) \cdot \cos(\lambda)]^2 \cdot [ch(\alpha \cdot \lambda) \cdot \sin(\alpha \cdot \lambda) - sh(\alpha \cdot \lambda) \cdot \cos(\alpha \cdot \lambda)] -$$

$$-8 \cdot [sh(\alpha \cdot \lambda) - \sin(\alpha \cdot \lambda)] \cdot [ch(\lambda) \cdot \sin(\lambda) - sh(\lambda) \cdot \cos(\lambda)]^2 -$$

$$-2 \cdot [sh(\lambda) - \sin(\lambda)]^2 \cdot [ch(\alpha \cdot \lambda) \cdot \sin(\alpha \cdot \lambda) - sh(\alpha \cdot \lambda) \cdot \cos(\alpha \cdot \lambda)] + 2 \cdot [sh(\lambda) - \sin(\lambda)]^2 \cdot [sh(\alpha \cdot \lambda) - \sin(\alpha \cdot \lambda)]\Big\} /$$

$$\Big/ \Big\{\ [1 - ch(\lambda) \cdot \cos(\lambda)]^2 \cdot [1 - ch(\alpha \cdot \lambda) \cdot \cos(\alpha \cdot \lambda)] \Big\} +$$

$$+ \beta^3 \cdot \alpha^3 \cdot \Big\{\ [ch(\alpha \cdot \lambda) \cdot \sin(\alpha \cdot \lambda) - sh(\alpha \cdot \lambda) \cdot \cos(\alpha \cdot \lambda)]^3 + 3 \cdot [sh(\alpha \cdot \lambda) - \sin(\alpha \cdot \lambda)]^2 \cdot [ch(\alpha \cdot \lambda) \cdot \sin(\alpha \cdot \lambda) - sh(\alpha \cdot \lambda) \cdot \cos(\alpha \cdot \lambda)] -$$

$$-3 \cdot [sh(\alpha \cdot \lambda) - \sin(\alpha \cdot \lambda)] \cdot [ch(\alpha \cdot \lambda) \cdot \sin(\alpha \cdot \lambda) - sh(\alpha \cdot \lambda) \cdot \cos(\alpha \cdot \lambda)]^2 - [sh(\alpha \cdot \lambda) - \sin(\alpha \cdot \lambda)]^3\ \Big\} \Big/$$

$$\Big/ \Big\{\ [1 - ch(\alpha \cdot \lambda) \cdot \cos(\alpha \cdot \lambda)]^3 \Big\} = 0$$, where:

$$\alpha = \frac{l}{h} \cdot \sqrt[4]{\frac{m_2 \cdot I_1}{m_1 \cdot I_2}}\ , \qquad \beta = \frac{h}{l} \cdot \frac{I_2}{I_1}\ ;$$

$$UY_{i1} = \Big\{\ [ch(\lambda_i) \cdot \sin(\lambda_i) - sh(\lambda_i) \cdot \cos(\lambda_i)] \cdot [1 - ch(\alpha \cdot \lambda_i) \cdot \cos(\alpha \cdot \lambda)_i] +$$

$$+ \beta \cdot \alpha \cdot \Big[\sin(\alpha \cdot \lambda_i) \cdot [ch(\alpha \cdot \lambda_i) + 1] - sh(\alpha \cdot \lambda_i) \cdot [\cos(\alpha \cdot \lambda_i) + 1]\ \Big] \cdot [1 - ch(\lambda_i) \cdot \cos(\lambda)_i]\ \Big\} \cdot UY_{i3} /$$

$$\Big/ \Big\{\ 2 \cdot [ch(\lambda_i) \cdot \sin(\lambda_i) - sh(\lambda_i) \cdot \cos(\lambda_i)] \cdot [1 - ch(\alpha \cdot \lambda_i) \cdot \cos(\alpha \cdot \lambda)_i] +$$

$$+ \beta \cdot \alpha \cdot \Big[\sin(\alpha \cdot \lambda_i) \cdot [ch(\alpha \cdot \lambda_i) + 1] - sh(\alpha \cdot \lambda_i) \cdot [\cos(\alpha \cdot \lambda_i) + 1]\ \Big] \cdot [1 - ch(\lambda_i) \cdot \cos(\lambda)_i]\ \Big\},$$

$$UY_{i2} = -\Big\{\ [ch(\lambda_i) \cdot \sin(\lambda_i) - sh(\lambda_i) \cdot \cos(\lambda_i)] \cdot [1 - ch(\alpha \cdot \lambda_i) \cdot \cos(\alpha \cdot \lambda)_i] +$$

$$+ \beta \cdot \alpha \cdot \Big[\sin(\alpha \cdot \lambda_i) \cdot [ch(\alpha \cdot \lambda_i) + 1] - sh(\alpha \cdot \lambda_i) \cdot [\cos(\alpha \cdot \lambda_i) + 1]\ \Big] \cdot [1 - ch(\lambda_i) \cdot \cos(\lambda)_i]\ \Big\} \cdot UY_{i3} /$$

$$\Big/ \Big\{\ [sh(\lambda_i) - \sin(\lambda_i)] \cdot [1 - ch(\alpha \cdot \lambda_i) \cdot \cos(\alpha \cdot \lambda)_i] \Big\},$$

$$UY_{i3} = 1.0\ ;$$

$$M = \sum_{i=1}^{N} M_i \cdot \sin(\omega_i \cdot t)$$ - without taking into account the phase shift of the harmonics,

t – are determined from the following expression:

$$\sum_{i=1}^{N} \omega_i \cdot M_i \cdot \cos(\omega_i \cdot t) = 0 \, ,$$

$$M = \sum_{i=1}^{q-1} |M_i| \cdot \sin\left(\frac{\pi \cdot \omega_i}{2 \cdot \omega_q}\right) + \sum_{i=q}^{N} |M_i| \, , \text{ at } \quad |M_q| > |M_i| \, , \ (i \neq q)$$ - taking into account the phase shift of the harmonics, where:

$$M_i = A_i \cdot M_{gi} \, , \quad A_i = \frac{A_{i1}}{A_{i2} \cdot \omega_i} \, ,$$

$$A_{i1} = \frac{2 \cdot s \cdot UY_{i1}}{\lambda_i^2} \cdot \left\{ h^2 \cdot \frac{ch(\lambda_i) - \cos(\lambda_i) - sh(\lambda_i) \cdot \sin(\lambda_i)}{1 - ch(\lambda_i) \cdot \cos(\lambda_i)} - \frac{l^2}{\alpha^2} \cdot \frac{ch(\alpha \cdot \lambda_i) - \cos(\alpha \cdot \lambda_i) - sh(\alpha \cdot \lambda_i) \cdot \sin(\alpha \cdot \lambda_i)}{1 - ch(\alpha \cdot \lambda_i) \cdot \cos(\alpha \cdot \lambda_i)} \right\} ,$$

$$A_{i2} = \frac{m_1 \cdot h^3}{\lambda_i^3} \cdot \left\{ \left(UY_{i1}^2 + UY_{i2}^2 + \frac{UY_{i3}^2}{2} \right) \cdot \left[\lambda_i \cdot \frac{[sh(\lambda_i) - \sin(\lambda_i)]^2}{[1 - ch(\lambda_i) \cdot \cos(\lambda_i)]^2} - \frac{ch(\lambda_i) \cdot \sin(\lambda_i) - sh(\lambda_i) \cdot \cos(\lambda_i)}{1 - ch(\lambda_i) \cdot \cos(\lambda_i)} \right] + \right.$$

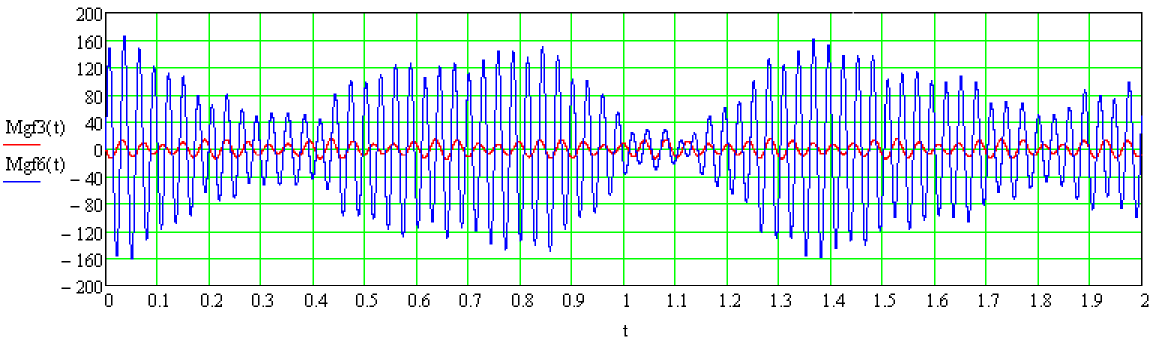
$$\left. (UY_{i1} \cdot UY_{i2} + UY_{i2} \cdot UY_{i3}) \cdot \left[\lambda_i \cdot \frac{[sh(\lambda_i) - \sin(\lambda_i)] \cdot [ch(\lambda_i) \cdot \sin(\lambda_i) - sh(\lambda_i) \cdot \cos(\lambda_i)]}{[1 - ch(\lambda_i) \cdot \cos(\lambda_i)]^2} - \lambda_i \cdot \frac{ch(\lambda_i) - \cos(\lambda_i)}{1 - ch(\lambda_i) \cdot \cos(\lambda_i)} - \frac{sh(\lambda_i) - \sin(\lambda_i)}{1 - ch(\lambda_i) \cdot \cos(\lambda_i)} \right] \right\} +$$

$$+ \frac{m_2 \cdot l^3}{2 \cdot \alpha^3 \cdot \lambda_i^3} \cdot \left\{ \left(UY_{i1}^2 + UY_{i2}^2 + UY_{i3}^2 \right) \cdot \left[\alpha \cdot \lambda_i \cdot \frac{[sh(\alpha \cdot \lambda_i) - \sin(\alpha \cdot \lambda_i)]^2}{[1 - ch(\alpha \cdot \lambda_i) \cdot \cos(\alpha \cdot \lambda_i)]^2} - \frac{ch(\alpha \cdot \lambda_i) \cdot \sin(\alpha \cdot \lambda_i) - sh(\alpha \cdot \lambda_i) \cdot \cos(\alpha \cdot \lambda_i)}{1 - ch(\alpha \cdot \lambda_i) \cdot \cos(\alpha \cdot \lambda_i)} - \right. \right.$$

$$- \alpha \cdot \lambda_i \cdot \frac{[sh(\alpha \cdot \lambda_i) - \sin(\alpha \cdot \lambda_i)] \cdot [ch(\alpha \cdot \lambda_i) \cdot \sin(\alpha \cdot \lambda_i) - sh(\alpha \cdot \lambda_i) \cdot \cos(\alpha \cdot \lambda_i)]}{[1 - ch(\alpha \cdot \lambda_i) \cdot \cos(\alpha \cdot \lambda_i)]^2} + \alpha \cdot \lambda_i \cdot \frac{ch(\alpha \cdot \lambda_i) - \cos(\alpha \cdot \lambda_i)}{1 - ch(\alpha \cdot \lambda_i) \cdot \cos(\alpha \cdot \lambda_i)} +$$

$$\left. \left. + \frac{sh(\alpha \cdot \lambda_i) - \sin(\alpha \cdot \lambda_i)}{1 - ch(\alpha \cdot \lambda_i) \cdot \cos(\alpha \cdot \lambda_i)} \right] \right\} .$$

$$M_{gi} = \frac{E \cdot l_2}{l} \cdot \alpha \cdot \lambda_i \cdot \frac{\sin(\alpha \cdot \lambda_i) \cdot (ch(\alpha \cdot \lambda_i) + 1) - sh(\alpha \cdot \lambda_i) \cdot (\cos(\alpha \cdot \lambda_i) + 1)}{1 - ch(\alpha \cdot \lambda_i) \cdot \cos(\alpha \cdot \lambda_i)} \cdot UY_{i2}$$

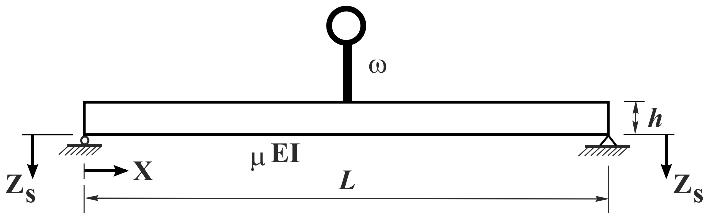


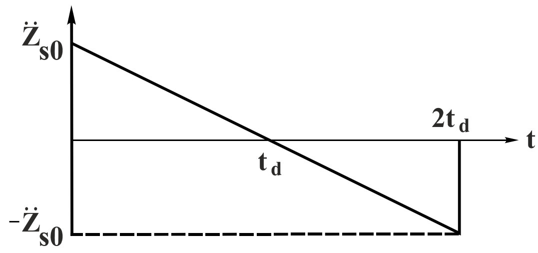
Graph of the variation of the bending moments M (tf·m) with time t (s)
in the girder of the first floor in the sections of its connections with the columns
from the total pulse load
taking into account 3 and 6 symmetric natural oscillation modes

The significant deviation of the results (>5%) in the amplitude values of the bending moment is due to the fact that the summation over the modes in the source *Analysis of Structures Subject to Pulse Actions* is performed without taking into account the phase shift. Later recommendations contain the requirement to take into account the phase shift. In this case the deviation from the theory is 1.25%.

Dynamic Analysis of Buildings and StructuresDynamic Analysis of Buildings and StructuresAnalysis of Structures Subject to Pulse ActionsAnalysis of Structures Subject to Pulse Actions

Seismic Response of a Beam according to the Linear Spectral Theory





Objective: The linear spectral method (determination of the response of a structure subjected to the seismic action given by the accelerogram)

***Initial data files*:** **LinSpectral.SPR** – design model
DIN_B_RS.SPC – accelerogram

Problem formulation: The simply supported beam of constant cross-section with the uniformly distributed mass μ is subjected to the kinematic excitation of supports according to the specified accelerogram:

$$\ddot{z}(t) = \ddot{z}_{s0} \cdot \left(1 - \frac{t}{t_d}\right).$$

It is necessary to determine (by the LST) seismic displacements and the corresponding maximum bending stress.

References: John M. Biggs, Introduction to Structural Dynamics, McGraw-Hill Book Companies, New York, 1964, p.262;

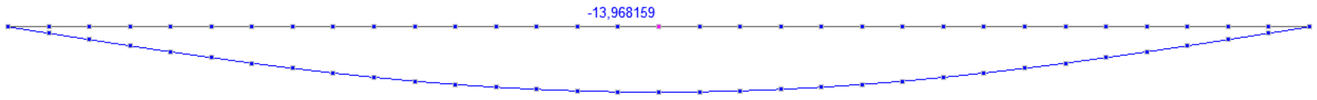
Initial data:

$E = 3.0 \cdot 10^7$ psi = $2.1092 \cdot 10^7$ tf/m^2	- elastic modulus;
$I = 333.333$ in^4 = $138.7448 \cdot 10^{-6}$ m^4	- cross-sectional moment of inertia of the beam.
h = 14 in = 0.3556 m	- height of the cross-section of the beam;
L = 240 in = 6.0960 m	- beam span length;
$\mu = 0.2$ lb·sec^2/in^2 = 0.1406 tf·s^2/m^2	- value of the uniformly distributed mass of the beam;
$\ddot{z}_{s0}$ = ±386.2200 in/sec^2 = ±9.81 m/s^2	- amplitude values of the acceleration of the supports according to the accelerogram;
t_d = 0.10 sec = 0.10 s	- half-interval of the kinematic excitation of supports;
g = 386.2200 in/sec^2 =9.81 m/s^2	- gravitational acceleration;

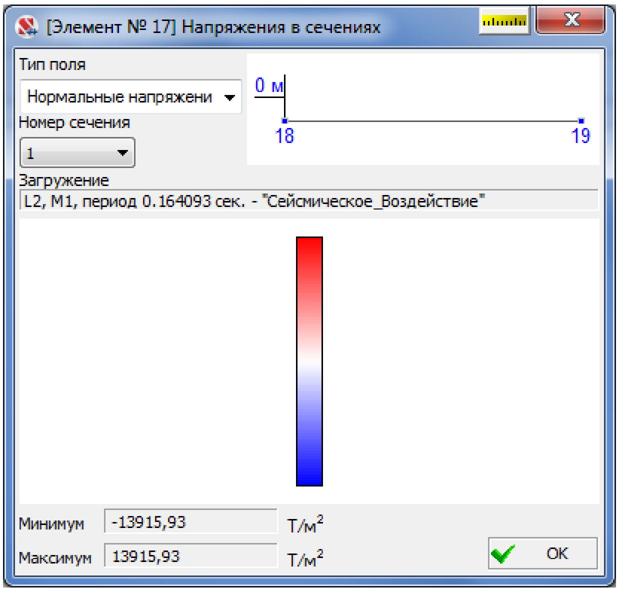
Finite element model: Design model – 32 bar elements of type 3. Boundary conditions of the simply supported ends of the beam are provided by imposing constraints in the direction of the degree of freedom Z. The dimensional stability of the design model is provided by imposing a constraint in the node of the cross-section along the symmetry axis of the beam in the direction of the degree of freedom UX. The distributed mass is specified by transforming the static load from the self-weight of the beam μ·g.
The kinematic excitation of supports is described by the graph of the acceleration variation with time (accelerogram) and is given in the form of the action along the Z axis of the global coordinate system (direction cosines to the X, Y, Z axes: 0.00, 0.00, 1.00) with the scale factor to the values of the accelerogram equal to 1.00. The height of the beam structure in the model is directed along the Z axis of the global coordinate system. The dissipation factor is taken as $\xi = 0.000001$. The intervals between the time points of the graph of the acceleration variation with time are equal to $\Delta t = 0.01$ s. When plotting the graph the acceleration is taken with the values $\ddot{z}(t) = \ddot{z}_{s0} \cdot (1 - n \cdot \Delta t / t_d)$ at the time points $n \cdot \Delta t$. The conversion factor for the added static loading is equal to k = 1.000 (mass generation). Number of nodes in the design model – 33. The determination of the natural oscillation modes and natural frequencies is performed by the method of subspace iteration. The matrix of concentrated masses is used in the calculation.

Results in SCAD

The 1-st natural frequency and the 1-st natural oscillation mode of the beam, seismic bending stresses on the bottom face of the beam and displacements are determined in the result of the calculation.



Design and deformed models



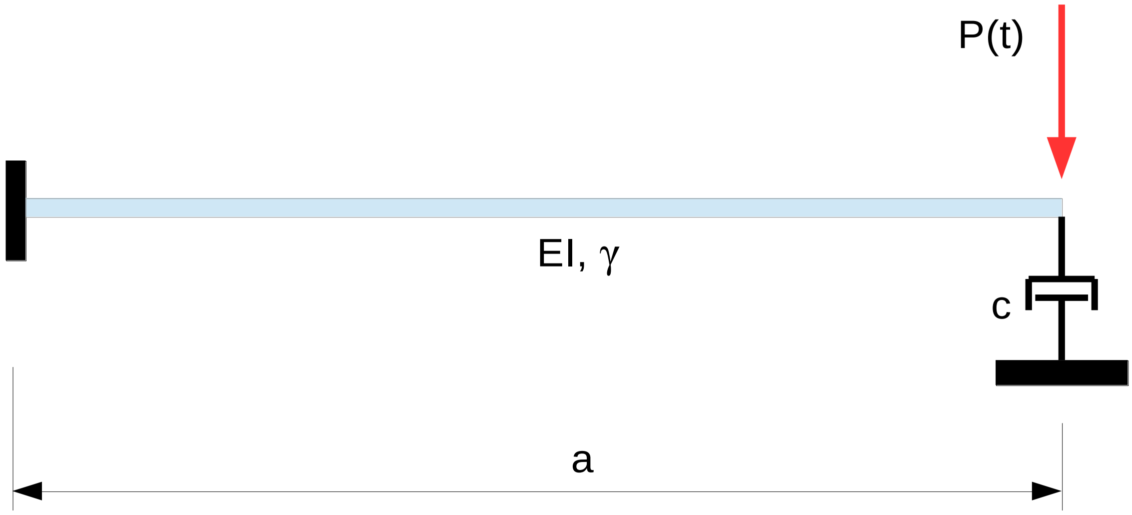
Normal stresses in the middle of the span

Comparison of solutions:

	Source	**SCAD**	**Deviation**
1-st natural frequency (Hz)	6,0979	6.0941	0,06 %
Displacement of the beam in the middle of the span (m)	0,01422	0,01397	1.75 %
Maximum normal stress (T/m^2)	14172,70	13915,93	1.85 %

Non-uniform Damping. Return to the Static Equilibrium Position

Objective: check that once the load ceases to change with time (we will call this value the static load component), the mechanical system subjected to the short-term loads and under damping returns to the static equilibrium position corresponding to the static load component.



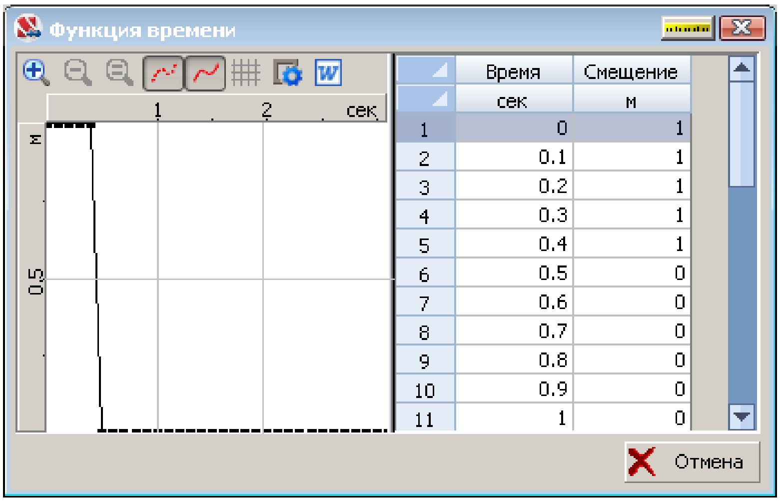
Cantilever beam with a local damper

Initial data files: **beam_local_damp_1.SPR** - design model
FileTimeFile.txt - time function

Problem formulation:

The cantilever beam with a 0.2 × 0.5 m rectangular cross-section, length of a = 3 m, and the elastic modulus of E = 23053.5 MN/m^2 is considered. The specific weight is γ = 0.0245 MN/m^3. The beam is divided into 3 finite elements. The matrix of concentrated masses is used. The maximum value of the force P is 0.01 MN.

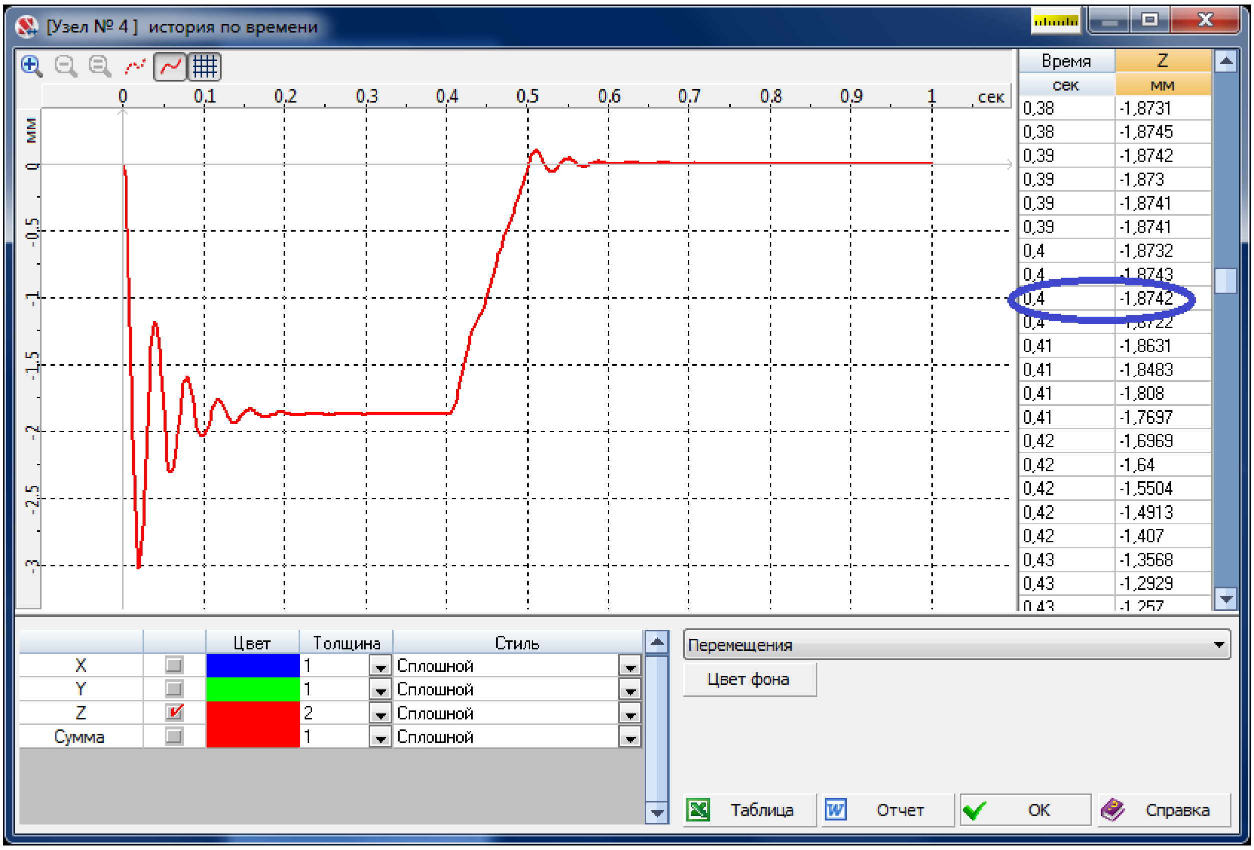
The load vs. time relationship is given in the figure:



Finite element model: Design model – general type system, 6 general type bar elements (type 5) and one single-node damper (type 56). Number of nodes in the design model – 8. The matrix of concentrated masses is used in the calculation.

Results in SCAD

The vertical displacement of the cantilever end vs. time relationship at c = 0.01 MN·s/m is given in the figure.



Vertical displacement of the cantilever end

Only the damping caused by the local damper is taken into account. When a load is suddenly applied, transverse oscillations of the beam appear and are rapidly damped. The value of the deflection corresponding to the state of static equilibrium at the force value of 0.01 MN is circled in the figure. The exact solution of the corresponding static problem is $w_{st} = Pa^3/(3EI) = -0.0018739$ m. When the dynamic problem is solved by the Newmark method, the integration step is taken as 0.001 s. The result is accurate to 3 significant digits. This suggests that after the damping of oscillations we come to a static solution of this problem. At $0.4 \text{ s} < t \leq 0.5$ s the load decreases to zero. Oscillations appear again and are rapidly damped. At $t = 1$ s the value of the normal deflection is $w = -1.533 \cdot 10^{-7}$ m, which is a good approximation of zero, the exact value of the no-load static deflection, in comparison with the maximum (absolute value) deflection $w_{max} = -3.024 \cdot 10^{-3}$ m.

Thus, the numerical solution obtained by the Newmark method, after the damping of oscillations, converges to the static solution of this problem, which confirms the reliability of the obtained results.

Non-uniform Damping

Objective: comparison of the solution of the problem (Fig. 1) by the Newmark method (SCAD) with the numerical solution obtained in MathCAD.

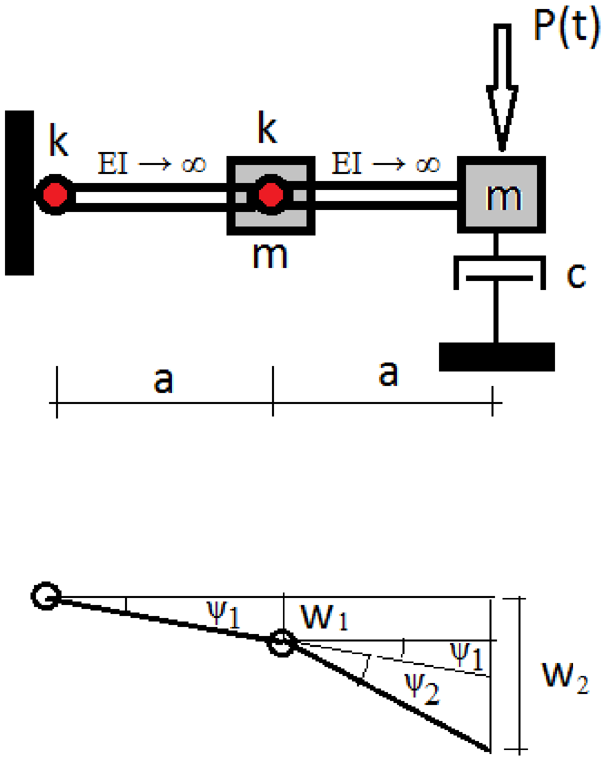


Figure 1. Two rigid weightless bodies are connected to each other and to the rigid support by springs with the stiffness *k*. The inertial properties of the system are represented by concentrated masses *m*. The local damper with the damping ratio *c* connects the end mass with the support.

Initial data files: **Test_local_damping.SPR**- design model
TimeHist_1.txt - time function
local_damping.xmcd – MathCAD file

Finite element model: Design model – general type system, two finite elements of the elastic constraint (type 55), two finite elements of the rigid body (type 100) and one single-node damper (type 56). Number of nodes in the design model – 5. The matrix of concentrated masses is used in the calculation.

Solution description:

A deformed model is shown in Fig. 1. The following kinematic relations follow from it:

$$\psi_1 = \frac{w_1}{a}; \quad \psi_2 = \frac{w_2 - w_1}{a} - \psi_1 = \frac{w_2 - 2w_1}{a} \ , \tag{1}$$

Here w_1, w_2 – normal deflections, and ψ_1, ψ_2 – deflection angles of rigid bars. The total potential energy of the system is given in the form

$$Э = \Pi + W = \frac{1}{2}k\psi_1^2 + \frac{1}{2}k\psi_2^2 - P\,w_2 \ , \tag{2}$$

where Π and W – potential energy of elastic deformations and change in the potential of external forces. The kinetic energy of the system T is given below:

$$T = \frac{1}{2}m\dot{w}_1^2 + \frac{1}{2}m\dot{w}_2^2 \ . \tag{3}$$

Applying Hamilton's variational principle

$$\delta\int_{t1}^{t2} L dt = 0 \ , \tag{4}$$

where $L = T - Э$, and adding the viscous friction forces we obtain the following equations of motion:

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{Kx} = \mathbf{p}(t) \ , \tag{5}$$

where

$$\mathbf{M} = \begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix}, \quad \mathbf{C} = \begin{pmatrix} 0 & 0 \\ 0 & c \end{pmatrix}, \quad \mathbf{K} = \frac{k}{a^2}\begin{pmatrix} 5 & -2 \\ -2 & 1 \end{pmatrix}, \quad \mathbf{x} = \begin{pmatrix} w_1 \\ w_2 \end{pmatrix}, \quad \mathbf{p}(t) = \begin{pmatrix} 0 \\ P(t) \end{pmatrix} . \tag{6}$$

The system of equations (5) is solved by MathCAD using the *rkfixed* procedure, which implements the fourth-order Runge-Kutta method. The function $P(t)$ is given by the following algorithm:

```
P(t) := | p ← 1  if  t ≤ 0.01
        | p ← −100·t + 2  if  t > 0.01 ∧ t ≤ 0.02
        | p ← 0  if  t > 0.02
        | return p
```

. (7)

The integration interval is taken as t ∈ [0, 1], a = 1 m, k = 1000 MN·m/rad, m = 10^6 kg, c = 10 MN·s/m, and the number of points approximating the unknown function is npoint = 1000. The Runge-Kutta method is an explicit integration method, therefore, it is conditionally stable. At npoint = 10 the unstable behavior of the solution is observed, the results for npoint = 100 and npoint = 1000 are slightly different at the right end of the interval, and the results for npoint = 1000 and npoint = 10000 are virtually identical. Therefore, we believe that the numerical solution for npoint = 1000 leads to virtually accurate results.

The SCAD design model is shown in Fig. 2, and the comparison of the above solution with the results obtained by the Newmark method (SCAD) is given in Fig. 4. When the Newmark method is used, the integration step is taken as Δt = 0.0001 s.

The load vs. time relationship corresponding to (7) is given in Fig. 3.

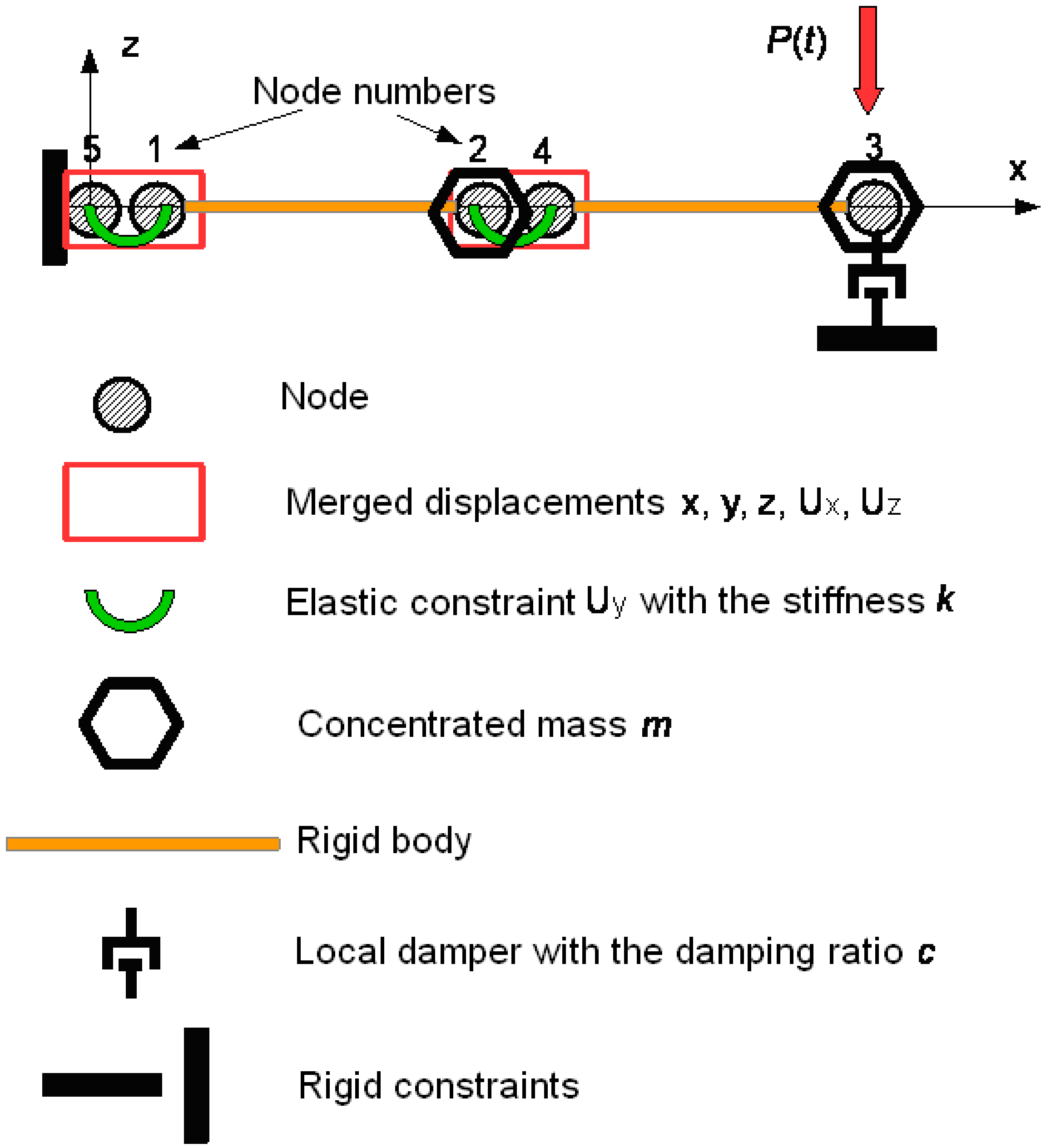
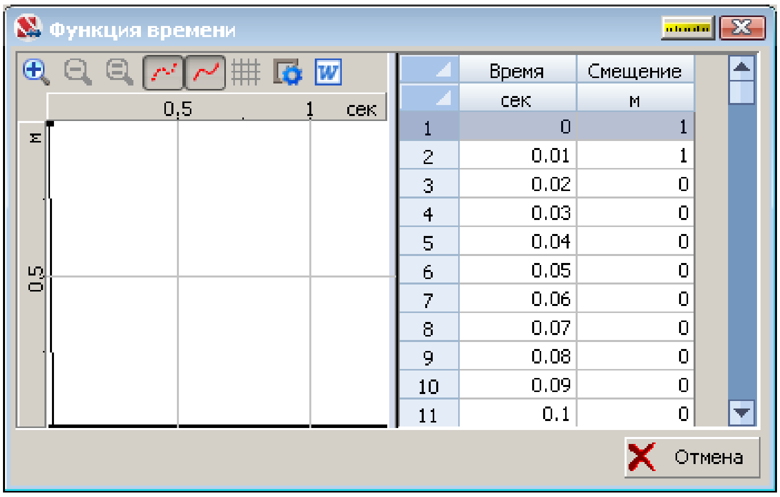


Figure 2. SCAD design model



	Время сек	Смещение м
1	0	1
2	0.01	1
3	0.02	0
4	0.03	0
5	0.04	0
6	0.05	0
7	0.06	0
8	0.07	0
9	0.08	0
10	0.09	0
11	0.1	0

Figure 3. Load vs. time relationship

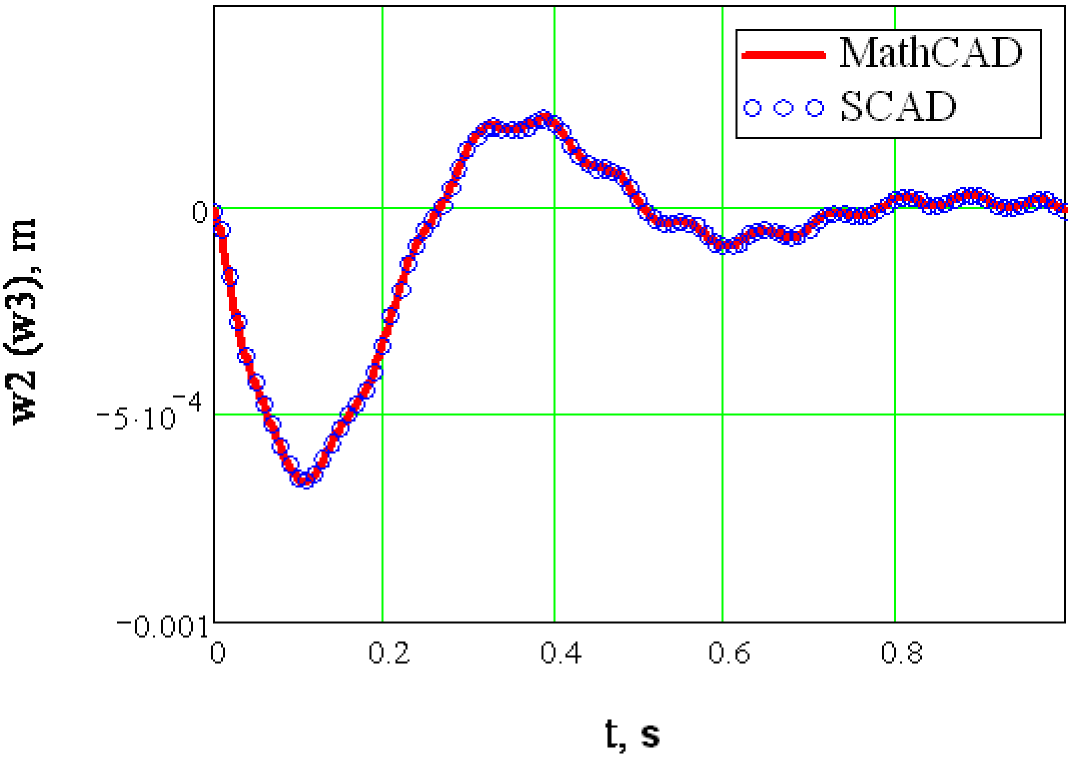


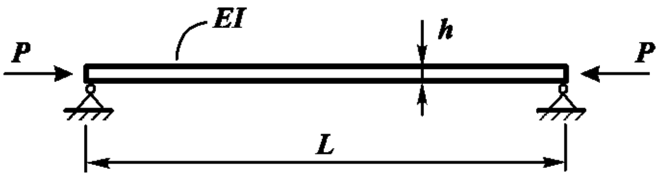
Figure 4. Vertical displacement of the cantilever end

At the time point t = 0.1 s the displacement of the cantilever end reaches a value close to the maximum (absolute value) one, and the displacement obtained by MathCAD is $w_2 = -6.5393 \cdot 10^{-4}$ (Fig. 1), and the displacement obtained by the Newmark method is $w_3 = -6.5170 \cdot 10^{-4}$ (Fig. 3).

Thus, the results of numerical solutions obtained by MathCAD and the Newmark method (SCAD) are virtually identical which confirms the reliability of both these methods.

Linear Stability

Stability of a Simply Supported Beam Subjected to a Concentrated Longitudinal Force



Objective: Determination of the critical value of a concentrated longitudinal force acting on a simply supported beam corresponding to the moment of its buckling.

Initial data file: **CB01_v11.3.SPR**

Problem formulation: The beam of square cross-section simply supported on both ends is subjected to a concentrated longitudinal force *P*. Determine the critical value of the concentrated longitudinal force P_{cr}, corresponding to the moment of the buckling of the beam.

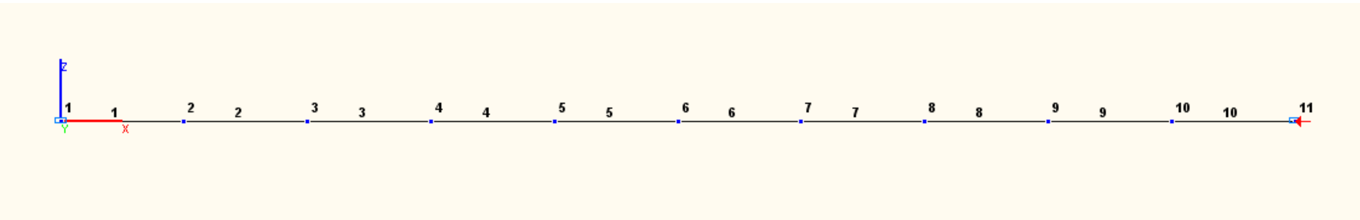
References: D. O. Brush and B. O. Almroth, Buckling of Bars, Plates and Shells, New York, McGraw-Hill Co., 1975, p. 22.

Initial data:

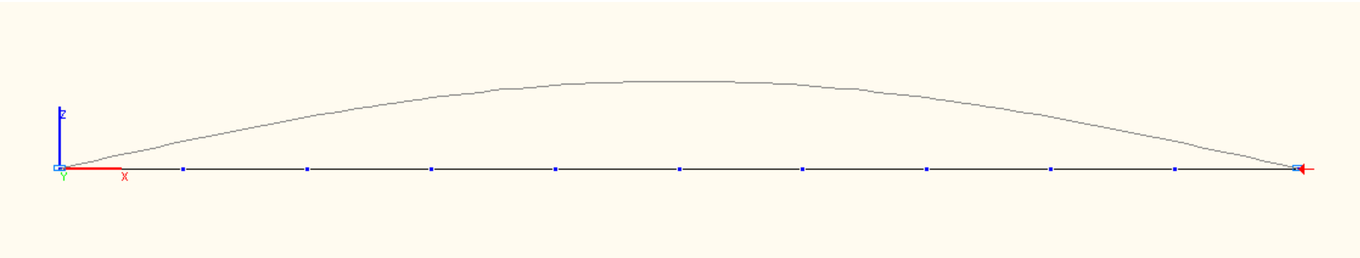
$E = 3.0 \cdot 10^7$ Pa	- elastic modulus,
L = 50.0 m	- beam length;
h = 1.0 m	- side of the cross-section of the beam;
$P = 1.0 \cdot 10^3$ N	- initial value of the concentrated longitudinal force.

Finite element model: Design model – plane frame, 10 elements of type 10. The spacing of the finite element mesh along the longitudinal axis (along the X axis of the global coordinate system) is 5.0 m. Boundary conditions of the roller supported (left) end are provided by imposing constraints in the directions of the degrees of freedom X, Z and those of the simply supported (right) end are provided by imposing constraints in the direction of the degree of freedom Z. The action with the initial value of the concentrated longitudinal force *P* is specified on the simply supported (right) end. Number of nodes in the design model – 11.

Results in SCAD



Design model



Buckling mode

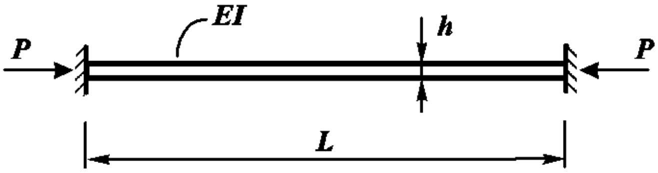
Comparison of solutions:

Parameter	**Theory**	**SCAD**	**Deviation, %**
Critical value of the concentrated longitudinal force P_{cr}, N	9869.6	9.8696·1000 = = 9869.6	0.00

Notes: In the analytical solution the critical value of the concentrated longitudinal force P_{cr} is determined according to the following formula:

$$P_{cr} = \frac{\pi^2 \cdot E \cdot I}{L^2}\text{, where:} \quad I = \frac{h^4}{12}.$$

Stability of a Clamped Beam Subjected to a Concentrated Longitudinal Force



Objective: Determination of the critical value of a concentrated longitudinal force acting on a clamped beam corresponding to the moment of its buckling.

Initial data file: **CB02_v11.3.SPR**

Problem formulation: The beam of square cross-section clamped on both ends is subjected to a concentrated longitudinal force *P*. Determine the critical value of the concentrated longitudinal force P_{cr}, corresponding to the moment of the buckling of the beam.

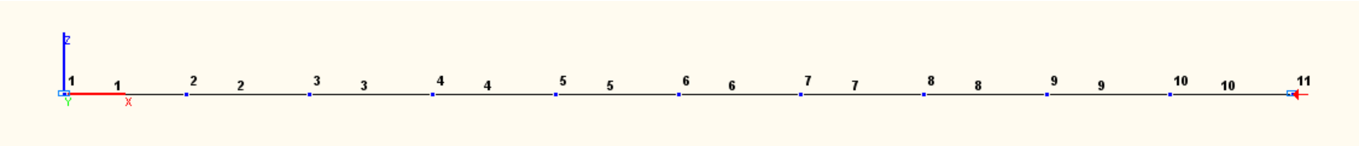
References: D. O. Brush and B. O. Almroth, Buckling of Bars, Plates and Shells, New York, McGraw-Hill Co., 1975, p. 22.

Initial data:

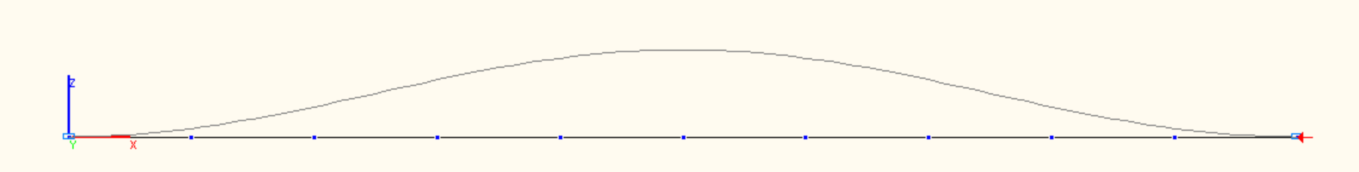
E = $3.0 \cdot 10^7$ Pa	- elastic modulus,
L = 50.0 m	- beam length;
h = 1.0 m	- side of the cross-section of the beam;
P = $1.0 \cdot 10^4$ N	- initial value of the concentrated longitudinal force.

Finite element model: Design model – plane frame, 10 elements of type 10. The spacing of the finite element mesh along the longitudinal axis (along the X axis of the global coordinate system) is 5.0 m. Boundary conditions of the clamped (left) end are provided by imposing constraints in the directions of the degrees of freedom X, Z, UY and those of the (right) end with a clamping floating along the beam axis are provided by imposing constraints in the directions of the degrees of freedom Z, UY. The action with the initial value of the concentrated longitudinal force *P* is specified on the (right) end with a clamping floating along the beam axis. Number of nodes in the design model – 11.

Results in SCAD



Design model



Buckling mode

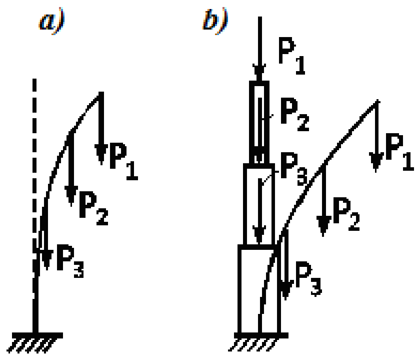
Comparison of solutions:

Parameter	Theory	SCAD	Deviation, %
Critical value of the concentrated longitudinal force P_{cr}, N	39478.4	3.94783·1000 = =39478.3	0.00

Notes: In the analytical solution the critical value of the concentrated longitudinal force P_{cr} is determined according to the following formula:

$$P_{cr} = \frac{4 \cdot \pi^2 \cdot E \cdot I}{L^2}, \text{ where: } \qquad I = \frac{h^4}{12}.$$

Stability of a Cantilever Column with a Step Change in Cross-Section Subjected to Longitudinal Compressive Forces Applied to the Intermediate and End Sections



Objective: Determination of the critical values of longitudinal compressive forces applied to the intermediate and end sections of the cantilever column with a step change in cross-section corresponding to the moment of its buckling. Determination of the unsupported lengths of the column steps.

Initial data files:

File name	Description
Leg_of_varying_section_Beam.SPR	Bar model
Leg_of_varying_section_Shell.SPR	Shell element model

- ***Problem formulation:*** The cantilever column with a step change in cross-section is subjected to longitudinal forces P_i, applied to the intermediate and end sections. Determine the critical values of the longitudinal compressive forces P_{cri}, corresponding to the moment of the buckling of the cantilever column. Determine the unsupported lengths of the column steps L_{0i}.

References: S. P. Timoshenko, Stability of Bars, Plates and Shells, Moscow, Nauka, 1971, p. 166.
S.D. Ponomarev, V.L. Biederman, K.K. Likharev, V.M. Makushin, N.N. Malinin, V.I. Feodos'yev, Fundamentals of Modern Methods for Strength Analysis in Mechanical Engineering. Dynamic Analysis. Stability. Creep. Moscow, Mashgiz, 1952, p. 543, 555.

Initial data:

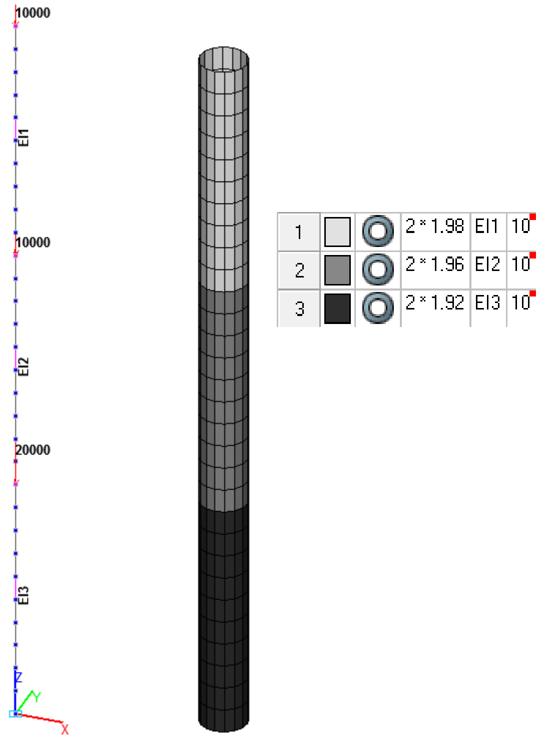
L_1 = 10.0 m	- length of the first (upper) step of the column;
L_2 = 10.0 m	- length of the second (middle) step of the column;
L_3 = 10.0 m	- length of the third (lower) step of the column;
D_1 = 2.0 m	- outer diameter of the circular hollow section of the first step of the column;
D_2 = 2.0 m	- outer diameter of the circular hollow section of the second step of the column;
D_3 = 2.0 m	- outer diameter of the circular hollow section of the third step of the column;
t_1 = 0.01 m	- thickness of the circular hollow section of the first step of the column;
t_2 = 0.02 m	- thickness of the circular hollow section of the second step of the column;
t_3 = 0.04 m	- thickness of the circular hollow section of the third step of the column;
$E = 2.06 \cdot 10^8$ kN/m^2	- elastic modulus of the column material;
$\nu = 0.3$	- Poisson's ratio;
$P_1 = 1.0 \cdot 10^4$ kN/m^2	- initial value of the compressive longitudinal force applied to the upper edge of the first step of the column;
$P_2 = 1.0 \cdot 10^4$ kN/m^2	- initial value of the compressive longitudinal force applied to the upper edge of the second step of the column;
$P_3 = 2.0 \cdot 10^4$ kN/m^2	- initial value of the compressive longitudinal force applied to the upper edge of the third step of the column.

Finite element model: Two design models are considered:
Bar model, design model – plane frame, 30 bar elements of the plane frame of type 2. The spacing of the finite element mesh along the longitudinal axis of the column (along the X1 axes of the local coordinate systems) is 1.0 m. Boundary conditions are provided by imposing constraints on the clamped node of the

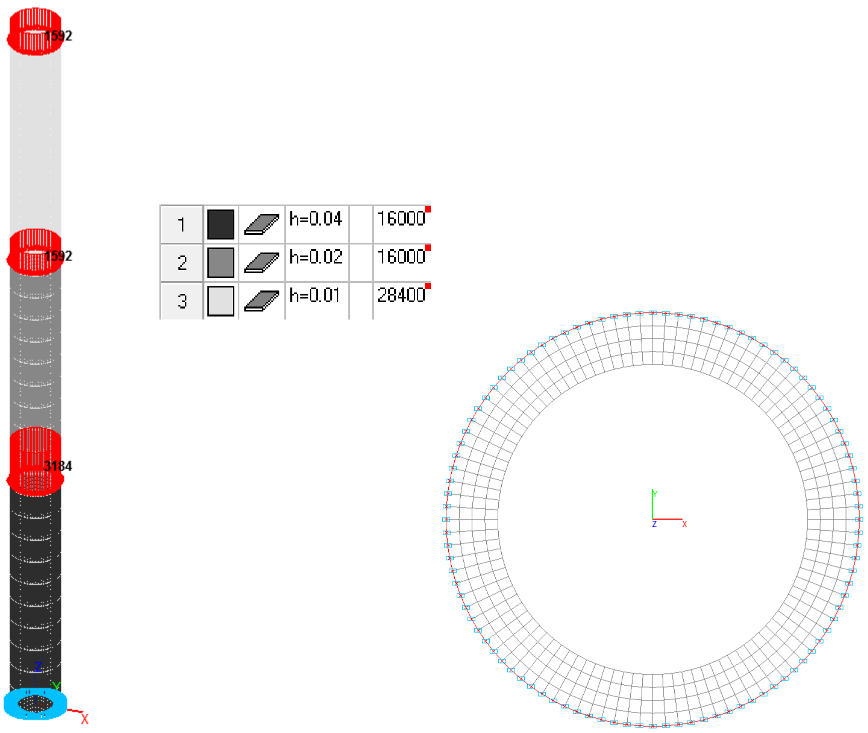
column in the directions of the degrees of freedom X, Y, Z, UX, UY, UZ. Concentrated forces with the initial values P_1, P_2, P_3 are specified in the nodes of the upper edges of the column steps. Number of nodes – 31.

Shell element model, design model – general type system, 60400 four-node shallow shell elements allowing for shear of type 150. The spacing of the finite element mesh of the column in the circumferential direction (along the X1 axes of the local coordinate systems) is 3.6°, and along the longitudinal axis (along the Y1 axes of the local coordinate systems) is 0.0625 m. Horizontal ring stiffeners 0.25 m wide are arranged with a vertical spacing of 1.00 m inside the column in order to prevent the local buckling of its shell. The spacing of the finite element mesh of the stiffeners in the radial direction (along the Y1 axes of the local coordinate systems) is 0.0625 m. Boundary conditions are provided by imposing constraints on the nodes of the clamped edge in the directions of the degrees of freedom X, Y, Z, UX, UY, UZ. Loads uniformly distributed along the line with the initial values $P_1/(\pi \cdot D_1)$, $P_2/(\pi \cdot D_2)$, $P_3/(\pi \cdot D_3)$ are specified in the nodes of the upper edges of the column steps. Number of nodes – 60500.

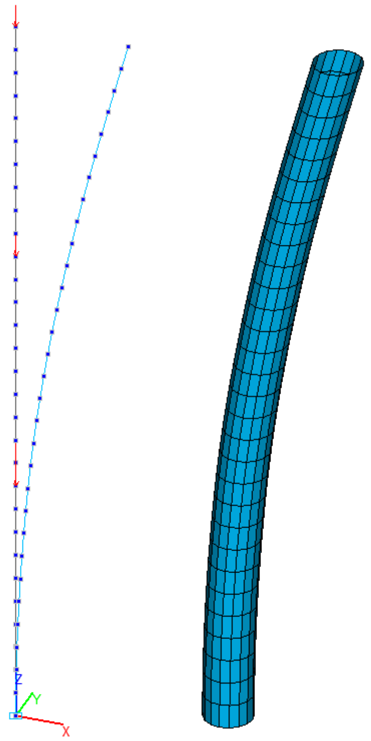
Results in SCAD



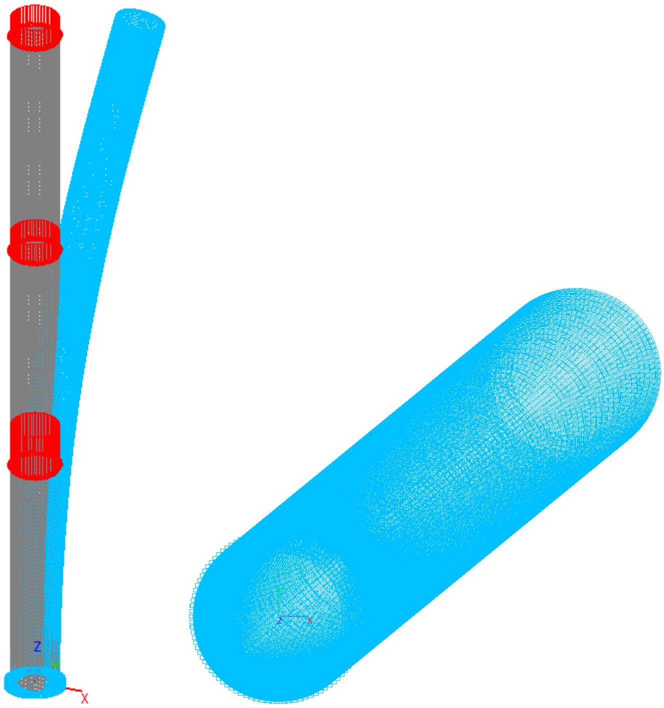
Bar model



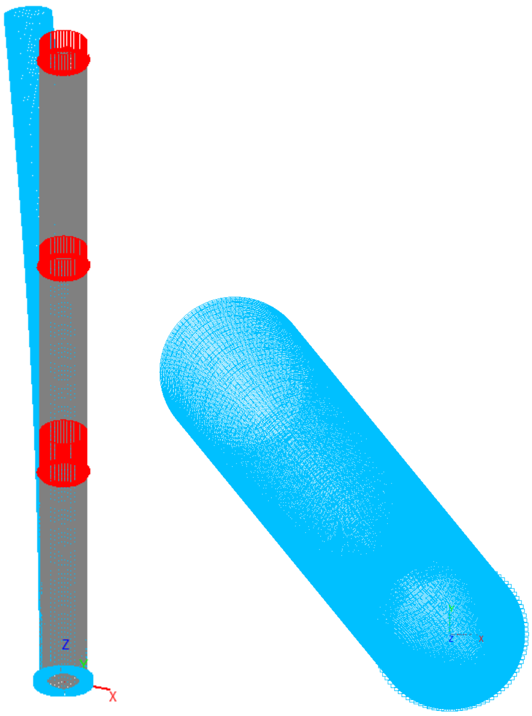
Shell element model



1-st buckling mode for the bar model



1-st buckling mode for the shell element model



2-nd buckling mode for the shell element model

Comparison of solutions:

Parameter	Theory	SCAD			
		Bar model	**Deviation, %**	**Shell element model**	**Deviation, %**
Critical value of the concentrated longitudinal force applied to the upper edge of the first step P_{cr1}, kN	32978	3.297920· ·10000 = = 32979	0.00	3.394470· ·10000 = = 33945	2.93
Critical value of the concentrated longitudinal force applied to the upper edge of the second step P_{cr2}, kN	32978	3.297920· ·10000 = = 32979	0.00	3.394470· ·10000 = = 33945	2.93
Critical value of the concentrated longitudinal force applied to the upper edge of the third step P_{cr3}, kN	65957	3.297920· ·20000 = = 65958	0.00	3.394470· ·20000 = = 67890	2.93
Unsupported length of the first column step L_{01}, m	43.680	43.681	0.00	—	—
Unsupported length of the second column step L_{02}, m	43.353	43.353	0.00	—	—
Unsupported length of the third column step L_{03}, m	42.704	42.705	0.00	—	—

Notes: In the analytical solution the critical values of the longitudinal compressive forces P_{cri}, corresponding to the moment of the buckling of the cantilever column and unsupported lengths of the column steps L_{0i} can be determined according to the following formulas:

$$P_{cr1} = k \cdot P_1; \qquad P_{cr2} = k \cdot P_2; \qquad P_{cr3} = k \cdot P_3;$$

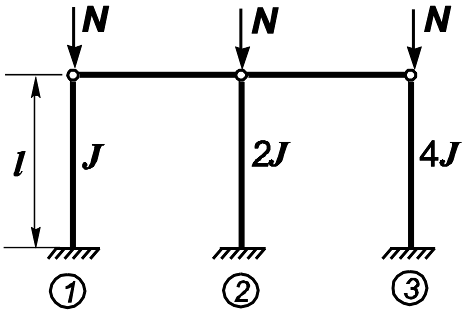
$$L_{01} = \pi \cdot \sqrt{\frac{E \cdot I_1}{k \cdot P_1}}; \qquad L_{02} = \pi \cdot \sqrt{\frac{E \cdot I_2}{k \cdot (P_1 + P_2)}}; \qquad L_{03} = \pi \cdot \sqrt{\frac{E \cdot I_3}{k \cdot (P_1 + P_2 + P_3)}}, \text{ where}$$

k – stability factor of safety of the system is determined on the basis of the condition of equality to zero of the determinant of the system of governing equations:

$$\left| \begin{array}{ccc}
sin\left[\sqrt{\frac{k \cdot P_1}{E \cdot I_1}} \cdot (L_1 + L_2 + L_3)\right] & cos\left[\sqrt{\frac{k \cdot P_1}{E \cdot I_1}} \cdot (L_1 + L_2 + L_3)\right] & 0 \\
0 & 0 & 0 \\
\frac{P_1}{P_1 + P_2} \cdot sin\left[\sqrt{\frac{k \cdot P_1}{E \cdot I_1}} \cdot (L_2 + L_3)\right] & \frac{P_1}{P_1 + P_2} \cdot cos\left[\sqrt{\frac{k \cdot P_1}{E \cdot I_1}} \cdot (L_2 + L_3)\right] & -sin\left[\sqrt{\frac{k \cdot (P_1 + P_2)}{E \cdot I_2}} \cdot (L_2 + L_3)\right] \\
\sqrt{\frac{k \cdot P_1}{E \cdot I_1}} \cdot cos\left[\sqrt{\frac{k \cdot P_1}{E \cdot I_1}} \cdot (L_2 + L_3)\right] & -\sqrt{\frac{k \cdot P_1}{E \cdot I_1}} \cdot sin\left[\sqrt{\frac{k \cdot P_1}{E \cdot I_1}} \cdot (L_2 + L_3)\right] & -\sqrt{\frac{k \cdot (P_1 + P_2)}{E \cdot I_2}} \cdot cos\left[\sqrt{\frac{k \cdot (P_1 + P_2)}{E \cdot I_2}} \cdot (L_2 + L_3)\right] \\
0 & 0 & \frac{P_1 + P_2}{P_1 + P_2 + P_3} \cdot sin\left[\sqrt{\frac{k \cdot (P_1 + P_2)}{E \cdot I_2}} \cdot L_3\right] \\
0 & 0 & \sqrt{\frac{k \cdot (P_1 + P_2)}{E \cdot I_2}} \cdot cos\left[\sqrt{\frac{k \cdot (P_1 + P_2)}{E \cdot I_2}} \cdot L_3\right]
\end{array} \right.$$

$$\left.\begin{matrix}
0 & 0 & 0 \\
0 & \sqrt{\dfrac{k\cdot(P_1+P_2+P_3)}{E\cdot I_3}} & 0 \\
-\cos\left[\sqrt{\dfrac{k\cdot(P_1+P_2)}{E\cdot I_2}}\cdot(L_2+L_3)\right] & 0 & 0 \\
\sqrt{\dfrac{k\cdot(P_1+P_2)}{E\cdot I_2}}\cdot\sin\left[\sqrt{\dfrac{k\cdot(P_1+P_2)}{E\cdot I_2}}\cdot(L_2+L_3)\right] & 0 & 0 \\
\dfrac{P_1+P_2}{P_1+P_2+P_3}\cdot\cos\left[\sqrt{\dfrac{k\cdot(P_1+P_2)}{E\cdot I_2}}\cdot L_3\right] & -\sin\left[\sqrt{\dfrac{k\cdot(P_1+P_2+P_3)}{E\cdot I_3}}\cdot L_3\right] & -\cos\left[\sqrt{\dfrac{k\cdot(P_1+P_2+P_3)}{E\cdot I_3}}\cdot L_3\right] \\
-\sqrt{\dfrac{k\cdot(P_1+P_2)}{E\cdot I_2}}\cdot\sin\left[\sqrt{\dfrac{k\cdot(P_1+P_2)}{E\cdot I_2}}\cdot L_3\right] & -\sqrt{\dfrac{k\cdot(P_1+P_2+P_3)}{E\cdot I_3}}\cdot\cos\left[\sqrt{\dfrac{k\cdot(P_1+P_2+P_3)}{E\cdot I_3}}\cdot L_3\right] & \sqrt{\dfrac{k\cdot(P_1+P_2+P_3)}{E\cdot I_3}}\cdot\sin\left[\sqrt{\dfrac{k\cdot(P_1+P_2+P_3)}{E\cdot I_3}}\cdot L_3\right]
\end{matrix}\right| = 0$$

Stability of the System of Three Equally Loaded Columns of Different Rigidity Hingedly Interconnected by Girders



Objective: Determination of the critical value of the concentrated longitudinal forces of the same value acting on the system of three columns of different rigidity hingedly interconnected by girders corresponding to the moment of its buckling. Determination of the unsupported lengths of the columns.

Initial data file: **Frame_5a1.spr**

Problem formulation: Three columns of different rigidity embedded into the foundation and hingedly interconnected into a system by girders are subjected to the action of concentrated longitudinal forces of the same value N. The axial stiffness values of the girders and columns are assumed to be significant in order to exclude their effect on the solution of the problem. Determine the critical value of the concentrated longitudinal forces N_{cr}, corresponding to the moment of buckling of the system. Determine the unsupported lengths of the columns H_0.

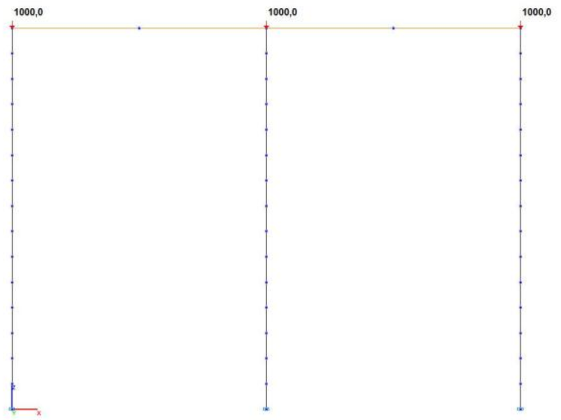
References: N. P. Melnikov, V. M. Vakhurkin, B. G. Lozhkin, Stability Analysis of Bar Systems. Reference data and examples, Moscow, Design Institute of Steel Structures, Issue 1395, 1954, p. 34.

Initial data:

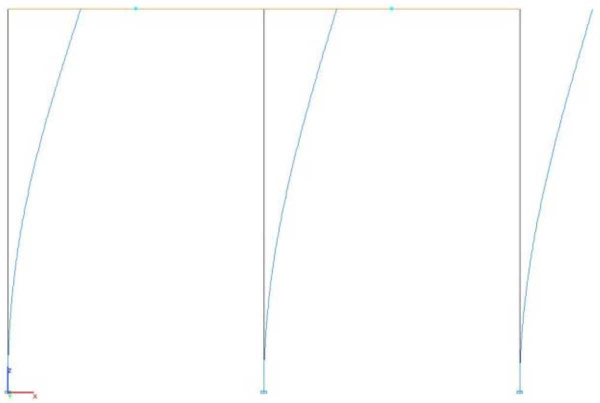
L = 5.0 m	- length of the girders of the frame;
H = 7.5 m	- height of the columns of the frame;
$EA = 1.0 \cdot 10^9$ kN	- axial stiffness of the columns;
$EI_{C1} = 1.14 \cdot 10^4$ kN·m^2	- bending stiffness of the left column;
$EI_{C2} = 2.28 \cdot 10^5$ kN·m^2	- bending stiffness of the middle column;
$EI_{C3} = 4.56 \cdot 10^5$ kN·m^2	- bending stiffness of the right column;
$N = 1.0 \cdot 10^3$ kN	- initial value of the concentrated longitudinal forces on the columns of the system.

Finite element model: Design model – plane frame, columns – 45 elements of type 2 (the spacing of the finite element mesh along the longitudinal axes is 0.5 m), girders – 2 elements of type 100 (three-node rigid bodies with the constraints in the directions X and Z, master nodes in the middle of the girder spans, and slave nodes on the connected columns). Boundary conditions are provided by imposing constraints on the support nodes of the columns in the directions of the degrees of freedom X, Z, UY. The action with the initial value of the concentrated longitudinal forces N is specified in the beam-to-column joints. Number of nodes in the design model – 50.

Results in SCAD



Design model



Buckling mode

Comparison of solutions:

Parameter	Theory	SCAD	Deviation, %
Critical value of the concentrated longitudinal forces N_{cr}, kN	1159.2 (1166.8)	1.1591·1000 = = 1159.1	0.01 (0.66)
Unsupported length of the left column (C1) H_0, m	9.8522 (9.8198)	9.8523	0.00 (0.33)
Unsupported length of the middle column (C2) H_0, m	13.9331 (13.8873)	13.9332	0.00 (0.33)
Unsupported length of the right column (C3) H_0, m	19.7043 (19.6396)	19.7042	0.00 (0.33)

The values of the approximate solution by the equivalent frame method are given in brackets.

Notes: In the exact analytical solution the critical value of the concentrated longitudinal forces N_{cr}, corresponding to the moment of buckling of the system, and the unsupported lengths of the columns H_0 can be determined according to the following formulas:

$$N_{cr} = \nu^2 \cdot \frac{EI_{C1}}{H^2},$$

where ν (critical load parameter) is determined by solving the transcendental equation:

$$\left(tg\left(\nu\right)-\nu\right)\cdot\left(tg\left(\frac{\sqrt{2}}{2}\cdot\nu\right)-\frac{\sqrt{2}}{2}\cdot\nu\right)+\sqrt{2}\cdot\left(tg\left(\nu\right)-\nu\right)\cdot\left(tg\left(\frac{1}{2}\cdot\nu\right)-\frac{1}{2}\cdot\nu\right)+$$

$$2\cdot\left(tg\left(\frac{\sqrt{2}}{2}\cdot\nu\right)-\frac{\sqrt{2}}{2}\cdot\nu\right)\cdot\left(tg\left(\frac{1}{2}\cdot\nu\right)-\frac{1}{2}\cdot\nu\right)=0;$$

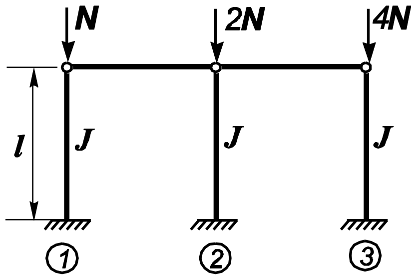
$$C1:\quad H_0=\frac{\pi\cdot H}{\nu};\quad C2:\quad H_0=\sqrt{2}\cdot\frac{\pi\cdot H}{\nu};\quad C3:\quad H_0=2\cdot\frac{\pi\cdot H}{\nu}.$$

In the approximate analytical solution the critical value of the concentrated longitudinal forces N_{cr}, corresponding to the moment of buckling of the system, and the unsupported lengths of the columns H_0 can be determined according to the following formulas:

$$N_{cr}=\frac{7}{3}\cdot\frac{\pi^2\cdot EI_{C1}}{\left(2\cdot H\right)^2};$$

$$C1:\quad H_0=\pi\cdot\sqrt{\frac{EI_{C1}}{N_{cr}}};\quad C2:\quad H_0=\sqrt{2}\cdot\sqrt{\frac{EI_{C1}}{N_{cr}}};\quad C3:\quad H_0=2\cdot\sqrt{\frac{EI_{C1}}{N_{cr}}}.$$

Stability of the System of Three Differently Loaded Columns of the Same Rigidity Hingedly Interconnected by Girders



Objective: Determination of the critical values of the concentrated longitudinal forces with different values corresponding to the moment of buckling of the system in the structure of three columns of the same rigidity hingedly interconnected by girders. Determination of the unsupported lengths of the columns.

Initial data file: **Frame_5a2.spr**

Problem formulation: Three columns of the same rigidity embedded into the foundation and hingedly interconnected into a system by girders are subjected to the action of concentrated longitudinal forces with different values k·N. The axial stiffness values of the girders and columns are assumed to be significant in order to exclude their effect on the solution of the problem. Determine the critical values of the concentrated longitudinal forces N_{cr}, corresponding to the moment of buckling of the system. Determine the unsupported lengths of the columns H_0.

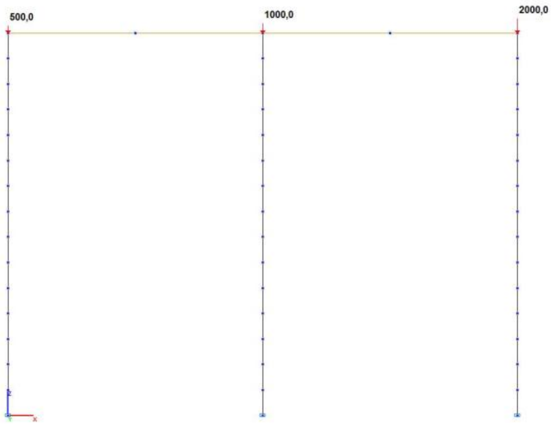
References: N. P. Melnikov, V. M. Vakhurkin, B. G. Lozhkin, Stability Analysis of Bar Systems. Reference data and examples, Moscow, Design Institute of Steel Structures, Issue 1395, 1954, p. 36.

Initial data:

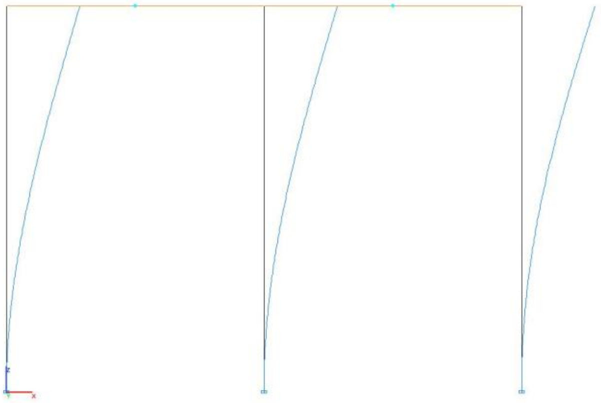
L = 5.0 m	- length of the girders of the frame;
H = 7.5 m	- height of the columns of the frame;
$EA = 1.0\cdot10^9$ kN	- axial stiffness of the columns;
$EI_C = 2.28\cdot10^5$ kN·m^2	- bending stiffness of the columns;
$1\cdot N = 0.5\cdot10^3$ kN	- initial value of the concentrated longitudinal force on the left column;
$2\cdot N = 1.0\cdot10^3$ kN	- initial value of the concentrated longitudinal force on the middle column;
$4\cdot N = 2.0\cdot10^3$ kN	- initial value of the concentrated longitudinal force on the right column.

Finite element model: Design model – plane frame, columns – 45 elements of type 2 (the spacing of the finite element mesh along the longitudinal axes is 0.5 m), girders – 2 elements of type 100 (three-node rigid bodies with the constraints in the directions X and Z, master nodes in the middle of the girder spans, and slave nodes on the connected columns). Boundary conditions are provided by imposing constraints on the support nodes of the columns in the directions of the degrees of freedom X, Z, UY. The action with the initial values of the concentrated longitudinal forces k·N is specified in the beam-to-column joints. Number of nodes in the design model – 50.

Results in SCAD



Design model



Buckling mode

Comparison of solutions:

Parameter	Theory	SCAD	Deviation, %
Critical value of the concentrated longitudinal force on the left column (C1) N_{cr}, kN	426.6 (428.6)	0.853157·500 = = 426.6	0.00 (0.47)
Critical value of the concentrated longitudinal force on the middle column (C2) N_{cr}, kN	853.2 (857.2)	0.853157·1000 = = 853.2	0.00 (0.47)
Critical value of the concentrated longitudinal force on the right column (C3) N_{cr}, kN	1706.3 (1714.5)	0.853157·2000 = = 1706.3	0.00 (0.48)
Unsupported length of the left column (C1) H_0, m	22.9676 (22.9129)	22.9677	0.00 (0.24)
Unsupported length of the middle column (C2) H_0, m	16.2405 (16.2019)	16.2406	0.00 (0.24)
Unsupported length of the right column (C3) H_0, m	11.4838 (11.4564)	11.4839	0.00 (0.24)

The values of the approximate solution by the equivalent frame method are given in brackets

Notes: In the exact analytical solution the critical values of the concentrated longitudinal forces N_{cr}, corresponding to the moment of buckling of the system, and the unsupported lengths of the columns H_0 can be determined according to the following formulas:

$$C1: \quad N_{cr} = \nu^2 \cdot \frac{EI_C}{H^2} \quad C2: \quad N_{cr} = 2 \cdot \nu^2 \cdot \frac{EI_C}{H^2} \quad C3: \quad 4 \cdot \nu^2 \cdot \frac{EI_C}{H^2},$$

where ν (critical load parameter) is determined by solving the transcendental equation:

$$(tg(\nu)-\nu)\cdot\left(tg\left(\sqrt{2}\cdot\nu\right)-\sqrt{2}\cdot\nu\right)+\frac{\sqrt{2}}{4}\cdot(tg(\nu)-\nu)\cdot(tg(2\cdot\nu)-2\cdot\nu)+$$
$$+\frac{1}{8}\cdot\left(tg\left(\sqrt{2}\cdot\nu\right)-\sqrt{2}\cdot\nu\right)\cdot(tg(2\cdot\nu)-2\cdot\nu)=0;$$

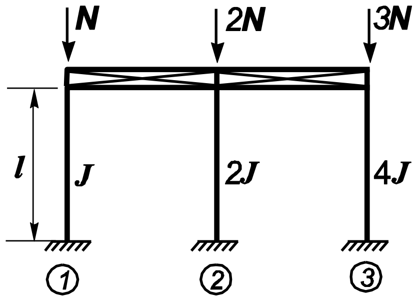
$$C1:\quad H_0=\frac{\pi\cdot H}{\nu};\quad C2:\quad H_0=\frac{\sqrt{2}}{2}\frac{\pi\cdot H}{\nu};\quad C3:\quad H_0=\frac{1}{2}\cdot\frac{\pi\cdot H}{\nu}.$$

In the approximate analytical solution the critical value of the concentrated longitudinal forces N_{cr}, corresponding to the moment of buckling of the system, and the unsupported lengths of the columns H_0 can be determined according to the following formulas:

$$C1:\quad N_{cr}=\frac{3}{7}\cdot\frac{\pi^2\cdot EI_C}{(2\cdot H)^2}\quad C2:\quad N_{cr}=\frac{6}{7}\cdot\frac{\pi^2\cdot EI_C}{(2\cdot H)^2}\quad C3:\quad N_{cr}=\frac{12}{7}\cdot\frac{\pi^2\cdot EI_C}{(2\cdot H)^2};$$

$$H_0=\pi\cdot\sqrt{\frac{EI_C}{N_{cr}}}.$$

Stability of the System of Three Differently Loaded Columns of Different Rigidity Interconnected by Girders Infinitely Rigid in Bending



Objective: Determination of the critical values of the concentrated longitudinal forces with different values acting on the system of three columns of different rigidity interconnected by girders infinitely rigid in bending, corresponding to the moment of its buckling. Determination of the unsupported lengths of the columns.

Initial data file: **Frame_5б.spr**

Problem formulation: Three columns of different rigidity embedded into the foundation and interconnected into a system by girders infinitely rigid in bending are subjected to the action of concentrated longitudinal forces with different values k·N. The axial stiffness values of the girders and columns are assumed to be significant in order to exclude their effect on the solution of the problem. Determine the critical values of the concentrated longitudinal forces N_{cr}, corresponding to the moment of buckling of the system. Determine the unsupported lengths of the columns H_0.

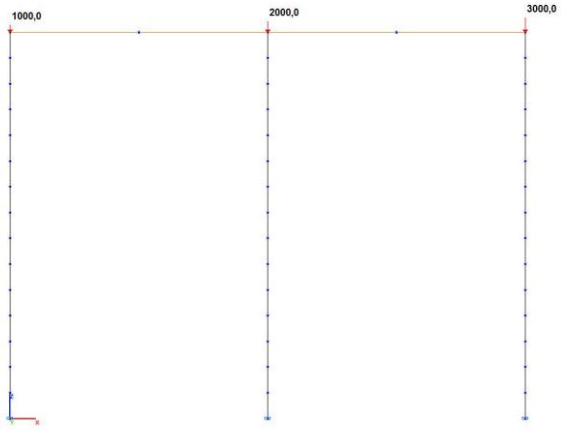
References: N. P. Melnikov, V. M. Vakhurkin, B. G. Lozhkin, Stability Analysis of Bar Systems. Reference data and examples, Moscow, Design Institute of Steel Structures, Issue 1395, 1954, p. 37.

Initial data:

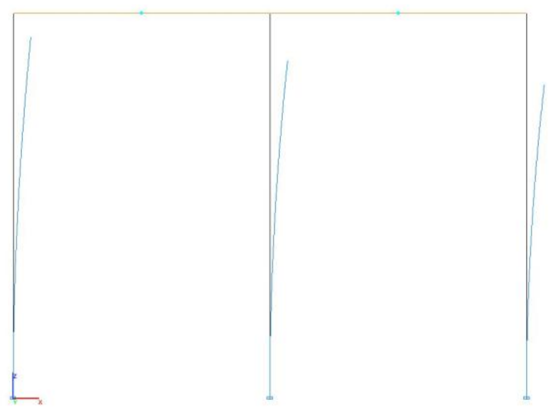
L = 5.0 m	- length of the girders of the frame;
H = 7.5 m	- height of the columns of the frame;
$EA = 1.0 \cdot 10^9$ kN	- axial stiffness of the columns;
$EI_{C1} = 1.14 \cdot 10^5$ kN·m^2	- bending stiffness of the left column;
$EI_{C2} = 2.28 \cdot 10^5$ kN·m^2	- bending stiffness of the middle column;
$EI_{C3} = 4.56 \cdot 10^5$ kN·m^2	- bending stiffness of the right column;
$1 \cdot N = 1.0 \cdot 10^3$ kN	- initial value of the concentrated longitudinal force on the left column;
$2 \cdot N = 2.0 \cdot 10^3$ kN	- initial value of the concentrated longitudinal force on the middle column;
$3 \cdot N = 3.0 \cdot 10^3$ kN	- initial value of the concentrated longitudinal force on the right column.

Finite element model: Design model – plane frame, columns – 45 elements of type 2 (the spacing of the finite element mesh along the longitudinal axes is 0.5 m), girders – 2 elements of type 100 (three-node rigid bodies with the constraints in the directions X, Z and UY, master nodes in the middle of the girder spans, and slave nodes on the connected columns). Boundary conditions are provided by imposing constraints on the support nodes of the columns in the directions of the degrees of freedom X, Z, UY. The action with the initial values of the concentrated longitudinal forces k·N is specified in the beam-to-column joints. Number of nodes in the design model – 50.

Results in SCAD



Design model



Buckling mode

Comparison of solutions:

Parameter	Theory	SCAD	Deviation, %
Critical value of the concentrated longitudinal force on the left column (C1) N_{cr}, kN	2332.8 (2333.6)	2.332764·1000 = = 2332.7	0.00 (0.04)
Critical value of the concentrated longitudinal force on the middle column (C2) N_{cr}, kN	4665.6 (4667.2)	2.332764·2000 = = 4665.5	0.00 (0.04)
Critical value of the concentrated longitudinal force on the right column (C3) N_{cr}, kN	6998.5 (7000.8)	2.332764·3000 = = 6998.3	0.00 (0.04)
Unsupported length of the left column (C1) H_0, m	6.9448 (6.9437)	6.9449	0.00 (0.02)
Unsupported length of the middle column (C2) H_0, m	6.9448 (6.9437)	6.9449	0.00 (0.02)
Unsupported length of the right column (C3) H_0, m	8.0192 (8.0178)	8.0193	0.00 (0.02)

The values of the approximate solution by the equivalent frame method are given in brackets

Notes: In the exact analytical solution the critical values of the concentrated longitudinal forces N_{cr}, corresponding to the moment of buckling of the system, and the unsupported lengths of the columns H_0 can be determined according to the following formulas:

$$C1: \quad N_{cr} = \nu^2 \cdot \frac{EI_{C1}}{H^2} \quad C2: \quad N_{cr} = 2 \cdot \nu^2 \cdot \frac{EI_{C1}}{H^2} \quad C3: \quad 3 \cdot \nu^2 \cdot \frac{EI_{C1}}{H^2},$$

where ν (critical load parameter) is determined by solving the transcendental equation:

$$6\cdot\nu\cdot\left(\frac{tg\left(\frac{\nu}{2}\right)}{2\cdot tg\left(\frac{\nu}{2}\right)-\nu}+\frac{2\cdot tg\left(\frac{\sqrt{3}\cdot\nu}{4}\right)}{4\cdot tg\left(\frac{\sqrt{3}\cdot\nu}{4}\right)-\sqrt{3}\cdot\nu}\right)=0;$$

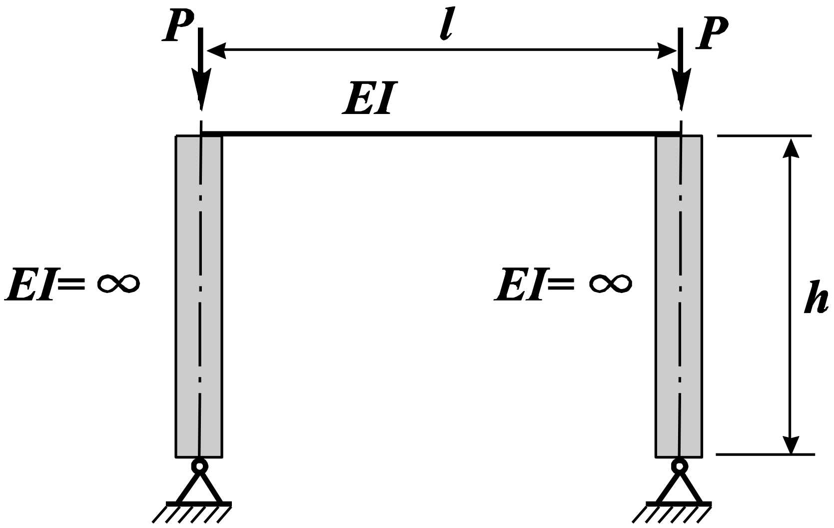
$$C1:\quad H_0=\frac{\pi\cdot H}{\nu};\quad C2:\quad H_0=\frac{\pi\cdot H}{\nu};\quad C3:\quad H_0=\frac{2}{\sqrt{3}}\cdot\frac{\pi\cdot H}{\nu}.$$

In the approximate analytical solution the critical value of the concentrated longitudinal forces N_{cr}, corresponding to the moment of buckling of the system, and the unsupported lengths of the columns H_0 can be determined according to the following formulas:

$$C1:\quad N_{cr}=\frac{7}{6}\cdot\frac{\pi^2\cdot EI_{C1}}{H^2}\quad C2:\quad N_{cr}=\frac{7}{3}\cdot\frac{\pi^2\cdot EI_{C1}}{H^2}\quad C3:\quad N_{cr}=\frac{7}{2}\cdot\frac{\pi^2\cdot EI_{C1}}{H^2};$$

$$C1:\quad H_0=\sqrt{\frac{6}{7}}\cdot H\quad C2:\quad H_0=\sqrt{\frac{6}{7}}\cdot H\quad C3:\quad H_0=\sqrt{\frac{8}{7}}\cdot H.$$

Stability of the Frame of Two Simply Supported Equally Loaded Rigid Columns Rigidly Interconnected by a Girder



Objective: Determination of the critical value of the concentrated longitudinal forces of the same value acting on two simply supported equally loaded rigid columns of the frame rigidly interconnected by a girder corresponding to the moment of buckling of the frame.

Initial data file:: Frame_leg_hard.SPR

Problem formulation: Two simply supported rigid columns of the frame rigidly interconnected by a girder are subjected to the action of concentrated longitudinal forces of the same value N. The axial stiffness of the girder is assumed to be significant in order to exclude its effect on the solution of the problem. Determine the critical value of the concentrated longitudinal forces N_{cr}, corresponding to the moment of buckling of the frame.

References: A. V. Perelmuter, V. I. Slivker, Handbook of Mechanical Stability in Engineering. Volume 2. Stability of Elastically Deformable Mechanical Systems, Moscow, SACD SOFT, 2010, p. 173.

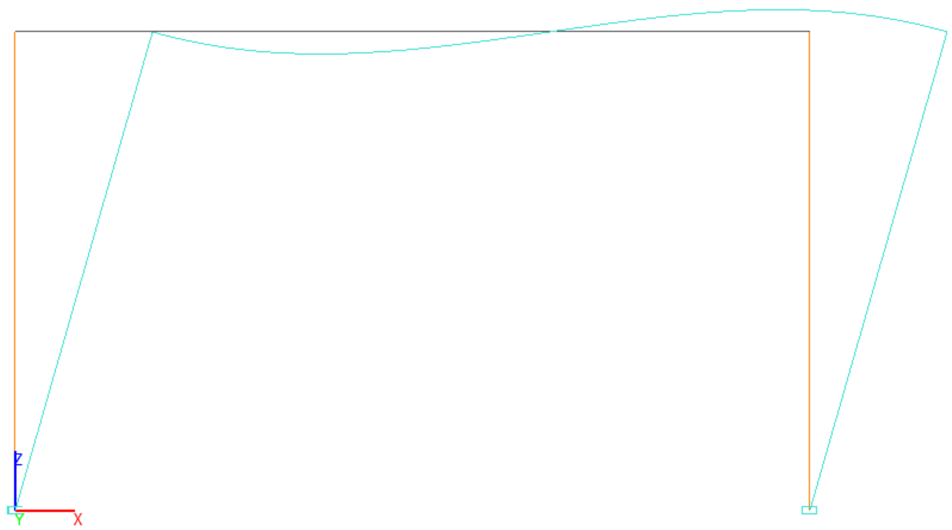
Initial data:

L = 10.0 m	- length of the girder of the frame;
H = 6.0 m	- height of the columns of the frame;
EA = $1.0\cdot10^8$ t	- axial stiffness of the girder;
EI = $1.0\cdot10^4$ t·m^2	- bending stiffness of the girder;
N = $1.0\cdot10^3$ t	- initial value of the concentrated longitudinal forces on the columns of the frame.

Finite element model: Design model – plane frame, columns – 2 elements of type 100 (two-node rigid bodies with the constraints in the directions X, Z, UY, support master nodes and slave nodes on the adjacent girder), girder – 10 elements of type 2 (the spacing of the finite element mesh along the longitudinal axes is 1.0 m). Boundary conditions are provided by imposing constraints on the support nodes of the columns in the directions of the degrees of freedom X and Z. The action with the initial value of the concentrated longitudinal forces N is specified in the beam-to-column joints. Number of nodes in the design model – 13.

Results in SCAD

Design model



Buckling mode

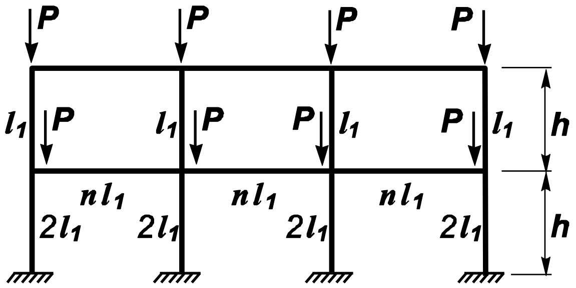
Comparison of solutions:

Parameter	Theory	SCAD	Deviation, %
Critical value of the concentrated longitudinal forces N_{cr}, t	1000	0.999975·1000 = = 1000	0.00

Notes: In the exact analytical solution the critical value of the concentrated longitudinal forces N_{cr}, corresponding to the moment of buckling of the frame can be determined according to the following formula:

$$N_{cr} = \frac{6 \cdot EI}{L \cdot H}.$$

Stability of a Three-Span Two-Storey Frame Subjected to Concentrated Longitudinal Forces Applied to the Columns in the Joints with Girders



Objective: Determination of the critical value of the concentrated longitudinal forces acting on the columns of a three-span two-storey frame in the joints with the girders corresponding to the moment of its buckling.

Initial data file: **6.1.spr**

Problem formulation: The three-span two-storey frame is subjected to the action of concentrated longitudinal forces P applied to the columns in the joints with the girders. The beam-to-column and column-to-foundation joints are rigid. The axial stiffness values of the girders and columns are assumed to be significant in order to exclude their effect on the solution of the problem. Determine the critical value of the concentrated longitudinal forces P_{cr}, corresponding to the moment of buckling of the frame.

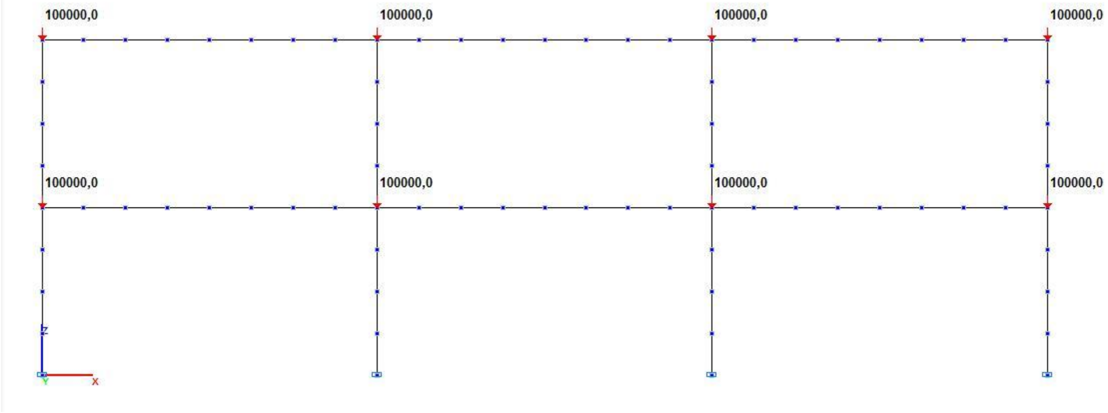
References: N. V. Kornoukhov, Strength and Stability of Framework Structures, Moscow, Stroyizdat Publ., 1949, p. 259.

Initial data:

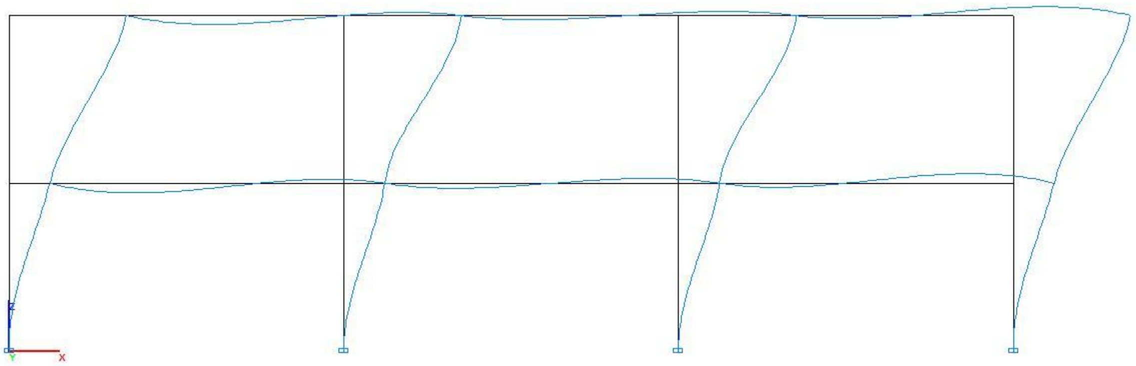
L = 8.0 m	- length of the girders of the frame;
H = 4.0 m	- height of the columns of the frame;
$EA = 1.0 \cdot 10^{10}$ kN	- axial stiffness of the structural members of the frame;
$EI_{C1} = 8.00 \cdot 10^5$ kN·m^2	- bending stiffness of the columns of the first storey;
$i_{C1} = EI_{C1}/H = 2.0 \cdot 10^5$ kN·m	- bending stiffness of the columns of the first storey per running meter;
$EI_{C2} = 4.00 \cdot 10^5$ kN·m^2	- bending stiffness of the columns of the second storey;
$i_{C2} = EI_{C2}/H = 1.00 \cdot 10^5$ kN·m	- bending stiffness of the columns of the second storey per running meter;
$EI_{P1} = 13.28 \cdot 10^5$ kN·m^2	- bending stiffness of the girders of the first storey;
$i_{P1} = EI_{P1}/L = 1.66 \cdot 10^5$ kN·m	- bending stiffness of the girders of the first storey per running meter;
$EI_{P2} = 8.00 \cdot 10^5$ kN·m^2	- bending stiffness of the girders of the second storey;
$i_{P2} = EI_{P2}/L = 1.00 \cdot 10^5$ kN·m	- bending stiffness of the girders of the second storey per running meter;
$P = 1.0 \cdot 10^5$ kN	- initial value of the concentrated longitudinal forces on the columns of the frame in the joints with the girders.

Finite element model: Design model – plane frame, 80 elements of type 2. The spacing of the finite element mesh along the longitudinal axes of the structural members (along the X1 axes of the local coordinate systems) is 1.0 m. Boundary conditions are provided by imposing constraints on the support nodes of the frame in the directions of the degrees of freedom X, Z, UY. The action with the initial value of the concentrated longitudinal forces P is specified in the beam-to-column joints. Number of nodes in the design model – 78.

Results in SCAD



Design model



Buckling mode

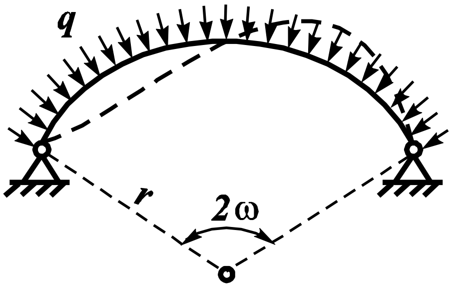
Comparison of solutions:

Parameter	Theory	SCAD	Deviation, %
Critical value of the concentrated longitudinal forces P_{cr}, kN	156250	1.5625·100000 = = 156250	0.00

Notes: In the analytical solution the critical value of the concentrated longitudinal forces P_{cr}, corresponding to the moment of buckling of the frame can be determined according to the following formula:

$$P_{cr} = 2.5000^2 \cdot \frac{i_{C1}}{H} .$$

Stability of a Circular Two-Hinged Arch of a Constant Cross-Section Subjected to Hydrostatic Pressure



Objective: Determination of the critical value of the hydrostatic pressure applied to a circular two-hinged arch of a constant cross-section corresponding to the moment of its buckling.

Initial data files:

File name	Description
Arch_hinged_alfa_30.SPR	Design model with the central angle of the arc $2·\omega = 2·30$ °
Arch_hinged_alfa_90.SPR	Design model with the central angle of the arc $2·\omega = 2·90$ °

Problem formulation: The circular two-hinged arch of a constant cross-section is subjected to the action of the uniformly distributed radial load q. Determine the critical value of the uniformly distributed radial load q_{cr}, corresponding to the moment of buckling of the arch. It is assumed that when the arch buckles, the load elements follow the axis of the arch staying parallel to their former directions, and therefore the displacement of the pressure line takes place at the buckling of the arch. Compare the result of the calculation with the solution (S.P. Timoshenko), when the load action lines do not change at the distortion of the arch axis and the pressure line does not move at the buckling of the arch.

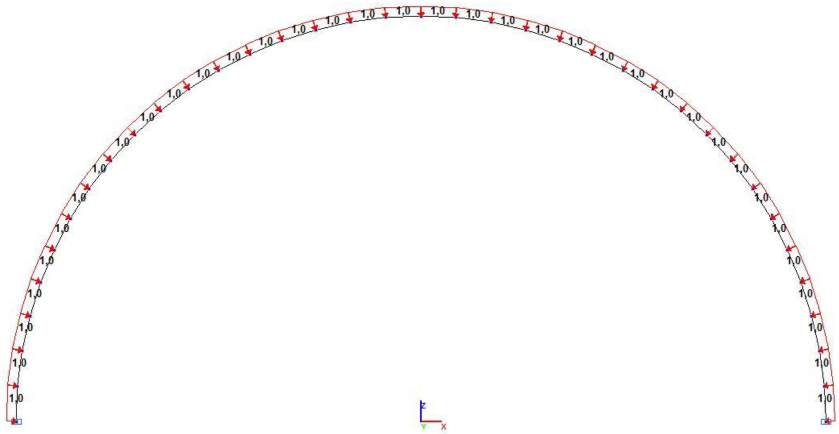
References: N. V. Kornoukhov, Strength and Stability of Framework Structures, Moscow, Stroyizdat Publ., 1949, p. 212.

Initial data:

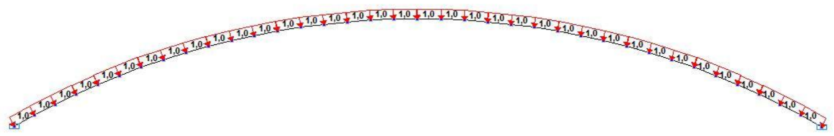
R = 60.0 m (120.0 m)	- radius of the longitudinal axis of the arch;
$2·\omega = 2·90$ ° ($2·30$ °)	- central angle of the arc;
$EA = 2.16·10^6$ kN	- axial stiffness of the arch;
$EI = 2.592·10^5$ kN·m^2	- bending stiffness of the arch;
q = 1.0 kN/m	- initial value of the uniformly distributed radial load on the arch.

Finite element model: Design model – plane frame, 36 elements of type 2. The arch is divided into finite elements along its longitudinal axis (along the X1 axes of the local coordinate systems) by the step of the central angle of 5.0 ° (1.667 °). Boundary conditions are provided by imposing constraints on the support nodes of the arch in the directions of the degrees of freedom X, Z. The action with the initial value of the uniformly distributed radial load q is specified in the directions opposite to the Z1 axes of the local coordinate systems of the elements. Number of nodes in the design model – 37.

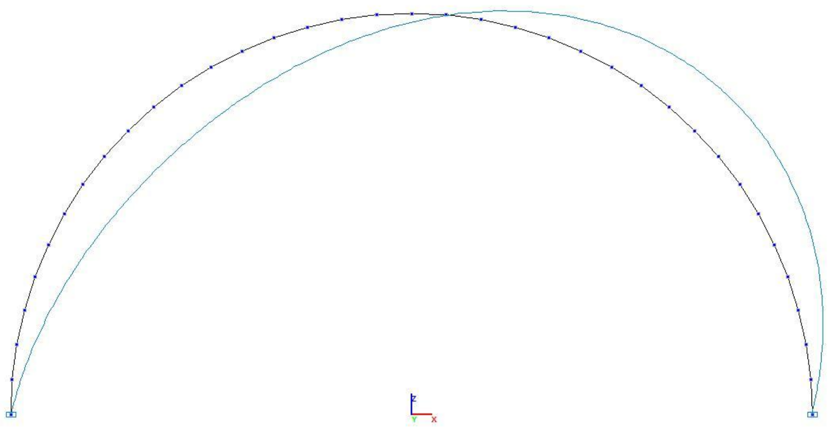
Results in SCAD



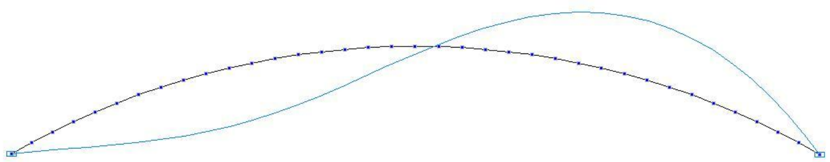
Design model with the central angle of the arc 2·90 °



Design model with the central angle of the arc 2·30 °



Buckling mode for the model with the central angle of the arc 2·90 °



Buckling mode for the model with the central angle of the arc 2·30 °

Comparison of solutions:

Critical value of the uniformly distributed radial load on the arch q_{cr}, kN/m

Design model	Theory	SCAD	Deviation, %
with the central angle of the arc 2·90 °	3.925 (3.600) [3.932]	3.933914·1.0 = = 3.934	0.23 (9.28) [0.05]
with the central angle of the arc 2·30 °	5.391 (5.250) [5.392]	5.393093·1.0 = = 5.393	0.04 (2.72) [0.02]

Theoretical values calculated according to the conditions of this example (according to N. V. Kornoukhov) are given without brackets;
Theoretical values calculated according to the conditions of S. P. Timoshenko are given in round brackets;
Theoretical values calculated for a two-hinged frame made up of 2·m=36 equal chords inscribed in an arc of a circle and subjected to the action of equal radial forces in all its nodes are given in square brackets.

Notes: In the analytical solution according to the conditions of N. V. Kornoukhov the critical value of the uniformly distributed radial load q_{cr}, corresponding to the moment of buckling of the arch can be determined according to the following formula:

$$q_{cr} = \eta^2 \cdot \frac{EI}{R^3},$$

where η (critical load parameter) is determined by solving the transcendental equation:

$$\frac{1}{(\eta^2-1)^2} \cdot \left[\eta^3 \cdot \left(\omega + \frac{1}{2} \cdot sin(2\cdot\omega) \right) - \eta \cdot \left(\omega + \frac{3}{2} \cdot sin(2\cdot\omega) \right) + \frac{1-cos(2\cdot\omega)}{tg(\eta\cdot\omega)} \right] = 0.$$

In the analytical solution according to the conditions of S. P. Timoshenko the critical value of the uniformly distributed radial load q_{cr}, corresponding to the moment of buckling of the arch can be determined according to the following formula:

$$q_{cr} = \frac{EI}{R^3} \cdot \left(\frac{\pi^2}{\omega^2} - 1 \right).$$

In the analytical solution for a two-hinged frame made up of equal chords inscribed in an arc of a circle, the critical value of the uniformly distributed radial load q_{cr}, corresponding to its moment of buckling can be determined according to the following formula:

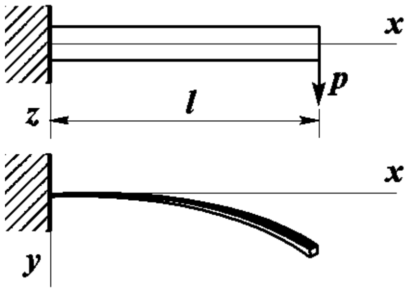
$$q_{cr} = 2 \cdot \upsilon^2 \cdot \frac{EI}{L^3} \cdot sin\left(\frac{A}{2} \right),$$

where υ (critical load parameter) is determined by solving the transcendental equation:

$$\left(1 - \frac{sin(\upsilon)}{\upsilon} \cdot \frac{1-cos(A)}{cos(\upsilon)-cos(A)}\right) \cdot \left(m + \frac{sin(2\cdot m\cdot A)}{2\cdot sin(A)} \right) + \left(\frac{2\cdot sin\left(\frac{A}{2}\right)\cdot sin\left(\frac{\upsilon}{2}\right)}{cos(\upsilon)-cos(A)} \cdot \frac{sin(\upsilon)}{\upsilon} \cdot \frac{sin(m\cdot A)}{sin(m\cdot\upsilon)} \right) \cdot \left(\frac{sin(m\cdot(A+\upsilon))}{2\cdot sin\left(\frac{A+\upsilon}{2}\right)} + \frac{sin(m\cdot(A-\upsilon))}{2\cdot sin\left(\frac{A-\upsilon}{2}\right)} \right) = 0.$$

2·m - number of chords of the frame,
A - central angle of one chord of the frame,
L - length of one chord of the frame:
$L = R \cdot \sqrt{2\cdot(1-cos(A))}$.

Stability of In-Plane Bending of a Cantilever Strip of a Rectangular Cross-Section by a Shear Force Applied at the Free End



Objective: Determination of the critical value of the concentrated shear force applied at the free end of a cantilever strip of a rectangular cross-section corresponding to the moment of its buckling.

Initial data files:

File name	Description
6.2_O_P_b_0.01.SPR	Thickness of the cantilever strip cross-section – 0.01 m
6.2_O_P_b_0.1.SPR	Thickness of the cantilever strip cross-section – 0.10 m
6.2_O_P_b_1.0.SPR	Thickness of the cantilever strip cross-section – 1.00 m

Problem formulation: The cantilever strip of a rectangular cross-section is subjected to the action of the concentrated shear force P, applied at its free end. Determine the critical value of the concentrated shear force P_{cr}, corresponding to the moment of buckling of the cantilever strip.

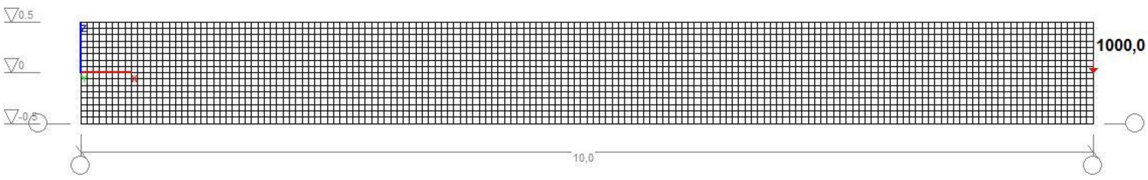
References: S. P. Timoshenko, Stability of Bars, Plates and Shells. — Moscow. — Nauka. — 1971. — p. 291.
A.S. Volmir. Stability of Deformable Systems. — Moscow. — Nauka. — 1967. — p.211;
A. V. Perelmuter, V. I. Slivker, Handbook of Mechanical Stability in Engineering. — Volume 1. — Moscow. — SCAD SOFT. — 2010. — p. 465;
A. V. Perelmuter, V. I. Slivker, Handbook of Mechanical Stability in Engineering. — Volume 2. — Moscow. — SCAD SOFT. — 2010. — p. 17.

Initial data:

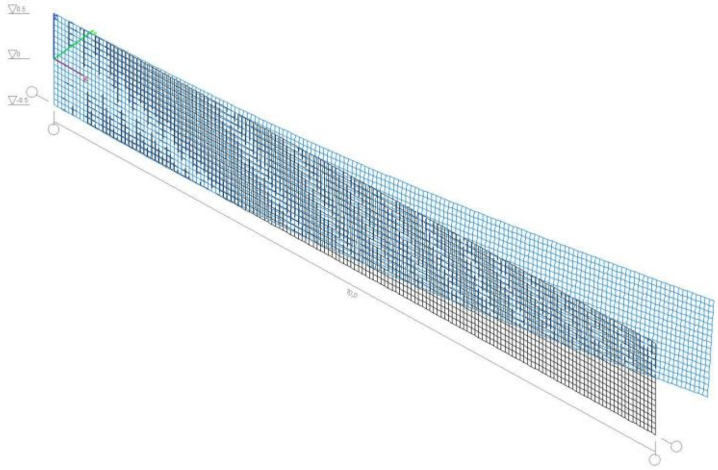
L = 10.0 m	- length of the cantilever strip;
h = 1.0 m	- height of the cantilever strip cross-section;
b = 0.01; 0.10; 1.00 m	- thickness of the cantilever strip cross-section;
$E = 3.0 \cdot 10^7$ kN/m^2	- elastic modulus of the cantilever strip material;
ν = 0.2	- Poisson’s ratio;
$P_1 = 1.0; 1.0 \cdot 10^3; 1.0 \cdot 10^5$ kN	- initial value of the concentrated shear force applied at the free end in the plane of the strip;
$P = 1.0; 1.0 \cdot 10^3; 1.0 \cdot 10^5$ kN	- initial value of the concentrated shear force applied at the free end out of the plane of the strip.

Finite element model: Design model – general type system. Reissner-Mindlin shell element model, 2560 eight-node elements of type 150, the spacing of the finite element mesh along the longitudinal axis and along the height of the strip is 0.0625 m. Boundary conditions are provided by imposing constraints on the nodes of the clamped end of the strip in the directions of the degrees of freedom X, Y, Z, UX, UY, UZ. The action with the initial value of the concentrated shear force P is specified in the node of the longitudinal axis of the strip on the free end. Number of nodes in the design model – 8033.
The stability of in-plane bending of the cantilever strip subjected to the shear force applied at the free end in the plane of the strip is checked.

Results in SCAD



Design model. Reissner-Mindlin shell element model



Buckling mode. Reissner-Mindlin shell element model

Comparison of solutions:

The critical value of the concentrated shear force P_{1cr} (kN), applied at the free end in the plane of the strip

Design model		Theory	SCAD	Deviation, %
Reissner-Mindlin shell element	b = 0.01 m	0.12901 (0.12901)	0.134811·1 = 0.13481	4.50 (4.50)
	b = 0.10 m	125.28 (124.66)	0.130559·1000 = 130.56	4.21 (4.73)
	b = 1.00 m	84048 (59431)	0.821978·100000 = 82198	2.20 (38.31)

Theoretical values calculated taking into account the effect of the bending stiffness in the shear force plane are given in brackets

Notes: In the analytical solution the critical value of the concentrated shear force P_{cr}, corresponding to the moment of buckling of the cantilever strip can be determined according to the following formulas:

without taking into account the effect of the bending stiffness in the shear force plane

$$P_{cr} = \frac{4.01}{l^2} \cdot \sqrt{B \cdot C}$$

taking into account the effect of the bending stiffness in the shear force plane

$$P_{cr} = \frac{4.01}{l^2} \cdot \sqrt{\frac{B \cdot C \cdot B_1}{B + B_1}} = \frac{k}{l^2} \cdot \sqrt{B \cdot C}\text{, where:}$$

$$k = \frac{4.01}{\sqrt{1 + \left(\frac{b}{h}\right)^2}}.$$

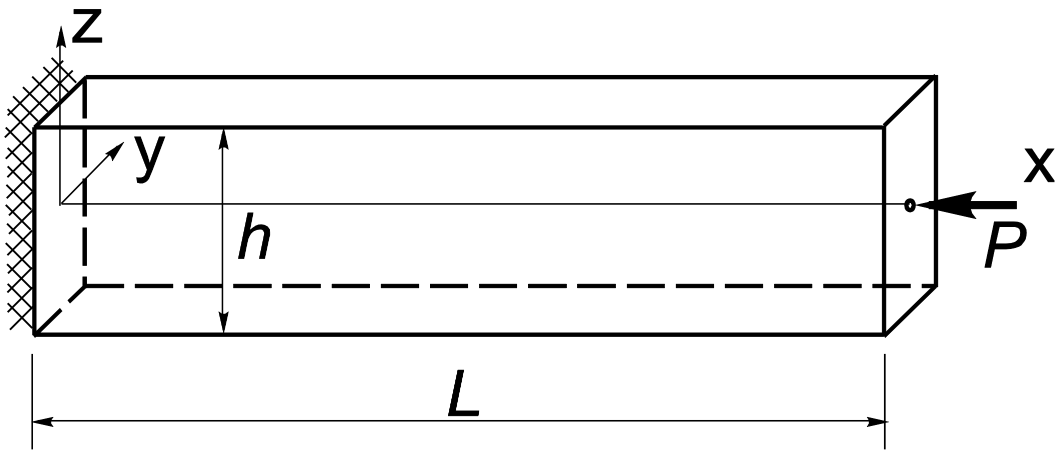
$B = E \cdot \frac{h \cdot b^3}{12}$ – minimum bending stiffness (out of the moment plane);

$B_1 = E \cdot \frac{b \cdot h^3}{12}$ – maximum bending stiffness (in the moment plane);

$C = \frac{E}{2 \cdot (1 + \nu)} \cdot k_f \cdot h \cdot b^3$ – free torsional stiffness, where:

$$k_f = \frac{1}{3} \cdot \left\{ 1 - \frac{192}{\pi^5} \cdot \frac{b}{h} \cdot \sum_{n=1}^{\infty} \left[sin^2\left(\frac{n \cdot \pi}{2}\right) \cdot \frac{1}{n^5} \cdot th\left(\frac{n \cdot \pi \cdot h}{2 \cdot b}\right) \right] \right\}.$$

Stability of a Cantilever Beam of a Square Cross-Section Subjected to a Concentrated Longitudinal Compressive Force Centrally Applied at the Free End (Central Compression)



Objective: Determination of the first two critical values of a concentrated longitudinal compressive force centrally applied at the free end of a cantilever beam of a square cross-section corresponding to the moments of its buckling.

Initial data files:

File name	Description
Stability_Bar_1_Bar.SPR	Bar model
Stability_Bar_1_Shell.SPR	Shell element model
Stability_Bar_1_Solid.SPR	Solid element model

Problem formulation: The cantilever beam of a square cross-section is subjected to the action of the concentrated longitudinal compressive force P, centrally applied at its free end. Determine the first two critical values of the concentrated longitudinal compressive force P_{cr1} and P_{cr2}, corresponding to the moments of buckling of the cantilever beam.

References: A.S. Volmir. Stability of Deformable Systems, Moscow, Nauka, 1967, p.23, 193;

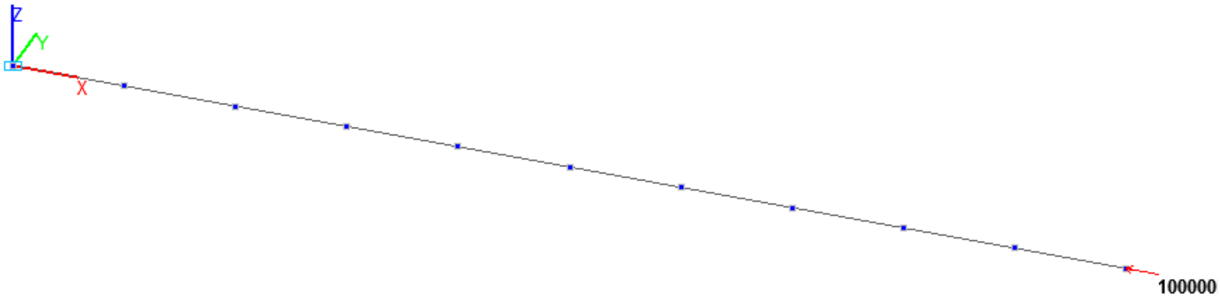
Initial data:

L = 10.0 m	- length of the cantilever beam;
h = b = 1.0 m	- side of the square cross-section of the cantilever beam;
$E = 3.0 \cdot 10^7$ kN/m^2	- elastic modulus of the cantilever beam material;
$\nu = 0.2$	- Poisson’s ratio;
$P = 10^5$ kN	- initial value of the concentrated longitudinal compressive force centrally applied at the free end of the beam.

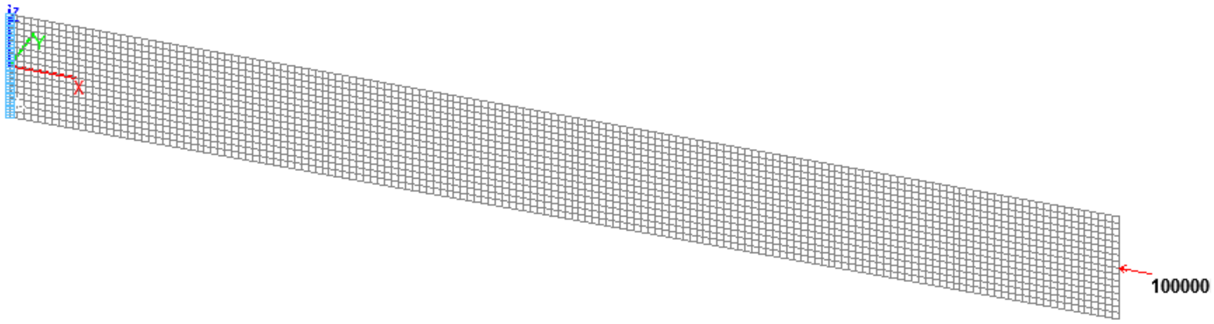
Finite element model: Design model – general type system. Three design models are considered:
Bar model (B), 10 elements of type 5, the spacing of the finite element mesh along the longitudinal axis is 1.0 m. Boundary conditions are provided by imposing constraints on the node of the clamped end of the beam in the directions of the degrees of freedom X, Y, Z, UX, UY, UZ. The action with the initial value of the concentrated longitudinal compressive force P is specified in the node of the free end of the beam. Number of nodes in the design model – 11;
Reissner-Mindlin shell element model (P), 2560 eight-node elements of type 150, the spacing of the finite element mesh along the longitudinal axis and along the height of the beam is 0.0625 m. Boundary conditions are provided by imposing constraints on the nodes of the clamped end of the beam in the directions of the degrees of freedom X, Y, Z, UX, UY, UZ. The action with the initial value of the concentrated longitudinal compressive force P is specified in the node of the longitudinal axis of the beam on the free end. Number of nodes in the design model – 8033.
Solid element model (S), 5120 twenty-node elements of type 37, the spacing of the finite element mesh along the longitudinal axis, width and height of the beam is 0.125 m. Boundary conditions are provided by

imposing constraints on the nodes of the clamped end of the beam in the directions of the degrees of freedom X, Y, Z, UX, UY, UZ. The action with the initial value of the concentrated longitudinal compressive force P is specified as a load uniformly distributed over the external faces of the elements of the beam end p = P/(h·b). Number of nodes in the design model – 24705.

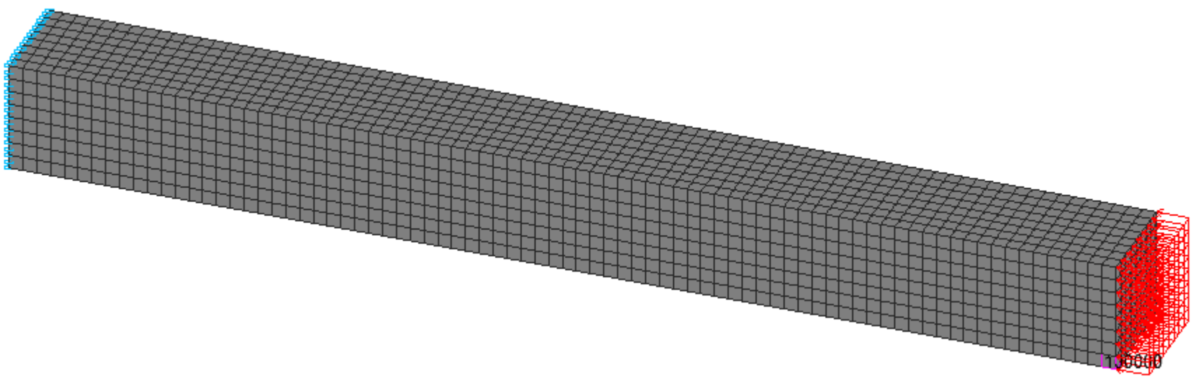
Results in SCAD



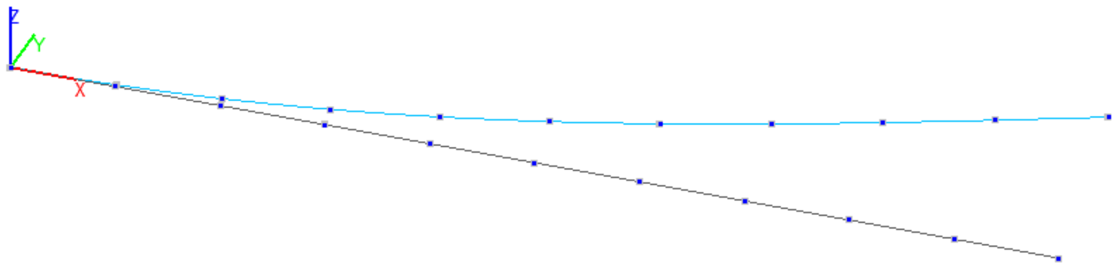
Design model. Bar model



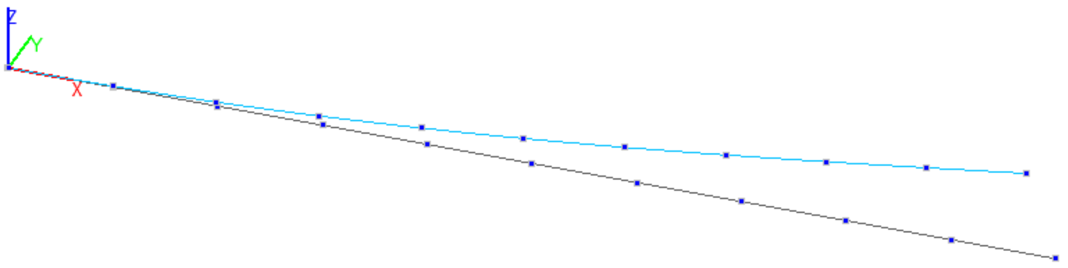
Design model. Reissner-Mindlin shell element model



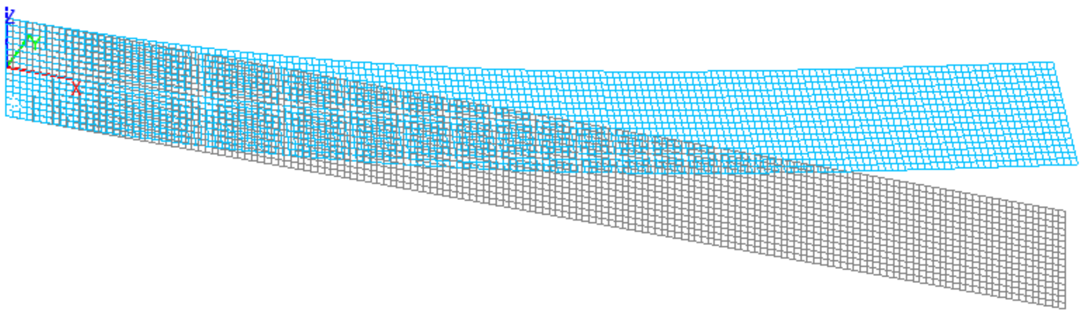
Design model. Solid element model



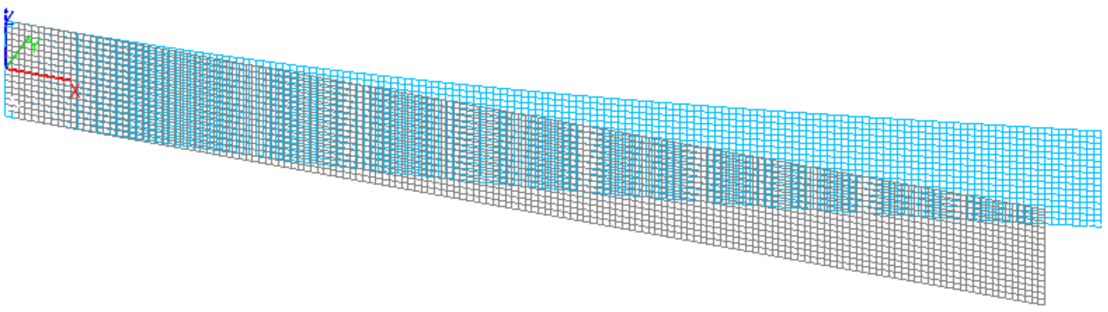
1-st buckling mode. Bar model



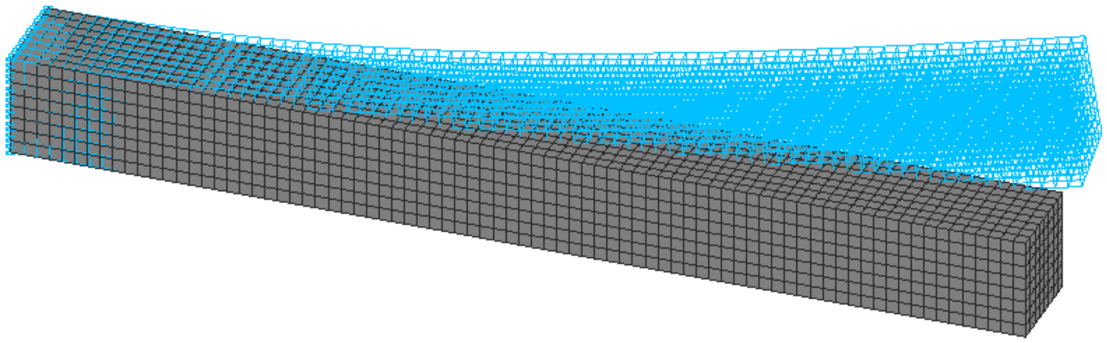
2-nd buckling mode. Bar model



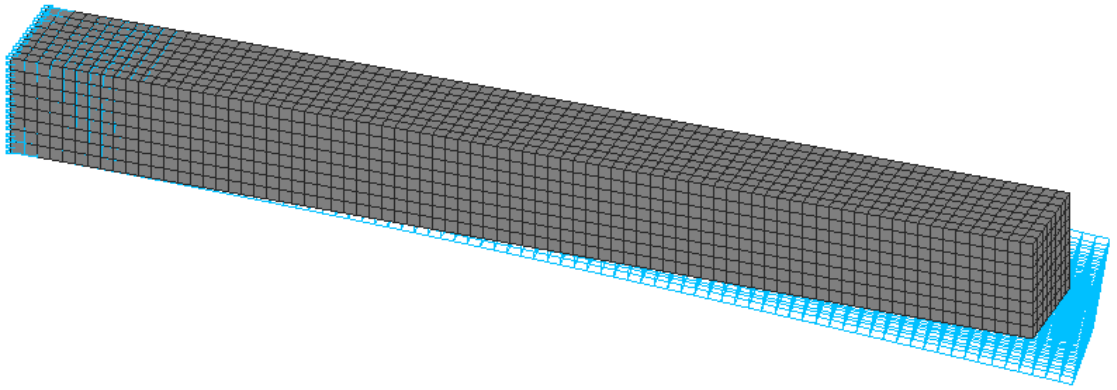
1-st buckling mode. Reissner-Mindlin shell element model



2-nd buckling mode. Reissner-Mindlin shell element model



1- st buckling mode. Solid element model



2- nd buckling mode. Solid element model

Comparison of solutions:

Critical values of the concentrated longitudinal compressive force P_{cr1} and P_{cr2} (kN), centrally applied at the free end of the beam

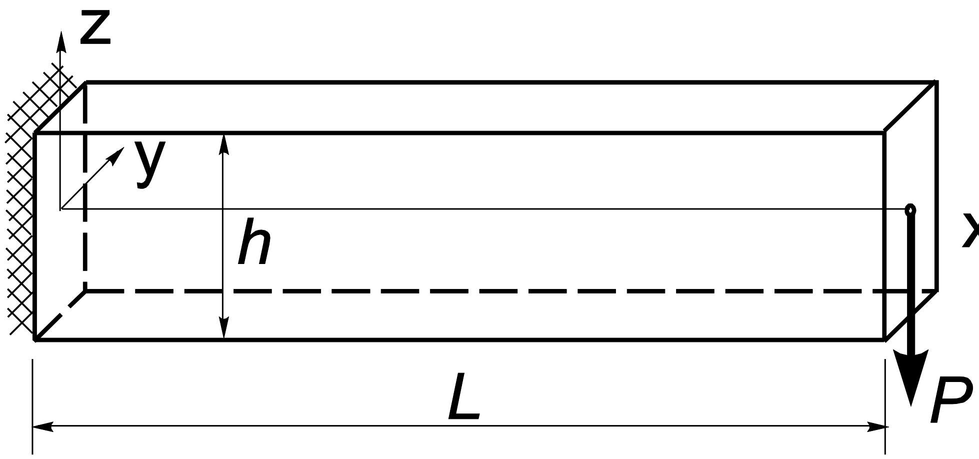
Design model	Buckling mode	Theory	SCAD	Deviation, %
Bar	1-st	61685	0,616821·100000=61682	0,01
	2-nd	61685	0,616821·100000= 61682	0,01
Reissner-Mindlin shell element	1-st	61685	0,613922·100000=61392	0,48
	2-nd	61685	0,617533·100000=61753	0,11
Solid element	1-st	61685	0,613281·100000=61328	0,58
	2-nd	61685	0,613281·100000=61328	0,58

Notes: In the analytical solution the critical values of the concentrated longitudinal compressive force P_{cr1} and P_{cr2}, corresponding to the moments of buckling of the cantilever beam can be determined according to the following formulas:

$$P_{cr1} = \frac{\pi^2 \cdot E \cdot I_y}{4 \cdot L^2} \qquad P_{cr2} = \frac{\pi^2 \cdot E \cdot I_z}{4 \cdot L^2}$$

$$I_y = \frac{b \cdot h^3}{12} \qquad I_z = \frac{h \cdot b^3}{12}$$

Stability of a Cantilever Beam of a Square Cross-Section Subjected to a Concentrated Transverse Bending Force Centrally Applied at the Free End



Objective: Determination of the critical value of the concentrated transverse bending force centrally applied at the free end of a cantilever beam of a square cross-section corresponding to the moment of its buckling.

Initial data files:

File name	Description
Stability_Bar_2_Bar.SPR	Bar model
Stability_Bar_2_Shell.SPR	Shell element model
Stability_Bar_2_Solid.SPR	Solid element model

Problem formulation: The cantilever beam of a square cross-section is subjected to the action of the concentrated transverse bending force P, centrally applied at its free end. Determine the critical value of the concentrated transverse bending force P_{cr}, corresponding to the moment of buckling of the cantilever beam.

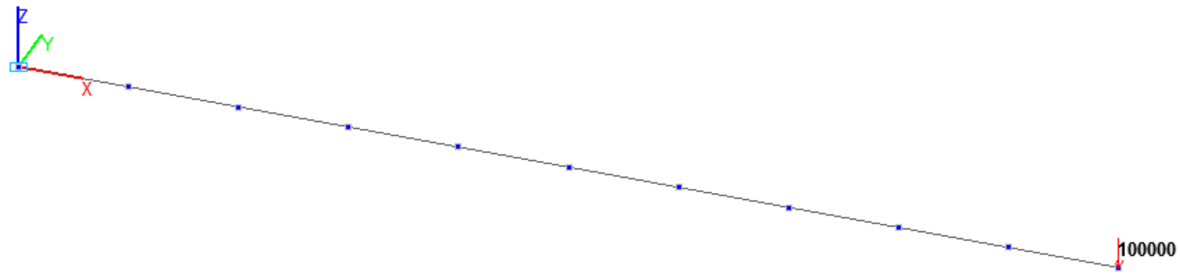
References: A.S. Volmir. Stability of Deformable Systems, Moscow, Nauka, 1967, p.214;

Initial data:

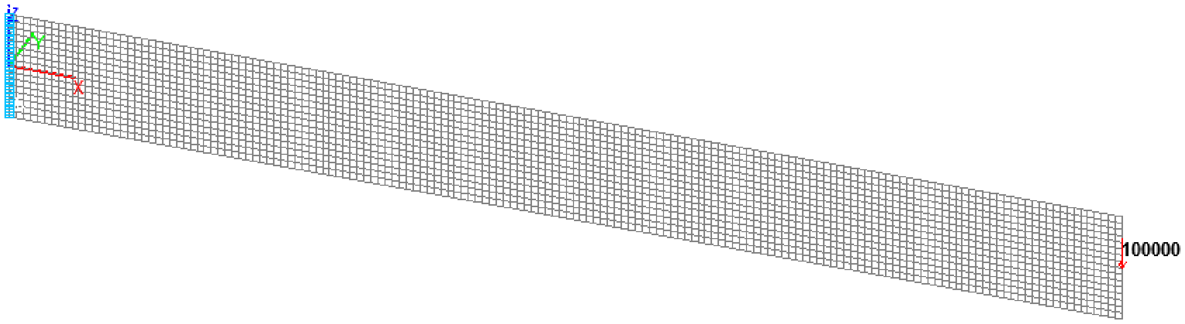
L = 10.0 m	- length of the cantilever beam;
h = b = 1.0 m	- side of the square cross-section of the cantilever beam;
$E = 3.0 \cdot 10^7$ kN/m^2	- elastic modulus of the cantilever beam material;
ν = 0.2	- Poisson's ratio;
$P = 10^5$ kN	- initial value of the concentrated transverse bending force centrally applied at the free end of the beam.

Finite element model: Design model – general type system. Three design models are considered:
Bar model (B), 10 elements of type 5, the spacing of the finite element mesh along the longitudinal axis is 1.0 m. Boundary conditions are provided by imposing constraints on the node of the clamped end of the beam in the directions of the degrees of freedom X, Y, Z, UX, UY, UZ. The action with the initial value of the concentrated transverse bending force P is specified in the node of the free end of the beam. Number of nodes in the design model – 11;
Reissner-Mindlin shell element model (P), 2560 eight-node elements of type 150, the spacing of the finite element mesh along the longitudinal axis and along the height of the beam is 0.0625 m. Boundary conditions are provided by imposing constraints on the nodes of the clamped end of the beam in the directions of the degrees of freedom X, Y, Z, UX, UY, UZ. The action with the initial value of the concentrated longitudinal compressive force P is specified in the node of the longitudinal axis of the beam on the free end. Number of nodes in the design model – 8033.
Solid element model (S), 5120 twenty-node elements of type 37, the spacing of the finite element mesh along the longitudinal axis, width and height of the beam is 0.125 m. Boundary conditions are provided by imposing constraints on the nodes of the clamped end of the beam in the directions of the degrees of freedom X, Y, Z, UX, UY, UZ. The action with the initial value of the concentrated transverse bending force P is specified as a load uniformly distributed over the external faces of the elements of the beam end $p = P/(h \cdot b)$. Number of nodes in the design model – 24705.

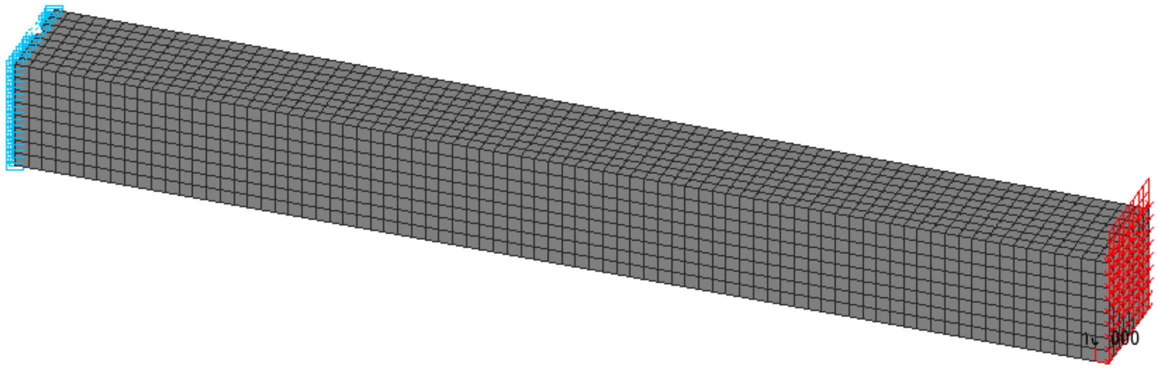
Results in SCAD



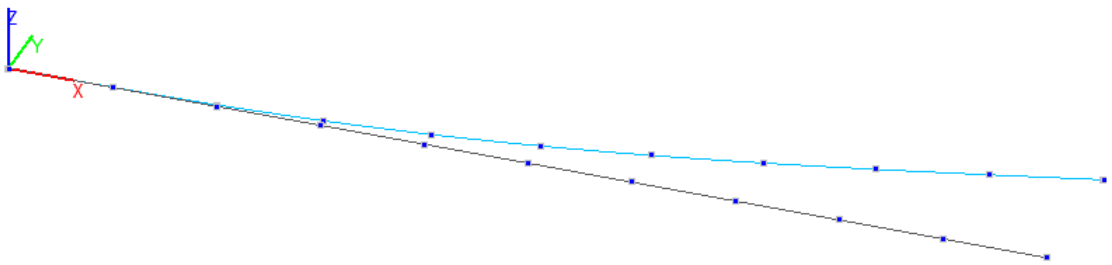
Design model. Bar model



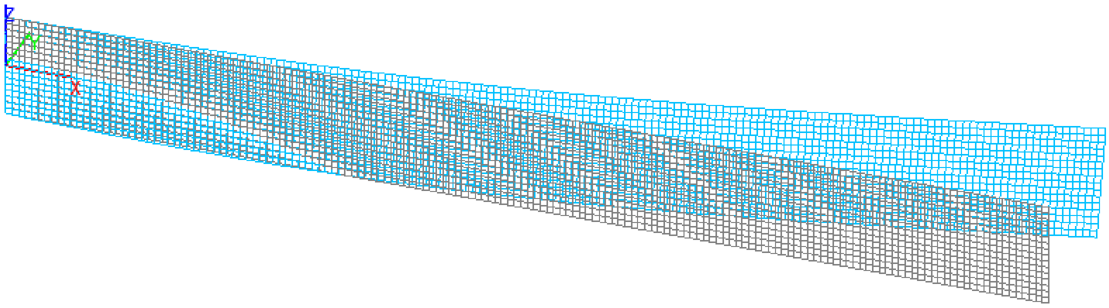
Design model. Reissner-Mindlin shell element model



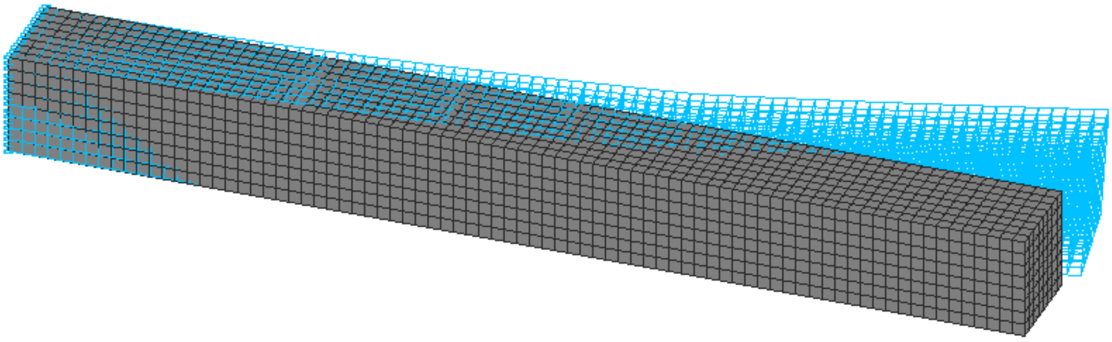
Design model. Solid element model



1-st buckling mode. Bar model



1-st buckling mode. Reissner-Mindlin shell element model



1-st buckling mode. Solid element model

Comparison of solutions:

Critical value of the concentrated transverse bending force P_{cr} (kN), centrally applied at the free end of the beam

Design model	Theory	SCAD	Deviation, %
Bar	84111	0,834694·100000=83469	0,76
Reissner-Mindlin shell element	84111	0,821972·100000=82197	2,28
Solid element	84111	0,843750·100000=84375	0,31

Notes: In the analytical solution the critical value of the concentrated transverse bending force P_{cr}, corresponding to the moment of buckling of the cantilever beam can be determined according to the following formula:

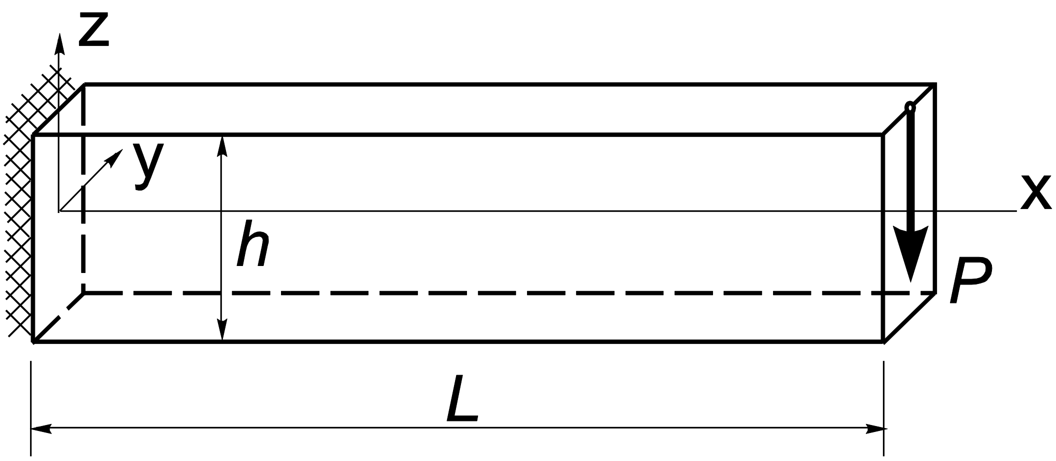
$$P = \frac{4,01 \cdot \sqrt{E \cdot I_z \cdot G \cdot I_x}}{L^2} \qquad G = \frac{E}{2 \cdot (1+\nu)}$$

$I_z = \frac{h \cdot b^3}{12}$ – minimum bending inertia moment (out of the moment plane);

$I_x = k_f \cdot h \cdot b^3$ – free torsional inertia moment, where:

$$k_f = \frac{1}{3} \cdot \left\{ 1 - \frac{192}{\pi^5} \cdot \frac{b}{h} \cdot \sum_{n=1}^{\infty} \left[sin^2\left(\frac{n \cdot \pi}{2}\right) \cdot \frac{1}{n^5} \cdot th\left(\frac{n \cdot \pi \cdot h}{2 \cdot b}\right) \right] \right\}$$

Stability of a Cantilever Beam of a Square Cross-Section Subjected to a Concentrated Transverse Bending Force Applied to the Upper Edge of the Free End



Objective: Determination of the critical value of the concentrated transverse bending force applied to the upper edge of the free end of a cantilever beam of a square cross-section corresponding to the moment of its buckling.

Initial data files:

File name	Description
Stability_Bar_3_Bar.SPR	Bar model
Stability_Bar_3_Shell.SPR	Shell element model
Stability_Bar_3_Solid.SPR	Solid element model

Problem formulation: The cantilever beam of a square cross-section is subjected to the action of the concentrated transverse bending force *P*, applied to the upper edge of its free end. Determine the critical value of the concentrated transverse bending force P_{cr}, corresponding to the moment of buckling of the cantilever beam.

References: .S. Volmir, Stability of Deformable Systems, Moscow, Nauka, 1967, p.216;

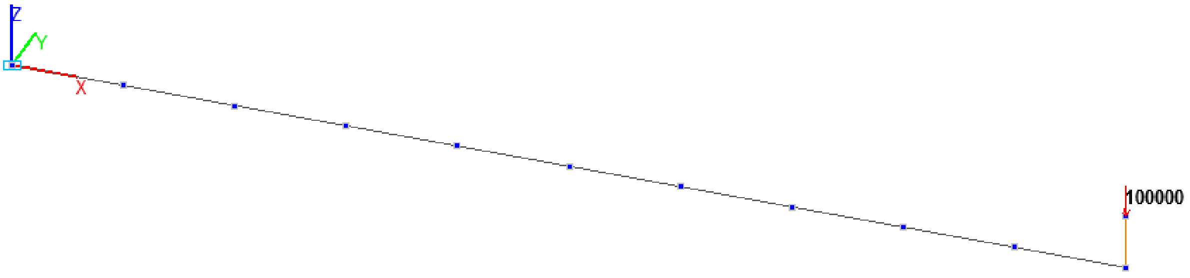
Initial data:

L = 10.0 m	- length of the cantilever beam;
h = b = 1.0 m	- side of the square cross-section of the cantilever beam;
h/2 = 0.5 m	- height of the application point of the concentrated transverse bending force with respect to the longitudinal axis of the beam (X axis of the global coordinate system);
$E = 3.0 \cdot 10^7$ kN/m^2	- elastic modulus of the cantilever beam material;
ν = 0.2	- Poisson's ratio;
$P = 10^5$ kN	- initial value of the concentrated transverse bending force applied to the upper edge of the free end of the beam.

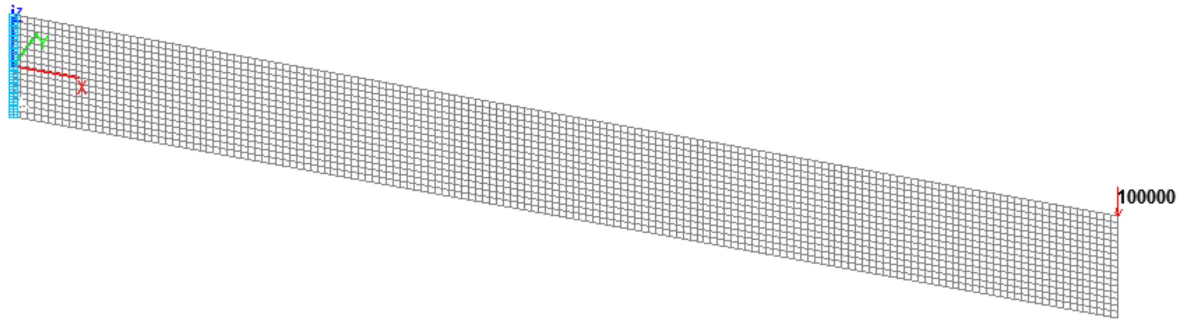
Finite element model: Design model – general type system. Three design models are considered:
Bar model (B), 10 elements of type 5, the spacing of the finite element mesh along the longitudinal axis is 1.0 m. Boundary conditions are provided by imposing constraints on the node of the clamped end of the beam in the directions of the degrees of freedom X, Y, Z, UX, UY, UZ. 1 vertical upward two-node element of type 100 (3D rigid body) with the length h/2 is adjacent to the node of the free end of the beam. The action with the initial value of the concentrated transverse bending force P is specified in the free node of the element of the rigid body (elevated application point). Number of nodes in the design model – 12.
Reissner-Mindlin shell element model (P), 2560 eight-node elements of type 150, the spacing of the finite element mesh along the longitudinal axis and along the height of the beam is 0.0625 m. Boundary conditions are provided by imposing constraints on the nodes of the clamped end of the beam in the directions of the degrees of freedom X, Y, Z, UX, UY, UZ. The action with the initial value of the

concentrated transverse bending force P is specified in the node on the free end at the height h/2 from the longitudinal axis of the beam. Number of nodes in the design model – 8033.
Solid element model (S), 5120 twenty-node elements of type 37, the spacing of the finite element mesh along the longitudinal axis, width and height of the beam is 0.125 m. Boundary conditions are provided by imposing constraints on the nodes of the clamped end of the beam in the directions of the degrees of freedom X, Y, Z, UX, UY, UZ. The action with the initial value of the concentrated transverse bending force P is specified as a group of nodal forces on the upper edge of the free end of the beam P_i = P·0.0625/1.0 = 6250 kN (3125 kN for corner nodes). Number of nodes in the design model – 24705.

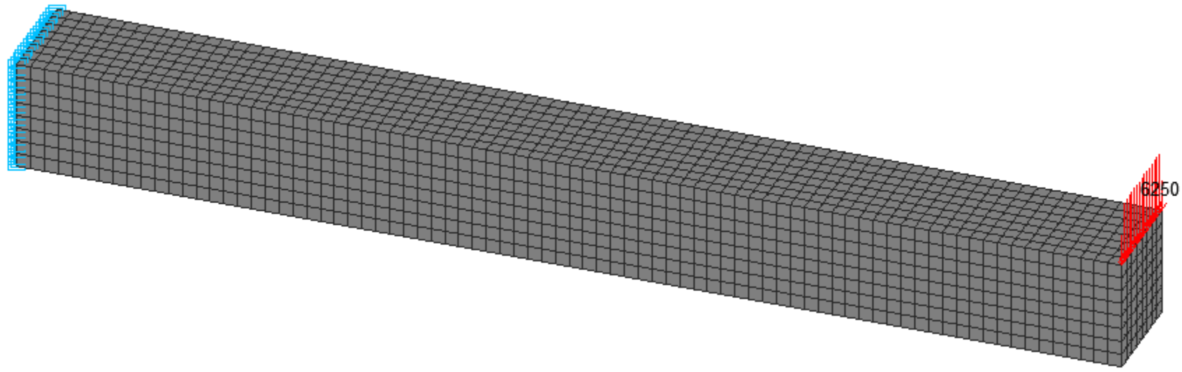
Results in SCAD



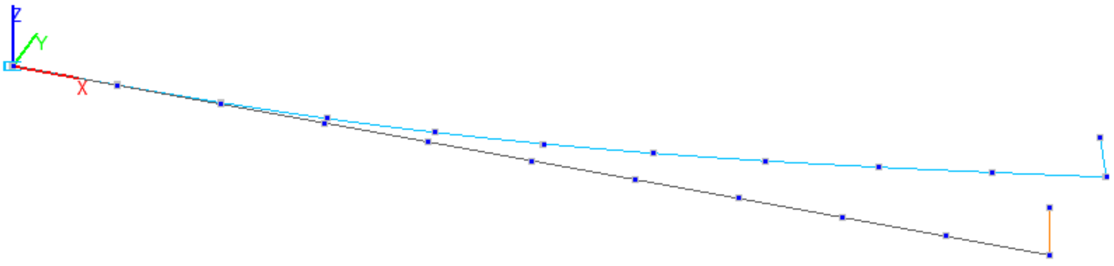
Design model. Bar model



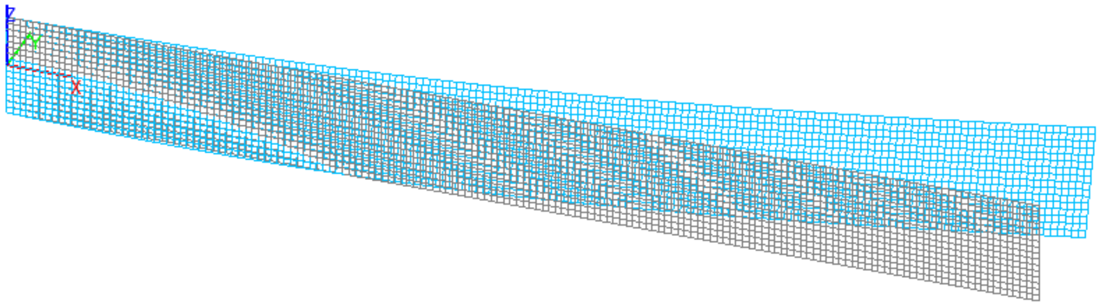
Design model. Reissner-Mindlin shell element model



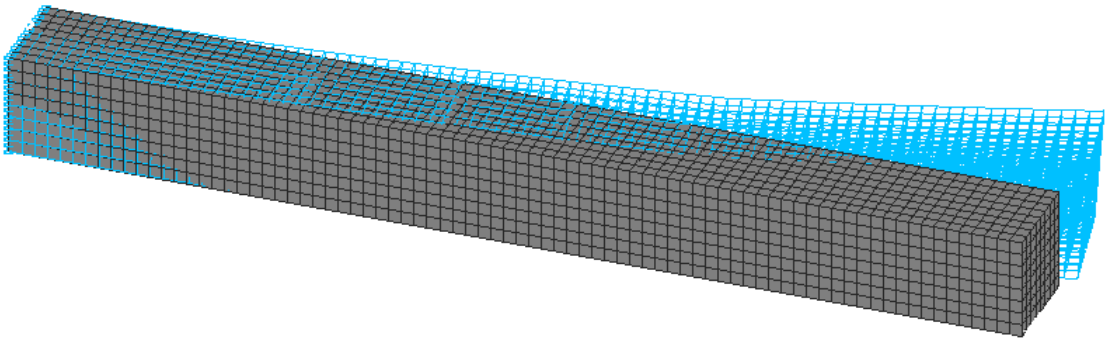
Design model. Solid element model



1-st buckling mode. Bar model



1-st buckling mode. Reissner-Mindlin shell element model



1-st buckling mode. Solid element model

Comparison of solutions:

Critical value of the concentrated transverse bending force P_{cr} (kN), applied to the upper edge of the free end of the beam

Design model	Theory	SCAD	Deviation, %
Bar	78305	0,778008·100000=77801	0,64
Reissner-Mindlin shell element	78305	0,768958·100000=76896	1,80
Solid element	78305	0,816406·100000=81641	4,26

Notes: In the analytical solution the critical value of the concentrated transverse bending force P_{cr}, corresponding to the moment of buckling of the cantilever beam can be determined according to the following formula:

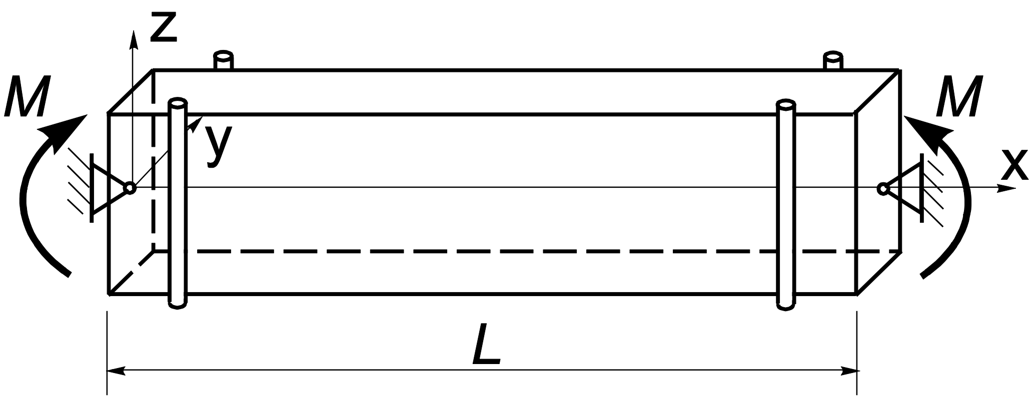
$$P = \frac{4,01 \cdot \sqrt{E \cdot I_z \cdot G \cdot I_x}}{L^2} \cdot k_h \qquad G = \frac{E}{2 \cdot (1+\nu)} \qquad k_h = f\left(\frac{h}{2 \cdot L} \cdot \sqrt{\frac{E \cdot I_z}{G \cdot I_x}}\right)$$

$I_z = \frac{h \cdot b^3}{12}$ – minimum bending inertia moment (out of the moment plane);

$I_x = k_f \cdot h \cdot b^3$ – free torsional inertia moment, where:

$$k_f = \frac{1}{3} \cdot \left\{1 - \frac{192}{\pi^5} \cdot \frac{b}{h} \cdot \sum_{n=1}^{\infty}\left[\sin^2\left(\frac{n \cdot \pi}{2}\right) \cdot \frac{1}{n^5} \cdot th\left(\frac{n \cdot \pi \cdot h}{2 \cdot b}\right)\right]\right\}$$

Stability of a Beam of a Square Cross-Section Simply Supported in and out of the Bending Plane Subjected to Concentrated Bending Moments Applied at the Ends and Equal in Value (Pure Bending)



Objective: Determination of the critical value of the concentrated bending moments equal in value and applied at the ends of a beam of a square cross-section simply supported in and out of the bending plane corresponding to the moment of its buckling.

Initial data files:

File name	Description
Stability_Bar_4_Bar.SPR	Bar model
Stability_Bar_4_Shell.SPR	Shell element model

Problem formulation: The beam of a square cross-section simply supported in and out of the bending plane is subjected to the action of the concentrated bending moments M, equal in value and applied at its ends. Determine the critical value of the concentrated bending moments M_{cr}, corresponding to the moment of buckling of the simply supported beam.

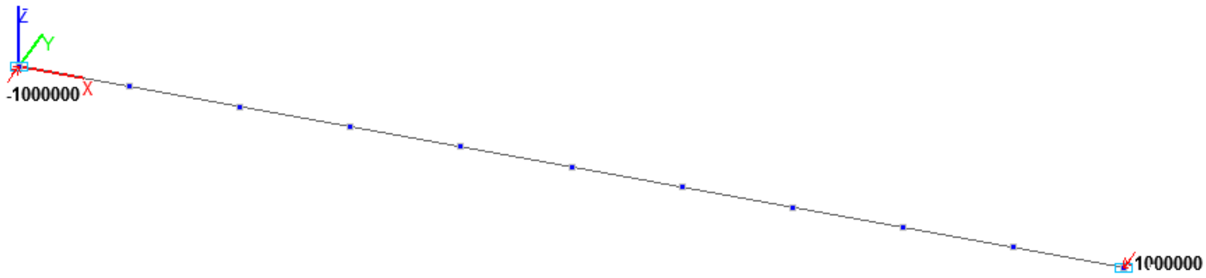
References: A.S. Volmir. Stability of Deformable Systems, Moscow, Nauka, 1967, p.204, 213;

Initial data:

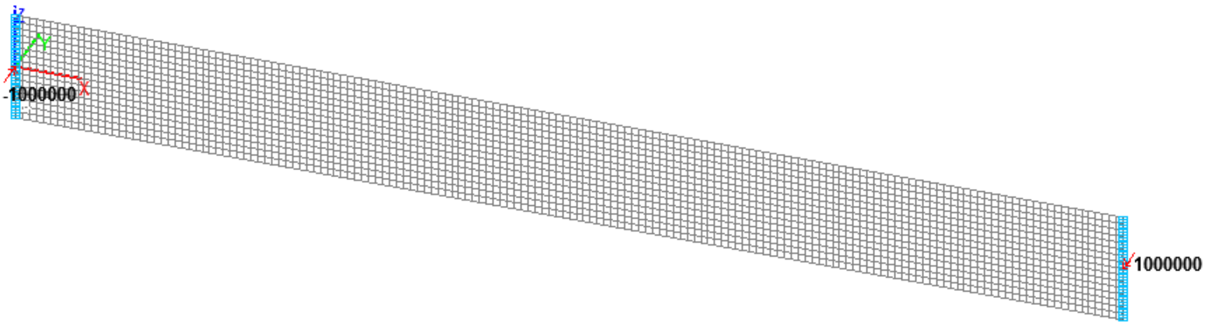
L = 10.0 m	- length of the simply supported beam;
h = b = 1.0 m	- side of the square cross-section of the simply supported beam;
$E = 3.0 \cdot 10^7$ kN/m^2	- elastic modulus of the simply supported cantilever beam material;
ν = 0.2	- Poisson's ratio;
$M = 10^6$ kN·m	- initial value of the concentrated bending moments applied at the ends of the beam.

Finite element model: Design model – general type system. Two design models are considered:
Bar model (B), 10 elements of type 5, the spacing of the finite element mesh along the longitudinal axis is 1.0 m. Boundary conditions are provided by imposing constraints on the nodes of the simply supported ends of the beam in the directions of the degrees of freedom X, Y, Z, UX. The action with the initial value of the concentrated bending moments M is specified in the nodes of the ends of the beam. Number of nodes in the design model – 11;
Reissner-Mindlin shell element model (P), 2560 eight-node elements of type 150, the spacing of the finite element mesh along the longitudinal axis and along the height of the beam is 0.0625 m. The shell is supported by vertical high-rigidity bars (h = b = 1.0 m; $E = 3.0 \cdot 10^9$ kN/m^2; ν = 0.2), 64 elements of type 5. Boundary conditions are provided by imposing constraints on the nodes of the ends of the beam lying on its longitudinal axis in the directions of the degrees of freedom X, Y, Z and on all other nodes of the ends of the beam in the direction of the degree of freedom Y. The action with the initial value of the concentrated bending moments M is specified on the nodes of the ends of the beam lying on its longitudinal axis. Number of nodes in the design model – 8033.

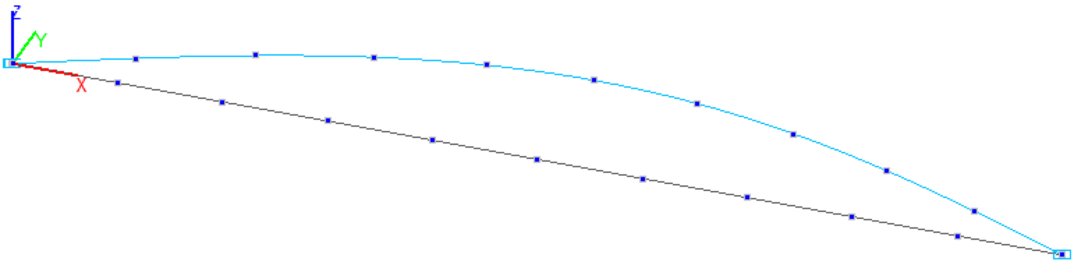
Results in SCAD



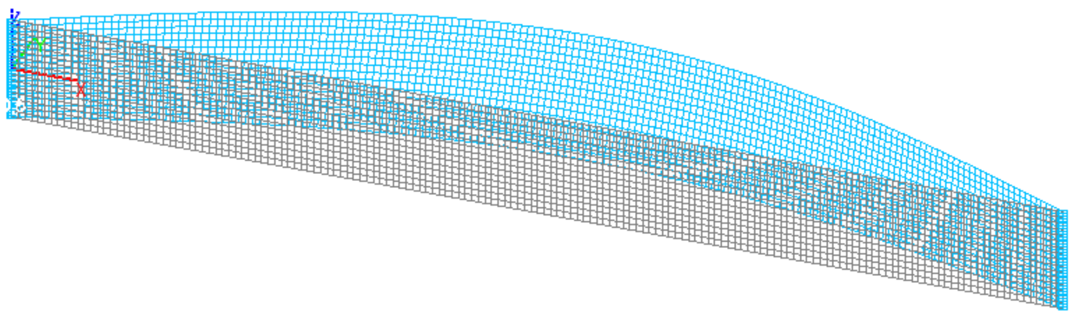
Design model. Bar model



Design model. Reissner-Mindlin shell element model



1-st buckling mode. Bar model



1- st buckling mode. Reissner-Mindlin shell element model

Comparison of solutions:

Critical value of the concentrated bending moments M_{cr} (kN·m), applied at the ends of the beam simply supported in and out of the bending plane

Design model	Theory	SCAD	Deviation, %
Bar	658464	0,654602·1000000=54602	0,59
Reissner-Mindlin shell element	658464	0,650024·1000000=650024	1,28

Notes: In the analytical solution the critical value of the concentrated bending moments M_{cr}, corresponding to the moment of buckling of the simply supported beam can be determined according to the following formula:

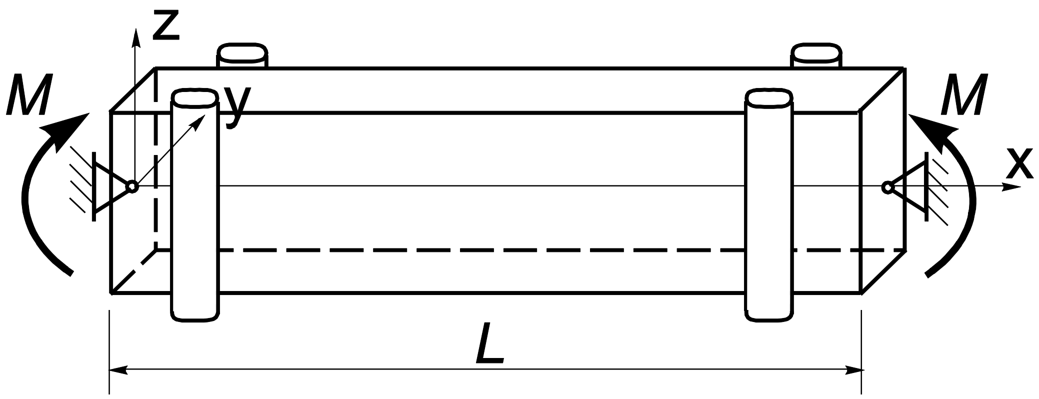
$$M = \frac{\pi \cdot \sqrt{E \cdot I_z \cdot G \cdot I_x}}{L} \qquad G = \frac{E}{2 \cdot (1+\nu)}$$

$I_z = \dfrac{h \cdot b^3}{12}$ – minimum bending inertia moment (out of the moment plane);

$I_x = k_f \cdot h \cdot b^3$ – free torsional inertia moment, where:

$$k_f = \frac{1}{3} \cdot \left\{ 1 - \frac{192}{\pi^5} \cdot \frac{b}{h} \cdot \sum_{n=1}^{\infty} \left[sin^2\left(\frac{n \cdot \pi}{2}\right) \cdot \frac{1}{n^5} \cdot th\left(\frac{n \cdot \pi \cdot h}{2 \cdot b}\right) \right] \right\}$$

Stability of a Beam of a Square Cross-Section Simply Supported in the Bending Plane and Clamped out of the Bending Plane Subjected to Concentrated Bending Moments Applied at the Ends and Equal in Value (Pure Bending)



Objective: Determination of the critical value of the concentrated bending moments equal in value and applied at the ends of a beam of a square cross-section simply supported in the bending plane and clamped out of the bending plane corresponding to the moment of its buckling.

Initial data files:

File name	Description
Stability_Bar_5_Bar.SPR	Bar model
Stability_Bar_5_Shell.SPR	Shell element model

Problem formulation: The beam of a square cross-section simply supported in the bending plane and clamped out of the bending plane is subjected to the action of the concentrated bending moments M, equal in value and applied at its ends. Determine the critical value of the concentrated bending moments M_{cr}, corresponding to the moment of buckling of the simply supported beam.

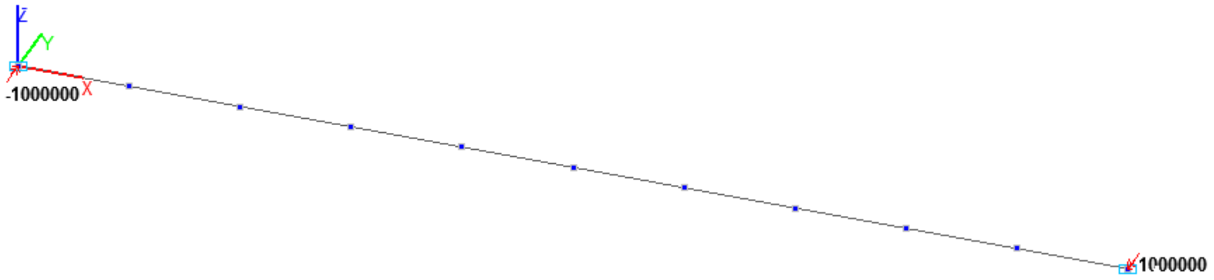
References: I.A. Birger, Ya.G. Panovko, Strength, Stability, Vibrations, Handbook in three volumes, Volume 3, Moscow, Mechanical engineering, 1968, p.68;

Initial data:

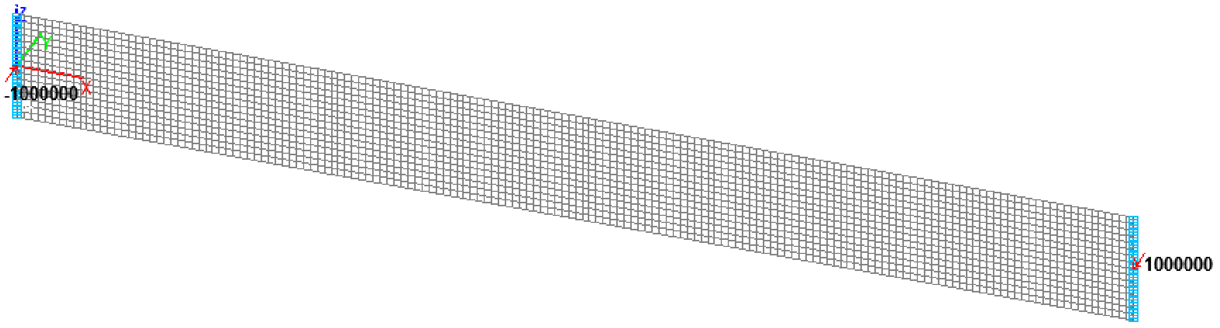
L = 10.0 m	- length of the simply supported beam;
h = b = 1.0 m	- side of the square cross-section of the simply supported beam;
$E = 3.0 \cdot 10^7$ kN/m^2	- elastic modulus of the simply supported beam material;
ν = 0.2	- Poisson’s ratio;
$M = 10^6$ kN·m	- initial value of the concentrated bending moments applied at the ends of the beam.

Finite element model: Design model – general type system. Two design models are considered:
Bar model (B), 10 elements of type 5, the spacing of the finite element mesh along the longitudinal axis is 1.0 m. Boundary conditions are provided by imposing constraints on the nodes of the simply supported ends of the beam in the directions of the degrees of freedom X, Y, Z, UX, UZ. The action with the initial value of the concentrated bending moments M is specified in the nodes of the ends of the beam. Number of nodes in the design model – 11;
Reissner-Mindlin shell element model (P), 2560 eight-node elements of type 150, the spacing of the finite element mesh along the longitudinal axis and along the height of the beam is 0.0625 m. The shell is supported by vertical high-rigidity bars (h = b = 1.0 m; $E = 3.0 \cdot 10^9$ kN/m^2; ν = 0.2), 64 elements of type 5. Boundary conditions are provided by imposing constraints on the nodes of the ends of the beam lying on its longitudinal axis in the directions of the degrees of freedom X, Y, Z and on all other nodes of the ends of the beam in the directions of the degrees of freedom Y. The action with the initial value of the concentrated bending moments M is specified on the nodes of the ends of the beam lying on its longitudinal axis. Number of nodes in the design model – 8033.

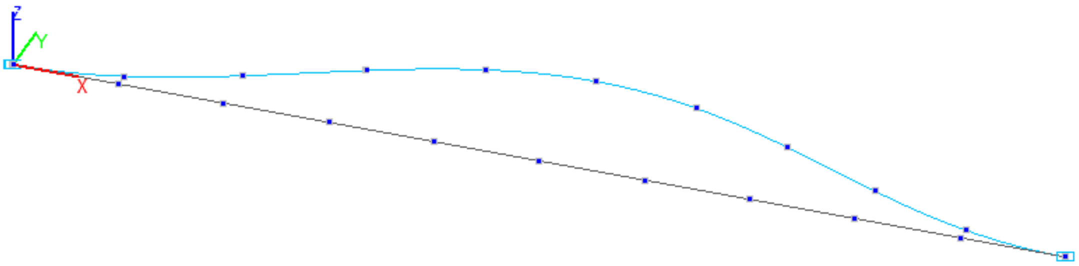
Results in SCAD



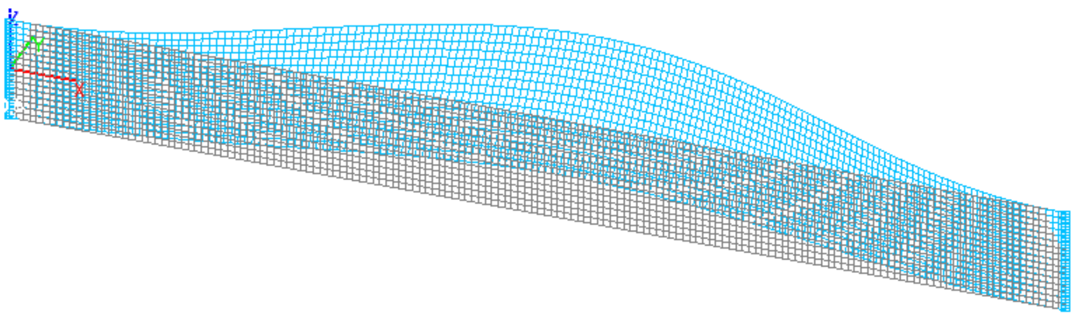
Design model. Bar model



Design model. Reissner-Mindlin shell element model



1-st buckling mode. Bar model



1-st buckling mode. Reissner-Mindlin shell element model

Comparison of solutions:

Critical value of the concentrated bending moments M_{cr} (kN·m), applied at the ends of the beam simply supported in the bending plane and clamped out of the bending plane

Deseign model	Theory	SCAD	Deviation, %
Bar	1316928	1,325369·1000000=1325369	0,64
Reissner-Mindlin shell element	1316928	1,246357·1000000=1246357	5,36

Notes: In the analytical solution the critical value of the concentrated bending moments M_{cr}, corresponding to the moment of buckling of the simply supported beam can be determined according to the following formula:

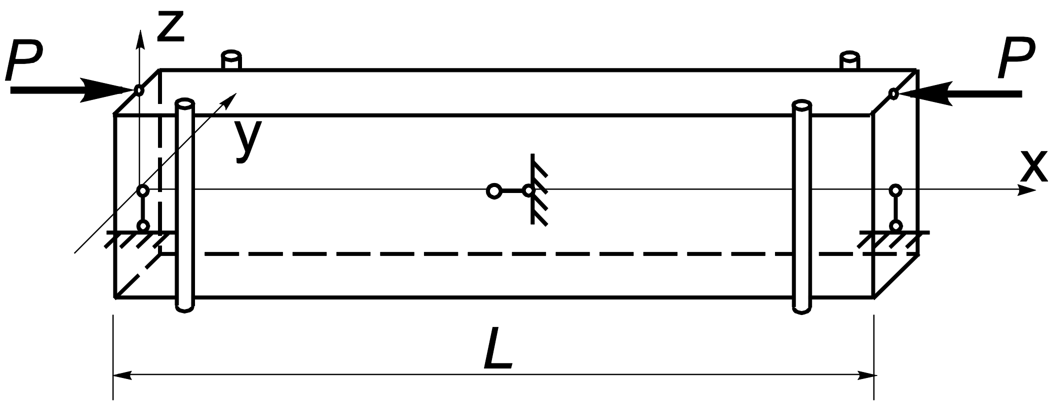
$$M = \frac{2 \cdot \pi \cdot \sqrt{E \cdot I_z \cdot G \cdot I_x}}{L} \qquad G = \frac{E}{2 \cdot (1+\nu)}$$

$I_z = \frac{h \cdot b^3}{12}$ – minimum bending inertia moment (out of the moment plane);

$I_x = k_f \cdot h \cdot b^3$ – free torsional inertia moment, where:

$$k_f = \frac{1}{3} \cdot \left\{ 1 - \frac{192}{\pi^5} \cdot \frac{b}{h} \cdot \sum_{n=1}^{\infty} \left[sin^2\left(\frac{n \cdot \pi}{2}\right) \cdot \frac{1}{n^5} \cdot th\left(\frac{n \cdot \pi \cdot h}{2 \cdot b}\right) \right] \right\}$$

Stability of a Beam of a Square Cross-Section Simply Supported in and out of the Bending Plane Subjected to Concentrated Longitudinal Bending Forces Applied to the Upper Edges of the Ends and Equal in Value (Longitudinal Bending)



Objective: Determination of the first two critical values of concentrated longitudinal bending forces equal in value and applied to the upper edges of the ends of a beam of a square cross-section simply supported in and out of the bending plane corresponding to the moment of its buckling.

Initial data files:

File name	Description
Stability_Bar_6_Bar.SPR	Bar model
Stability_Bar_6_Shell.SPR	Shell element model

Problem formulation: The beam of a square cross-section simply supported in and out of the bending plane is subjected to the action of the concentrated longitudinal bending forces P, equal in value and applied to the upper edges of its ends. Determine first two critical values of the concentrated longitudinal bending forces P_{cr1} and P_{cr2}, corresponding to the moment of buckling of the simply supported beam.

References: S. P. Timoshenko, Stability of Bars, Plates and Shells, Moscow, Nauka, 1971, p.291

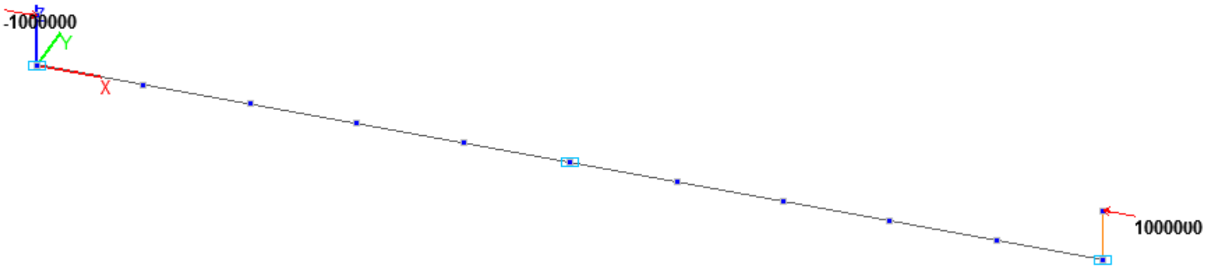
Initial data:

L = 10.0 m	- length of the simply supported beam;
h = b = 1.0 m	- side of the square cross-section of the simply supported beam;
$E = 3.0 \cdot 10^7$ kN/m^2	- elastic modulus of the simply supported beam material;
ν = 0.2	- Poisson's ratio;
$P = 10^6$ kN	- initial value of the concentrated longitudinal bending forces applied to the upper edges of the ends of the beam.

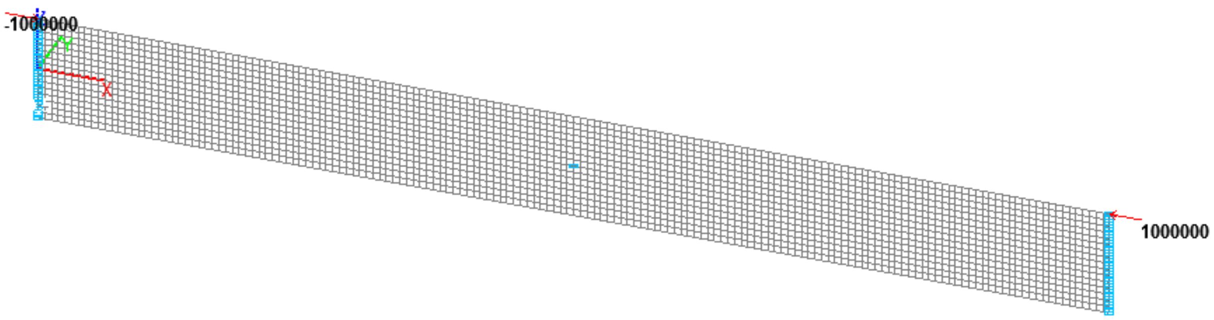
Finite element model: Design model – general type system. Two design models are considered:
Bar model (B), 10 elements of type 5, the spacing of the finite element mesh along the longitudinal axis is 1.0 m. Boundary conditions are provided by imposing constraints on the nodes of the simply supported ends of the beam in the directions of the degrees of freedom Y, Z. The dimensional stability is provided by imposing constraints on the node in the middle of the beam span in the directions of the degrees of freedom X, UX. 2 vertical upward two-node elements of type 100 (3D rigid body) with the length h/2 are adjacent to the nodes of the ends of the beam. The action with the initial value of the concentrated longitudinal bending forces P is specified in the free nodes of the elements of the rigid bodies (elevated application points). Number of nodes in the design model – 13.
Reissner-Mindlin shell element model (P), 2560 eight-node elements of type 150, the spacing of the finite element mesh along the longitudinal axis and along the height of the beam is 0.0625 m. The shell is supported by vertical high-rigidity bars (h = b = 1.0 m; $E = 3.0 \cdot 10^9$ kN/m^2; ν = 0.2), 64 elements of type 5. Boundary conditions are provided by imposing constraints on the nodes of the ends of the beam lying on its longitudinal axis in the directions of the degrees of freedom Y, Z and on all other nodes of the ends of the beam in the direction of the degree of freedom Y. The dimensional stability is provided by imposing a constraint on the node in the middle of the beam span along its longitudinal axis in the direction of the degree of freedom X. The action with the initial value of the concentrated longitudinal bending forces P is

specified in the nodes on the ends at the height h/2 from the longitudinal axis of the beam. Number of nodes in the design model – 8033.

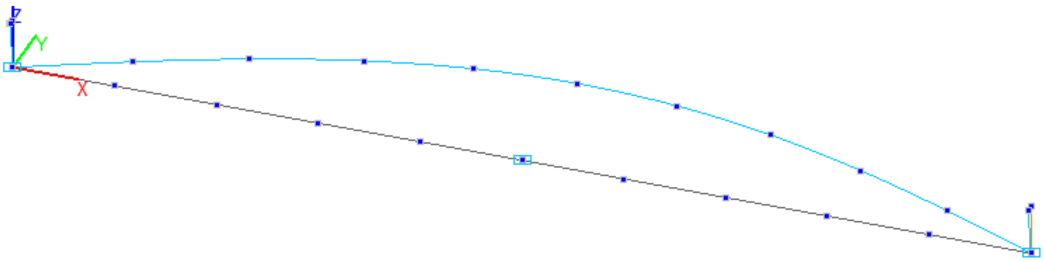
Results in SCAD



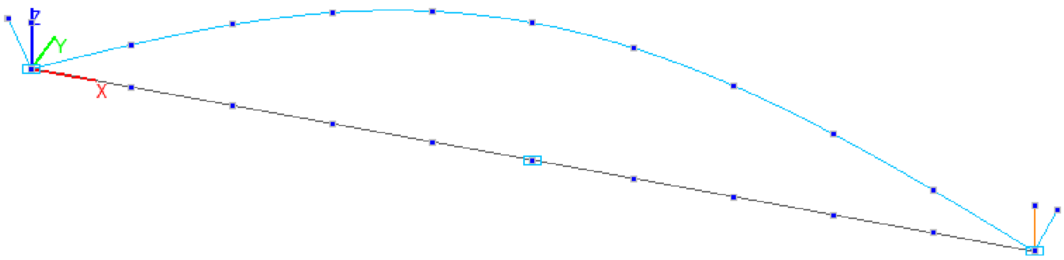
Design model. Bar model



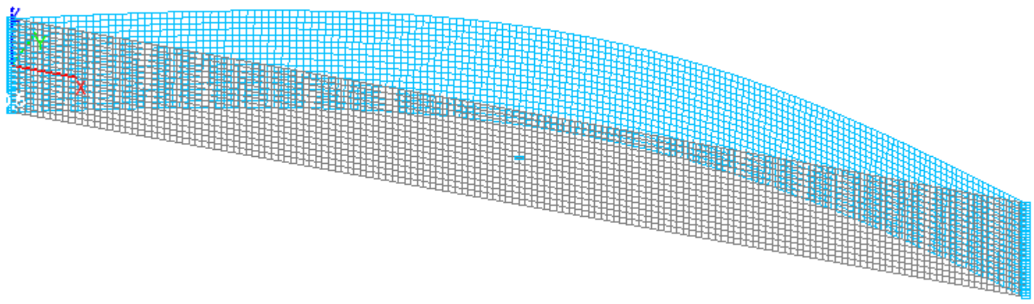
Design model. Reissner-Mindlin shell element model



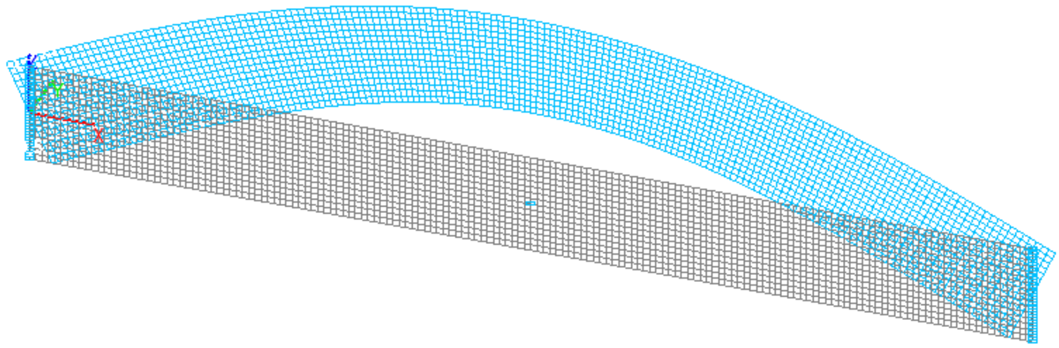
1- st buckling mode. Bar model



2- nd buckling mode. Bar model



1- st buckling mode. Reissner-Mindlin shell element model



2-nd buckling mode. Reissner-Mindlin shell element model

Comparison of solutions:

Critical values of the concentrated longitudinal bending forces P_{cr1} and P_{cr2} (kN), applied to the upper edges of the ends of the beam simply supported in and out of the bending plane

Design model	Buckling mode	Theory	SCAD	Deviation, %
Bar	1-st	61685	0,616821·100000=61682	0,01
	2-nd	61685	0,616821·100000=61682	0,01
Reissner-Mindlin shell element	1-st	61685	0,613922·100000=61392	0,48
	2-nd	61685	0,617533·100000=61753	0,11

Notes: In the analytical solution the critical values of the concentrated longitudinal bending forces P_{cr1} and P_{cr2}, corresponding to the moments of buckling of the simply supported beam can be determined according to the following formulas:

$$P_1 = \frac{2 \cdot G \cdot I_x}{h^2} \cdot \left(-1 + \sqrt{1 + \frac{\pi^2 \cdot h^2}{L^2} \cdot \frac{E \cdot I_z}{G \cdot I_x}} \right) \qquad P_2 = \frac{\pi^2 \cdot E \cdot I_y}{L^2} \qquad G = \frac{E}{2 \cdot (1 + \nu)}$$

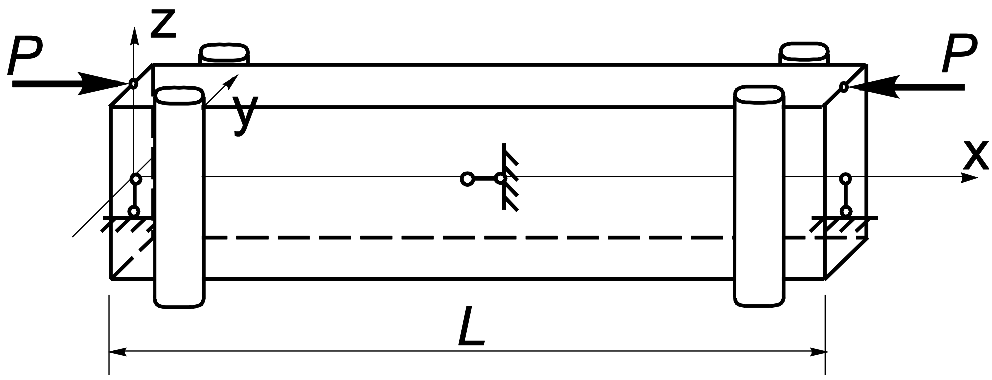
$I_z = \frac{h \cdot b^3}{12}$ – minimum bending inertia moment (out of the moment plane);

$I_y = \frac{b \cdot h^3}{12}$ – maximum bending inertia moment (in the moment plane);

$I_x = k_f \cdot h \cdot b^3$ – free torsional inertia moment, where:

$$k_f = \frac{1}{3} \cdot \left\{ 1 - \frac{192}{\pi^5} \cdot \frac{b}{h} \cdot \sum_{n=1}^{\infty} \left[sin^2\left(\frac{n \cdot \pi}{2} \right) \cdot \frac{1}{n^5} \cdot th\left(\frac{n \cdot \pi \cdot h}{2 \cdot b} \right) \right] \right\}$$

Stability of a Beam of a Square Cross-Section Simply Supported in the Bending Plane and Clamped out of the Bending Plane Subjected to Concentrated Longitudinal Bending Forces Applied to the Upper Edges of the Ends and Equal in Value (Longitudinal Bending)



Objective: Determination of the first two critical values of concentrated longitudinal bending forces equal in value and applied to the upper edges of the ends of a beam of a square cross-section simply supported in the bending plane and clamped out of the bending plane corresponding to the moment of its buckling.

Initial data files:

File name	Description
Stability_Bar_7_Bar.SPR	Bar model
Stability_Bar_7_Shell.SPR	Shell element model

Problem formulation: The beam of a square cross-section simply supported in the bending plane and clamped out of the bending plane is subjected to the action of the concentrated longitudinal bending forces P, equal in value and applied to the upper edges of its ends. Determine first two critical values of the concentrated longitudinal bending forces P_{cr1} and P_{cr2}, corresponding to the moment of buckling of the simply supported beam.

References: S. P. Timoshenko, Stability of Bars, Plates and Shells, Moscow, Nauka, 1971, p.291

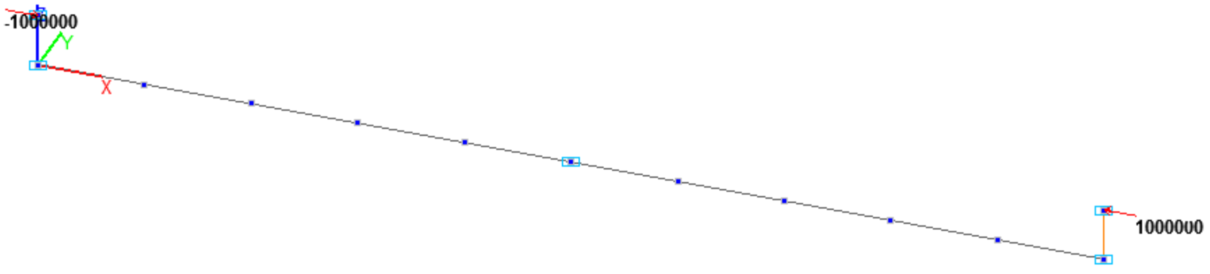
Initial data:

L = 10.0 m	- length of the simply supported beam;
h = b = 1.0 m	- side of the square cross-section of the simply supported beam;
$E = 3.0 \cdot 10^7$ kN/m^2	- elastic modulus of the simply supported beam material;
$\nu = 0.2$	- Poisson's ratio;
$P = 10^6$ kN	- initial value of the concentrated longitudinal bending forces applied to the upper edges of the ends of the beam.

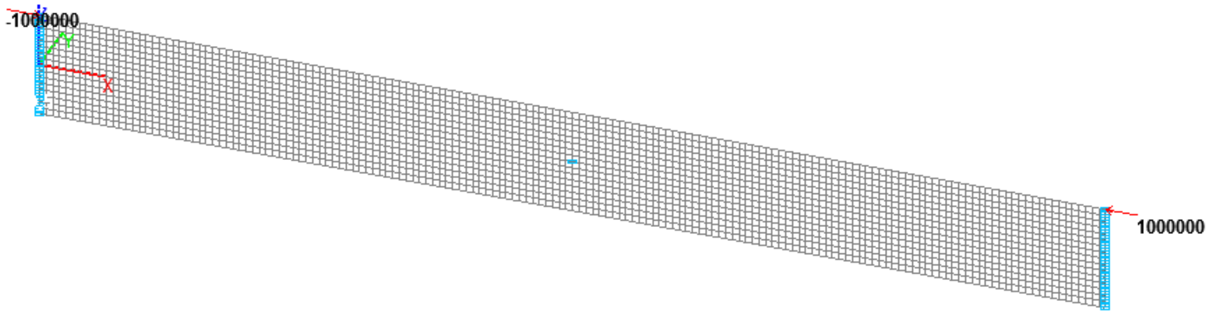
Finite element model: Design model – general type system. Two design models are considered:
Bar model (B), 10 elements of type 5, the spacing of the finite element mesh along the longitudinal axis is 1.0 m. Boundary conditions are provided by imposing constraints on the nodes of the simply supported ends of the beam in the directions of the degrees of freedom Y, Z, UZ. The dimensional stability is provided by imposing constraints on the node in the middle of the beam span in the directions of the degrees of freedom X, UX. 2 vertical upward two-node elements of type 100 (3D rigid body) with the length h/2 are adjacent to the nodes of the ends of the beam. A constraint in the UZ direction is imposed on the upper nodes of the elements of rigid bodies. The action with the initial value of the concentrated longitudinal bending forces P is specified in the upper nodes of the elements of the rigid bodies (elevated application points). Number of nodes in the design model – 13;
Reissner-Mindlin shell element model (P), 2560 eight-node elements of type 150, the spacing of the finite element mesh along the longitudinal axis and along the height of the beam is 0.0625 m. The shell is supported by vertical high-rigidity bars (h = b = 1.0 m; $E = 3.0 \cdot 10^9$ kN/m^2; $\nu = 0.2$), 64 elements of type 5. Boundary conditions are provided by imposing constraints on the nodes of the ends of the beam lying on its longitudinal axis in the directions of the degrees of freedom Y, Z, UZ and on all other nodes of the ends of the beam in the directions of the degrees of freedom Y, UZ. The dimensional stability is provided by

imposing a constraint on the node in the middle of the beam span along its longitudinal axis in the direction of the degree of freedom X. The action with the initial value of the concentrated longitudinal bending forces P is specified in the nodes on the ends at the height h/2 from the longitudinal axis of the beam. Number of nodes in the design model – 8033.

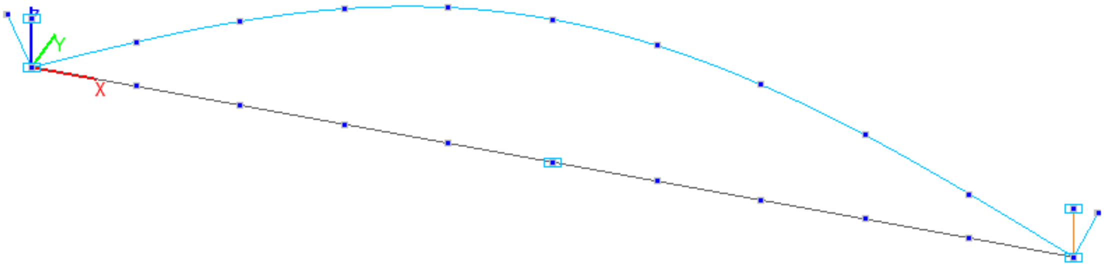
Results in SCAD



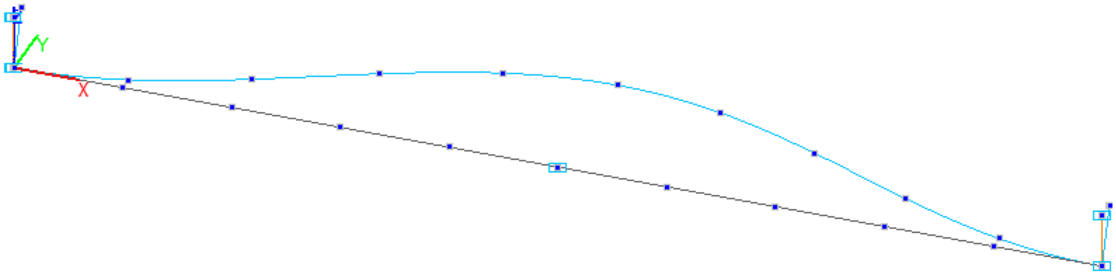
Design model. Bar model



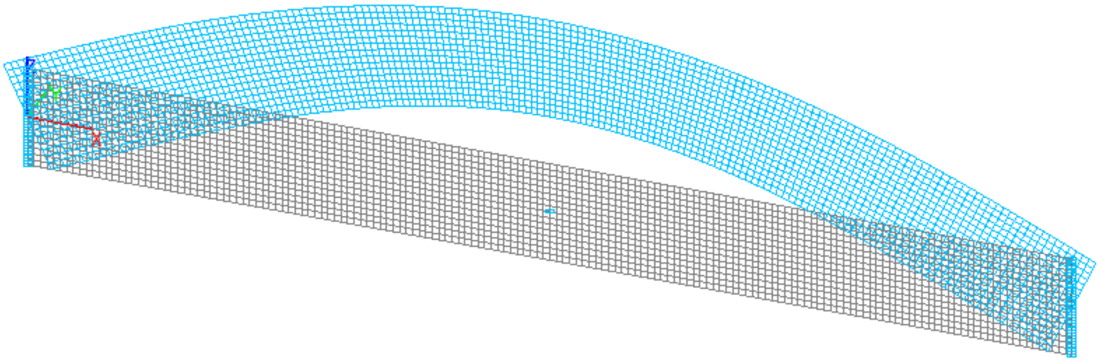
Design model. Reissner-Mindlin shell element model



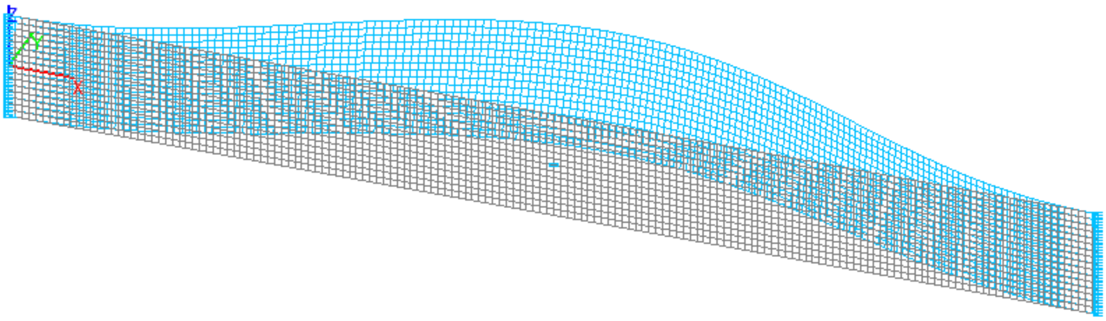
1-st buckling mode. Bar model



2-nd buckling mode. Bar model



1-st buckling mode. Reissner-Mindlin shell element model



2-nd buckling mode. Reissner-Mindlin shell element model

Comparison of solutions:

Critical values of the concentrated longitudinal bending forces P_{cr1} and P_{cr2} (kN), applied to the upper edges of the ends of the beam simply supported in the bending plane and clamped out of the bending plane

Design model	Buckling mode	Theory	SCAD	Deviation, %
Bar	1-st	246741	0,246740·1000000=246740	0,00
	2-nd	877429	0,877630·1000000=877630	0,02
Reissner-Mindlin shell element	1-st	246741	0,241230·1000000=241230	2,23
	2-nd	877429	0,805670·1000000=61753	8,18

Notes: In the analytical solution the critical values of the concentrated longitudinal bending forces P_{cr1} and P_{cr2}, corresponding to the moments of buckling of the simply supported beam can be determined according to the following formulas:

$$P_1 = \frac{\pi^2 \cdot E \cdot I_y}{L^2} \qquad P_2 = \frac{2 \cdot G \cdot I_x}{h^2} \cdot \left(-1 + \sqrt{1 + \frac{4 \cdot \pi^2 \cdot h^2}{L^2} \cdot \frac{E \cdot I_z}{G \cdot I_x}}\right) \qquad G = \frac{E}{2 \cdot (1 + \nu)}$$

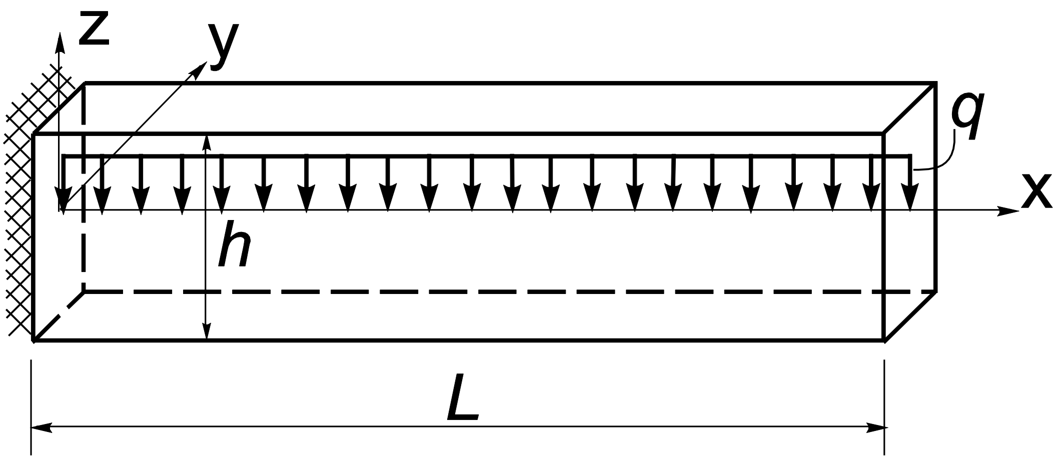
$I_z = \dfrac{h \cdot b^3}{12}$ – minimum bending inertia moment (out of the moment plane);

$I_y = \dfrac{b \cdot h^3}{12}$ – maximum bending inertia moment (in the moment plane);

$I_x = k_f \cdot h \cdot b^3$ – free torsional inertia moment, where:

$$k_f = \frac{1}{3} \cdot \left\{1 - \frac{192}{\pi^5} \cdot \frac{b}{h} \cdot \sum_{n=1}^{\infty} \left[sin^2\left(\frac{n \cdot \pi}{2}\right) \cdot \frac{1}{n^5} \cdot th\left(\frac{n \cdot \pi \cdot h}{2 \cdot b}\right)\right]\right\}$$

Stability of a Cantilever Beam of a Square Cross-Section Subjected to a Load Uniformly Distributed along Its Longitudinal Axis



Objective: Determination of the critical value of the load uniformly distributed along the longitudinal axis of a cantilever beam of a square cross-section corresponding to the moment of its buckling.

Initial data files:

File name	Description
Stability_Bar_8_Bar.SPR	Bar model
Stability_Bar_8_Shell.SPR	Shell element model
Stability_Bar_8_Solid.SPR	Solid element model

Problem formulation: The cantilever beam of a square cross-section is subjected to the action of the load q, uniformly distributed along its longitudinal axis. Determine the critical value of the uniformly distributed load q_{cr}, corresponding to the moment of buckling of the cantilever beam.

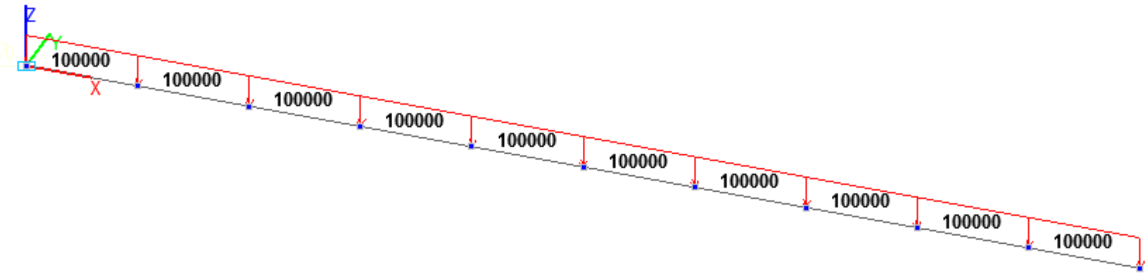
References: A.S. Volmir. Stability of Deformable Systems, Moscow, Nauka, 1967, p.217;

Initial data:

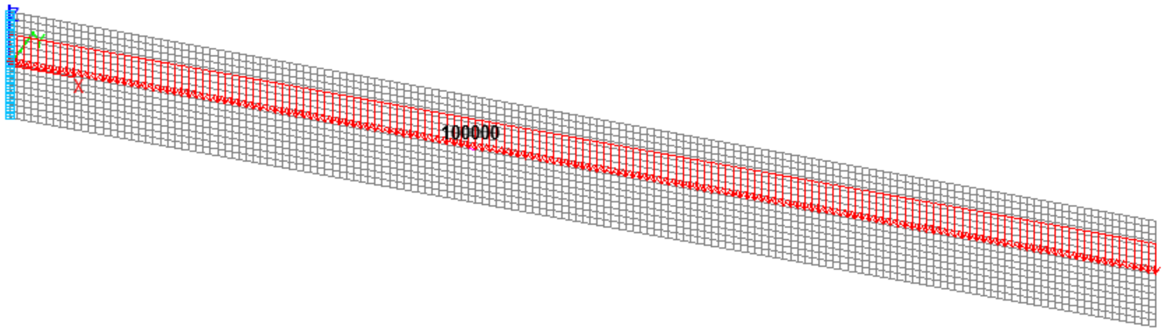
L = 10.0 m	- length of the cantilever beam;
h = b = 1.0 m	- side of the square cross-section of the cantilever beam;
$E = 3.0 \cdot 10^7$ kN/m^2	- elastic modulus of the cantilever beam material;
ν = 0.2	- Poisson's ratio;
$q = 10^5$ kN/m	- initial value of the load uniformly distributed along the longitudinal axis of the beam.

Finite element model: Design model – general type system. Three design models are considered:
Bar model (B), 10 elements of type 5, the spacing of the finite element mesh along the longitudinal axis is 1.0 m. Boundary conditions are provided by imposing constraints on the node of the clamped end of the beam in the directions of the degrees of freedom X, Y, Z, UX, UY, UZ. The action with the initial value of the uniformly distributed load q is specified on all elements of the beam. Number of nodes in the design model – 11;
Reissner-Mindlin shell element model (P), 2560 eight-node elements of type 150, the spacing of the finite element mesh along the longitudinal axis and along the height of the beam is 0.0625 m. Boundary conditions are provided by imposing constraints on the nodes of the clamped end of the beam in the directions of the degrees of freedom X, Y, Z, UX, UY, UZ. The action with the initial value of the load q uniformly distributed along the line is specified on the upper sides of all beam elements located under the longitudinal axis of the beam. Number of nodes in the design model – 8033.
Solid element model (S), 5120 twenty-node elements of type 37, the spacing of the finite element mesh along the longitudinal axis, width and height of the beam is 0.125 m. Boundary conditions are provided by imposing constraints on the nodes of the clamped end of the beam in the directions of the degrees of freedom X, Y, Z, UX, UY, UZ. The action with the initial value of the load uniformly distributed over the face q_A = q/ b is specified on the upper faces of all beam elements located under the longitudinal axis of the beam. Number of nodes in the design model – 24705.

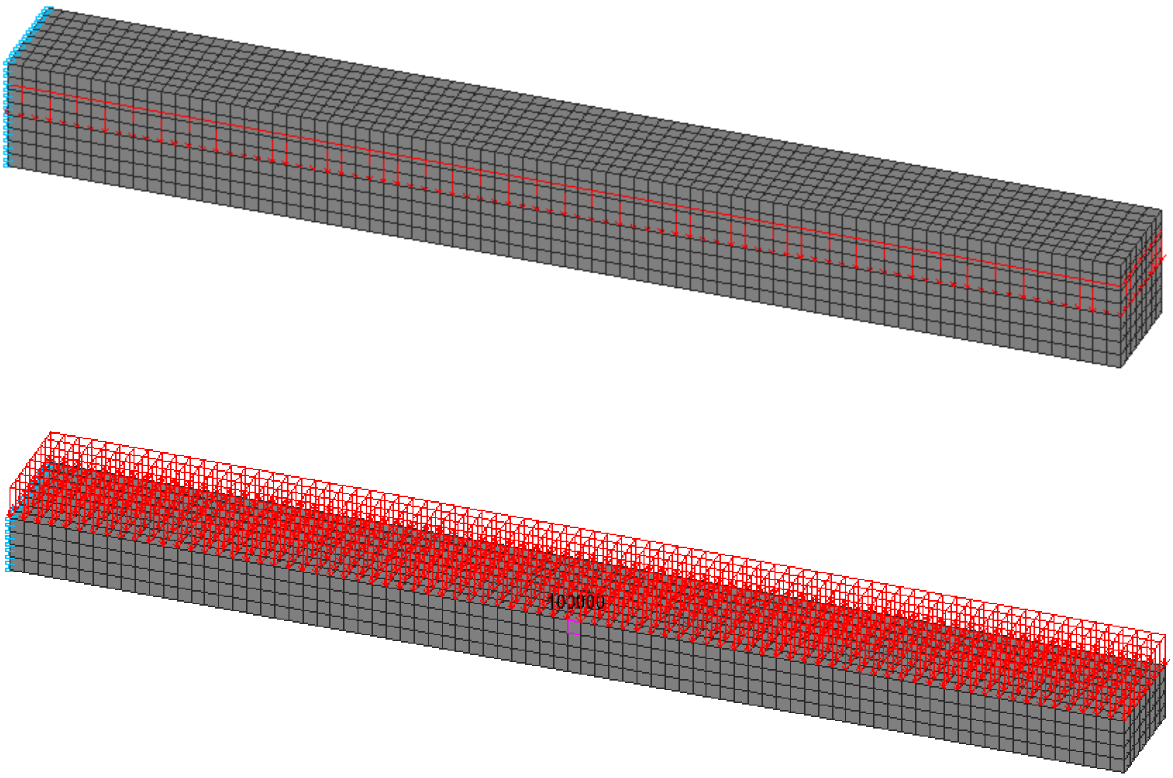
Results in SCAD



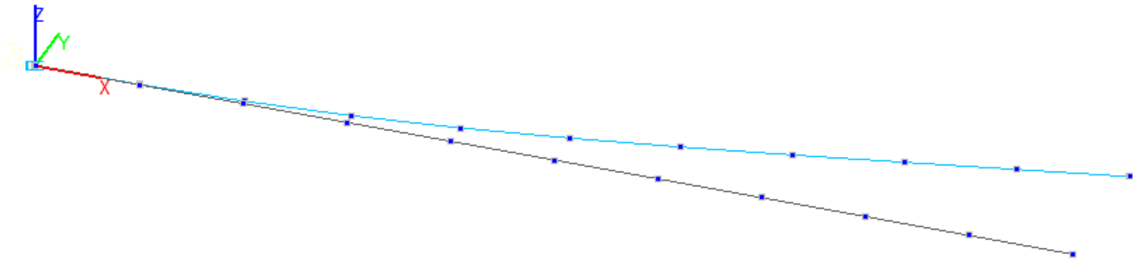
Design model. Bar model



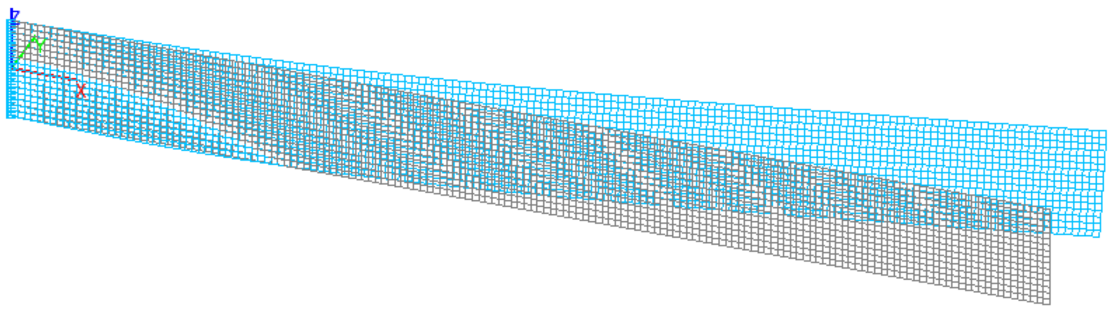
Design model. Reissner-Mindlin shell element model



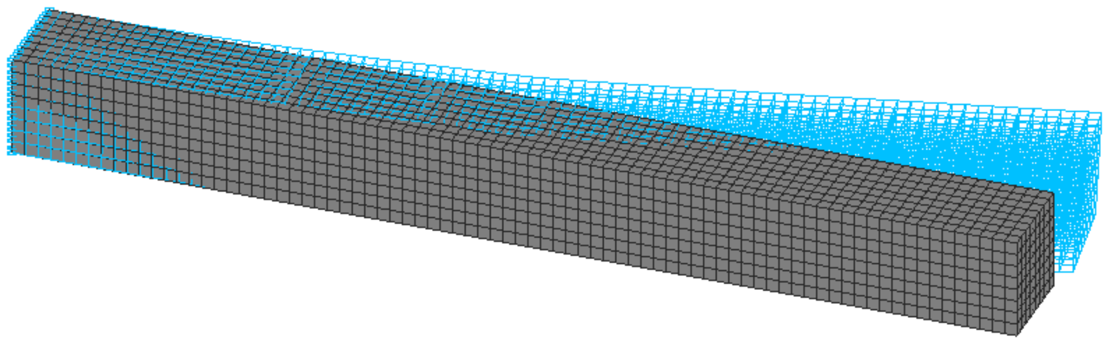
Design model. Solid element model



1-st buckling mode. Bar model



1-st buckling mode. Reissner-Mindlin shell element model



1-st buckling mode. Solid element model

Comparison of solutions:

Critical value of the load q_{cr},
uniformly distributed along the longitudinal axis of the cantilever beam

Design model	Theory	SCAD	Deviation, %
Bar	26933	0,268111·100000=26811	0,45
Reissner-Mindlin shell element	26933	0,260448·100000=26045	3,30
Solid element	26933	0,253906·100000=25391	5,73

Notes: In the analytical solution the critical value of the uniformly distributed load q_{cr}, corresponding to the moment of buckling of the cantilever beam can be determined according to the following formula:

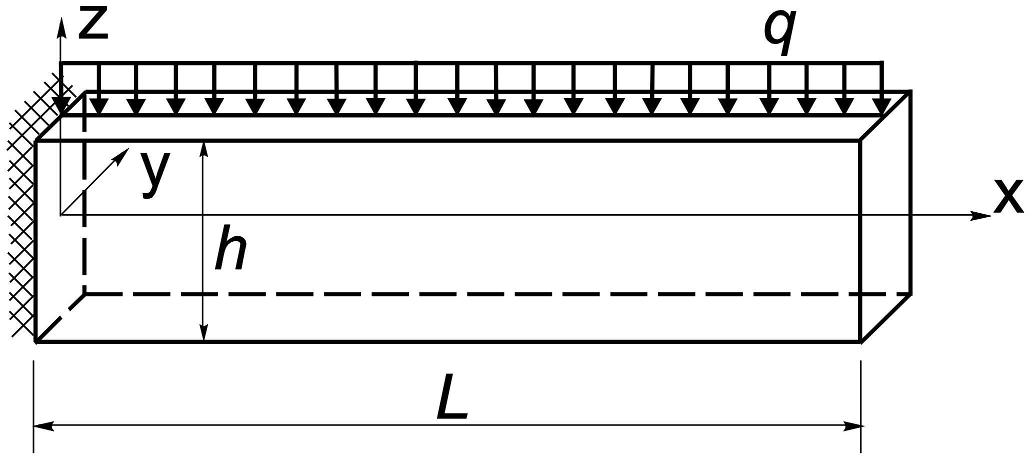
$$q = \frac{12{,}85 \cdot \sqrt{E \cdot I_z \cdot G \cdot I_x}}{L^3} \qquad G = \frac{E}{2 \cdot (1 + \nu)}$$

$I_z = \frac{h \cdot b^3}{12}$ – minimum bending inertia moment (out of the moment plane);

$I_x = k_f \cdot h \cdot b^3$ – free torsional inertia moment, where:

$$k_f = \frac{1}{3} \cdot \left\{ 1 - \frac{192}{\pi^5} \cdot \frac{b}{h} \cdot \sum_{n=1}^{\infty} \left[sin^2\left(\frac{n \cdot \pi}{2} \right) \cdot \frac{1}{n^5} \cdot th\left(\frac{n \cdot \pi \cdot h}{2 \cdot b} \right) \right] \right\}$$

Stability of a Cantilever Beam of a Square Cross-Section Subjected to a Load Uniformly Distributed along the Longitudinal Axis of Its Upper Face



Objective: Determination of the critical value of the load uniformly distributed along the longitudinal axis of the upper face of a cantilever beam of a square cross-section corresponding to the moment of its buckling.

Initial data files:

File name	Description
Stability_Bar_9_Bar.SPR	Bar model
Stability_Bar_9_Shell.SPR	Shell element model
Stability_Bar_9_Solid.SPR	Solid element model

Problem formulation: The cantilever beam of a square cross-section is subjected to the action of the load q, uniformly distributed along the longitudinal axis of its upper face. Determine the critical value of the uniformly distributed load q_{cr}, corresponding to the moment of buckling of the cantilever beam.

References: S. P. Timoshenko, Stability of Bars, Plates and Shells, Moscow, Nauka, 1971, p.303

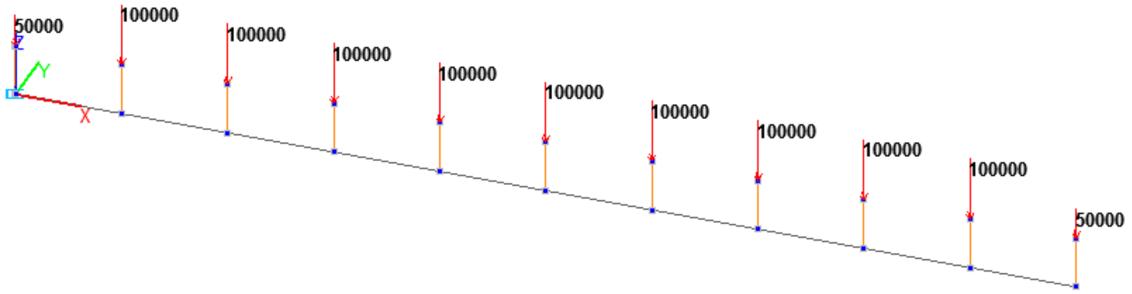
Initial data:

L = 10.0 m	- length of the cantilever beam;
h = b = 1.0 m	- side of the square cross-section of the cantilever beam;
$E = 3.0 \cdot 10^7$ kN/m^2	- elastic modulus of the cantilever beam material;
$\nu = 0.2$	- Poisson's ratio;
$q = 10^5$ kN/m	- initial value of the load uniformly distributed along the longitudinal axis of the upper face of the beam.

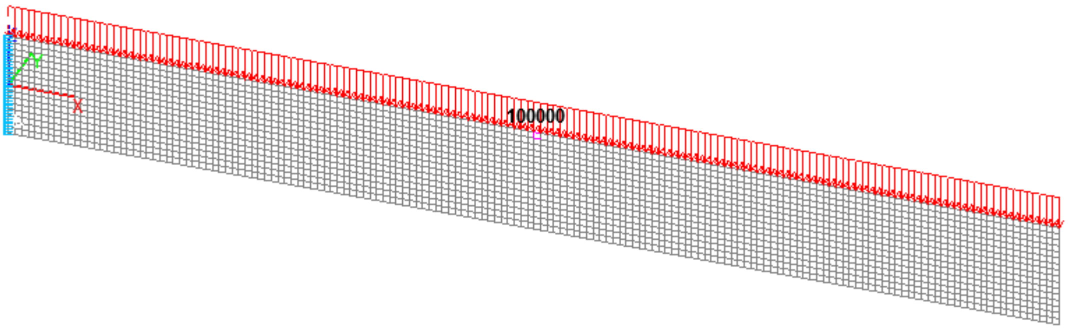
Finite element model: Design model – general type system. Three design models are considered:
Bar model (B), 10 elements of type 5, the spacing of the finite element mesh along the longitudinal axis is 1.0 m. Boundary conditions are provided by imposing constraints on the node of the clamped end of the beam in the directions of the degrees of freedom X, Y, Z, UX, UY, UZ. 11 vertical upward two-node elements of type 100 (3D rigid body) with the length h/2 are adjacent to the nodes of the beam. The action with the initial value of the uniformly distributed load q is specified in the free nodes of the elements of the rigid body (elevated application point) as concentrated forces $P = q \cdot b \cdot 1.0 = 10^5$ kN ($0.5 \cdot 10^5$ kN for the end nodes). Number of nodes in the design model – 22;
Reissner-Mindlin shell element model (P), 2560 eight-node elements of type 150, the spacing of the finite element mesh along the longitudinal axis and along the height of the beam is 0.0625 m. Boundary conditions are provided by imposing constraints on the nodes of the clamped end of the beam in the directions of the degrees of freedom X, Y, Z, UX, UY, UZ. The action with the initial value of the load q uniformly distributed along the line is specified on the upper sides of all beam elements located under the upper face of the beam. Number of nodes in the design model – 8033.
Solid element model (S), 5120 twenty-node elements of type 37, the spacing of the finite element mesh along the longitudinal axis, width and height of the beam is 0.125 m. Boundary conditions are provided by imposing constraints on the nodes of the clamped end of the beam in the directions of the degrees of

freedom X, Y, Z, UX, UY, UZ. The action with the initial value of the load uniformly distributed over the face $q_A = q/b$ is specified on the upper faces of all beam elements located under the upper face of the beam. Number of nodes in the design model – 24705.

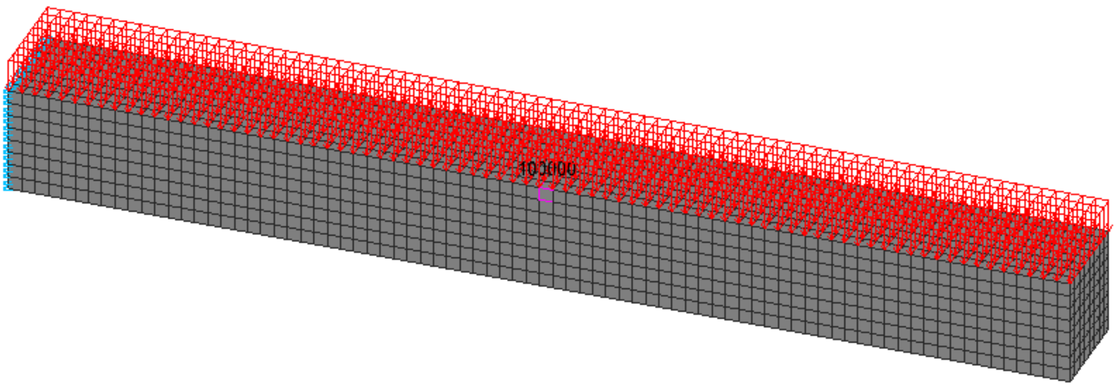
Results in SCAD



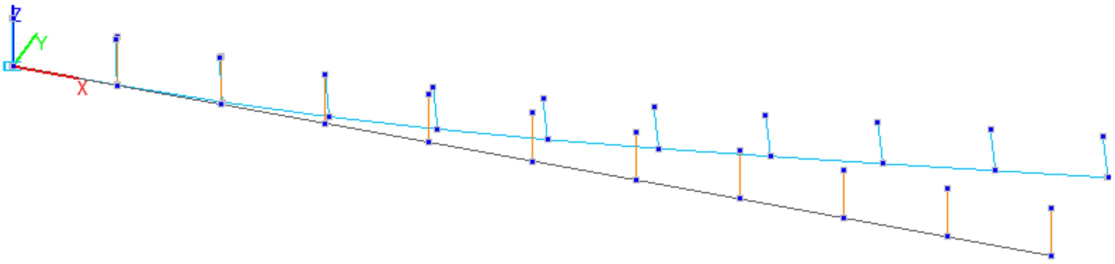
Design model. Bar model



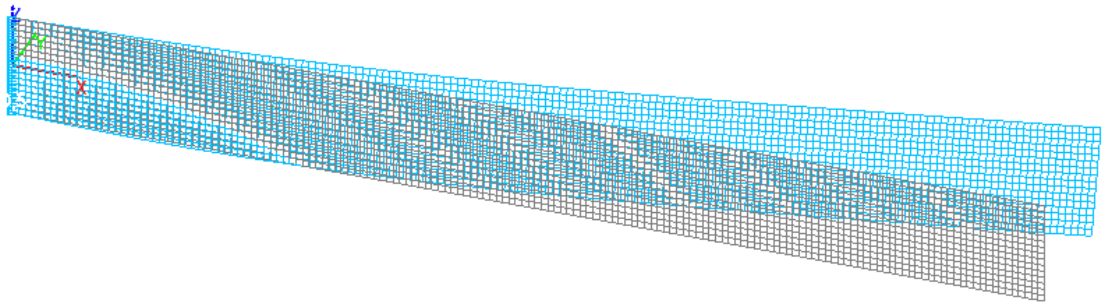
Design model. Reissner-Mindlin shell element model



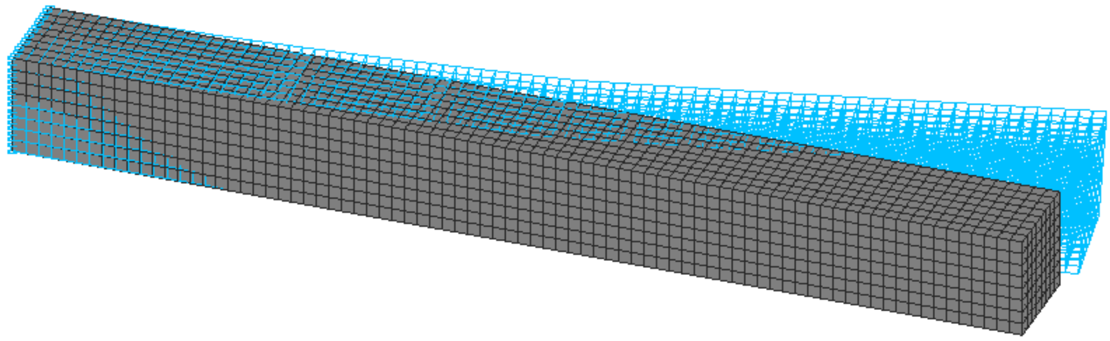
Design model. Solid element model



1-st buckling mode. Bar model



1-st buckling mode. Reissner-Mindlin shell element model



1-st buckling mode. Solid element model

Comparison of solutions:

Critical value of the load q_{cr},
uniformly distributed along the longitudinal axis of the upper face of the cantilever beam

Design model	Theory	SCAD	Deviation, %
Bar	23737	0,236895·100000=23690	0,20
Reissner-Mindlin shell element	23737	0,233316·100000=23332	1,71
Solid element	23737	0,246094·100000=24609	3,67

Notes: In the analytical solution the critical value of the uniformly distributed load q_{cr}, corresponding to the moment of buckling of the cantilever beam can be determined according to the following formula:

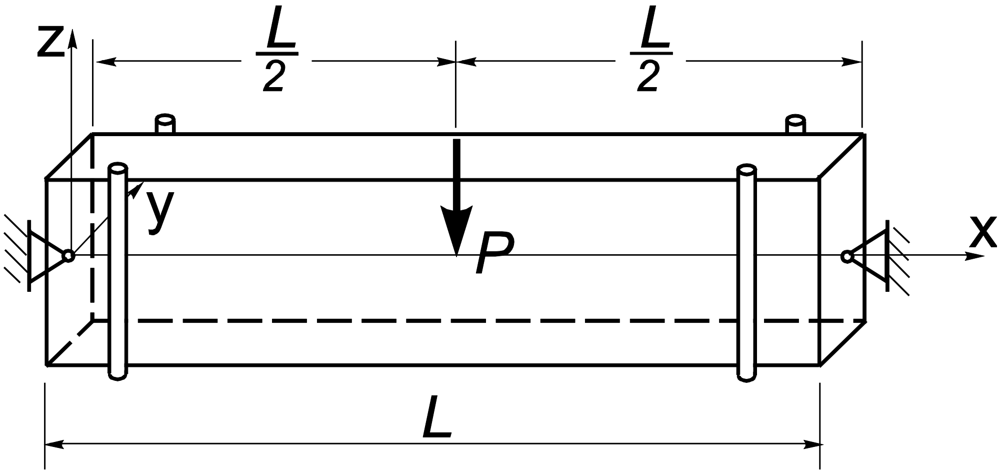
$$q = \frac{12{,}85 \cdot \sqrt{E \cdot I_z \cdot G \cdot I_x}}{L^3} \cdot k_h \qquad k_h = f\left(\frac{h}{2 \cdot L} \cdot \sqrt{\frac{E \cdot I_z}{G \cdot I_x}} \right) \qquad G = \frac{E}{2 \cdot (1+\nu)}$$

$I_z = \frac{h \cdot b^3}{12}$ – minimum bending inertia moment (out of the moment plane);

$I_x = k_f \cdot h \cdot b^3$ – free torsional inertia moment, where:

$$k_f = \frac{1}{3} \cdot \left\{ 1 - \frac{192}{\pi^5} \cdot \frac{b}{h} \cdot \sum_{n=1}^{\infty} \left[sin^2\left(\frac{n \cdot \pi}{2}\right) \cdot \frac{1}{n^5} \cdot th\left(\frac{n \cdot \pi \cdot h}{2 \cdot b}\right) \right] \right\}$$

Stability of a Beam of a Square Cross-Section Simply Supported in and out of the Bending Plane Subjected to a Concentrated Transverse Bending Force Applied in the Middle of the Span at the Level of the Longitudinal Axis (Transverse Bending)



Objective: Determination of the critical value of the concentrated transverse bending force applied in the middle of the span at the level of the longitudinal axis of a beam of a square cross-section simply supported in and out of the bending plane corresponding to the moment of its buckling.

Initial data files:

File name	Description
Stability_Bar_10_Bar.SPR	Bar model
Stability_Bar_10_Shell.SPR	Shell element model

Problem formulation: The beam of a square cross-section simply supported in and out of the bending plane is subjected to the action of the concentrated transverse bending force P, applied in the middle of its span at the level of the longitudinal axis. Determine the critical value of the concentrated transverse bending force P, corresponding to the moment of buckling of the simply supported beam.

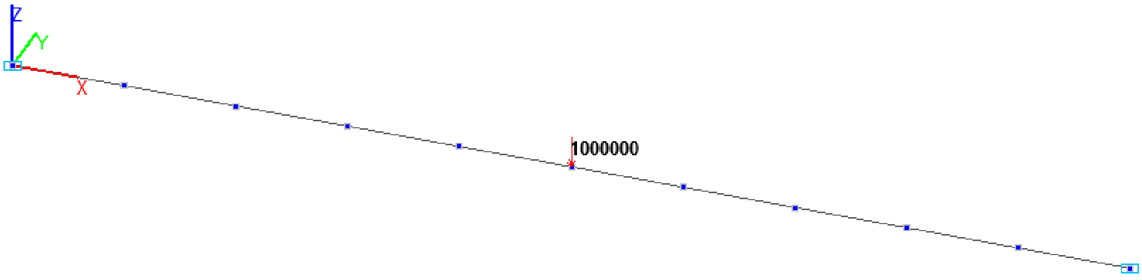
References: A.S. Volmir. Stability of Deformable Systems, Moscow, Nauka, 1967, p.218

Initial data:

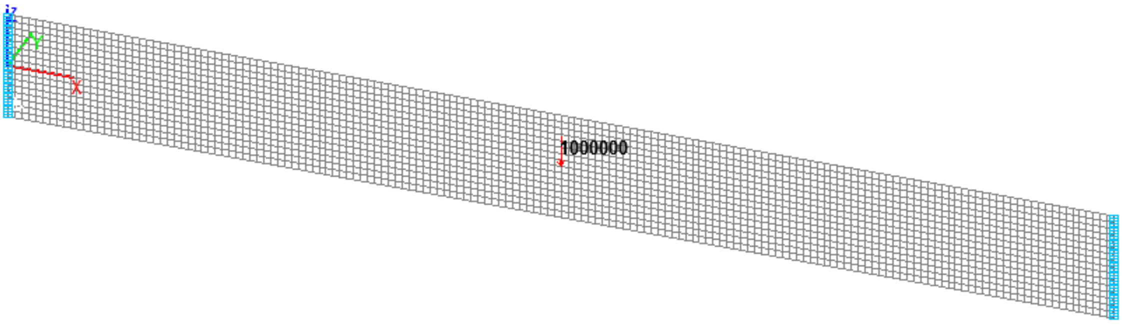
L = 10.0 m	- length of the simply supported beam;
h = b = 1.0 m	- side of the square cross-section of the simply supported beam;
$E = 3.0\cdot10^7$ kN/m^2	- elastic modulus of the simply supported beam material;
ν = 0.2	- Poisson’s ratio;
$P = 10^6$ kN	- initial value of the concentrated transverse bending force applied in the middle of the span at the level of the longitudinal axis of the beam.

Finite element model: Design model – general type system. Two design models are considered:
Bar model (B), 10 elements of type 5, the spacing of the finite element mesh along the longitudinal axis is 1.0 m. Boundary conditions are provided by imposing constraints on the nodes of the simply supported ends of the beam in the directions of the degrees of freedom X, Y, Z, UX. The action with the initial value of the concentrated transverse bending force P is specified in the node in the middle of the beam span. Number of nodes in the design model – 11;
Reissner-Mindlin shell element model (P), 2560 eight-node elements of type 150, the spacing of the finite element mesh along the longitudinal axis and along the height of the beam is 0.0625 m. The shell is supported by vertical high-rigidity bars (h = b = 1.0 m; $E = 3.0\cdot10^9$ kN/m^2; ν = 0.2), 64 elements of type 5. Boundary conditions are provided by imposing constraints on the nodes of the ends of the beam lying on its longitudinal axis in the directions of the degrees of freedom X, Y, Z and on all other nodes of the ends of the beam in the direction of the degree of freedom Y. The action with the initial value of the concentrated transverse bending force P is specified in the node in the middle of the beam span at the level of the longitudinal axis of the beam. Number of nodes in the design model – 8033.

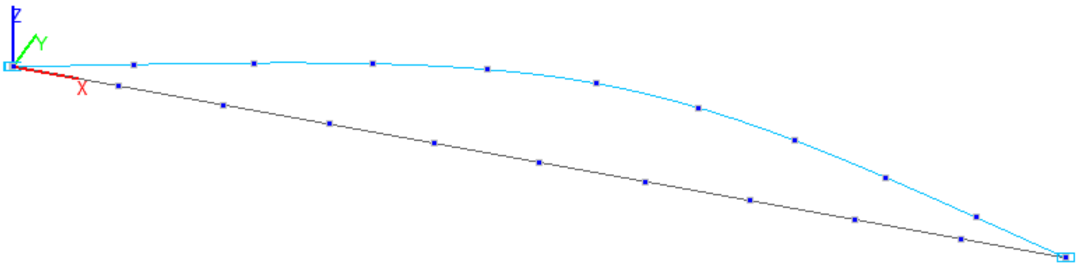
Results in SCAD



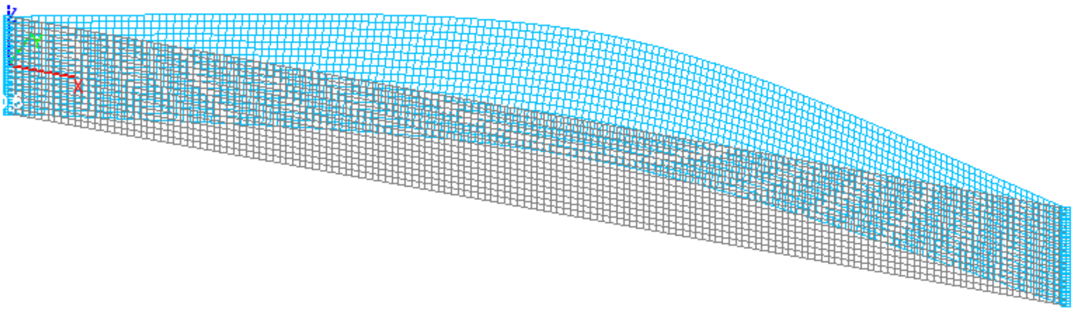
Design model. Bar model



Design model. Reissner-Mindlin shell element model



1-st buckling mode. Bar model



1- st Buckling mode. Reissner-Mindlin shell element model

Comparison of solutions:

Critical value of the concentrated transverse bending force P_{cr} (kN), applied in the middle of the span at the level of the longitudinal axis of the beam simply supported in and out of the bending plane

Design model	Theory	SCAD	Deviation, %
Bar	355055	0,353193·1000000=353193	0,52
Reissner-Mindlin shell element	355055	0,344706·1000000=344706	2,91

Notes: In the analytical solution the critical value of the concentrated transverse bending force P_{cr}, corresponding to the moment of buckling of the simply supported beam can be determined according to the following formula:

$$P = \frac{16{,}94 \cdot \sqrt{E \cdot I_z \cdot G \cdot I_x}}{L^2} \qquad G = \frac{E}{2 \cdot (1+\nu)}$$

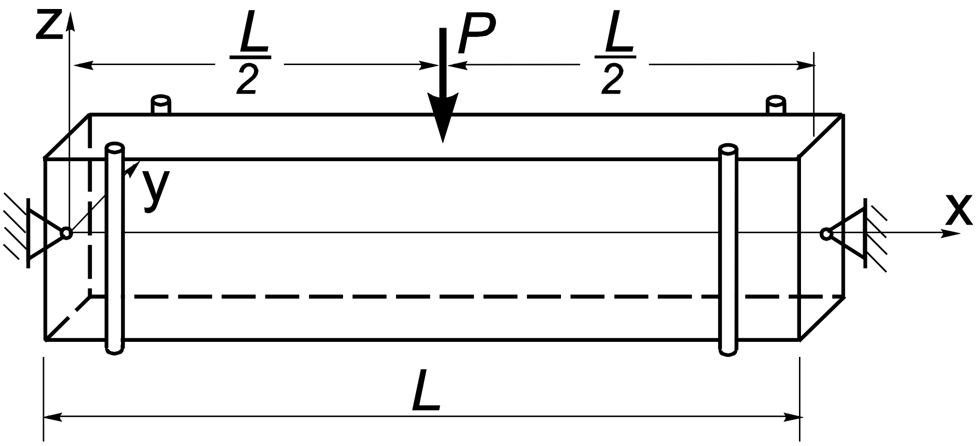
$I_z = \frac{h \cdot b^3}{12}$ – minimum bending inertia moment (out of the moment plane);

$I_y = \frac{b \cdot h^3}{12}$ – maximum bending inertia moment (in the moment plane);

$I_x = k_f \cdot h \cdot b^3$ – free torsional inertia moment, where:

$$k_f = \frac{1}{3} \cdot \left\{ 1 - \frac{192}{\pi^5} \cdot \frac{b}{h} \cdot \sum_{n=1}^{\infty} \left[sin^2\left(\frac{n \cdot \pi}{2}\right) \cdot \frac{1}{n^5} \cdot th\left(\frac{n \cdot \pi \cdot h}{2 \cdot b}\right) \right] \right\}$$

Stability of a Beam of a Square Cross-Section Simply Supported in and out of the Bending Plane Subjected to a Concentrated Transverse Bending Force Applied in the Middle of the Span at the Level of the Longitudinal Axis of the Upper Face (Transverse Bending)



Objective: Determination of the critical value of the concentrated transverse bending force applied in the middle of the span at the level of the longitudinal axis of the upper face of a beam of a square cross-section simply supported in and out of the bending plane corresponding to the moment of its buckling.

Initial data files:

File name	Description
Stability_Bar_11_Bar.SPR	Bar model
Stability_Bar_11_Shell.SPR	Shell element model

Problem formulation: The beam of a square cross-section simply supported in and out of the bending plane is subjected to the action of the concentrated transverse bending force P, applied in the middle of its span at the level of the longitudinal axis of the upper face. Determine the critical value of the concentrated transverse bending force P, corresponding to the moment of buckling of the simply supported beam.

References: A.S. Volmir. Stability of Deformable Systems, Moscow, Nauka, 1967, p.219

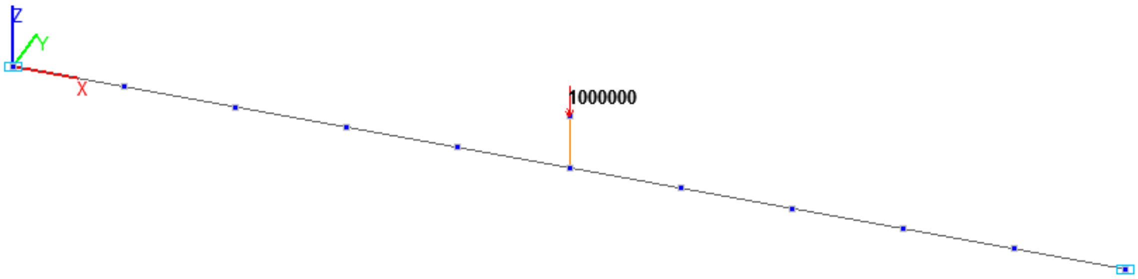
Initial data:

$L = 10.0$ m	- length of the simply supported beam;
$h = b = 1.0$ m	- side of the square cross-section of the simply supported beam;
$E = 3.0 \cdot 10^7$ kN/m^2	- elastic modulus of the simply supported beam material;
$\nu = 0.2$	- Poisson's ratio;
$P = 10^6$ kN	- initial value of the concentrated transverse bending force applied in the middle of the span at the level of the longitudinal axis of the upper face of the beam.

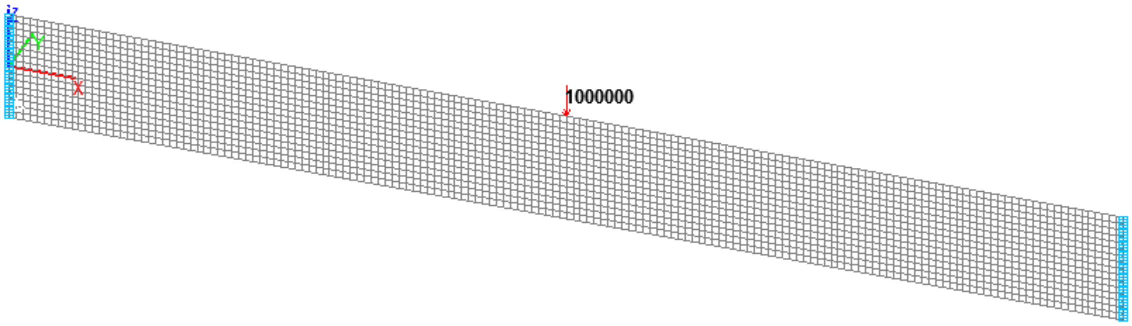
Finite element model: Design model – general type system. Two design models are considered:
Bar model (B), 10 elements of type 5, the spacing of the finite element mesh along the longitudinal axis is 1.0 m. Boundary conditions are provided by imposing constraints on the nodes of the simply supported ends of the beam in the directions of the degrees of freedom X, Y, Z, UX. 1 vertical upward two-node element of type 100 (3D rigid body) with the length h/2 is adjacent to the node in the middle of the beam span. The action with the initial value of the concentrated transverse bending force P is specified in the free node of the element of the rigid body (elevated application point). Number of nodes in the design model – 12;
Reissner-Mindlin shell element model (P), 2560 eight-node elements of type 150, the spacing of the finite element mesh along the longitudinal axis and along the height of the beam is 0.0625 m. The shell is supported by vertical high-rigidity bars ($h = b = 1.0$ m; $E = 3.0 \cdot 10^9$ kN/m^2; $\nu = 0.2$), 64 elements of type 5. Boundary conditions are provided by imposing constraints on the nodes of the ends of the beam lying on its longitudinal axis in the directions of the degrees of freedom X, Y, Z and on all other nodes of the ends of the beam in the direction of the degree of freedom Y. The action with the initial value of the concentrated

transverse bending force P is specified in the node in the middle of the beam span at the height h/2 from the longitudinal axis of the beam. Number of nodes in the design model – 8033.

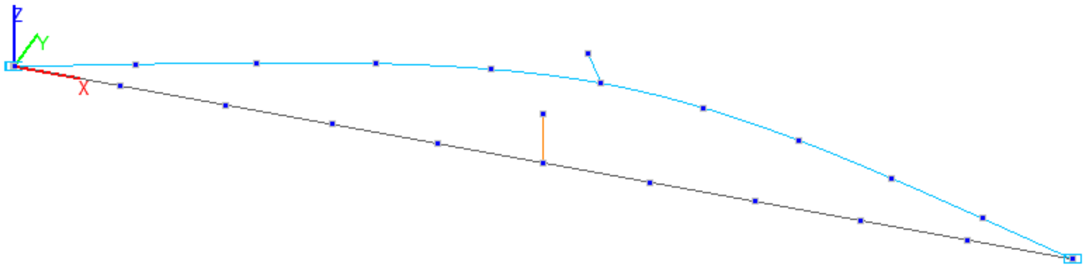
Results in SCAD



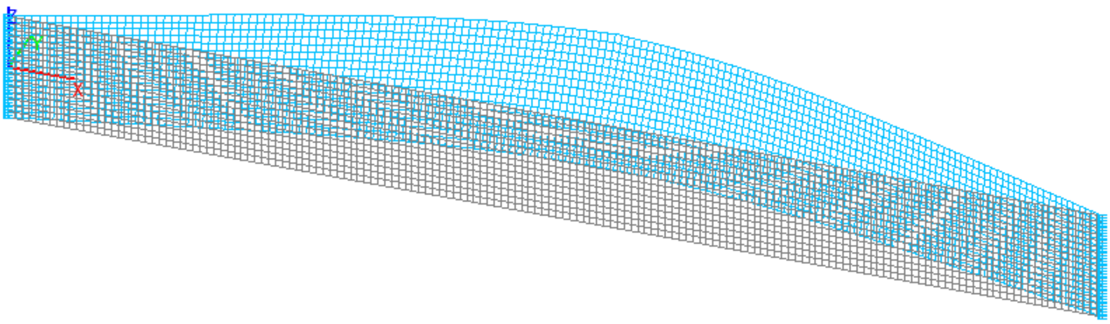
Design model. Bar model



Design model. Reissner-Mindlin shell element model



1-st buckling mode. Bar model



1-st buckling mode. Reissner-Mindlin shell element model

Comparison of solutions:

Critical value of the concentrated transverse bending force P_{cr} (kN), applied in the middle of the span at the level of the longitudinal axis of the upper face of the beam simply supported in and out of the bending plane

Design model	Theory	SCAD	Deviation, %
Bar	317747	0,313904·1000000=313904	1,21
Reissner-Mindlin shell element	317747	0,304932·1000000=304932	4,03

Notes: In the analytical solution the critical value of the concentrated transverse bending force P_{cr}, corresponding to the moment of buckling of the simply supported beam can be determined according to the following formula:

$$P = \frac{16{,}94 \cdot \sqrt{E \cdot I_z \cdot G \cdot I_x}}{L^2} \cdot k_h \qquad k_h = f\left(\frac{h}{2 \cdot L} \cdot \sqrt{\frac{E \cdot I_z}{G \cdot I_x}}\right) \qquad G = \frac{E}{2 \cdot (1+\nu)}$$

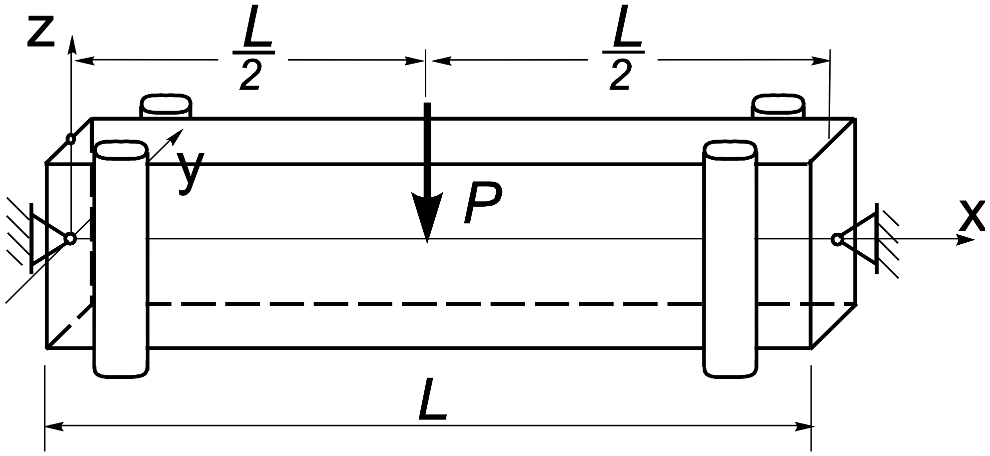
$I_z = \frac{h \cdot b^3}{12}$ – minimum bending inertia moment (out of the moment plane);

$I_y = \frac{b \cdot h^3}{12}$ – maximum bending inertia moment (in the moment plane);

$I_x = k_f \cdot h \cdot b^3$ – free torsional inertia moment, where:

$$k_f = \frac{1}{3} \cdot \left\{1 - \frac{192}{\pi^5} \cdot \frac{b}{h} \cdot \sum_{n=1}^{\infty}\left[sin^2\left(\frac{n \cdot \pi}{2}\right) \cdot \frac{1}{n^5} \cdot th\left(\frac{n \cdot \pi \cdot h}{2 \cdot b}\right)\right]\right\}$$

Stability of a Beam of a Square Cross-Section Simply Supported in the Bending Plane and Clamped out of the Bending Plane Subjected to a Concentrated Transverse Bending Force Applied in the Middle of the Span at the Level of the Longitudinal Axis (Transverse Bending)



Objective: Determination of the critical value of the concentrated transverse bending force applied in the middle of the span at the level of the longitudinal axis of a beam of a square cross-section simply supported in the bending plane and clamped out of the bending plane corresponding to the moment of its buckling.

Initial data files:

File name	Description
Stability_Bar_12_Bar.SPR	Bar model
Stability_Bar_12_Shell.SPR	Shell element model

Problem formulation: The beam of a square cross-section simply supported in the bending plane and clamped out of the bending plane is subjected to the action of the concentrated transverse bending force P, applied in the middle of its span at the level of the longitudinal axis. Determine the critical value of the concentrated transverse bending force P, corresponding to the moment of buckling of the simply supported beam applied in the middle of its span at the level of the longitudinal axis.

References: A.S. Volmir. Stability of Deformable Systems, Moscow, Nauka, 1967, p.220

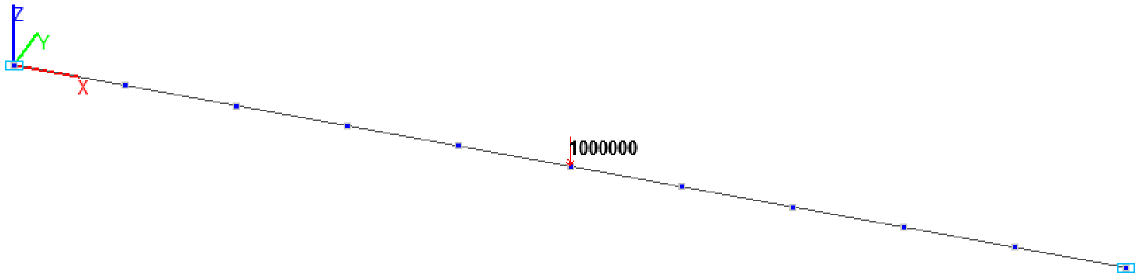
Initial data:

L = 10.0 m	- length of the simply supported beam;
h = b = 1.0 m	- side of the square cross-section of the simply supported beam;
$E = 3.0 \cdot 10^7$ kN/m^2	- elastic modulus of the simply supported beam material;
ν = 0.2	- Poisson's ratio;
$P = 10^6$ kN	- initial value of the concentrated transverse bending force applied in the middle of the span at the level of the longitudinal axis of the beam.

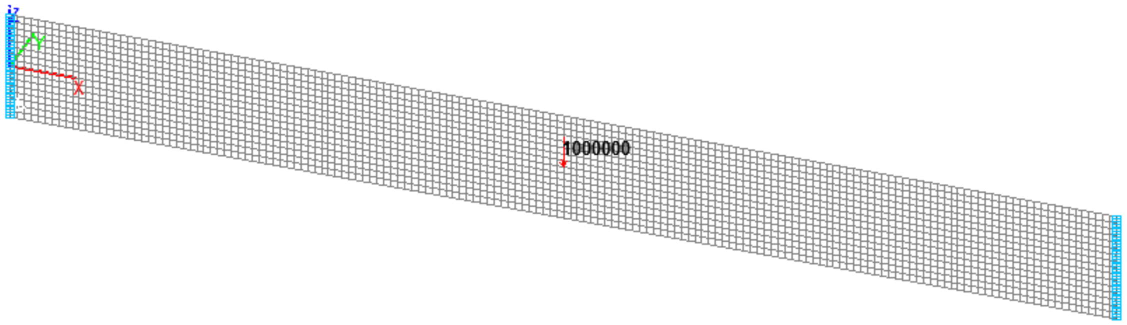
Finite element model: Design model – general type system. Two design models are considered:
Bar model (B), 10 elements of type 5, the spacing of the finite element mesh along the longitudinal axis is 1.0 m. Boundary conditions are provided by imposing constraints on the nodes of the simply supported ends of the beam in the directions of the degrees of freedom X, Y, Z, UX, UZ. The action with the initial value of the concentrated transverse bending force P is specified in the node in the middle of the beam span. Number of nodes in the design model – 11.
Reissner-Mindlin shell element model (P), 2560 eight-node elements of type 150, the spacing of the finite element mesh along the longitudinal axis and along the height of the beam is 0.0625 m. The shell is supported by vertical high-rigidity bars (h = b = 1.0 m; $E = 3.0 \cdot 10^9$ kN/m^2; ν = 0.2), 64 elements of type 5. Boundary conditions are provided by imposing constraints on the nodes of the ends of the beam lying on its longitudinal axis in the directions of the degrees of freedom X, Y, Z, UZ and on all other nodes of the ends of the beam in the directions of the degrees of freedom Y, UZ. The action with the initial value of the

concentrated transverse bending force P is specified in the node in the middle of the beam span at the level of the longitudinal axis of the beam. Number of nodes in the design model – 8033.

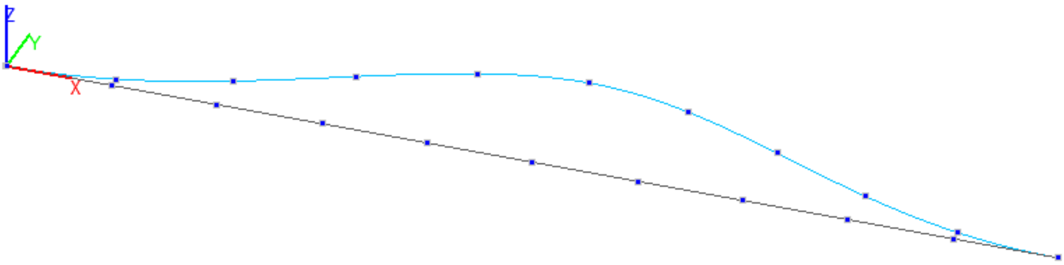
Results in SCAD



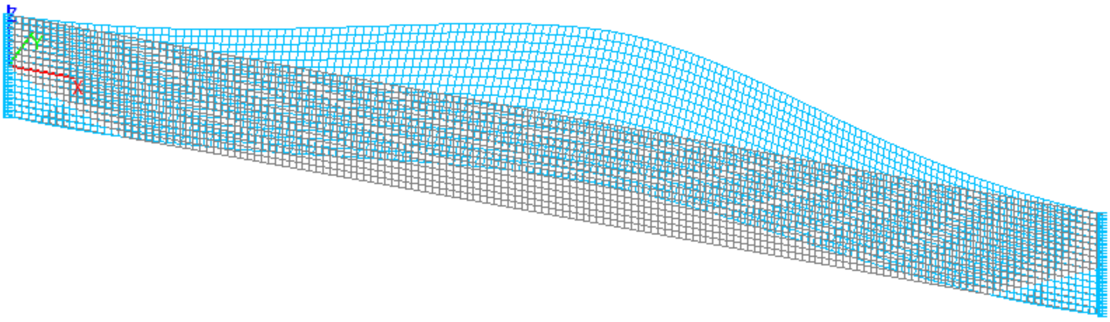
Design model. Bar model



Design model. Reissner-Mindlin shell element model



1-st buckling mode. Bar model



1-st buckling mode. Reissner-Mindlin shell element model

Comparison of solutions:

Critical value of the concentrated transverse bending force P_{cr} (kN), applied in the middle of the span at the level of the longitudinal axis of the beam simply supported in the bending plane and clamped out of the bending plane

Design model	Theory	SCAD	Deviation, %
Bar	559620	0,541779·1000000=541779	3,19
Reissner-Mindlin shell element	559620	0,506897·1000000=506897	9,42

Notes: In the analytical solution the critical value of the concentrated transverse bending force P_{cr}, corresponding to the moment of buckling of the simply supported beam can be determined according to the following formula:

$$P = \frac{26{,}70 \cdot \sqrt{E \cdot I_z \cdot G \cdot I_x}}{L^2} \qquad G = \frac{E}{2 \cdot (1+\nu)}$$

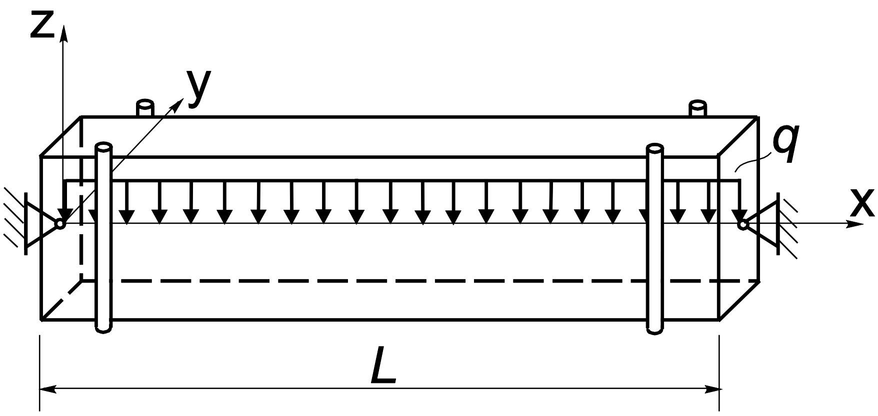
$I_z = \dfrac{h \cdot b^3}{12}$ – minimum bending inertia moment (out of the moment plane);

$I_y = \dfrac{b \cdot h^3}{12}$ – maximum bending inertia moment (in the moment plane);

$I_x = k_f \cdot h \cdot b^3$ – free torsional inertia moment, where:

$$k_f = \frac{1}{3} \cdot \left\{ 1 - \frac{192}{\pi^5} \cdot \frac{b}{h} \cdot \sum_{n=1}^{\infty} \left[sin^2\left(\frac{n \cdot \pi}{2}\right) \cdot \frac{1}{n^5} \cdot th\left(\frac{n \cdot \pi \cdot h}{2 \cdot b}\right) \right] \right\}$$

Stability of a Beam of a Square Cross-Section Simply Supported in and out of the Bending Plane Subjected to a Transverse Load Uniformly Distributed along Its Longitudinal Axis



Objective: Determination of the critical value of the transverse load uniformly distributed along the longitudinal axis of a beam of a square cross-section simply supported in and out of the bending plane corresponding to the moment of its buckling.

Initial data files:

File name	Description
Stability_Bar_13_Bar.SPR	Bar model
Stability_Bar_13_Shell.SPR	Shell element model

Problem formulation: The beam of a square cross-section simply supported in and out of the bending plane is subjected to the action of the transverse load q, uniformly distributed along its longitudinal axis. Determine the critical value of the transverse uniformly distributed load q_{cr}, corresponding to the moment of buckling of the simply supported beam.

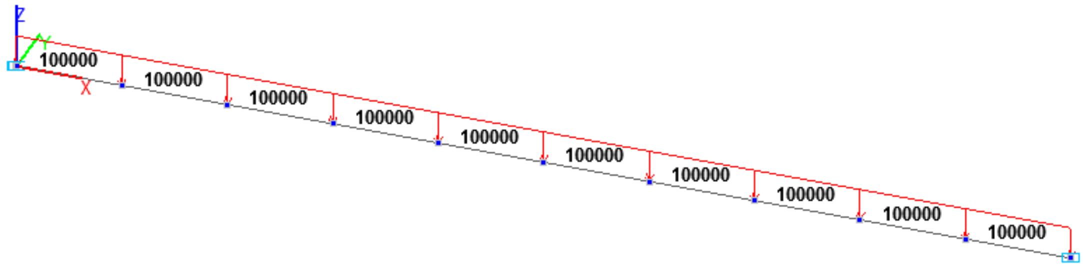
References: A.S. Volmir. Stability of Deformable Systems, Moscow, Nauka, 1967, p.220

Initial data:

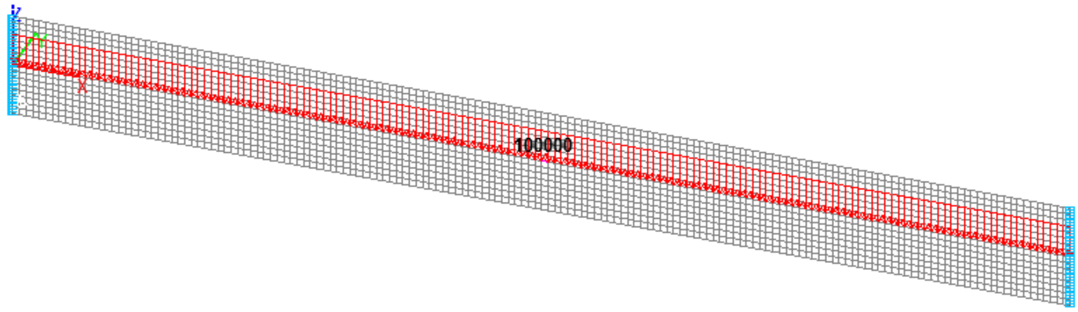
L = 10.0 m	- length of the simply supported beam;
h = b = 1.0 m	- side of the square cross-section of the simply supported beam;
$E = 3.0 \cdot 10^7$ kN/m^2	- elastic modulus of the simply supported beam material;
$\nu = 0.2$	- Poisson’s ratio;
$q = 10^5$ kN/m	- initial value of the transverse load uniformly distributed along the longitudinal axis of the beam.

Finite element model: Design model – general type system. Two design models are considered:
Bar model (B), 10 elements of type 5, the spacing of the finite element mesh along the longitudinal axis is 1.0 m. Boundary conditions are provided by imposing constraints on the nodes of the simply supported ends of the beam in the directions of the degrees of freedom X, Y, Z, UX. The action with the initial value of the transverse uniformly distributed load q is specified on all elements of the beam. Number of nodes in the design model – 11;
Reissner-Mindlin shell element model (P), 2560 eight-node elements of type 150, the spacing of the finite element mesh along the longitudinal axis and along the height of the beam is 0.0625 m. The shell is supported by vertical high-rigidity bars (h = b = 1.0 m; $E = 3.0 \cdot 10^9$ kN/m^2; $\nu = 0.2$), 64 elements of type 5. Boundary conditions are provided by imposing constraints on the nodes of the ends of the beam lying on its longitudinal axis in the directions of the degrees of freedom X, Y, Z and on all other nodes of the ends of the beam in the direction of the degree of freedom Y. The action with the initial value of the transverse load q uniformly distributed along the line is specified on the lower sides of all beam elements located above its longitudinal axis. Number of nodes in the design model – 8033.

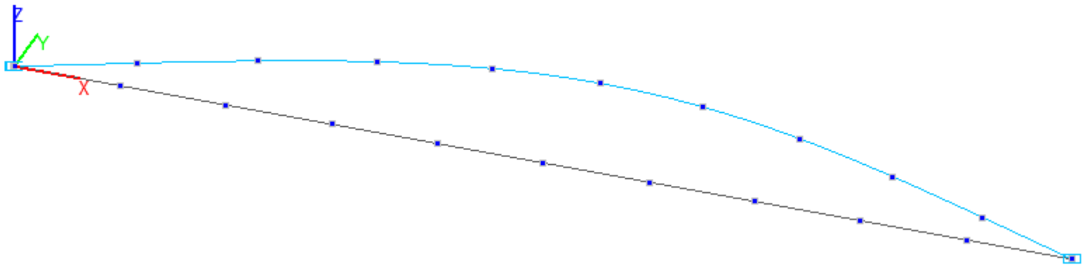
Results in SCAD



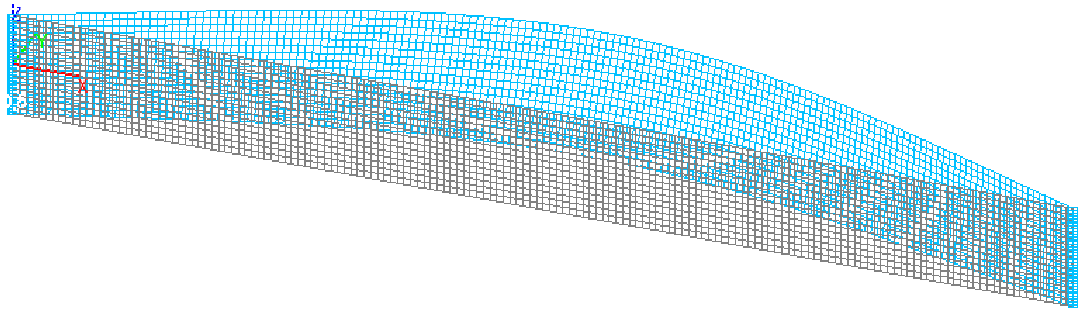
Design model. Bar model



Design model. Reissner-Mindlin shell element model



1-st buckling mode. Bar model



1- st buckling mode. Reissner-Mindlin shell element model

Comparison of solutions:

Critical value of the transverse load q_{cr} (kN/m), uniformly distributed along the longitudinal axis of the beam simply supported in and out of the bending plane

Design model	Theory	SCAD	Deviation, %
Bar	59337	0,590213·100000=59021	0,53
Reissner-Mindlin shell element	59337	0,578880·100000=57888	2,44

Notes: In the analytical solution the critical value of the transverse uniformly distributed load q_{cr}, corresponding to the moment of buckling of the simply supported beam can be determined according to the following formula:

$$q = \frac{28{,}31 \cdot \sqrt{E \cdot I_z \cdot G \cdot I_x}}{L^3} \qquad G = \frac{E}{2 \cdot (1+\nu)}$$

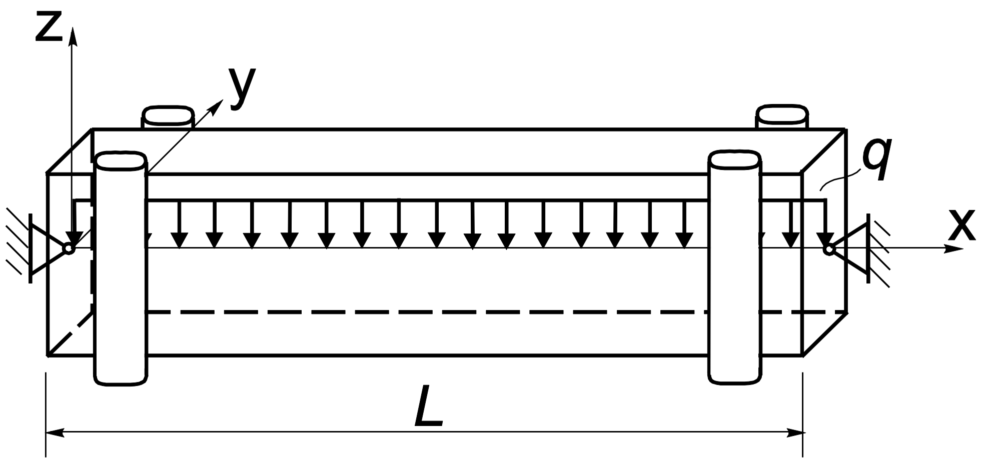
$I_z = \dfrac{h \cdot b^3}{12}$ – minimum bending inertia moment (out of the moment plane);

$I_y = \dfrac{b \cdot h^3}{12}$ – maximum bending inertia moment (in the moment plane);

$I_x = k_f \cdot h \cdot b^3$ – free torsional inertia moment, where:

$$k_f = \frac{1}{3} \cdot \left\{ 1 - \frac{192}{\pi^5} \cdot \frac{b}{h} \cdot \sum_{n=1}^{\infty} \left[sin^2\left(\frac{n \cdot \pi}{2}\right) \cdot \frac{1}{n^5} \cdot th\left(\frac{n \cdot \pi \cdot h}{2 \cdot b}\right) \right] \right\}$$

Stability of a Beam of a Square Cross-Section Simply Supported in the Bending Plane and Clamped out of the Bending Plane Subjected to a Transverse Load Uniformly Distributed along Its Longitudinal Axis



Objective: Determination of the critical value of the transverse load uniformly distributed along the longitudinal axis of a beam of a square cross-section simply supported in the bending plane and clamped out of the bending plane corresponding to the moment of its buckling.

Initial data files:

File name	Description
Stability_Bar_14_Bar.SPR	Bar model
Stability_Bar_14_Shell.SPR	Shell element model

Problem formulation: The beam of a square cross-section simply supported in the bending plane and clamped out of the bending plane is subjected to the action of the transverse load q, uniformly distributed along its longitudinal axis. Determine the critical value of the transverse uniformly distributed load q_{cr}, corresponding to the moment of buckling of the simply supported beam.

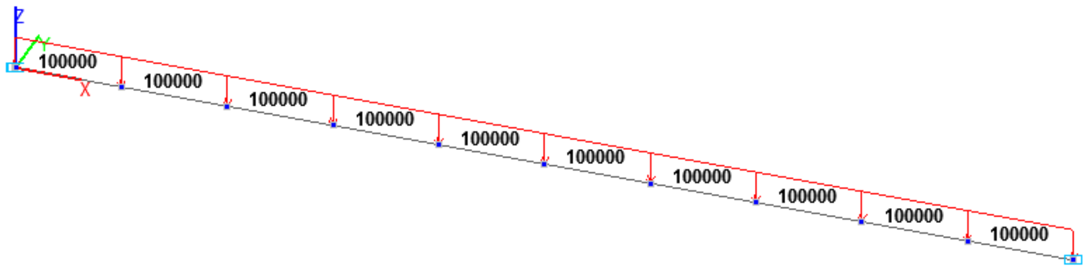
References: I.A. Birger, Ya.G. Panovko, Strength, Stability, Vibrations, Handbook in three volumes, Volume 3, Moscow, Mechanical engineering, 1968, p.72

Initial data:

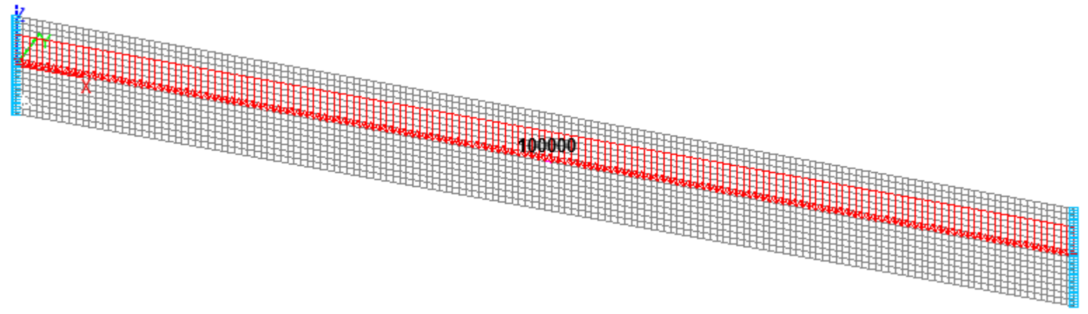
L = 10.0 m	- length of the simply supported beam;
h = b = 1.0 m	- side of the square cross-section of the simply supported beam;
$E = 3.0 \cdot 10^7$ kN/m^2	- elastic modulus of the simply supported beam material;
ν = 0.2	- Poisson’s ratio;
$q = 10^5$ kN/m	- initial value of the transverse load uniformly distributed along the longitudinal axis of the beam.

Finite element model: Design model – general type system. Two design models are considered:
Bar model (B), 10 elements of type 5, the spacing of the finite element mesh along the longitudinal axis is 1.0 m. Boundary conditions are provided by imposing constraints on the nodes of the simply supported ends of the beam in the directions of the degrees of freedom X, Y, Z, UX, UZ. The action with the initial value of the transverse uniformly distributed load q is specified on all elements of the beam. Number of nodes in the design model – 11;
Reissner-Mindlin shell element model (P), 2560 eight-node elements of type 150, the spacing of the finite element mesh along the longitudinal axis and along the height of the beam is 0.0625 m. The shell is supported by vertical high-rigidity bars (h = b = 1.0 m; $E = 3.0 \cdot 10^9$ kN/m^2; ν = 0.2), 64 elements of type 5. Boundary conditions are provided by imposing constraints on the nodes of the ends of the beam lying on its longitudinal axis in the directions of the degrees of freedom X, Y, Z, UZ and on all other nodes of the ends of the beam in the directions of the degrees of freedom Y, UZ. The action with the initial value of the transverse load q uniformly distributed along the line is specified on the lower sides of all beam elements located above its longitudinal axis. Number of nodes in the design model – 8033.

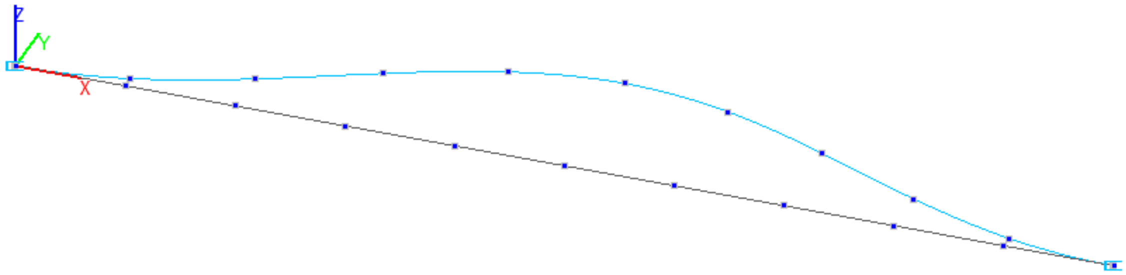
Results in SCAD



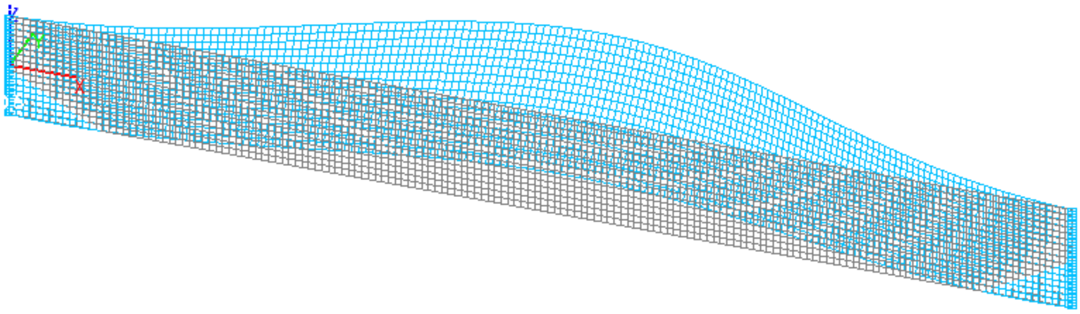
Design model. Bar model



Design model. Reissner-Mindlin shell element model



1-st buckling mode. Bar model



1- st buckling mode. Reissner-Mindlin shell element model

Comparison of solutions:

Critical value of the transverse load q_{cr} (kN/m), uniformly distributed along the longitudinal axis of the beam simply supported in the bending plane and clamped out of the bending plane

Design model	Theory	SCAD	Deviation, %
Bar	101863	0,995488·100000 = 99549	2,27
Reissner-Mindlin shell element	101863	0,944805·100000= 94481	7,25

Notes: In the analytical solution the critical value of the transverse uniformly distributed load q_{cr}, corresponding to the moment of buckling of the simply supported beam can be determined according to the following formula:

$$q = \frac{48{,}60 \cdot \sqrt{E \cdot I_z \cdot G \cdot I_x}}{L^3} \qquad G = \frac{E}{2 \cdot (1+\nu)}$$

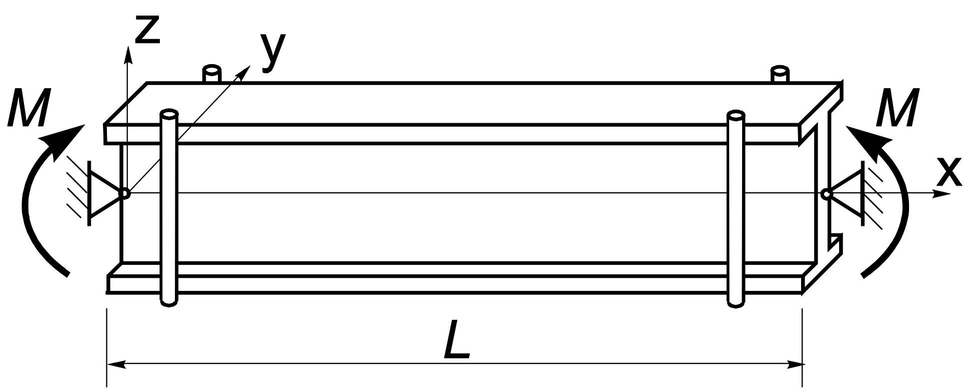
$I_z = \frac{h \cdot b^3}{12}$ – minimum bending inertia moment (out of the moment plane);

$I_y = \frac{b \cdot h^3}{12}$ – maximum bending inertia moment (in the moment plane);

$I_x = k_f \cdot h \cdot b^3$ – free torsional inertia moment, where:

$$k_f = \frac{1}{3} \cdot \left\{ 1 - \frac{192}{\pi^5} \cdot \frac{b}{h} \cdot \sum_{n=1}^{\infty} \left[sin^2\left(\frac{n \cdot \pi}{2}\right) \cdot \frac{1}{n^5} \cdot th\left(\frac{n \cdot \pi \cdot h}{2 \cdot b}\right) \right] \right\}$$

Stability of an I-beam Simply Supported in and out of the Bending Plane Subjected to Concentrated Bending Moments Applied at the Ends and Equal in Value (Pure Bending)



Objective: Determination of the critical value of the concentrated bending moments equal in value and applied at the ends of an I-beam simply supported in and out of the bending plane corresponding to the moment of its buckling.

Initial data files:

File name	Description
Stability_Flanged_Beam_1_Bar.SPR Flanged_Beam.tns	Bar model Thin-walled beam cross-section
Stability_Flanged_Beam_1_Shell.SPR	Shell element model

Problem formulation: The I-beam simply supported in and out of the bending plane is subjected to the action of the concentrated bending moments M, equal in value and applied at its ends. Determine the critical value of the concentrated bending moments M_{cr}, corresponding to the moment of buckling of the simply supported beam.

References: A.S. Volmir. Stability of Deformable Systems, Moscow, Nauka, 1967, p.222;

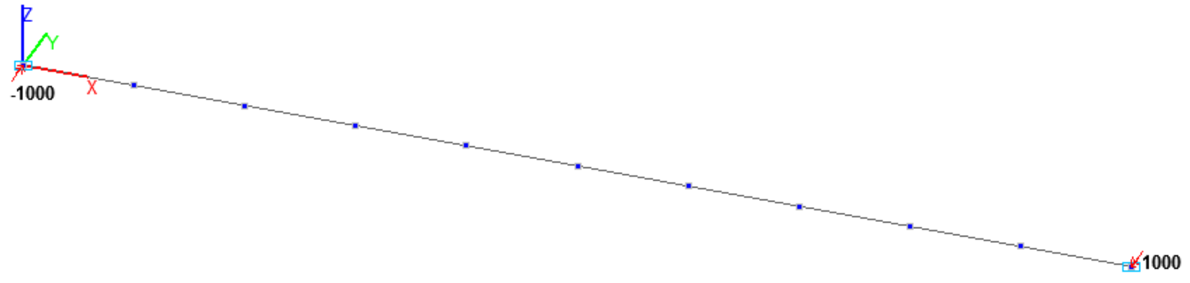
Initial data:

$L = 10.0$ m - length of the simply supported beam;
$E = 3.0 \cdot 10^7$ kN/m^2 - elastic modulus of the simply supported beam material;
$\nu = 0.2$ - Poisson's ratio;
$b = b_f = 0.5$ m - width of the flanges of the cross-section of the simply supported beam;
$t = t_f = 0.04$ m - thickness of the flanges of the cross-section of the simply supported beam;
$h_w = 1.0$ m - height of the web of the cross-section of the simply supported beam;
$t_w = 0.02$ m - thickness of the web of the cross-section of the simply supported beam;
$M = 10^3$ kN·m - initial value of the concentrated bending moments applied at the ends of the beam.

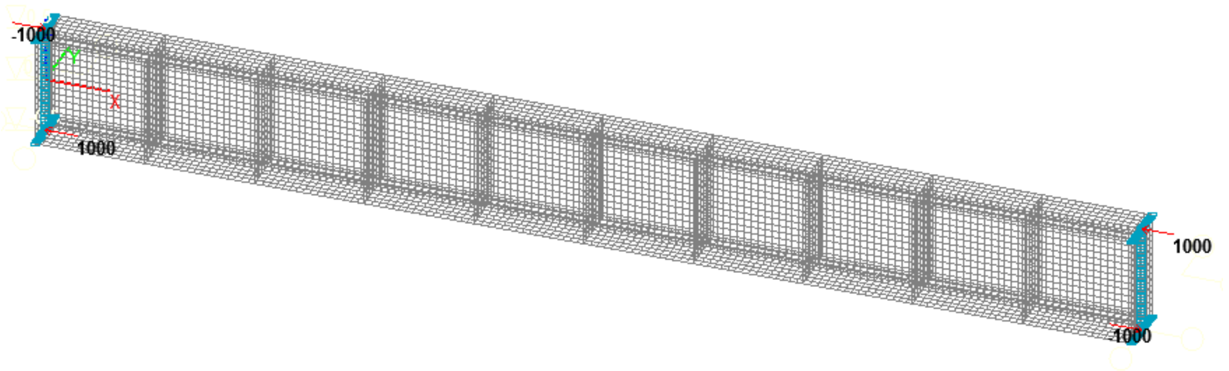
Finite element model: Design model – general type system. Two design models are considered:
Bar model (B), 10 elements of type 5, the spacing of the finite element mesh along the longitudinal axis of the beam is 1.0 m. The reduced free torsional stiffness of the cross-section of the simply supported beam taking into account the warping effect is calculated according to the following formula: $G \cdot I_{x\_red} = G \cdot I_x + \frac{\pi^2}{L^2} \cdot E \cdot I_\omega$. Boundary conditions are provided by imposing constraints on the nodes of the simply supported ends of the beam in the directions of the degrees of freedom X, Y, Z, UX. The action with the initial value of the concentrated bending moments M is specified in the nodes of the ends of the beam. Number of nodes in the design model – 11;
Reissner-Mindlin shell element model (P), 2560 eight-node beam elements of type 150, the spacing of the finite element mesh along the longitudinal axis and along the height of the beam is 0.0625 m. Vertical stiffeners are arranged with a spacing of 1.0 m along the length in order to prevent the local buckling of the web and the flanges of the beam ($h_w = 1.0$ m; $b_w = 0.5$ m; $t_w = 0.02$ m; $E = 3.0 \cdot 10^7$ kN/m^2; $\nu = 0.2$), 3968 elements of type 150. Boundary conditions are provided by imposing constraints on the nodes of the ends

of the beam lying on its longitudinal axis in the directions of the degrees of freedom X, Y, Z, and on all other nodes of the ends of the beam in the direction of the degree of freedom Y. The action with the initial value of the concentrated bending moments M is specified as a pair of forces $P = M/h_w = 10^3$ kN on the nodes of the ends of the beam lying on the longitudinal axes of its flanges. Number of nodes in the design model – 19793.

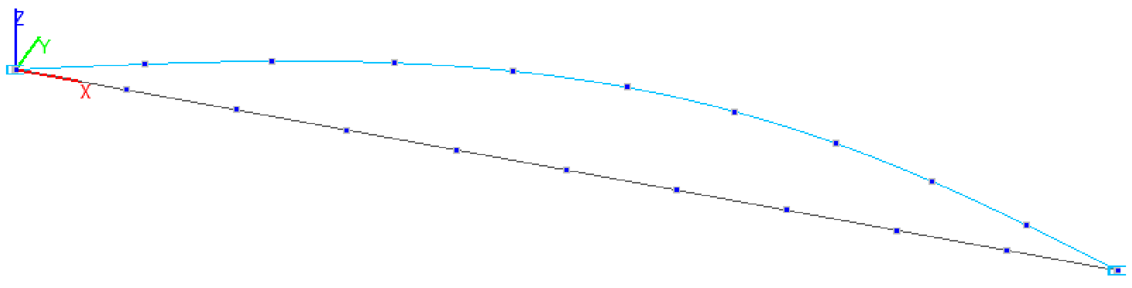
Results in SCAD



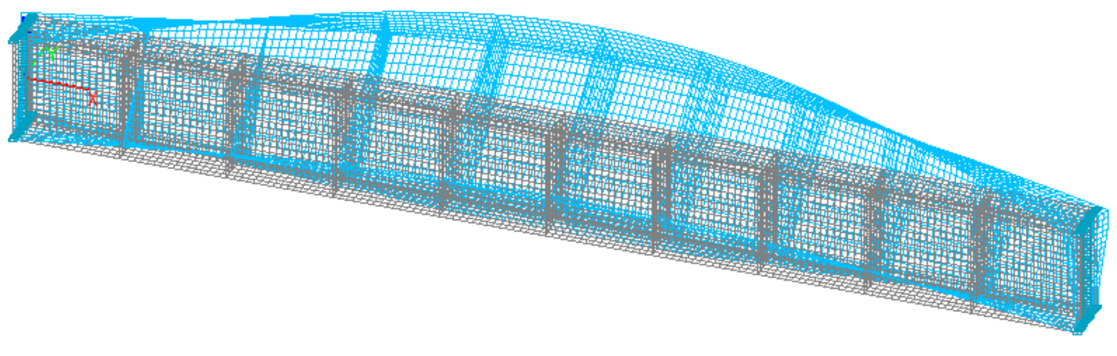
Design model. Bar model



Design model. Reissner-Mindlin shell element model



1-st buckling mode. Bar model



1-st buckling mode. Reissner-Mindlin shell element model

Comparison of solutions:

Critical value of the concentrated bending moments M_{cr} (kN·m),
applied at the ends of the beam simply supported in and out of the bending plane

Design model	Theory	SCAD	Deviation, %
Bar	1493	1,510993·1000= 1511	1,19
Reissner-Mindlin shell element	1493	1, 545837·1000= 1546	3,52

Notes: In the analytical solution the critical value of the concentrated bending moments M_{cr}, corresponding to the moment of buckling of the simply supported beam can be determined according to the following formula:

$$M = \frac{\pi \cdot \sqrt{E \cdot I_z \cdot G \cdot I_x}}{L} \cdot \chi \qquad \chi = \sqrt{1 + \frac{\pi^2}{L^2} \cdot \frac{E \cdot I_\omega}{G \cdot I_x}} \qquad G = \frac{E}{2 \cdot (1+\nu)}$$

$$I_z = \frac{h_w \cdot t_w^3}{12} + 2 \cdot \frac{b_f^3 \cdot t_f}{12}$$ – minimum bending inertia moment (out of the moment plane);

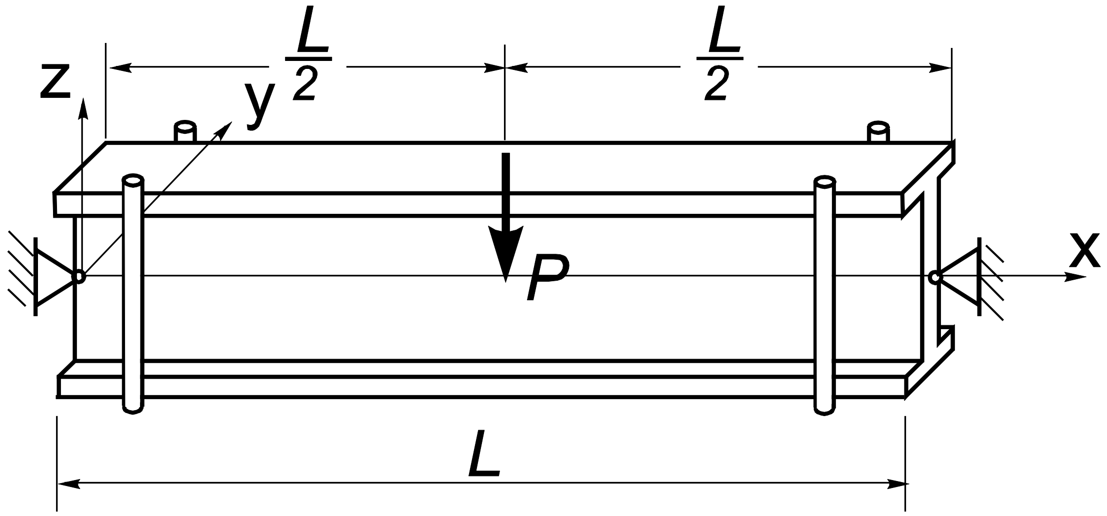
$$I_\omega = \frac{h_w^2 \cdot b_f^3 \cdot t_f}{24}$$ – sectorial constrained torsional inertia moment;

$$I_x = 2 \cdot k_f \cdot b_f \cdot t_f^3 + k_w \cdot h_w \cdot t_w^3$$ – free torsional inertia moment, where:

$$k_f = \frac{1}{3} \cdot \left\{ 1 - \frac{192}{\pi^5} \cdot \frac{t_f}{b_f} \cdot \sum_{n=1}^{\infty} \left[sin^2\left(\frac{n \cdot \pi}{2}\right) \cdot \frac{1}{n^5} \cdot th\left(\frac{n \cdot \pi \cdot h}{2 \cdot b}\right)\right]\right\},$$

$$k_w = \frac{1}{3} \cdot \left\{ 1 - \frac{192}{\pi^5} \cdot \frac{t_w}{h_w} \cdot \sum_{n=1}^{\infty} \left[sin^2\left(\frac{n \cdot \pi}{2}\right) \cdot \frac{1}{n^5} \cdot th\left(\frac{n \cdot \pi \cdot h}{2 \cdot b}\right)\right]\right\}$$

Stability of an I-beam Simply Supported in and out of the Bending Plane Subjected to a Concentrated Transverse Bending Force Applied in the Middle of the Span at the Level of the Longitudinal Axis (Transverse Bending)



Objective: Determination of the critical value of the concentrated transverse bending force applied in the middle of the span at the level of the longitudinal axis of an I-beam simply supported in and out of the bending plane corresponding to the moment of its buckling.

Initial data files:

File name	Description
Stability_Flanged_Beam_2_Bar.SPR Flanged_Beam.tns	Bar model Thin-walled beam cross-section
Stability_Flanged_Beam_2_Shell.SPR	Shell element model

Problem formulation: The I-beam simply supported in and out of the bending plane is subjected to the action of the concentrated transverse bending force P, applied in the middle of its span at the level of the longitudinal axis. Determine the critical value of the concentrated transverse bending force P_{cr}, corresponding to the moment of buckling of the simply supported beam.

References: A.S. Volmir. Stability of Deformable Systems, Moscow, Nauka, 1967, p.222;

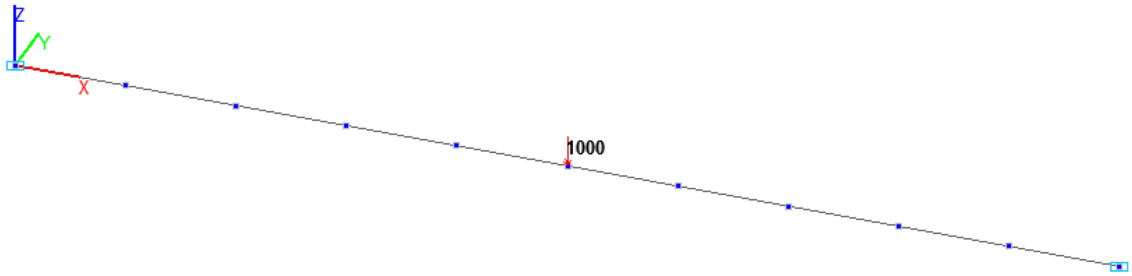
Initial data:

$L = 10.0$ m - length of the simply supported beam;
$E = 3.0 \cdot 10^7$ kN/m^2 - elastic modulus of the simply supported beam material;
$\nu = 0.2$ - Poisson's ratio;
$b = b_f = 0.5$ m - width of the flanges of the cross-section of the simply supported beam;
$t = t_f = 0.04$ m - thickness of the flanges of the cross-section of the simply supported beam;
$h_w = 1.0$ m - height of the web of the cross-section of the simply supported beam;
$t_w = 0.02$ m - thickness of the web of the cross-section of the simply supported beam;
$P = 10^3$ kN - initial value of the concentrated transverse bending force applied in the middle of the span at the level of the longitudinal axis of the beam.

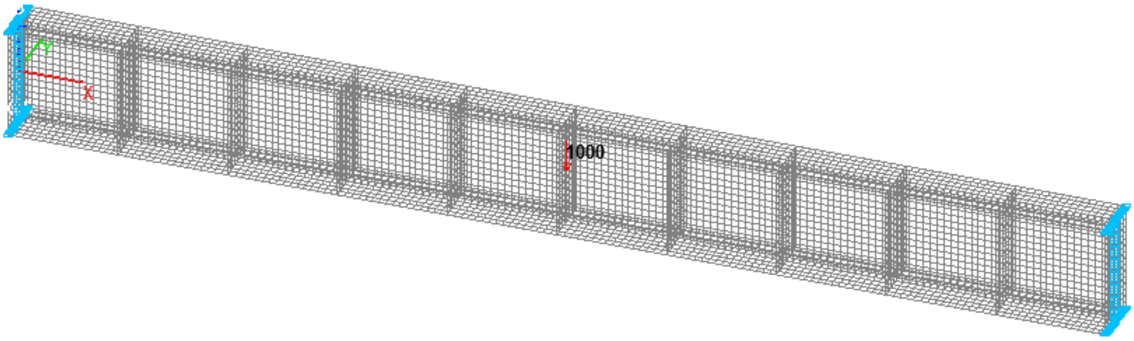
Finite element model: Design model – general type system. Two design models are considered:
Bar model (B), 10 elements of type 5, the spacing of the finite element mesh along the longitudinal axis of the beam is 1.0 m. The reduced free torsional stiffness of the cross-section of the simply supported beam taking into account the warping effect is calculated according to the following formula: $G \cdot I_{x\_red} = G \cdot I_x + \frac{\pi^2}{L^2} \cdot E \cdot I_\omega$. Boundary conditions are provided by imposing constraints on the nodes of the simply supported ends of the beam in the directions of the degrees of freedom X, Y, Z, UX. The action with the initial value of the concentrated transverse bending force P is specified in the node in the middle of the beam span. Number of nodes in the design model – 11;

Reissner-Mindlin shell element model (P), 2560 eight-node beam elements of type 150, the spacing of the finite element mesh along the longitudinal axis and along the height of the beam is 0.0625 m. Vertical stiffeners are arranged with a spacing of 1.0 m along the length in order to prevent the local buckling of the web and the flanges of the beam (h_w = 1.0 m; b_w = 0.5 m; t_w = 0.02 m; $E = 3.0\cdot10^7$ kN/m^2; ν = 0.2), 3968 elements of type 150. Boundary conditions are provided by imposing constraints on the nodes of the ends of the beam lying on its longitudinal axis in the directions of the degrees of freedom X, Y, Z, and on all other nodes of the ends of the beam in the direction of the degree of freedom Y. The action with the initial value of the concentrated transverse bending force P is specified in the node in the middle of the beam span at the level of the longitudinal axis of the beam. Number of nodes in the design model – 19793.

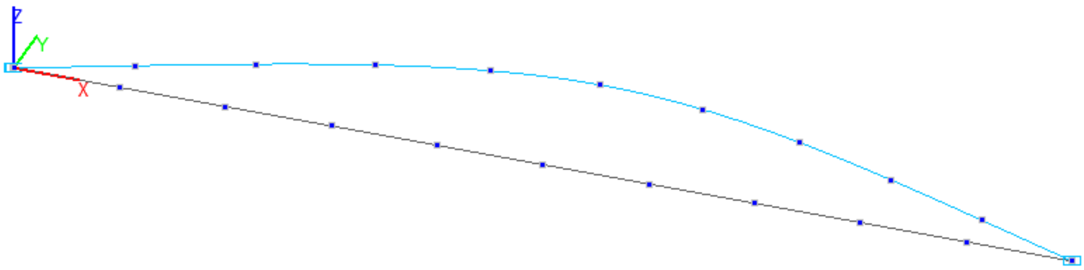
Results in SCAD



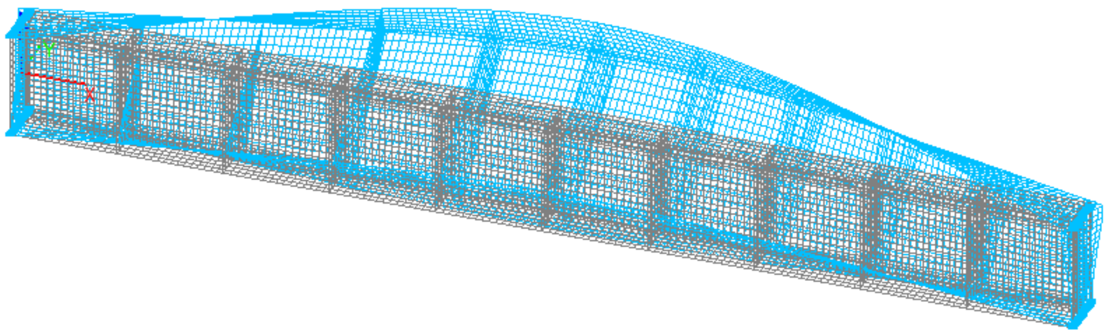
Design model. Bar model



Design model. Reissner-Mindlin shell element model



1-st buckling mode. Bar model



1- st buckling mode. Reissner-Mindlin shell element model

Comparison of solutions:

Critical value of the concentrated transverse bending force P_{cr} (kN), applied in the middle of the span at the level of the longitudinal axis of the beam simply supported in and out of the bending plane

Design model	Theory	SCAD	Deviation, %
Bar	804	0,815304·1000= 815	1,38
Reissner-Mindlin shell element	804	0,817535·1000= 818	1,65

Notes: In the analytical solution the critical value of the concentrated transverse bending force P_{cr}, corresponding to the moment of buckling of the simply supported beam can be determined according to the following formula:

$$P = \frac{16{,}92 \cdot \sqrt{E \cdot I_z \cdot G \cdot I_x}}{L^2} \cdot \chi \qquad \chi = \sqrt{1 + \frac{\pi^2}{L^2} \cdot \frac{E \cdot I_\omega}{G \cdot I_x}} \qquad G = \frac{E}{2 \cdot (1+\nu)}$$

$I_z = \frac{h_w \cdot t_w^3}{12} + 2 \cdot \frac{b_f^3 \cdot t_f}{12}$ – minimum bending inertia moment (out of the moment plane);

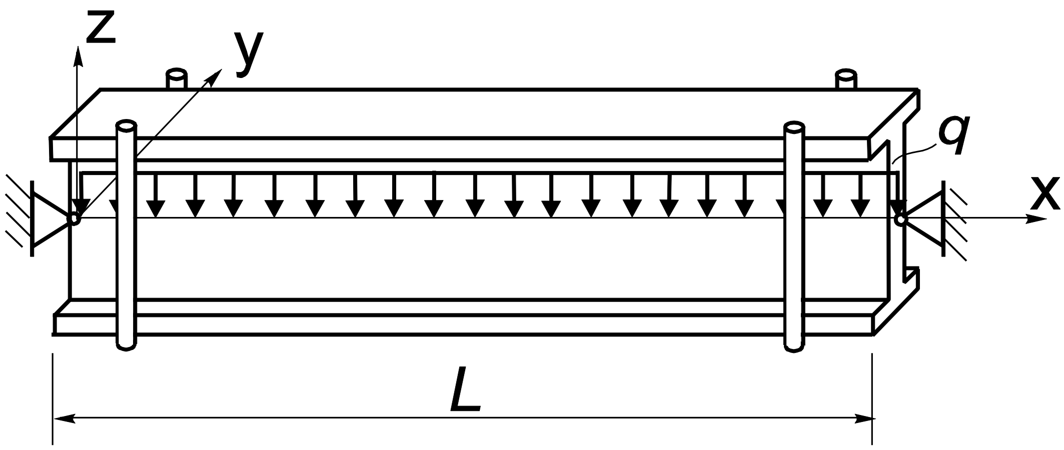
$I_\omega = \frac{h_w^2 \cdot b_f^3 \cdot t_f}{24}$ – sectorial constrained torsional inertia moment;

$I_x = 2 \cdot k_f \cdot b_f \cdot t_f^3 + k_w \cdot h_w \cdot t_w^3$ – free torsional inertia moment, where:

$$k_f = \frac{1}{3} \cdot \left\{ 1 - \frac{192}{\pi^5} \cdot \frac{t_f}{b_f} \cdot \sum_{n=1}^{\infty} \left[sin^2\left(\frac{n \cdot \pi}{2}\right) \cdot \frac{1}{n^5} \cdot th\left(\frac{n \cdot \pi \cdot h}{2 \cdot b}\right) \right] \right\},$$

$$k_w = \frac{1}{3} \cdot \left\{ 1 - \frac{192}{\pi^5} \cdot \frac{t_w}{h_w} \cdot \sum_{n=1}^{\infty} \left[sin^2\left(\frac{n \cdot \pi}{2}\right) \cdot \frac{1}{n^5} \cdot th\left(\frac{n \cdot \pi \cdot h}{2 \cdot b}\right) \right] \right\}$$

Stability of an I-beam Simply Supported in and out of the Bending Plane Subjected to a Transverse Load Uniformly Distributed along Its Longitudinal Axis



Objective: Determination of the critical value of the transverse load uniformly distributed along the longitudinal axis of an I-beam simply supported in and out of the bending plane corresponding to the moment of its buckling.

Initial data files:

File name	Description
Stability_Flanged_Beam_3_Bar.SPR Flanged_Beam.tns	Bar model Thin-walled beam cross-section
Stability_Flanged_Beam_3_Shell.SPR	Shell element model

Problem formulation: The I-beam simply supported in and out of the bending plane is subjected to the action of the transverse load q, uniformly distributed along its longitudinal axis. Determine the critical value of the transverse uniformly distributed load q_{cr}, corresponding to the moment of buckling of the simply supported beam.

References: A.S. Volmir. Stability of Deformable Systems, Moscow, Nauka, 1967, p.222;

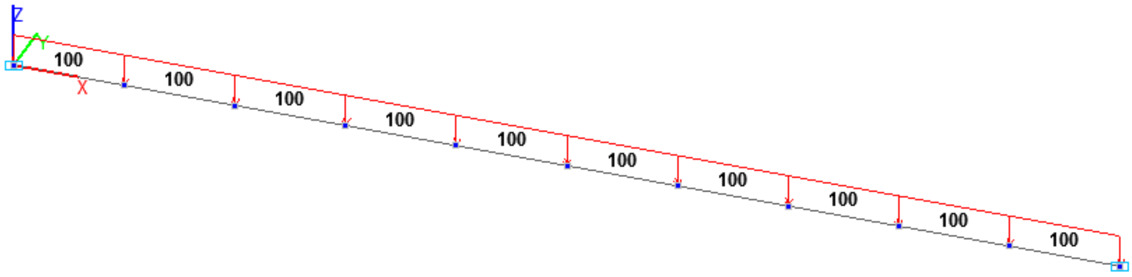
Initial data:

$L = 10.0$ m - length of the simply supported beam;
$E = 3.0 \cdot 10^7$ kN/m^2 - elastic modulus of the simply supported beam material;
$\nu = 0.2$ - Poisson's ratio;
$b = b_f = 0.5$ m - width of the flanges of the cross-section of the simply supported beam;
$t = t_f = 0.04$ m - thickness of the flanges of the cross-section of the simply supported beam;
$h_w = 1.0$ m - height of the web of the cross-section of the simply supported beam;
$t_w = 0.02$ m - thickness of the web of the cross-section of the simply supported beam;
$q = 10^2$ kN/m - initial value of the transverse load uniformly distributed along the longitudinal axis of the beam.

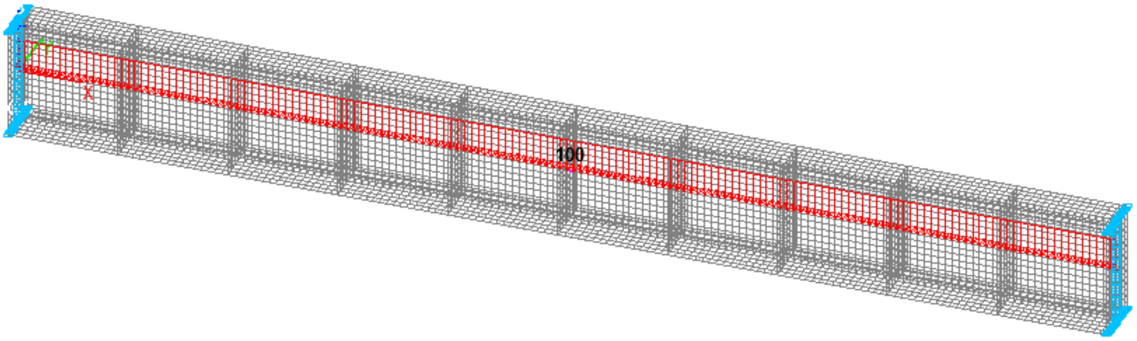
Finite element model: Design model – general type system. Two design models are considered:
Bar model (B), 10 elements of type 5, the spacing of the finite element mesh along the longitudinal axis of the beam is 1.0 m. The reduced free torsional stiffness of the cross-section of the simply supported beam taking into account the warping effect is calculated according to the following formula: $G \cdot I_{x\_red} = G \cdot I_x + \frac{\pi^2}{L^2} \cdot E \cdot I_\omega$. Boundary conditions are provided by imposing constraints on the nodes of the simply supported ends of the beam in the directions of the degrees of freedom X, Y, Z, UX. The action with the initial value of the transverse uniformly distributed load q is specified on all elements of the beam. Number of nodes in the design model – 11;
Reissner-Mindlin shell element model (P), 2560 eight-node beam elements of type 150, the spacing of the finite element mesh along the longitudinal axis and along the height of the beam is 0.0625 m. Vertical stiffeners are arranged with a spacing of 1.0 m along the length in order to prevent the local buckling of the web and the flanges of the beam ($h_w = 1.0$ m; $b_w = 0.5$ m; $t_w = 0.02$ m; $E = 3.0 \cdot 10^7$ kN/m^2; $\nu = 0.2$), 3968

elements of type 150. Boundary conditions are provided by imposing constraints on the nodes of the ends of the beam lying on its longitudinal axis in the directions of the degrees of freedom X, Y, Z, and on all other nodes of the ends of the beam in the direction of the degree of freedom Y. The action with the initial value of the transverse load q uniformly distributed along the line is specified on the lower sides of all beam elements located above the longitudinal axis of the beam. Number of nodes in the design model – 19793.

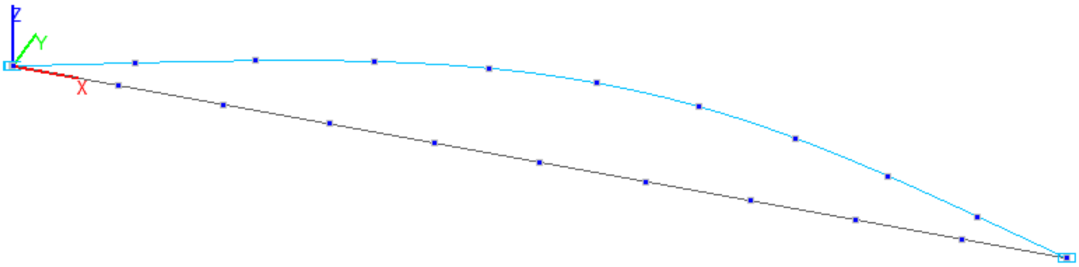
Results in SCAD



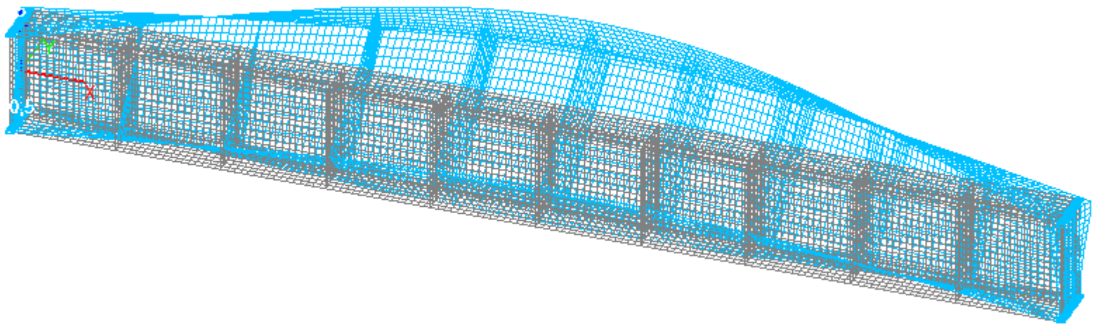
Design model. Bar model



Design model. Reissner-Mindlin shell element model



1-st buckling mode. Bar model



1-st buckling mode. Reissner-Mindlin shell element model

Comparison of solutions:

Critical value of the transverse load q_{cr} (kN/m), uniformly distributed along the longitudinal axis of the beam simply supported in and out of the bending plane

Design model	Theory	SCAD	Deviation, %
Bar	135	1,362356·100= 136	1,21
Reissner-Mindlin shell element	135	1,359283·100= 136	0,98

Notes: In the analytical solution the critical value of the transverse uniformly distributed load q_{cr}, corresponding to the moment of buckling of the simply supported beam can be determined according to the following formula:

$$q = \frac{28{,}32 \cdot \sqrt{E \cdot I_z \cdot G \cdot I_x}}{L^3} \cdot \chi \qquad \chi = \sqrt{1 + \frac{\pi^2}{L^2} \cdot \frac{E \cdot I_\omega}{G \cdot I_x}} \qquad G = \frac{E}{2 \cdot (1+\nu)}$$

$$I_z = \frac{h_w \cdot t_w^3}{12} + 2 \cdot \frac{b_f^3 \cdot t_f}{12}$$ – minimum bending inertia moment (out of the moment plane);

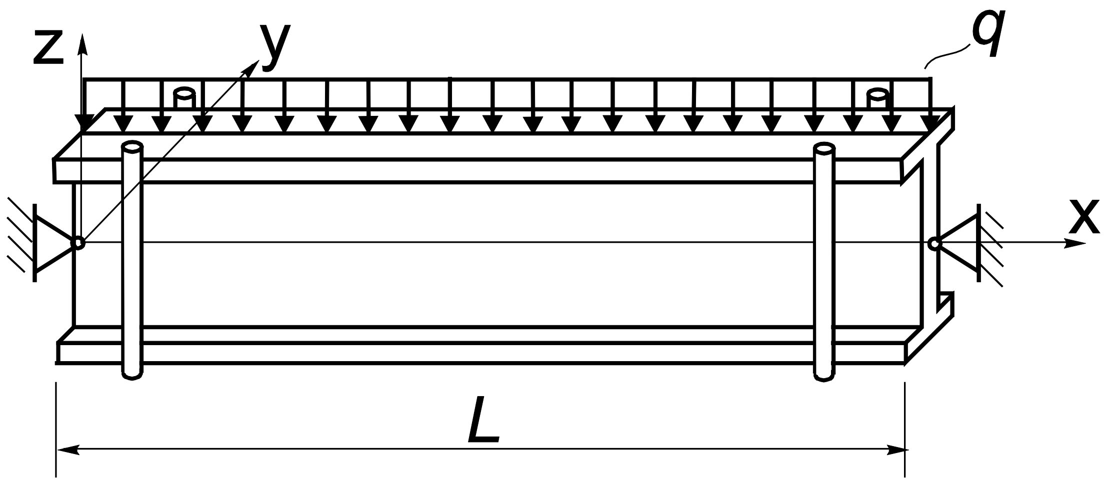
$$I_\omega = \frac{h_w^2 \cdot b_f^3 \cdot t_f}{24}$$ – sectorial constrained torsional inertia moment;

$$I_x = 2 \cdot k_f \cdot b_f \cdot t_f^3 + k_w \cdot h_w \cdot t_w^3$$ – free torsional inertia moment, where:

$$k_f = \frac{1}{3} \cdot \left\{ 1 - \frac{192}{\pi^5} \cdot \frac{t_f}{b_f} \cdot \sum_{n=1}^{\infty} \left[sin^2\left(\frac{n \cdot \pi}{2}\right) \cdot \frac{1}{n^5} \cdot th\left(\frac{n \cdot \pi \cdot h}{2 \cdot b}\right) \right] \right\},$$

$$k_w = \frac{1}{3} \cdot \left\{ 1 - \frac{192}{\pi^5} \cdot \frac{t_w}{h_w} \cdot \sum_{n=1}^{\infty} \left[sin^2\left(\frac{n \cdot \pi}{2}\right) \cdot \frac{1}{n^5} \cdot th\left(\frac{n \cdot \pi \cdot h}{2 \cdot b}\right) \right] \right\}$$

Stability of an I-beam Simply Supported in and out of the Bending Plane Subjected to a Load Uniformly Distributed along the Longitudinal Axis of Its Upper Flange



Objective: Determination of the critical value of the load uniformly distributed along the longitudinal axis of the upper flange of an I-beam simply supported in and out of the bending plane corresponding to the moment of its buckling.

Initial data files:

File name	Description
Stability_Flanged_Beam_4_Bar.SPR Flanged_Beam.tns	Bar model Thin-walled beam cross-section
Stability_Flanged_Beam_4_Shell.SPR	Shell element model

Problem formulation: The I-beam simply supported in and out of the bending plane is subjected to the action of the load q, uniformly distributed along the longitudinal axis of its upper flange. Determine the critical value of the uniformly distributed load q_{cr}, corresponding to the moment of buckling of the simply supported beam.

References: A.S. Volmir. Stability of Deformable Systems, Moscow, Nauka, 1967, p.222;

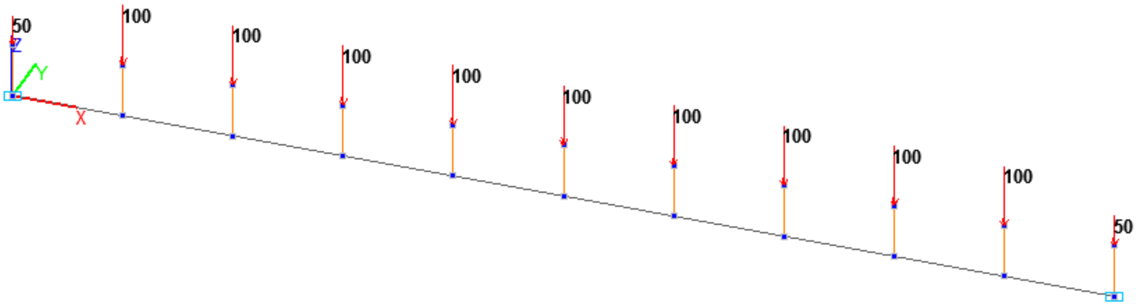
Initial data:

$L = 10.0$ m - length of the simply supported beam;
$E = 3.0 \cdot 10^7$ kN/m^2 - elastic modulus of the simply supported beam material;
$\nu = 0.2$ - Poisson's ratio;
$b = b_f = 0.5$ m - width of the flanges of the cross-section of the simply supported beam;
$t = t_f = 0.04$ m - thickness of the flanges of the cross-section of the simply supported beam;
$h_w = 1.0$ m - height of the web of the cross-section of the simply supported beam;
$t_w = 0.02$ m - thickness of the web of the cross-section of the simply supported beam;
$q = 10^2$ kN/m - initial value of the transverse load uniformly distributed along the longitudinal axis of the upper flange of the beam.

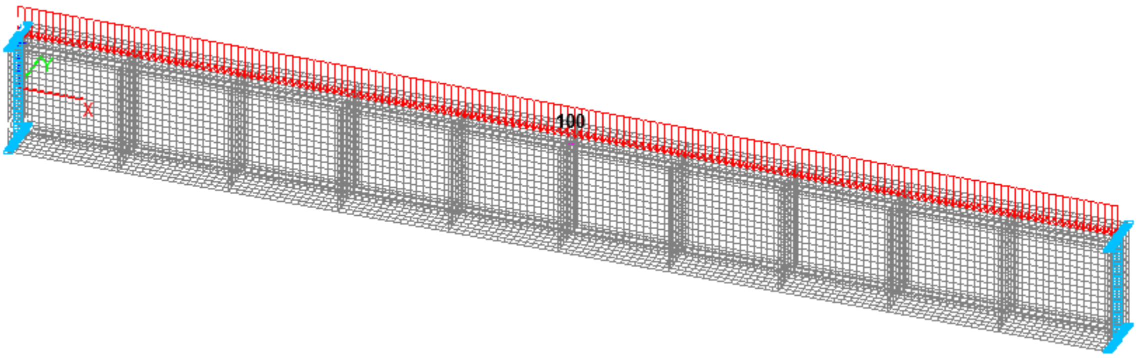
Finite element model: Design model – general type system. Two design models are considered:
Bar model (B), 10 elements of type 5, the spacing of the finite element mesh along the longitudinal axis of the beam is 1.0 m. The reduced free torsional stiffness of the cross-section of the simply supported beam taking into account the warping effect is calculated according to the following formula: $G \cdot I_{x\_red} = G \cdot I_x + \frac{\pi^2}{L^2} \cdot E \cdot I_\omega$. Boundary conditions are provided by imposing constraints on the nodes of the simply supported ends of the beam in the directions of the degrees of freedom X, Y, Z, UX. 11 vertical upward two-node elements of type 100 (3D rigid body) with the length h/2 are adjacent to the nodes of the beam. The action with the initial value of the uniformly distributed load q is specified in the free nodes of

the elements of the rigid bodies (elevated application points) as concentrated forces P = q·1.0 = 10^2 kN ($0.5·10^2$ kN for end nodes). Number of nodes in the design model – 22;
Reissner-Mindlin shell element model (P), 2560 eight-node beam elements of type 150, the spacing of the finite element mesh along the longitudinal axis and along the height of the beam is 0.0625 m. Vertical stiffeners are arranged with a spacing of 1.0 m along the length in order to prevent the local buckling of the web and the flanges of the beam (h_w = 1.0 m; b_w = 0.5 m; t_w = 0.02 m; E = $3.0·10^7$ kN/m^2; ν = 0.2), 3968 elements of type 150. Boundary conditions are provided by imposing constraints on the nodes of the ends of the beam lying on its longitudinal axis in the directions of the degrees of freedom X, Y, Z, and on all other nodes of the ends of the beam in the direction of the degree of freedom Y. The action with the initial value of the load q uniformly distributed along the line is specified on the upper sides of all elements of the beam web located under the upper flange of the beam. Number of nodes in the design model – 19793.

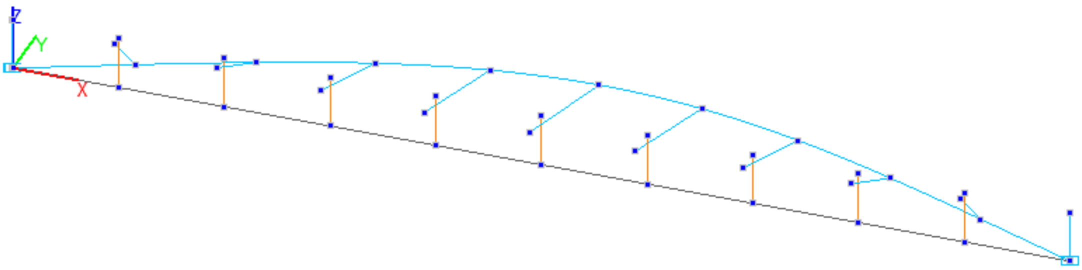
Results in SCAD



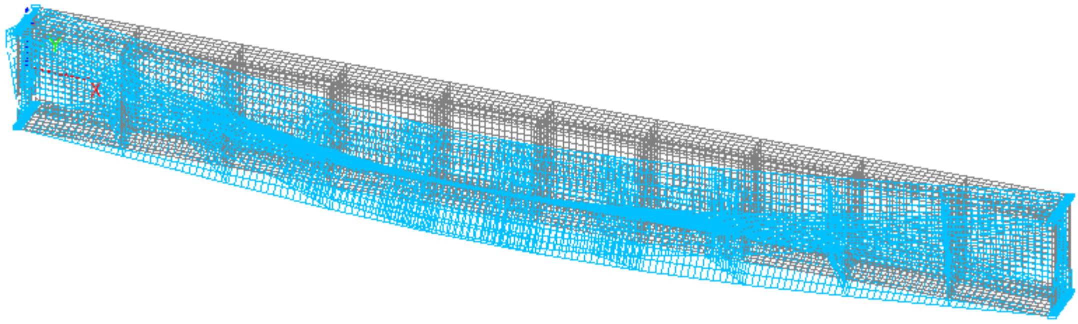
Design model. Bar model



Design model. Reissner-Mindlin shell element model



1-st buckling mode. Bar model



1-st buckling mode. Reissner-Mindlin shell element model

Comparison of solutions:

Critical value of the load q_{cr} (kN/m), uniformly distributed along the longitudinal axis of the upper flange of the beam simply supported in and out of the bending plane

Design model	Theory	SCAD	Deviation, %
Bar	93	0,943201·100= 94	1,54
Reissner-Mindlin shell element	93	0,949310·100= 95	1,87

Notes: In the analytical solution the critical value of the transverse uniformly distributed load q_{cr}, corresponding to the moment of buckling of the simply supported beam can be determined according to the following formula:

$$q = \frac{28{,}32 \cdot \sqrt{E \cdot I_z \cdot G \cdot I_x}}{L^3} \cdot \chi \cdot k_h \qquad \chi = \sqrt{1 + \frac{\pi^2}{L^2} \cdot \frac{E \cdot I_\omega}{G \cdot I_x}} \qquad G = \frac{E}{2 \cdot (1+\nu)}$$

$$k_h = \sqrt{1 + \frac{20{,}32}{\pi^2} \cdot \frac{E \cdot I_\omega}{G \cdot I_x \cdot L^2 + \pi^2 \cdot E \cdot I_\omega}} - \frac{4{,}50}{\pi} \cdot \sqrt{\frac{E \cdot I_\omega}{G \cdot I_x \cdot L^2 + \pi^2 \cdot E \cdot I_\omega}}$$

$I_z = \frac{h_w \cdot t_w^3}{12} + 2 \cdot \frac{b_f^3 \cdot t_f}{12}$ – minimum bending inertia moment (out of the moment plane);

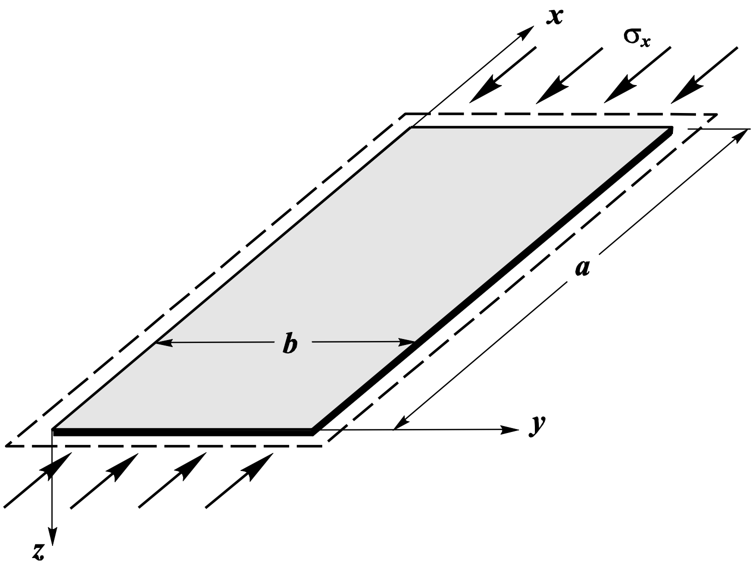
$I_\omega = \frac{h_w^2 \cdot b_f^3 \cdot t_f}{24}$ – sectorial constrained torsional inertia moment;

$I_x = 2 \cdot k_f \cdot b_f \cdot t_f^3 + k_w \cdot h_w \cdot t_w^3$ – free torsional inertia moment, where:

$$k_f = \frac{1}{3} \cdot \left\{ 1 - \frac{192}{\pi^5} \cdot \frac{t_f}{b_f} \cdot \sum_{n=1}^{\infty} \left[sin^2\left(\frac{n \cdot \pi}{2}\right) \cdot \frac{1}{n^5} \cdot th\left(\frac{n \cdot \pi \cdot h}{2 \cdot b}\right) \right] \right\},$$

$$k_w = \frac{1}{3} \cdot \left\{ 1 - \frac{192}{\pi^5} \cdot \frac{t_w}{h_w} \cdot \sum_{n=1}^{\infty} \left[sin^2\left(\frac{n \cdot \pi}{2}\right) \cdot \frac{1}{n^5} \cdot th\left(\frac{n \cdot \pi \cdot h}{2 \cdot b}\right) \right] \right\}$$

Stability of a Simply Supported Rectangular Plate Uniformly Compressed in One Direction



Objective: Determination of the critical value of the compressive forces uniformly distributed along two opposite sides of a simply supported rectangular plate corresponding to the moment of its buckling.

Initial data files:

File name	Description
6.6_a_4_n_4.SPR	Design model with the ratios of the sides of the plate a/b = 0.5 from four-node shell elements of type 44
6.6_a_4_n_8.SPR	Design model with the ratios of the sides of the plate a/b = 0.5 from eight- node shell elements of type 50
6.6_a_8_n_4.SPR	Design model with the ratios of the sides of the plate a/b = 1.0 from four-node shell elements of type 44
6.6_a_8_n_8.SPR	Design model with the ratios of the sides of the plate a/b = 1.0 from eight- node shell elements of type 50
6.6_a_12_n_4.SPR	Design model with the ratios of the sides of the plate a/b = 1.5 from four-node shell elements of type 44
6.6_a_12_n_8.SPR	Design model with the ratios of the sides of the plate a/b = 1.5 from eight- node shell elements of type 50

Problem formulation: The simply supported rectangular plate is subjected to the action of compressive forces σ, uniformly distributed along two opposite sides. Determine the critical value of the compressive forces σ_{cr}, corresponding to the moment of buckling of the rectangular plate.

References:

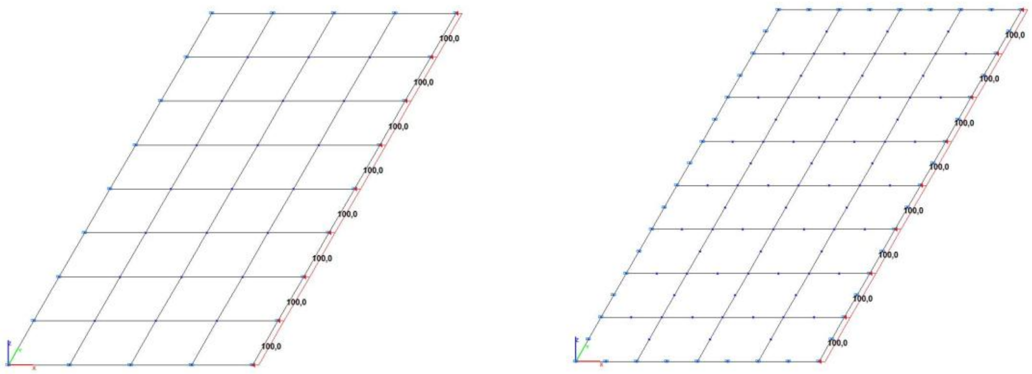
S. P. Timoshenko, Stability of Bars, Plates and Shells. — Moscow. Nauka. — 1971. — p. 621.
A.S. Volmir. Stability of Deformable Systems. — Moscow. — Nauka. — 1967. — p. 328.

Initial data:

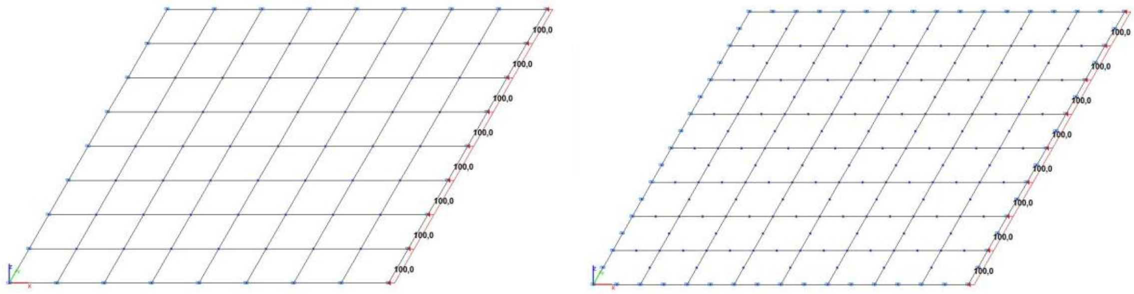
a = 4.0; 8.0; 12.0 m	- side of the rectangular plate free from forces (along the X axis of the global coordinate system);
b = 8.0 m	- side of the rectangular plate subjected to the compressive forces (along the Y axis of the global coordinate system);
h = 0.08 m	- thickness of the rectangular plate;
$E = 1.0\cdot10^7$ kN/m^2	- elastic modulus of the rectangular plate material;
ν = 1/3	- Poisson's ratio;
$\sigma = 1.25\cdot10^3$ kN/m^2	- initial value of the compressive forces.

Finite element model: Design model – general type system. Two design models with four-node shell elements of type 44 and eight-node shell elements of type 50 are considered for three cases with the ratios of the sides of the plate a/b = 0.5; 1.0; 1.5. The spacing of the finite element mesh along the sides of the plate (along the X and Y axes of the global coordinate system) is 1.0 m. Number of elements in the models – 32; 64; 96. Boundary conditions are provided by imposing constraints on the nodes of the support contour of the plate in the direction of the degree of freedom Z. A load uniformly distributed along the line with the initial value p = σ·h = 100 kN/m is specified on one of the two opposite sides of the plate subjected to the compressive forces, and the constraints in the respective direction (along the X axis of the global coordinate system) are imposed on the nodes of the other one. The dimensional stability of the design model is provided by imposing constraints in the normal direction (along the Y axis of the global coordinate system) on the nodes of one of the two opposite sides of the plate free from forces, and by imposing constraints in the UZ direction of the global coordinate system on the node of one of the corners of the plate. Number of nodes in the models – 45 (121); 81 (225); 117 (329).

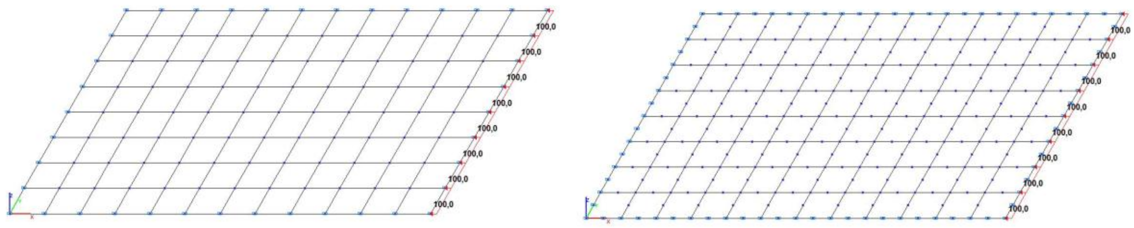
Results in SCAD



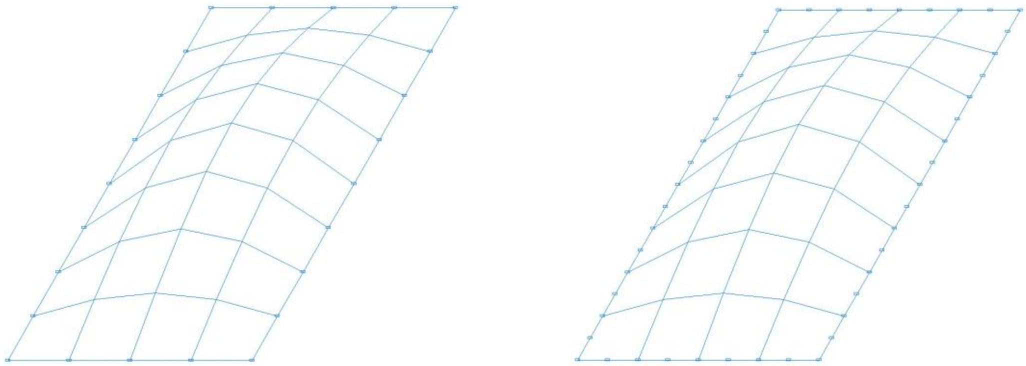
Design models with the ratio of the sides of the plate a/b = 0.5



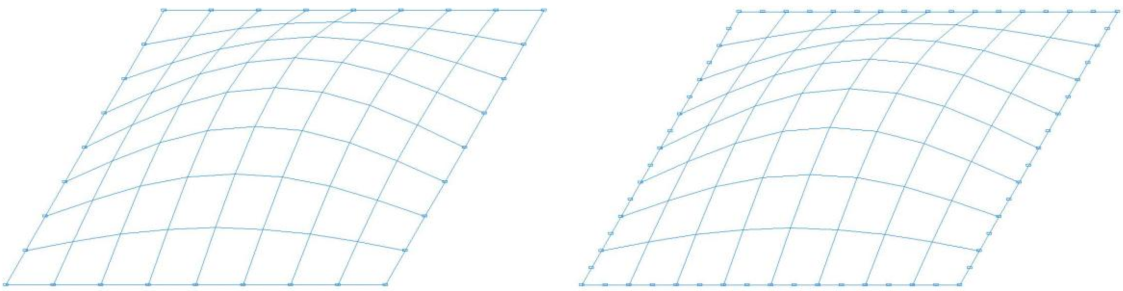
Design models with the ratio of the sides of the plate a/b = 1.0



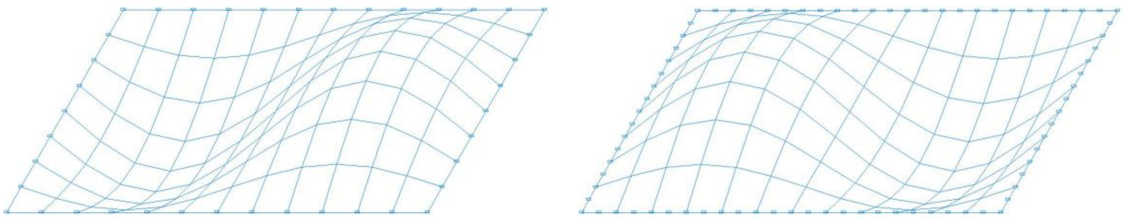
Design models with the ratio of the sides of the plate a/b = 1.5



Buckling modes for the design models with the ratio of the sides of the plate a/b = 0.5



Buckling modes for the design models with the ratio of the sides of the plate a/b = 1.0



Buckling modes for the design models with the ratio of the sides of the plate a/b = 1.5

Comparison of solutions:

Critical value of the compressive forces σ_{cr}, kN/m^2

Plate sides ratio	Design model	Theory	SCAD	Deviation, %
a/b = 0.5	Member type 44 n = 4 nodes	5783	4.716991·100/0.08 = = 5896	1.95
	Member type 50 n = 8 nodes		4.626558·100/0.08 = = 5783	0.00
a/b = 1.0	Member type 44 n = 4 nodes	3701	2.998497·100/0.08 = = 3748	1.27
	Member type 50 n = 8 nodes		2.960899·100/0.08 = = 3701	0.00
a/b = 1.5	Member type 44 n = 4 nodes	4016	3.264680·100/0.08 = = 4081	1.62
	Member type 50 n = 8 nodes		3.212803·100/0.08 = = 4016	0.00

Notes: In the analytical solution the critical value of the compressive forces σ_{cr}, corresponding to the moment of buckling of the rectangular plate can be determined according to the following formula:

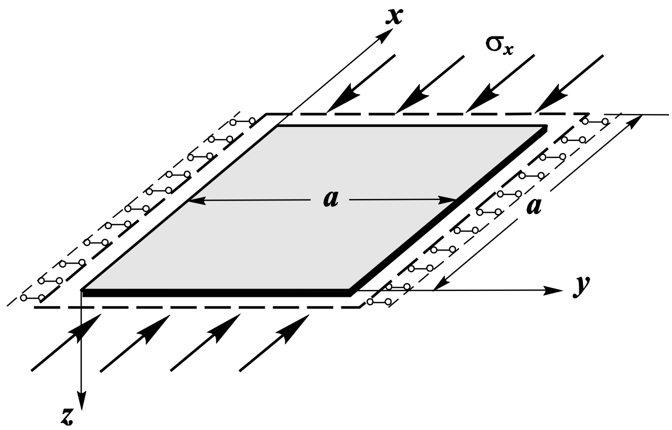
$$\sigma_{cr} = k \cdot \frac{\pi^2 \cdot D}{b^2 \cdot h}, \text{ where:}$$

$$D = \frac{E \cdot h^3}{12 \cdot (1 - \nu^2)}, \qquad k = \left(\frac{m \cdot b}{a} + \frac{a}{m \cdot b} \right)^2,$$

m = 1, 2, 3 … – number of half waves of the buckling mode in the direction of the compression of the plate; its minimum value is determined from the following expression:

$$\frac{a}{b} \le \sqrt{m \cdot (m + 1)}.$$

Stability of a Simply Supported Square Plate Uniformly Compressed in One Direction



Objective: Determination of the critical value of the compressive forces uniformly distributed along two opposite sides of a simply supported square plate corresponding to the moment of its buckling.

Initial data files:

File name	Description
6.7_n_4.SPR	Design model with four-node shell elements of type 44
6.7_n_8.SPR	Design model with eight-node shell elements of type 50

Problem formulation: The square plate is subjected to the action of compressive forces σ, uniformly distributed along two opposite roller supported sides. Two other opposite sides of the plate free from forces are pinned. Determine the critical value of the compressive forces σ_{cr}, corresponding to the moment of buckling of the square plate.

References:

J.H. Argyris, P.C. Dunne, G.A. Malejannakis, E. Schelkle. A simple triangular facet shell element with applications to linear and non-linear equilibrium and elastic stability problems. Computer methods in applied mechanics and engineering, 11. — 1977.— p. 97-131.

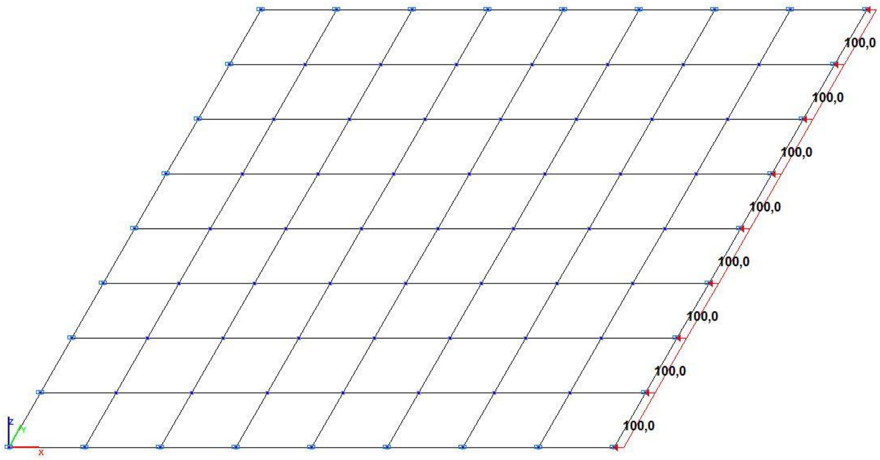
S.P. Timoshenko, J.M. Gere. Theory of elastic stability. McGraw-Hill. — New York. — 1963. — p. 356.

Initial data:

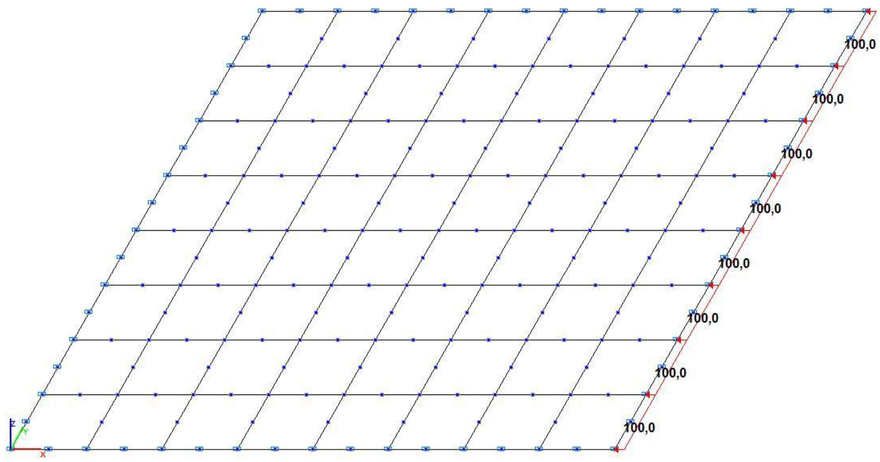
a = 8.0 m	- side of the square plate;
h = 0.08 m	- thickness of the square plate;
$E = 1.0\cdot10^7$ kN/m^2	- elastic modulus of the square plate material;
ν = 1/3	- Poisson's ratio;
$\sigma = 1.25\cdot10^3$ kN/m^2	- initial value of the compressive forces.

Finite element model: Design model – general type system. Two design models with four-node shell elements of type 44 and eight-node shell elements of type 50 are considered. The spacing of the finite element mesh along the sides of the plate (along the X and Y axes of the global coordinate system) is 1.0 m. Number of elements in the models – 64. Boundary conditions are provided by imposing constraints on the nodes of the support contour of the plate in the direction of the degree of freedom Z, and by imposing constraints in the normal direction along the Y axis of the global coordinate system on the nodes of one of the two opposite sides of the plate free from forces. A load uniformly distributed along the line with the initial value $p = \sigma\cdot h = 100$ kN/m is specified on one of the two opposite sides of the plate subjected to the compressive forces, and the constraints in the respective direction (along the X axis of the global coordinate system) are imposed on the nodes of the other one. The dimensional stability of the design model is provided by imposing a constraint in the UZ direction of the global coordinate system on the node of the support contour of the plate. Number of nodes in the models – 81; 225.

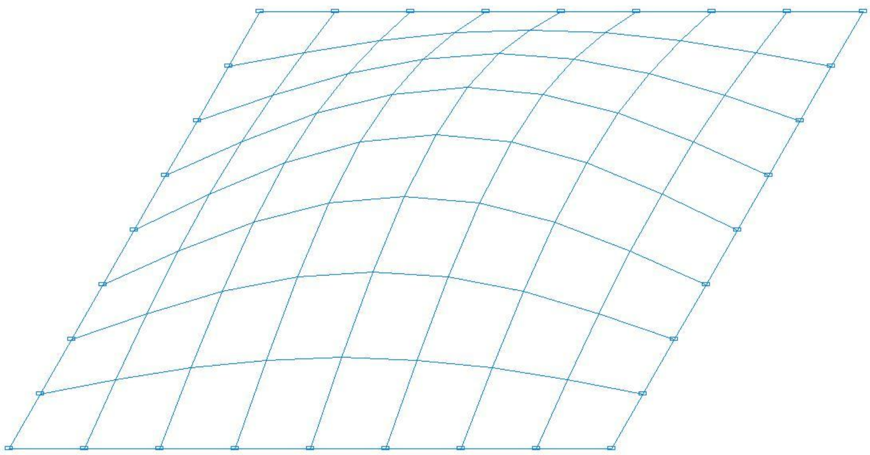
Results in SCAD



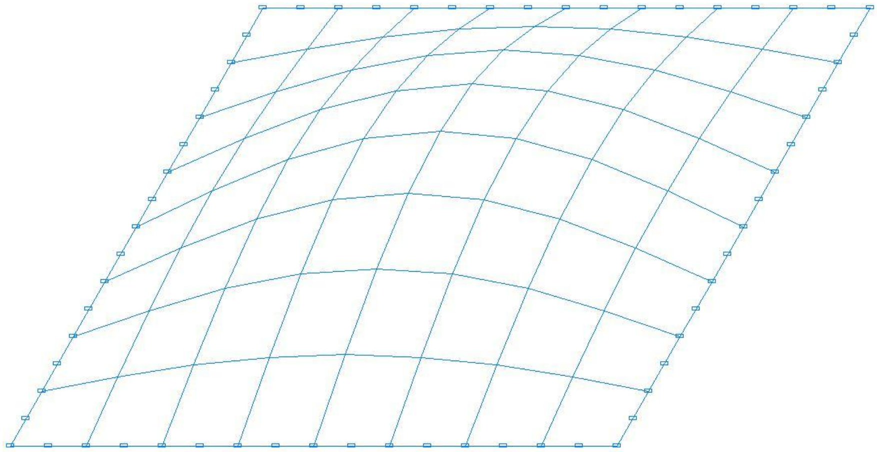
Design model. Model with four-node shell elements



Design model. Model with eight-node shell elements



Buckling mode. Model with four-node shell elements



Buckling mode. Model with eight-node shell elements

Comparison of solutions:

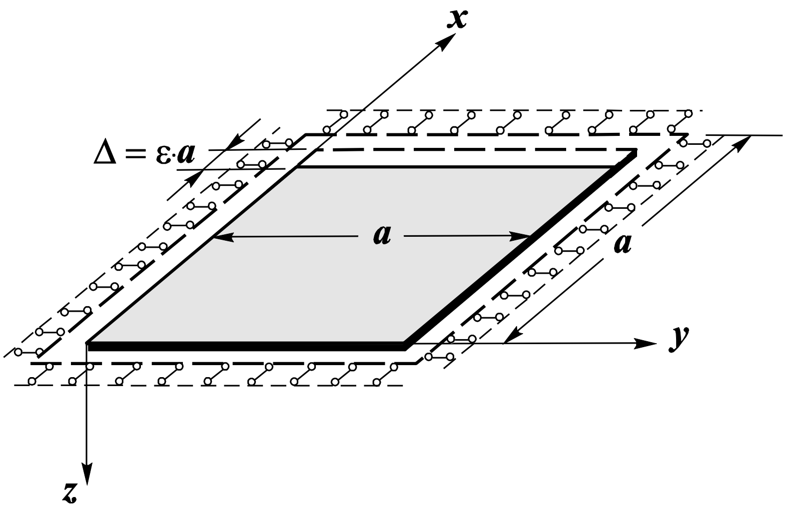
Critical value of the compressive forces σ_{cr}, kN/m²

Design model	Theory	SCAD	Deviation, %
Member type 44 n = 4 nodes	2776	2.248923·100/0.08 = = 2811	1.26
Member type 50 n = 8 nodes		2.220676·100/0.08 = = 2776	0.00

Notes: In the analytical solution the critical value of the compressive forces σ_{cr}, corresponding to the moment of buckling of the square plate can be determined according to the following formula:

$$\sigma_{cr} = \frac{4 \cdot \pi^2 \cdot D}{(1+\nu) \cdot a^2 \cdot h}, \text{ where: } \quad D = \frac{E \cdot h^3}{12 \cdot (1-\nu^2)}.$$

Stability of a Simply Supported Square Plate Uniformly Compressed in One Direction under Kinematic Action



Objective: Determination of the critical value of the approach of two opposite sides of a simply supported square plate corresponding to the moment of its buckling.

Initial data files:

File name	Description
6.8_n_4.SPR	Design model with four-node shell elements of type 44
6.8_n_8.SPR	Design model with eight-node shell elements of type 50

Problem formulation: The square plate is subjected to the action of the approach Δ of two opposite roller supported sides. Two other opposite sides of the plate free from actions are pinned. Determine the critical value of the approach Δ_{cr}, corresponding to the moment of buckling of the square plate.

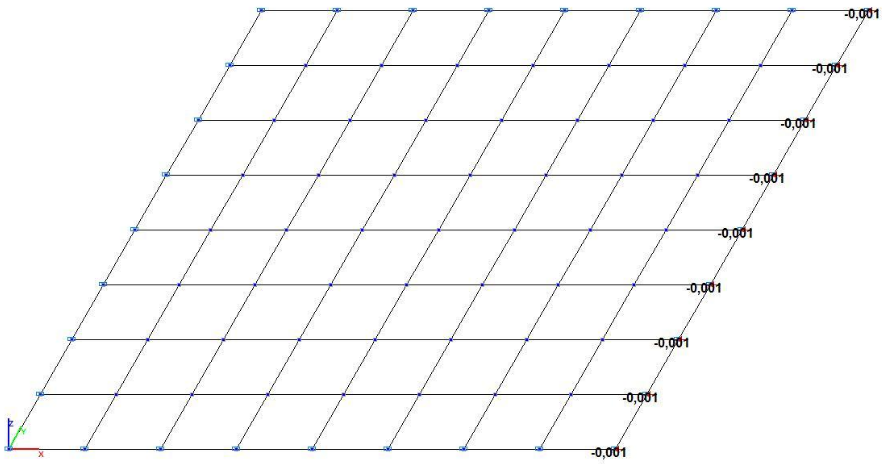
References: J.H. Argyris, P.C. Dunne, G.A. Malejannakis, E. Schelkle, A simple triangular facet shell element with applications to linear and non-linear equilibrium and elastic stability problems, Computer methods in applied mechanics and engineering, 11 (1977), p. 97-131.
S.P. Timoshenko, J.M. Gere, Theory of elastic stability, McGraw-Hill, New York, 1963, p. 356.

Initial data:

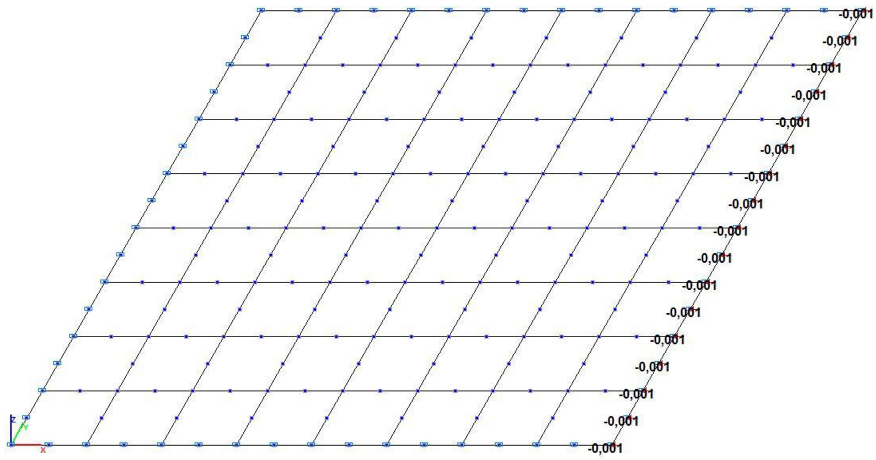
a = 8.0 m	- side of the square plate;
h = 0.08 m	- thickness of the square plate;
$E = 1.0\cdot10^7$ kN/m^2	- elastic modulus of the square plate material;
ν = 1/3	- Poisson's ratio;
$\Delta = 1.0\cdot10^{-3}$ m	- initial value of the approach.

Finite element model: Design model – general type system. Two design models with four-node shell elements of type 44 and eight-node shell elements of type 50 are considered. The spacing of the finite element mesh along the sides of the plate (along the X and Y axes of the global coordinate system) is 1.0 m. Number of elements in the models – 64. Boundary conditions are provided by imposing constraints on the nodes of the support contour of the plate in the direction of the degree of freedom Z, and by imposing constraints in the normal direction along the Y axis of the global coordinate system on the nodes of one of the two opposite sides of the plate free from actions. Constraints in the respective direction (along the X axis of the global coordinate system) are imposed on the nodes of two opposite sides of the plate subjected to the kinematic action. The action is specified by the displacement of the constraints of one of these sides with the initial value $\Delta = 1.0\cdot10^{-3}$ m. The dimensional stability of the design model is provided by imposing a constraint in the UZ direction of the global coordinate system on the node of the support contour of the plate. Number of nodes in the models – 81; 225.

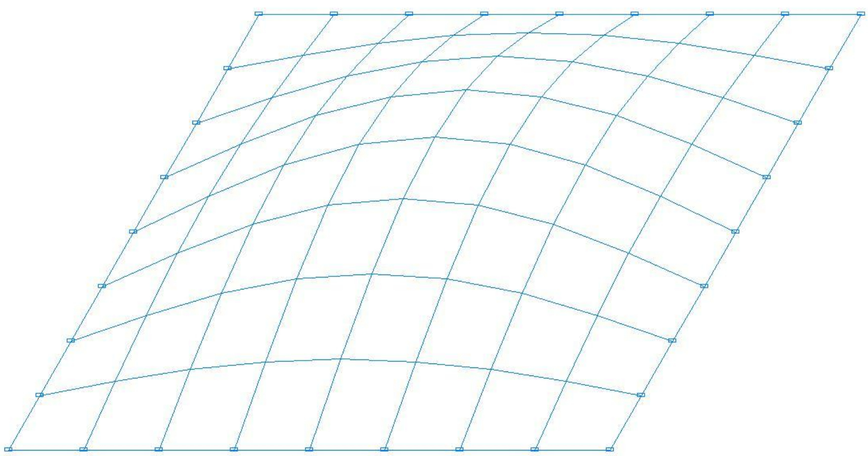
Results in SCAD



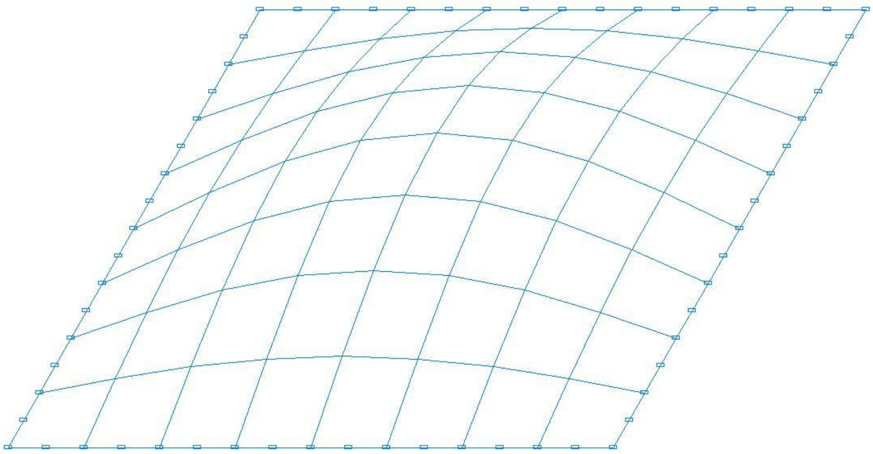
Design model. Model with four-node shell elements



Design model. Model with eight-node shell elements



Buckling mode. Model with four-node shell elements



Buckling mode. Model with eight-node shell elements

Comparison of solutions:

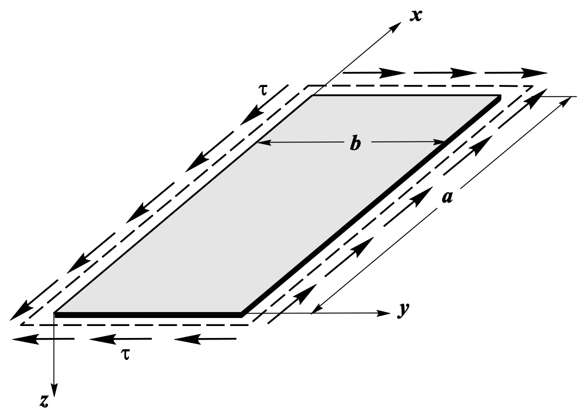
Critical value of the compressive forces σ_{cr}, kN/m^2

Design model	Theory	SCAD	Deviation, %
Member type 44 n = 4 nodes	$1.974\cdot10^{-3}$	$1.999043\cdot1.0\cdot10^{-3}$ = = $1.999\cdot10^{-3}$	1.27
Member type 50 n = 8 nodes		$1.973935\cdot1.0\cdot10^{-3}$ = = $1.974\cdot10^{-3}$	0.00

Notes: In the analytical solution the critical value of the approach Δ_{cr} of two opposite sides of the simply supported square plate corresponding to the moment of its buckling can be determined according to the following formula:

$$\Delta_{cr} = \frac{\pi^2 \cdot h^2}{3 \cdot (1+\nu) \cdot a}.$$

Stability of a Rectangular Simply Supported Plate under Pure Shear



Objective: Determination of the critical value of the shear forces uniformly distributed along two opposite sides of a simply supported rectangular plate corresponding to the moment of its buckling.

Initial data files:

File name	Description
6.9_a_8_n_4.SPR	Design model with the ratios of the sides of the plate a/b = 1.0 5 from four-node shell elements of type 44
6.9_a_8_n_8.SPR	Design model with the ratios of the sides of the plate a/b = 1.0 from eight- node shell elements of type 50
6.9_a_16_n_4.SPR	Design model with the ratios of the sides of the plate a/b = 2.0 5 from four-node shell elements of type 44
6.9_a_16_n_8.SPR	Design model with the ratios of the sides of the plate a/b = 2.0 from eight- node shell elements of type 50

Problem formulation: The simply supported rectangular plate is subjected to the action of shear forces τ, uniformly distributed along two opposite sides. Determine the critical value of the shear forces τ_{cr}, corresponding to the moment of buckling of the rectangular plate.

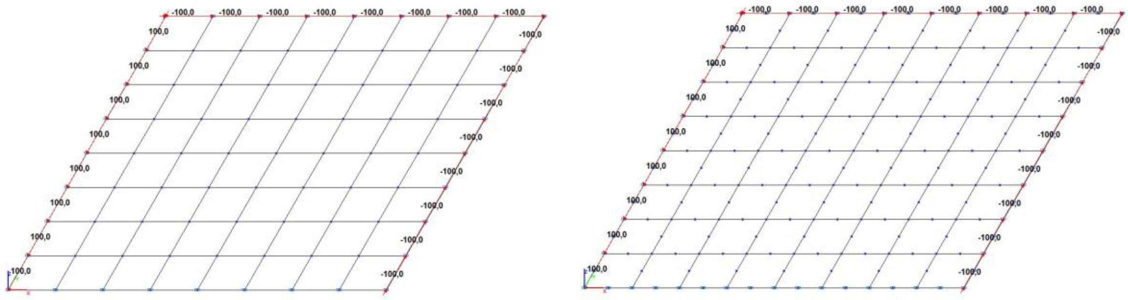
References: S. P. Timoshenko, Stability of Bars, Plates and Shells, Moscow, Nauka, 1971, p. 626.
A.S. Volmir. Stability of Deformable Systems, Moscow, Nauka, 1967, p. 344.

Initial data:

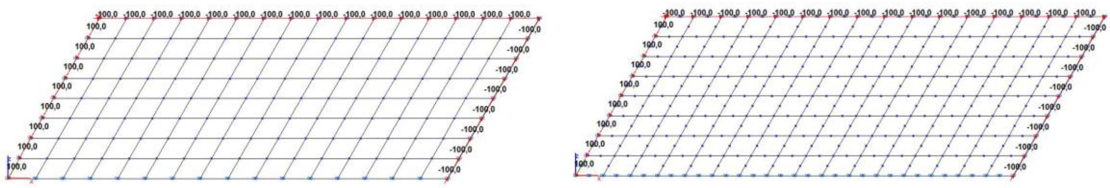
a = 8.0; 16.0 m	- side of the rectangular plate along the X axis of the global coordinate system;
b = 8.0 m	- side of the rectangular plate along the Y axis of the global coordinate system;
h = 0.08 m	- thickness of the rectangular plate;
$E = 1.0 \cdot 10^7$ kN/m^2	- elastic modulus of the rectangular plate material;
$\nu = 1/3$	- Poisson's ratio;
$\sigma = 1.25 \cdot 10^3$ kN/m^2	- initial value of the shear forces.

Finite element model: Design model – general type system. Two design models with four-node shell elements of type 44 and eight-node shell elements of type 50 are considered for two cases with the ratios of the sides of the plate a/b = 1.0; 2.0. The spacing of the finite element mesh along the sides of the plate (along the X and Y axes of the global coordinate system) is 1.0 m. Number of elements in the models – 64; 128. Boundary conditions are provided by imposing constraints on the nodes of the support contour of the plate in the direction of the degree of freedom Z. A load uniformly distributed along the line with the initial value $p = -\tau \cdot h = -100$ kN/m is specified on one of the two opposite sides of the plate parallel to the X axis of the global coordinate system, and the constraints in the directions of the degrees of freedom X and Y are imposed on the nodes of the other one (lying on the X axis). A load uniformly distributed along the line with the initial value $p = -\tau \cdot h = -100$ kN/m is specified on one of the two opposite sides of the plate parallel to the Y axis of the global coordinate system, and a load uniformly distributed along the line with the initial value $p = \tau \cdot h = 100$ kN/m is specified on the other one (lying on the Y axis). The dimensional stability of the design model is provided by imposing a constraint in the UZ direction of the global coordinate system on the node of one of the corners of the plate. Number of nodes in the models – 81 (225); 153 (433).

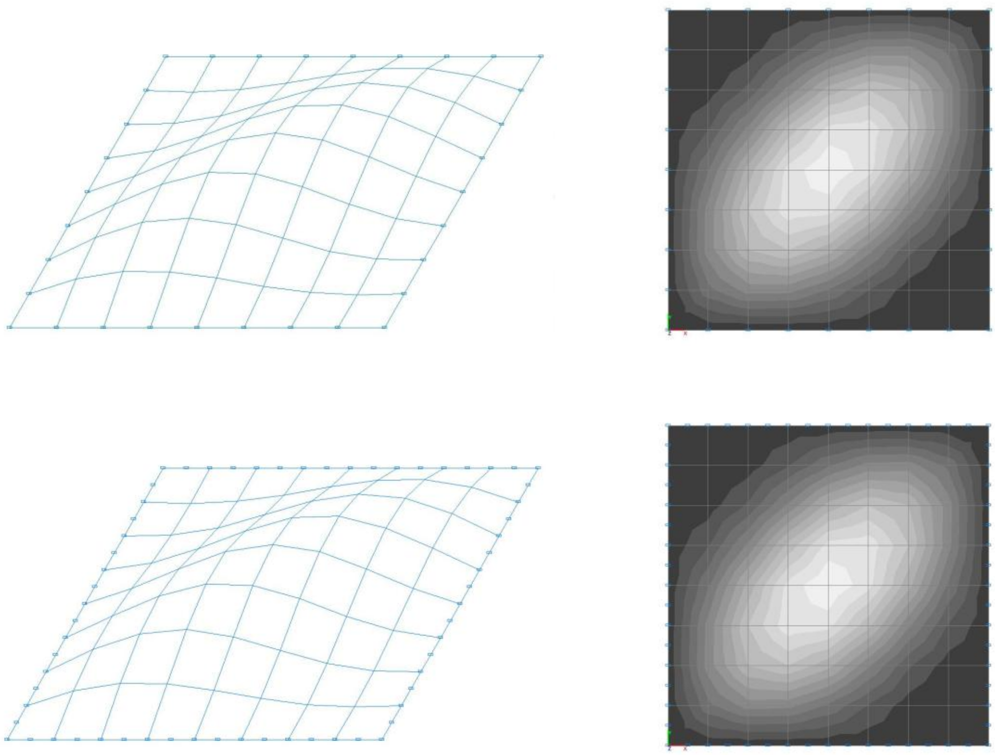
Results in SCAD



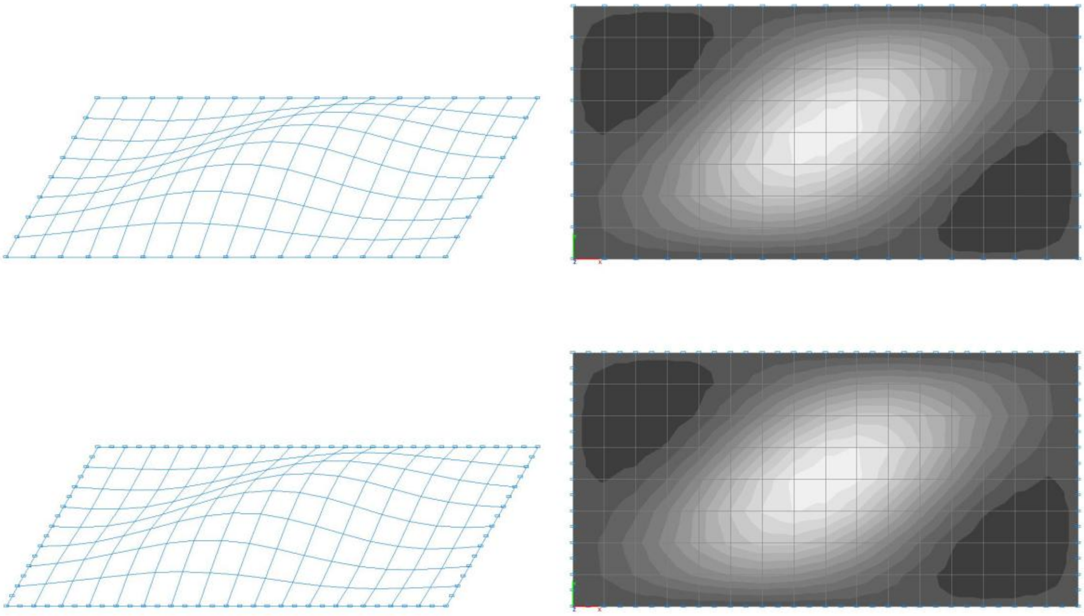
Design models with the ratio of the sides of the plate a/b = 1.0



Design models with the ratio of the sides of the plate a/b = 2.0



Buckling modes for the design models with the ratio of the sides of the plate a/b = 1.0



Buckling modes for the design models with the ratio of the sides of the plate a/b = 2.0

Comparison of solutions:

Critical value of the shear forces τ_{cr}, kN/m^2

Plate sides ratio	Design model	Theory	SCAD	Deviation, %
a/b = 1.0	Member type 44 n = 4 nodes	8631	7.129409·100/0.08 = = 8912	3.26
	Member type 50 n = 8 nodes		6.903095·100/0.08 = = 8629	0.02
a/b = 2.0	Member type 44 n = 4 nodes	6060	4.930113·100/0.08 = = 6163	1.70
	Member type 50 n = 8 nodes		4.845765·100/0.08 = = 6057	0.05

Notes: In the analytical solution the critical value of the shear forces τ_{cr}, corresponding to the moment of buckling of the rectangular plate can be determined according to the following formula:

$$\sigma_{cr} = k \cdot \frac{\pi^2 \cdot D}{b^2 \cdot h}, \text{ where:}$$

$$D = \frac{E \cdot h^3}{12 \cdot \left(1 - \nu^2\right)}, \qquad k = \frac{\pi^2}{32 \cdot \alpha^3 \cdot \lambda}, \qquad \alpha = \frac{a}{b}.$$

Parameter λ is determined on the basis of the condition of equality to zero of the determinant of the system of equations:

$$\lambda \cdot \left(m^2 + \alpha^2 \cdot n^2\right) \cdot A_{mn} - \sum_i \sum_j \frac{m \cdot n \cdot i \cdot j}{\left(m^2 - i^2\right) \cdot \left(n^2 - j^2\right)} \cdot A_{ij} = 0,$$

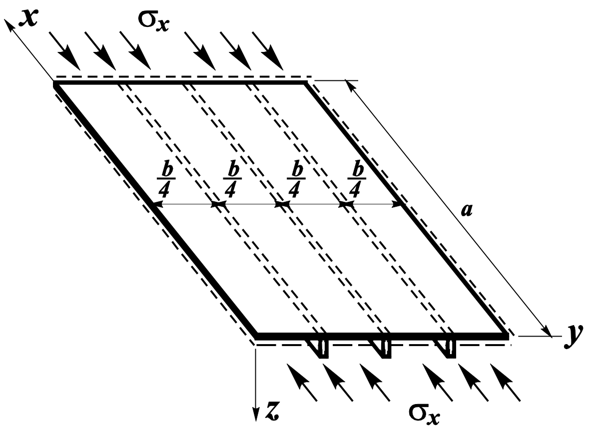
with the following combinations of indices:

$$m+i \quad odd$$
$$n+j \quad odd$$
$$m+n \quad even.$$

At m, n, i, j = 1, 2, 3, 4, 5, 6, 7, 8 (the determinant dimension is 32·32) we have:

$$\frac{a}{b}=1.0 \quad k=9.328$$
$$\frac{a}{b}=2.0 \quad k=6.549.$$

Stability of a Rectangular Simply Supported Plate with Longitudinal Stiffeners Uniformly Compressed in the Longitudinal Direction (Model 1)



Objective: Determination of the critical value of the compressive forces uniformly distributed along two opposite transverse sides of a rectangular simply supported plate reinforced by longitudinal stiffeners corresponding to the moment of its buckling.

Initial data files:

File name	Description
6.10_shell_beam_lambda_1.SPR	Design model with the ratios of the sides of the plate a/b = 1.0
6.10_shell_beam_lambda_4.SPR	Design model with the ratios of the sides of the plate a/b = 4.0

Problem formulation: The rectangular simply supported plate reinforced by longitudinal stiffeners is subjected to the action of compressive forces σ, uniformly distributed along two opposite transverse sides. Determine the critical value of the compressive forces σ_{cr}, corresponding to the moment of buckling of the rectangular reinforced plate taking into account the following assumptions made when deriving the analytical solution:

- The stiffeners are symmetric with respect to the midplane of the reinforced plate;
- Torsional stiffness of the stiffeners is not taken into account;
- The stiffeners and the plate are subjected to the uniform compression.

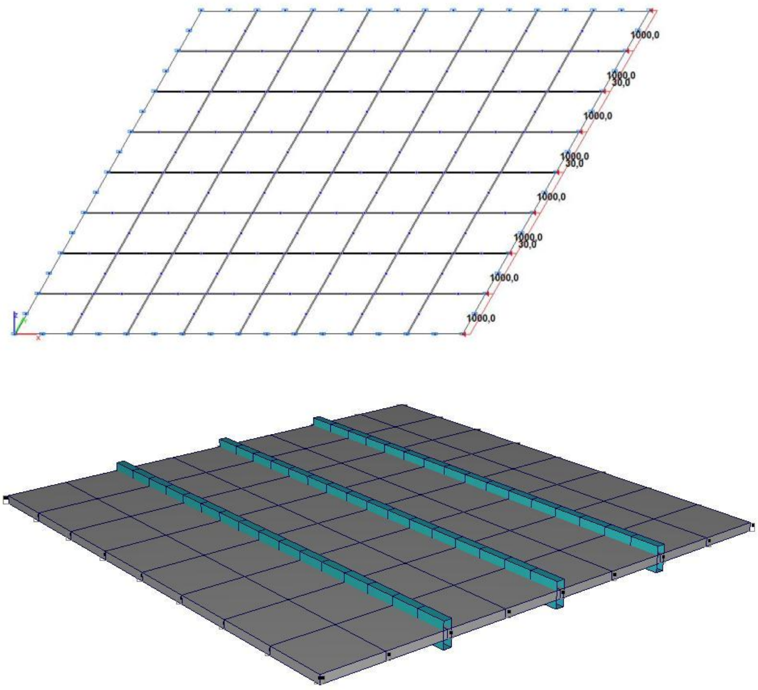
References: S. P. Timoshenko, Stability of Bars, Plates and Shells, Moscow, Nauka, 1971, p. 507.
A.S. Volmir. Stability of Deformable Systems, Moscow, Nauka, 1967, p. 377.

Initial data:

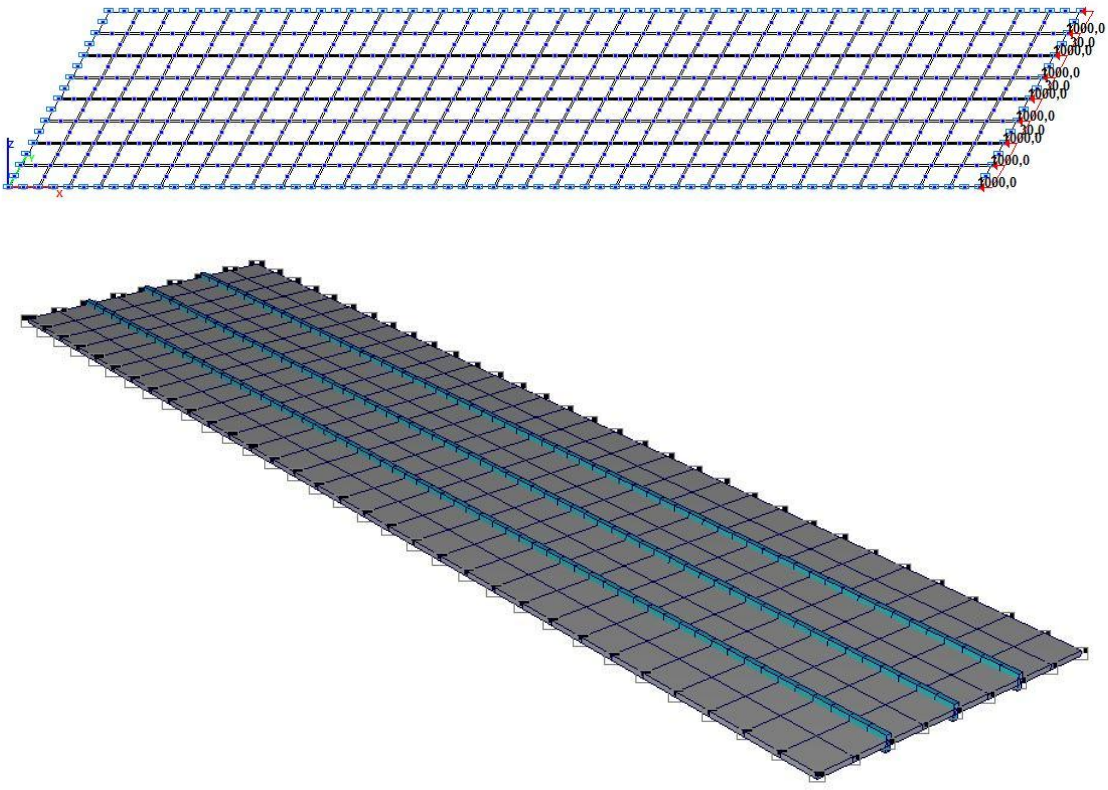
a = 0.6; 2.4 m	- side of the rectangular plate free from forces (along the X axis of the global coordinate system);
b = 0.6 m	- side of the rectangular plate subjected to the compressive forces (along the Y axis of the global coordinate system);
h = 0.01 m	- thickness of the rectangular plate;
$F = 0.01\cdot0.03 = 3\cdot10^{-4}$ m^2	- cross-sectional area of the stiffeners;
$I = 0.01\cdot0.03^3/12 = 2.25\cdot10^{-8}$ m^4	- cross-sectional moment of inertia of the stiffeners;
s = 3	- number of stiffeners arranged uniformly along the width of the plate;
$E = 2.0\cdot10^8$ kN/m^2	- elastic modulus of the material of the plate and stiffeners;
ν = 0.3	- Poisson's ratio;
$\sigma = 1.0\cdot10^5$ kN/m^2	- initial value of the compressive forces.

Finite element model: Design model – general type system. Two design models with the ratios of the sides of the plate a/b = 1.0; 4.0 are considered. The plate is modeled by eight-node shell elements of type 50. The spacing of the finite element mesh along the sides of the plate (along the X and Y axes of the global coordinate system) is 0.075 m. Number of plate elements in the models – 64; 256. The stiffeners are modeled by spatial bar elements of type 5. The spacing of the finite element mesh along the longitudinal axes of the stiffeners (along the X1 axes of the local coordinate systems) is 0.0375 m. Number of stiffener elements in the models – 48; 192. Boundary conditions are provided by imposing constraints on the nodes of the support contour of the plate in the direction of the degree of freedom Z. The load uniformly distributed along the line on the plate with the initial value p = σ·h = 1000 kN/m and nodal loads on the stiffeners with the initial value P = σ·F = 30 kN are specified on one of the two opposite transverse sides of the plate subjected to the compressive forces, and constraints in the respective direction (along the X axis of the global coordinate system) are imposed on the nodes of the other one. The dimensional stability of the design model is provided by imposing constraints in the normal direction (along the Y axis of the global coordinate system) on the nodes of one of the two opposite longitudinal sides of the plate free from forces, and by imposing a constraint in the UZ direction of the global coordinate system on the node of one of the corners of the plate. Number of nodes in the models – 225; 849.

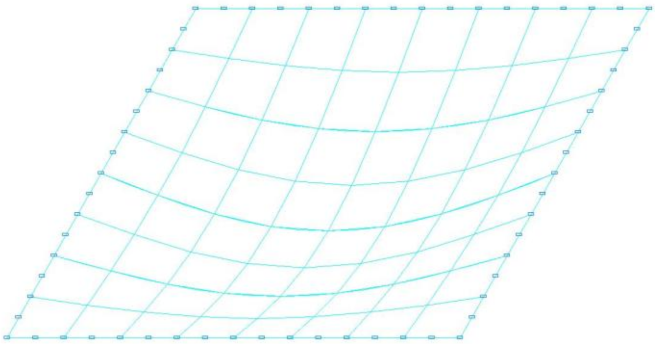
Results in SCAD



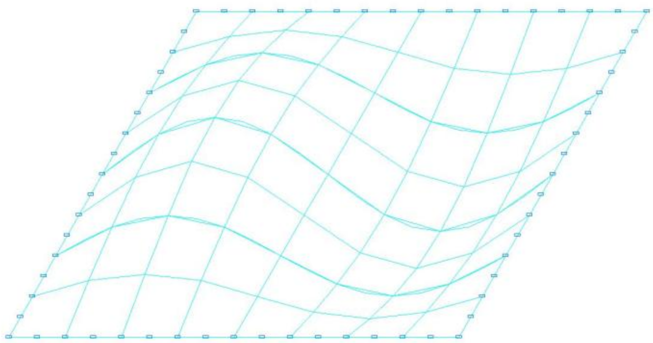
Design model with the ratio of the sides of the plate a/b = 1.0



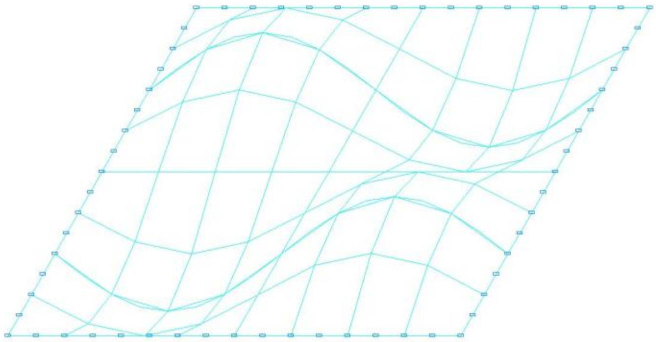
Design model with the ratio of the sides of the plate a/b = 4.0



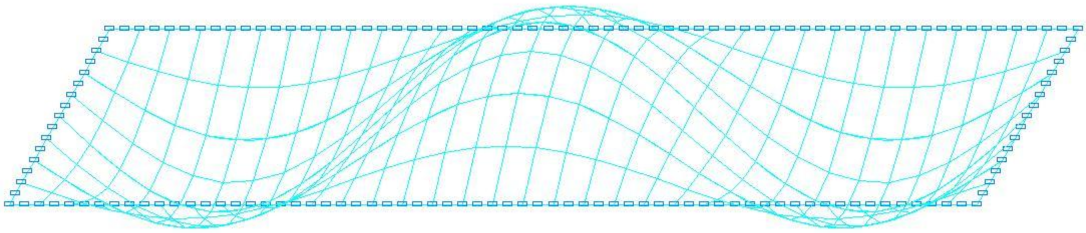
1-st buckling mode for the design model with the ratio of the sides of the plate a/b = 1.0



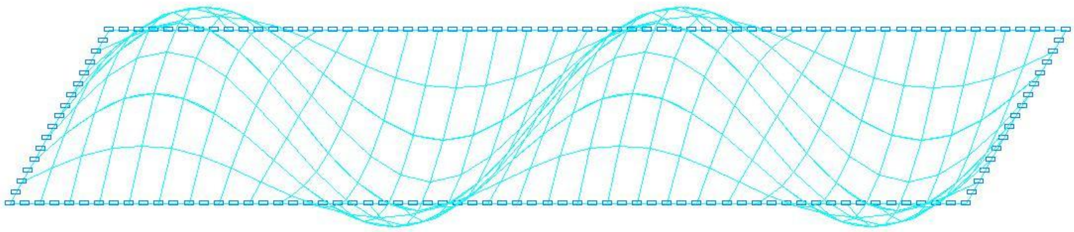
2-nd buckling mode for the design model with the ratio of the sides of the plate a/b = 1.0



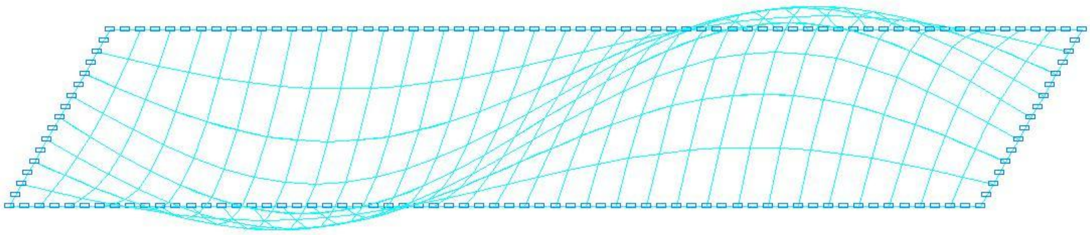
3-rd buckling mode for the design model with the ratio of the sides of the plate a/b = 1.0



1-st buckling mode for the design model with the ratio of the sides of the plate a/b = 4.0



2-nd buckling mode for the design model with the ratio of the sides of the plate a/b = 4.0



3-rd buckling mode for the design model with the ratio of the sides of the plate a/b = 4.0

Comparison of solutions:

Critical value of the compressive forces σ_{cr}, kN/m^2

Plate sides ratio	Buckling mode	Number of half waves in the transverse n and in the longitudinal m directions	Theory	SCAD	Deviation, %
a/b = 1.0	1	1; 1	235900 (235911)	2.359001·1000/0.01 = = 235900	0.00
	2	1; 2	533934 (535675)	5.339341·1000/0.01 = = 533934	0.01
	3	2; 2	942681 (943645)	9.426809·1000/0.01 = = 942681	0.10
a/b = 4.0	1	1; 3	220165 (220164)	2.201645·1000/0.01 = = 220165	0.00

Plate sides ratio	Buckling mode	Number of half waves in the transverse n and in the longitudinal m directions	Theory	SCAD	Deviation, %
	2	1; 4	235900 (235911)	2.359002·1000/0.01 = = 235900	0.00
	3	1; 2	278652 (278654)	2.786517·1000/0.01 = = 278652	0.00

Theoretical values calculated in the fourth approximation are given without brackets;
Theoretical values calculated in the first approximation are given in brackets

Notes: In the analytical solution the critical value of the compressive forces σ_{cr1} in the first approximation corresponding to the moment of buckling of the rectangular reinforced plate can be determined according to the following formula:

$$\sigma_{cr1} = \frac{\pi^2 \cdot D \cdot m^2}{b^2 \cdot h \cdot \lambda^2} \cdot \frac{\left[1 + \left(\frac{n \cdot \lambda}{m}\right)^2\right]^2 + 2 \cdot \gamma \cdot \sum_{i=1}^{s} sin^2\left(\frac{\pi \cdot i}{s+1}\right)}{1 + 2 \cdot \delta \cdot \sum_{i=1}^{s} sin^2\left(\frac{\pi \cdot i}{s+1}\right)},$$

at s =3 $$\sigma_{cr1} = \frac{\pi^2 \cdot D \cdot m^2}{b^2 \cdot h \cdot \lambda^2} \cdot \frac{\left[1 + \left(\frac{n \cdot \lambda}{m}\right)^2\right]^2 + 4 \cdot \gamma}{1 + 4 \cdot \delta},$$ where:

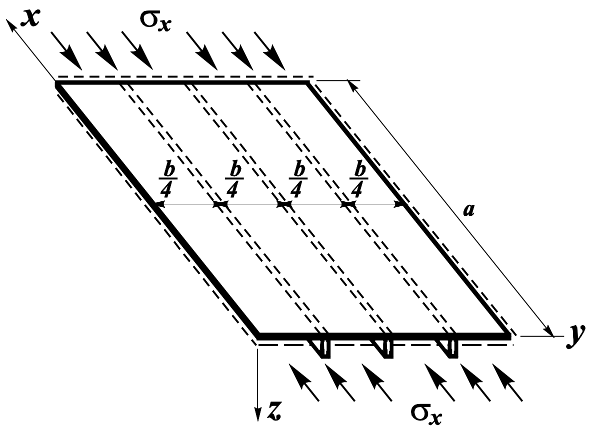
$$D = \frac{E \cdot h^3}{12 \cdot \left(1 - \nu^2\right)}, \quad \lambda = \frac{a}{b}, \quad \gamma = \frac{E \cdot I}{b \cdot D}, \quad \delta = \frac{F}{b \cdot h},$$

n, m = 1, 2, 3 … – number of half waves of the buckling mode in the transverse and longitudinal directions with respect to the compression of the plate.

In the analytical solution the critical value of the compressive forces σ_{cr1} in the fourth approximation corresponding to the moment of buckling of the rectangular reinforced plate is determined on the basis of the condition of equality to zero of the determinant of the system of governing equations:

$$\begin{vmatrix}
\frac{\pi^2 \cdot D \cdot m^2}{b^2 \cdot h \cdot \lambda^2} \cdot \left[\left[1 + \left(\frac{n \cdot \lambda}{m}\right)^2\right]^2 + 4 \cdot \gamma\right] - \sigma_{cr4} \cdot (1 + 4 \cdot \delta) & -\frac{\pi^2 \cdot D \cdot m^2}{b^2 \cdot h \cdot \lambda^2} \cdot 4 \cdot \gamma + \sigma_{cr4} \cdot 4 \cdot \delta & \frac{\pi^2 \cdot D \cdot m^2}{b^2 \cdot h \cdot \lambda^2} \cdot 4 \cdot \gamma - \sigma_{cr4} \cdot 4 \cdot \delta & -\frac{\pi^2 \cdot D \cdot m^2}{b^2 \cdot h \cdot \lambda^2} \cdot 4 \cdot \gamma + \sigma_{cr4} \cdot 4 \cdot \delta \\
-\frac{\pi^2 \cdot D \cdot m^2}{b^2 \cdot h \cdot \lambda^2} \cdot 4 \cdot \gamma + \sigma_{cr4} \cdot 4 \cdot \delta & \frac{\pi^2 \cdot D \cdot m^2}{b^2 \cdot h \cdot \lambda^2} \cdot \left[\left[1 + 49 \cdot \left(\frac{n \cdot \lambda}{m}\right)^2\right]^2 + 4 \cdot \gamma\right] - \sigma_{cr4} \cdot (1 + 4 \cdot \delta) & -\frac{\pi^2 \cdot D \cdot m^2}{b^2 \cdot h \cdot \lambda^2} \cdot 4 \cdot \gamma + \sigma_{cr4} \cdot 4 \cdot \delta & \frac{\pi^2 \cdot D \cdot m^2}{b^2 \cdot h \cdot \lambda^2} \cdot 4 \cdot \gamma - \sigma_{cr4} \cdot 4 \cdot \delta \\
\frac{\pi^2 \cdot D \cdot m^2}{b^2 \cdot h \cdot \lambda^2} \cdot 4 \cdot \gamma - \sigma_{cr4} \cdot 4 \cdot \delta & -\frac{\pi^2 \cdot D \cdot m^2}{b^2 \cdot h \cdot \lambda^2} \cdot 4 \cdot \gamma + \sigma_{cr4} \cdot 4 \cdot \delta & \frac{\pi^2 \cdot D \cdot m^2}{b^2 \cdot h \cdot \lambda^2} \cdot \left[\left[1 + 81 \cdot \left(\frac{n \cdot \lambda}{m}\right)^2\right]^2 + 4 \cdot \gamma\right] - \sigma_{cr4} \cdot (1 + 4 \cdot \delta) & -\frac{\pi^2 \cdot D \cdot m^2}{b^2 \cdot h \cdot \lambda^2} \cdot 4 \cdot \gamma + \sigma_{cr4} \cdot 4 \cdot \delta \\
-\frac{\pi^2 \cdot D \cdot m^2}{b^2 \cdot h \cdot \lambda^2} \cdot 4 \cdot \gamma + \sigma_{cr4} \cdot 4 \cdot \delta & \frac{\pi^2 \cdot D \cdot m^2}{b^2 \cdot h \cdot \lambda^2} \cdot 4 \cdot \gamma - \sigma_{cr4} \cdot 4 \cdot \delta & -\frac{\pi^2 \cdot D \cdot m^2}{b^2 \cdot h \cdot \lambda^2} \cdot 4 \cdot \gamma + \sigma_{cr4} \cdot 4 \cdot \delta & \frac{\pi^2 \cdot D \cdot m^2}{b^2 \cdot h \cdot \lambda^2} \cdot \left[\left[1 + 225 \cdot \left(\frac{n \cdot \lambda}{m}\right)^2\right]^2 + 4 \cdot \gamma\right] - \sigma_{cr4} \cdot (1 + 4 \cdot \delta)
\end{vmatrix} = 0$$

Stability of a Rectangular Simply Supported Plate with Longitudinal Stiffeners Uniformly Compressed in the Longitudinal Direction (Model 2)



Objective: Determination of the critical value of the compressive forces uniformly distributed along two opposite transverse sides of a rectangular simply supported plate reinforced by longitudinal stiffeners corresponding to the moment of its buckling.

Initial data files:

File name	Description
6.10_shell_shell_lambda_1.SPR	Design model with the ratios of the sides of the plate a/b = 1.0
6.10_shell_shell_lambda_4.SPR	Design model with the ratios of the sides of the plate a/b = 4.0

Problem formulation: The rectangular simply supported plate reinforced by longitudinal stiffeners is subjected to the action of compressive forces σ, uniformly distributed along two opposite transverse sides. Determine the critical value of the compressive forces σ_{cr}, corresponding to the moment of buckling of the rectangular reinforced plate taking into account the following assumptions made when deriving the analytical solution:

- The stiffeners are symmetric with respect to the midplane of the reinforced plate;
- Torsional stiffness of the stiffeners is not taken into account
- The stiffeners and the plate are subjected to the uniform compression

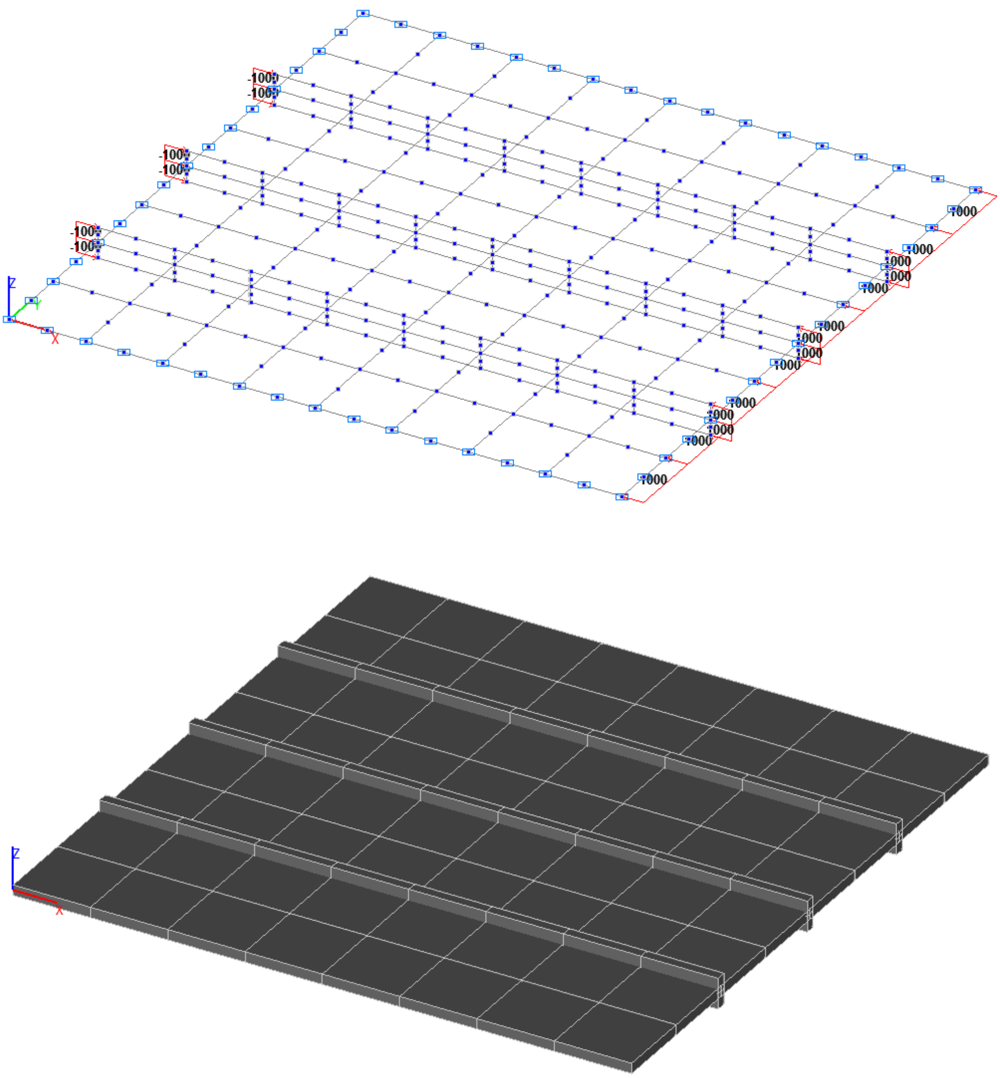
References: S. P. Timoshenko, Stability of Bars, Plates and Shells, Moscow, Nauka 1971, p. 507.
A.S. Volmir. Stability of Deformable Systems, Moscow, Nauka, 1967, p. 377.

Initial data:

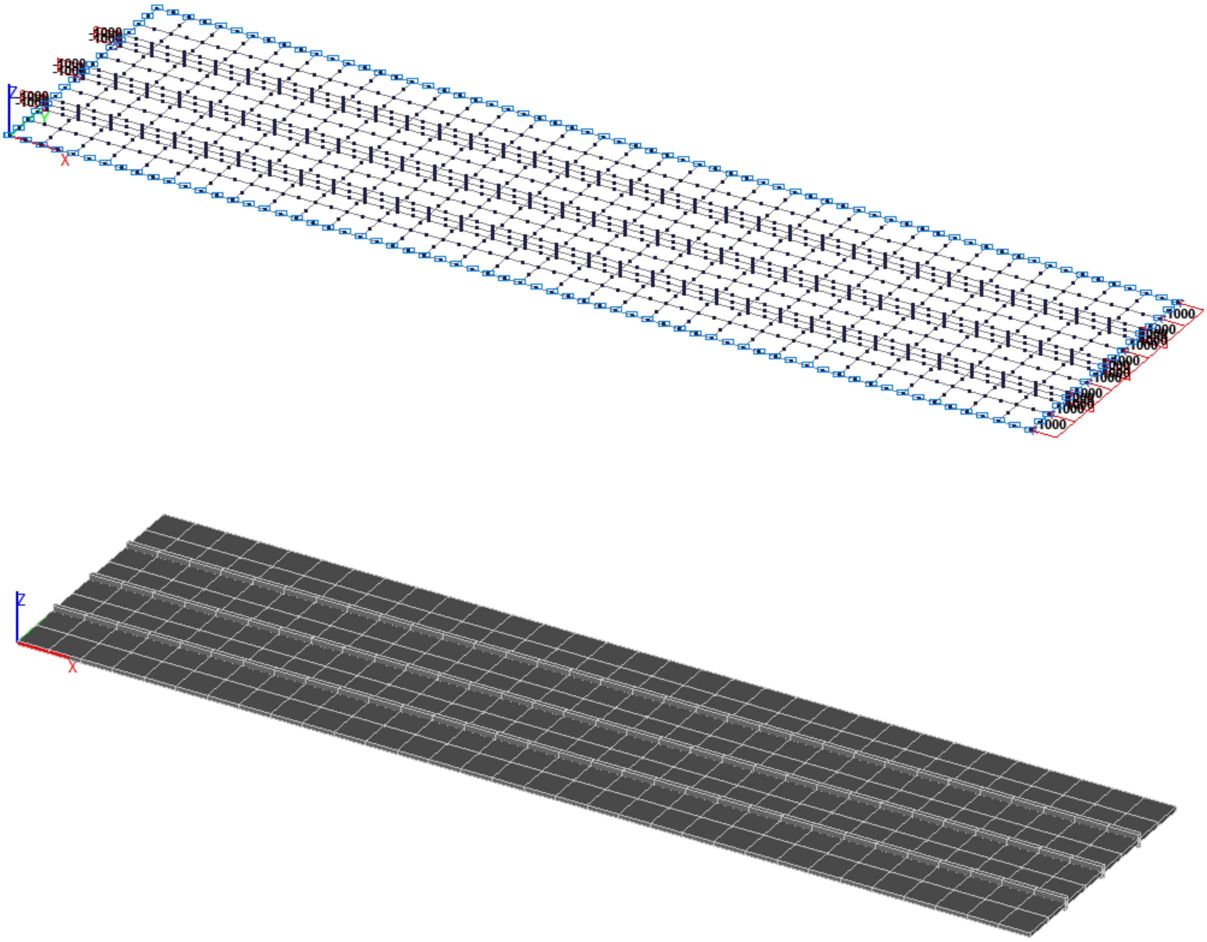
a = 0.6; 2.4 m	- side of the rectangular plate free from forces (along the X axis of the global coordinate system)
b = 0.6 m	- side of the rectangular plate subjected to the compressive forces (along the Y axis of the global coordinate system);
h = 0.01 m	- thickness of the rectangular plate;
$F = 0.01 \cdot 0.03 = 3 \cdot 10^{-4}$ m²	- cross-sectional area of the stiffeners;
$I = 0.01 \cdot 0.03^3/12 = 2.25 \cdot 10^{-8}$ m⁴	- cross-sectional moment of inertia of the stiffeners;
s = 3	- number of stiffeners arranged uniformly along the width of the plate;
$E = 2.0 \cdot 10^8$ kN/m²	- elastic modulus of the material of the plate and stiffeners;
ν = 0.3	- Poisson's ratio;
$\sigma = 1.0 \cdot 10^5$ kN/m²	- initial value of the compressive forces.

Finite element model: Design model – general type system. Two design models with the ratios of the sides of the plate a/b = 1.0; 4.0 are considered. The plate is modeled by eight-node shell elements of type 50. The spacing of the finite element mesh along the sides of the plate (along the X and Y axes of the global coordinate system) is 0.075 m. Number of plate elements in the models – 64; 256. The stiffeners are modeled by eight-node shell elements of type 50. The spacing of the finite element mesh along the longitudinal axes of the stiffeners (along the X1 axes of the local coordinate systems) is 0.075 m, and along the height of the stiffeners (along the Y1 axes of the local coordinate systems) is 0.015 m. Number of stiffener elements in the models – 48; 192. Boundary conditions are provided by imposing constraints on the nodes of the support contour of the plate in the direction of the degree of freedom Z. The load uniformly distributed along the line on the plate and on the stiffeners with the initial value p = σ·h = 1000 kN/m is specified on one of the two opposite transverse sides of the plate subjected to the compressive forces, and constraints in the respective direction on the plate (along the X axis of the global coordinate system) are imposed on the nodes of the other one, and the load uniformly distributed along the line on the stiffeners with the initial value p = σ·h = 1000 kN/m is specified on it. The dimensional stability of the design model is provided by imposing constraints in the normal direction (along the Y axis of the global coordinate system) on the nodes of one of the two opposite longitudinal sides of the plate free from forces, and by imposing a constraint in the UZ direction of the global coordinate system on the node of one of the corners of the plate. Number of nodes in the models – 381; 1437.

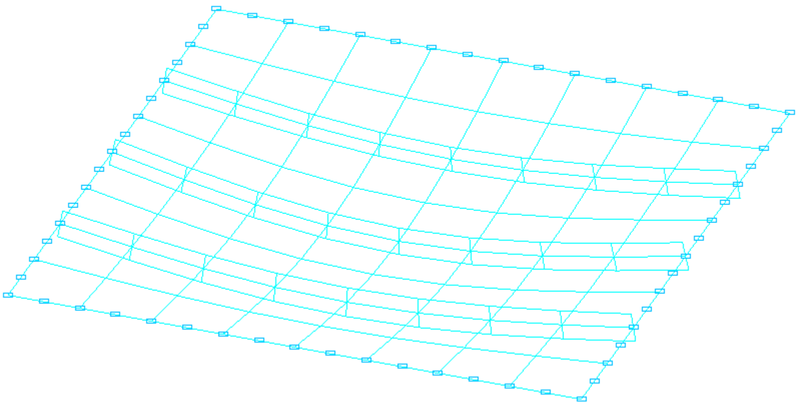
Results in SCAD



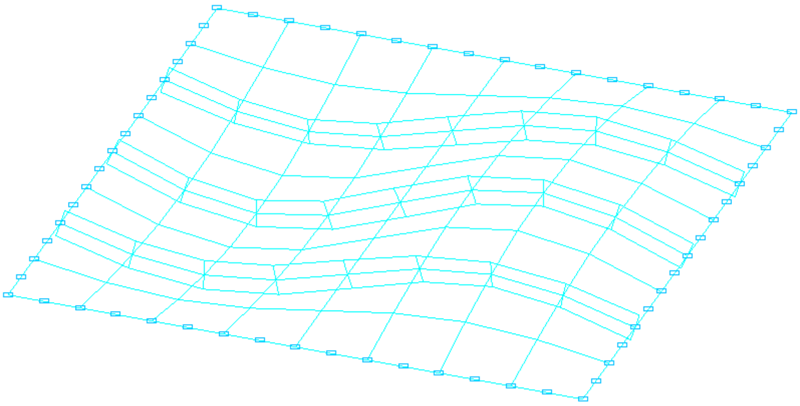
Design model with the ratio of the sides of the plate a/b = 1.0



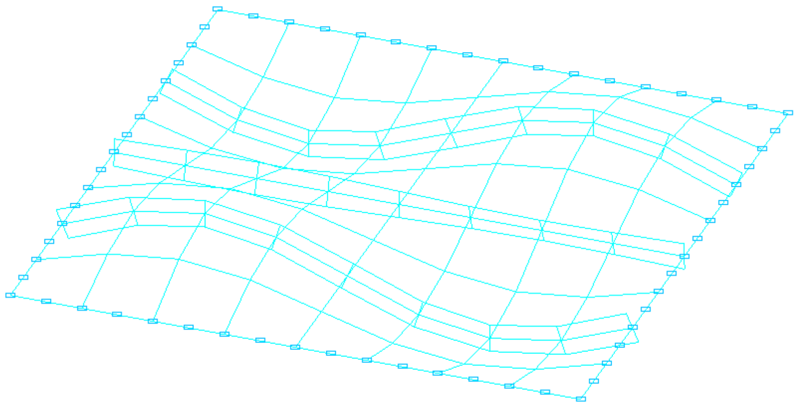
Design model with the ratio of the sides of the plate a/b = 4.0



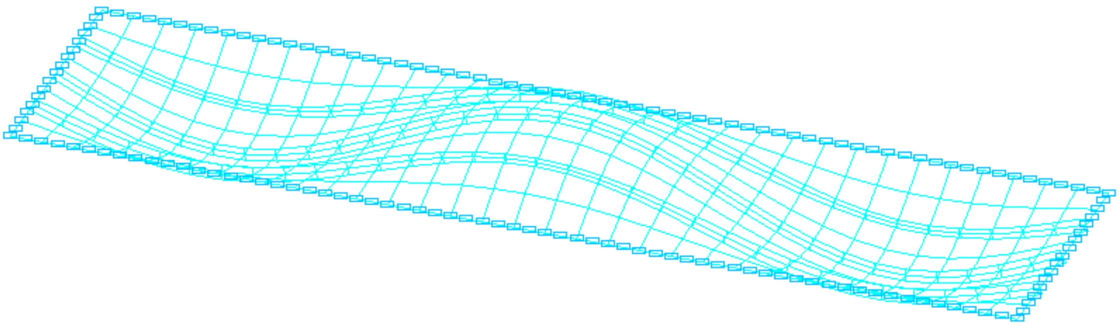
1-st buckling mode for the design model with the ratio of the sides of the plate a/b = 1.0



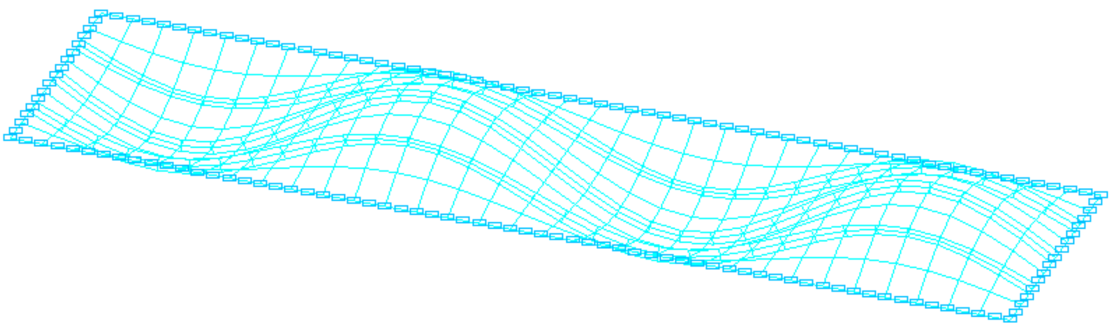
2-nd buckling mode for the design model with the ratio of the sides of the plate a/b = 1.0



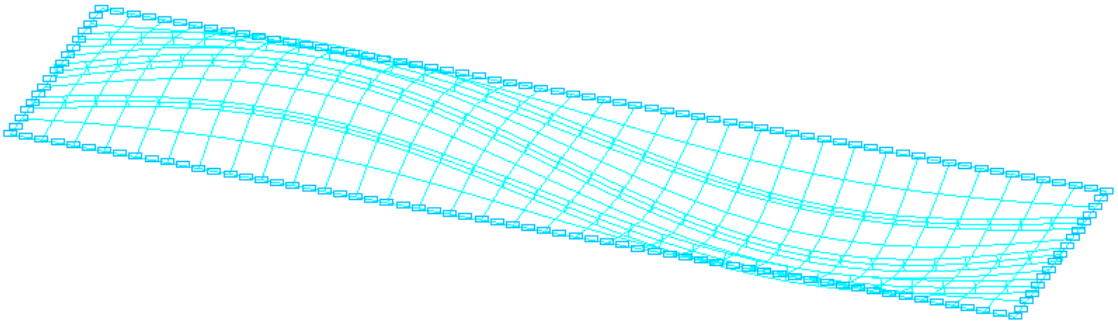
3-rd buckling mode for the design model with the ratio of the sides of the plate a/b = 1.0



1-st buckling mode for the design model with the ratio of the sides of the plate a/b = 4.0



2-nd buckling mode for the design model with the ratio of the sides of the plate a/b = 4.0



3-rd buckling mode for the design model with the ratio of the sides of the plate a/b = 4.0

Comparison of solutions:

Critical value of the compressive forces σ_{cr}, kN/m^2

Plate sides ratio	Buckling mode	Number of half waves in the transverse n and in the longitudinal m directions	Theory	SCAD	Deviation, %
a/b = 1.0	1	1; 1	235900 (235911)	2.410318·1000/0.01 = = 241032	2.18
	2	1; 2	533934 (535675)	5.369516·1000/0.01 = = 536952	0.57
	3	2; 2	942681 (943645)	9.604025·1000/0.01 = = 960403	1.88
a/b = 4.0	1	1; 3	220165 (220164)	2.257856·1000/0.01 = = 225786	2.55
	2	1; 4	235900 (235911)	2.414278·1000/0.01 = = 241428	2.34
	3	1; 2	278652 (278654)	2.842984·1000/0.01 = = 284298	2.03

Theoretical values calculated in the fourth approximation are given without brackets;
Theoretical values calculated in the first approximation are given in brackets

Notes: In the analytical solution the critical value of the compressive forces σ_{cr1} in the first approximation corresponding to the moment of buckling of the rectangular reinforced plate can be determined according to the following formula:

$$\sigma_{cr1} = \frac{\pi^2 \cdot D \cdot m^2}{b^2 \cdot h \cdot \lambda^2} \cdot \frac{\left[1 + \left(\frac{n \cdot \lambda}{m}\right)^2\right]^2 + 2 \cdot \gamma \cdot \sum_{i=1}^{s} \sin^2\left(\frac{\pi \cdot i}{s+1}\right)}{1 + 2 \cdot \delta \cdot \sum_{i=1}^{s} \sin^2\left(\frac{\pi \cdot i}{s+1}\right)},$$

at s =3 $$\sigma_{cr1} = \frac{\pi^2 \cdot D \cdot m^2}{b^2 \cdot h \cdot \lambda^2} \cdot \frac{\left[1 + \left(\frac{n \cdot \lambda}{m}\right)^2\right]^2 + 4 \cdot \gamma}{1 + 4 \cdot \delta},$$ where:

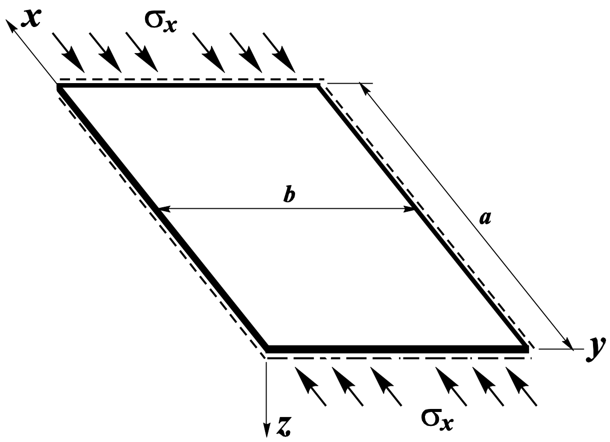
$$D = \frac{E \cdot h^3}{12 \cdot (1 - \nu^2)}, \quad \lambda = \frac{a}{b}, \quad \gamma = \frac{E \cdot I}{b \cdot D}, \quad \delta = \frac{F}{b \cdot h},$$

n, *m* = 1, 2, 3 … – number of half waves of the buckling mode in the transverse and longitudinal directions with respect to the compression of the plate.

In the analytical solution the critical value of the compressive forces σ_{cr1} in the fourth approximation corresponding to the moment of buckling of the rectangular reinforced plate is determined on the basis of the condition of equality to zero of the determinant of the system of governing equations:

$$\begin{vmatrix}
\frac{\pi^2 \cdot D \cdot m^2}{b^2 \cdot h \cdot \lambda^2} \cdot \left[\left[1 + \left(\frac{n \cdot \lambda}{m} \right)^2 \right]^2 + 4 \cdot \gamma \right] - \sigma_{cr4} \cdot (1 + 4 \cdot \delta) & -\frac{\pi^2 \cdot D \cdot m^2}{b^2 \cdot h \cdot \lambda^2} \cdot 4 \cdot \gamma + \sigma_{cr4} \cdot 4 \cdot \delta & \frac{\pi^2 \cdot D \cdot m^2}{b^2 \cdot h \cdot \lambda^2} \cdot 4 \cdot \gamma - \sigma_{cr4} \cdot 4 \cdot \delta & -\frac{\pi^2 \cdot D \cdot m^2}{b^2 \cdot h \cdot \lambda^2} \cdot 4 \cdot \gamma + \sigma_{cr4} \cdot 4 \cdot \delta \\
-\frac{\pi^2 \cdot D \cdot m^2}{b^2 \cdot h \cdot \lambda^2} \cdot 4 \cdot \gamma + \sigma_{cr4} \cdot 4 \cdot \delta & \frac{\pi^2 \cdot D \cdot m^2}{b^2 \cdot h \cdot \lambda^2} \cdot \left[\left[1 + 49 \cdot \left(\frac{n \cdot \lambda}{m} \right)^2 \right]^2 + 4 \cdot \gamma \right] - \sigma_{cr4} \cdot (1 + 4 \cdot \delta) & -\frac{\pi^2 \cdot D \cdot m^2}{b^2 \cdot h \cdot \lambda^2} \cdot 4 \cdot \gamma + \sigma_{cr4} \cdot 4 \cdot \delta & \frac{\pi^2 \cdot D \cdot m^2}{b^2 \cdot h \cdot \lambda^2} \cdot 4 \cdot \gamma - \sigma_{cr4} \cdot 4 \cdot \delta \\
\frac{\pi^2 \cdot D \cdot m^2}{b^2 \cdot h \cdot \lambda^2} \cdot 4 \cdot \gamma - \sigma_{cr4} \cdot 4 \cdot \delta & -\frac{\pi^2 \cdot D \cdot m^2}{b^2 \cdot h \cdot \lambda^2} \cdot 4 \cdot \gamma + \sigma_{cr4} \cdot 4 \cdot \delta & \frac{\pi^2 \cdot D \cdot m^2}{b^2 \cdot h \cdot \lambda^2} \cdot \left[\left[1 + 81 \cdot \left(\frac{n \cdot \lambda}{m} \right)^2 \right]^2 + 4 \cdot \gamma \right] - \sigma_{cr4} \cdot (1 + 4 \cdot \delta) & -\frac{\pi^2 \cdot D \cdot m^2}{b^2 \cdot h \cdot \lambda^2} \cdot 4 \cdot \gamma + \sigma_{cr4} \cdot 4 \cdot \delta \\
-\frac{\pi^2 \cdot D \cdot m^2}{b^2 \cdot h \cdot \lambda^2} \cdot 4 \cdot \gamma + \sigma_{cr4} \cdot 4 \cdot \delta & \frac{\pi^2 \cdot D \cdot m^2}{b^2 \cdot h \cdot \lambda^2} \cdot 4 \cdot \gamma - \sigma_{cr4} \cdot 4 \cdot \delta & -\frac{\pi^2 \cdot D \cdot m^2}{b^2 \cdot h \cdot \lambda^2} \cdot 4 \cdot \gamma + \sigma_{cr4} \cdot 4 \cdot \delta & \frac{\pi^2 \cdot D \cdot m^2}{b^2 \cdot h \cdot \lambda^2} \cdot \left[\left[1 + 225 \cdot \left(\frac{n \cdot \lambda}{m} \right)^2 \right]^2 + 4 \cdot \gamma \right] - \sigma_{cr4} \cdot (1 + 4 \cdot \delta)
\end{vmatrix} = 0$$

Stability of a Rectangular Simply Supported Orthotropic Plate Uniformly Compressed in One Direction



Objective: Determination of the critical value of the compressive forces uniformly distributed along two opposite transverse sides of a rectangular simply supported orthotropic plate corresponding to the moment of its buckling.

Initial data files:

File name	Description
6.10_shell_orthotropic_lambda_1.SPR	Design model with the ratios of the sides of the plate a/b = 1.0
6.10_shell_orthotropic_lambda_4.SPR	Design model with the ratios of the sides of the plate a/b = 4.0

Problem formulation: The rectangular simply supported orthotropic plate is subjected to the action of compressive forces σ, uniformly distributed along two opposite transverse sides. Determine the critical value of the compressive forces σ_{crort}, corresponding to the moment of buckling of the rectangular orthotropic plate.

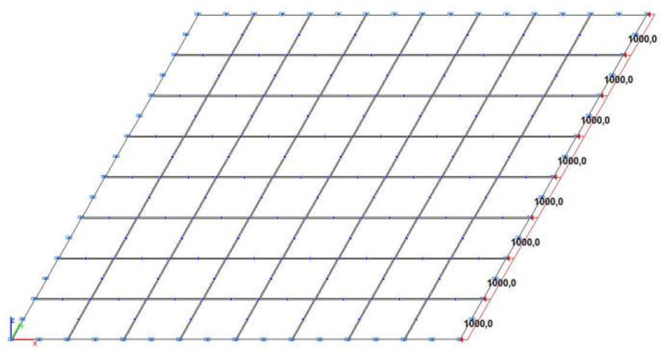
References: A.S. Volmir. Stability of Deformable Systems, Moscow, Nauka, 1967, p. 374.

Initial data:

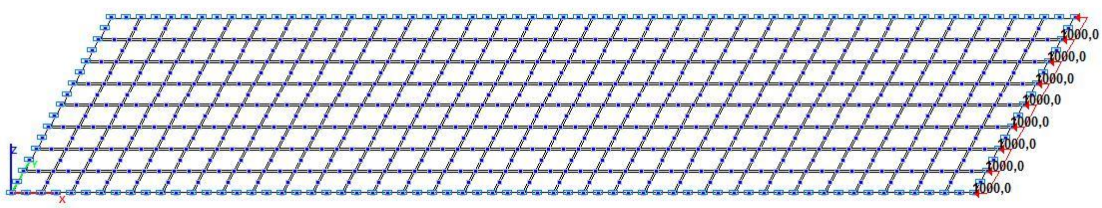
a = 0.6; 2.4 m	- side of the rectangular plate free from forces (along the X axis of the global coordinate system);
b = 0.6 m	- side of the rectangular plate subjected to the compressive forces (along the Y axis of the global coordinate system);
h = 0.01 m	- thickness of the rectangular plate;
$E_x = 5.600 \cdot 10^8$ kN/m^2	- elastic modulus of the plate material corresponding to longitudinal deformations along the X axis of the global coordinate system;
$\nu_{yx} = 0.300$	- Poisson’s ratio corresponding to transverse deformations along the Y axis of the global coordinate system;
$E_y = 2.123 \cdot 10^8$ kN/m^2	- elastic modulus of the plate material corresponding to longitudinal deformations along the Y axis of the global coordinate system;
$\nu_{xy} = 0.114$	- Poisson’s ratio corresponding to transverse deformations along the X axis of the global coordinate system;
$G_{xy} = 0.769 \cdot 10^8$ kN/m^2	- shear modulus of the plate material;
$\sigma = 1.0 \cdot 10^5$ kN/m^2	- initial value of the compressive forces.

Finite element model: Design model – general type system. Two design models with the ratios of the sides of the plate a/b = 1.0; 4.0 are considered. The plate is modeled by eight-node shell elements of type 50. The spacing of the finite element mesh along the sides of the plate (along the X and Y axes of the global coordinate system) is 0.075 m. Number of plate elements in the models – 64; 256. Boundary conditions are provided by imposing constraints on the nodes of the support contour of the plate in the direction of the degree of freedom Z. A load uniformly distributed along the line with the initial value $p = \sigma \cdot h = 1000$ kN/m is specified on one of the two opposite sides of the plate subjected to the compressive forces, and the constraints in the respective direction (along the X axis of the global coordinate system) are imposed on the nodes of the other one. The dimensional stability of the design model is provided by imposing constraints in the normal direction (along the Y axis of the global coordinate system) on the nodes of one of the two opposite sides of the plate free from forces, and by imposing a constraint in the UZ direction of the global coordinate system on the node of one of the corners of the plate. The dimensional stability of the design model is provided by imposing constraints in the normal direction (along the Y axis of the global coordinate system) on the nodes of one of the two opposite longitudinal sides of the plate free from forces, and by imposing a constraint in the UZ direction of the global coordinate system on the node of one of the corners of the plate. Number of nodes in the models – 225; 849.

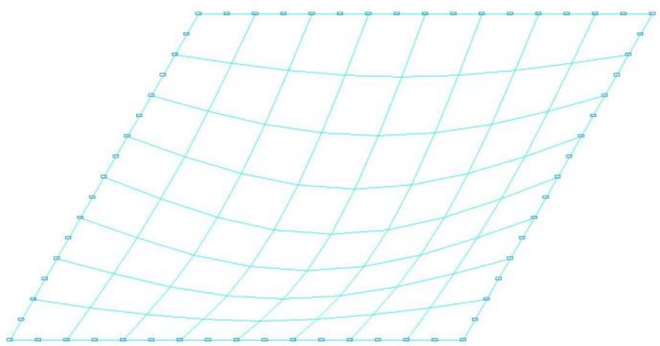
Results in SCAD



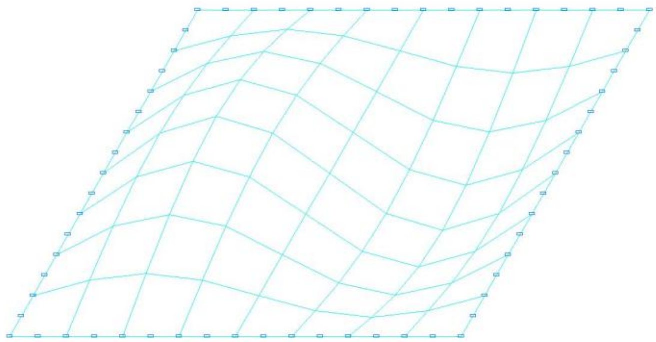
Design model with the ratio of the sides of the plate a/b = 1.0



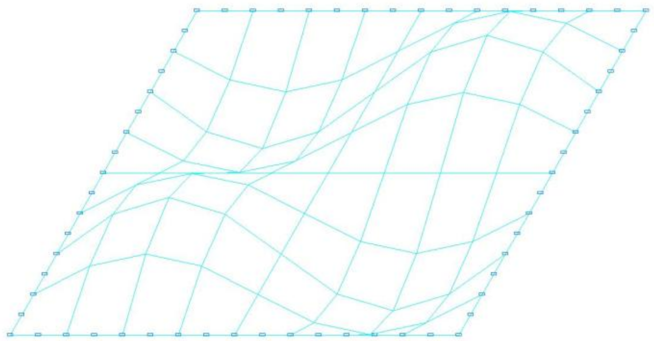
Design model with the ratio of the sides of the plate a/b = 4.0



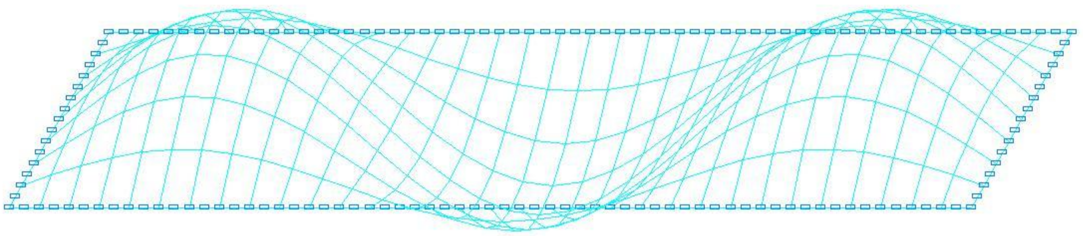
1-st buckling mode for the design model with the ratio of the sides of the plate a/b = 1.0



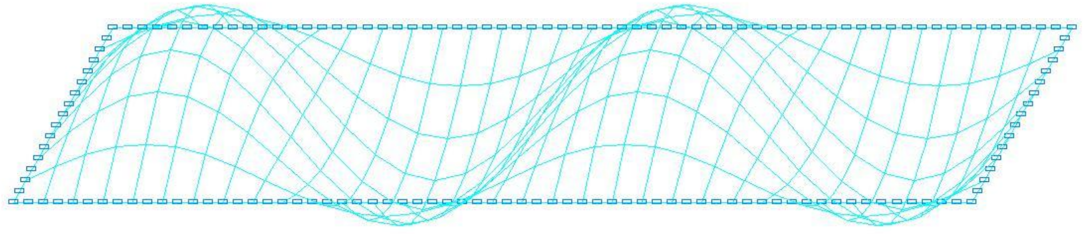
2-nd buckling mode for the design model with the ratio of the sides of the plate a/b = 1.0



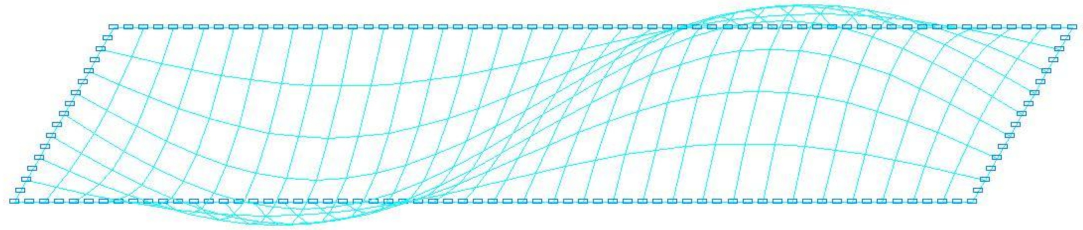
3-rd buckling mode for the design model with the ratio of the sides of the plate a/b = 1.0



1-st buckling mode for the design model with the ratio of the sides of the plate a/b = 4.0



2-nd buckling mode for the design model with the ratio of the sides of the plate a/b = 4.0



3-rd buckling mode for the design model with the ratio of the sides of the plate a/b = 4.0

Comparison of solutions:

Critical value of the compressive forces σ_{crort}, kN/m^2

Plate sides ratio	**Buckling mode**	**Number of half waves in the transverse n and in the longitudinal m directions**	**Theory**	**SCAD**	**Deviation, %**
a/b = 1.0	1	1; 1	283093	2.831349 ·1000/0.01 = = 283135	0.01
	2	1; 2	642810	6.428985 ·1000/0.01 = = 642899	0.01
	3	2; 2	1132373	11.326625 ·1000/0.01= = 1132663	0.03
a/b = 4.0	1	1; 3	264196	2.642394 ·1000/0.01 = = 264239	0.02
	2	1; 4	283093	2.831351 ·1000/0.01 = = 283135	0.01
	3	1; 2	334385	3.344432 ·1000/0.01 = = 334443	0.02

Notes: In the analytical solution the critical value of the compressive forces σ_{cr}, corresponding to the moment of buckling of the rectangular orthotropic plate can be determined according to the following formula:

$$\sigma_{crort} = \frac{\pi^2 \cdot \sqrt{D_1 \cdot D_2}}{b^2 \cdot h} \cdot \left[\sqrt{\frac{D_1}{D_2}} \cdot \left(\frac{m \cdot b}{a} \right)^2 + \frac{2 \cdot D_3 \cdot n^2}{\sqrt{D_1 \cdot D_2}} + \sqrt{\frac{D_2}{D_1}} \cdot \left(\frac{n^2 \cdot a}{m \cdot b} \right)^2 \right], \text{ where:}$$

$$D_1 = \frac{E_x \cdot h^3}{12 \cdot \left(1 - \nu_{yx} \cdot \nu_{xy}\right)}, \qquad D_2 = \frac{E_y \cdot h^3}{12 \cdot \left(1 - \nu_{yx} \cdot \nu_{xy}\right)},$$

$$D_3 = \frac{1}{2} \cdot \left(D_1 \cdot \nu_{xy} + D_2 \cdot \nu_{yx} + 4 \cdot D_t\right), \qquad D_t = \frac{G_{xy} \cdot h^3}{12},$$

n, *m* = 1, 2, 3 … – number of half waves of the buckling mode in the transverse and longitudinal directions with respect to the compression of the plate.

Stiffness properties of the orthotropic plate were taken on the basis of the conditions of equivalence to the stiffness properties of the reinforced plate from the Example 6.10 a:

$$D_1 = \frac{E \cdot I}{\frac{b}{s+1}} + \frac{E \cdot h^3}{12 \cdot \left(1 - \nu^2\right)}, \qquad D_2 = \frac{E \cdot h^3}{12 \cdot \left(1 - \nu^2\right)}, \qquad D_3 = \frac{E \cdot h^3}{12 \cdot \left(1 - \nu^2\right)},$$

and were determined according to the following formulas:

$$E_x = \frac{E}{1-\nu^2} \cdot \left[\frac{12 \cdot \left(1-\nu^2\right) \cdot I}{h^3 \cdot \frac{b}{s+1}} + 1 \right] \cdot \left[1 - \frac{\nu^2}{\frac{12 \cdot \left(1-\nu^2\right) \cdot I}{h^3 \cdot \frac{b}{s+1}} + 1} \right], \qquad E_y = \frac{E}{1-\nu^2} \cdot \left[1 - \frac{\nu^2}{\frac{12 \cdot \left(1-\nu^2\right) \cdot I}{h^3 \cdot \frac{b}{s+1}} + 1} \right],$$

$$\nu_{yx} = \nu, \qquad \nu_{xy} = \frac{\nu}{\dfrac{12 \cdot (1-\nu^2) \cdot I}{h^3 \cdot \dfrac{b}{s+1}} + 1}, \qquad G_{xy} = \frac{E}{2 \cdot (1+\nu)}.$$

The critical values of the compressive forces σ_{cr} for the reinforced plate have to be reduced by a factor k with respect to the critical values of the compressive forces σ_{cr} for the orthotropic plate, because when determining the latter the component acting on the stiffeners of the reinforced plate is not taken into account:

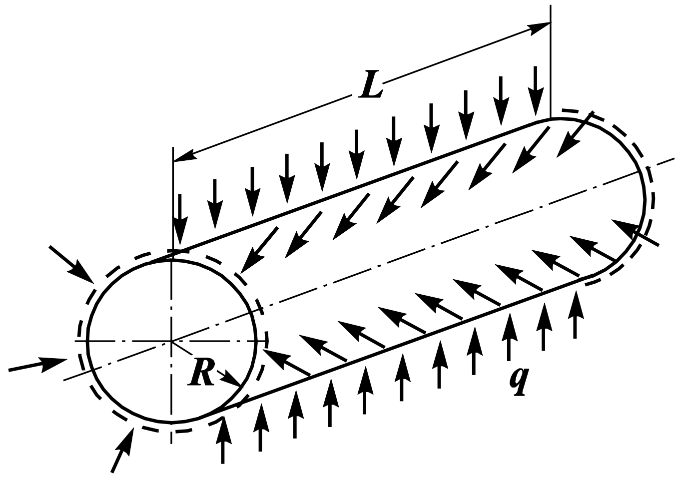
$$k = \frac{b \cdot h}{b \cdot h + F \cdot s} = 0.869565,$$

where *F* is stiffener's area, *s* is the quantity of stiffeners.

Critical value of the compressive forces σ_{cr}, kN/m^2

Plate sides ratio	Buckling mode	Number of half waves in the transverse n and in the longitudinal m directions	Theory	SCAD	Deviation, %
a/b = 1.0	1	1; 1	235900	283135·0.869565 = = 246204	4.37
	2	1; 2	533934	642899·0.869565 = = 559043	4.70
	3	2; 2	942681	1132663·0.869565 = = 984924	4.48
a/b = 4.0	1	1; 3	220165	264239·0.869565 = = 229773	4.36
	2	1; 4	235900	283135·0.869565 = = 246204	4.37
	3	1; 2	278652	334443·0.869565 = = 290820	4.37

Stability of a Cylindrical Thin-Walled Shell with Simply Supported Edges Subjected to Uniform External Pressure



Objective: Determination of the critical value of the external pressure uniformly distributed over the lateral surface of a cylindrical thin-walled shell with simply supported edges corresponding to the moment of its buckling.

Initial data file: **6.11_S.SPR**

Problem formulation: The cylindrical thin-walled shell with simply supported edges is subjected to the action of the uniform external pressure q. Determine the critical value of the uniform external pressure q_{cr}, corresponding to the moment of buckling of the cylindrical thin-walled shell.

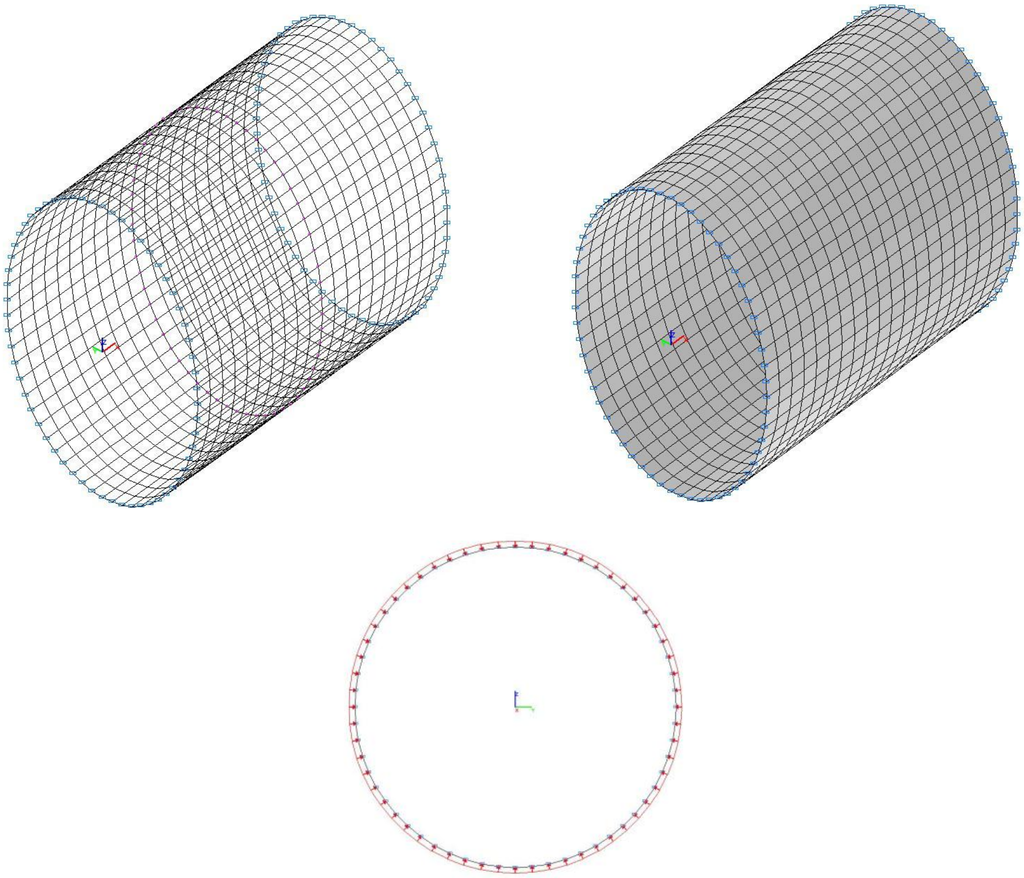
References: E.I. Grigolyuk, V.V. Kabanov, Stability of Shells, Moscow, Nauka, 1978, p. 137.
A.S. Volmir. Stability of Deformable Systems, Moscow, Nauka, 1967, p. 545.

Initial data:

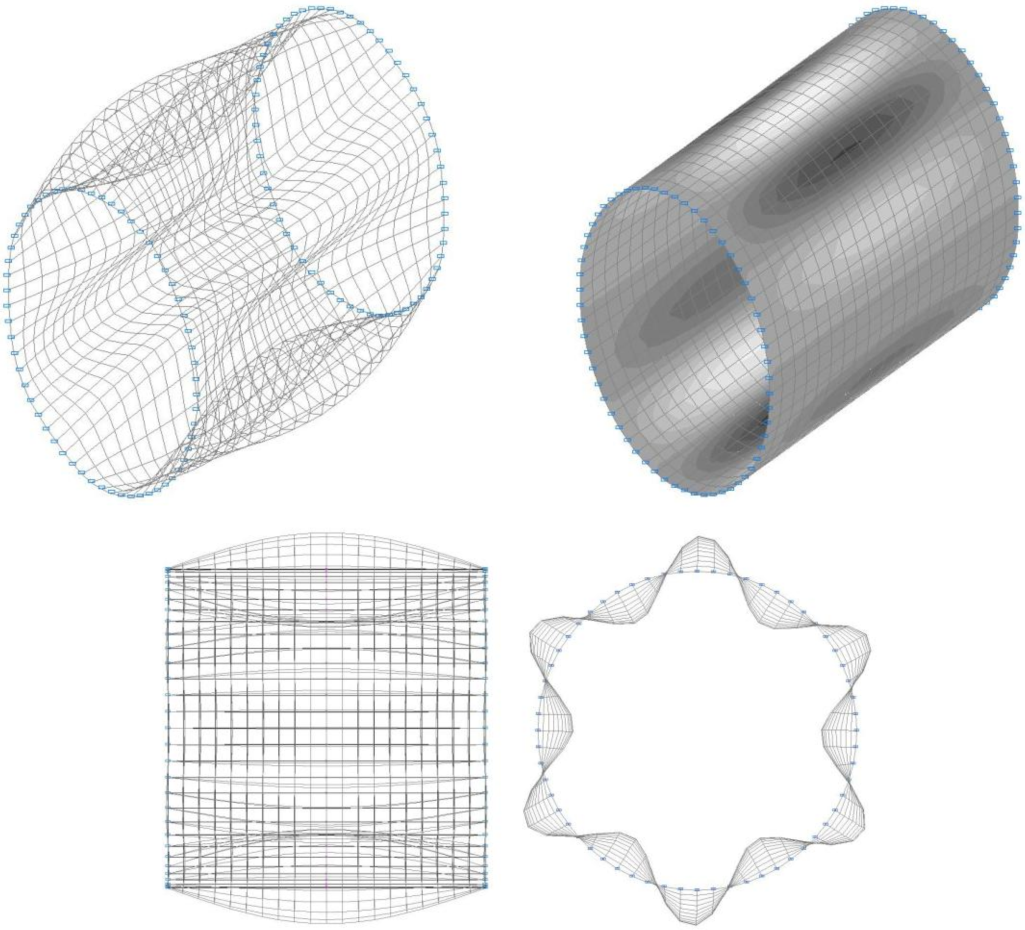
$E = 2.0\cdot10^8$ kPa	- elastic modulus of the shell material;
$\nu = 0.3$	- Poisson's ratio;
h = 0.005 m	- thickness of the shell;
R = 0.5 m	- radius of the midsurface of the shell;
L = 1.0 m	- length of the shell;
$q = 1.0\cdot10^3$ kPa	- initial value of the external pressure.

Finite element model: Design model – general type system, 1200 four-node shell elements of type 50. The spacing of the finite element mesh in the meridian direction is 0.05 m (20 elements) and in the circumferential is 6.0º (60 elements). Boundary conditions of the simply supported edges are provided by imposing constraints in the directions of the linear displacements in their plane (degrees of freedom Y, Z). The dimensional stability of the design model is provided by imposing constraints of finite rigidity (60 elements of type 51) in the nodes of the cross-section on the symmetry plane of the cylindrical shell in the meridian direction (k_x = 1.0 kN/m). The uniformly distributed load (along the Z1 axis of the local coordinate system) with the initial value $q = 1.0\cdot10^3$ kPa is specified on the lateral surface of the cylindrical shell. Number of nodes in the design model – 1260.

Results in SCAD



Design model



1-st buckling mode

Comparison of solutions:

Critical value of the uniform external pressure q_{cr}, kPa

Buckling mode	Number of half waves in the meridian direction m and number of waves in the circumferential n direction	Theory	SCAD	Deviation, %
1	1; 6	981 (917)	0.999898 ·1000 = = 1000	1.94 (9.05)

Theoretical values calculated according to the shallow shell theory for the membrane initial state are given without brackets;
Theoretical values calculated according to the general shell theory for the membrane initial state are given in round brackets.

Notes: In the analytical solution the critical value of the uniform external pressure q_{cr}, corresponding to the moment of buckling of the cylindrical thin-walled shell is determined in accordance with the shallow shell theory by the following formula:

$$q_{cr} = \frac{E \cdot h}{R \cdot n^2} \cdot \frac{\left(\frac{m \cdot \pi \cdot R}{L}\right)^4}{\left[\left(\frac{m \cdot \pi \cdot R}{L}\right)^2 + n^2\right]^2} + \frac{D}{R^3 \cdot n^2} \cdot \left[\left(\frac{m \cdot \pi \cdot R}{L}\right)^2 + n^2\right]^2, \text{ where:}$$

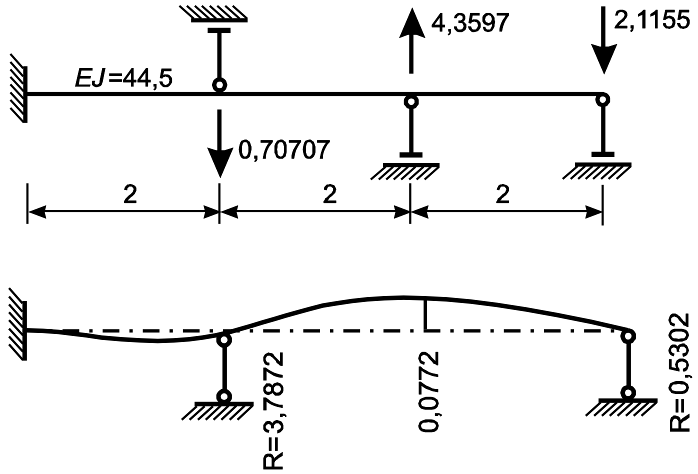
$$D = \frac{E \cdot h^3}{12 \cdot \left(1 - \nu^2\right)}.$$

In the analytical solution the critical value of the uniform external pressure q_{cr}, corresponding to the moment of buckling of the cylindrical thin-walled shell is determined in accordance with the general shell theory on the basis of the condition of equality to zero of the determinant of the system of governing equations:

$$\begin{vmatrix} \left(\frac{\pi \cdot R}{L}\right)^2 + \frac{1-\nu}{2} \cdot n^2 + \frac{(1-\nu)^2 \cdot R}{E \cdot h} \cdot n^2 \cdot q_{cr} & \frac{1+\nu}{2} \cdot \left(\frac{\pi \cdot R}{L}\right) \cdot n & \nu \cdot \left(\frac{\pi \cdot R}{L}\right) \\ \frac{1+\nu}{2} \cdot \left(\frac{\pi \cdot R}{L}\right) \cdot n & \left(1 + \frac{h^2}{12 \cdot R^2}\right) \cdot \left(\frac{1-\nu}{2} \cdot \left(\frac{\pi \cdot R}{L}\right)^2 + n^2\right) - \frac{(1-\nu)^2 \cdot R}{E \cdot h} \cdot n^2 \cdot q_{cr} & \left(1 + \frac{h^2}{12 \cdot R^2} \cdot \left(\left(\frac{\pi \cdot R}{L}\right)^2 + n^2\right)\right) \cdot n \\ \nu \cdot \left(\frac{\pi \cdot R}{L}\right) & \left(1 + \frac{h^2}{12 \cdot R^2} \cdot \left(\left(\frac{\pi \cdot R}{L}\right)^2 + n^2\right)\right) \cdot n & 1 + \frac{h^2}{12 \cdot R^2} \cdot \left(\left(\frac{\pi \cdot R}{L}\right)^2 + n^2\right)^2 - \frac{(1-\nu)^2 \cdot R}{E \cdot h} \cdot n^2 \cdot q_{cr} \end{vmatrix} = 0$$

Nonlinear Statics

Three-Span Beam with One Clamped End and Three Rigid One-Sided Supports Subjected to Concentrated Forces above Them



Objective: Determination of the reactions of one-sided supports of a three-span beam or deflections of the beam in the direction of installation of the supports in the structurally nonlinear formulation.

Initial data file: **Contact_1.SPR**

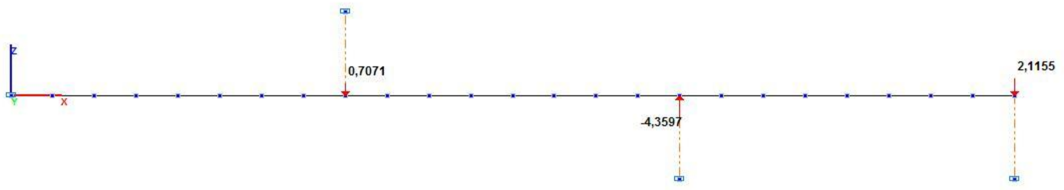
Problem formulation: The three-span beam with one clamped end and three rigid one-sided supports working in compression is subjected to concentrated shear forces above them.
Determine the reactions of the one-sided supports R_i or the deflections of the beam Z_i in the direction of installation of the supports.

References: A.V. Perelmuter, V.I. Slivker, Design Models of Structures and a Possibility of Their Analysis, Moscow, SCAD SOFT, 2011, p. 146

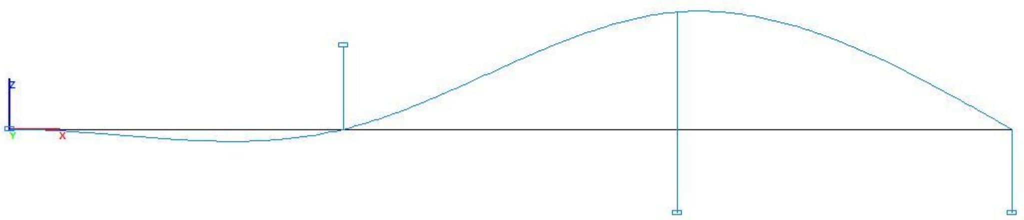
Initial data:
$EF = 1.00 \cdot 10^8$ kN - axial stiffness of the beam cross-section;
$EI = 44.50$ kN·m^2 - bending stiffness of the beam cross-section;
L = 2.00 m - beam span length;
$k = 1.00 \cdot 10^6$ kN/m - axial stiffness of the one-sided supports;
$P_1 = 0.7071$ kN - value of the concentrated force applied above the first (from the clamping) intermediate one-sided support and stretching it;
$P_2 = 4.3597$ kN - value of the concentrated force applied above the second (from the clamping) intermediate one-sided support and stretching it;
$P_3 = 2.1155$ kN - value of the concentrated force applied above the third (from the clamping) end one-sided support and compressing it.

***Finite element model*:** Design model – plane frame. Elements of the beam – 24 bar elements of type 2. The spacing of the finite element mesh along the beam length (along the X1 axes of the local coordinate systems) is 0.25 m. Elements of the one-sided supports – 3 two-node elements of unilateral constraints of type 352. Boundary conditions are provided by imposing constraints on the support node of the clamped end of the beam in the directions of the degrees of freedom X, Z, UY and on the support nodes of the one-sided supports in the directions of the degrees of freedom X, Z. The actions are specified as transverse nodal loads P (in the direction of the Z axis of the global coordinate system). The nonlinear loading was generated for the incremental-iterative method with a loading factor - 1, number of steps - 1, number of iterations - 10 for the linear loading P. Number of nodes in the design model – 28.

Results in SCAD



Design model



Deformed model

Values of deflections of the beam Z_i, m

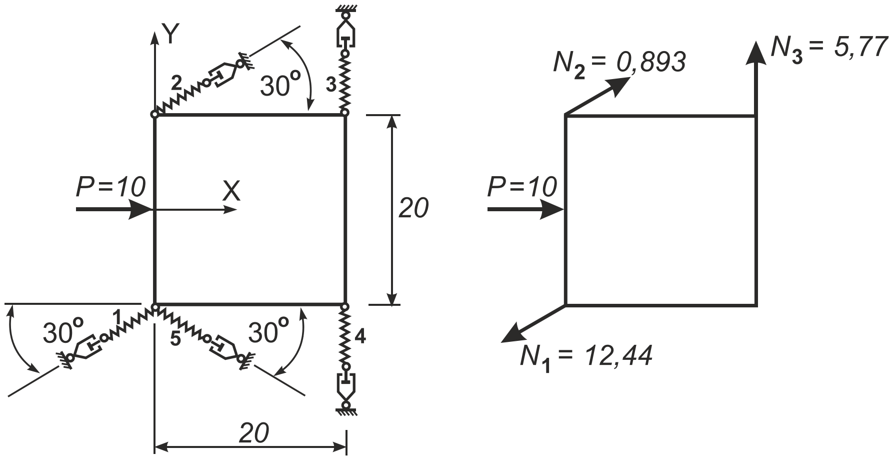
Values of reactions of the one-sided supports, kN

Comparison of solutions:

Parameter	**Theory**	**SCAD**	**Deviation, %**
R_1, kN	3.7872	3.8506	1.67
R_3, kN	0.5302	0.5301	0.02
Z_2, m	0.0772	0.0772	0.00

Notes: In the analytical solution the reactions of the one-sided supports R_i or the deflections of the beam Z_i in the direction of installation of the supports are determined by the quadratic programming method.

Rigid Body Restrained by Five Springs of the Same Rigidity Working Only in Tension Subjected to a Concentrated Force



Objective: Determination of the reactions of springs of the same rigidity working only in tension and restraining a rigid body from the action of a concentrated force applied to it in the structurally nonlinear formulation.

***Initial data file*: Contact_2.SPR**

***Problem formulation*:** The rigid body in the shape of a square with the sides parallel to the coordinate axes is restrained at the corners by five springs of the same rigidity working only in tension as follows:
two springs (1 and 5) are installed in the lower left corner of the square, the angles between their longitudinal axes and the lower side of the square are 150° and 30° respectively;
one spring is installed in the upper left corner of the square (2), the angle between its longitudinal axis and the upper side of the square is 30°;
springs (4 and 3) are installed in the lower right and in the upper right corners of the square, the angle between their longitudinal axes and the lower and upper sides of the square respectively is 90°.
The concentrated force P is applied perpendicular to the middle of the left side of the square of the rigid body.
Determine the reactions in the springs R_i.

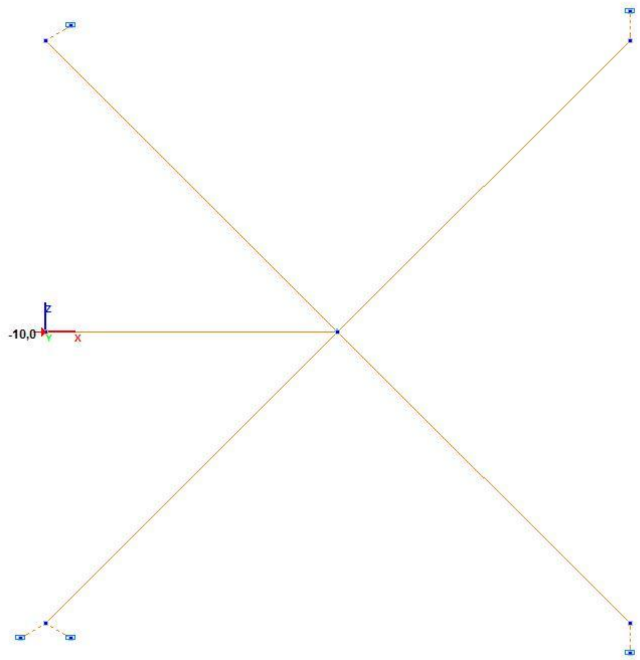
References: A.V. Perelmuter, V.I. Slivker, Design Models of Structures and a Possibility of Their Analysis, Moscow, SCAD SOFT, 2011, p. 147

Initial data:

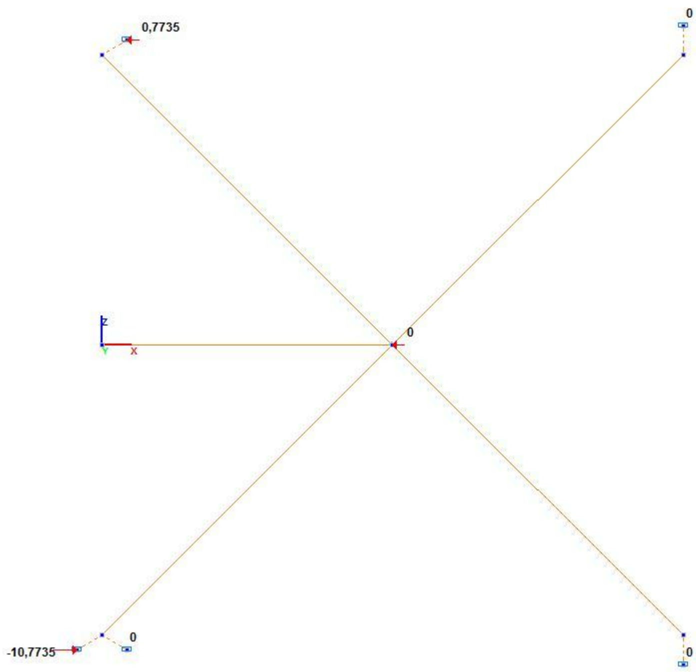
L = 20 m	- side of the square of the rigid body;
α_1 = 150°	- angle between the axis of the spring 1 and the lower side of the square;
α_2 = 30°	- angle between the axis of the spring 2 and the upper side of the square;
α_3 = 90°	- angle between the axis of the spring 3 and the upper side of the square;
α_4 = 90°	- angle between the axis of the spring 4 and the lower side of the square;
α_5 = 30°	- angle between the axis of the spring 5 and the lower side of the square;
$k = 1.00 \cdot 10^6$ kN/m	- axial stiffness of the springs;
P = 10.0 kN	- value of the concentrated force acting perpendicular to the middle of the left side of the square.

***Finite element model*:** Design model – plane frame. Element of the rigid body – 1 3D six-node rigid body element of type 100 (one master node lying at the intersection of the diagonals of the square, four slave nodes lying at the corners of the square, one slave node lying in the middle of the left side of the square). Elements of the springs – 5 two-node elements of unilateral constraints of type 352. Boundary conditions are provided by imposing constraints on the support nodes of the springs in the directions of the degrees of freedom X, Z. An element of the constraint of finite rigidity (type 51) of small value 0.001 kN/m in the direction of the X axis of the global coordinate system is introduced in the master node of the rigid body to provide the dimensional stability of the system during the nonlinear calculation. The results of the calculation are correct if there are no reactions in this constraint. The action is specified as a nodal load P (in the direction of the X axis of the global coordinate system). The nonlinear loading was generated for the incremental-iterative method with a loading factor - 1, number of steps - 1, number of iterations - 10 for the linear loading P. Number of nodes in the design model – 17.

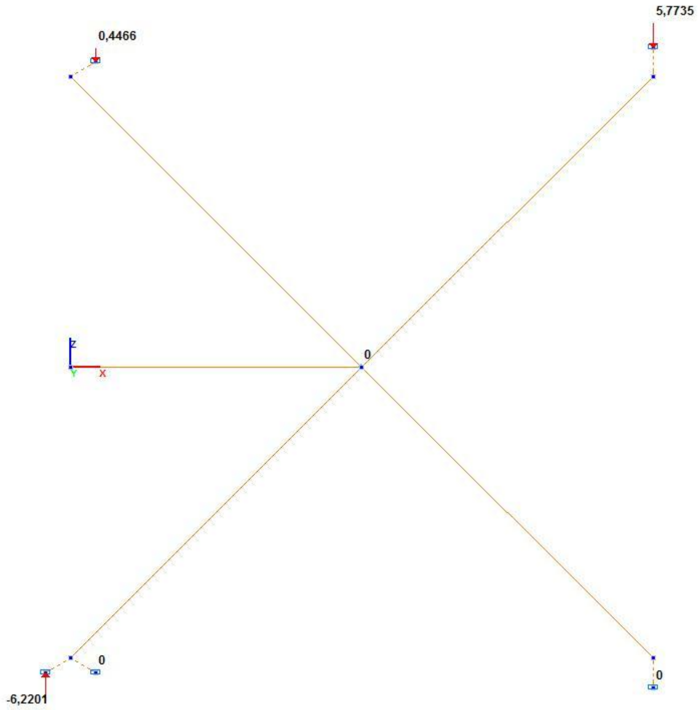
Results in SCAD



Design model



Values of reactions in the support nodes of the springs along the X axis of the global coordinate system R_x, kN



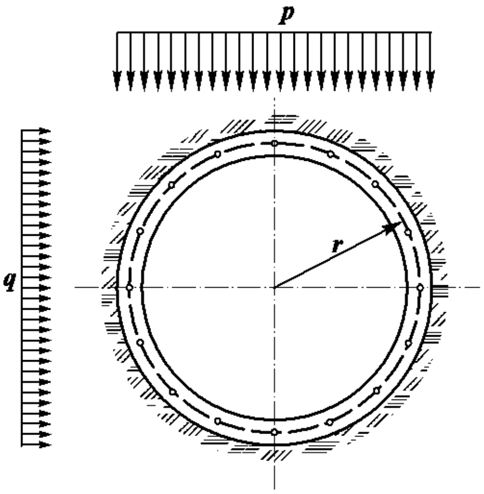
Values of reactions in the support nodes of the springs along the Z axis of the global coordinate system R_z, kN

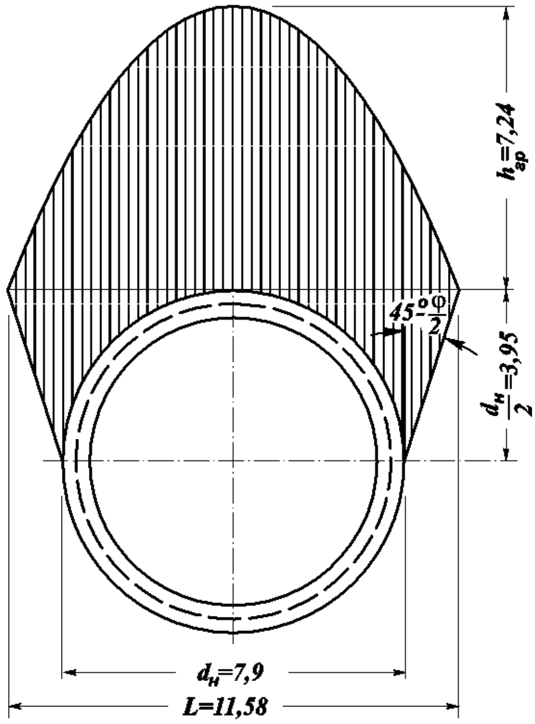
Comparison of solutions:

Parameter	Theory	SCAD	Deviation, %
R_1, kN	12.440	-10.7735·cos150° + 6.2201·sin150° = 12.440	0.00
R_2, kN	0.893	0.7735·cos30° + 0.4466·sin30° = 0.893	0.00
R_3, kN	5.770	5.7735·sin90° = 5.774	0.07
R_4, kN	0.000	0.000	0.00
R_5, kN	0.000	0.000	0.00

Notes: In the analytical solution the reactions in the springs R_i are determined by the quadratic programming method.

Circular Tunnel Lining Subjected to the Given Active Vertical And Horizontal Earth Pressure and Passive Lateral Earth Pressure in the Contact Area





Objective: Determination of the internal forces in the structure of a circular tunnel lining and the elastic reactions of soil in the contact area from the action of the given vertical and horizontal earth pressure in the structurally nonlinear formulation.

Initial data file: **Tunnel_lining.SPR**

Problem formulation: The circular tunnel lining is subjected to the action of the given active vertical p and horizontal q arching earth pressure and passive lateral earth pressure in the contact area. Determine the internal forces (longitudinal forces N and bending moments M) in the structure of the circular tunnel lining and the elastic reactions of soil R in the contact area.

References: M.M. Archangelsky, D.I. Jincharadze, A.S. Kurisko, Calculation of Tunnel Lining, Moscow, TRANSZHELDORIZDAT, 1960, p. 217

Initial data:

$E = 3.4 \cdot 10^6$ t/m^2 - elastic modulus of the tunnel lining material;
$\gamma_b = 2.6$ t/m^3 - specific weight of the tunnel lining material;
$d_{int} = 7.1$ m - inner diameter of the tunnel lining ring;
$h = 0.4$ - thickness of the rectangular cross-section of the tunnel lining;
$b = 1.0$ - width of the rectangular cross-section of the tunnel lining;
$\alpha = \pi/8$ rad - central angle of the side of the regular polygon of the frame replacing the circle of the design radius r of the tunnel lining;
$k = 5.0 \cdot 10^3$ t/m^3 - coefficient of lateral earth pressure in the area of contact with the tunnel lining;
$f = 0.8$ - Protodyakonov hardness coefficient;
$\varphi = 2 \cdot \pi/9$ rad - angle of internal friction of soil;
$\gamma_g = 1.9$ t/m^3 - specific weight of soil;
$d_{ext} = d_{int} + 2 \cdot h = 7.9$ m - outer diameter of the tunnel lining ring;
$r = (d_{ext} + d_{int})/4 = 3.75$ m - design radius of the tunnel lining;
$S = 2 \cdot r \cdot \sin(0.5 \cdot \alpha) = 1.463177$ m - side of the regular polygon of the replacement frame;
$I = b \cdot h^3/12 = 0.005333$ m^4 - cross-sectional moment of inertia of the tunnel lining;
$F = b \cdot h = 0.4$ m^2 - cross-sectional area of the tunnel lining;

$D = k \cdot S \cdot b = 7315.887$ t/m - stiffness of the elastic supports modeling the lateral earth pressure and radially arranged at the vertices of the polygon of the replacement frame;

$L_{arch} = d_{ext} \cdot (1 + tg(\pi/4 - \varphi/2)) = 11.584$ m - span of the earth pressure arch;

$H_{arch} = L_{arch}/(2 \cdot f) = 7.240$ m - height of the earth pressure arch above the excavation;

$p = H_{arch} \cdot \gamma_g + h \cdot \gamma_b = 14.796$ t/m^2 - vertical uniformly distributed active earth pressure;

$q = (H_{arch} + d_{ext}/2) \cdot \gamma_g \cdot tg^2(\pi/4 - \varphi/2) = 4.623$ t/m^2 - horizontal uniformly distributed active earth pressure.

Vertical concentrated forces in the nodes of the polygon of the frame replacing the distributed load

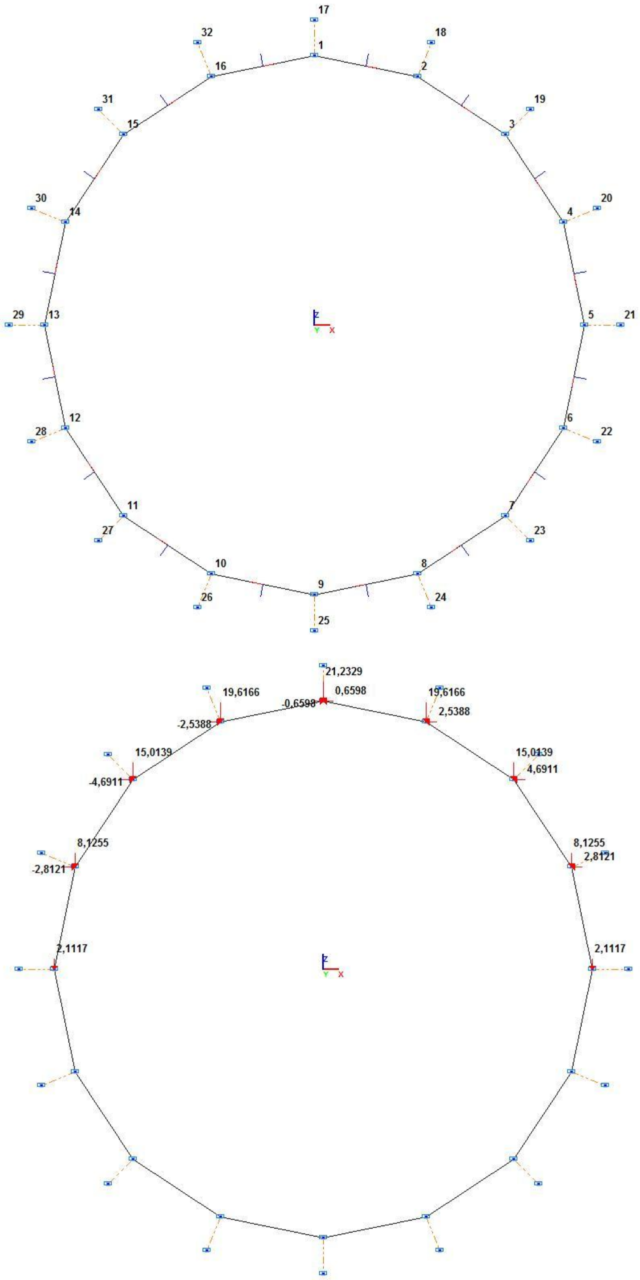
$P_1 = (S/2) \cdot p \cdot (\cos(-0.5 \cdot \alpha) + \cos(0.5 \cdot \alpha)) = 21.2329$ t; $P_2 = (S/2) \cdot p \cdot (\cos(0.5 \cdot \alpha) + \cos(1.5 \cdot \alpha)) = 19.6166$ t;
$P_3 = (S/2) \cdot p \cdot (\cos(1.5 \cdot \alpha) + \cos(2.5 \cdot \alpha)) = 15.0139$ t; $P_4 = (S/2) \cdot p \cdot (\cos(2.5 \cdot \alpha) + \cos(3.5 \cdot \alpha)) = 8.1255$ t;
$P_5 = (S/2) \cdot p \cdot \cos(3.5 \cdot \alpha) = 2.1117$ t.

Horizontal concentrated forces in the nodes of the polygon of the frame replacing the distributed load

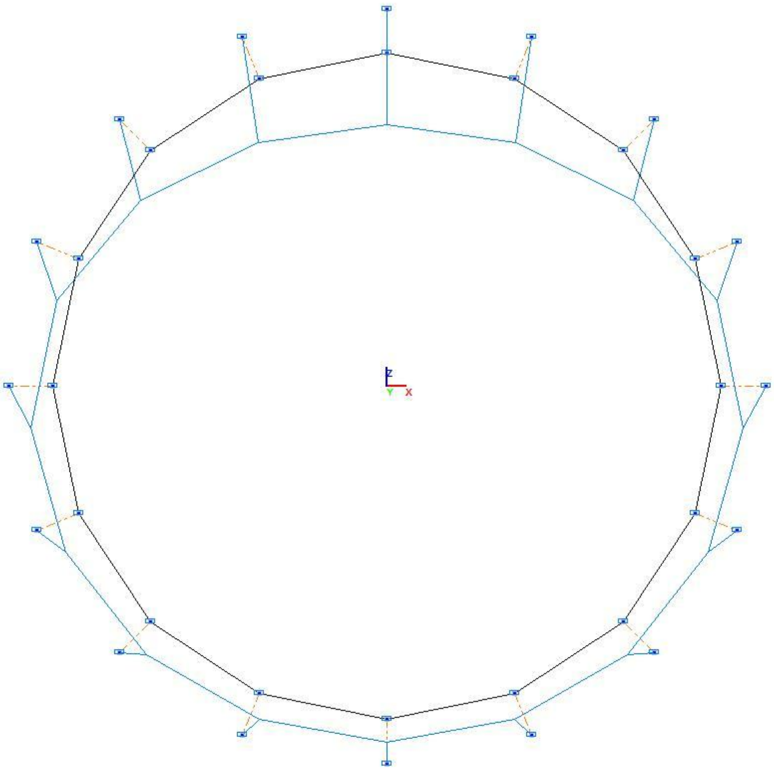
$Q_1 = (S/2) \cdot q \cdot \sin(0.5 \cdot \alpha) = 0.6598$ t; $Q_2 = (S/2) \cdot q \cdot (\sin(0.5 \cdot \alpha) + \sin(1.5 \cdot \alpha)) = 2.5388$ t;
$Q_3 = (S/2) \cdot q \cdot (\sin(1.5 \cdot \alpha) + \sin(2.5 \cdot \alpha)) = 4.6911$ t; $Q_4 = (S/2) \cdot q \cdot \sin(2.5 \cdot \alpha) = 2.8121$ t.

Finite element model: Design model – general type system. Elements of the tunnel lining – 16 bar elements of type 5. The tunnel lining is divided into finite elements along the circle of radius r = 3.75 m, lying in the XOZ plane of the global coordinate system by the step of the central angle of $\alpha = \pi/8$ rad. The origin of the global coordinate system is in the center of the circle. The X1 axes of the local coordinate systems of the elements are directed along the chords of the circle in the clockwise direction around the Y axis of the global coordinate system when viewed from the origin. The Z1 axes of the local coordinate systems of the elements are directed from the center of the circle. Elements modeling the lateral earth pressure – 16 two-node elements of unilateral constraints working in compression of type 352. Finite elements are directed along the radii of the circle from the center and are adjacent to the nodes between the elements of the tunnel lining. Boundary conditions are provided by imposing constraints on the support nodes of the elements modeling the lateral earth pressure in the directions of the degrees of freedom X, Y, Z, and on the elements of the tunnel lining in the direction of the degree of freedom Y. The dimensional stability of the design model is provided by imposing constraints in the direction of the degree of freedom X on the nodes of the elements of the tunnel lining located along the vertical axis of symmetry. The action of the active vertical and horizontal earth pressure is specified as vertical P_i and horizontal Q_i concentrated forces in the nodes between the elements of the tunnel lining. The nonlinear loading was generated by the simple incremental method with a loading factor – 0.01 and a number of steps – 100 for the linear loading. Number of nodes in the design model – 32.

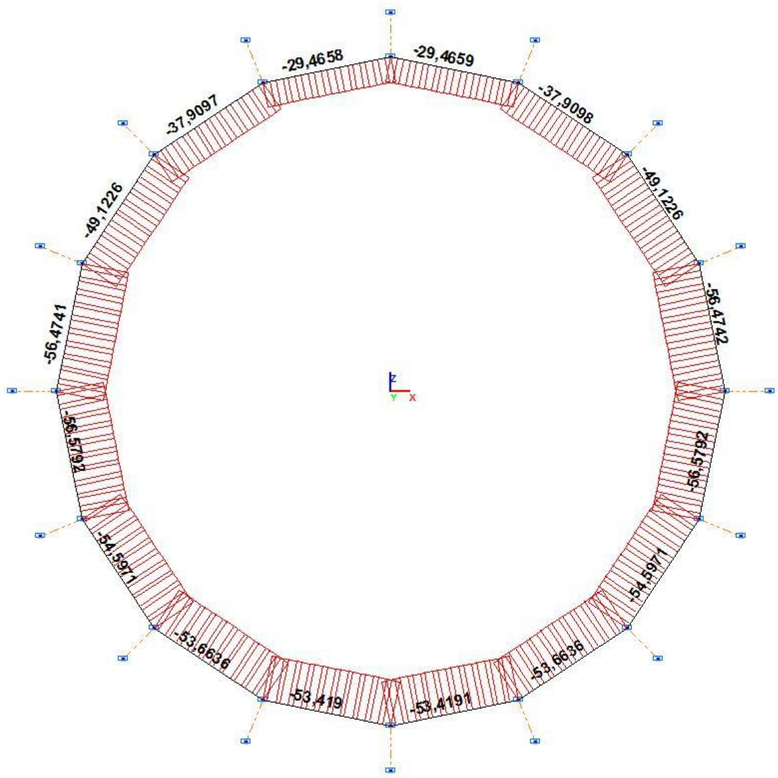
Results in SCAD



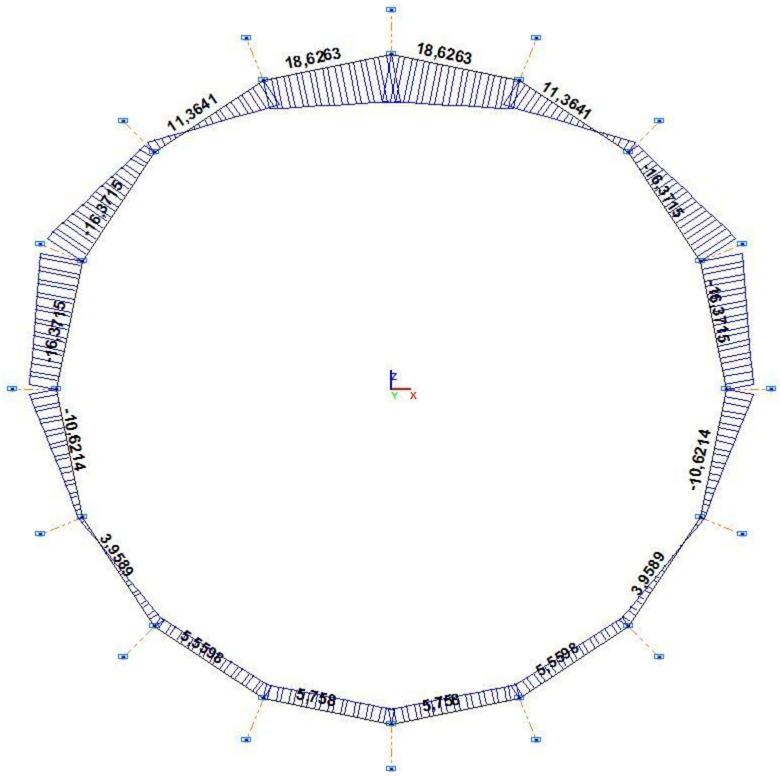
Design model



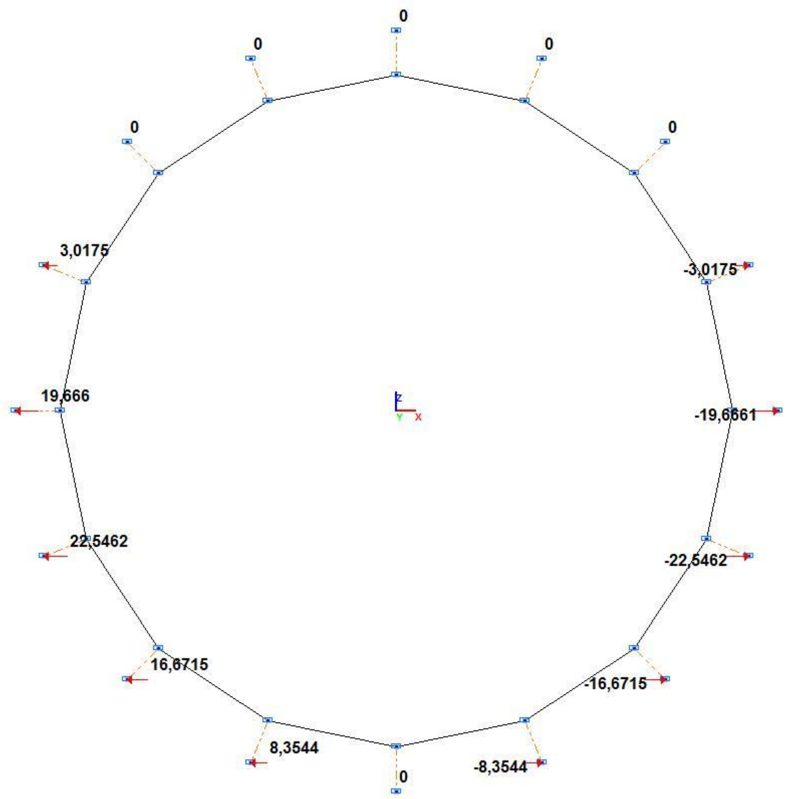
Deformed model



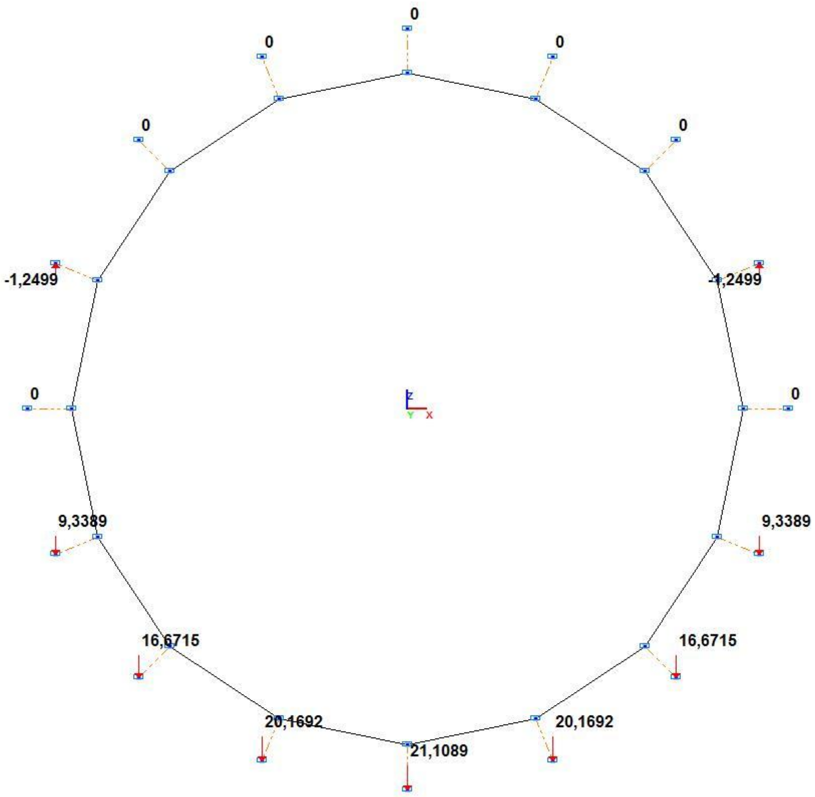
Longitudinal force diagram N, m



Bending moment diagram M, t·m



Values of reactions in the support nodes along the X axis of the global coordinate system R_x, m



Values of reactions in the support nodes along the Z axis of the global coordinate system R_z, m

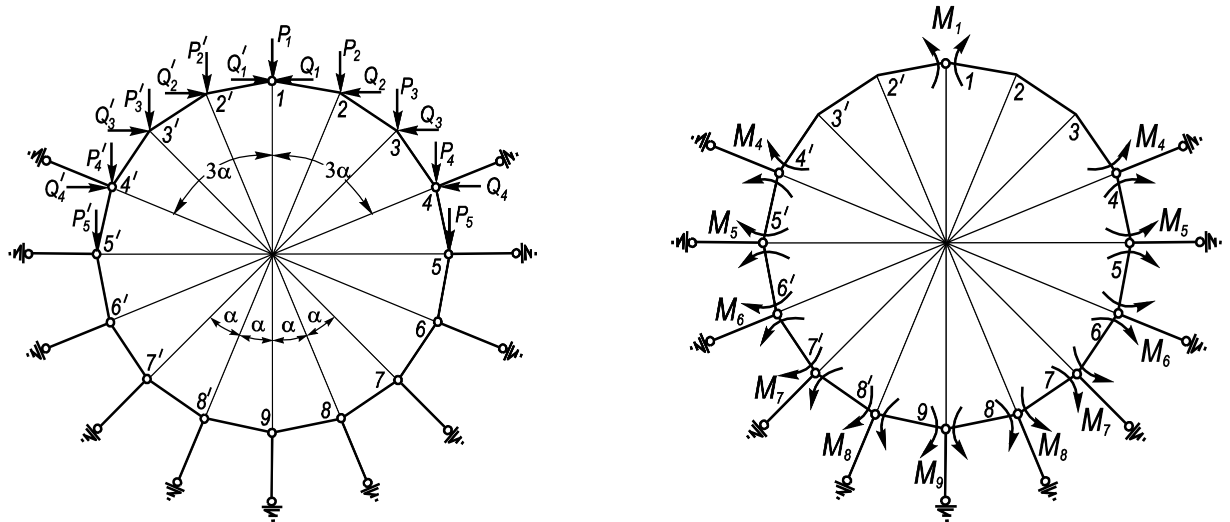
Comparison of solutions:

Parameter	Theory	SCAD	Deviation, %
N_{12}, t	-29.4660	-29.4659	0.00
N_{23}, t	-37.9098	-37.9098	0.00
N_{34}, t	-49.1226	-49.1226	0.00
N_{45}, t	-56.4742	-56.4742	0.00
N_{56}, t	-56.5793	-56.5792	0.00
N_{67}, t	-54.5971	-54.5971	0.00
N_{78}, t	-53.6637	-53.6636	0.00
N_{89}, t	-53.4191	-53.4191	0.00
M_1, t·m	18.6263	18.6263	0.00
M_2, t·m	11.3641	11.3641	0.00
M_3, t·m	-4.7755	-4.7755	0.00
M_4, t·m	-16.3715	-16.3715	0.00
M_5, t·m	-10.6215	-10.6214	0.00
M_6, t·m	-1.3066	-1.3065	0.01
M_7, t·m	3.9589	3.9589	0.00
M_8, t·m	5.5598	5.5598	0.00
M_9, t·m	5.7581	5.7580	0.00
R_1, t	0.0000	0.0000	0.00
R_2, t	0.0000	0.0000	0.00
R_3, t	0.0000	0.0000	0.00
R_4, t	-3.2661	$-3.0175 \cdot \cos(\pi/8) - 1.2499 \cdot \sin(\pi/8) = -3.2661$	0.00
R_5, t	-19.6660	-19.6661	0.00
R_6, t	-24.4038	$-22.5462 \cdot \cos(\pi/8) - 9.3389 \cdot \sin(\pi/8) = -24.4038$	0.00
R_7, t	-23.5771	$-16.6715 \cdot \cos(\pi/4) – 16.6715 \cdot \sin(\pi/4) = -23.5770$	0.00
R_8, t	-21.8310	$-8.3544 \cdot \cos(3 \cdot \pi/8) - 20.1692 \cdot \sin(3 \cdot \pi/8) = -21.8310$	0.00
R_9, t	-21.1089	-21.1089	0.00

Notes: The method of calculating tunnel linings proposed by the Metroproject, which takes into account the dependence of the stress state of the structure on the elastic properties of the continuum, is used in the analytical solution. The calculation procedure is as follows:

- The area of contact of the structure with the soil is specified; the circular contour of the lining is replaced by a regular polygon; all the active loads reduced to the nodal ones and the necessary geometric properties are calculated.
- The assumed primary system of the force method has the form of a polygon with hinges in all nodes with elastic supports, and also in the central angle of the detachment area, and as a result the upper part of the polygon turns into a three-hinged arch; moments which have to be applied in the hinges to eliminate the possibility of the relative rotation of the sides of the polygon are taken as the unknowns; unit moments are applied in all hinges (the action of the pairs of unknowns acting in the symmetric nodes is considered for a symmetric system); forces in the elements of the hinged chain and reactions of the elastic supports in all unit states are determined by successively cutting out the nodes and projecting forces in the directions of the bars, and on the bisector of the angle.
- The upper part of the polygon in the detachment area is considered as a three-hinged arch, and its support vertical and horizontal pressures from the external load on the rest of the hinged polygon are determined; forces for all elements of the hinged chain and reactions of the elastic supports caused by the support forces and the active loads applied in other nodes are determined by successively cutting out the nodes.
- The unit and loading displacements are determined by the Maxwell-Mohr formulas using the approximate summation methods.
- A system of canonical equations is compiled and solved using the Gauss algorithm in order to determine the redundants.
- The longitudinal forces, bending moments and reactions of the elastic constraints are determined.
- The correctness of the specification of the contact area between the structure and the soil is checked based on the values of the reactions of the elastic supports.

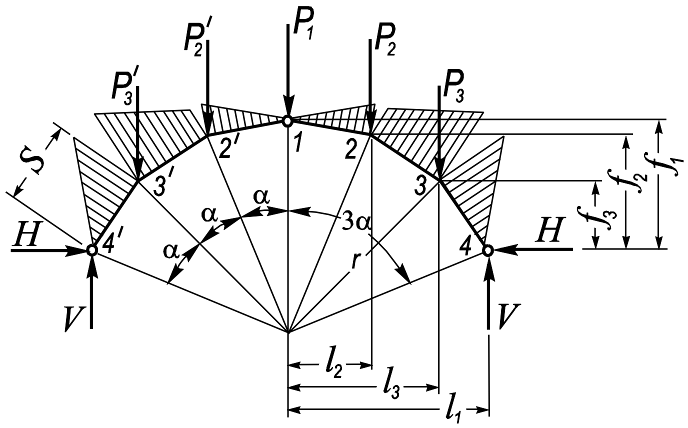
The formulas for the calculation are given below.



Primary system of the force method

Determination of forces in the primary system from the external loads

Determination of forces in the three-hinged arch from the vertical loads



Design model for the determination of forces in the three-hinged arch from the vertical loads

$$L_1 = r \cdot \sin(3 \cdot \alpha); \qquad L_2 = r \cdot \sin(\alpha); \qquad L_3 = r \cdot \sin(2 \cdot \alpha);$$
$$F_1 = r \cdot (1 - \cos(3 \cdot \alpha)); \qquad F_2 = r \cdot (\cos(\alpha) - \cos(3 \cdot \alpha)); \qquad F_3 = r \cdot (\cos(2 \cdot \alpha) - \cos(3 \cdot \alpha));$$

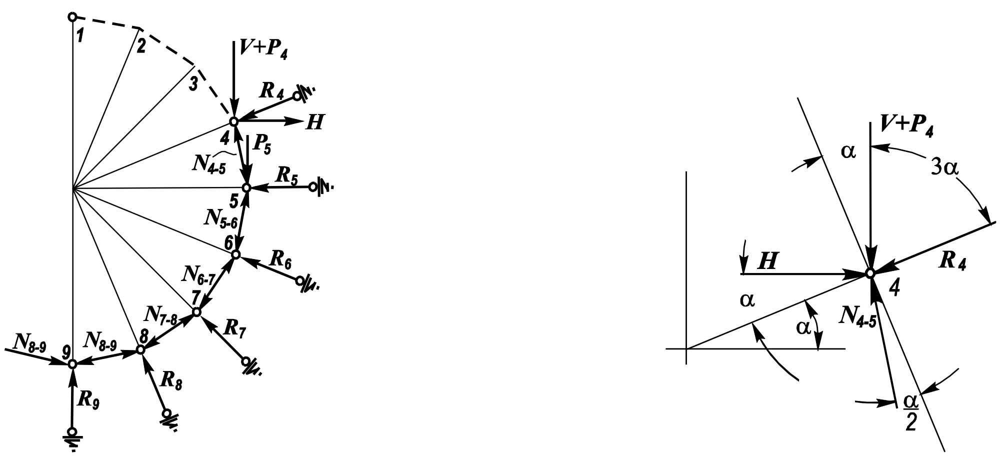
$$V = 0.5 \cdot P_1 + P_2 + P_3; \qquad H = \frac{V \cdot L_1 - P_2 \cdot L_2 - P_3 \cdot L_3}{F_1};$$

$$M_{3p} = V \cdot (L_1 - L_3) - H \cdot F_3; \qquad M_{2p} = V \cdot (L_1 - L_3) - H \cdot F_2 - P_3 \cdot (L_3 - L_2);$$

$$N_{12p} = H \cdot cos(0.5 \cdot \alpha) + \frac{P_1}{2} \cdot sin(0.5 \cdot \alpha); \qquad N_{23p} = H \cdot cos(1.5 \cdot \alpha) + \left(\frac{P_1}{2} + P_2\right) \cdot sin(1.5 \cdot \alpha);$$

$$N_{34p} = H \cdot cos(2.5 \cdot \alpha) + \left(\frac{P_1}{2} + P_2 + P_3\right) \cdot sin(2.5 \cdot \alpha).$$

Determination of forces in the hinged chain from the vertical loads



Design model for the determination of forces in the hinged chain from the vertical loads

$$N_{45p} = \frac{(V + P_4) \cdot sin(3 \cdot \alpha) + H \cdot sin(\alpha)}{cos(0.5 \cdot \alpha)}; \qquad N_{56p} = N_{45p} + \frac{P_5}{cos(0.5 \cdot \alpha)};$$

$$N_{67p} = N_{78p} = N_{89p} = N_{56p};$$

$$R_{4p} = H \cdot cos(\alpha) + N_{45p} \cdot sin(0.5 \cdot \alpha) - (V + P_4) \cdot cos(3 \cdot \alpha);$$

$R_{5p} = (N_{45p} + N_{56p}) \cdot sin(0.5 \cdot \alpha);$ $R_{6p} = (N_{56p} + N_{67p}) \cdot sin(0.5 \cdot \alpha);$

$R_{7p} = R_{8p} = R_{9p} = R_{6p}.$

Determination of forces in the three-hinged arch from the horizontal loads

$$H_{cr} = \frac{Q_1 \cdot F_1 + Q_2 \cdot F_2 + Q_3 \cdot F_3}{F_1}; \qquad H_{sk} = \frac{Q_2 \cdot (F_1 - F_2) + Q_3 \cdot (F_1 - F_3)}{F_1};$$

$$M_{2q} = (H_{cr} - Q_1) \cdot (F_1 - F_2); \qquad M_{3q} = H_{sk} \cdot F_3;$$

$$N_{12q} = (H_{cr} - Q_1) \cdot cos(0.5 \cdot \alpha); \qquad N_{23q} = (H_{cr} - Q_1 - Q_2) \cdot cos(1.5 \cdot \alpha);$$

$$N_{34q} = -H_{sk} \cdot cos(2.5 \cdot \alpha).$$

Determination of forces in the hinged chain from the horizontal loads

$$N_{45q} = -\frac{(H_{sk} + Q_4) \cdot cos(3 \cdot \alpha)}{cos(0.5 \cdot \alpha)}; \qquad N_{56q} = N_{67q} = N_{78q} = N_{89q} = N_{45q};$$

$$R_{4q} = -(H_{sk} + Q_4) \cdot sin(3 \cdot \alpha) + N_{45q} \cdot sin(0.5 \cdot \alpha); \qquad R_{5q} = (N_{45q} + N_{56q}) \cdot sin(0.5 \cdot \alpha);$$

$$R_{6q} = R_{7q} = R_{8q} = R_{9q} = R_{5q}.$$

Determination of forces in the primary system from the unit moments

Determination of forces in the three-hinged arch from the unit moment applied in the point 1

$$M_{11} = 1; \qquad H_1 = \frac{M_{11}}{F_1};$$

$$M_{21} = H_1 \cdot F_2; \qquad M_{31} = H_1 \cdot F_3;$$

$$N_{121} = -H_1 \cdot cos(0.5 \cdot \alpha); \qquad N_{231} = -H_1 \cdot cos(1.5 \cdot \alpha); \qquad N_{341} = -H_1 \cdot cos(2.5 \cdot \alpha).$$

Determination of forces in the hinged chain from the unit moment applied in the point 1

$$N_{451} = -\frac{H_1 \cdot sin(\alpha)}{cos(0.5 \cdot \alpha)}; \qquad N_{561} = N_{671} = N_{781} = N_{891} = N_{451};$$

$$R_{41} = N_{451} \cdot sin(0.5 \cdot \alpha) - H_1 \cdot cos(\alpha); \quad R_{51} = 2 \cdot N_{451} \cdot sin(0.5 \cdot \alpha); \quad R_{61} = R_{71} = R_{81} = R_{91} = R_{51}.$$

Determination of forces in the three-hinged arch from the unit moment applied in the point 4

$$M_{44} = 1 \quad ; \qquad H_4 = \frac{M_{44}}{F_1};$$

$$M_{24} = M_{44} - H_4 \cdot F_2; \qquad M_{34} = M_{44} - H_4 \cdot F_3;$$

$$N_{124} = H_4 \cdot cos(0.5 \cdot \alpha); \qquad N_{234} = H_4 \cdot cos(1.5 \cdot \alpha); \qquad N_{344} = H_4 \cdot cos(2.5 \cdot \alpha).$$

Determination of forces in the hinged chain from the unit moment applied in the point 4

$$N_{454} = \frac{H_4 \cdot sin(\alpha)}{cos(0.5 \cdot \alpha)} + \frac{M_{44} \cdot sin(0.5 \cdot \alpha)}{S \cdot cos(0.5 \cdot \alpha)}; \qquad N_{564} = N_{454} + \frac{M_{44} \cdot sin(0.5 \cdot \alpha)}{S \cdot cos(0.5 \cdot \alpha)};$$

$$N_{674} = N_{784} = N_{894} = N_{564};$$

$$R_{41} = N_{451} \cdot sin(0.5 \cdot \alpha) + H_4 \cdot cos(\alpha) + \frac{M_{44} \cdot cos(\alpha)}{S};$$

$$R_{51} = -\frac{M_{44} \cdot cos(0.5 \cdot \alpha)}{S} + (N_{454} + N_{564}) \cdot sin(0.5 \cdot \alpha);$$

$$R_{64} = 2 \cdot N_{564} \cdot sin(0.5 \cdot \alpha); \qquad R_{74} = R_{84} = R_{94} = R_{64}.$$

Determination of forces in the hinged chain from the unit moment applied in the point 5

$$M_{55} = 1;$$

$$N_{455} = -\frac{M_{55} \cdot sin(0.5 \cdot \alpha)}{S \cdot cos(0.5 \cdot \alpha)}; \qquad N_{565} = N_{455};$$

$$R_{45} = -\frac{M_{55}}{S \cdot cos(0.5 \cdot \alpha)};$$

$$R_{55} = \frac{2 \cdot M_{55} \cdot cos(0.5 \cdot \alpha)}{S} + 2 \cdot N_{455} \cdot sin(0.5 \cdot \alpha); R_{65} = -\frac{M_{55}}{S \cdot cos(0.5 \cdot \alpha)}.$$

Determination of forces in the hinged chain from the unit moment applied in the point 6

$$M_{66} = 1;$$

$$N_{566} = -\frac{M_{66} \cdot sin(0.5 \cdot \alpha)}{S \cdot cos(0.5 \cdot \alpha)}; \qquad N_{676} = N_{566};$$

$$R_{56} = -\frac{M_{66}}{S \cdot cos(0.5 \cdot \alpha)}; \qquad R_{66} = \frac{2 \cdot M_{66} \cdot cos(0.5 \cdot \alpha)}{S} + 2 \cdot N_{566} \cdot sin(0.5 \cdot \alpha);$$

$$R_{76} = -\frac{M_{66}}{S \cdot cos(0.5 \cdot \alpha)}.$$

Determination of forces in the hinged chain from the unit moment applied in the point 7

$$M_{77} = 1;$$

$$N_{677} = -\frac{M_{77} \cdot sin(0.5 \cdot \alpha)}{S \cdot cos(0.5 \cdot \alpha)}; \qquad N_{787} = N_{677};$$

$$R_{67} = -\frac{M_{77}}{S \cdot cos(0.5 \cdot \alpha)}; \qquad R_{77} = \frac{2 \cdot M_{77} \cdot cos(0.5 \cdot \alpha)}{S} + 2 \cdot N_{677} \cdot sin(0.5 \cdot \alpha);$$

$$R_{87} = -\frac{M_{77}}{S \cdot cos(0.5 \cdot \alpha)}.$$

Determination of forces in the hinged chain from the unit moment applied in the point 8

$$M_{88} = 1;$$

$$N_{788} = -\frac{M_{88} \cdot sin(0.5 \cdot \alpha)}{S \cdot cos(0.5 \cdot \alpha)}; \qquad N_{898} = N_{788};$$

$$R_{78} = -\frac{M_{88}}{S \cdot cos(0.5 \cdot \alpha)}; \qquad R_{88} = \frac{2 \cdot M_{88} \cdot cos(0.5 \cdot \alpha)}{S} + 2 \cdot N_{788} \cdot sin(0.5 \cdot \alpha);$$

$$R_{98} = -\frac{2 \cdot M_{88}}{S \cdot cos(0.5 \cdot \alpha)}.$$

Determination of forces in the hinged chain from the unit moment applied in the point 9

$M_{99} = 1$;

$N_{899} = -\dfrac{M_{99} \cdot sin(0.5 \cdot \alpha)}{S \cdot cos(0.5 \cdot \alpha)}$;

$R_{89} = -\dfrac{M_{99}}{S \cdot cos(0.5 \cdot \alpha)}$; $\quad R_{99} = \dfrac{2 \cdot M_{99} \cdot cos(0.5 \cdot \alpha)}{S} + 2 \cdot N_{899} \cdot sin(0.5 \cdot \alpha)$.

Determination of displacements

$$\delta_{11R} = 2 \cdot \frac{1}{D} \cdot \left(R_{41}{}^2 + R_{51}{}^2 + R_{61}{}^2 + R_{71}{}^2 + R_{81}{}^2 + R_{91}{}^2 \cdot 0.5\right);$$

$$\delta_{11M} = 2 \cdot \frac{S}{3 \cdot E \cdot I} \cdot \left(M_{11}{}^2 + M_{11} \cdot M_{21} + 2 \cdot M_{21}{}^2 + M_{21} \cdot M_{31} + 2 \cdot M_{31}{}^2\right);$$

$$\delta_{11N} = 2 \cdot \frac{S}{E \cdot F} \cdot \left(N_{121}{}^2 + N_{231}{}^2 + N_{341}{}^2 + N_{451}{}^2 + N_{561}{}^2 + N_{671}{}^2 + N_{781}{}^2 + N_{891}{}^2\right);$$

$$\delta_{11} = \delta_{11R} + \delta_{11M} + \delta_{11N};$$

$$\delta_{14R} = 2 \cdot \frac{1}{D} \cdot \left(R_{41} \cdot R_{44} + R_{51} \cdot R_{54} + R_{61} \cdot R_{64} + R_{71} \cdot R_{74} + R_{81} \cdot R_{84} + R_{91} \cdot R_{94} \cdot 0.5\right);$$

$$\delta_{14M} = 2 \cdot \frac{S}{6 \cdot E \cdot I} \cdot \left(M_{11} \cdot M_{24} + 4 \cdot M_{21} \cdot M_{24} + M_{21} \cdot M_{34} + M_{31} \cdot M_{24} + 4 \cdot M_{31} \cdot M_{34} + M_{31} \cdot M_{44}\right);$$

$$\delta_{14N} = 2 \cdot \frac{S}{E \cdot F} \cdot \left(N_{121} \cdot N_{124} + N_{231} \cdot N_{234} + N_{341} \cdot N_{344} + N_{451} \cdot N_{454} + \right.$$
$$\left. N_{561} \cdot N_{564} + N_{671} \cdot N_{674} + N_{781} \cdot N_{784} + N_{891} \cdot N_{894}\right);$$

$$\delta_{14} = \delta_{14R} + \delta_{14M} + \delta_{14N};$$

$$\delta_{15R} = 2 \cdot \frac{1}{D} \cdot \left(R_{41} \cdot R_{45} + R_{51} \cdot R_{55} + R_{61} \cdot R_{65}\right); \quad \delta_{15M} = 0;$$

$$\delta_{15N} = 2 \cdot \frac{S}{E \cdot F} \cdot \left(N_{451} \cdot N_{455} + N_{561} \cdot N_{565}\right); \quad \delta_{15} = \delta_{15R} + \delta_{15M} + \delta_{15N};$$

$$\delta_{16R} = 2 \cdot \frac{1}{D} \cdot \left(R_{51} \cdot R_{56} + R_{61} \cdot R_{66} + R_{71} \cdot R_{76}\right); \quad \delta_{16M} = 0;$$

$$\delta_{16N} = 2 \cdot \frac{S}{E \cdot F} \cdot \left(N_{561} \cdot N_{566} + N_{671} \cdot N_{676}\right); \quad \delta_{16} = \delta_{16R} + \delta_{16M} + \delta_{16N};$$

$$\delta_{17R} = 2 \cdot \frac{1}{D} \cdot \left(R_{61} \cdot R_{67} + R_{71} \cdot R_{77} + R_{81} \cdot R_{87}\right); \quad \delta_{17M} = 0;$$

$$\delta_{17N} = 2 \cdot \frac{S}{E \cdot F} \cdot \left(N_{671} \cdot N_{677} + N_{781} \cdot N_{787}\right); \quad \delta_{17} = \delta_{17R} + \delta_{17M} + \delta_{17N};$$

$$\delta_{18R} = 2 \cdot \frac{1}{D} \cdot \left(R_{71} \cdot R_{78} + R_{81} \cdot R_{88} + R_{91} \cdot R_{98} \cdot 0.5\right); \quad \delta_{18M} = 0;$$

$$\delta_{18N} = 2 \cdot \frac{S}{E \cdot F} \cdot \left(N_{781} \cdot N_{788} + N_{891} \cdot N_{898}\right); \quad \delta_{18} = \delta_{18R} + \delta_{18M} + \delta_{18N};$$

$$\delta_{19R} = 2 \cdot \frac{1}{D} \cdot \left(R_{81} \cdot R_{89} + R_{91} \cdot R_{99} \cdot 0.5\right); \quad \delta_{19M} = 0; \quad \delta_{19N} = 2 \cdot \frac{S}{E \cdot F} \cdot N_{891} \cdot N_{899};$$

$$\delta_{19} = \delta_{19R} + \delta_{19M} + \delta_{19N};$$

$$\delta_{44R} = 2 \cdot \frac{1}{D} \cdot \left(R_{44}{}^2 + R_{54}{}^2 + R_{64}{}^2 + R_{74}{}^2 + R_{84}{}^2 + R_{94}{}^2 \cdot 0.5\right);$$

$$\delta_{44M} = 2 \cdot \frac{S}{3 \cdot E \cdot I} \cdot \left(2 \cdot M_{24}{}^2 + 2 \cdot M_{34}{}^2 + M_{24} \cdot M_{34} + 2 \cdot M_{44}{}^2 + M_{34} \cdot M_{44}\right);$$

$$\delta_{44N} = 2 \cdot \frac{S}{E \cdot F} \cdot \left(N_{124}{}^2 + N_{234}{}^2 + N_{344}{}^2 + N_{454}{}^2 + N_{564}{}^2 + N_{674}{}^2 + N_{784}{}^2 + N_{894}{}^2\right);$$

$$\delta_{44} = \delta_{44R} + \delta_{44M} + \delta_{44N};$$

$$\delta_{45R} = 2 \cdot \frac{1}{D} \cdot \left(R_{44} \cdot R_{45} + R_{54} \cdot R_{55} + R_{64} \cdot R_{65}\right); \qquad \delta_{45M} = 2 \cdot \frac{S}{6 \cdot E \cdot I} \cdot M_{44} \cdot M_{55};$$

$$\delta_{45N} = 2 \cdot \frac{S}{E \cdot F} \cdot \left(N_{454} \cdot N_{455} + N_{564} \cdot N_{565}\right);$$

$$\delta_{45} = \delta_{45R} + \delta_{45M} + \delta_{45N};$$

$$\delta_{46R} = 2 \cdot \frac{1}{D} \cdot \left(R_{54} \cdot R_{56} + R_{64} \cdot R_{66} + R_{74} \cdot R_{76}\right); \qquad \delta_{46M} = 0;$$

$$\delta_{46N} = 2 \cdot \frac{S}{E \cdot F} \cdot \left(N_{564} \cdot N_{566} + N_{674} \cdot N_{676}\right);$$

$$\delta_{46} = \delta_{46R} + \delta_{46M} + \delta_{46N};$$

$$\delta_{47R} = 2 \cdot \frac{1}{D} \cdot \left(R_{64} \cdot R_{67} + R_{74} \cdot R_{77} + R_{84} \cdot R_{87}\right); \qquad \delta_{47M} = 0;$$

$$\delta_{47N} = 2 \cdot \frac{S}{E \cdot F} \cdot \left(N_{674} \cdot N_{677} + N_{784} \cdot N_{787}\right);$$

$$\delta_{47} = \delta_{47R} + \delta_{47M} + \delta_{47N};$$

$$\delta_{48R} = 2 \cdot \frac{1}{D} \cdot \left(R_{74} \cdot R_{78} + R_{84} \cdot R_{88} + R_{94} \cdot R_{98} \cdot 0.5\right); \qquad \delta_{48M} = 0;$$

$$\delta_{48N} = 2 \cdot \frac{S}{E \cdot F} \cdot \left(N_{784} \cdot N_{788} + N_{894} \cdot N_{898}\right);$$

$$\delta_{48} = \delta_{48R} + \delta_{48M} + \delta_{48N};$$

$$\delta_{49R} = 2 \cdot \frac{1}{D} \cdot \left(R_{84} \cdot R_{89} + R_{94} \cdot R_{99} \cdot 0.5\right); \qquad \delta_{49M} = 0; \qquad \delta_{49N} = 2 \cdot \frac{S}{E \cdot F} \cdot N_{894} \cdot N_{899};$$

$$\delta_{49} = \delta_{49R} + \delta_{49M} + \delta_{49N};$$

$$\delta_{55R} = 2 \cdot \frac{1}{D} \cdot \left(R_{45}{}^2 + R_{55}{}^2 + R_{65}{}^2\right); \qquad \delta_{55M} = 2 \cdot \frac{S}{3 \cdot E \cdot I} \cdot 2 \cdot M_{55}{}^2; \; \delta_{55N} = 2 \cdot \frac{S}{E \cdot F} \cdot \left(N_{455}{}^2 + N_{565}{}^2\right);$$

$$\delta_{55} = \delta_{55R} + \delta_{55M} + \delta_{55N};$$

$$\delta_{56R} = 2 \cdot \frac{1}{D} \cdot \left(R_{55} \cdot R_{56} + R_{65} \cdot R_{66}\right); \qquad \delta_{56M} = 2 \cdot \frac{S}{6 \cdot E \cdot I} \cdot M_{55} \cdot M_{66}; \; \delta_{56N} = 2 \cdot \frac{S}{E \cdot F} \cdot N_{565} \cdot N_{566};$$

$$\delta_{56} = \delta_{56R} + \delta_{56M} + \delta_{56N};$$

$$\delta_{57R} = 2 \cdot \frac{1}{D} \cdot R_{65} \cdot R_{67}; \qquad \delta_{57M} = 0; \qquad \delta_{57N} = 0;$$

$$\delta_{57} = \delta_{57R} + \delta_{57M} + \delta_{57N};$$

$$\delta_{58R} = 0; \qquad \delta_{58M} = 0; \qquad \delta_{58N} = 0;$$

$$\delta_{58} = \delta_{58R} + \delta_{58M} + \delta_{58N};$$

$$\delta_{59R} = 0; \qquad \delta_{59M} = 0; \qquad \delta_{59N} = 0;$$

$$\delta_{59} = \delta_{59R} + \delta_{59M} + \delta_{59N};$$

$$\delta_{66R} = 2 \cdot \frac{1}{D} \cdot \left(R_{56}{}^2 + R_{66}{}^2 + R_{76}{}^2\right); \qquad \delta_{66M} = 2 \cdot \frac{S}{3 \cdot E \cdot I} \cdot 2 \cdot M_{66}{}^2; \qquad \delta_{66N} = 2 \cdot \frac{S}{E \cdot F} \cdot \left(N_{566}{}^2 + N_{676}{}^2\right);$$

$$\delta_{66} = \delta_{66R} + \delta_{66M} + \delta_{66N};$$

$$\delta_{67R} = 2 \cdot \frac{1}{D} \cdot \left(R_{66} \cdot R_{67} + R_{76} \cdot R_{77}\right); \qquad \delta_{67M} = 2 \cdot \frac{S}{6 \cdot E \cdot I} \cdot M_{66} \cdot M_{77}; \; \delta_{67N} = 2 \cdot \frac{S}{E \cdot F} \cdot N_{676} \cdot N_{677};$$

$$\delta_{67} = \delta_{67R} + \delta_{67M} + \delta_{67N};$$

$$\delta_{68R} = 2 \cdot \frac{1}{D} \cdot R_{76} \cdot R_{78}; \qquad \delta_{68M} = 0; \qquad \delta_{68N} = 0;$$

$$\delta_{68} = \delta_{68R} + \delta_{68M} + \delta_{68N};$$

$$\delta_{69R} = 0; \qquad \delta_{69M} = 0; \qquad \delta_{69N} = 0;$$

$$\delta_{69} = \delta_{69R} + \delta_{69M} + \delta_{69N};$$

$$\delta_{77R} = 2 \cdot \frac{1}{D} \cdot \left(R_{67}^{2} + R_{77}^{2} + R_{87}^{2}\right); \qquad \delta_{77M} = 2 \cdot \frac{S}{3 \cdot E \cdot I} \cdot 2 \cdot M_{77}^{2}; \qquad \delta_{77N} = 2 \cdot \frac{S}{E \cdot F} \cdot \left(N_{677}^{2} + N_{787}^{2}\right);$$

$$\delta_{77} = \delta_{77R} + \delta_{77M} + \delta_{77N};$$

$$\delta_{78R} = 2 \cdot \frac{1}{D} \cdot \left(R_{77} \cdot R_{78} + R_{87} \cdot R_{88}\right); \qquad \delta_{78M} = 2 \cdot \frac{S}{6 \cdot E \cdot I} \cdot M_{77} \cdot M_{88}; \qquad \delta_{78N} = 2 \cdot \frac{S}{E \cdot F} \cdot N_{787} \cdot N_{788};$$

$$\delta_{78} = \delta_{78R} + \delta_{78M} + \delta_{78N};$$

$$\delta_{79R} = 2 \cdot \frac{1}{D} \cdot R_{87} \cdot R_{89}; \qquad \delta_{79M} = 0; \qquad \delta_{79N} = 0;$$

$$\delta_{79} = \delta_{79R} + \delta_{79M} + \delta_{79N};$$

$$\delta_{88R} = 2 \cdot \frac{1}{D} \cdot \left(R_{78}^{2} + R_{88}^{2} + R_{98}^{2} \cdot 0.5\right); \qquad \delta_{88M} = 2 \cdot \frac{S}{3 \cdot E \cdot I} \cdot 2 \cdot M_{88}^{2}; \qquad \delta_{88N} = 2 \cdot \frac{S}{E \cdot F} \cdot \left(N_{788}^{2} + N_{898}^{2}\right);$$

$$\delta_{88} = \delta_{88R} + \delta_{88M} + \delta_{88N};$$

$$\delta_{89R} = 2 \cdot \frac{1}{D} \cdot \left(R_{88} \cdot R_{89} + R_{98} \cdot R_{99} \cdot 0.5\right); \qquad \delta_{89M} = 2 \cdot \frac{S}{6 \cdot E \cdot I} \cdot M_{88} \cdot M_{99};$$

$$\delta_{89N} = 2 \cdot \frac{S}{E \cdot F} \cdot N_{898} \cdot N_{899};$$

$$\delta_{89} = \delta_{89R} + \delta_{89M} + \delta_{89N};$$

$$\delta_{99R} = 2 \cdot \frac{1}{D} \cdot \left(R_{89}^{2} + R_{99}^{2} \cdot 0.5\right); \qquad \delta_{99M} = 2 \cdot \frac{S}{3 \cdot E \cdot I} \cdot M_{99}^{2}; \qquad \delta_{99N} = 2 \cdot \frac{S}{E \cdot F} \cdot N_{899}^{2};$$

$$\delta_{99} = \delta_{99R} + \delta_{99M} + \delta_{99N};$$

$$\delta_{1pR} = 2 \cdot \frac{1}{D} \cdot \left(R_{41} \cdot R_{4p} + R_{51} \cdot R_{5p} + R_{61} \cdot R_{6p} + R_{71} \cdot R_{7p} + R_{81} \cdot R_{8p} + R_{91} \cdot R_{9p} \cdot 0.5\right);$$

$$\delta_{1pM} = 2 \cdot \frac{S}{6 \cdot E \cdot I} \cdot \left(4 \cdot M_{2p} \cdot M_{21} + M_{2p} \cdot M_{11} + 4 \cdot M_{3p} \cdot M_{31} + M_{3p} \cdot M_{21} + M_{2p} \cdot M_{31}\right);$$

$$\delta_{1pN} = 2 \cdot \frac{S}{E \cdot F} \cdot \left(N_{12p} \cdot N_{121} + N_{23p} \cdot N_{231} + N_{34p} \cdot N_{341} + N_{45p} \cdot N_{451} + \right.$$

$$\left. N_{56p} \cdot N_{561} + N_{67p} \cdot N_{671} + N_{78p} \cdot N_{781} + N_{89p} \cdot N_{891} \right);$$

$$\delta_{1p} = \delta_{1pR} + \delta_{1pM} + \delta_{1pN};$$

$$\delta_{4pR} = 2 \cdot \frac{1}{D} \cdot \left(R_{44} \cdot R_{4p} + R_{54} \cdot R_{5p} + R_{64} \cdot R_{6p} + R_{74} \cdot R_{7p} + R_{84} \cdot R_{8p} + R_{94} \cdot R_{9p} \cdot 0.5\right);$$

$$\delta_{4pM} = 2 \cdot \frac{S}{6 \cdot E \cdot I} \cdot \left(4 \cdot M_{2p} \cdot M_{24} + 4 \cdot M_{3p} \cdot M_{34} + M_{3p} \cdot M_{24} + M_{2p} \cdot M_{34} + M_{3p} \cdot M_{44}\right);$$

$$\delta_{4pN} = 2 \cdot \frac{S}{E \cdot F} \cdot \left(N_{12p} \cdot N_{124} + N_{23p} \cdot N_{234} + N_{34p} \cdot N_{344} + N_{45p} \cdot N_{454} + \right.$$

$$\left. N_{56p} \cdot N_{564} + N_{67p} \cdot N_{674} + N_{78p} \cdot N_{784} + N_{89p} \cdot N_{894} \right);$$

$$\delta_{4p} = \delta_{4pR} + \delta_{4pM} + \delta_{4pN};$$

$$\delta_{5pR} = 2 \cdot \frac{1}{D} \cdot \left(R_{45} \cdot R_{4p} + R_{55} \cdot R_{5p} + R_{65} \cdot R_{6p} \right); \qquad \delta_{5pM} = 0;$$

$$\delta_{5pN} = 2 \cdot \frac{S}{E \cdot F} \cdot \left(N_{45p} \cdot N_{455} + N_{56p} \cdot N_{565} \right);$$

$$\delta_{5p} = \delta_{5pR} + \delta_{5pM} + \delta_{5pN};$$

$$\delta_{6pR} = 2 \cdot \frac{1}{D} \cdot \left(R_{56} \cdot R_{5p} + R_{66} \cdot R_{6p} + R_{76} \cdot R_{7p} \right); \qquad \delta_{6pM} = 0;$$

$$\delta_{6pN} = 2 \cdot \frac{S}{E \cdot F} \cdot \left(N_{56p} \cdot N_{566} + N_{67p} \cdot N_{676} \right);$$

$$\delta_{6p} = \delta_{6pR} + \delta_{6pM} + \delta_{6pN};$$

$$\delta_{7pR} = 2 \cdot \frac{1}{D} \cdot \left(R_{67} \cdot R_{6p} + R_{77} \cdot R_{7p} + R_{87} \cdot R_{8p} \right); \qquad \delta_{7pM} = 0;$$

$$\delta_{7pN} = 2 \cdot \frac{S}{E \cdot F} \cdot \left(N_{67p} \cdot N_{677} + N_{78p} \cdot N_{787} \right);$$

$$\delta_{7p} = \delta_{7pR} + \delta_{7pM} + \delta_{7pN};$$

$$\delta_{8pR} = 2 \cdot \frac{1}{D} \cdot \left(R_{78} \cdot R_{7p} + R_{88} \cdot R_{8p} + R_{98} \cdot R_{9p} \cdot 0.5 \right); \qquad \delta_{8pM} = 0;$$

$$\delta_{8pN} = 2 \cdot \frac{S}{E \cdot F} \cdot \left(N_{78p} \cdot N_{788} + N_{89p} \cdot N_{898} \right);$$

$$\delta_{8p} = \delta_{8pR} + \delta_{8pM} + \delta_{8pN};$$

$$\delta_{9pR} = 2 \cdot \frac{1}{D} \cdot \left(R_{89} \cdot R_{8p} + R_{99} \cdot R_{9p} \cdot 0.5 \right); \qquad \delta_{9pM} = 0; \qquad \delta_{9pN} = 2 \cdot \frac{S}{E \cdot F} \cdot N_{89p} \cdot N_{899};$$

$$\delta_{9p} = \delta_{9pR} + \delta_{9pM} + \delta_{9pN};$$

$$\delta_{1qR} = 2 \cdot \frac{1}{D} \cdot \left(R_{41} \cdot R_{4q} + R_{51} \cdot R_{5q} + R_{61} \cdot R_{6q} + R_{71} \cdot R_{7q} + R_{81} \cdot R_{8q} + R_{91} \cdot R_{9q} \cdot 0.5 \right);$$

$$\delta_{1qM} = 2 \cdot \frac{S}{6 \cdot E \cdot I} \cdot \left(4 \cdot M_{2q} \cdot M_{21} + M_{2q} \cdot M_{11} + 4 \cdot M_{3q} \cdot M_{31} + M_{3q} \cdot M_{21} + M_{2q} \cdot M_{31} \right);$$

$$\delta_{1qN} = 2 \cdot \frac{S}{E \cdot F} \cdot \left(N_{12q} \cdot N_{121} + N_{23q} \cdot N_{231} + N_{34q} \cdot N_{341} + N_{45q} \cdot N_{451} + \right.$$
$$\left. N_{56q} \cdot N_{561} + N_{67q} \cdot N_{671} + N_{78q} \cdot N_{781} + N_{89q} \cdot N_{891} \right);$$

$$\delta_{1q} = \delta_{1qR} + \delta_{1qM} + \delta_{1qN};$$

$$\delta_{4qR} = 2 \cdot \frac{1}{D} \cdot \left(R_{44} \cdot R_{4q} + R_{54} \cdot R_{5q} + R_{64} \cdot R_{6q} + R_{74} \cdot R_{7q} + R_{84} \cdot R_{8q} + R_{94} \cdot R_{9q} \cdot 0.5 \right);$$

$$\delta_{4qM} = 2 \cdot \frac{S}{6 \cdot E \cdot I} \cdot \left(4 \cdot M_{qp} \cdot M_{24} + 4 \cdot M_{qp} \cdot M_{34} + M_{3q} \cdot M_{24} + M_{2q} \cdot M_{34} + M_{3q} \cdot M_{44} \right);$$

$$\delta_{4qN} = 2 \cdot \frac{S}{E \cdot F} \cdot \left(N_{12q} \cdot N_{124} + N_{23q} \cdot N_{234} + N_{34q} \cdot N_{344} + N_{45q} \cdot N_{454} + \right.$$
$$\left. N_{56q} \cdot N_{564} + N_{67q} \cdot N_{674} + N_{78q} \cdot N_{784} + N_{89q} \cdot N_{894} \right);$$

$$\delta_{4q} = \delta_{4qR} + \delta_{4qM} + \delta_{4qN};$$

$$\delta_{5qR} = 2 \cdot \frac{1}{D} \cdot \left(R_{45} \cdot R_{4q} + R_{55} \cdot R_{5q} + R_{65} \cdot R_{6q} \right); \qquad \delta_{5qM} = 0;$$

$$\delta_{5qN} = 2 \cdot \frac{S}{E \cdot F} \cdot \left(N_{45q} \cdot N_{455} + N_{56q} \cdot N_{565} \right);$$

$$\delta_{5q} = \delta_{5qR} + \delta_{5qM} + \delta_{5qN};$$

$$\delta_{6qR}=2\cdot\frac{1}{D}\cdot\left(R_{56}\cdot R_{5q}+R_{66}\cdot R_{6q}+R_{76}\cdot R_{7q}\right);\qquad \delta_{6qM}=0;$$

$$\delta_{6qN}=2\cdot\frac{S}{E\cdot F}\cdot\left(N_{56q}\cdot N_{566}+N_{67q}\cdot N_{676}\right);$$

$$\delta_{6q}=\delta_{6qR}+\delta_{6qM}+\delta_{6qN};$$

$$\delta_{7qR}=2\cdot\frac{1}{D}\cdot\left(R_{67}\cdot R_{6q}+R_{77}\cdot R_{7q}+R_{87}\cdot R_{8q}\right);\qquad \delta_{7qM}=0;$$

$$\delta_{7qN}=2\cdot\frac{S}{E\cdot F}\cdot\left(N_{67q}\cdot N_{677}+N_{78q}\cdot N_{787}\right);$$

$$\delta_{7q}=\delta_{7qR}+\delta_{7qM}+\delta_{7qN};$$

$$\delta_{8qR}=2\cdot\frac{1}{D}\cdot\left(R_{78}\cdot R_{7q}+R_{88}\cdot R_{8q}+R_{98}\cdot R_{9q}\cdot 0.5\right);\qquad \delta_{8qM}=0;$$

$$\delta_{8qN}=2\cdot\frac{S}{E\cdot F}\cdot\left(N_{78q}\cdot N_{788}+N_{89q}\cdot N_{898}\right);$$

$$\delta_{8q}=\delta_{8qR}+\delta_{8qM}+\delta_{8qN};$$

$$\delta_{9qR}=2\cdot\frac{1}{D}\cdot\left(R_{89}\cdot R_{8q}+R_{99}\cdot R_{9q}\cdot 0.5\right);\qquad \delta_{9qM}=0;\qquad \delta_{9qN}=2\cdot\frac{S}{E\cdot F}\cdot N_{89q}\cdot N_{899};$$

$$\delta_{9q}=\delta_{9qR}+\delta_{9qM}+\delta_{9qN}.$$

Determination of redundants

$$\Delta_1=\begin{bmatrix}\delta_{11}&\delta_{14}&\delta_{15}&\delta_{16}&\delta_{17}&\delta_{18}&\delta_{19}\\ \delta_{14}&\delta_{44}&\delta_{45}&\delta_{46}&\delta_{47}&\delta_{48}&\delta_{49}\\ \delta_{15}&\delta_{45}&\delta_{55}&\delta_{56}&\delta_{57}&\delta_{58}&\delta_{59}\\ \delta_{16}&\delta_{46}&\delta_{56}&\delta_{66}&\delta_{67}&\delta_{68}&\delta_{69}\\ \delta_{17}&\delta_{47}&\delta_{57}&\delta_{67}&\delta_{77}&\delta_{78}&\delta_{79}\\ \delta_{18}&\delta_{48}&\delta_{58}&\delta_{68}&\delta_{78}&\delta_{88}&\delta_{89}\\ \delta_{19}&\delta_{49}&\delta_{59}&\delta_{69}&\delta_{79}&\delta_{89}&\delta_{99}\end{bmatrix}\qquad \Delta_{pq}=\begin{bmatrix}\delta_{1p}+\delta_{1q}\\ \delta_{4p}+\delta_{4q}\\ \delta_{5p}+\delta_{5q}\\ \delta_{6p}+\delta_{6q}\\ \delta_{7p}+\delta_{7q}\\ \delta_{8p}+\delta_{8q}\\ \delta_{9p}+\delta_{9q}\end{bmatrix}\qquad X=-\Delta_1^{-1}\cdot\Delta_{pq}=\begin{bmatrix}X_1\\ X_4\\ X_5\\ X_6\\ X_7\\ X_8\\ X_9\end{bmatrix}$$

Determination of internal forces

$$M_1=M_{11}\cdot X_1;\qquad N_{12}=N_{121}\cdot X_1+N_{124}\cdot X_4+N_{12p}+N_{12q};$$

$$M_2=M_{21}\cdot X_1+M_{24}\cdot X_4+M_{2p}+M_{2q};\qquad N_{23}=N_{231}\cdot X_1+N_{234}\cdot X_4+N_{23p}+N_{23q};$$

$$M_3=M_{31}\cdot X_1+M_{34}\cdot X_4+M_{3p}+M_{3q};\qquad N_{34}=N_{341}\cdot X_1+N_{344}\cdot X_4+N_{34p}+N_{34q};$$

$$M_4=M_{44}\cdot X_4;\qquad N_{45}=N_{451}\cdot X_1+N_{454}\cdot X_4+N_{455}\cdot X_5+N_{45p}+N_{45q};$$

$$M_5=M_{55}\cdot X_5;\qquad N_{56}=N_{561}\cdot X_1+N_{564}\cdot X_4+N_{565}\cdot X_5+N_{566}\cdot X_6+N_{56p}+N_{56q};$$

$$M_6=M_{66}\cdot X_6;\qquad N_{67}=N_{671}\cdot X_1+N_{674}\cdot X_4+N_{676}\cdot X_6+N_{677}\cdot X_7+N_{67p}+N_{67q};$$

$$M_7=M_{77}\cdot X_7;\qquad N_{78}=N_{781}\cdot X_1+N_{784}\cdot X_4+N_{787}\cdot X_7+N_{788}\cdot X_8+N_{78p}+N_{78q};$$

$$M_8=M_{88}\cdot X_8;\qquad N_{89}=N_{891}\cdot X_1+N_{894}\cdot X_4+N_{898}\cdot X_8+N_{899}\cdot X_9+N_{89p}+N_{89q};$$

$$M_9=M_{99}\cdot X_9.$$

Determination of elastic reactions

$$R_1=R_2=R_3=0;$$

$$R_4=R_{41}\cdot X_1+R_{44}\cdot X_4+R_{45}\cdot X_5+R_{4p}+R_{4q};$$

$$R_5=R_{51}\cdot X_1+R_{54}\cdot X_4+R_{55}\cdot X_5+R_{56}\cdot X_6+R_{5p}+R_{5q};$$

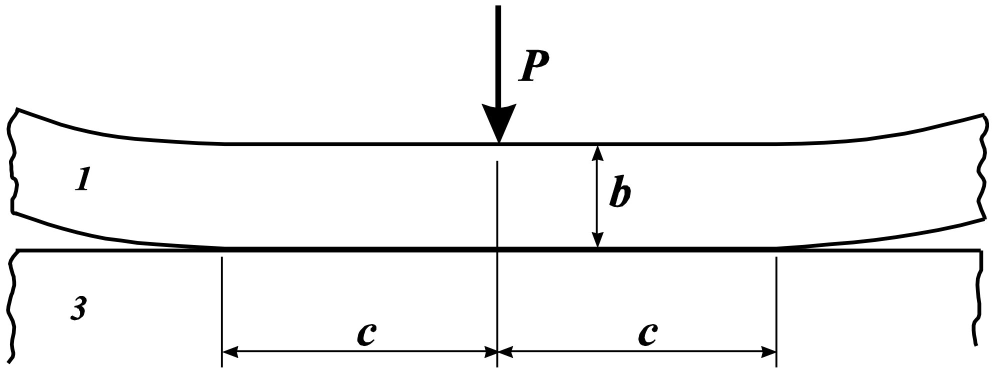
$$R_6=R_{61}\cdot X_1+R_{64}\cdot X_4+R_{65}\cdot X_5+R_{66}\cdot X_6+R_{67}\cdot X_7+R_{6p}+R_{6q};$$

$$R_7 = R_{71} \cdot X_1 + R_{74} \cdot X_4 + R_{76} \cdot X_6 + R_{77} \cdot X_7 + R_{78} \cdot X_8 + R_{7p} + R_{7q};$$

$$R_8 = R_{81} \cdot X_1 + R_{84} \cdot X_4 + R_{87} \cdot X_7 + R_{88} \cdot X_8 + R_{89} \cdot X_9 + R_{8p} + R_{8q};$$

$$R_9 = R_{91} \cdot X_1 + R_{94} \cdot X_4 + R_{98} \cdot X_8 + R_{99} \cdot X_9 + R_{9p} + R_{9q}.$$

Contact with Detachment for a Layer and Subgrade with a Concentrated Shear Force Applied to the Layer



Objective: Determination of the size of a contact area of a layer with the subgrade, when a concentrated shear force is applied to the layer, in the structurally nonlinear formulation.

Initial data file: **Contact_3_731.SPR**

Problem formulation: The elastic layer of height b lies on the elastic subgrade with the possibility of slipping and is subjected to the action of the concentrated shear force P applied to the upper surface. Determine the size of the area of contact of the layer with the subgrade $2 \cdot c$.

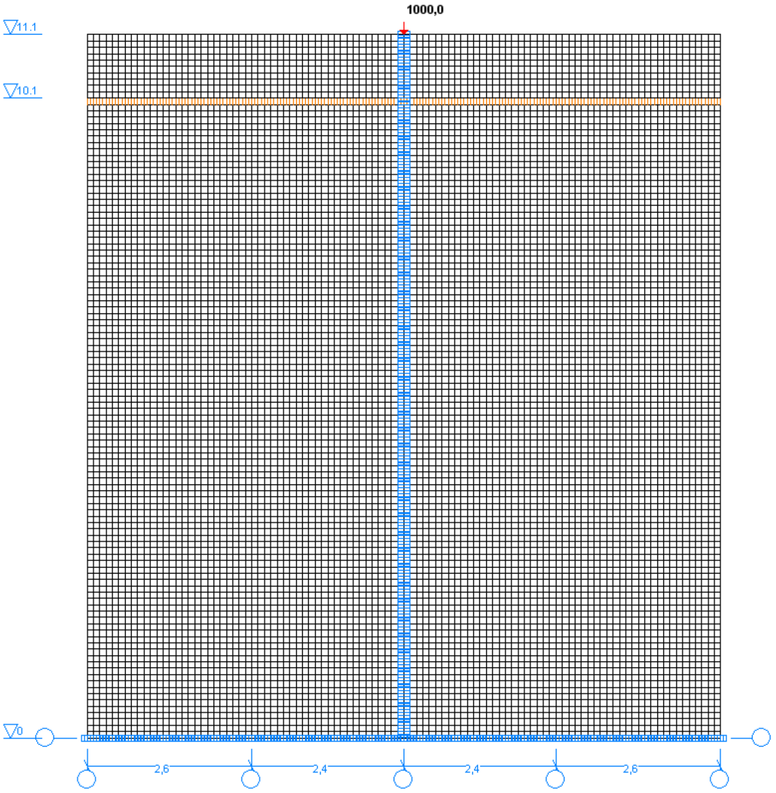
References: K. Johnson, Mechanics of Contact Interaction, Moscow, Mir, 1989, p. 163

Initial data:

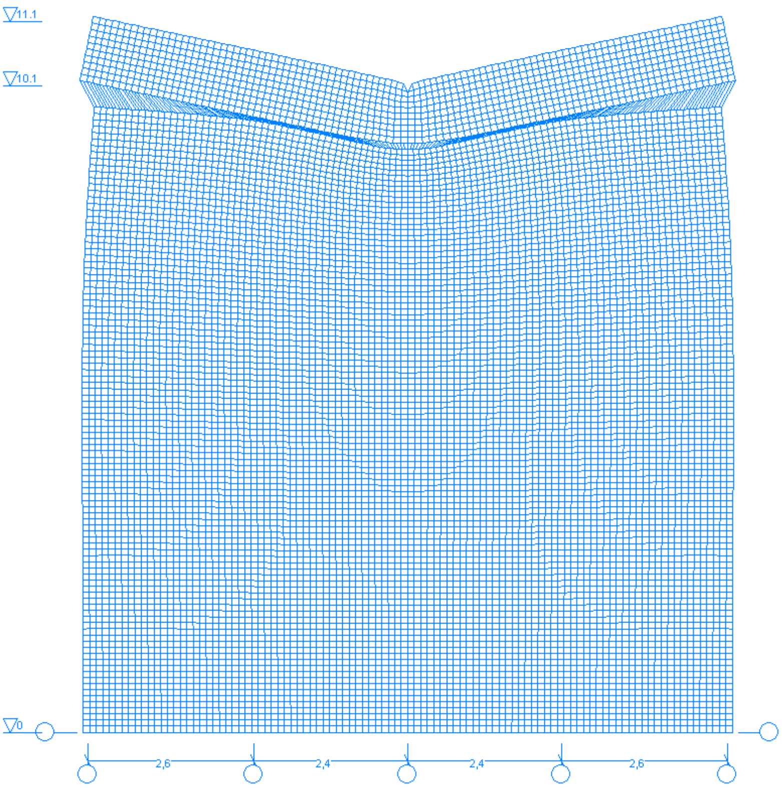
$E_1 = 21.0 \cdot 10^7$ kN/m^2	- elastic modulus of the layer material;
$\nu_1 = 0.3$	- Poisson's ratio of the layer material;
$E_3 = 3.0 \cdot 10^7$ kN/m^2	- elastic modulus of the subgrade material;
$\nu_3 = 0.2$	- Poisson's ratio of the subgrade material;
b = 1.00 m	- height of the layer;
L = 10.00 m	- length of the layer and subgrade in the model;
H = 10.00 m	- height of the subgrade in the model;
P = 1000 kN	- value of the concentrated force applied to the upper surface of the layer.

Finite element model: Design model – plane frame. Elements of the layer – 1000 eight-node grade beam elements of type 30. The spacing of the finite element mesh along the height and length of the layer is 0.1 m. Elements of the subgrade – 10000 eight-node grade beam elements of type 30. The spacing of the finite element mesh along the height and length of the subgrade is 0.1 m. 201 two-node elements of unilateral constraints of type 352 of increased stiffness $k = 1.0 \cdot 10^9$ kN/m are introduced to model the contact with detachment between the lower surface of the layer and the upper surface of the subgrade. Each element vertically joins the nodes of the layer and the subgrade. Boundary conditions are provided by imposing constraints on the lower surface of the subgarde in the direction of the degree of freedom Z. The dimensional stability of the design model is provided by imposing constraints in the direction of the degree of freedom X along the vertical axis of symmetry of the layer and the subgrade (along the force P). The action is specified as a transverse nodal load P (in the direction of the Z axis of the global coordinate system). The nonlinear loading was generated for the incremental-iterative method with a loading factor - 1, number of steps - 1, number of iterations - 10 for the linear loading P. Number of nodes in the design model – 33622.

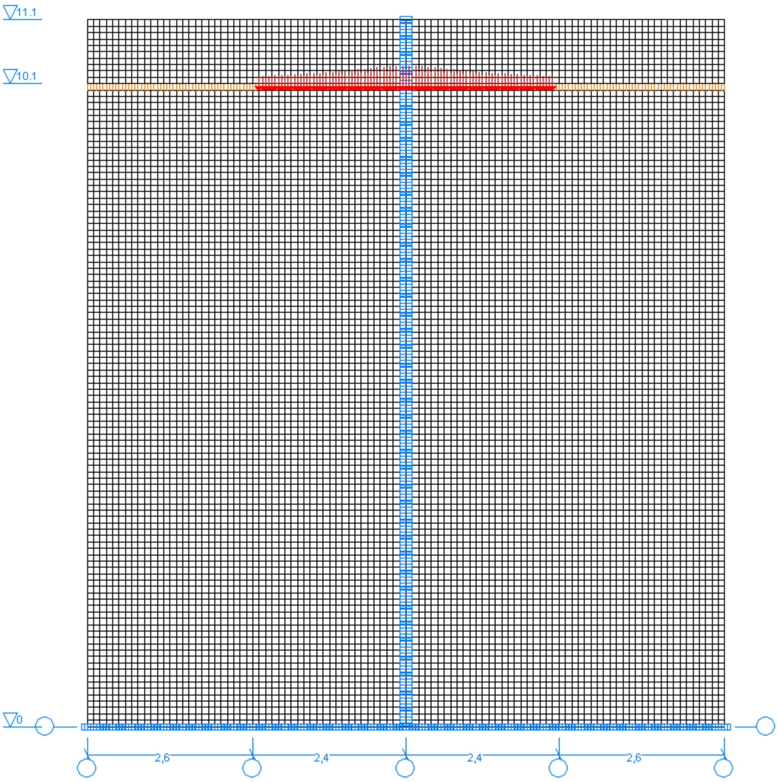
Results in SCAD



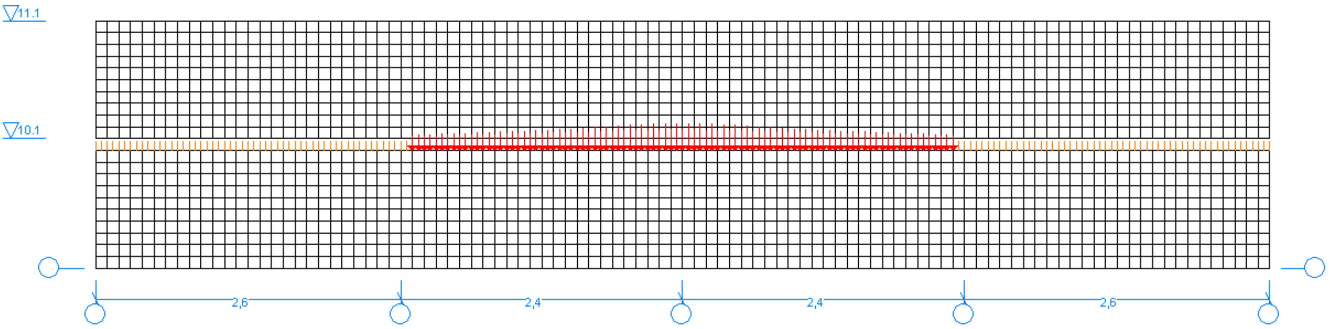
Design model



Deformed model



Area of contact with the subgrade, contact stresses



Area of contact with the subgrade, contact stresses
(fragment of the model)

Comparison of solutions:

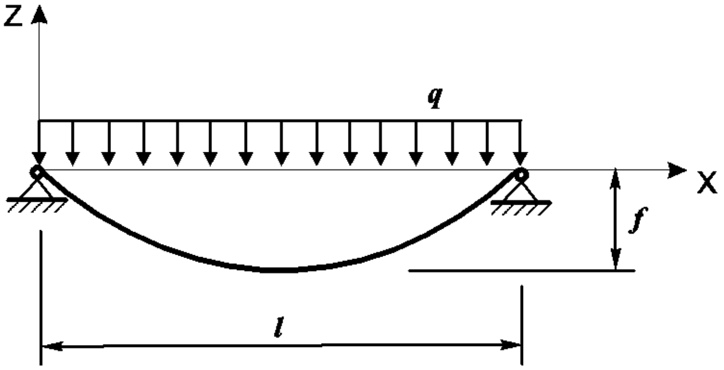
Area of contact with the subgrade 2·*c*, m

Theory	**SCAD**	**Deviation, %**
4.78	4.60	3.77

Notes: In the analytical solution the area of contact with the subgrade 2·*c* can be determined according to the following formula:

$$2 \cdot c = 2 \cdot b \cdot \sqrt[3]{1.845 \cdot \frac{1-\nu_3^{\,2}}{E_3} \cdot \frac{E_1}{1-\nu_1^{\,2}}}.$$

Flexible Thread with Supports in One Level Subjected to a Uniformly Distributed Transverse Load



Objective: Determination of the stress-strain state of a flexible thread with supports in one level subjected to a uniformly distributed transverse load *q*.

Initial data file: **NL_CANAT_v11.3.SPR**

Problem formulation: The flexible thread with supports in one level is subjected to the uniformly distributed transverse load q from the self-weight γ. Determine the sag *f* and the strain σ of the flexible thread.

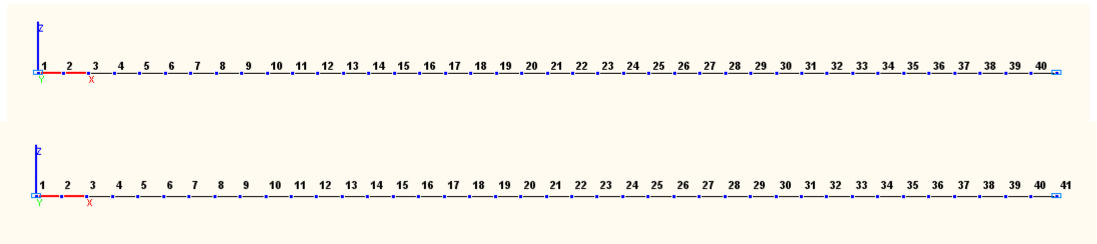
References: S.P. Fesik, Reference Book on Strength of Materials, 2-nd, Kiev, Budivelnik, 1982, p. 33.

Initial data:

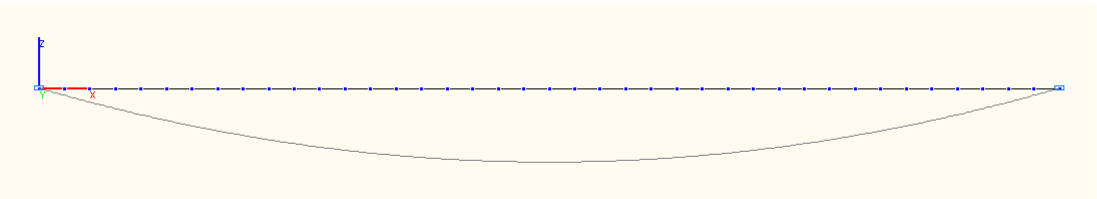
$E = 1.0 \cdot 10^7$ tf/m^2	- elastic modulus of the thread;
l = 40.0 m	- length of the span of the flexible thread;
d = 0.04 m	- diameter of the cross-section of the flexible thread;
$\gamma = 8.0$ tf/m^3	- value of the specific weight of the flexible thread material.

Finite element model: Design model – plane frame, 40 elements of type 302. Boundary conditions are provided by imposing constraints in the support nodes of the flexible thread in the directions of the degrees of freedom X, Z. The action of the uniformly distributed transverse load is specified as $q = \gamma \cdot F$, where $F = \pi \cdot d^2/4$. Number of nodes in the design model – 41. The calculation is performed in the geometrically nonlinear formulation by the simple incremental method with the following parameters: loading factor – 0.01, number of steps – 100.

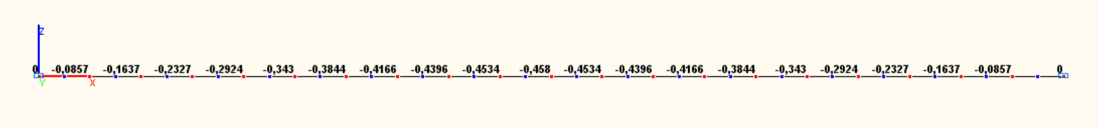
Results in SCAD



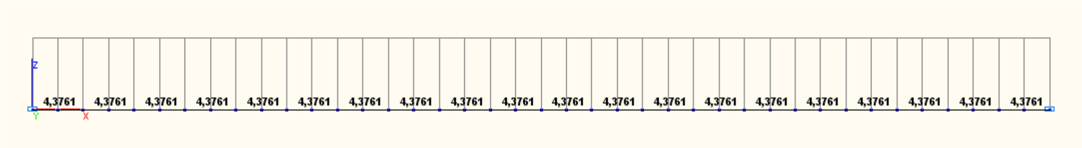
Design model



Deformed model



Values of vertical displacements Z (m)



Longitudinal force diagram N (tf)

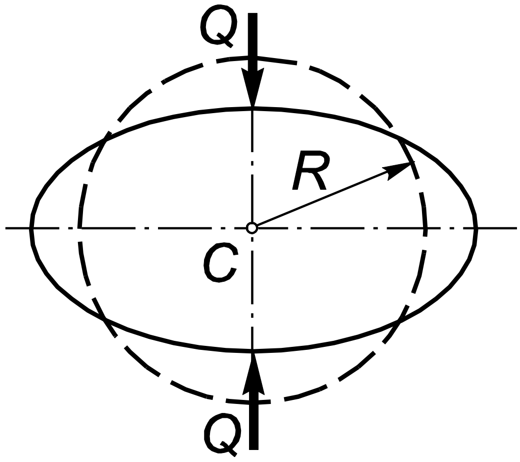
Comparison of solutions:

Parameter	Theory	SCAD	Deviations, %
Sag *f* of the flexible thread, m	-0.4579	-0.4580	0.02
Strain σ of the flexible thread, tf/m^2	3494.3	4.3761 / (3.1416 · 0.04^2/4) = 3482.4	0.34

Notes: In the analytical solution the sag *f* and the strain σ of the flexible thread are determined according to the following formulas:

$$f = \frac{l}{2} \cdot \sqrt[3]{\frac{3 \cdot \gamma \cdot l}{8 \cdot E}}\,; \qquad \sigma = \frac{\gamma \cdot l^2}{8 \cdot f}\,.$$

Flexible Ring Subjected to Two Mutually Balanced Radially Compressive Forces



Objective: Determination of maximum displacements and bending moments in a flexible ring subjected to two mutually balanced radially compressive forces in the geometrically nonlinear formulation.

Initial data files:

File name	Description
Кольцо_Q_50.SPR	The flexible ring is subjected to the radially compressive forces Q = 50 kN

Problem formulation: The flexible ring of constant cross-section is subjected to two mutually balanced radially compressive forces Q. Determine: the transverse displacements w and the bending moments M in the compressive force application points.

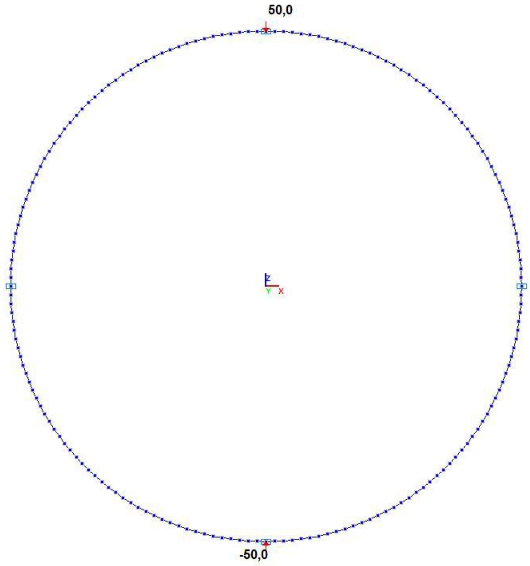
References: E. P. Popov, Theory and Calculation of Flexible Elastic Bars, Moscow, Nauka, 1986, p. 154

Initial data:

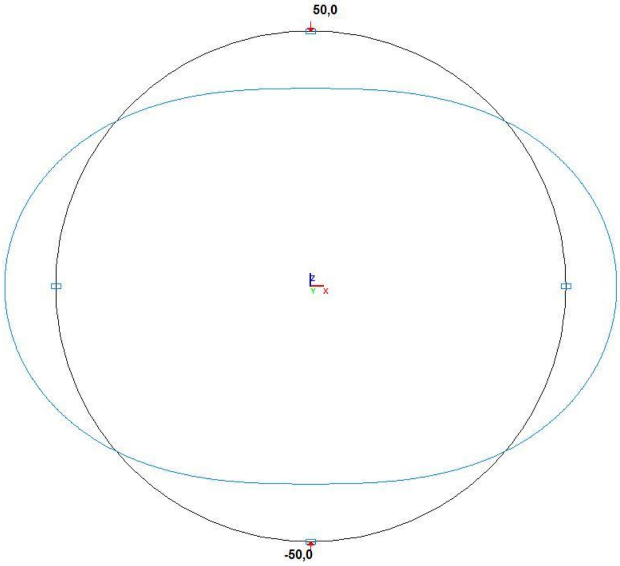
$EF = 1.5 \cdot 10^7$ kN	- axial stiffness of the cross-section of the ring;
$EI_y = 3.125 \cdot 10^5$ kN·m^2	- bending stiffness of the cross-section of the ring in its plane;
$EI_z = 1.250 \cdot 10^6$ kN·m^2	- bending stiffness of the cross-section of the ring out of its plane;
$GI_x = 3.533 \cdot 10^5$ kN·m^2	- torsional stiffness of the cross-section of the ring;
R = 50.0 m	- radius of the ring;
Q = 50 kN	- value of the compressive forces.

Finite element model: Design model – general type system. Elements of the plate - 180 bar elements taking into account the geometric nonlinearity of type 310. The spacing of the finite element mesh along the longitudinal axis of the ring is 2.0º. The dimensional stability of the design model is provided by imposing constraints according to its symmetry conditions. The nonlinear loading was generated for the incremental-iterative method with a loading factor - 1, number of steps - 1, number of iterations - 7 for the linear loading Q. Number of nodes in the design model – 180.

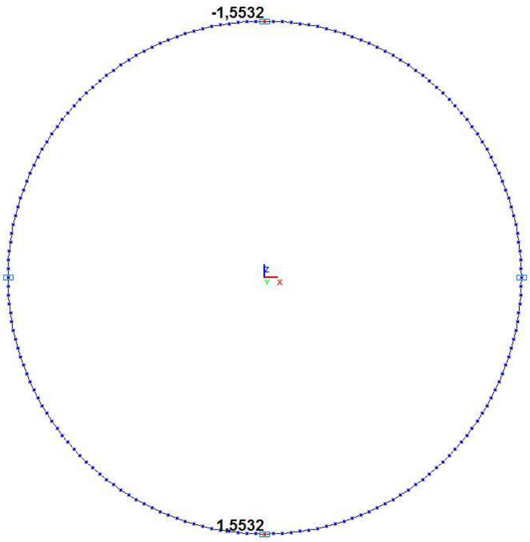
Results in SCAD



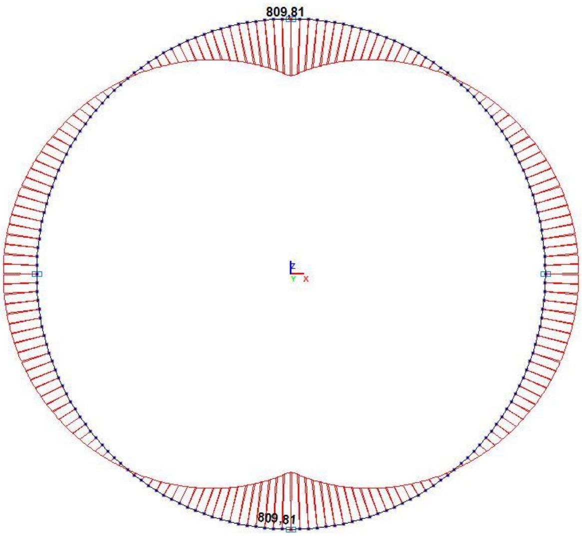
Design model



Deformed model



Values of transverse displacements w (m)



Bending moment diagrams M (kN·m)

Comparison of solutions:

Parameter	Theory	SCAD	Deviation, %
The transverse displacement of the ring section w, m in the points of the application of the compressive forces Q = 50 kN	±1.6060	±1.5532	3.29
The bending moment for the ring section M, kN·m in the points of the application of the compressive forces Q = 50 kN	809.37	809.81	0.05

Notes: In the analytical solution the transverse displacements *w* and the bending moments M in the compressive force application points can be determined according to the following formulas:

At $0 \le Q \le 0.6297 \cdot \frac{EI}{R^2}$:

$$w = \left[\frac{2}{k} \cdot \sqrt{\frac{2 \cdot EI}{Q \cdot R^2}} \cdot E\left(\frac{\pi}{4}\right) - \left(\frac{2}{k^2} - 1\right) \cdot \frac{\pi}{2} \right] \cdot R; \qquad M = \frac{2}{k} \cdot \sqrt{1 - \frac{k^2}{2}} \cdot \sqrt{\frac{Q \cdot EI}{2}} - \frac{EI}{R},$$

where k is determined by solving the equation: $k \cdot F\left(\frac{\pi}{4}\right) = \frac{\pi \cdot R}{2} \cdot \sqrt{\frac{Q}{2 \cdot EI}}$;

$$F\left(\frac{\pi}{4}\right) = \int_0^{\frac{\pi}{4}} \frac{d\varphi}{\sqrt{1 - k^2 \cdot \sin^2(\varphi)}} -$$ Legendre elliptic integral of the first kind,

$$E\left(\frac{\pi}{4}\right) = \int_0^{\frac{\pi}{4}} \sqrt{1 - k^2 \cdot \sin^2(\varphi)} \cdot d\varphi -$$ Legendre elliptic integral of the second kind.

At $0.6297 \cdot \frac{EI}{R^2} \le Q \le 2.7865 \cdot \frac{EI}{R^2}$:

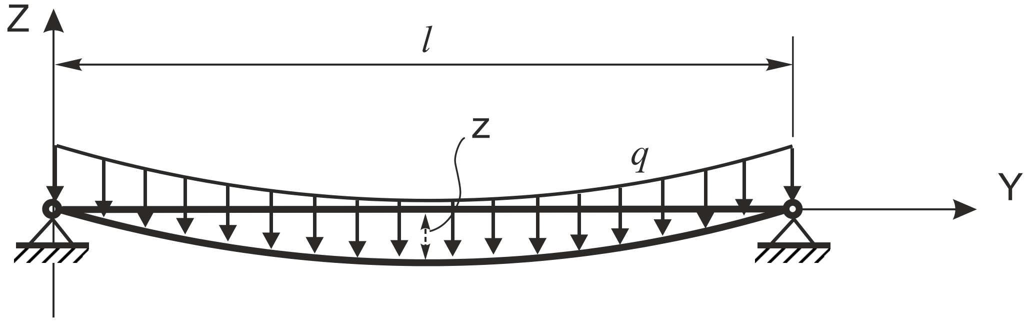
$$w = \left[2 \cdot \sqrt{\frac{2 \cdot EI}{Q \cdot R^2}} \cdot E(\Psi) - \frac{\pi}{2} \right] \cdot R; \qquad M = 2 \cdot k \cdot cos(\Psi) \cdot \sqrt{\frac{Q \cdot EI}{2}} - \frac{EI}{R},$$

where k and Ψ are determined by solving the system of equations: $\begin{cases} k \cdot sin(\Psi) = \frac{\sqrt{2}}{2} \\ F(\Psi) = \frac{\pi \cdot R}{2} \cdot \sqrt{\frac{Q}{2 \cdot EI}} \end{cases};$

$F(\Psi) = \int_0^{\Psi} \frac{d\psi}{\sqrt{1 - k^2 \cdot sin^2(\psi)}} -$ Legendre elliptic integral of the first kind,

$E(\Psi) = \int_0^{\Psi} \sqrt{1 - k^2 \cdot sin^2(\psi)} \cdot d\psi -$ Legendre elliptic integral of the second kind.

Flexible Long Rectangular Plate Simply Supported along the Longitudinal Edges Subjected to a Uniformly Distributed Transverse Load



Objective: Determination of the stress-strain state of a flexible long rectangular plate simply supported along the longitudinal edges subjected to a uniformly distributed transverse load.

Initial data file: **NEL.SPR**

Problem formulation: The flexible long rectangular plate simply supported along the longitudinal edges is subjected to the transverse load *q* uniformly distributed over its area. Determine the transverse displacement Z of the deformed midsurface, as well as the maximum σ_{yd} and minimum σ_{yt} normal stresses over the cross-section in the half of the plate span.

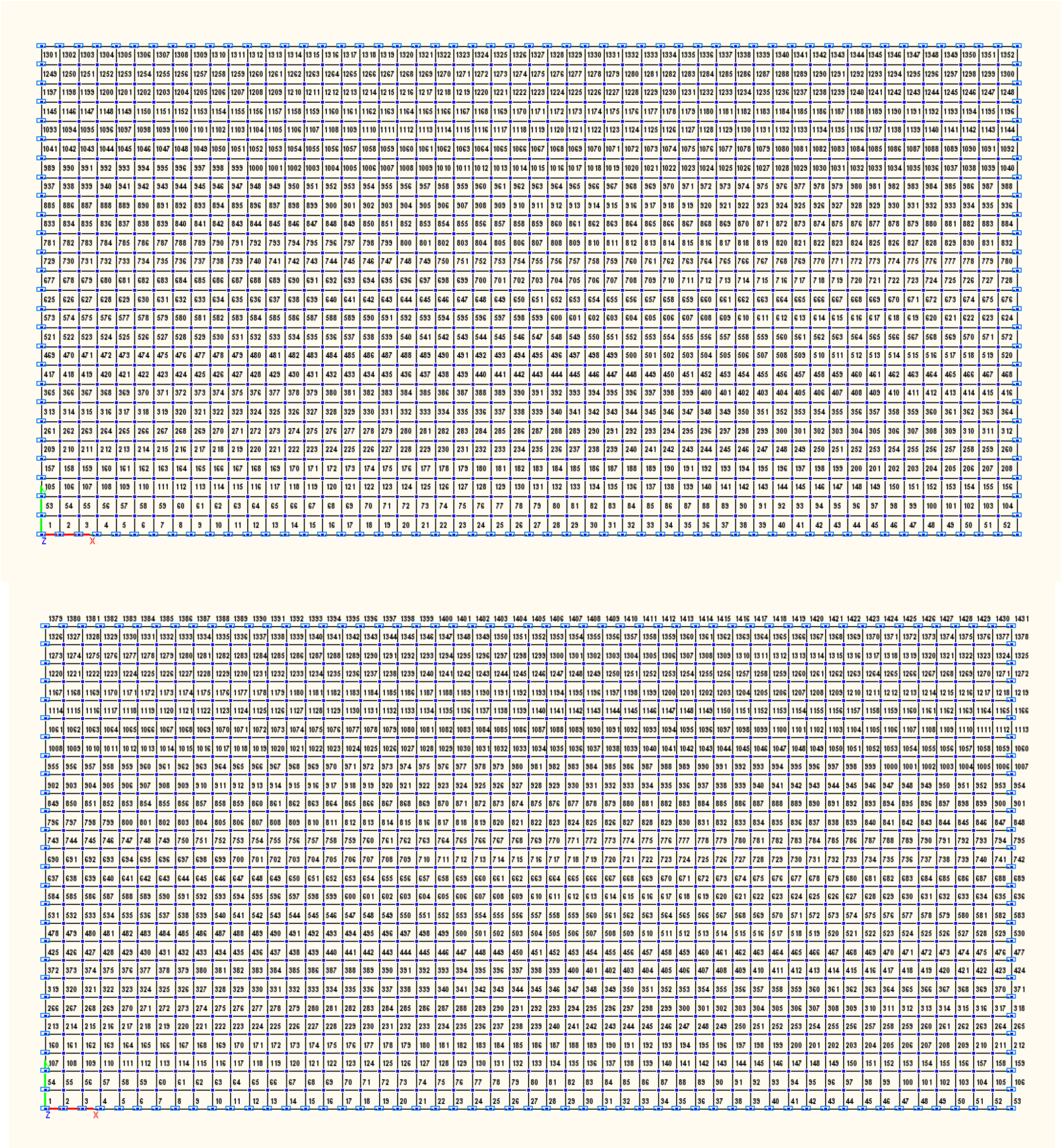
References: S. Timoshenko, S. Woinowsky-Krieger, Theory of Plates and Shells, Moscow, Fizmatgis, 1963, p. 20.

Initial data:

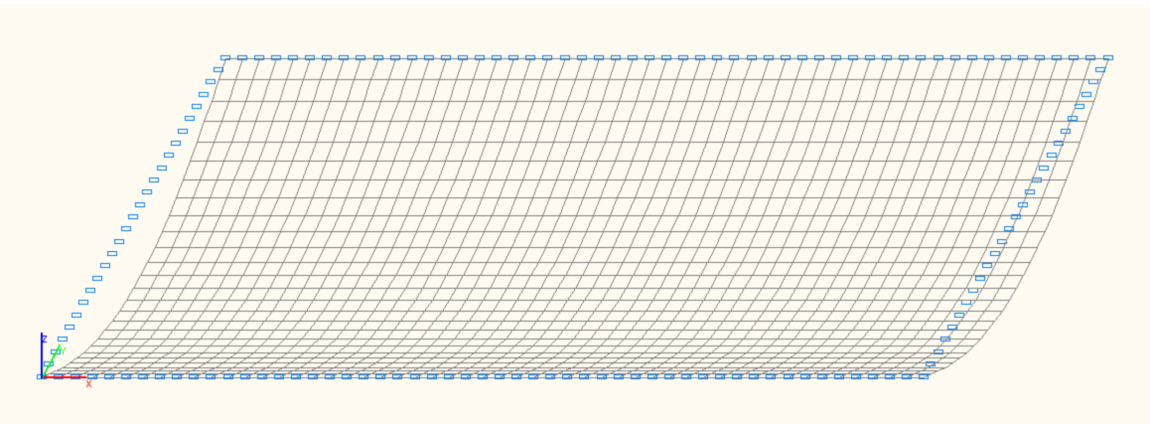
$E = 2.1 \cdot 10^6$ kgf/cm^2	- elastic modulus;
$\nu = 0.3$	- Poisson's ratio;
h = 1.3 cm	- thickness of the plate;
l = 130.0 cm	- short side of the plate (along the Y axis of the global coordinate system);
b = 260.0 cm	- size of the elementary strip of the long side of the plate (along the X axis of the global coordinate system);
q = 1.4 kgf/cm^2	- value of the uniformly distributed transverse load.

Finite element model: Design model – general type system, 1352 plate elements of type 341. The spacing of the finite element mesh along the sides of the plate (along the X, Y axes of the global coordinate system) is 5.0 cm. Boundary conditions are provided by imposing constraints in the directions of the degrees of freedom X, Y, Z for the long edges parallel to the X axis of the global coordinate system based on the simply supported conditions, and in the directions of the degrees of freedom X, UY for the short edges parallel to the Y axis of the global coordinate system based on the conditions of cylindrical bending of the elementary strip of the long side of the plate. Number of nodes in the design model 1431. The calculation is performed in the geometrically nonlinear formulation by the incremental-iterative method with the following parameters: loading factor – 0.1, number of steps – 10, . number of iterations – 30.

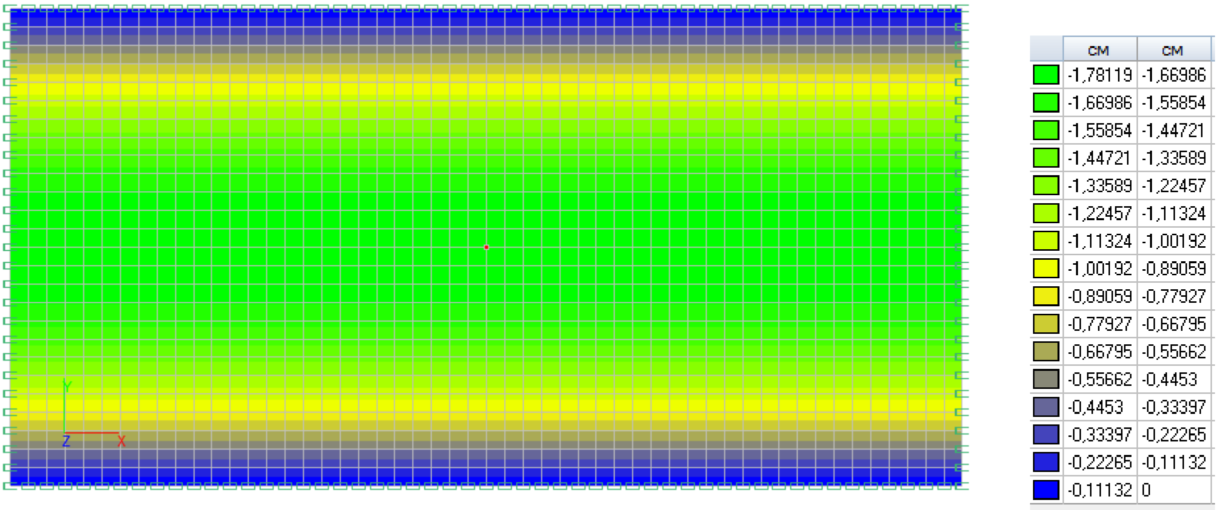
Results in SCAD



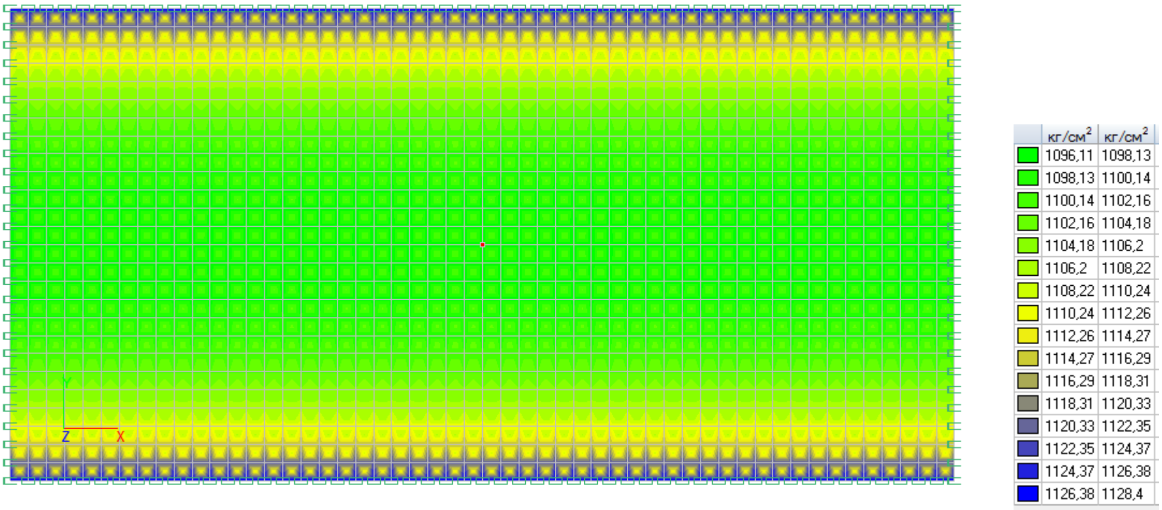
Design model



Deformed model

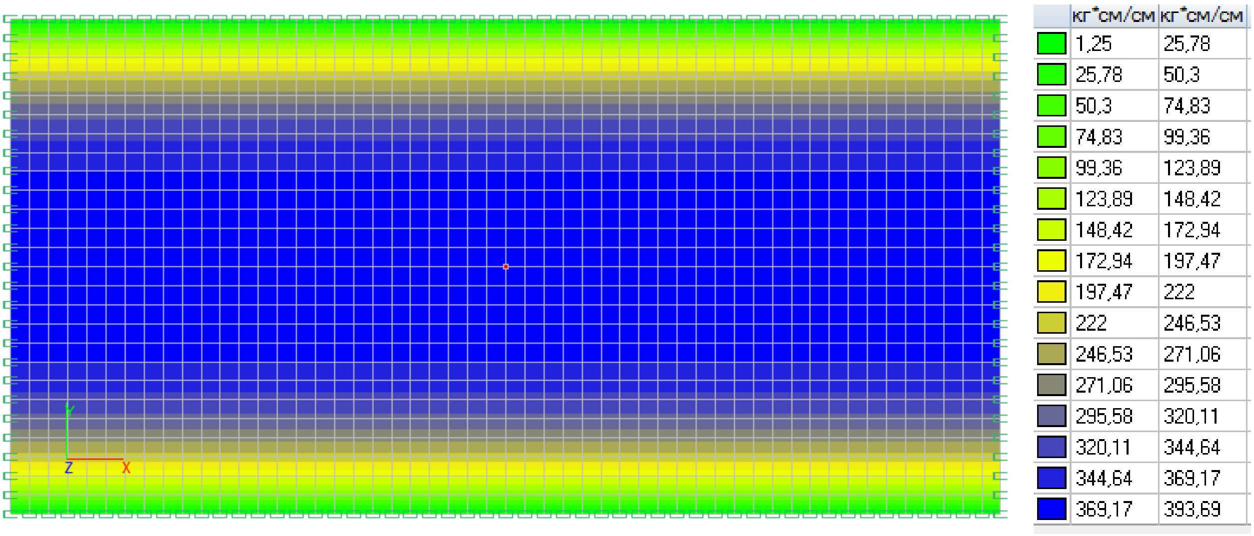


Values of transverse displacements Z (cm)

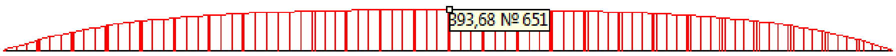


Values of longitudinal stresses N_y (kgf/cm^2)

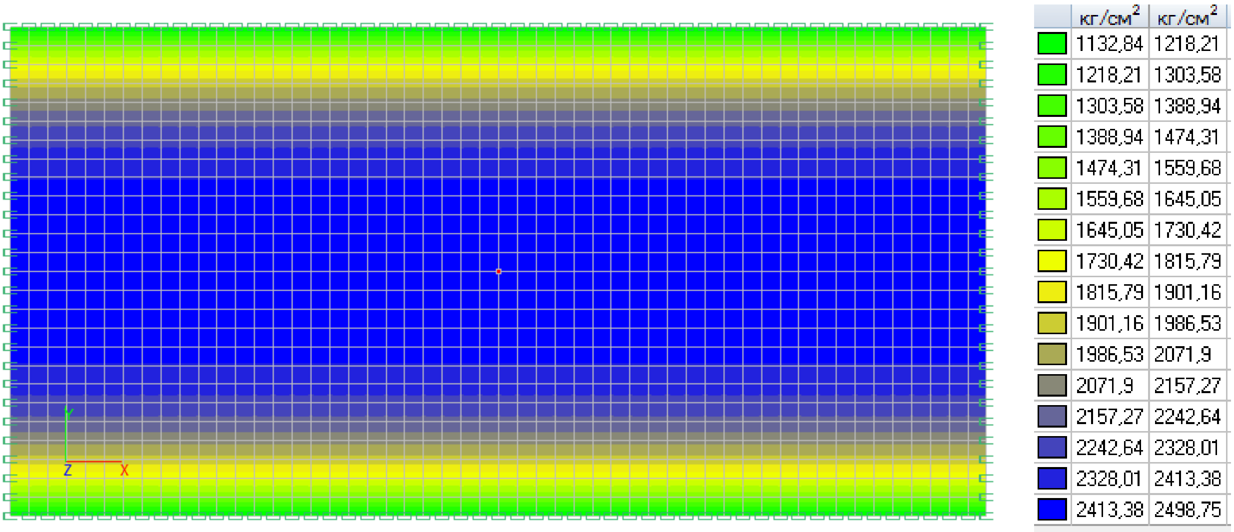
Longitudinal stress diagram N_y (kgf/cm^2)



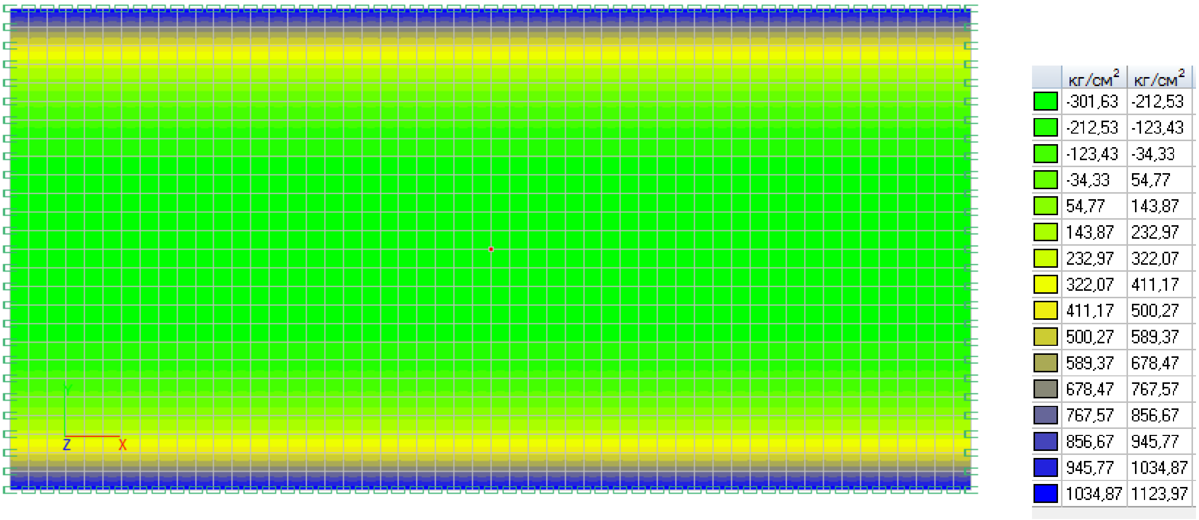
Values of bending moments M_y (kgf·cm/cm)



Bending moment diagram M_y (kgf·cm/cm)



Values of normal stresses s_{yd} (kgf/cm^2)



Values of normal stresses s_{yt} (kgf/cm^2)

Comparison of solutions:

Parameter	Theory	SCAD	Deviations, %
Transverse displacement Z of the deformed midsurface in the half of the plate span, cm	-1.782	-1.781	0.06
maximum normal stresses over the cross-section in the half of the plate span s_{yd}, kgf/cm^2	2503	2498.7	0,17
minimum normal stresses over the cross-section in the half of the plate span s_{yt}, kgf/cm^2	-287	-294.0	2.44

Notes: In the analytical solution the displacement Z of the deformed midsurface, as well as the maximum s_{yd} and minimum s_{yt} normal stresses over the cross-section in the half of the plate span can be calculated according to the following formulas:

$$Z = \frac{5 \cdot q \cdot l^4}{384 \cdot D} \cdot \frac{\frac{1}{ch(u)} - 1 + \frac{u^2}{2}}{\frac{5 \cdot u^4}{24}}; \qquad s_{yd} = s_{yN} + s_{yM}; \qquad s_{yt} = s_{yN} - s_{yM}, \text{ where:}$$

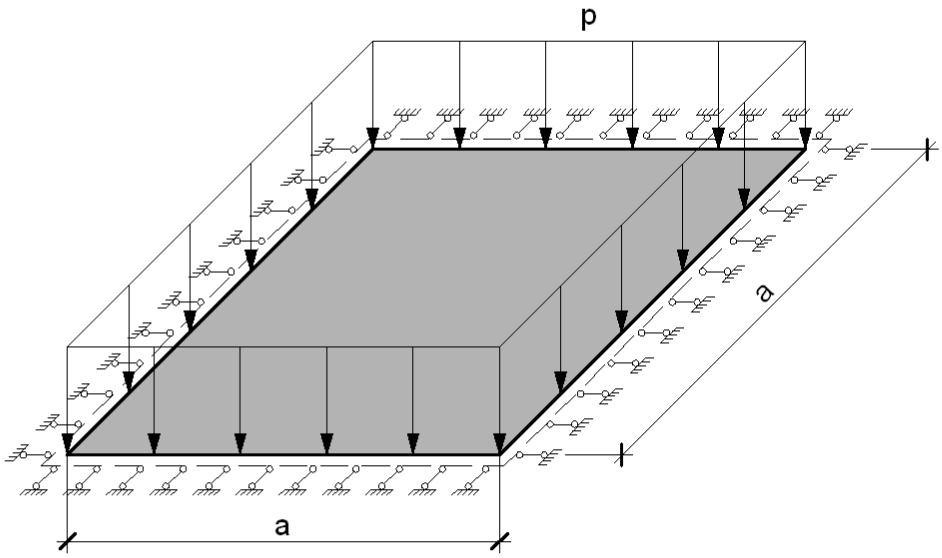
$$s_{yN} = \frac{N_y}{h}; \qquad s_{yM} = \frac{6 \cdot M_y}{h^2}; \qquad N_y = \frac{4 \cdot u^2 \cdot D}{l^2}; \qquad M_y = \frac{q \cdot l^2}{8} \cdot \frac{1 - \frac{1}{ch(u)}}{\frac{u^2}{2}};$$

$$D = \frac{E \cdot h^3}{12 \cdot (1 - \nu^2)}.$$

The value *u* is determined from the following expression:

$$\frac{E^2 \cdot h^8}{(1 - \nu^2)^2 \cdot q^2 \cdot l^8} = \frac{135}{16} \cdot \frac{th(u)}{u^9} + \frac{27}{16} \cdot \frac{th^2(u)}{u^8} - \frac{135}{16 \cdot u^8} + \frac{9}{8 \cdot u^6}.$$

Flexible Square Plate Simply Supported along the Perimeter Subjected to a Uniformly Distributed Transverse Load



Objective: Determination of maximum displacements and longitudinal stresses in a flexible square plate simply supported along the perimeter and subjected to a uniformly distributed transverse load in the geometrically nonlinear formulation.

Initial data file: **7.6.SPR**

Problem formulation: The flexible square isotropic plate of constant thickness is simply supported along the perimeter and subjected to the uniformly distributed transverse load p. Determine: the transverse displacements w and longitudinal stresses N_x and N_y for the center of the plate.

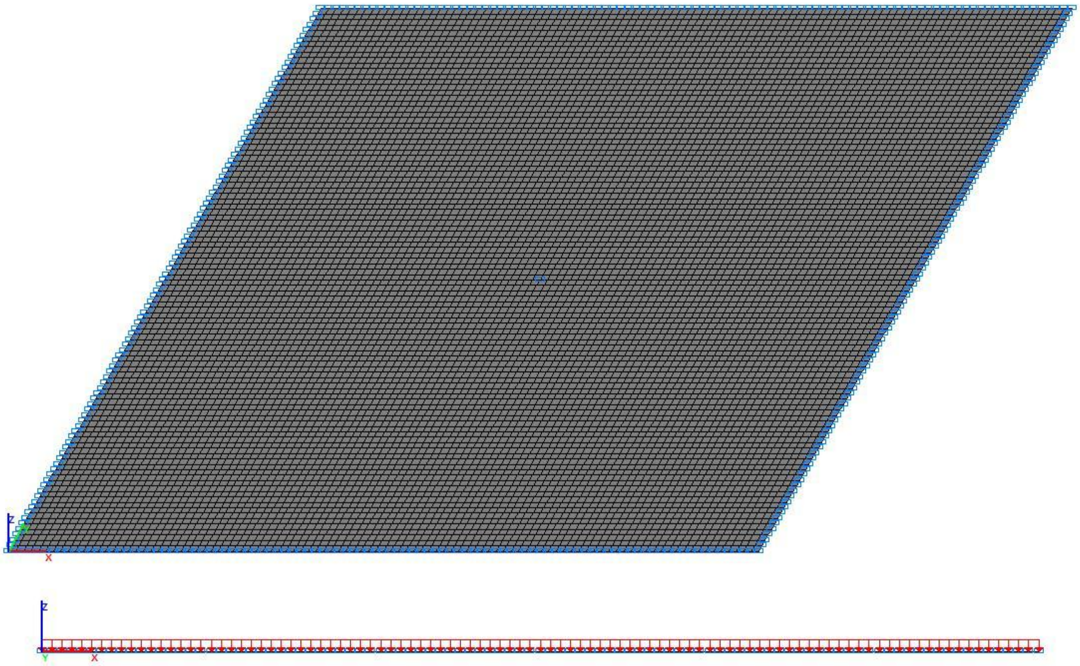
References: S. Levy, Bending of rectangular plates with large deflections, Washington, National advisory committee for aeronautics, Technical note No 846, May 1942.
H. Hencky, Die berechnung dünner rechteckiger platten mit verschwindender biegungsteifigkeit, Dresden, Zeitschrift für angewandte mathematic und mechanic, April 1921.
I.A. Birger, Ya.G. Panovko, Strength, Stability, Vibrations, Handbook in three volumes, Volume 1, Moscow, Mechanical engineering, 1968, p. 606

Initial data:

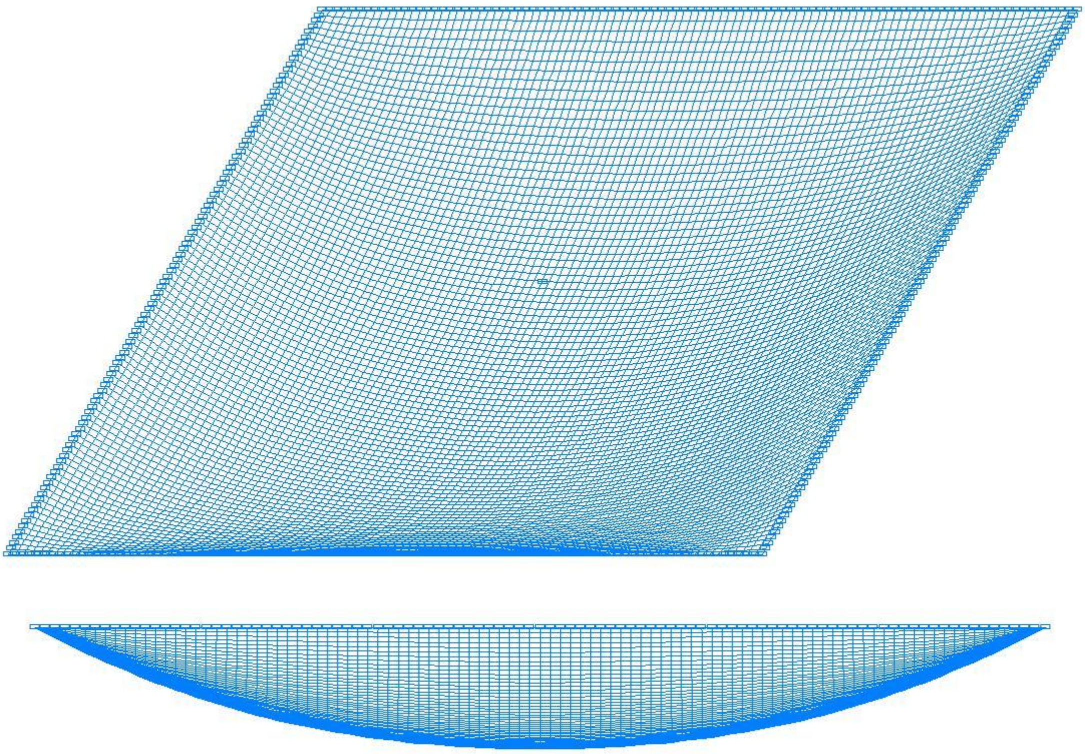
$E = 2.0 \cdot 10^8$ kPa	- elastic modulus of the plate material;
$\nu = 0.3$	- Poisson's ratio;
h = 0.01 m	- thickness of the plate;
a = 10.0 m	- side of the plate;
p = 10 kPa	- value of the uniformly distributed load.

Finite element model: Design model – general type system. Plate elements - 10000 four-node shell elements taking into account the geometric nonlinearity of type 344. The spacing of the finite element mesh along the sides of the plate (along the X, Y axes of the global coordinate system) is 0.10 m. Boundary conditions are provided by imposing constraints on the nodes of the support contour of the plate in the direction of the degree of freedom Z, and by imposing constraints on the nodes of the sides of the plate in the direction normal to them (for two opposite sides parallel to the X axis of the global coordinate system – along the Y axis, for two opposite sides parallel to the Y axis of the global coordinate system – along the X axis). The dimensional stability of the design model is provided by imposing a constraint in the node of the center of the plate in the UZ direction of the global coordinate system. The nonlinear loading was generated for the incremental-iterative method with a loading factor - 1, number of steps - 1, number of iterations - 100 for the linear loading p. Number of nodes in the design model – 10201.

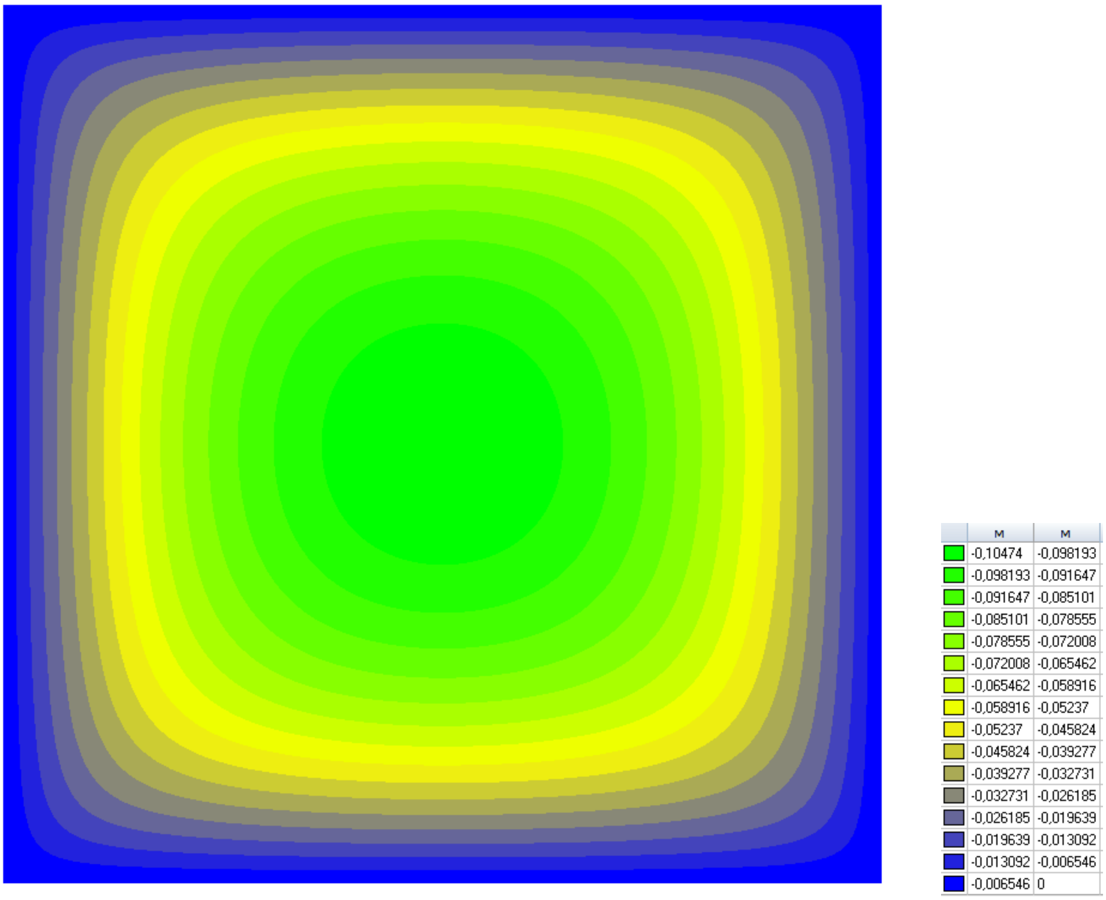
Results in SCAD



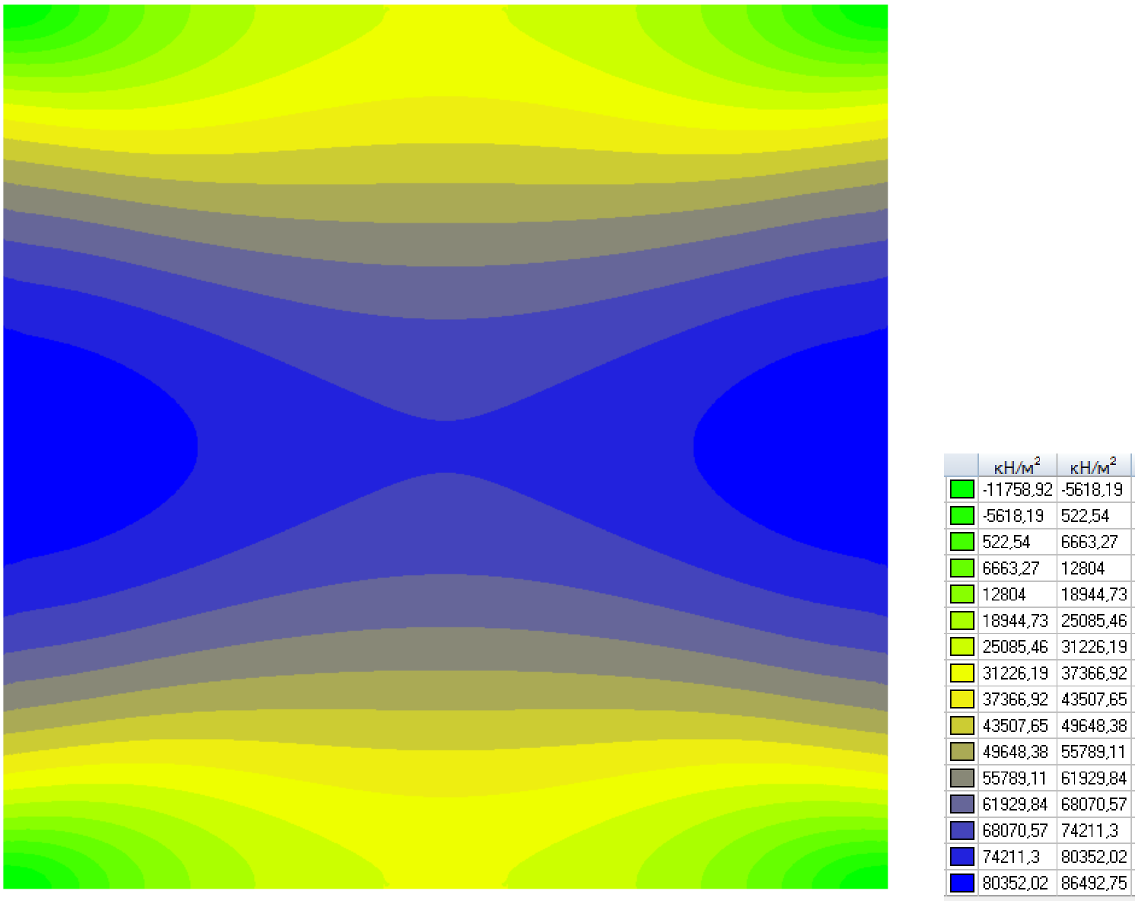
Design model



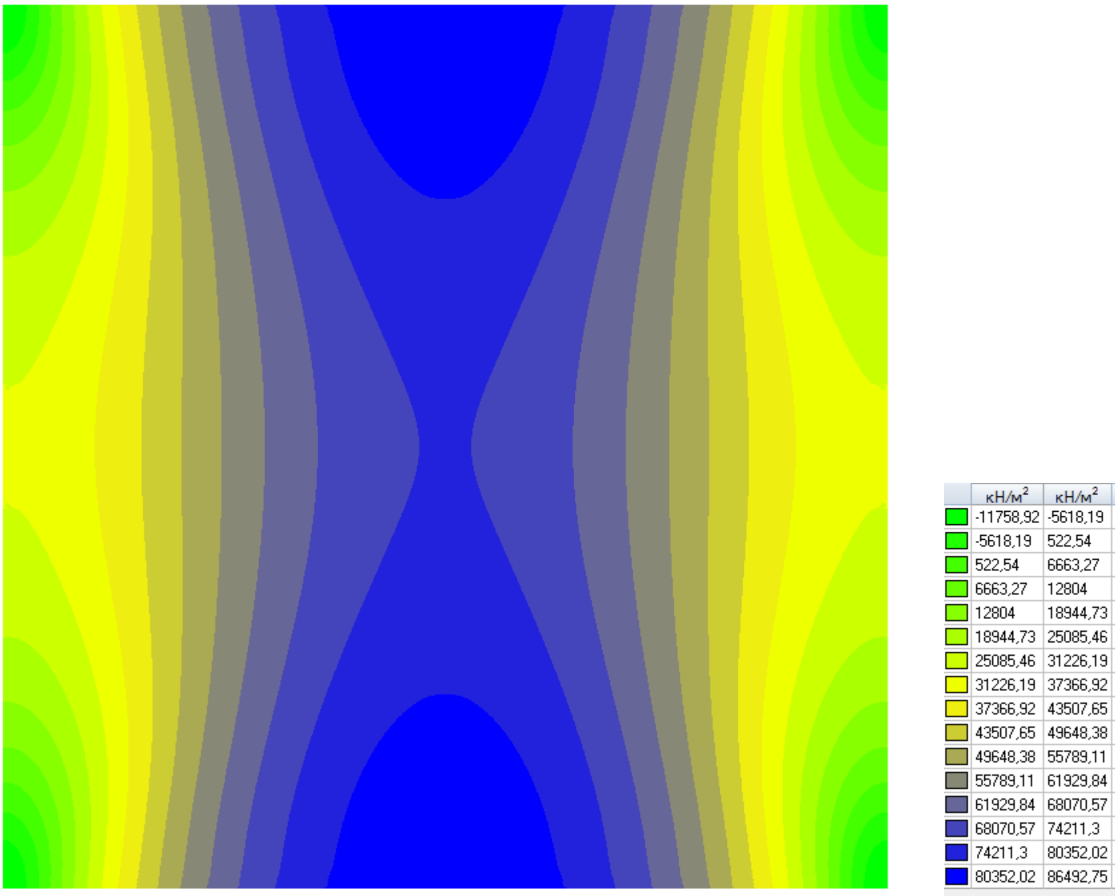
Deformed model



Values of transverse displacements w (m)



Values of longitudinal stresses N_x (kN/m^2)



Values of longitudinal stresses N_y (kN/m^2)

Comparison of solutions:

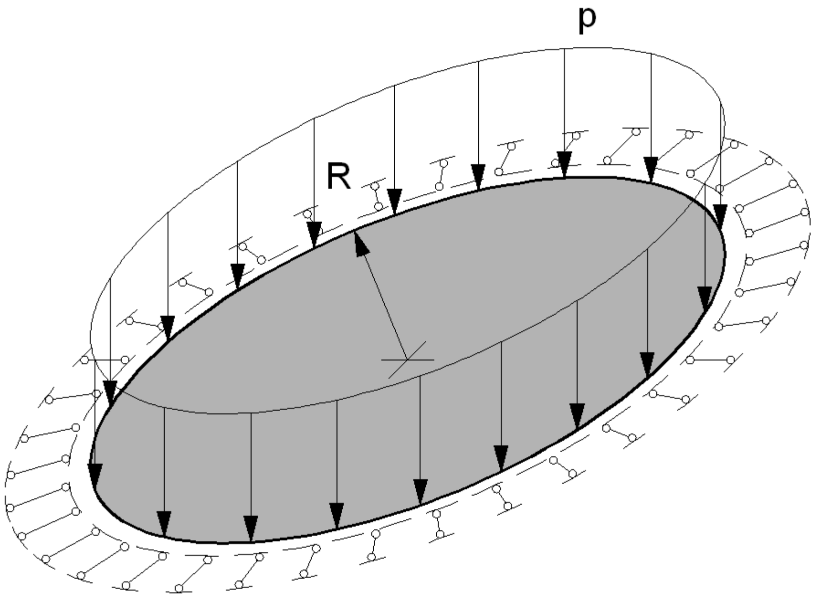
Parameter	Theory	SCAD	Deviation, %
Transverse displacement in the center of the plate w, m	0.1050 (0.1067)	0.1047	0.29 (1.87)
Longitudinal stress in the center of the plate N_x, kN/m^2	74963 (75830)	74480	0.64 (1,7)
Longitudinal stress in the center of the plate N_y, kN/m^2	74963 (75830)	74480	0.64 (1,7)

The values of the approximate Hencky solution for the Karman theory are given without brackets;
The values of the refined Levy solution for the Karman theory are given in brackets

Notes: In the analytical approximate Hencky solution the transverse displacements w and the longitudinal stresses N_x and N_y for the center of the plate can be determined according to the following formulas (Poisson's ratio ν = 0.3):

$$w = 0.285 \cdot a \cdot \sqrt[3]{\frac{p}{E} \cdot \frac{a}{h}}; \qquad N_x = N_y = 3.4 \cdot E \cdot \left(\frac{w}{a}\right)^2.$$

Simply Supported Flexible Circular Plate Subjected to a Uniformly Distributed Transverse Load



Objective: Determination of maximum displacements and longitudinal radial tangential stresses in a flexible circular plate simply supported along the contour and subjected to a uniformly distributed transverse load in the geometrically nonlinear formulation.

Initial data file: **7.7.SPR**

Problem formulation: The flexible circular isotropic plate of constant thickness is simply supported along the contour and subjected to the uniformly distributed transverse load p. Determine: the transverse displacements w and longitudinal radial tangential stresses N_r and N_t for the center of the plate.

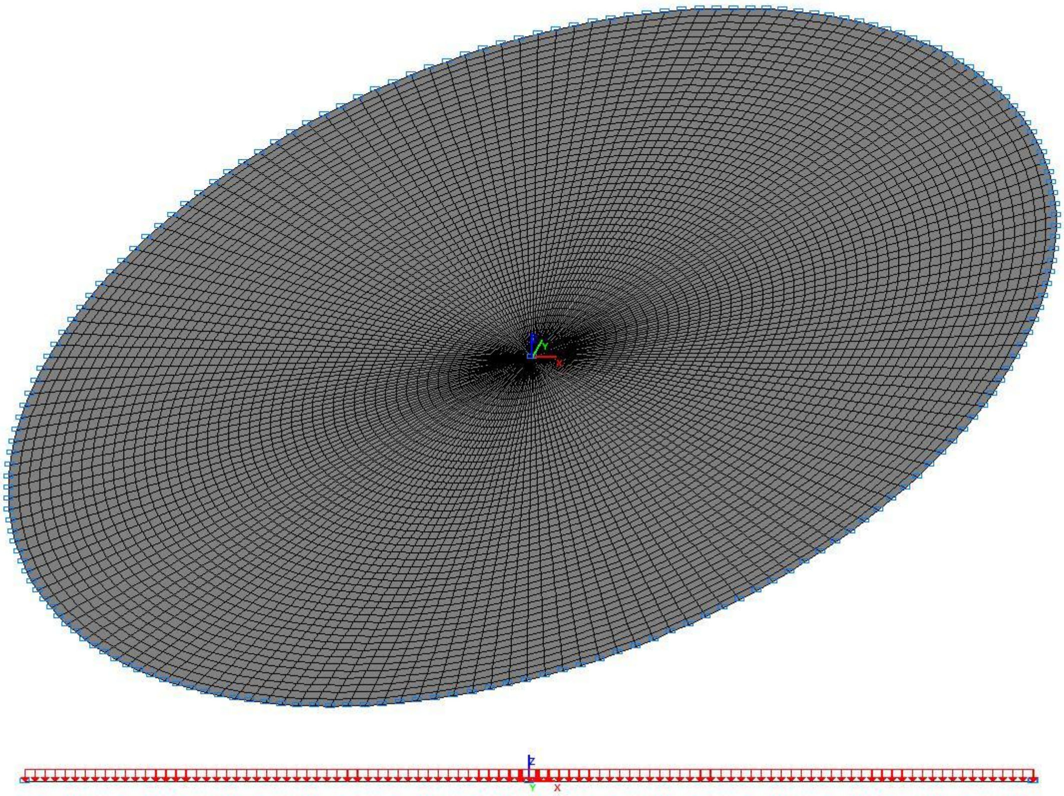
References: S. Way, Bending of circular plates with large deflections, New York, ASME, v.56 N 8, 1934, p. 627-636.
H. Hencky, Uber den spannungsztand in kreisrunden platten mit verschwindender biegungssteifigkeit, Dresden, Zeitschrift für angewandte mathematic und physik, v.63, 1915, p. 311-317.
I.A. Birger, Ya.G. Panovko, Strength, Stability, Vibrations, Handbook in three volumes, Volume 1, Moscow, Mechanical engineering, 1968, p. 614

Initial data:

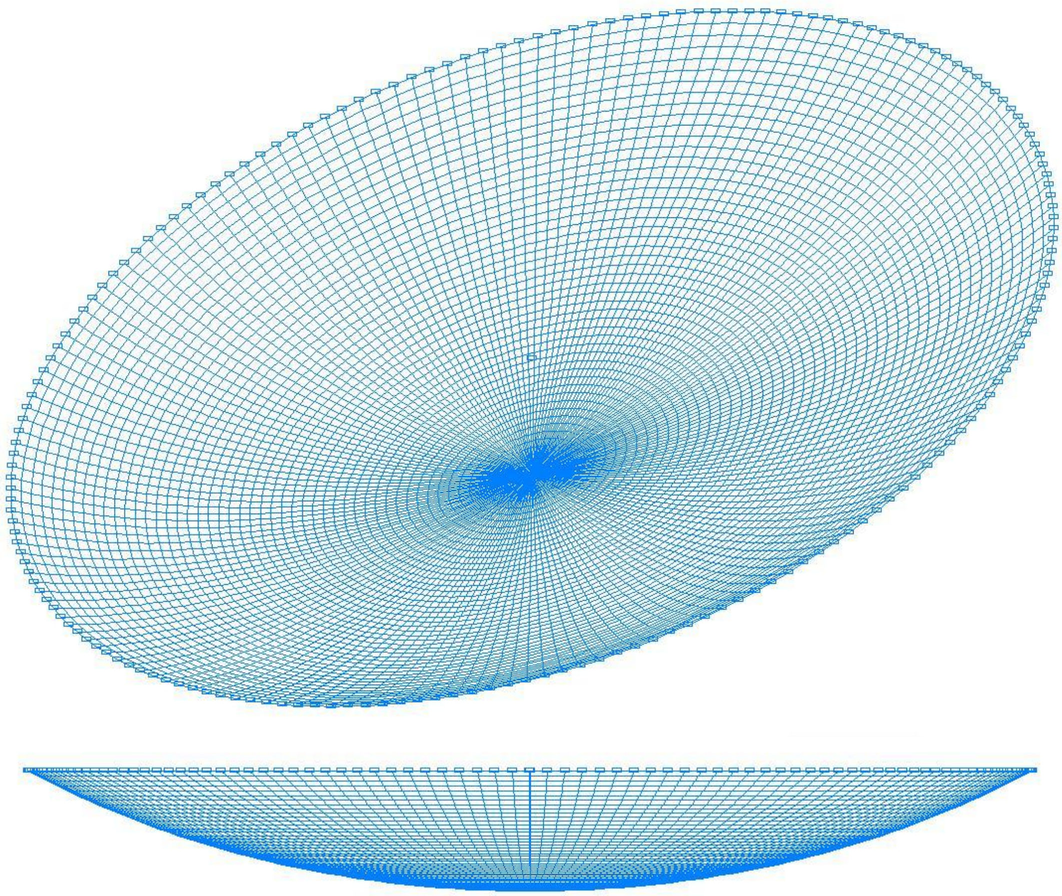
$E = 2.0 \cdot 10^8$ kPa	- elastic modulus of the plate material;
$\nu = 0.3$	- Poisson's ratio;
h = 0.01 m	- thickness of the plate;
R = 5.0 m	- outer radius of the plate;
p = 10 kPa	- value of the uniformly distributed load.

Finite element model: Design model – general type system. Elements of the plate - 8820 four-node shell elements taking into account the geometric nonlinearity of type 344 and 180 three-node shell elements taking into account the geometric nonlinearity of type 342. The spacing of the finite element mesh in the radial direction is 0.10 m and in the tangential direction is 2.0º. The direction of the output of internal forces is radial tangential. Boundary conditions are provided by imposing constraints in the directions of the degrees of freedom X, Y and Z along the external contour of the plate. The dimensional stability of the design model is provided by imposing a constraint in the node of the center of the plate in the UZ direction of the global coordinate system. The nonlinear loading was generated for the incremental-iterative method with a loading factor - 1, number of steps - 1, number of iterations - 100 for the linear loading p. Number of nodes in the design model – 9001.

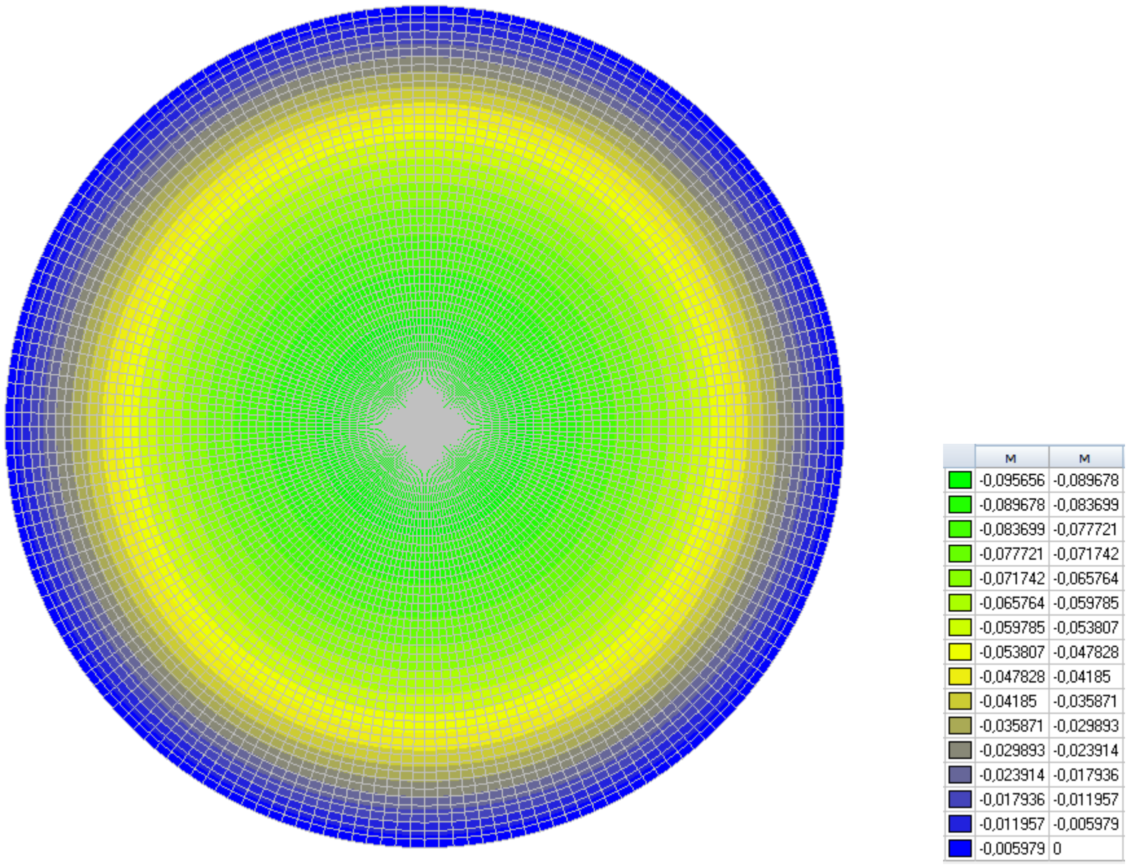
Results in SCAD



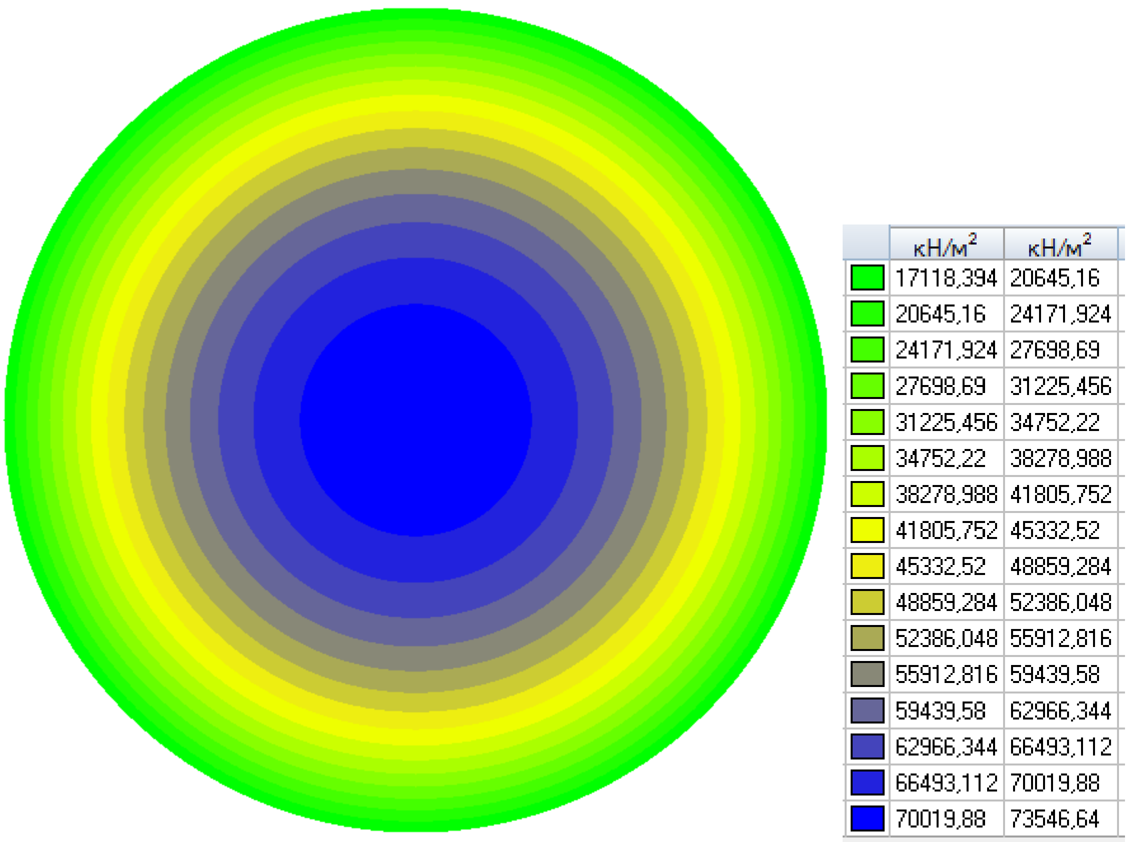
Design model



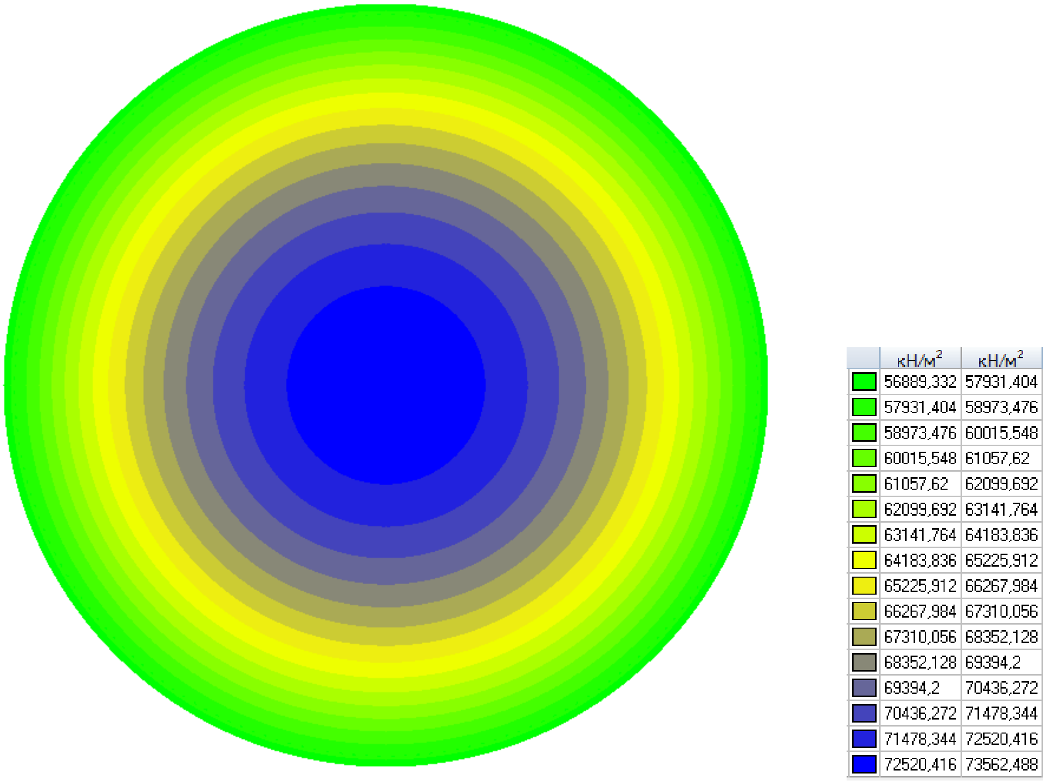
Deformed model



Values of transverse displacements w (m)



Values of longitudinal radial stresses N_r (kN/m^2)



Values of longitudinal tangential stresses N_t (kN/m^2)

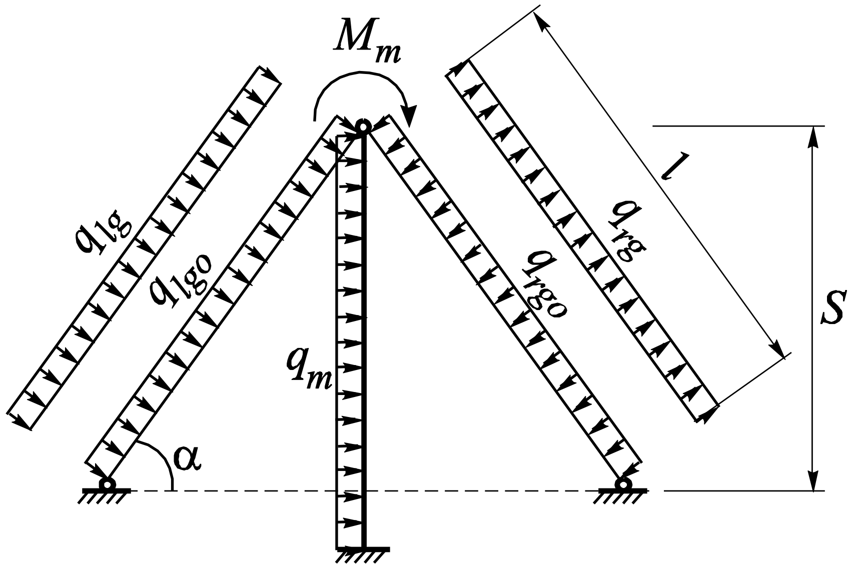
Comparison of solutions:

Parameter	**Theory**	**SCAD**	**Deviation, %**
Transverse displacement in the center of the plate w, m	0.0968	0.0957	1.14
Longitudinal radial stress in the center of the plate N_r, kN/m^2	72316	73540	1.69
Longitudinal tangential stress in the center of the plate N_t, kN/m^2	72316	73540	1.69

Notes: In the analytical approximate Hencky solution according to the Karman theory the transverse displacements w and the longitudinal radial tangential stresses N_r and N_t for the center of the plate can be determined according to the following formulas (Poisson's ratio ν = 0.3):

$$w = 0.662 \cdot R \cdot \sqrt[3]{\frac{p}{E} \cdot \frac{R}{h}}; \qquad N_r = N_t = 0.965 \cdot E \cdot \left(\frac{w}{R}\right)^2.$$

Double-Guyed Mast Subjected to Static Loads and Prestressing Forces



Objective: Determination of the stress state of a double-guyed mast subjected to static loads and prestressing forces in the physically nonlinear formulation.

Initial data file: **Mast.spr**

Problem formulation: The double-guyed mast with a trunk clamped in the support and cable stays symmetrically descending from its top at an angle α to the horizon is subjected to the following actions (in the plane of the mast structure):

- In the initial state the cable stays are subjected to the uniformly distributed shear load $q_{lg0} = q_{rg0}$ and are prestressed with the force H_0;
- In the operating state the mast trunk is subjected to the uniformly distributed load q_m and to the moment M_m applied at its top, the windward and leeward cable stays are subjected to the uniformly distributed loads q_{lg} and q_{rg}, the temperature of the system does not change.

Determine the longitudinal forces N_{lg}, N_{rg} and N_m in the windward and leeward cable stays and in the mast trunk, as well as the bending moments M_m in the cross-sections of the mast trunk.

References: A. V. Perelmuter, Principles of Analysis of Cable-Bar Systems, Stroyizdat, 1969, p. 61

Initial data:

$EF_g = 0.58 \cdot 10^5$ t	- axial stiffness of the cable stays;
$EI_m = 0.92 \cdot 10^7$ t·m^2	- bending stiffness of the mast trunk;
S = 93.0 m	- height of the mast trunk;
L = 115.5 m	- length of the chord of the cable stays;
α = 45°	- angle of the cable stays to the horizon;
$q_{lg0} = q_{rg0} = 22.75 \cdot 10^{-3} \cdot \cos(\alpha) = 16.087$ t/m	- uniformly distributed shear load on the cable stays in the initial state;
$q_{lg} = 37.40 \cdot 10^{-3} - q_{lg0}/\cos(\alpha) = 14.650$ t/m	- uniformly distributed shear load on the windward cable stay in the operating state;
$q_{rg} = q_{rg0}/\cos(\alpha) - 8.10 \cdot 10^{-3} = 14.650$ t/m	- uniformly distributed shear load on the leeward cable stay in the operating state;
$q_m = 950.00 \cdot 10^{-3}$ t/m	- uniformly distributed shear load on the mast trunk in the operating state;
$M_m = 401.00$ t·m	- moment at the top of the mast trunk;
$H_0 = 19.40$ t	- prestressing forces of the cable stays.

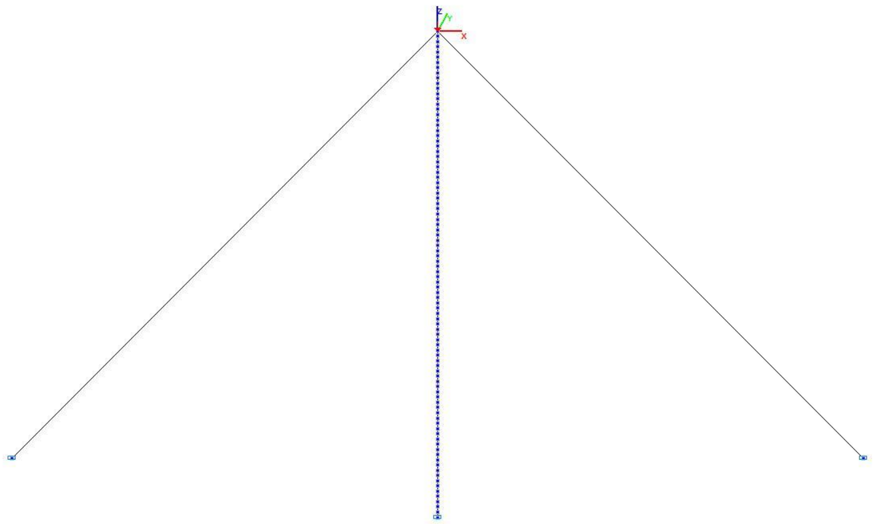
Finite element model: Design model – general type system. Elements of the mast trunk – 93 bar elements of type 5. The spacing of the finite element mesh along the height of the mast trunk (along the X1 axes of the local coordinate systems) is 1.0 m. Stiffness properties of the elements of the mast trunk:
$EF = 1.00 \cdot 10^8$ t;$EI_y = EI_z = GI_x = 0.92 \cdot 10^7$ t·m^2.

Elements of the cable stays – 2 cable-stayed elements of type 308. Boundary conditions are provided by imposing constraints on the support nodes of the cable stays in the directions of the degrees of freedom X, Z and on the support node of the mast trunk in the directions of the degrees of freedom X, Z, UY.
Actions in the initial state are defined by the stiffness properties of the cable-stayed elements:

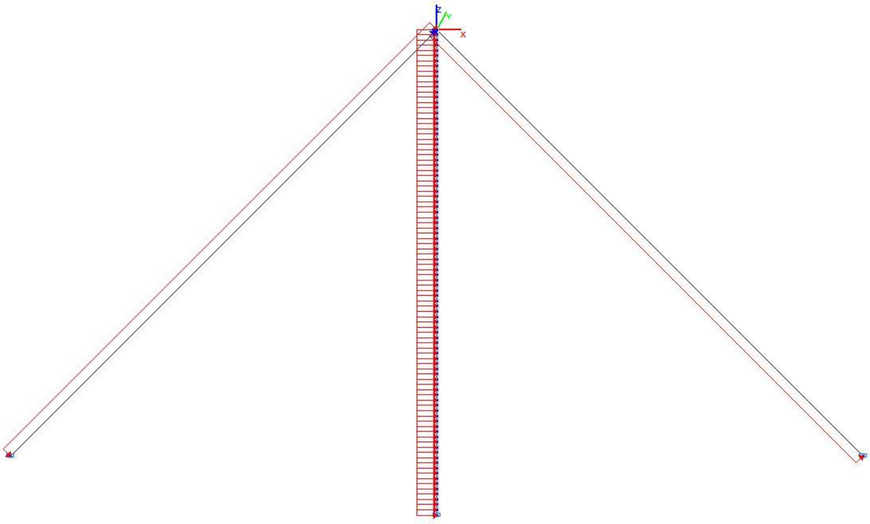
γ = 7.84483 t/m^3	- specific weight of the cable stays;
E = 2.00·10^7 t/m^2	- elastic modulus of the material of the cable stays;
ν = 0.3	- Poisson's ratio;
H_0 = 19.40 t	- prestressing forces of the cable stays;
D = 6.0675 cm	- outer diameter of the ring cross-section of the cable stays;
d = 0.0001 cm	- inner diameter of the ring cross-section of the cable stays.

A separate loading with a vertical concentrated load of the minimum value $P_0 = 1.00 \cdot 10^{-4}$ t applied at the top of the mast trunk is created to control the values of the internal forces in the initial state.
The actions in the operating state are specified as the following loads:
uniformly distributed shear load in the local coordinate system along the Z1 axis applied to the cable-stayed elements;
uniformly distributed shear load in the global coordinate system along the X axis applied to the elements of the mast trunk;
concentrated moment about the Y axis of the global coordinate system applied to the top node of the mast trunk.
The nonlinear loading was generated for the simple incremental method with a loading factor – 0.1 and a number of steps – 10 for the actions of the initial state, with a loading factor – 0.01 and a number of steps – 100 for the actions of the operating state.
Number of nodes in the design model – 96.

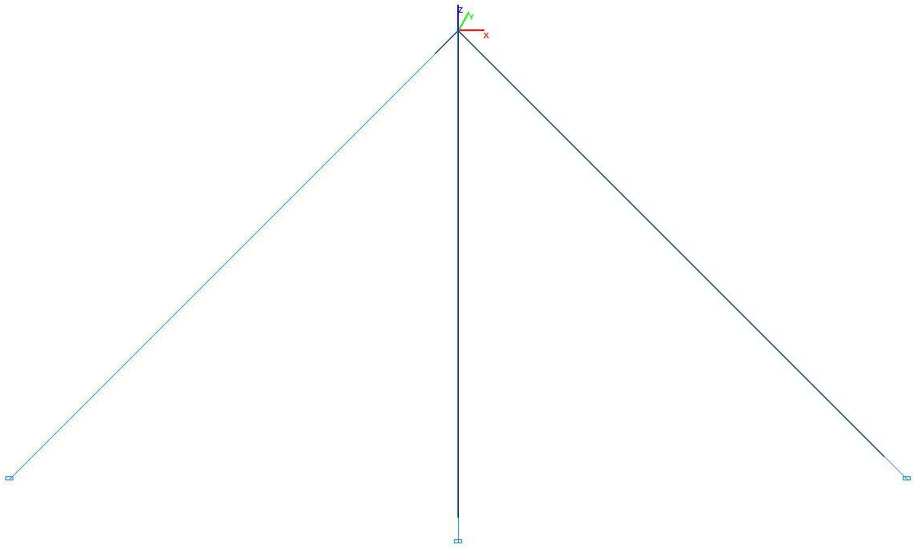
Results in SCAD



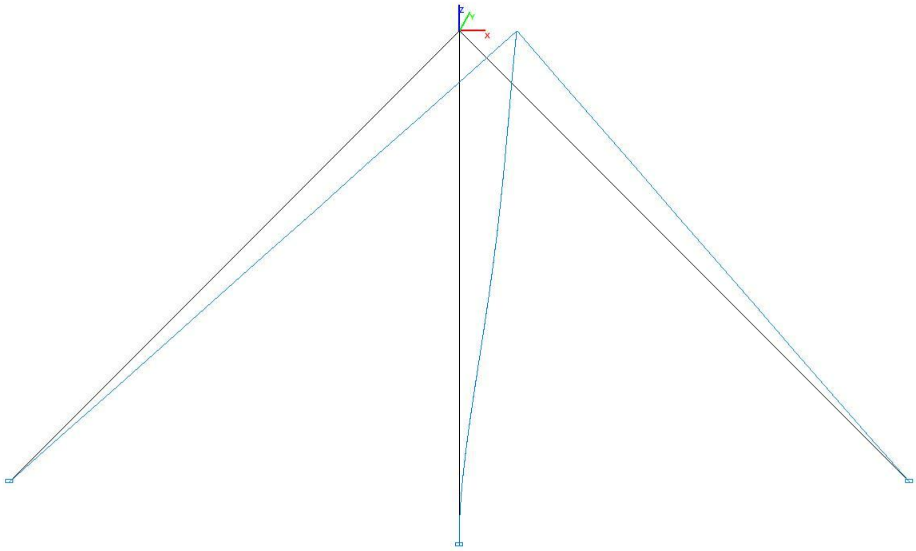
Design model in the initial state



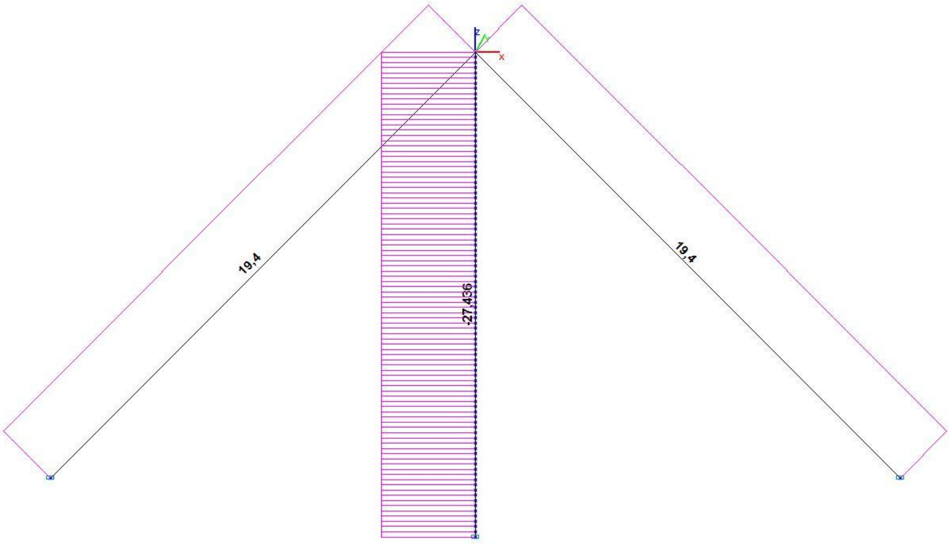
Design model in the operating state



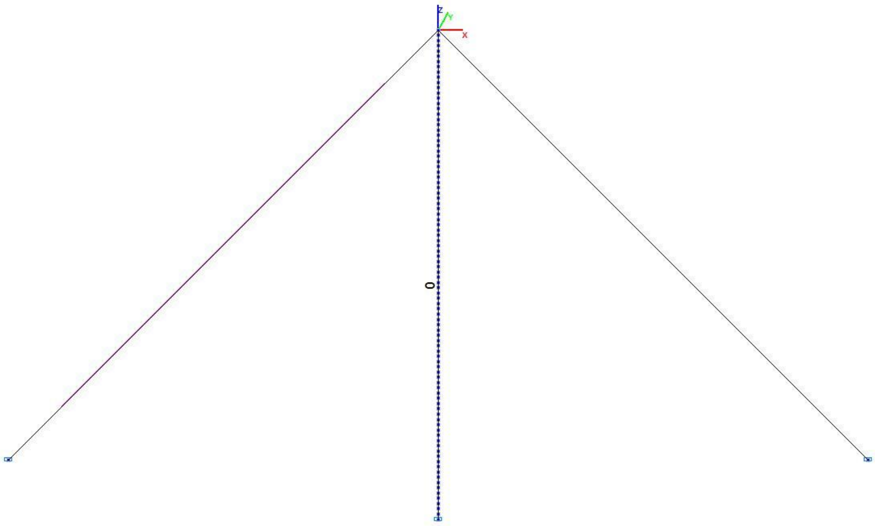
Deformed model in the initial state



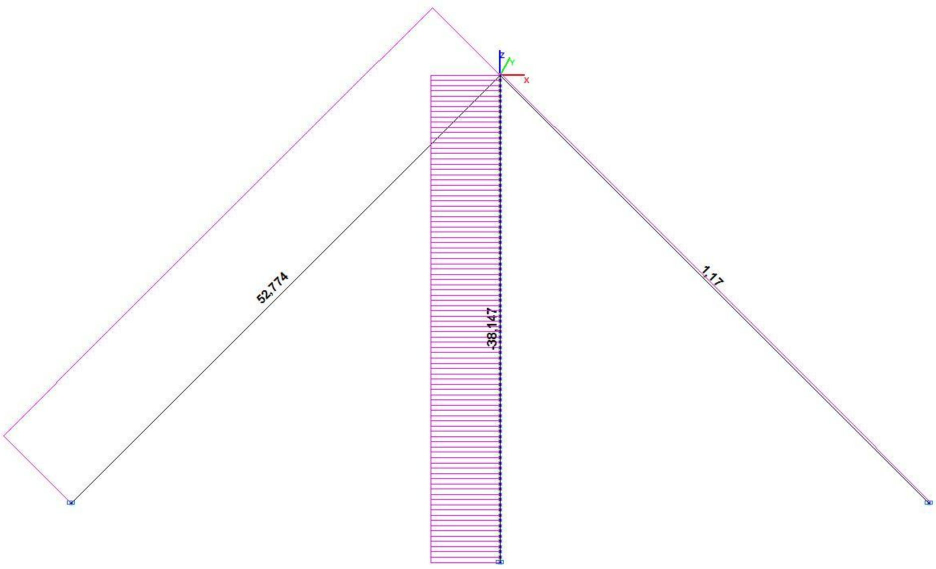
Deformed model in the operating state



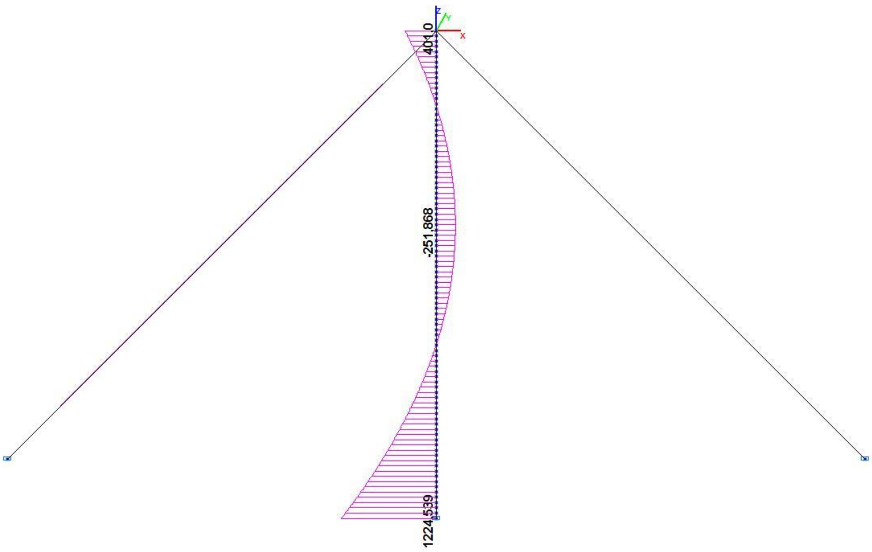
Longitudinal force diagrams in the initial state (m)



Bending moment diagrams in the initial state (t·m)



Longitudinal force diagrams in the operating state (t)

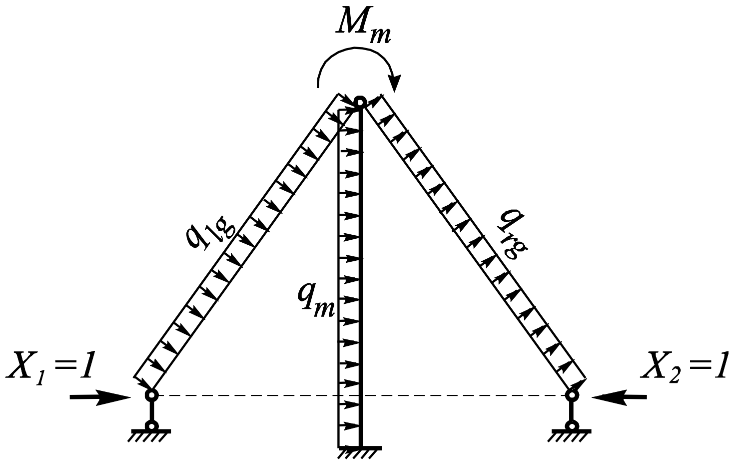


Bending moment diagrams in the operating state (t·m)

Comparison of solutions:

Parameter	Theory	SCAD	Deviation, %
N_{lg}, t	52.769	52.774	0.01
N_{rg}, t	1.171	1.170	0.09
N_m, t	-38.142	-38.147	0.01
$M_m(0)$, t·m	1227.376	1224.539	0.23
$M_m(S)$, t·m	401.000	401.000	0.00
S_{extr}, m	55.853	56.000	—
$M_m(S_{extr})$, t·m	-254.437	-251.868	1.01

Notes: In the analytical solution the internal forces in the twice statically indeterminate mast structure are determined by the force method, and the thrust reactions X_1 and X_2 of the support nodes of the cable stays are taken as the unknowns.



$$N_{lg} = H_0 + N_{lg\,1} \cdot X_1 + N_{lg\,q}$$
$$N_{rg} = H_0 + N_{rg\,2} \cdot X_2 + N_{rgq}$$
$$N_m = -2 \cdot H_0 \cdot sin(\alpha) + N_{m1} \cdot X_1 + N_{m2} \cdot X_2 + N_{mq}$$
$$M_m(0) = M_{m1}(0) \cdot X_1 + M_{m2}(0) \cdot X_2 + M_{mq}(0)$$
$$M_m(S) = M_{m1}(S) \cdot X_1 + M_{m2}(S) \cdot X_2 + M_{mq}(S)$$
$$S_{extr} = S + \frac{q_{lg} + q_{rg}}{q_m} \cdot L \cdot sin(\alpha) + \frac{X_1 - X_2}{q_m}$$
$$M_m(S_{extr}) = \frac{q_m}{2} \cdot S_{extr}{}^2 - \left[q_m \cdot S + (q_{lg} + q_{rg}) \cdot L \cdot sin(\alpha) + X_1 - X_2\right] \cdot S_{extr} +$$
$$+ \frac{q_m}{2} \cdot S^2 + (q_{lg} + q_{rg}) \cdot S \cdot L \cdot sin(\alpha) + M_m + (X_1 - X_2) \cdot S$$

The values of the unknowns X_1 and X_2 are determined by solving the system of linear equations:

$$\left.\begin{array}{l}
\left(\frac{N_{lg\,1}{}^2 \cdot L}{EF_g} + \frac{M_{m1}{}^2(0) \cdot S}{3 \cdot EI_m}\right) \cdot X_1 + \frac{M_{m1}(0) \cdot M_{m2}(0) \cdot S}{3 \cdot EI_m} \cdot X_2 + \frac{N_{lg\,1} \cdot N_{lg\,q} \cdot L}{EF_g} + \\
+ \frac{S}{6 \cdot EI_m} \cdot \left(M_{m1}(0) \cdot M_{mq}(0) + 4 \cdot M_{m1}\left(\frac{S}{2}\right) \cdot M_{mq}\left(\frac{S}{2}\right) + M_{m1}(S) \cdot M_{mq}(S)\right) - \\
- \frac{1}{2} \cdot N_{lg\,1} \cdot \left(\frac{(q_{lg} + q_{lg\,0})^2 \cdot L^3}{12 \cdot (H_0 + N_{lg\,q} + N_{lg\,1} \cdot X_1)^2} - \frac{q_{lg\,0}{}^2 \cdot L^3}{12 \cdot H_0{}^2}\right) = 0 \\
\frac{M_{m1}(0) \cdot M_{m2}(0) \cdot S}{3 \cdot EI_m} \cdot X_1 + \left(\frac{N_{rg\,2}{}^2 \cdot L}{EF_g} + \frac{M_{m2}{}^2(0) \cdot S}{3 \cdot EI_m}\right) \cdot X_2 + \frac{N_{rg1} \cdot N_{rgq} \cdot L}{EF_g} + \\
+ \frac{S}{6 \cdot EI_m} \cdot \left(M_{m2}(0) \cdot M_{mq}(0) + 4 \cdot M_{m2}\left(\frac{S}{2}\right) \cdot M_{mq}\left(\frac{S}{2}\right) + M_{m2}(S) \cdot M_{mq}(S)\right) - \\
- \frac{1}{2} \cdot N_{rg\,2} \cdot \left(\frac{(-q_{rg} + q_{rg0})^2 \cdot L^3}{12 \cdot (H_0 + N_{rgq} + N_{rg\,2} \cdot X_2)^2} - \frac{q_{rg0}{}^2 \cdot L^3}{12 \cdot H_0{}^2}\right) = 0
\end{array}\right\}, \text{where:}$$

$$N_{lg\,1} = -\frac{1}{cos(\alpha)}; \qquad N_{m1} = tg(\alpha);$$
$$M_{m1}(0) = 1 \cdot S; \qquad M_{m1}\left(\frac{S}{2}\right) = \frac{S}{2}; \qquad M_{m1}(S) = 0 \cdot S;$$
$$N_{rg\,2} = -\frac{1}{cos(\alpha)}; \qquad N_{m2} = tg(\alpha);$$
$$M_{m2}(0) = -1 \cdot S; \qquad M_{m2}\left(\frac{S}{2}\right) = -\frac{S}{2}; \qquad M_{m2}(S) = -0 \cdot S;$$
$$N_{lg\,q} = -\frac{q_{lg} \cdot L}{2} \cdot tg(\alpha); \qquad N_{rgq} = \frac{q_{rg} \cdot L}{2} \cdot tg(\alpha);$$
$$N_{mq} = -\frac{(q_{lg} - q_{rg}) \cdot L}{2} \cdot \frac{cos(2 \cdot \alpha)}{cos(\alpha)};$$
$$M_{mq}(0) = M_m + \frac{q_m \cdot S^2}{2} + (q_{lg} + q_{rg}) \cdot S \cdot L \cdot sin(\alpha);$$
$$M_{mq}\left(\frac{S}{2}\right) = M_m + \frac{q_m \cdot S^2}{8} + \frac{(q_{lg} + q_{rg}) \cdot S \cdot L}{2} \cdot sin(\alpha);$$
$$M_{mq}(S) = M_m.$$

Square Membrane with a Compliant Contour

Objective: Comparison of the results of the geometrically nonlinear analysis with the experimental studies.

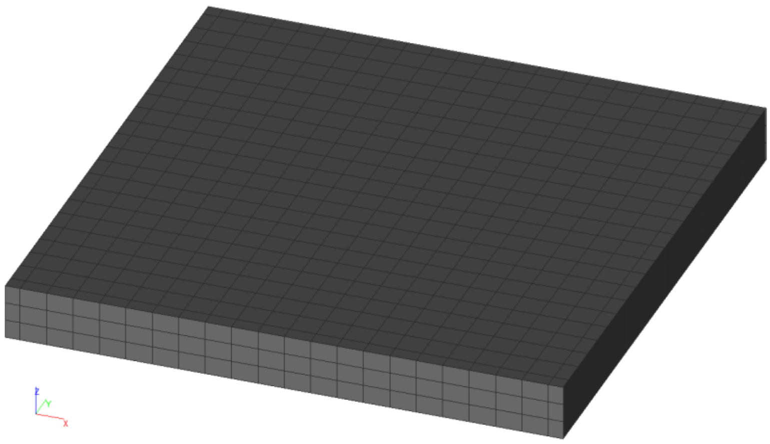
Initial data file: Плита-мембрана 4.SPR

Problem formulation:
The behavior of a square membrane with a support contour compliant in its plane subjected to the load uniformly distributed over the surface. It is necessary to compare the calculated data with the experimental one, when the deflection in the center and the overall picture are known.

References: G.L. Anikeev, A.Ya. Pritsker, I.N. Lebedich, Experience in Designing, Manufacturing and Testing Roofing Panels of Aluminum Alloys // Building Structures from Aluminum Alloys (Design, Research, Production) - M.: Stroyizdat, 1963

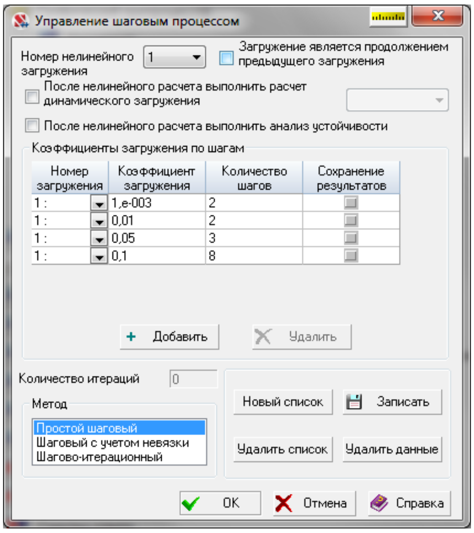
Initial data:
A membrane structure 3×3 m made of AMG-6M alloy was tested. The thickness of the membrane sheet is 1 mm, the contour is made of a bent channel 80×300×3 mm. The measurement of the prototype was performed before the tests, the initial sag of the membrane center was 1,5 mm. The test load is 100 kgf/m^2. The displacements in the center were measured by the Maximov deflectometer. A very characteristic deflection pattern was noted, in which the level lines deviate far from the oval shape and are closer to a rectangular form in the vicinity of the contour.

Finite element model: The design model is assembled from shell finite elements (FE 341), the model contains 832 elements. Constraints in the Z direction were provided at the corners of the structure, and constraints along X and along Y were provided at the centers of the sides of the support contour parallel to the X and Y axes respectively.
The initial imperfection of the membrane is taken into account in the design model.



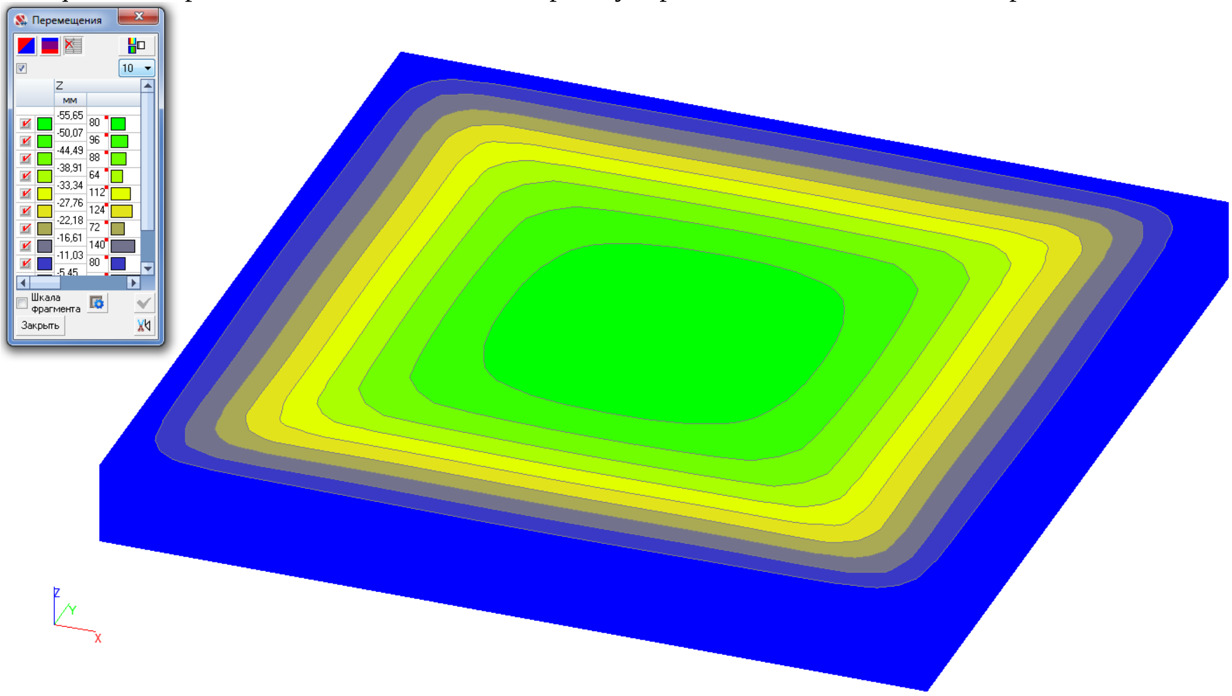
Design model

The nonlinear problem was solved by the incremental method with the steps shown in the following screenshot:



Results in SCAD

The qualitative picture of the deformation completely repeated that observed in the experiment



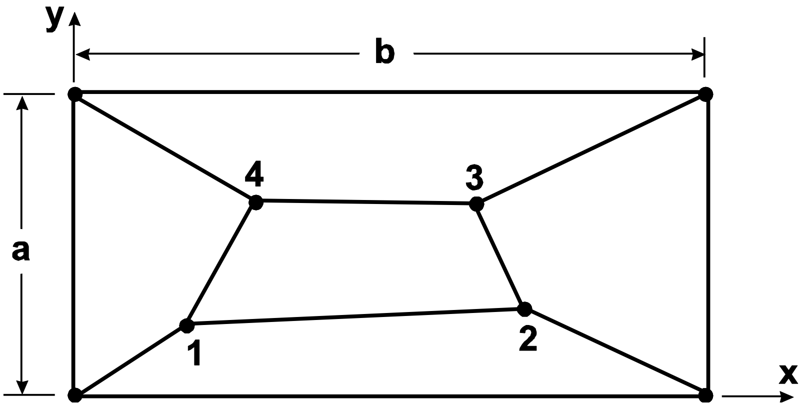
Isofields of displacements

Comparison of solutions:

Parameter	Experiment	SCAD	Deviation, %
Maximum displacement in the vertical direction (mm)	59,1	55,65	5,84

Pathological Tests

Rectangular Plate under the Constant Stresses on the Midsurface



Objective: Check of the obtained values of the constant stresses on the midsurface of a rectangular plate at an irregular coarse finite element mesh.

Initial data files:

File name	Description
Patch_test_Constant_stress_Shell_42.SPR	Design model with the elements of type 42
Patch_test_Constant_stress_Shell_44.SPR	Design model with the elements of type 44
Patch_test_Constant_stress_Shell_45.SPR	Design model with the elements of type 45
Patch_test_Constant_stress_Shell_50.SPR	Design model with the elements of type 50

Problem formulation: The rectangular isotropic plate of constant thickness is subjected to the displacements of the outer edges providing the conditions of constant stresses on the midsurface. Check that the conditions of constant normal σ_x, σ_y and tangential τ_{xy} stresses on the midsurface are provided.

References: R. H. Macneal, R. L. Harder, A proposed standard set of problems to test finite element accuracy, North-Holland, Finite elements in analysis and design, 1, 1985, p. 3-20.
J. Robinson, S. Blackham, An evaluation of lower order membranes as contained in MSC/NASTRAN, ASAS and PARFEC FEM system, Dorset, Robinson and associates, 1979.

Initial data:

$E = 1.0 \cdot 10^6$ kPa - elastic modulus of the plate material;
$\nu = 0.25$ - Poisson's ratio;
t = 0.001 m - thickness of the plate;
a = 0.12 m - short side of the plate;
b = 0.24 m - long side of the plate;

Boundary conditions:

$u = 10^{-3} \cdot (x + y/2)$ - displacement of the outer edges along the long side of the plate;
$v = 10^{-3} \cdot (x/2 + y)$ - displacement of the outer edges along the short side of the plate;

Location of internal nodes of the finite element mesh:

Numbers of nodes in the Figure 1	x	y
1	0.04	0.02
2	0.18	0.03
3	0.16	0.08
4	0.08	0.08

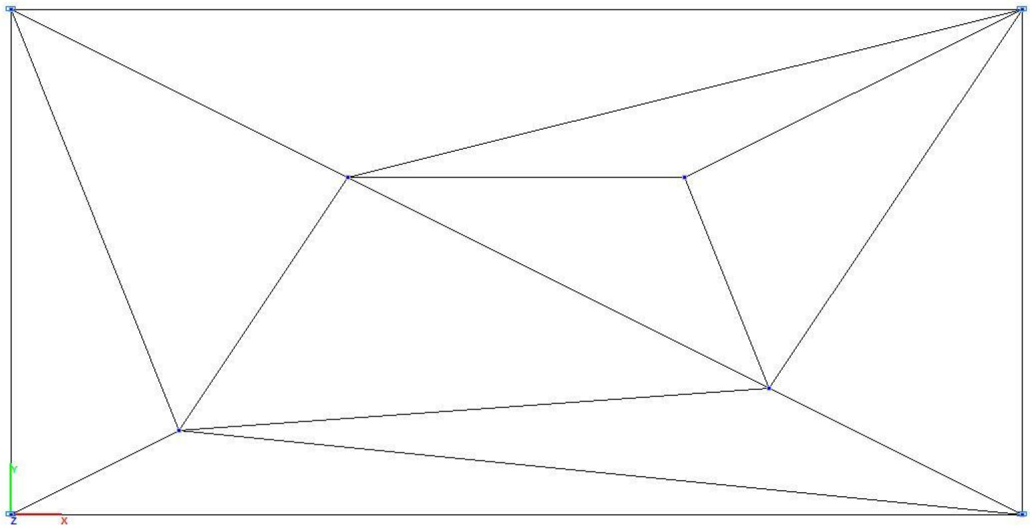
***Finite element model*:** Design model – general type system. Four design models are considered:
Model 1 - 10 three-node shell elements of type 42. Boundary conditions are provided by imposing constraints on the nodes of the outer edges of the plate in the directions of the degrees of freedom X, Y, Z, UX, UY, UZ and their displacement in accordance with the specified values u and v. Number of nodes in the model – 8.

Model 2 - 5 four-node shell elements of type 44. Boundary conditions are provided by imposing constraints on the nodes of the outer edges of the plate in the directions of the degrees of freedom X, Y, Z, UX, UY, UZ and their displacement in accordance with the specified values u and v. Number of nodes in the model – 8.

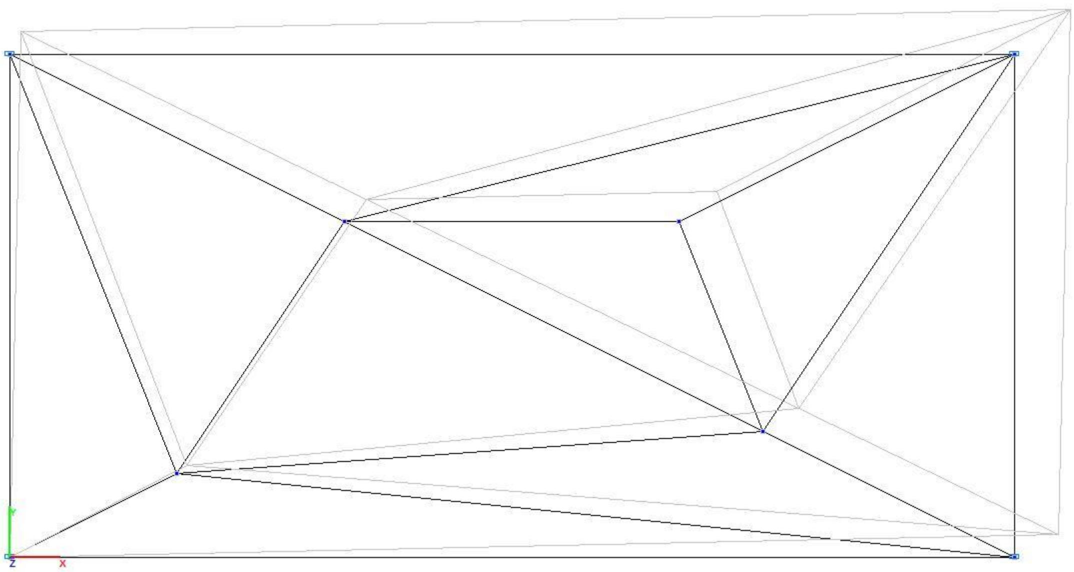
Model 3 - 10 six-node shell elements of type 45. Boundary conditions are provided by imposing constraints on the nodes of the outer edges of the plate in the directions of the degrees of freedom X, Y, Z, UX, UY, UZ and their displacement in accordance with the specified values u and v. Number of nodes in the model – 25.

Model 4 - 5 eight-node shell elements of type 50. Boundary conditions are provided by imposing constraints on the nodes of the outer edges of the plate in the directions of the degrees of freedom X, Y, Z, UX, UY, UZ and their displacement in accordance with the specified values u and v. Number of nodes in the model – 20.

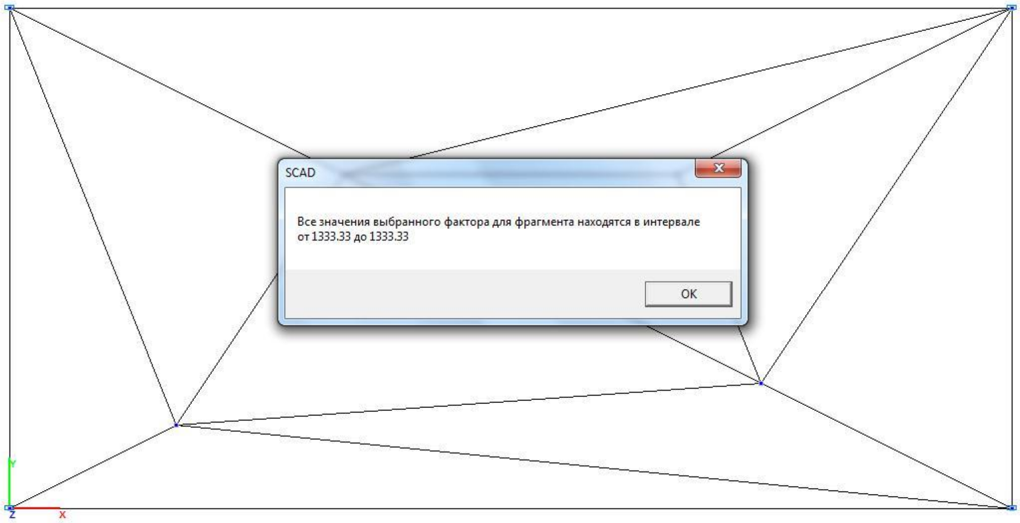
Results in SCAD



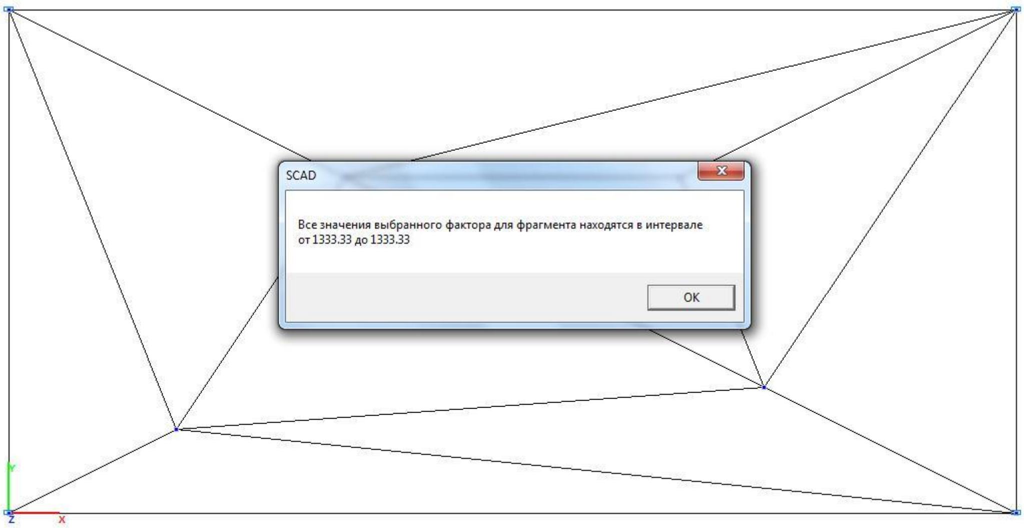
Model 1. Design model



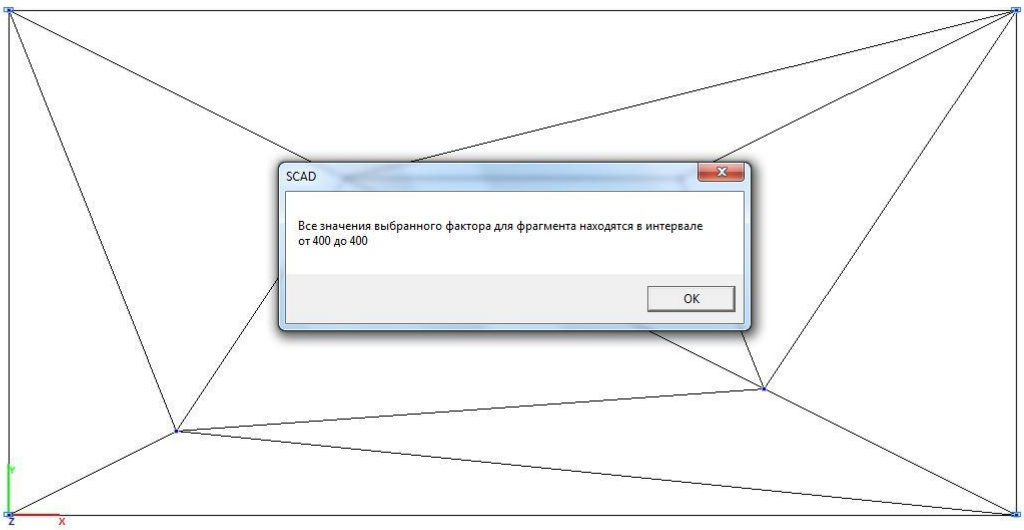
Model 1. Deformed model



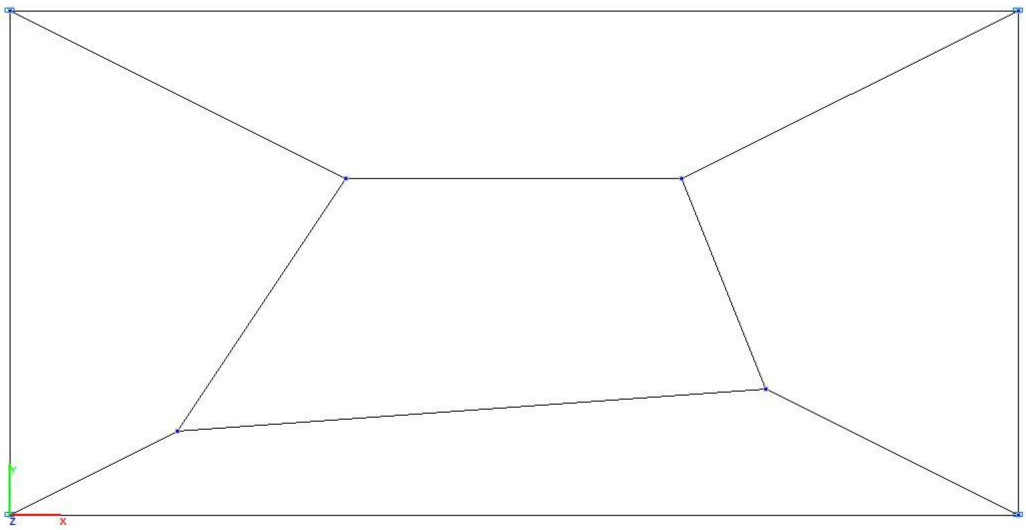
Model 1. Values of normal stresses σ_x (kN/m^2)



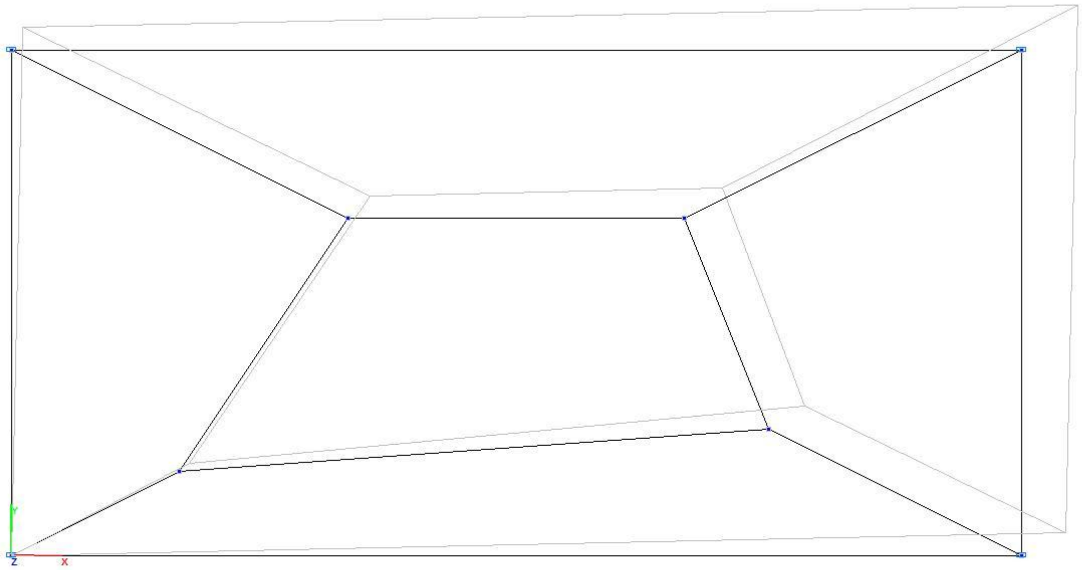
Model 1. Values of normal stresses σ_y (kN/m^2)



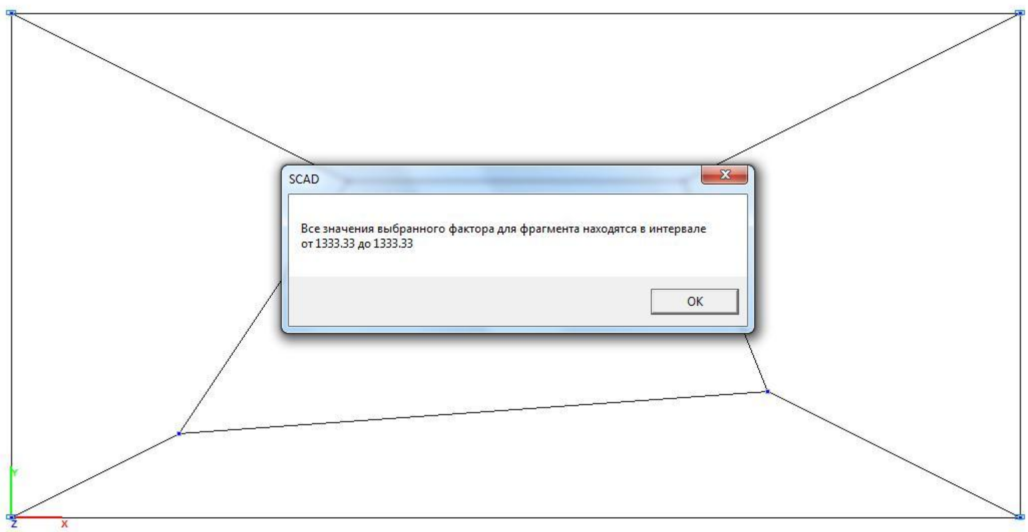
Model 1. Values of tangential stresses τ_{xy} (kN/m^2)



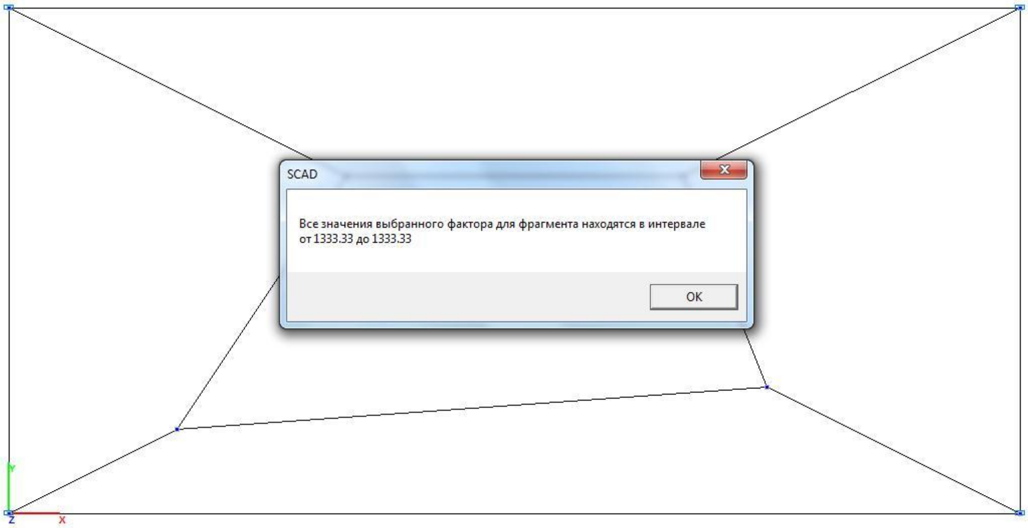
Model 2. Design model



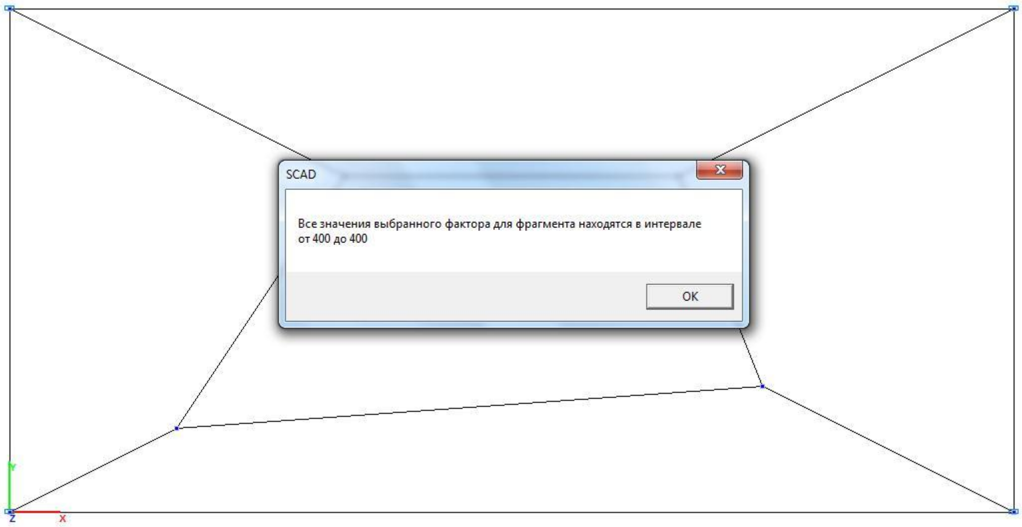
Model 2. Deformed model



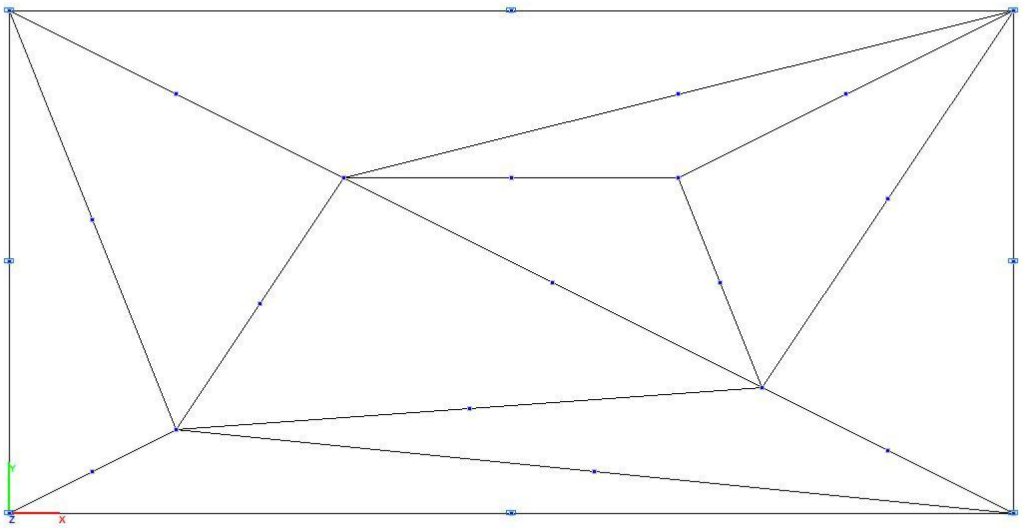
Model 2. Values of normal stresses σ_x (kN/m^2)



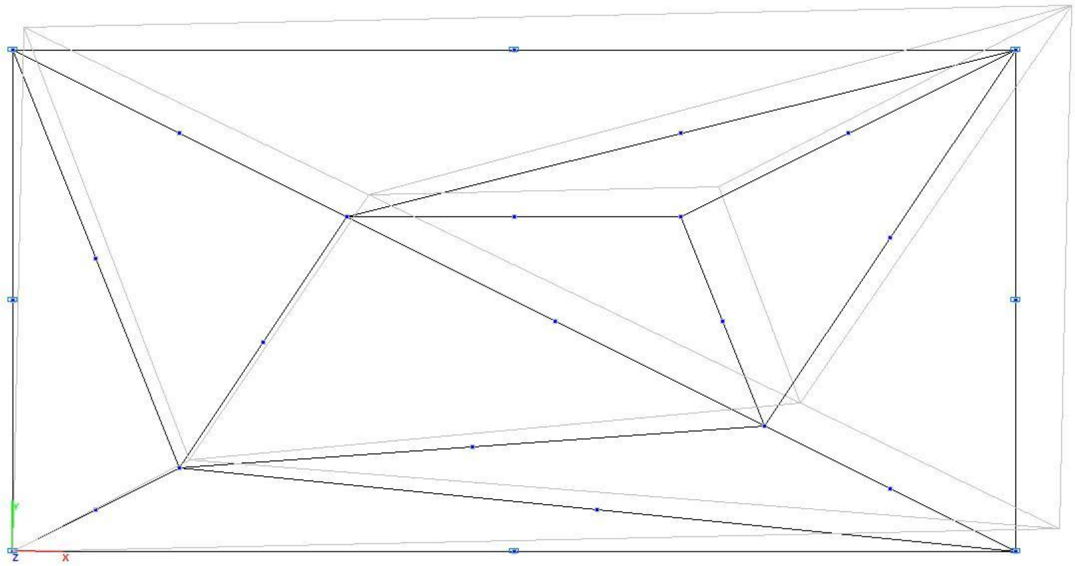
Model 2. Values of normal stresses σ_y (kN/m^2)



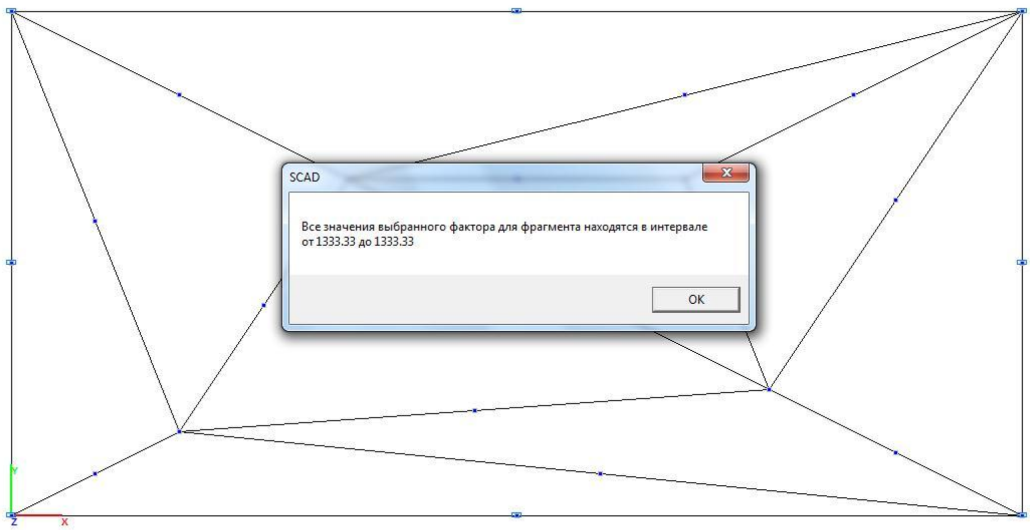
Model 2. Values of tangential stresses τ_{xy} (kN/m^2)



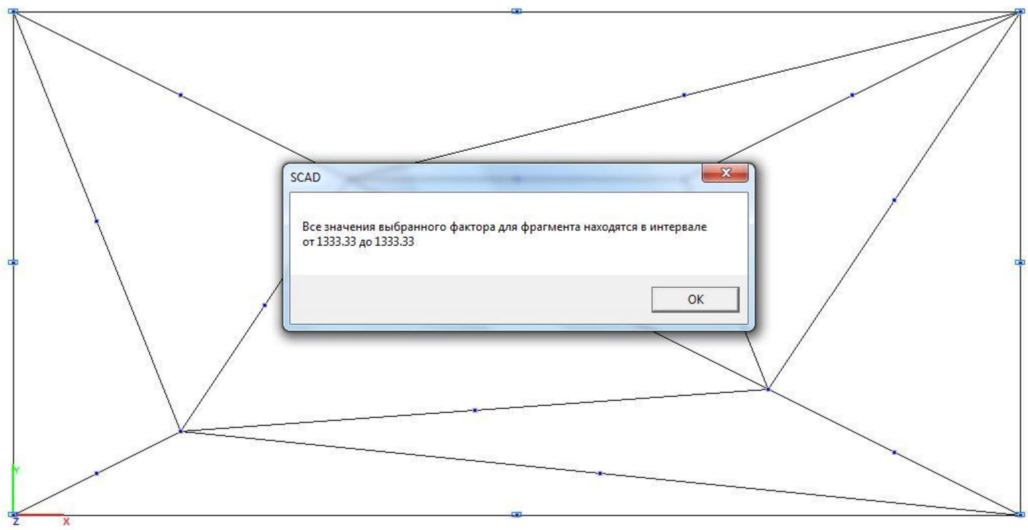
Model 3. Design model



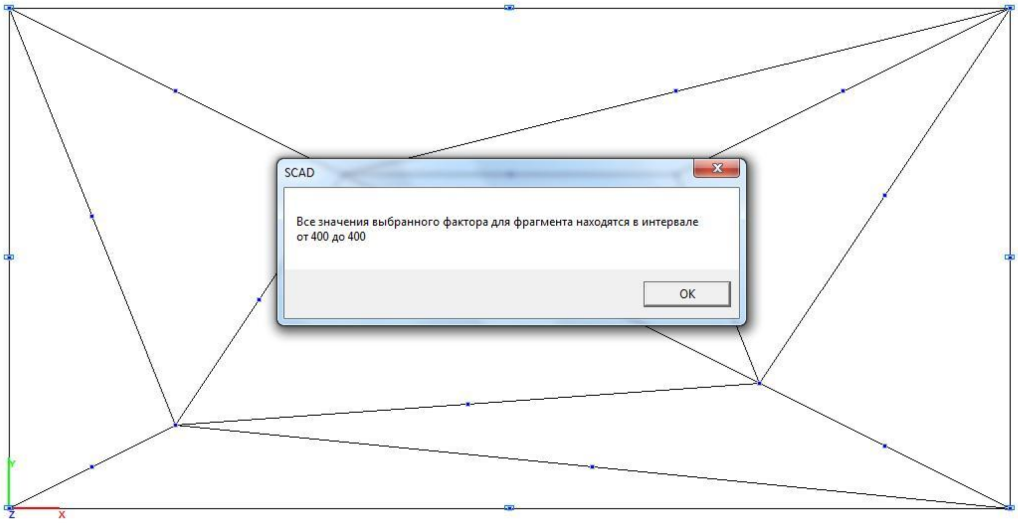
Model 3. Deformed model



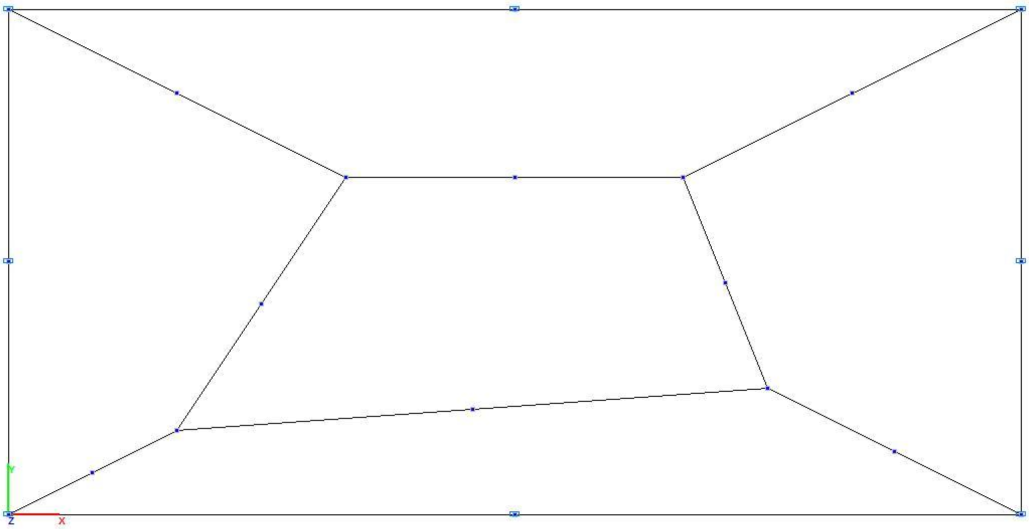
Model 3. Values of normal stresses σ_x (kN/m^2)



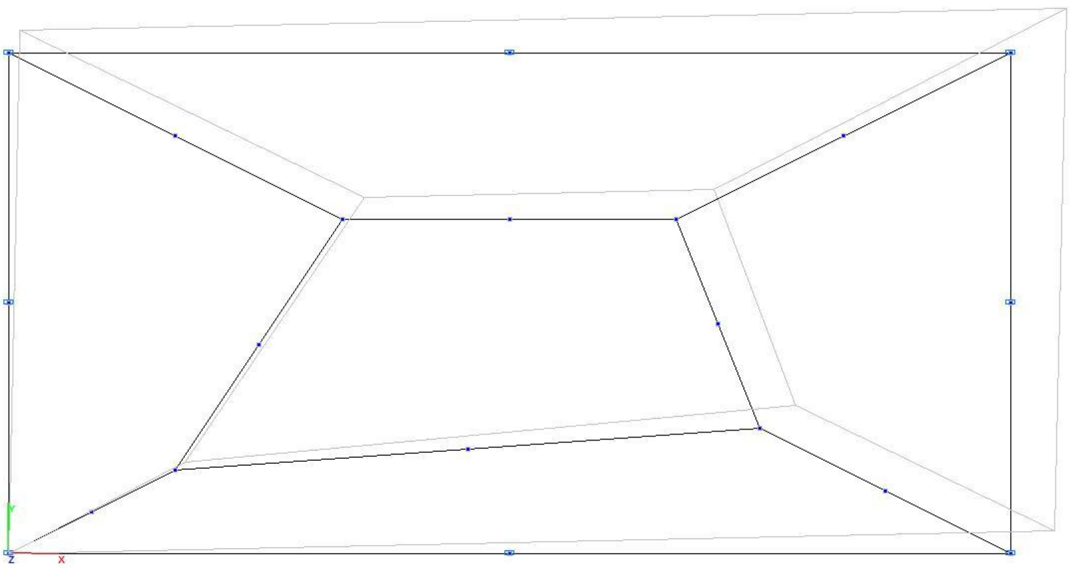
Model 3. Values of normal stresses σ_y (kN/m^2)



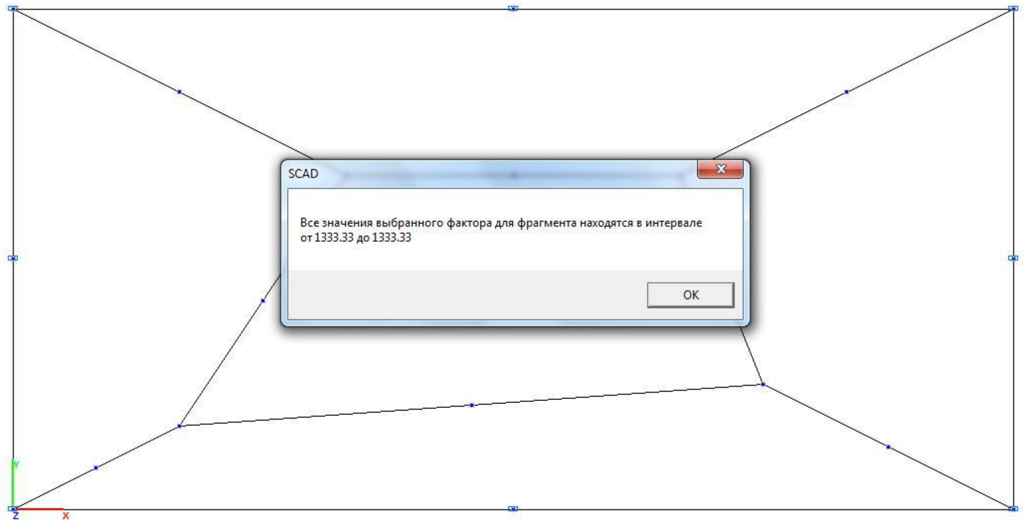
Model 3. Values of tangential stresses τ_{xy} (kN/m^2)



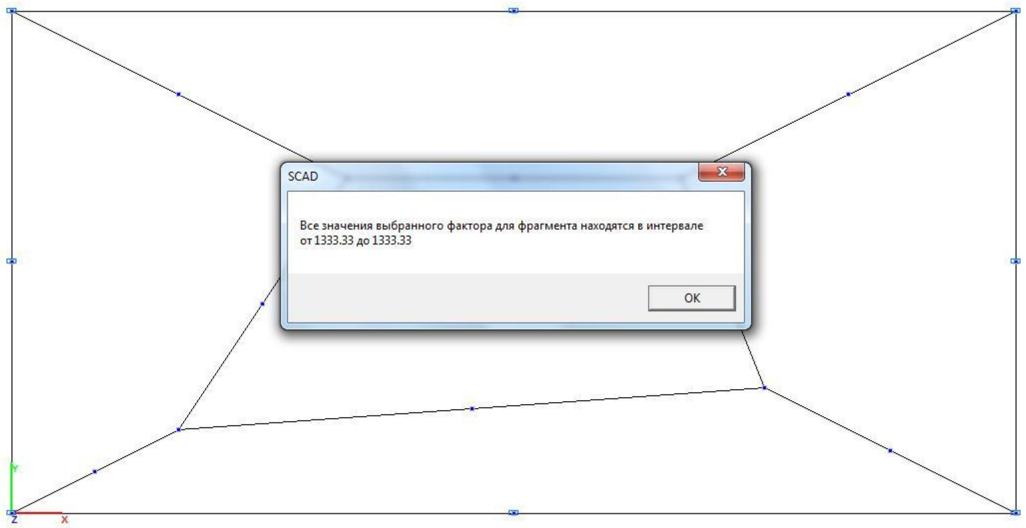
Model 4. Design model



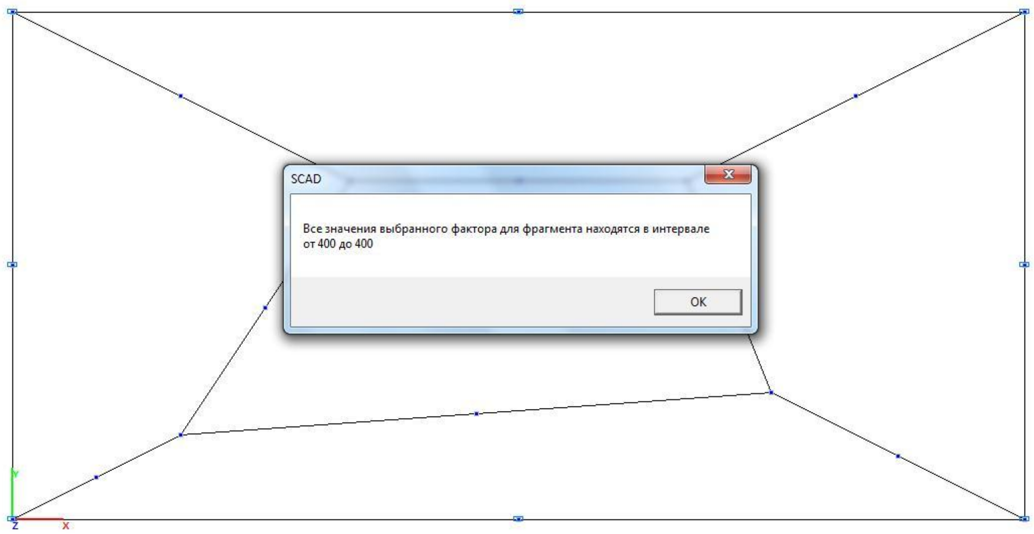
Model 4. Deformed model



Model 4. Values of normal stresses σ_x (kN/m^2)



Model 4. Values of normal stresses σ_y (kN/m^2)



Model 4. Values of tangential stresses τ_{xy} (kN/m^2)

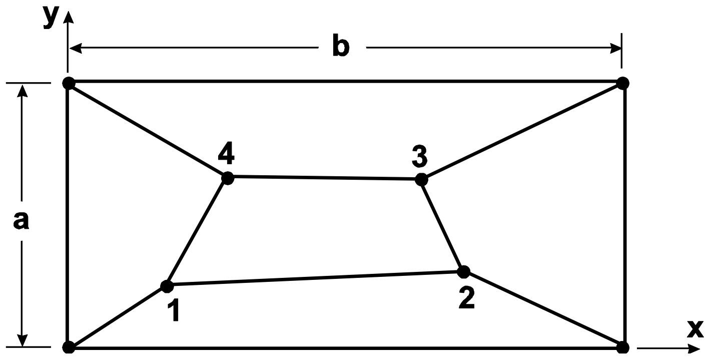
Comparison of solutions:

Model	Parameter	Theory	SCAD	Deviation, %
1	Normal stresses σ_x, kN/m^2	1333	1333	0.00
	Normal stresses σ_y, kN/m^2	1333	1333	0.00
	Tangential stresses τ_{xy}, kN/m^2	400	400	0.00
2	Normal stresses σ_x, kN/m^2	1333	1333	0.00
	Normal stresses σ_y, kN/m^2	1333	1333	0.00
	Tangential stresses τ_{xy}, kN/m^2	400	400	0.00
3	Normal stresses σ_x, kN/m^2	1333	1333	0.00
	Normal stresses σ_y, kN/m^2	1333	1333	0.00
	Tangential stresses τ_{xy}, kN/m^2	400	400	0.00
4	Normal stresses σ_x, kN/m^2	1333	1333	0.00
	Normal stresses σ_y, kN/m^2	1333	1333	0.00
	Tangential stresses τ_{xy}, kN/m^2	400	400	0.00

Notes: In the analytical solution the normal σ_x, σ_y and tangential τ_{xy} stresses on the midsurface of the plate are determined according to the following formulas:

$$\sigma_x = 10^{-3} \cdot \frac{E}{1-\nu}; \qquad \sigma_y = 10^{-3} \cdot \frac{E}{1-\nu}; \qquad \tau_{xy} = 10^{-3} \cdot \frac{E}{2 \cdot (1+\nu)}.$$

Rectangular Plate with Constant Curvature



Objective: Check of the obtained values of the stresses on the external surface for a rectangular plate at an irregular coarse finite element mesh.

Initial data files:

File name	Description
Patch_test_Constant_curvature_Shell_42.SPR	Design model with the elements of type 42
Patch_test_Constant_curvature_Shell_44.SPR	Design model with the elements of type 44
Patch_test_Constant_curvature_Shell_45.SPR	Design model with the elements of type 45
Patch_test_Constant_curvature_Shell_50.SPR	Design model with the elements of type 50

Problem formulation: The rectangular isotropic plate of constant thickness is subjected to the displacements and rotations of the outer edges providing the constant curvature (stresses on the external surface). Check that the constant curvature κ_x, κ_y, κ_{xy} (stresses on the external surface σ_x, σ_y, τ_{xy}) is provided.

References: R. H. Macneal, R. L. Harder, A proposed standard set of problems to test finite element accuracy, North-Holland, Finite elements in analysis and design, 1, 1985, p. 3-20.
J. Robinson, S. Blackham, An evaluation of plate bending elements: MSC/NASTRAN, ASAS, PARFEC, ANSYS and SAP4, Dorset, Robinson and associates, 1981.

Initial data:

$E = 1.0 \cdot 10^6$ kPa - elastic modulus of the plate material;
$\nu = 0.25$ - Poisson's ratio;
$t = 0.001$ m - thickness of the plate;
$a = 0.12$ m - short side of the plate;
$b = 0.24$ m - long side of the plate;

Boundary conditions:

$w = 10^{-3} \cdot (x^2 + x \cdot y + y^2)/2$ - displacement of the outer edges along the normal to the surface of the plate;
$\theta_x = 10^{-3} \cdot (x/2 + y)$ - rotation of the outer edges about the short sides of the plate;
$\theta_y = 10^{-3} \cdot (-x - y/2)$ - rotation of the outer edges about the long sides of the plate.

Location of internal nodes of the finite element mesh:

Numbers of nodes in the Figure 1	x	y
1	0.04	0.02
2	0.18	0.03
3	0.16	0.08
4	0.08	0.08

Finite element model: Design model – general type system. Four design models are considered:
Model 1 - 10 three-node shell elements of type 42. Boundary conditions are provided by imposing constraints on the nodes of the outer edges of the plate in the directions of the degrees of freedom X, Y, Z,

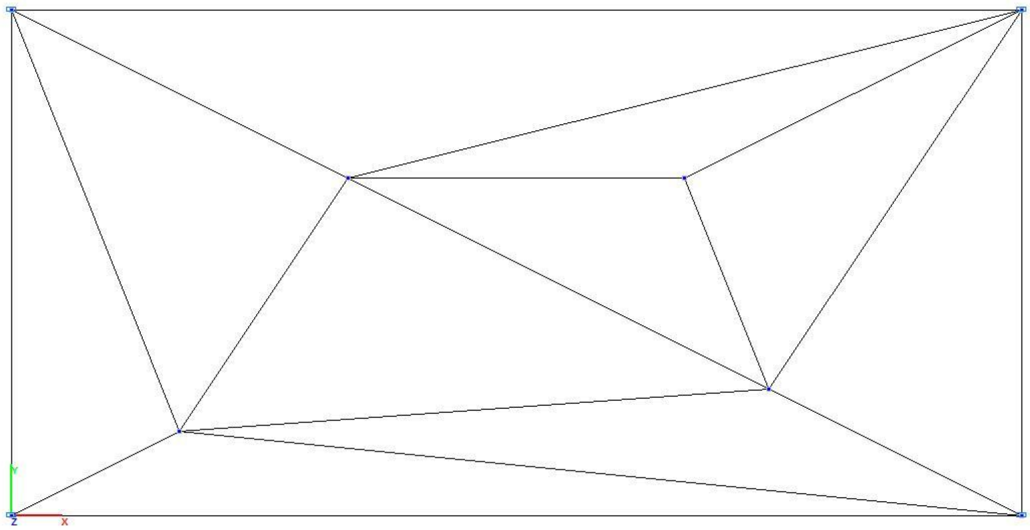
UX, UY, UZ and their displacement (rotation) in accordance with the specified values w, θ_x and θ_y. Number of nodes in the model – 8.

Model 2 - 5 four-node shell elements of type 44. Boundary conditions are provided by imposing constraints on the nodes of the outer edges of the plate in the directions of the degrees of freedom X, Y, Z, UX, UY, UZ and their displacement (rotation) in accordance with the specified values w, θ_x and θ_y.. Number of nodes in the model – 8.

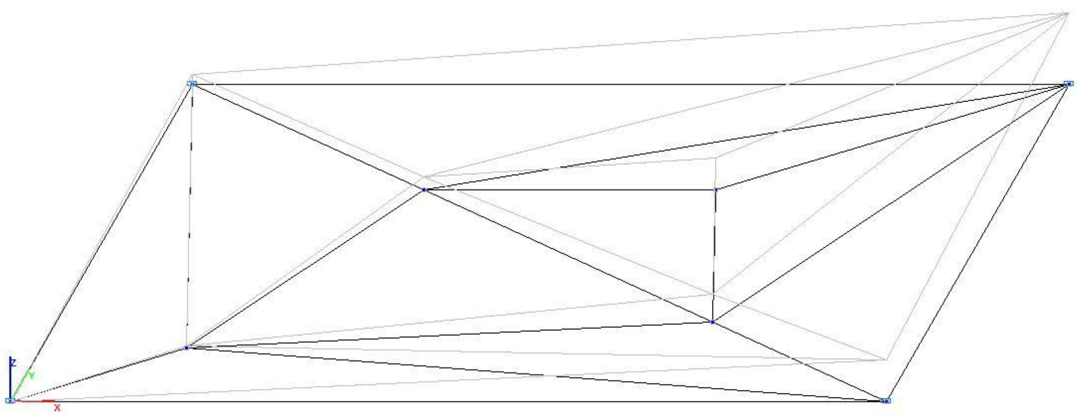
Model 3 – 10 six-node shell elements of type 45. Boundary conditions are provided by imposing constraints on the nodes of the outer edges of the plate in the directions of the degrees of freedom X, Y, Z, UX, UY, UZ and their displacement (rotation) in accordance with the specified values w, θ_x and θ_y. Number of nodes in the model – 25.

Model 4 - 5 eight-node shell elements of type 50. Boundary conditions are provided by imposing constraints on the nodes of the outer edges of the plate in the directions of the degrees of freedom X, Y, Z, UX, UY, UZ and their displacement (rotation) in accordance with the specified values w, θ_x and θ_y. Number of nodes in the model – 20.

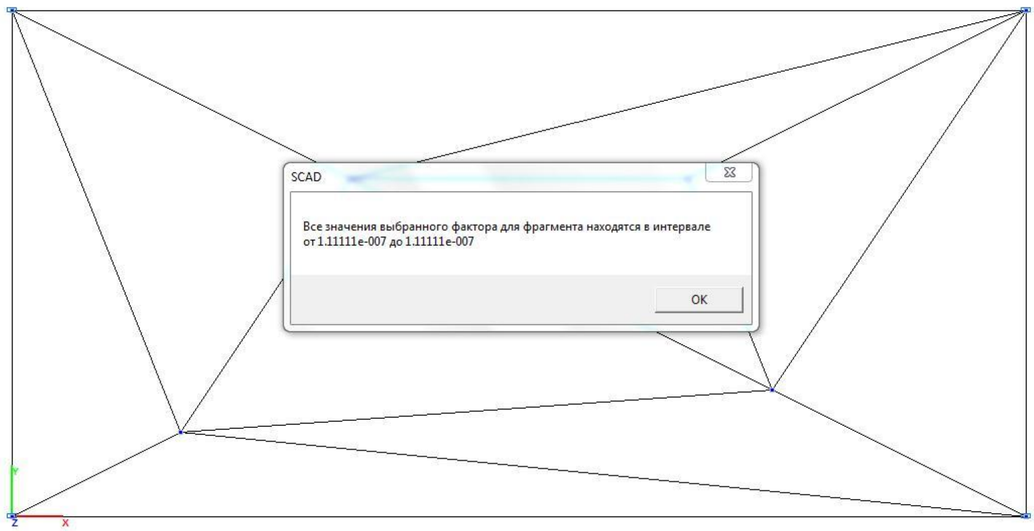
Results in SCAD



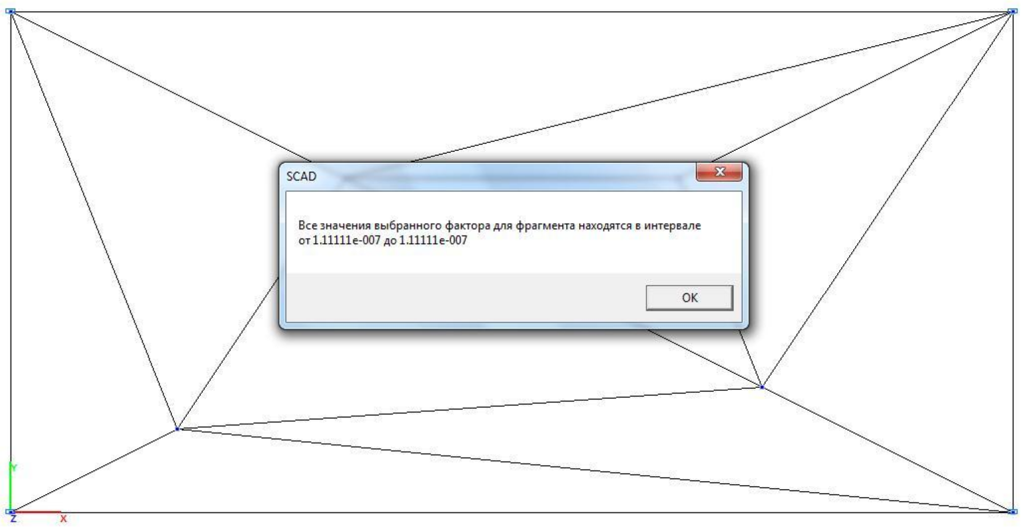
Model 1. Design model



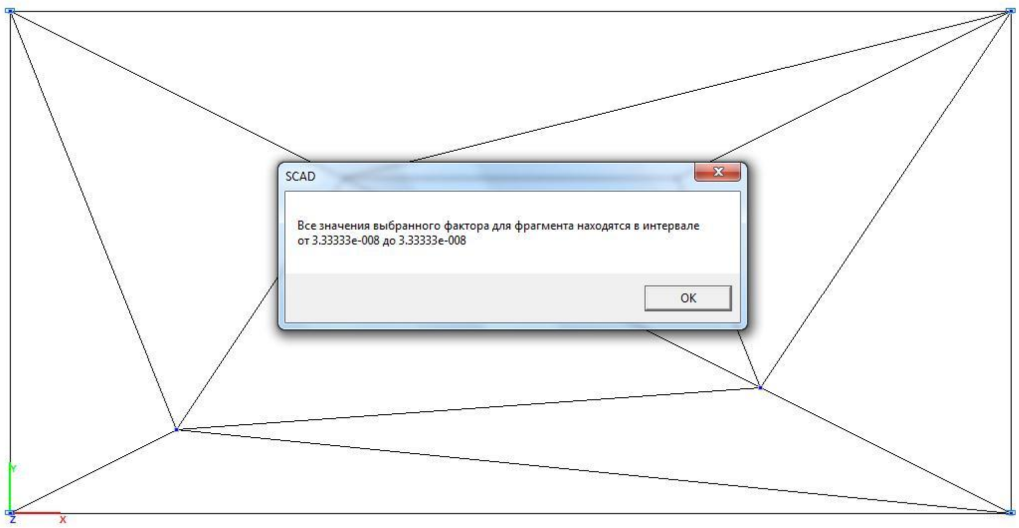
Model 1. Deformed model



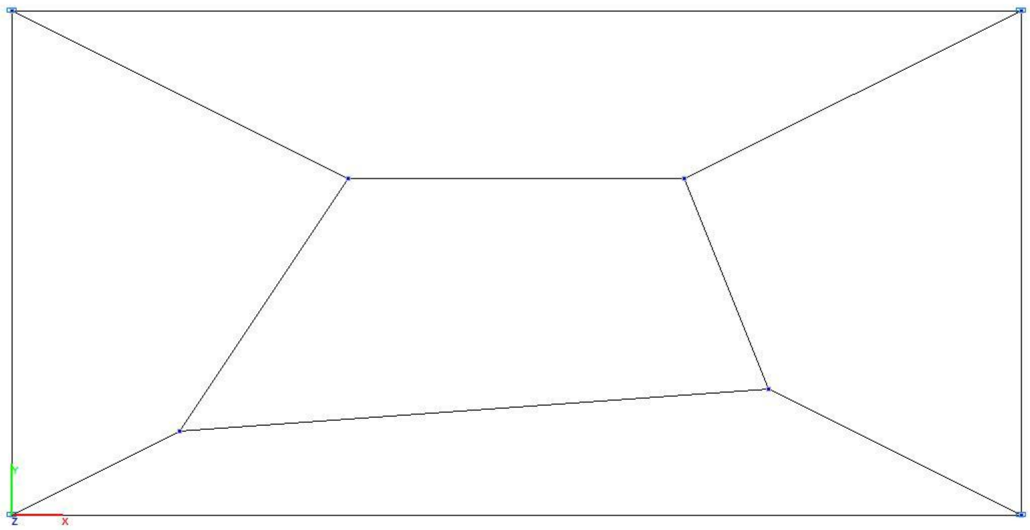
Model 1. Values of the bending moment M_x (kN·m/m)



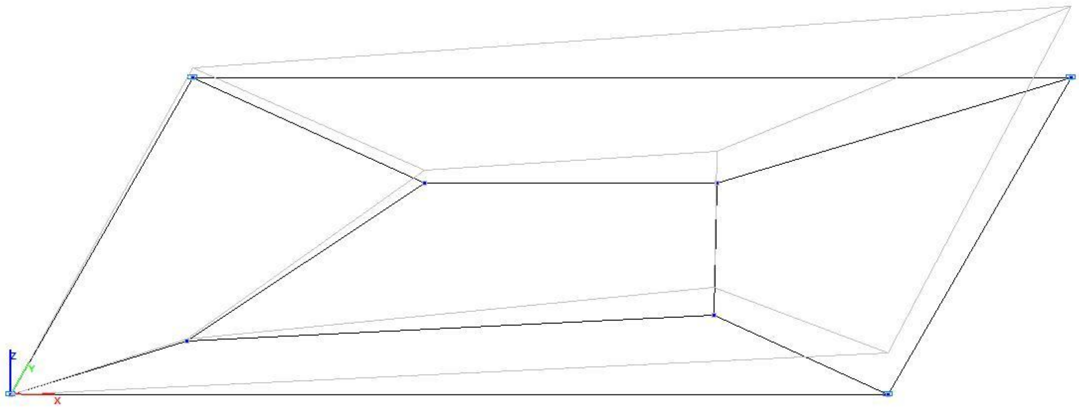
Model 1. Values of the bending moment M_y (kN·m/m)



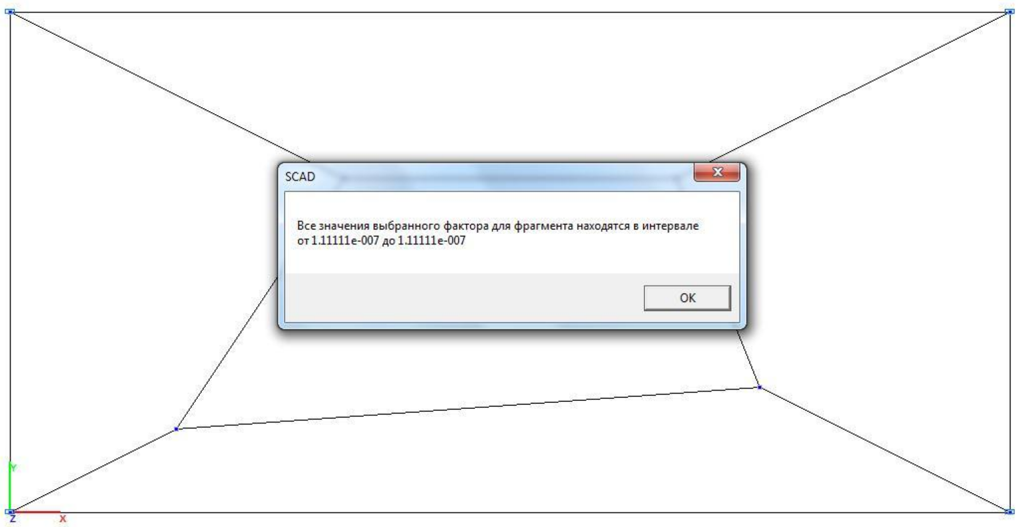
Model 1. Values of the torque M_{xy} (kN·m/m)



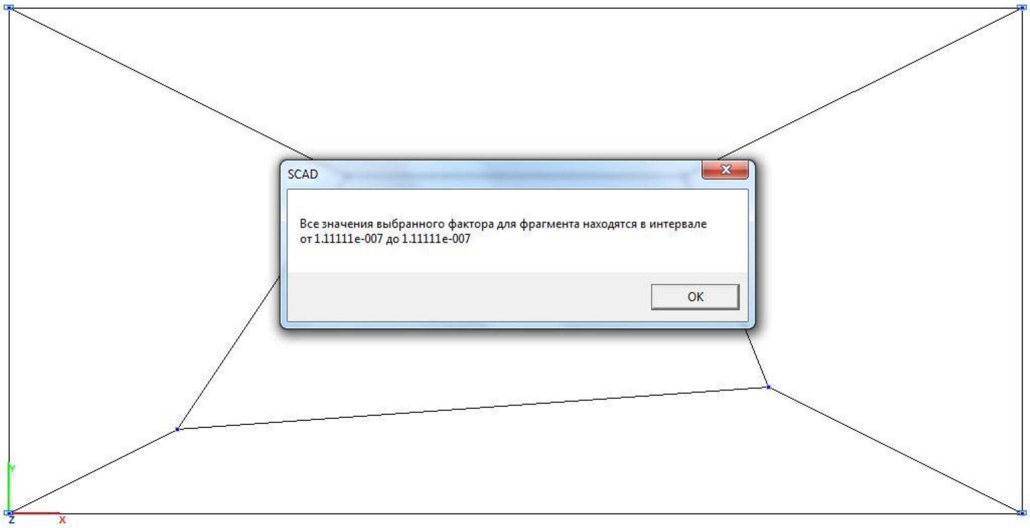
Model 2. Design model



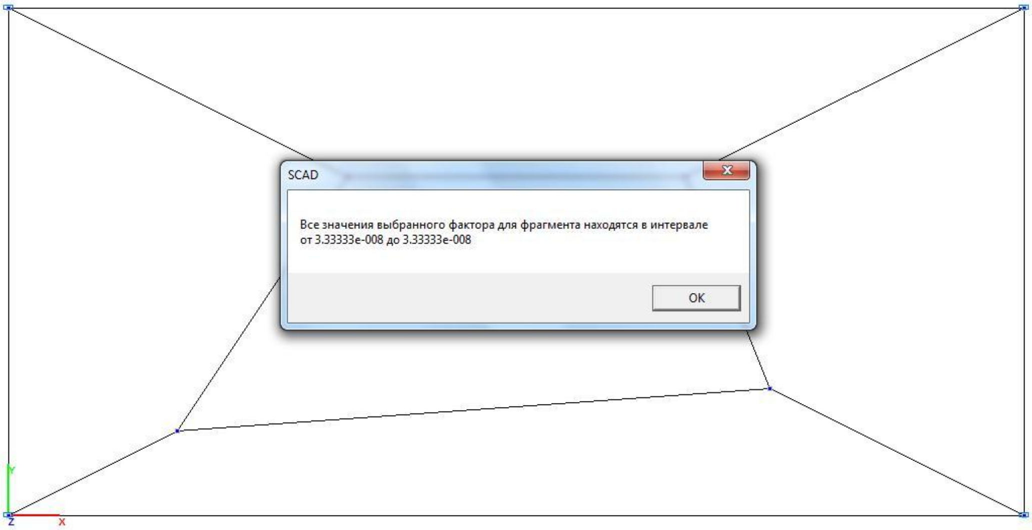
Model 2. Deformed model



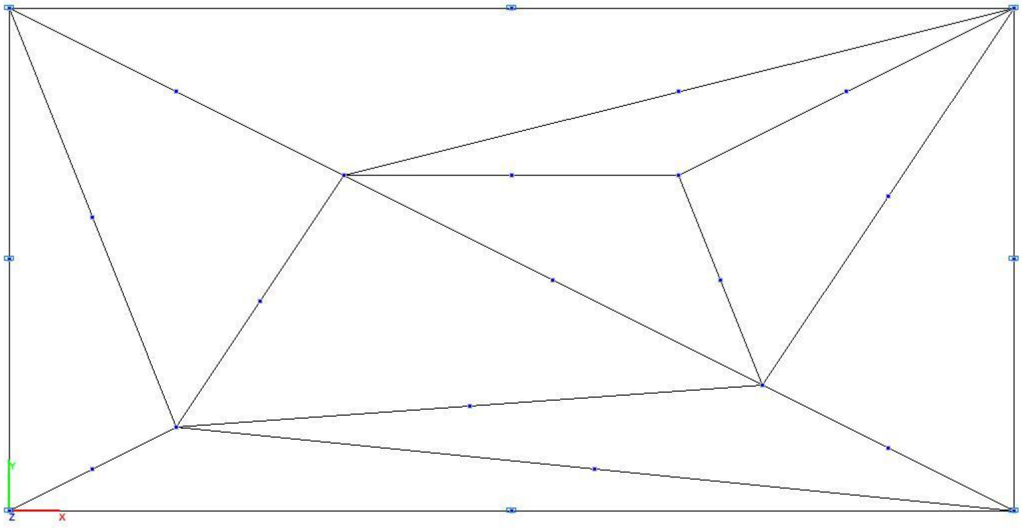
Model 2. Values of the bending moment M_x (kN·m/m)



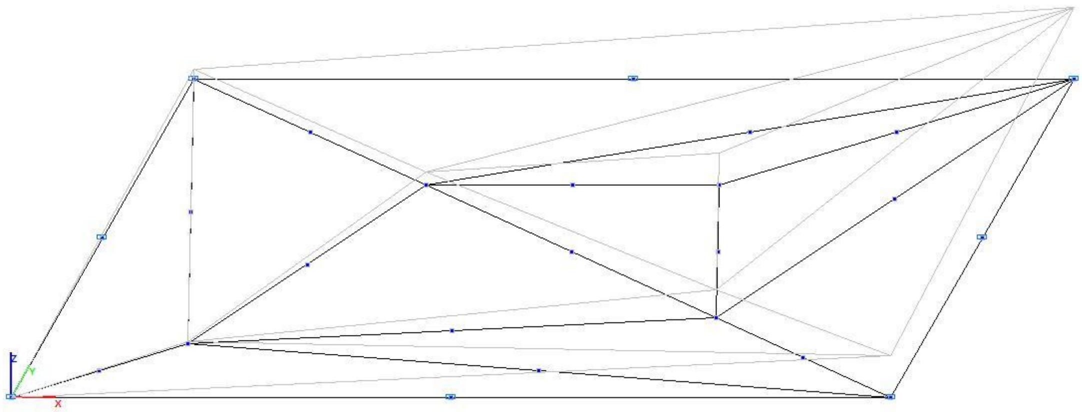
Model 2. Values of the bending moment M_y (kN·m/m)



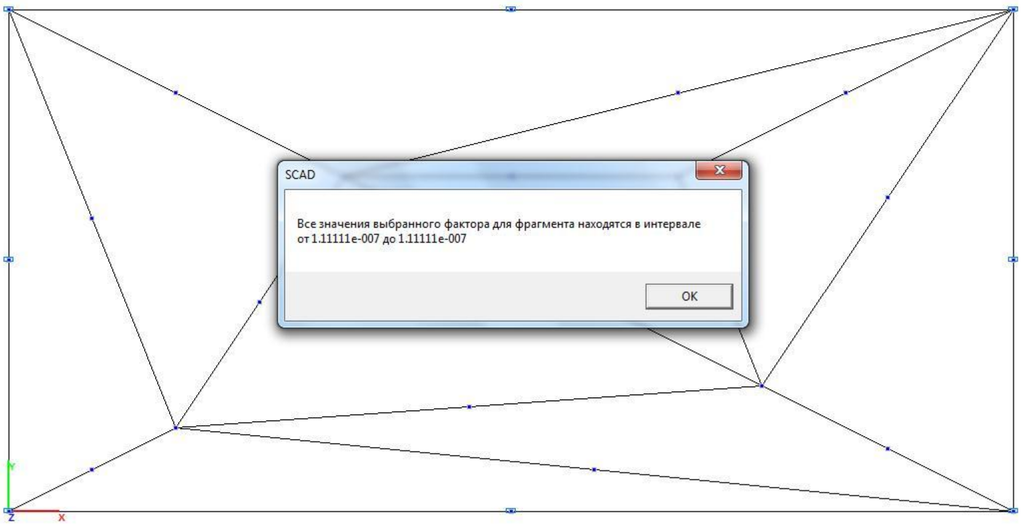
Model 2. Values of the torque M_{xy} (kN·m/m)



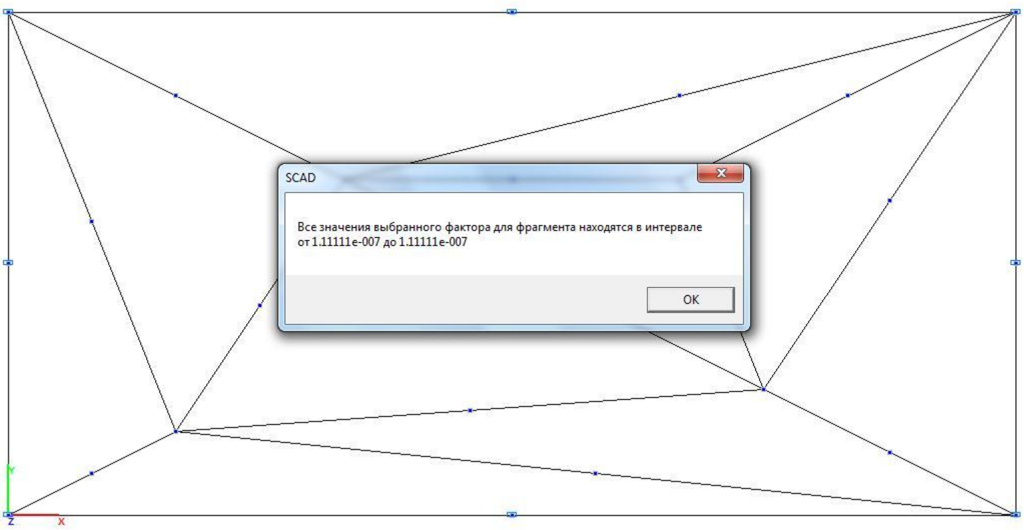
Model 3. Design model



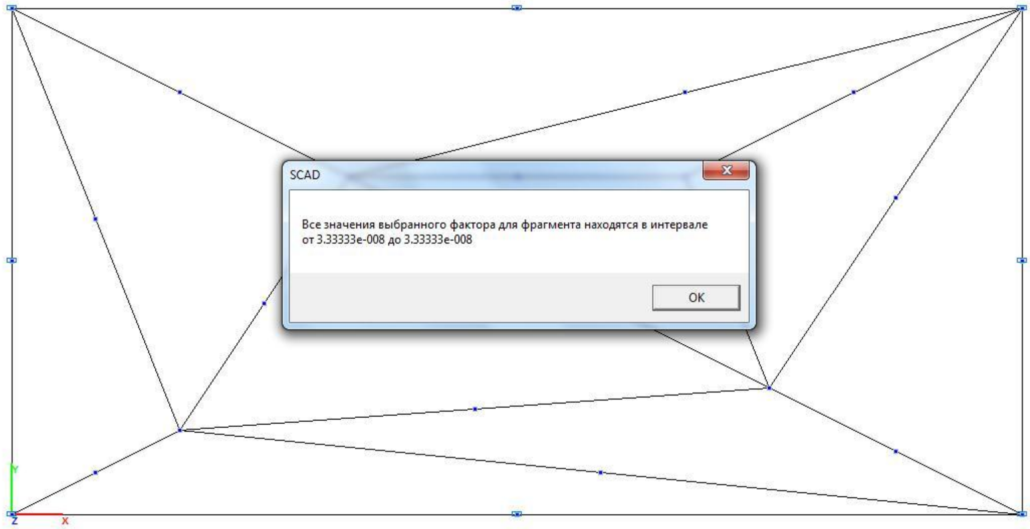
Model 3. Deformed model



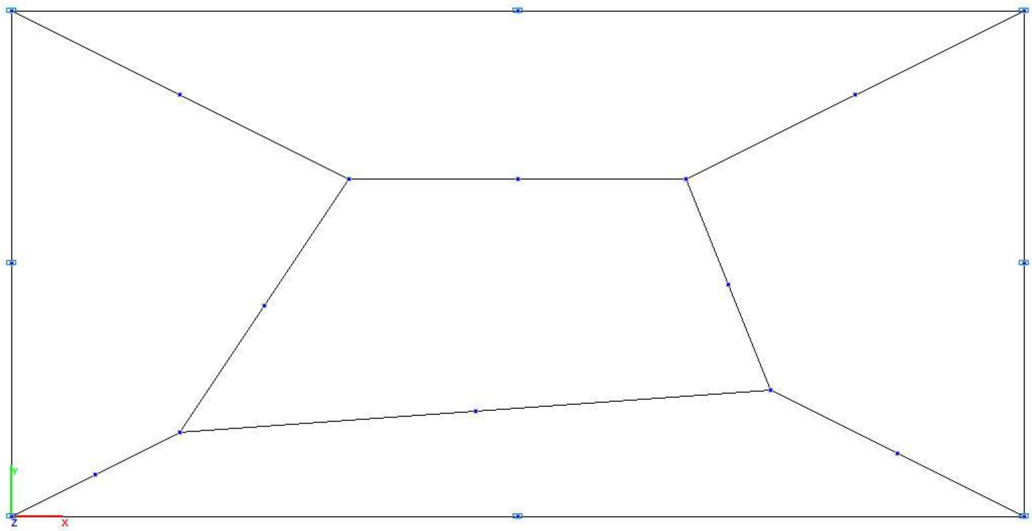
Model 3. Values of the bending moment M_x (kN·m/m)



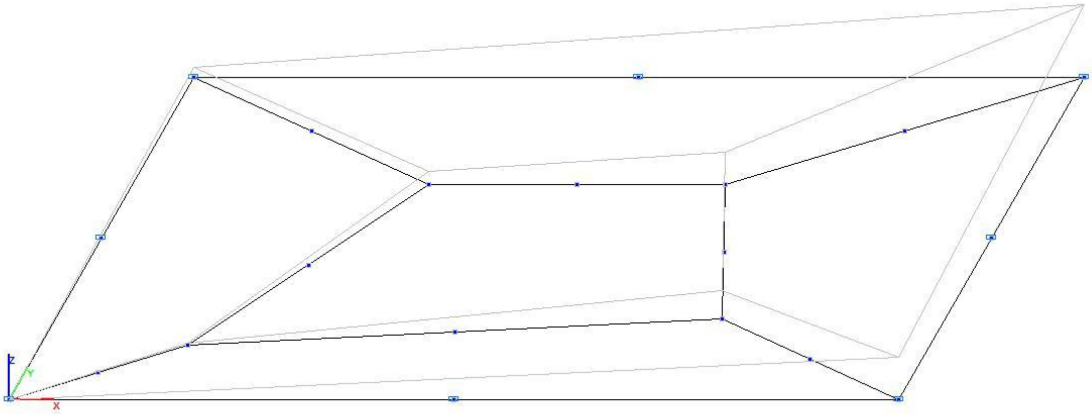
Model 3. Values of the bending moment M_y (kN·m/m)



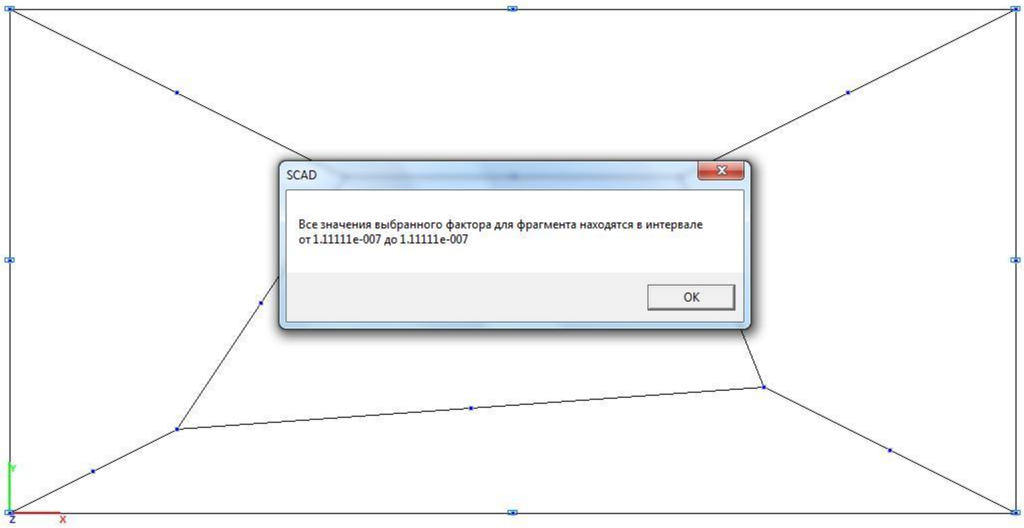
Model 3. Values of the torque M_{xy} (kN·m/m)



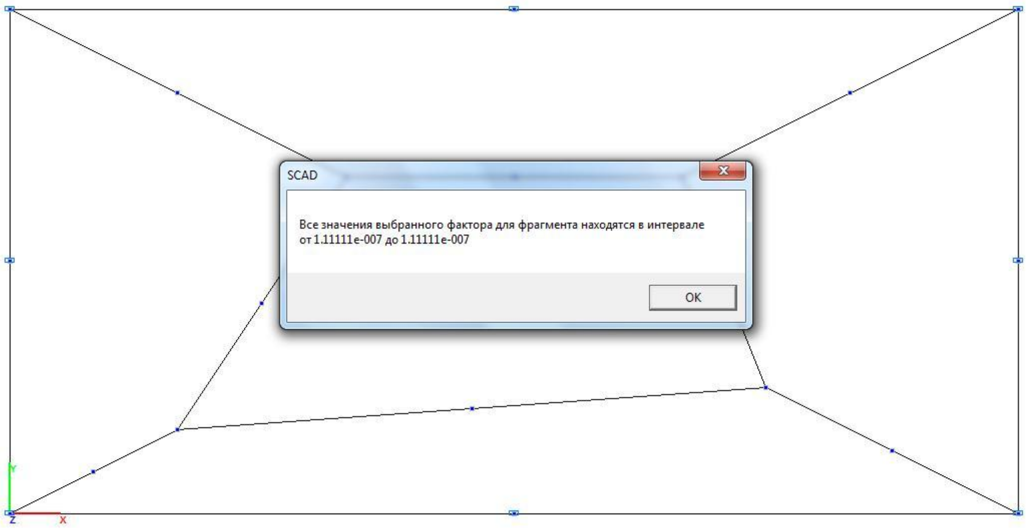
Model 4. Design model



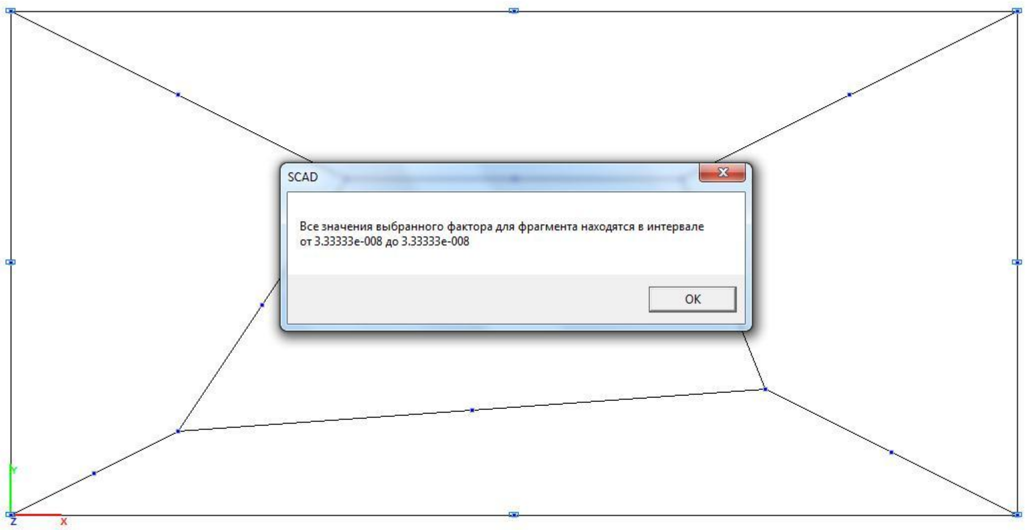
Model 4. Deformed model



Model 4. Values of the bending moment M_x (kN·m/m)



Model 4. Values of the bending moment M_y (kN·m/m)



Model 4. Values of the torque M_{xy} (kN·m/m)

Comparison of solutions:

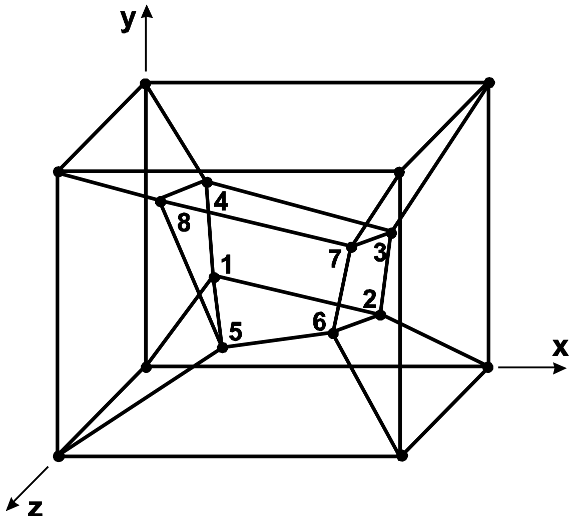
Model	Parameter	Theory	SCAD	Deviation, %
1	Normal stresses σ_x, kN/m^2	0.667	$6\cdot1.111\cdot10^{-7}/0.001^2$ = = 0.667	0.00
	Normal stresses σ_y, kN/m^2	0.667	$6\cdot1.111\cdot10^{-7}/0.001^2$ = = 0.667	0.00
	Tangential stresses τ_{xy}, kN/m^2	0.200	$6\cdot0.333\cdot10^{-7}/0.001^2$ = = 0.200	0.00
2	Normal stresses σ_x, kN/m^2	0.667	$6\cdot1.111\cdot10^{-7}/0.001^2$ = = 0.667	0.00
	Normal stresses σ_y, kN/m^2	0.667	$6\cdot1.111\cdot10^{-7}/0.001^2$ = = 0.667	0.00
	Tangential stresses τ_{xy}, kN/m^2	0.200	$6\cdot0.333\cdot10^{-7}/0.001^2$ = = 0.200	0.00
3	Normal stresses σ_x, kN/m^2	0.667	$6\cdot1.111\cdot10^{-7}/0.001^2$ = = 0.667	0.00
	Normal stresses σ_y, kN/m^2	0.667	$6\cdot1.111\cdot10^{-7}/0.001^2$ = = 0.667	0.00
	Tangential stresses τ_{xy}, kN/m^2	0.200	$6\cdot0.333\cdot10^{-7}/0.001^2$ = = 0.200	0.00
4	Normal stresses σ_x, kN/m^2	0.667	$6\cdot1.111\cdot10^{-7}/0.001^2$ = = 0.667	0.00
	Normal stresses σ_y, kN/m^2	0.667	$6\cdot1.111\cdot10^{-7}/0.001^2$ = = 0.667	0.00
	Tangential stresses τ_{xy}, kN/m^2	0.200	$6\cdot0.333\cdot10^{-7}/0.001^2$ = = 0.200	0.00

Notes: In the analytical solution the normal σ_x, σ_y and tangential τ_{xy} stresses on the external surface of the plate are determined according to the following formulas:

$$\sigma_x = 10^{-3} \cdot \frac{E \cdot t}{2 \cdot (1-\nu)} = \frac{6 \cdot M_x}{t^2}; \qquad \sigma_y = 10^{-3} \cdot \frac{E \cdot t}{2 \cdot (1-\nu)} = \frac{6 \cdot M_y}{t^2};$$

$$\tau_{xy} = 10^{-3} \cdot \frac{E \cdot t}{4 \cdot (1+\nu)} = \frac{6 \cdot M_{xy}}{t^2}.$$

Cube under the Constant Stresses throughout the Volume



Objective: Check of the obtained values of the constant stresses throughout the volume of the cube at an irregular coarse finite element mesh.

Initial data files:

File name	Description
Patch_test_Constant_stress_Solid_32.SPR	Design model with the elements of type 32
Patch_test_Constant_stress_Solid_34.SPR	Design model with the elements of type 34
Patch_test_Constant_stress_Solid_36.SPR	Design model with the elements of type 36
Patch_test_Constant_stress_Solid_37.SPR	Design model with the elements of type 37

Problem formulation: The unit isotropic cube is subjected to the displacements of the external surfaces providing the conditions of the constant stresses throughout the volume. Check that the conditions of constant normal σ_x, σ_y, σ_z and tangential τ_{xy}, τ_{xz}, τ_{yz} stresses throughout the volume are provided.

References: R. H. Macneal, R. L. Harder, A proposed standard set of problems to test finite element accuracy, North-Holland, Finite elements in analysis and design, 1, 1985, p. 3-20.

Initial data:

$E = 1.0 \cdot 10^6$ kPa - elastic modulus of the plate material;
$\nu = 0.25$ - Poisson's ratio;
a = 1.00 m - side of the cube;

Boundary conditions:

$u = 10^{-3} \cdot (2 \cdot x + y + z)/2$ - displacement of the external surfaces along the X axis of the global coordinate system;
$v = 10^{-3} \cdot (x + 2 \cdot y + z)/2$ - displacement of the external surfaces along the Y axis of the global coordinate system;
$w = 10^{-3} \cdot (x + y + 2 \cdot z)/2$ - displacement of the external surfaces along the Z axis of the global coordinate system;

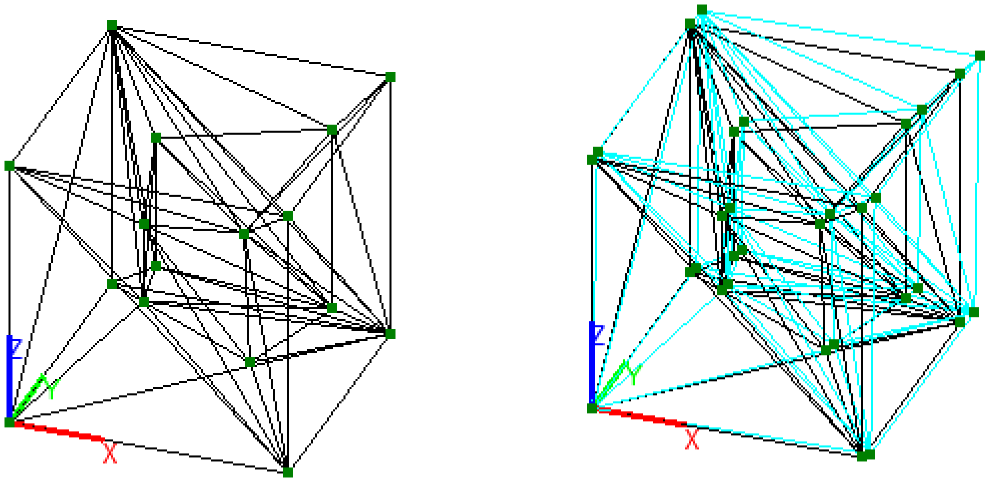
Location of internal nodes of the finite element mesh:

Numbers of nodes in the Figure 1	x	y	z
1	0.35	0.35	0.35
2	0.75	0.25	0.25
3	0.85	0.85	0.15
4	0.25	0.75	0.25
5	0.35	0.35	0.65
6	0.75	0.25	0.75

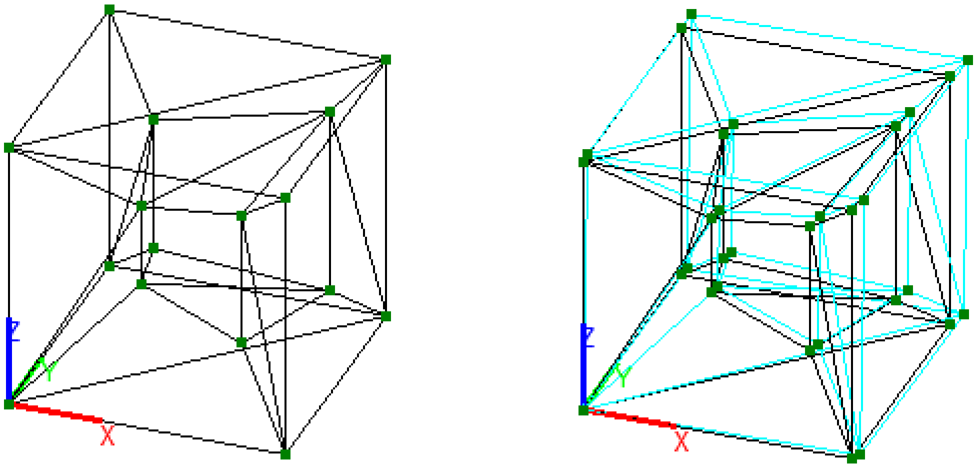
Numbers of nodes in the Figure 1	x	y	z
7	0.85	0.85	0.85
8	0.25	0.75	0.75

Finite element model: Design model – general type system. Four design models are considered:
Model 1 - 42 four-node pyramid elements of type 32. Boundary conditions are provided by imposing constraints on the nodes of the external surfaces of the cube in the directions of the degrees of freedom X, Y, Z and their displacement in accordance with the specified values u, v, w. Number of nodes in the model – 16.
Model 2 - 14 six-node isoparametric solid elements of type 34. Boundary conditions are provided by imposing constraints on the nodes of the external surfaces of the cube in the directions of the degrees of freedom X, Y, Z and their displacement in accordance with the specified values u, v, w. Number of nodes in the model – 16.
Model 3 - 7 eight-node isoparametric solid elements of type 36. Boundary conditions are provided by imposing constraints on the nodes of the external surfaces of the cube in the directions of the degrees of freedom X, Y, Z and their displacement in accordance with the specified values u, v, w. Number of nodes in the model – 16.
Model 4 - 7 twenty-node isoparametric solid elements of type 37. Boundary conditions are provided by imposing constraints on the nodes of the external surfaces of the cube in the directions of the degrees of freedom X, Y, Z and their displacement in accordance with the specified values u, v, w. Number of nodes in the model – 48.

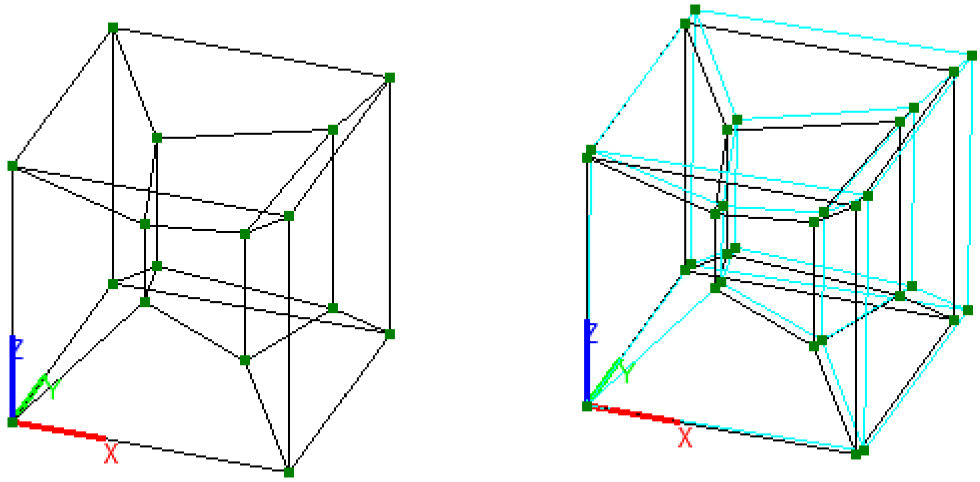
Results in SCAD



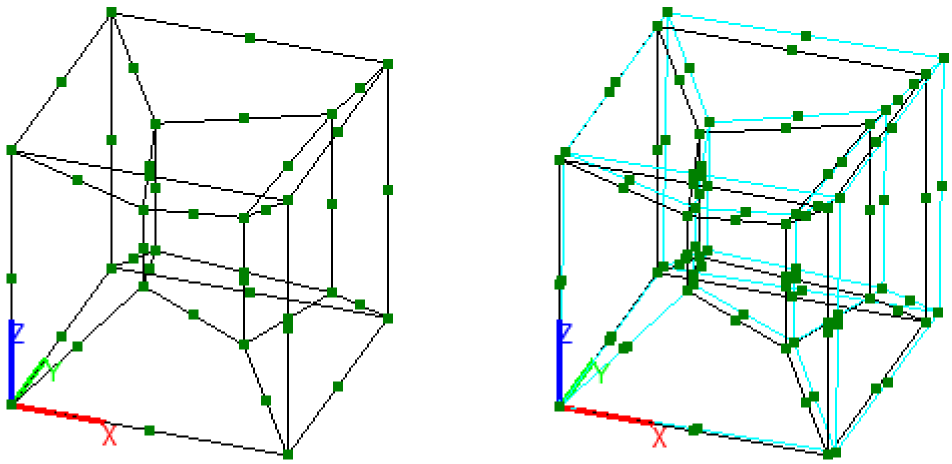
Model 1. Design and deformed models



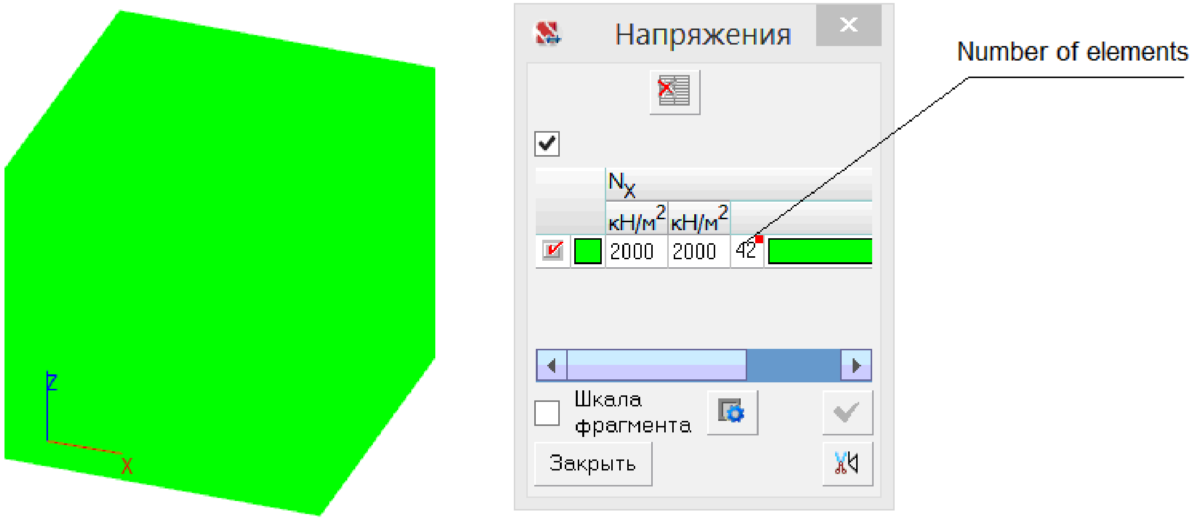
Model 2. Design and deformed models



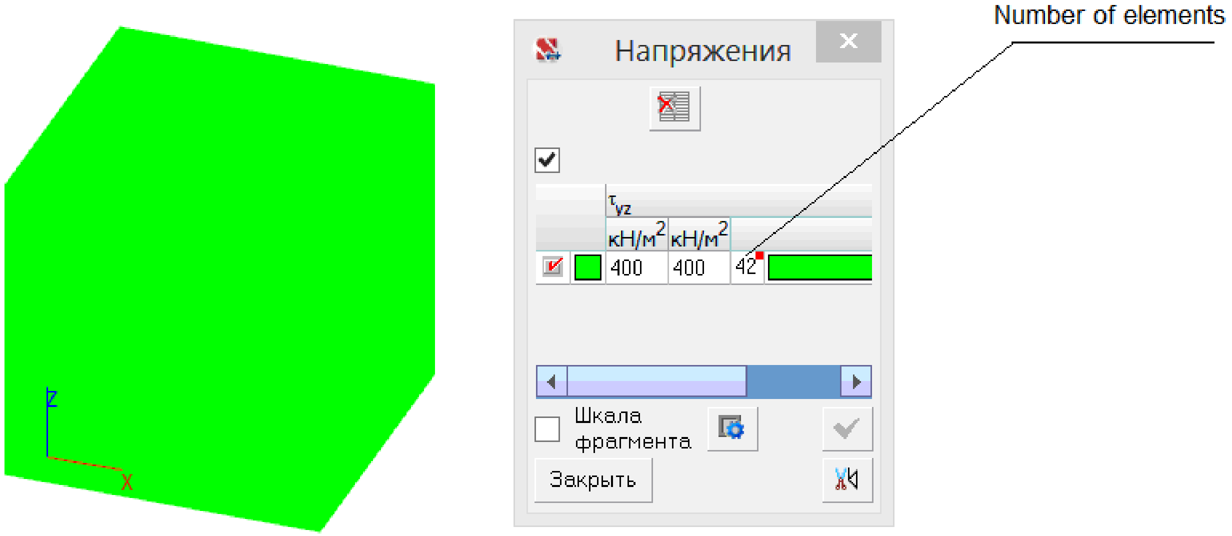
Model 3. Design and deformed models



Model 4. Design and deformed models



Values of normal stresses for all models σ_x, σ_y σ_z (kN/m²)



Values of tangential stresses for all models τ_{xz}, τ_{xy}, τ_{yz} (kN/m²)

Comparison of solutions:

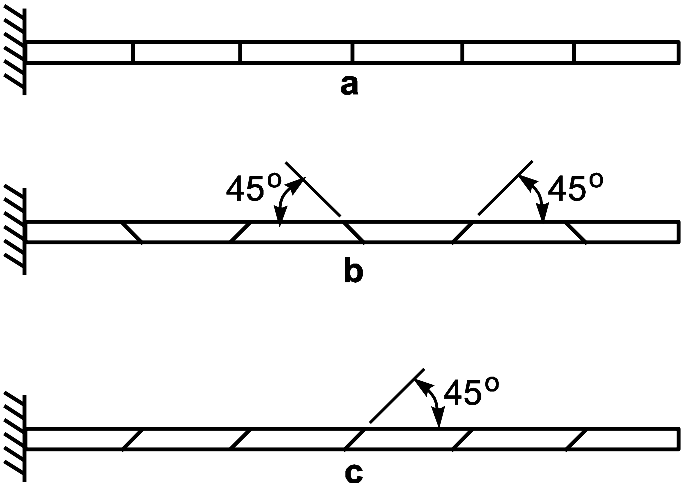
Model	Parameter	Theory	SCAD	Deviation, %
1-4	Normal stresses σ_x, kN/m²	2000	2000	0.00
	Normal stresses σ_y, kN/m²	2000	2000	0.00
	Normal stresses σ_z, kN/m²	2000	2000	0.00
	Tangential stresses τ_{xy}, kN/m²	400	400	0.00
	Tangential stresses τ_{xz}, kN/m²	400	400	0.00
	Tangential stresses τ_{yz}, kN/m²	400	400	0.00

Notes: In the analytical solution the normal σ_x, σ_y, σ_z and tangential τ_{xy}, τ_{xz}, τ_{yz} stresses throughout the volume of the cube are determined according to the following formulas:

$$\sigma_x = 10^{-3} \cdot \frac{E}{1-2\cdot\nu}; \qquad \sigma_y = 10^{-3} \cdot \frac{E}{1-2\cdot\nu}; \qquad \sigma_z = 10^{-3} \cdot \frac{E}{1-2\cdot\nu};$$

$$\tau_{xy} = 10^{-3} \cdot \frac{E}{2 \cdot (1+\nu)}; \qquad \tau_{xz} = 10^{-3} \cdot \frac{E}{2 \cdot (1+\nu)}; \qquad \tau_{yz} = 10^{-3} \cdot \frac{E}{2 \cdot (1+\nu)}.$$

Rectilinear Cantilever Beam with Concentrated Longitudinal and Shear Forces and a Torque at Its Free End



Objective: Check of the obtained values of the longitudinal and transverse displacements and the torsional angle of the free end of a rectilinear cantilever beam subjected to concentrated longitudinal and shear forces and a torque under different distortions of the finite element mesh.

Initial data files:

File name	Description
Straight_cantilever_beam_Regular_shape_Shell_42.SPR	Design model with the elements of type 42 at a regular mesh
Straight_cantilever_beam_Regular_shape_Shell_142.SPR	Design model with the elements of type 142 at a regular mesh
Straight_cantilever_beam_Regular_shape_Shell_44.SPR	Design model with the elements of type 44 at a regular mesh
Straight_cantilever_beam_Regular_shape_Shell_144.SPR	Design model with the elements of type 144 at a regular mesh
Straight_cantilever_beam_Regular_shape_Shell_45.SPR	Design model with the elements of type 45 at a regular mesh
Straight_cantilever_beam_Regular_shape_Shell_145.SPR	Design model with the elements of type 145 at a regular mesh
Straight_cantilever_beam_Regular_shape_Shell_50.SPR	Design model with the elements of type 50 at a regular mesh
Straight_cantilever_beam_Regular_shape_Shell_150.SPR	Design model with the elements of type 150 at a regular mesh
Straight_cantilever_beam_Regular_shape_ Solid _36.SPR	Design model with the elements of type 36 at a regular mesh
Straight_cantilever_beam_Regular_shape_ Solid _37.SPR	Design model with the elements of type 37 at a regular mesh
Straight_cantilever_beam_Trapezoidal_shape_Shell_42.SPR	Design model with the elements of type 42 at a trapezoidal mesh
Straight_cantilever_beam_Trapezoidal_shape_Shell_142.SPR	Design model with the elements of type 142 at a trapezoidal mesh
Straight_cantilever_beam_Trapezoidal_shape_Shell_44.SPR	Design model with the elements of type 44 at a trapezoidal mesh
Straight_cantilever_beam_Trapezoidal_shape_Shell_144.SPR	Design model with the elements of type 144 at a trapezoidal mesh
Straight_cantilever_beam_Trapezoidal_shape_Shell_45.SPR	Design model with the elements of type 45 at a trapezoidal mesh
Straight_cantilever_beam_Trapezoidal_shape_Shell_145.SPR	Design model with the elements of type 145 at a trapezoidal mesh
Straight_cantilever_beam_Trapezoidal_shape_Shell_50.SPR	Design model with the elements of type 50 at a trapezoidal mesh
Straight_cantilever_beam_Trapezoidal_shape_Shell_150.SPR	Design model with the elements of type 150 at a trapezoidal mesh

File name	Description
Straight_cantilever_beam_Trapezoidal_shape_ Solid _36.SPR	Design model with the elements of type 36 at a trapezoidal mesh
Straight_cantilever_beam_Trapezoidal_shape_ Solid _37.SPR	Design model with the elements of type 37 at a trapezoidal mesh
Straight_cantilever_beam_Parallelogram_shape_Shell_42.SPR	Design model with the elements of type 42 at a parallelogram mesh
Straight_cantilever_beam_Parallelogram_shape_Shell_142.SPR	Design model with the elements of type 142 at a parallelogram mesh
Straight_cantilever_beam_Parallelogram_shape_Shell_44.SPR	Design model with the elements of type 44 at a parallelogram mesh
Straight_cantilever_beam_Parallelogram_shape_Shell_144.SPR	Design model with the elements of type 144 at a parallelogram mesh
Straight_cantilever_beam_Parallelogram_shape_Shell_45.SPR	Design model with the elements of type 45 at a parallelogram mesh
Straight_cantilever_beam_Parallelogram_shape_Shell_145.SPR	Design model with the elements of type 145 at a parallelogram mesh
Straight_cantilever_beam_Parallelogram_shape_Shell_50.SPR	Design model with the elements of type 50 at a parallelogram mesh
Straight_cantilever_beam_Parallelogram_shape_Shell_150.SPR	Design model with the elements of type 150 at a parallelogram mesh
Straight_cantilever_beam_Parallelogram_shape_ Solid _36.SPR	Design model with the elements of type 36 at a parallelogram mesh
Straight_cantilever_beam_Parallelogram_shape_ Solid _37.SPR	Design model with the elements of type 37 at a parallelogram mesh

Problem formulation: The rectilinear isotropic cantilever beam of a rectangular cross-section is subjected to the concentrated longitudinal P_x and shear P_y, P_z forces and the torque M_x applied at its free end. Check the obtained values of the longitudinal X and transverse displacements Y, Z and the torsional angle UX of the free end of the rectilinear cantilever beam from the respective actions.

References: R. H. Macneal, R. L. Harder, A proposed standard set of problems to test finite element accuracy, North-Holland, Finite elements in analysis and design, 1, 1985, p. 3-20.

Initial data:

$E = 1.0 \cdot 10^7$ kPa	- elastic modulus of the beam material;
$\nu = 0.30$	- Poisson's ratio;
b = 0. 1 m	- width of the beam;
h = 0. 2 m	- height of the beam;
L = 6.0 m	- length of the beam;
$P_x = 1.0$ kN	- value of the longitudinal force;
$P_y = 1.0$ kN	- value of the shear force acting along the height of the beam;
$P_z = 1.0$ kN	- value of the shear force acting along the width of the beam;
$M_x = 1.0$ kN·m	- value of the torque.

Finite element model: Design models – general type systems. Ten design models with regular, trapezoidal and parallelogram finite element meshes are considered:

Model 1 - 12 three-node shell elements of type 42. Boundary conditions are provided by imposing constraints on the nodes of the clamped end of the beam in the directions of the degrees of freedom X, Y, Z, UX, UY, UZ. The concentrated longitudinal P_x and shear P_y, P_z forces and the torque M_x are given in the form of two nodal forces ($P_x = 2 \cdot 0.5$ kN, $P_y = 2 \cdot 0.5$ kN, $P_z = 2 \cdot 0.5$ kN, $M_x = 2 \cdot 5.0 \cdot 0.2/2$ kN·m). Number of nodes in the model – 14.

Model 2 - 12 three-node shell elements allowing for shear of type 142. Boundary conditions are provided by imposing constraints on the nodes of the clamped end of the beam in the directions of the degrees of freedom X, Y, Z, UX, UY, UZ. The concentrated longitudinal P_x and shear P_y, P_z forces and the torque M_x are given in the form of two nodal forces ($P_x = 2 \cdot 0.5$ kN, $P_y = 2 \cdot 0.5$ kN, $P_z = 2 \cdot 0.5$ kN, $M_x = 2 \cdot 5.0 \cdot 0.2/2$ kN·m). Number of nodes in the model – 14.

Model 3 - 6 four-node shell elements of type 44. Boundary conditions are provided by imposing constraints on the nodes of the clamped end of the beam in the directions of the degrees of freedom X, Y, Z, UX, UY, UZ. The concentrated longitudinal P_x and shear P_y, P_z forces and the torque M_x are given in the form of two nodal forces (P_x = 2·0.5 kN, P_y = 2·0.5 kN, P_z = 2·0.5 kN, M_x = 2·5.0·0.2/2 kN·m). Number of nodes in the model – 14.

Model 4 - 6 four-node shell elements allowing for shear of type 144. Boundary conditions are provided by imposing constraints on the nodes of the clamped end of the beam in the directions of the degrees of freedom X, Y, Z, UX, UY, UZ. The concentrated longitudinal P_x and shear P_y, P_z forces and the torque M_x are given in the form of two nodal forces (P_x = 2·0.5 kN, P_y = 2·0.5 kN, P_z = 2·0.5 kN, M_x = 2·5.0·0.2/2 kN·m). Number of nodes in the model – 14.

Model 5 - 12 six-node shell elements of type 45. Boundary conditions are provided by imposing constraints on the nodes of the clamped end of the beam in the directions of the degrees of freedom X, Y, Z, UX, UY, UZ. The concentrated longitudinal P_x and shear P_y, P_z forces and the torque M_x are given in the form of two nodal forces (P_x = 2·0.5 kN, P_y = 2·0.5 kN, P_z = 2·0.5 kN, M_x = 2·5.0·0.2/2 kN·m). Number of nodes in the model – 39.

Model 6 - 12 six-node shell elements allowing for shear of type 145. Boundary conditions are provided by imposing constraints on the nodes of the clamped end of the beam in the directions of the degrees of freedom X, Y, Z, UX, UY, UZ. The concentrated longitudinal P_x and shear P_y, P_z forces and the torque M_x are given in the form of two nodal forces (P_x = 2·0.5 kN, P_y = 2·0.5 kN, P_z = 2·0.5 kN, M_x = 2·5.0·0.2/2 kN·m). Number of nodes in the model – 39.

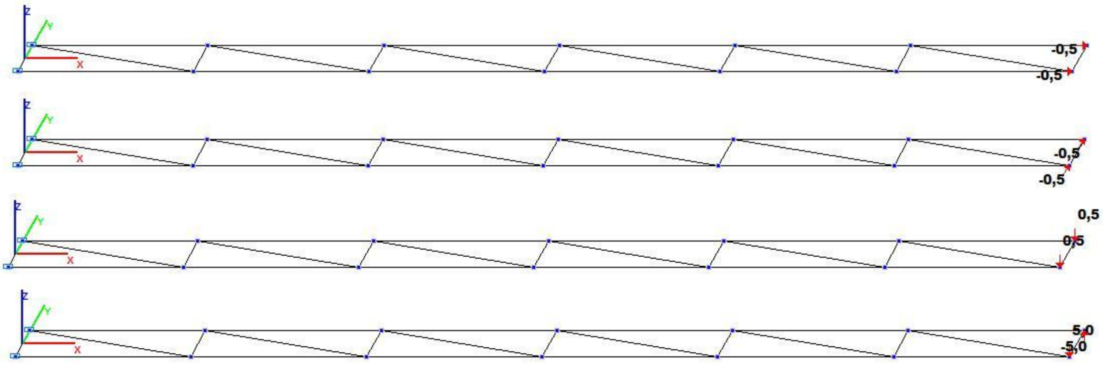
Model 7 - 6 eight-node shell elements of type 50. Boundary conditions are provided by imposing constraints on the nodes of the clamped end of the beam in the directions of the degrees of freedom X, Y, Z, UX, UY, UZ. The concentrated longitudinal P_x and shear P_y, P_z forces and the torque M_x are given in the form of two nodal forces (P_x = 2·0.5 kN, P_y = 2·0.5 kN, P_z = 2·0.5 kN, M_x = 2·5.0·0.2/2 kN·m). Number of nodes in the model – 33.

Model 8 - 6 eight-node shell elements allowing for shear of type 150. Boundary conditions are provided by imposing constraints on the nodes of the clamped end of the beam in the directions of the degrees of freedom X, Y, Z, UX, UY, UZ. The concentrated longitudinal P_x and shear P_y, P_z forces and the torque M_x are given in the form of two nodal forces (P_x = 2·0.5 kN, P_y = 2·0.5 kN, P_z = 2·0.5 kN, M_x = 2·5.0·0.2/2 kN·m). Number of nodes in the model – 33.

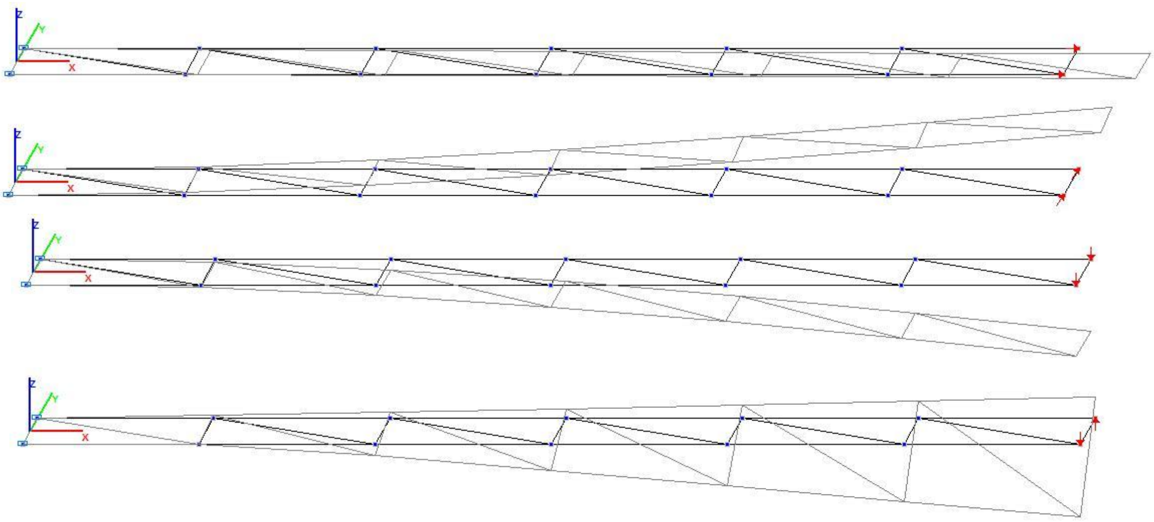
Model 9 - 6 eight-node isoparametric solid elements of type 36. Boundary conditions are provided by imposing constraints on the nodes of the clamped end of the beam in the directions of the degrees of freedom X, Y, Z, UX, UY, UZ. The concentrated longitudinal P_x and shear P_y, P_z forces and the torque M_x are given in the form of four nodal forces (P_x = 4·0.25 kN, P_y = 4·0.25 kN, P_z = 4·0.25 kN, M_x = 4·2.5·0.2/2 kN·m). Number of nodes in the model – 28.

Model 10 - 6 twenty-node isoparametric solid elements of type 37. Boundary conditions are provided by imposing constraints on the nodes of the clamped end of the beam in the directions of the degrees of freedom X, Y, Z, UX, UY, UZ. The concentrated longitudinal P_x and shear P_y, P_z forces and the torque M_x are given in the form of four nodal forces (P_x = 4·0.25 kN, P_y = 4·0.25 kN, P_z = 4·0.25 kN, M_x = 4·2.5·0.2/2 kN·m). Number of nodes in the model – 80.

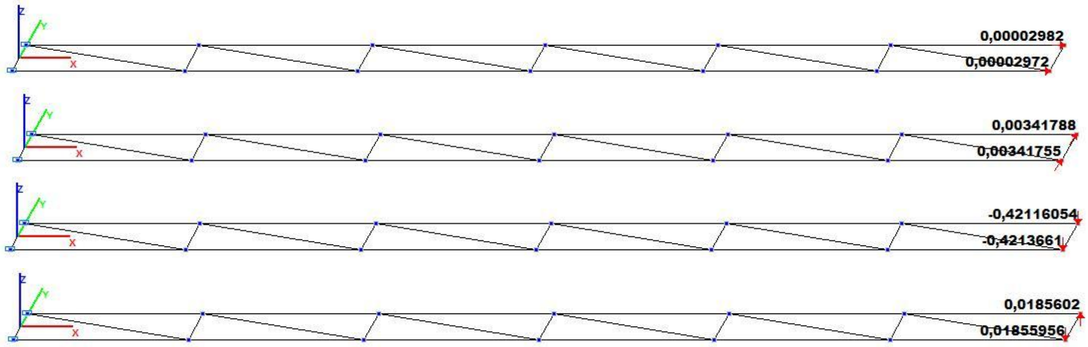
Results in SCAD



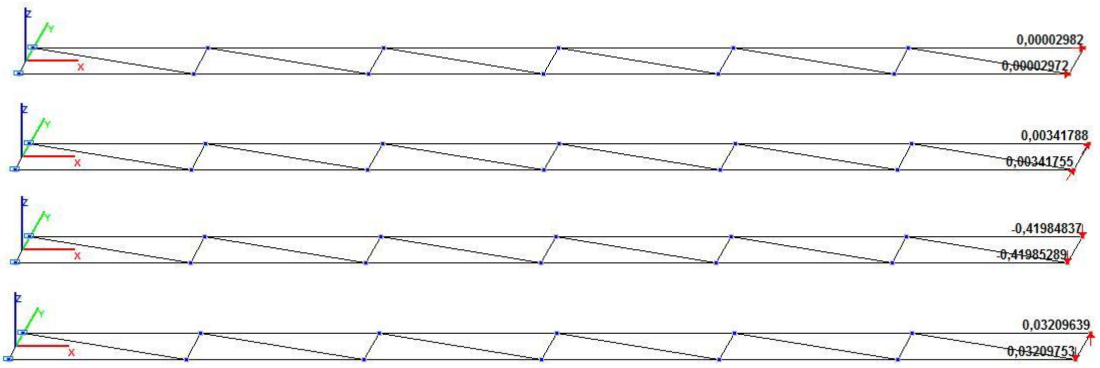
Models 1 and 2.
Design model with a regular finite element mesh



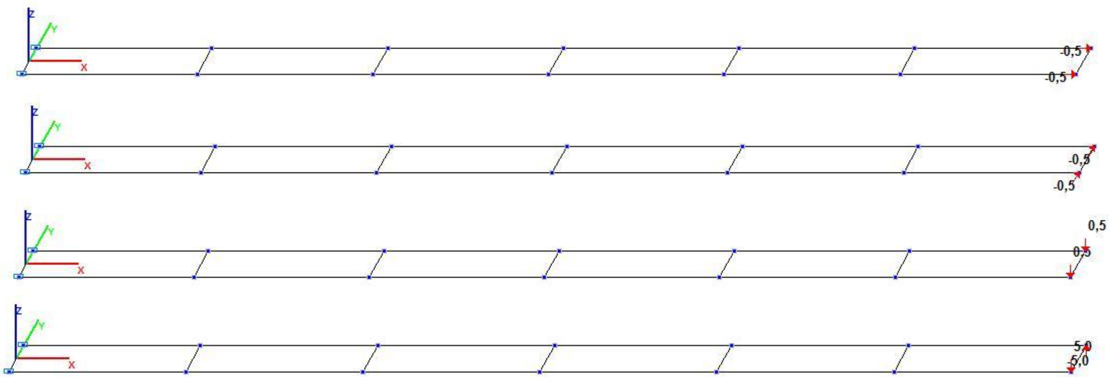
Models 1 and 2.
Deformed model with a regular finite element mesh



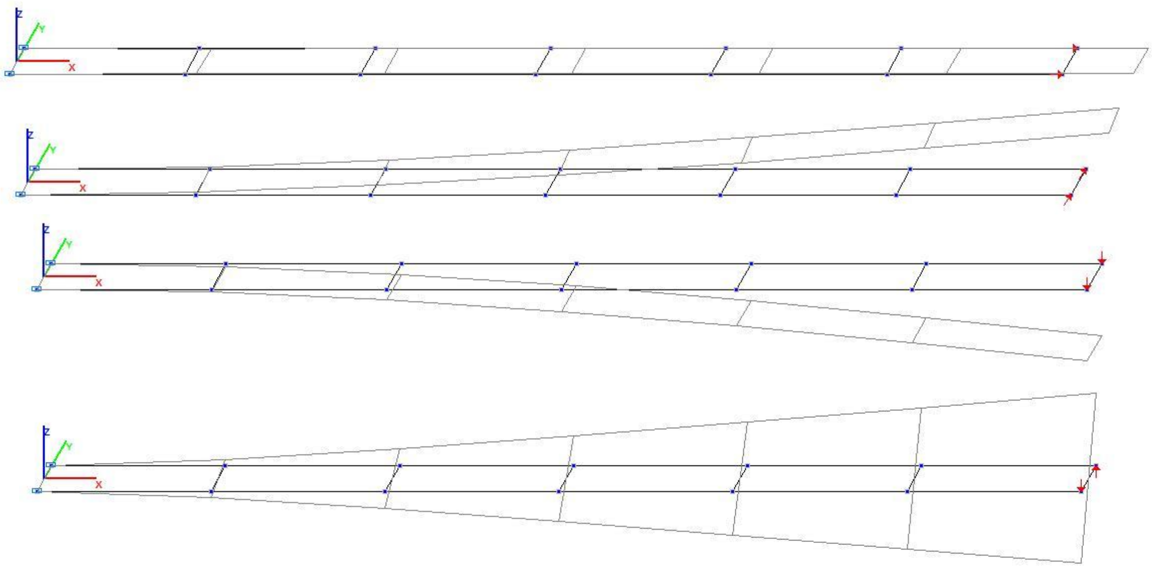
Model 1.
Values of the longitudinal displacement X, transverse displacements Y, Z and the torsional angle UX of the free end of the rectilinear cantilever beam (m, m, m, rad)



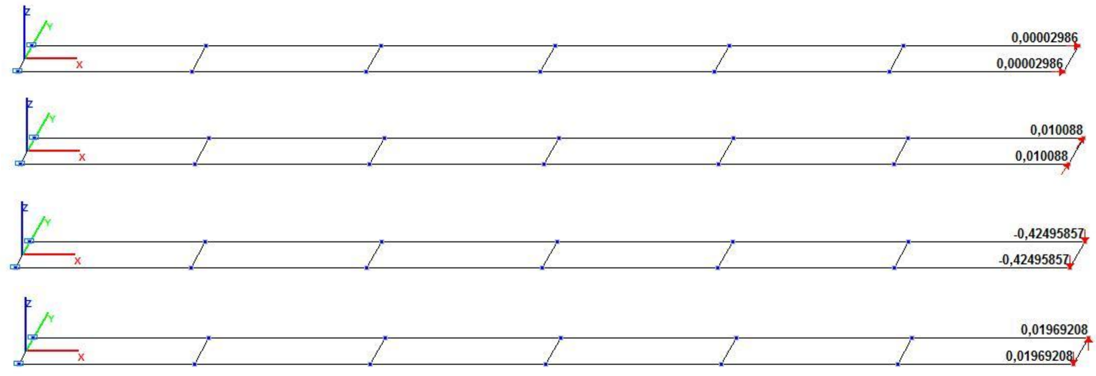
Model 2.
Values of the longitudinal displacement X, transverse displacements Y, Z and the torsional angle UX of the free end of the rectilinear cantilever beam (m, m, m, rad)



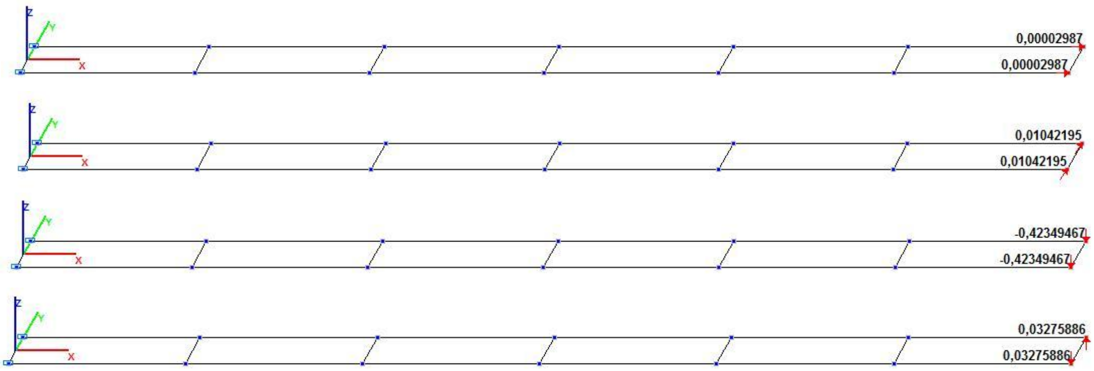
Models 3 and 4.
Design model with a regular finite element mesh



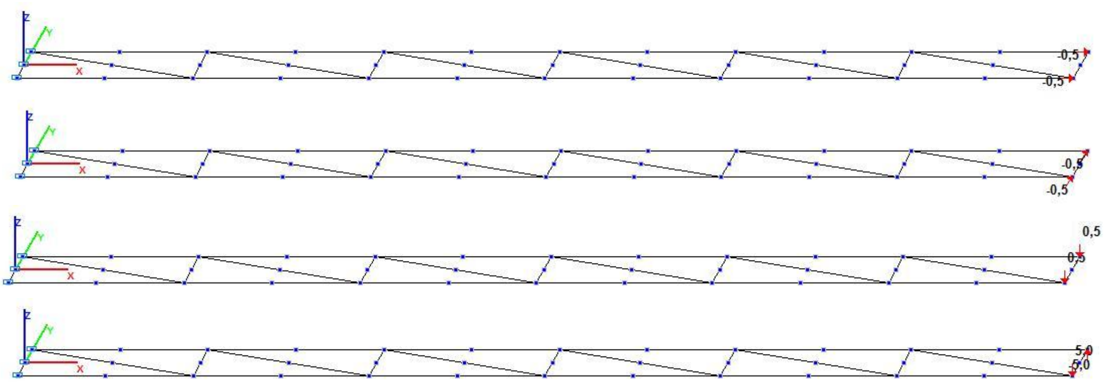
Models 3 and 4.
Deformed model with a regular finite element mesh



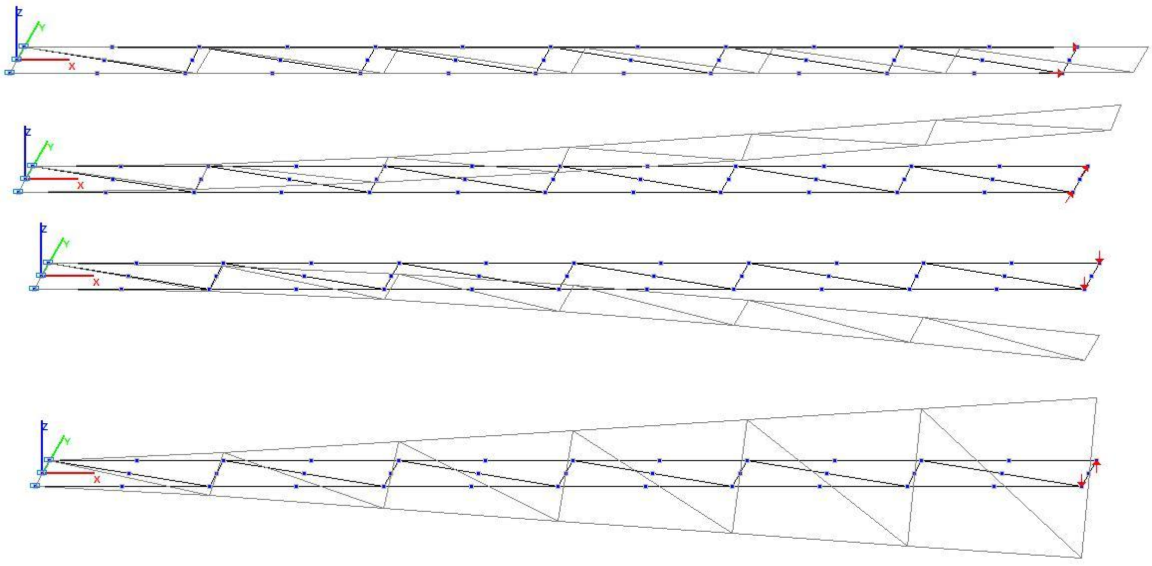
Model 3.
Values of the longitudinal displacement X, transverse displacements Y, Z and the torsional angle UX of the free end of the rectilinear cantilever beam (m, m, m, rad)



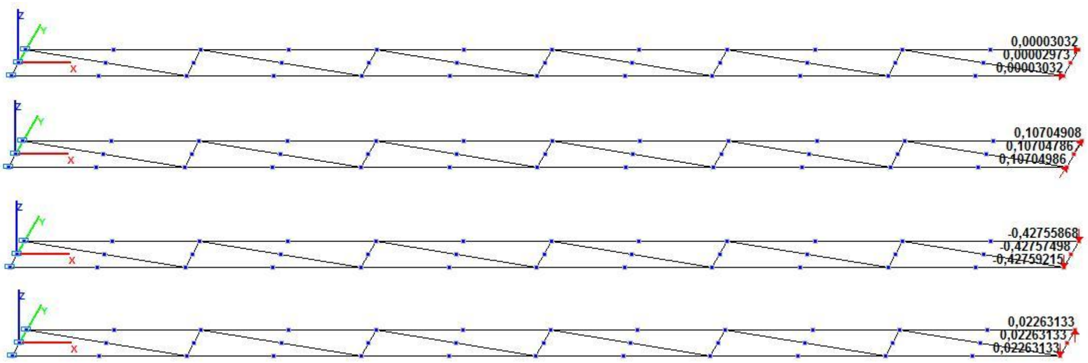
Model 4.
Values of the longitudinal displacement X, transverse displacements Y, Z and the torsional angle UX of the free end of the rectilinear cantilever beam (m, m, m, rad)



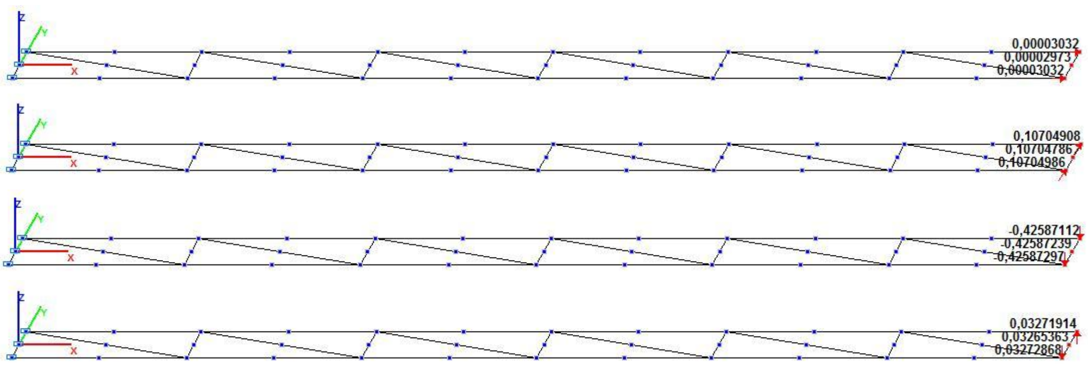
Models 5 and 6.
Design model with a regular finite element mesh



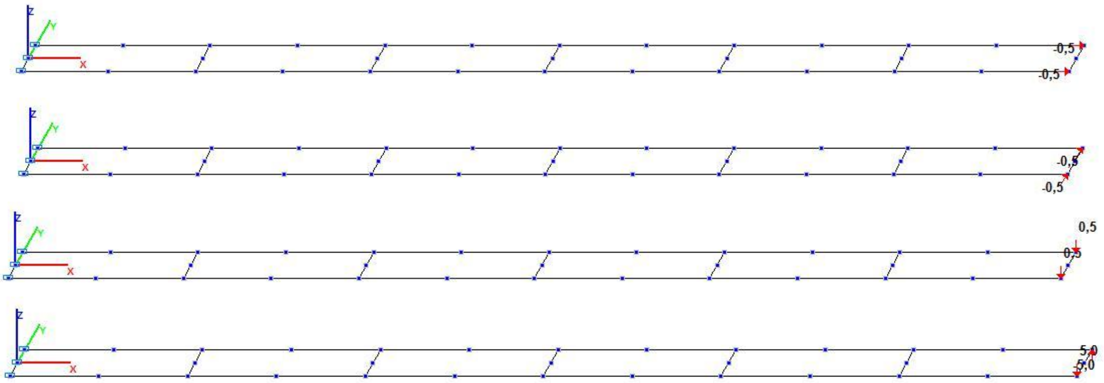
Models 5 and 6.
Deformed model with a regular finite element mesh



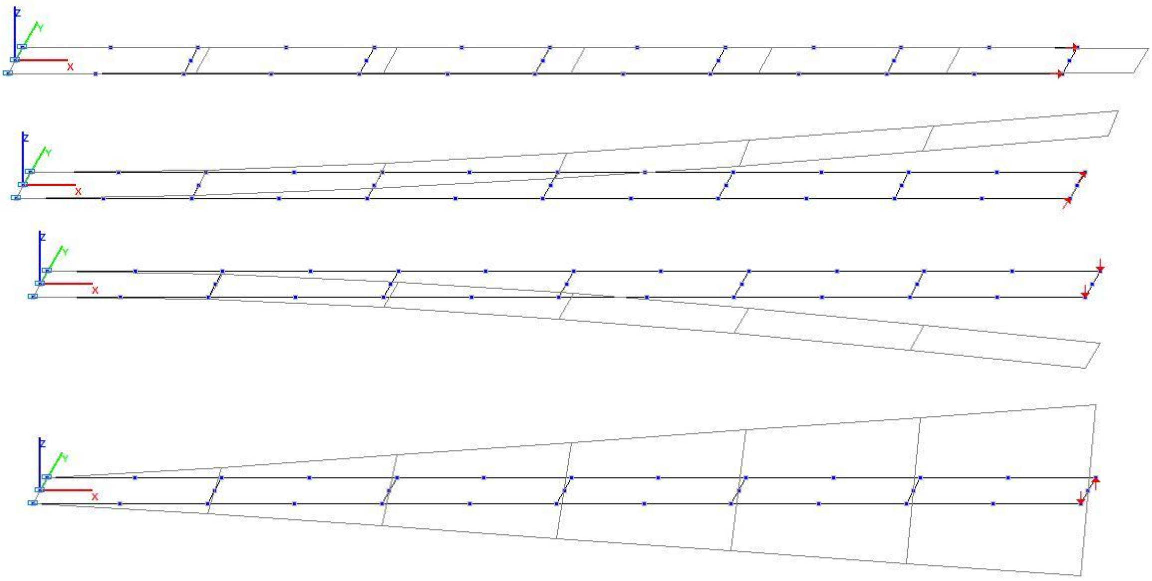
Model 5.
Values of the longitudinal displacement X, transverse displacements Y, Z and the torsional angle UX of the free end of the rectilinear cantilever beam (m, m, m, rad)



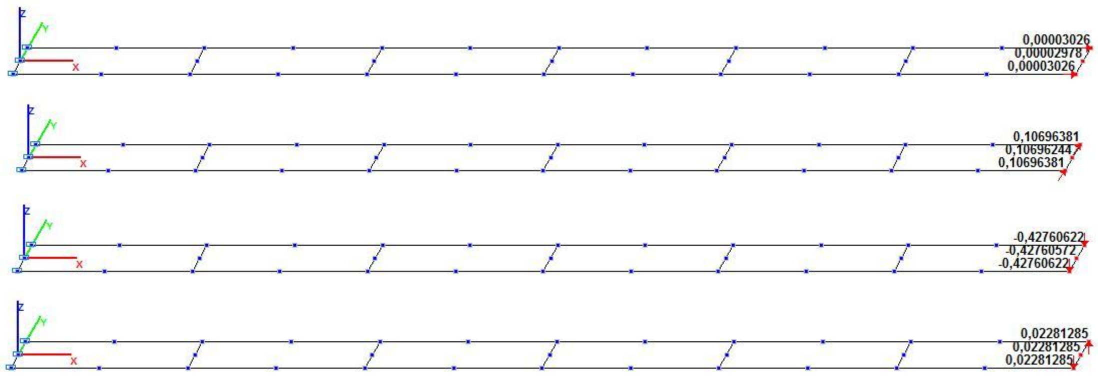
Model 6.
Values of the longitudinal displacement X, transverse displacements Y, Z and the torsional angle UX of the free end of the rectilinear cantilever beam (m, m, m, rad)



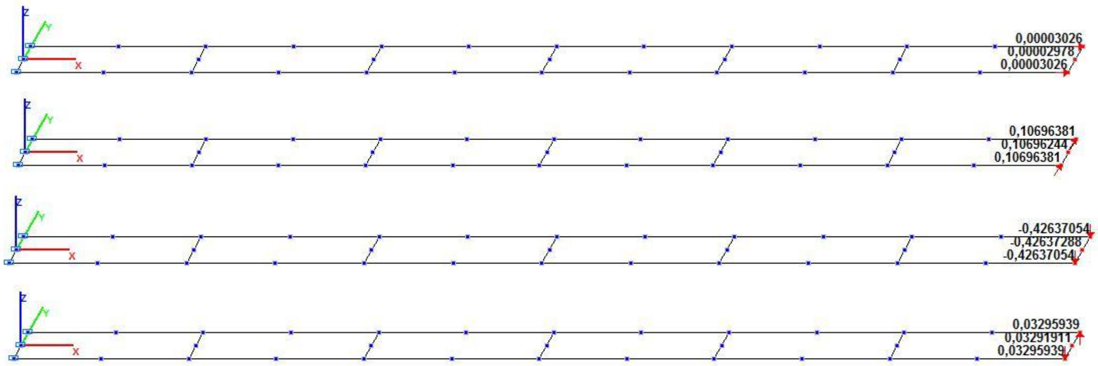
Models 7 and 8.
Design model with a regular finite element mesh



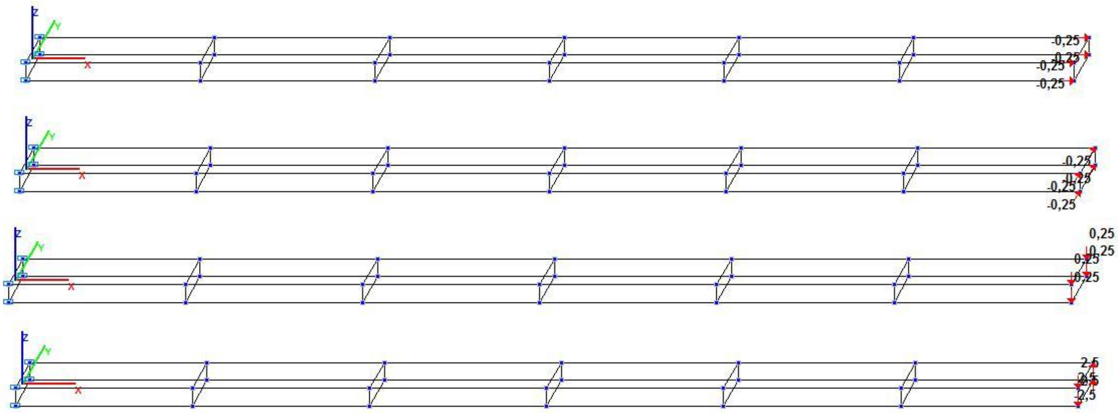
Models 7 and 8.
Deformed model with a regular finite element mesh



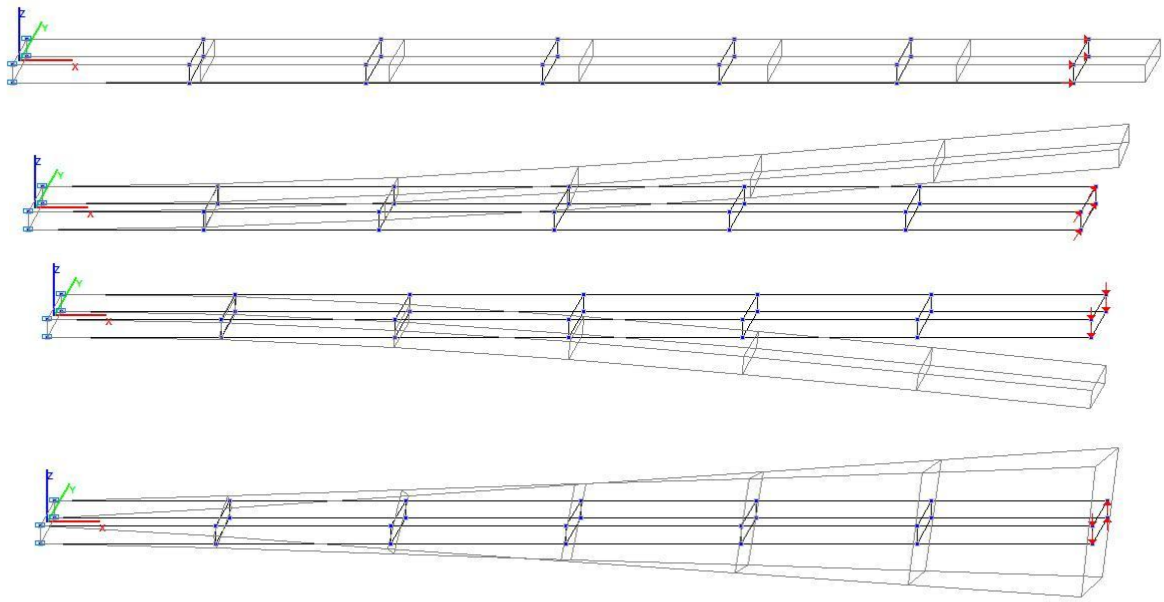
Model 7.
Values of the longitudinal displacement X, transverse displacements Y, Z and the torsional angle UX of the free end of the rectilinear cantilever beam (m, m, m, rad)



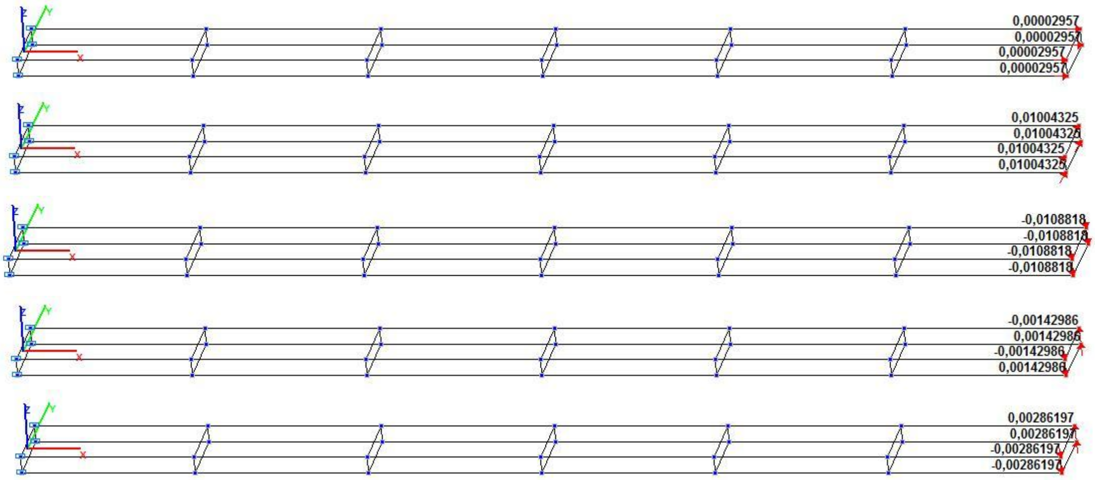
Model 8.
Values of the longitudinal displacement X, transverse displacements Y, Z and the torsional angle UX of the free end of the rectilinear cantilever beam (m, m, m, rad)



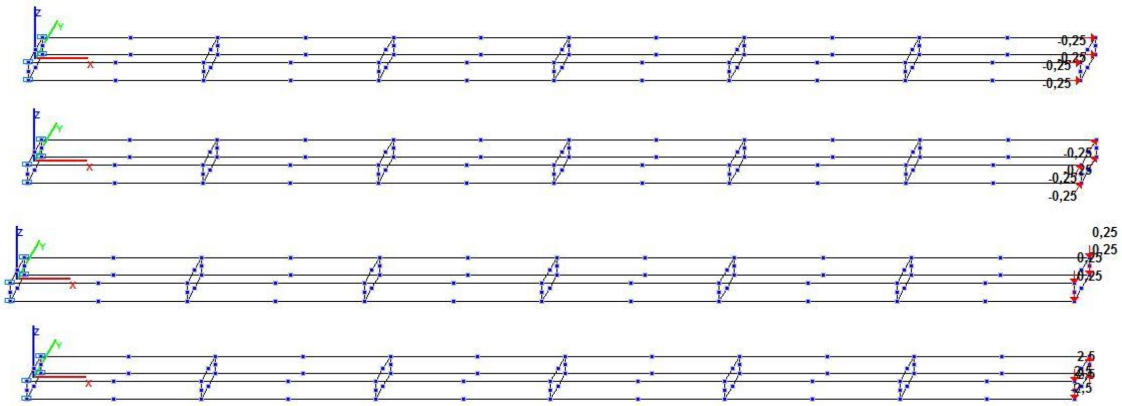
Model 9.
Design model with a regular finite element mesh



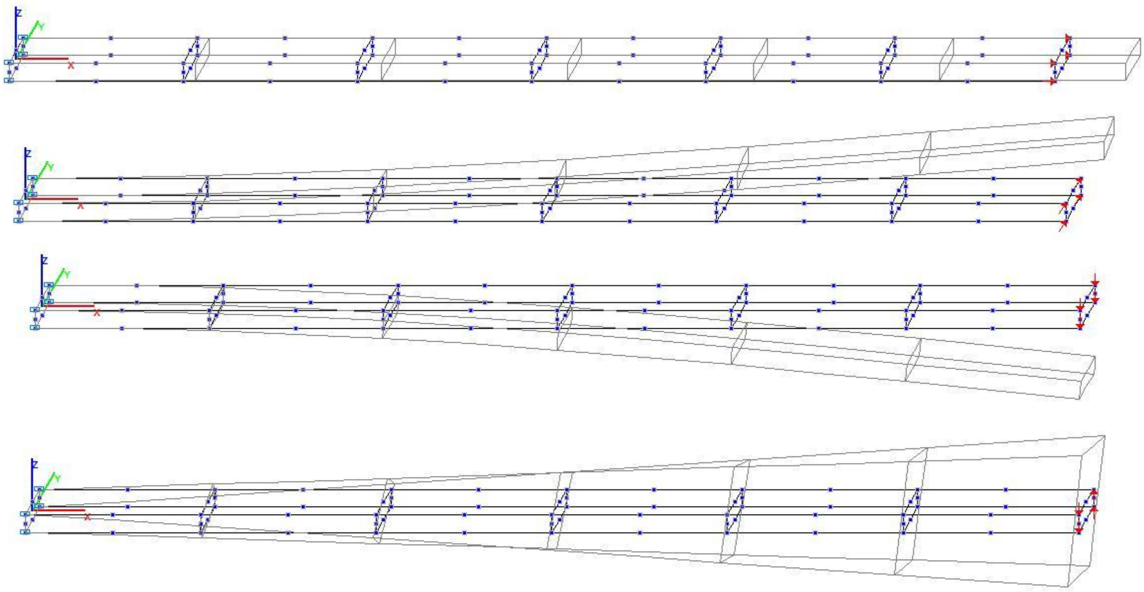
Model 9.
Deformed model with a regular finite element mesh



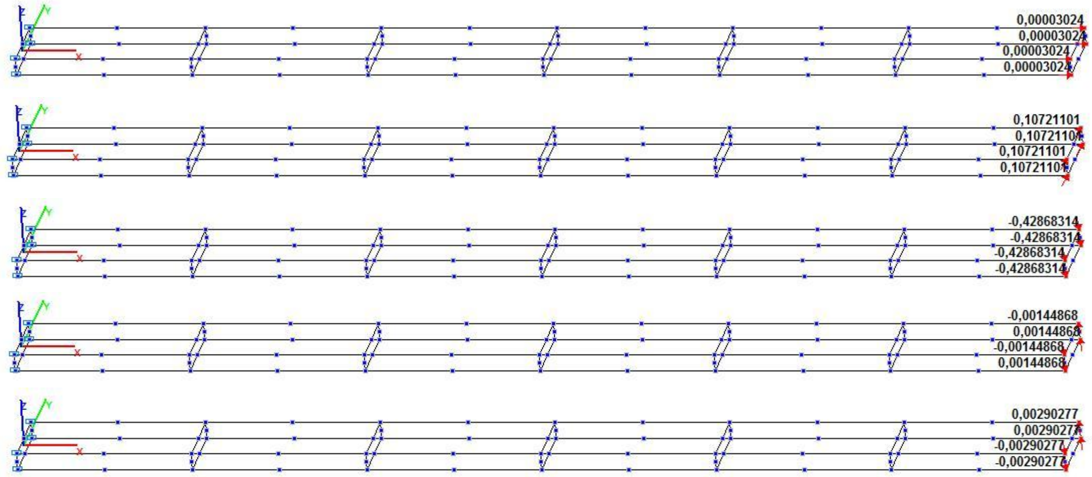
Model 9.
Values of the longitudinal displacement X from the action P_x,
transverse displacements Y, Z from the actions P_y, P_z
and transverse displacements Y, Z from the action M_x
of the free end of the rectilinear cantilever beam (m, m, m, m, m)



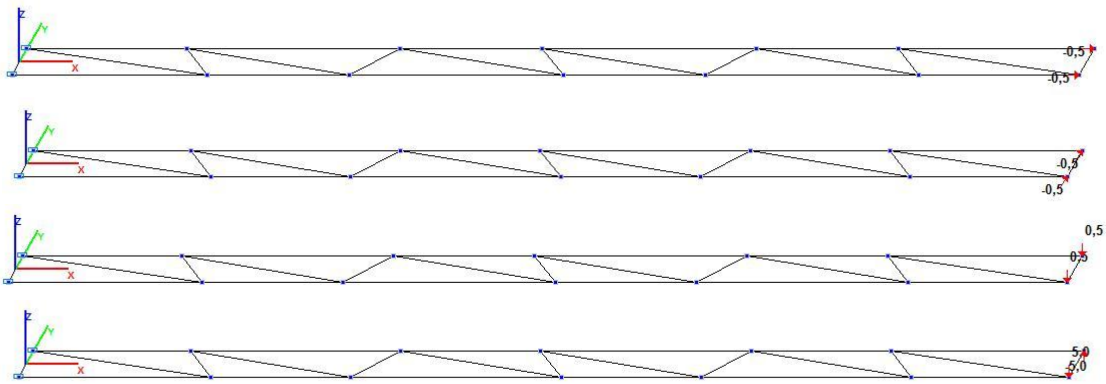
Model 10.
Design model with a regular finite element mesh



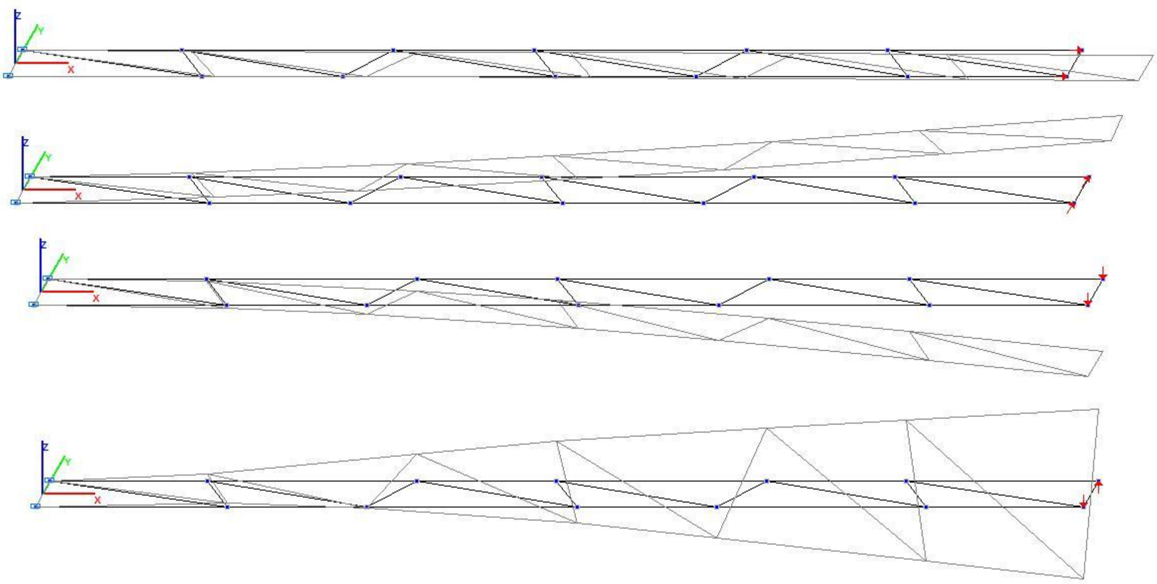
Model 10.
Deformed model with a regular finite element mesh



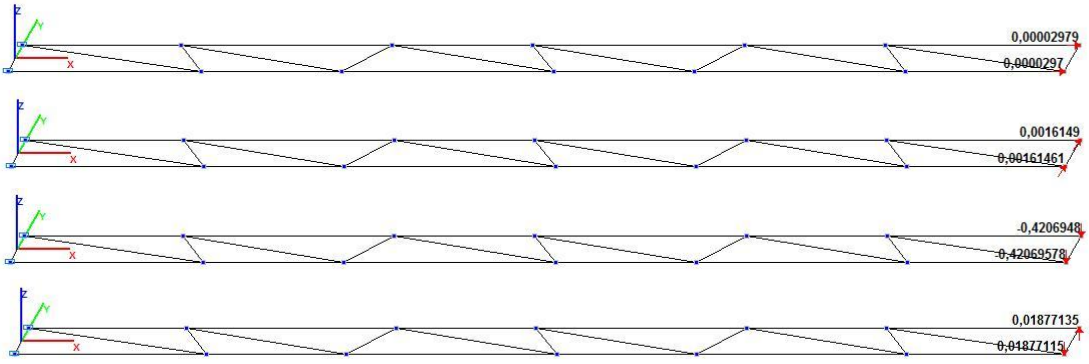
Model 10.
Values of the longitudinal displacement X from the action P_x,
transverse displacements Y, Z from the actions P_y, P_z
and transverse displacements Y, Z from the action M_x
of the free end of the rectilinear cantilever beam (m, m, m, m, m)



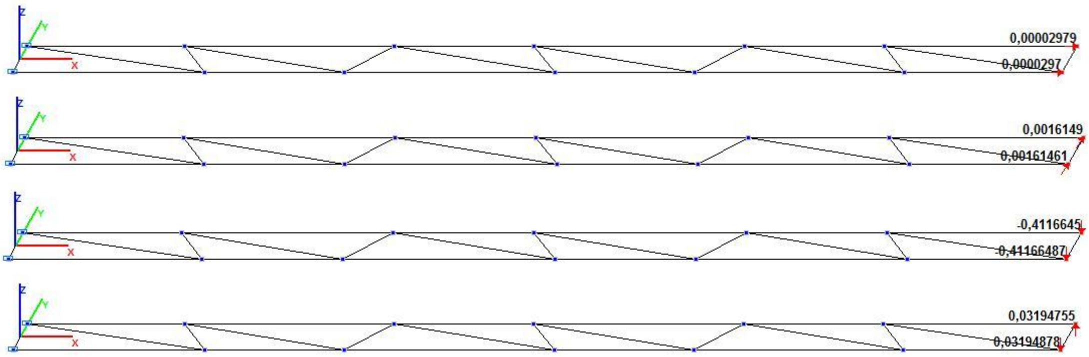
Models 1 and 2.
Design model with a trapezoidal finite element mesh



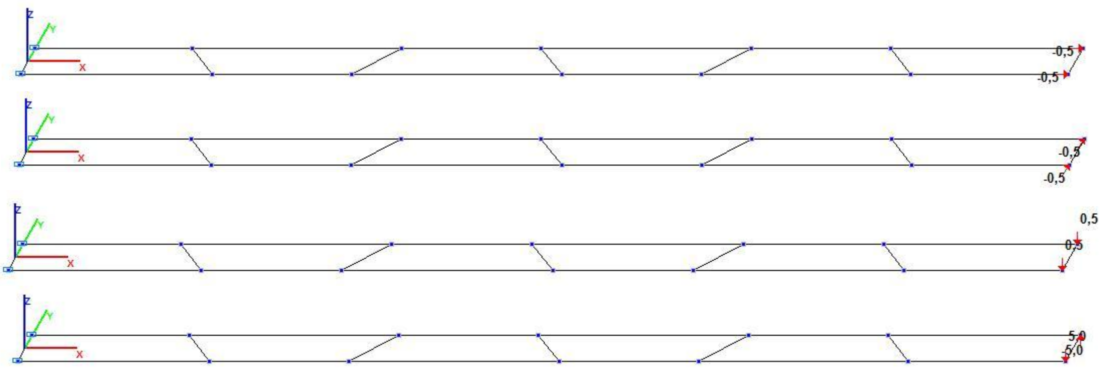
Models 1 and 2.
Deformed model with a trapezoidal finite element mesh



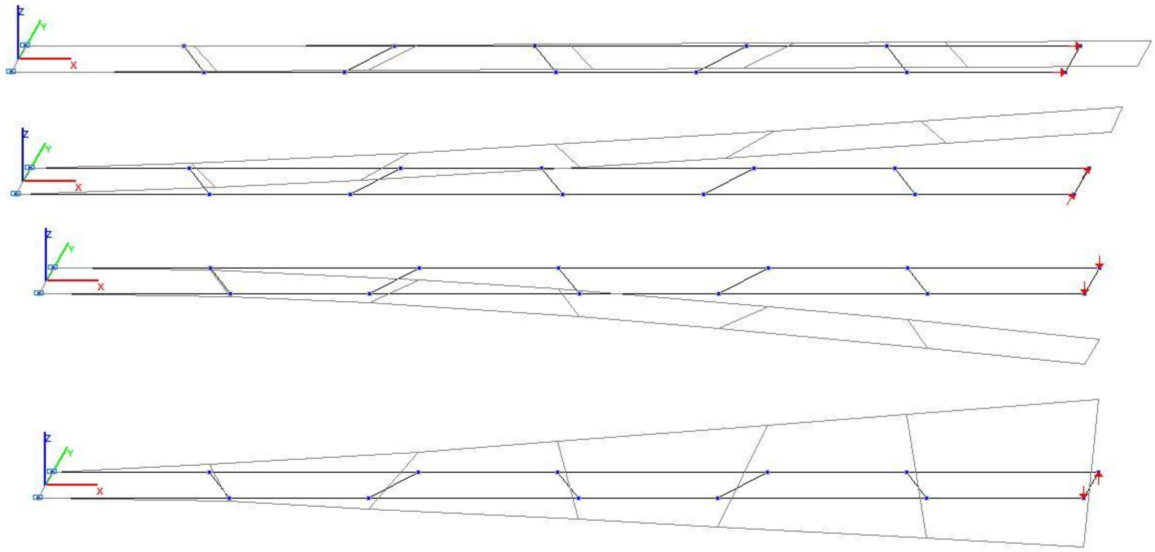
Model 1.
Values of the longitudinal displacement X, transverse displacements Y, Z and the torsional angle UX of the free end of the rectilinear cantilever beam (m, m, m, rad)



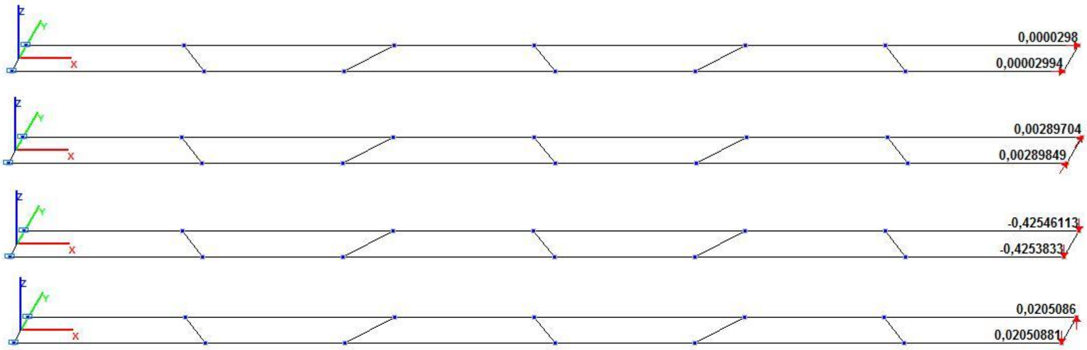
Model 2.
Values of the longitudinal displacement X, transverse displacements Y, Z and the torsional angle UX of the free end of the rectilinear cantilever beam (m, m, m, rad)



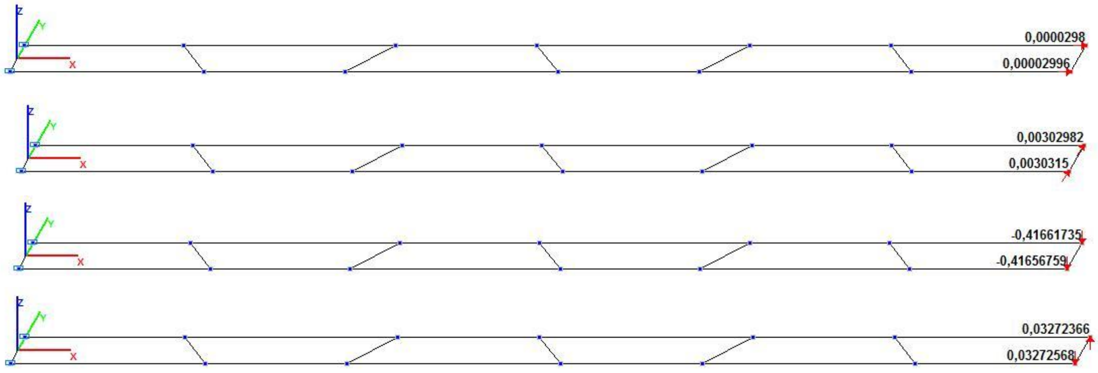
Models 3 and 4.
Design model with a trapezoidal finite element mesh



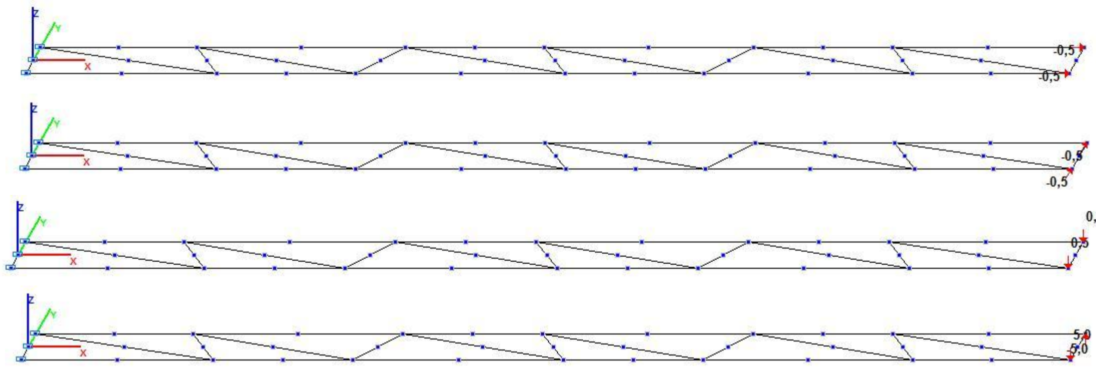
Models 3 and 4.
Deformed model with a trapezoidal finite element mesh



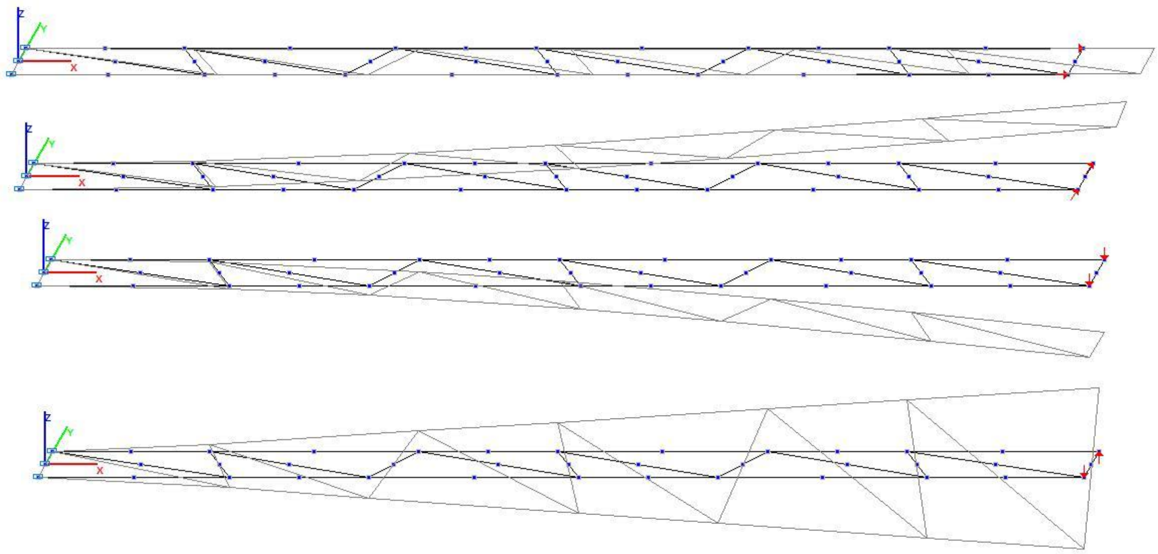
Model 3.
Values of the longitudinal displacement X, transverse displacements Y, Z and the torsional angle UX of the free end of the rectilinear cantilever beam (m, m, m, rad)



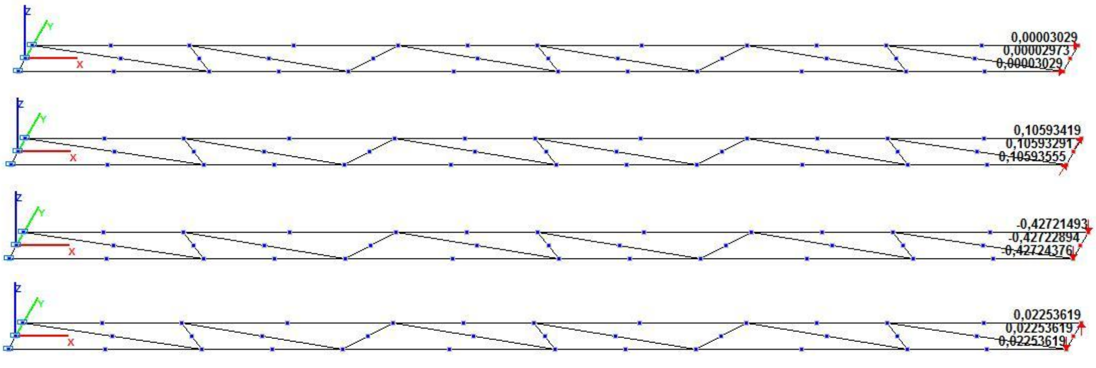
Model 4.
Values of the longitudinal displacement X, transverse displacements Y, Z and the torsional angle UX of the free end of the rectilinear cantilever beam (m, m, m, rad)



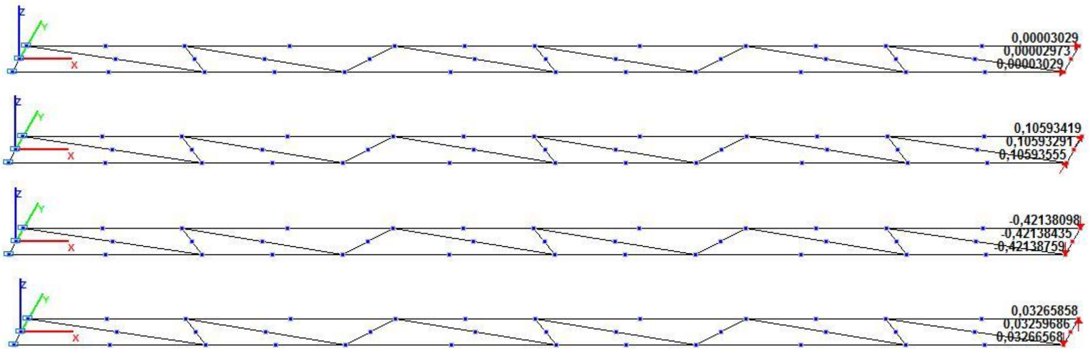
Models 5 and 6.
Design model with a trapezoidal finite element mesh



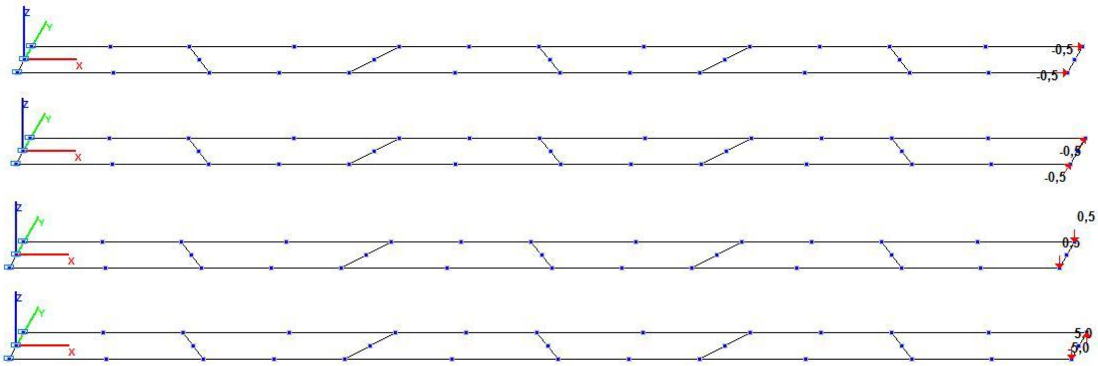
Models 5 and 6.
Deformed model with a trapezoidal finite element mesh



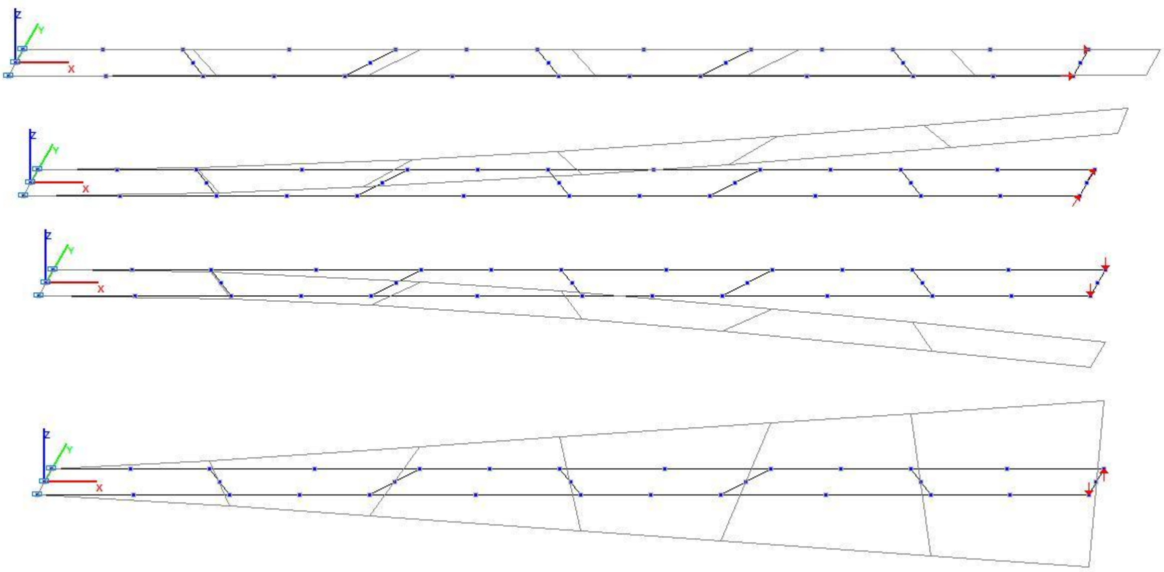
Model 5.
Values of the longitudinal displacement X, transverse displacements Y, Z and the torsional angle UX of the free end of the rectilinear cantilever beam (m, m, m, rad)



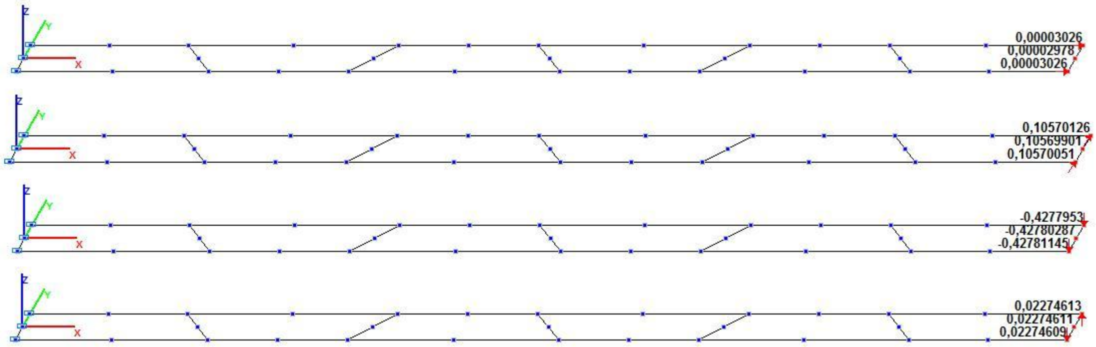
Model 6.
Values of the longitudinal displacement X, transverse displacements Y, Z and the torsional angle UX of the free end of the rectilinear cantilever beam (m, m, m, rad)



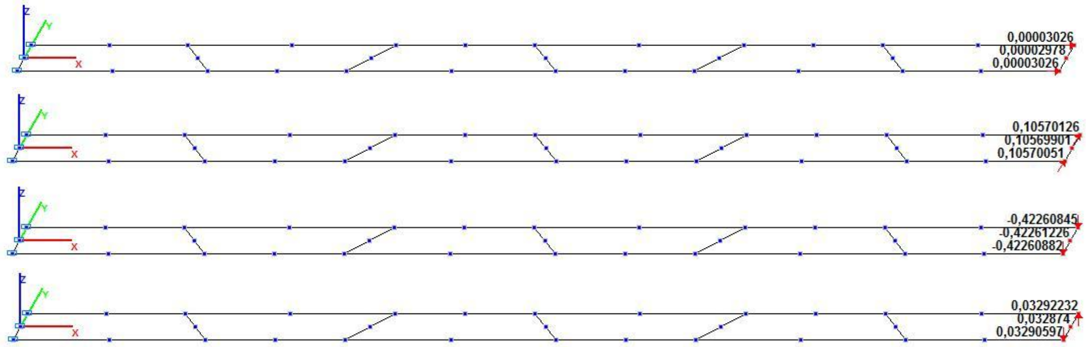
Models 7 and 8.
Design model with a trapezoidal finite element mesh



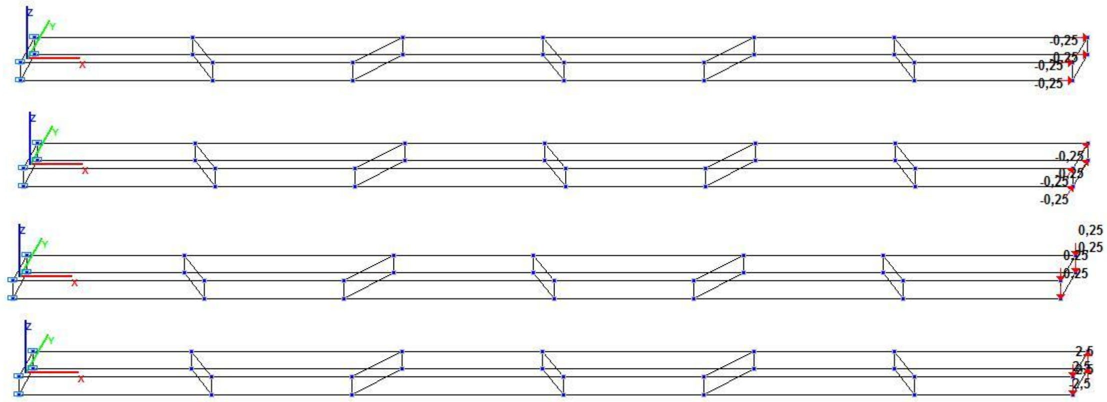
Models 7 and 8.
Deformed model with a trapezoidal finite element mesh



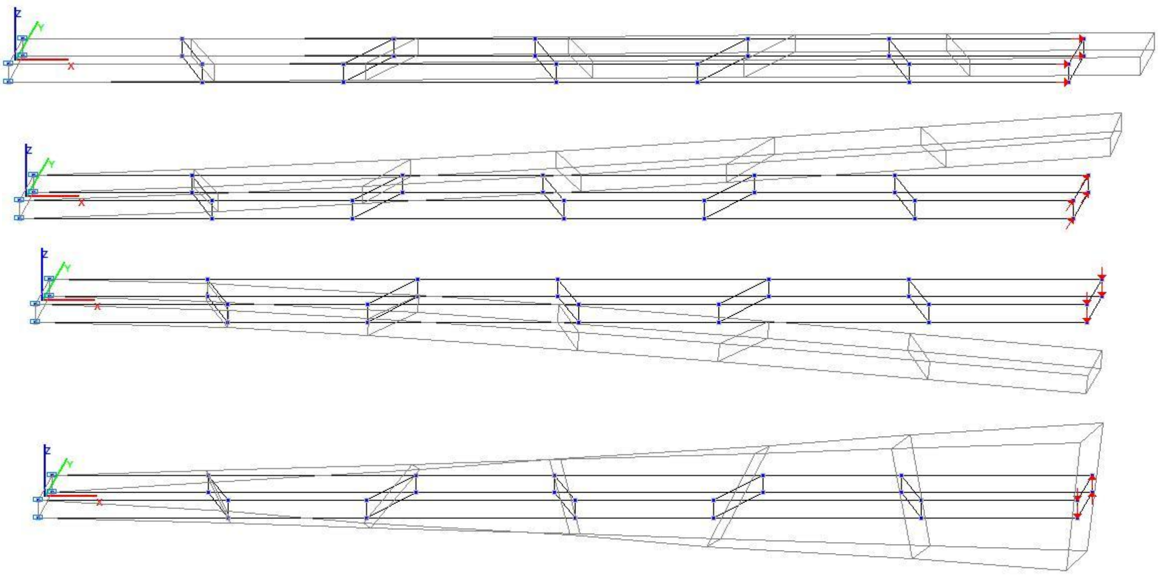
Model 7.
Values of the longitudinal displacement X, transverse displacements Y, Z and the torsional angle UX of the free end of the rectilinear cantilever beam (m, m, m, rad)



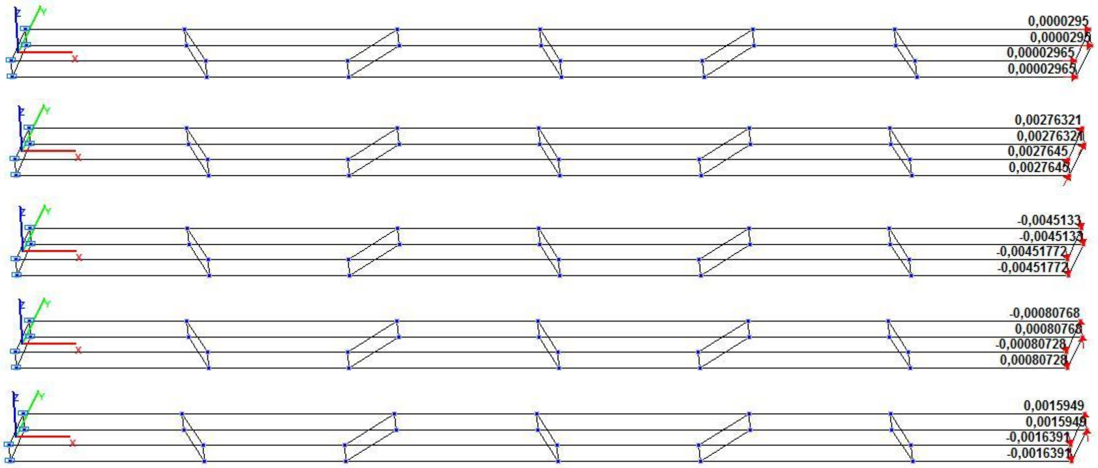
Model 8.
Values of the longitudinal displacement X, transverse displacements Y, Z and the torsional angle UX of the free end of the rectilinear cantilever beam (m, m, m, rad)



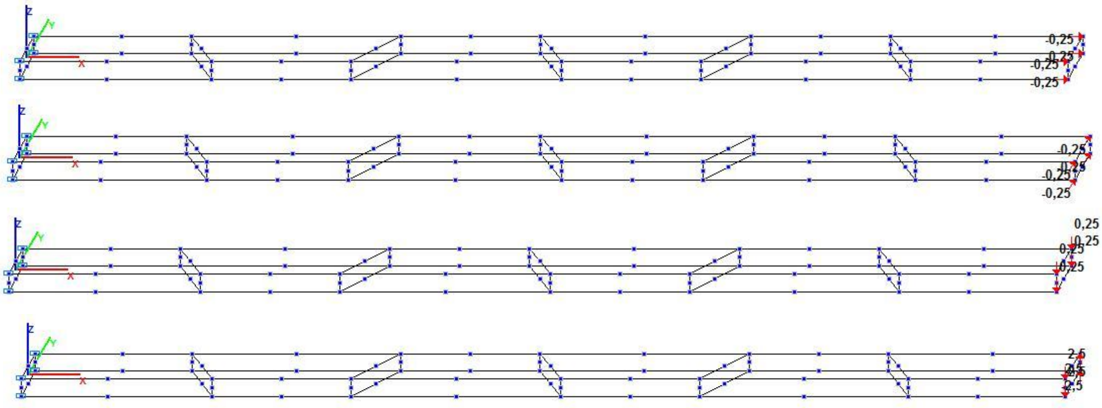
Model 9.
Design model with a trapezoidal finite element mesh



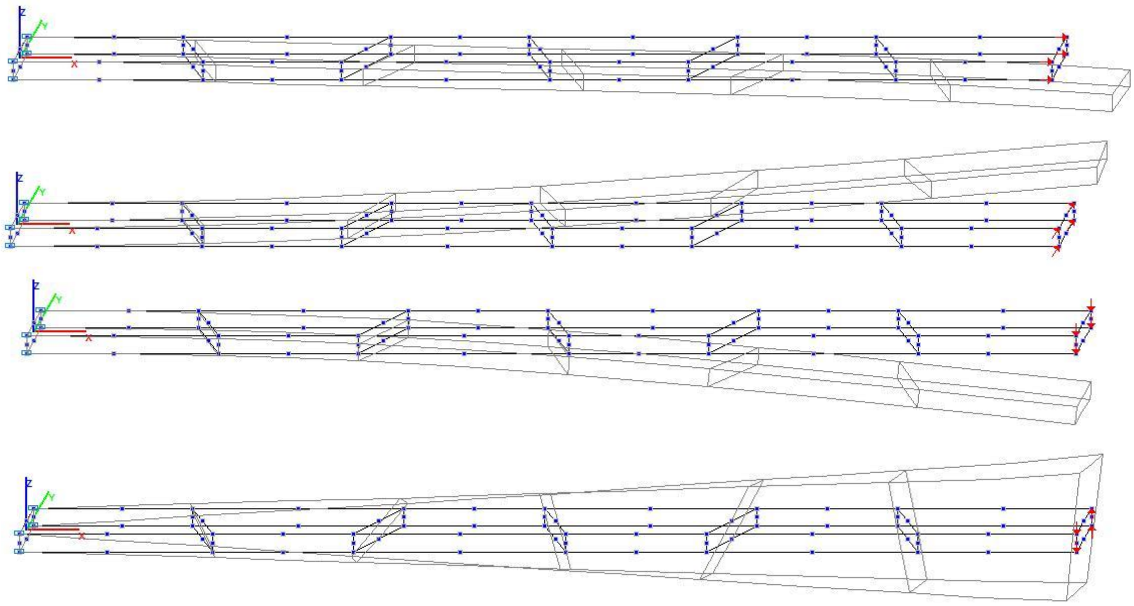
Model 9.
Deformed model with a trapezoidal finite element mesh



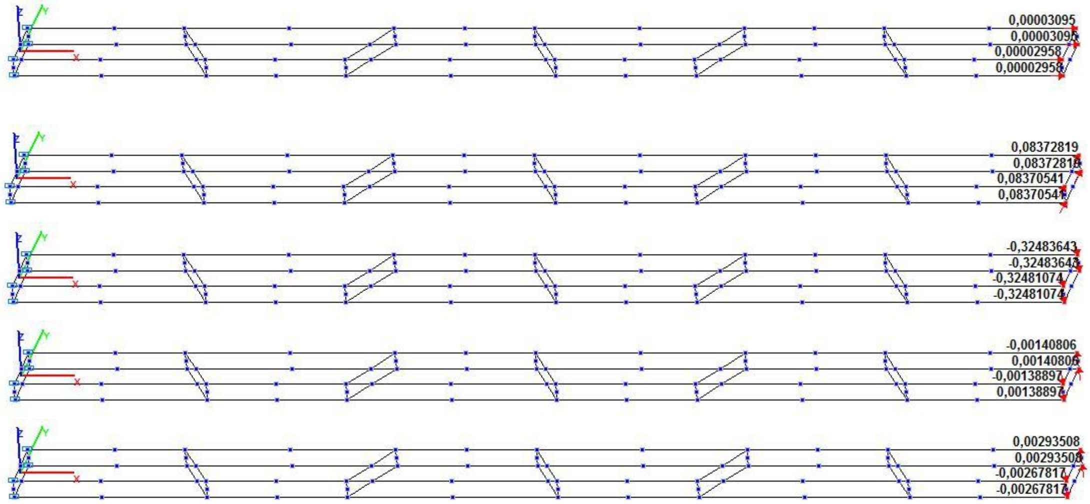
Model 9.
Values of the longitudinal displacement X from the action P_x,
transverse displacements Y, Z from the actions P_y, P_z
and transverse displacements Y, Z from the action M_x
of the free end of the rectilinear cantilever beam (m, m, m, m, m)



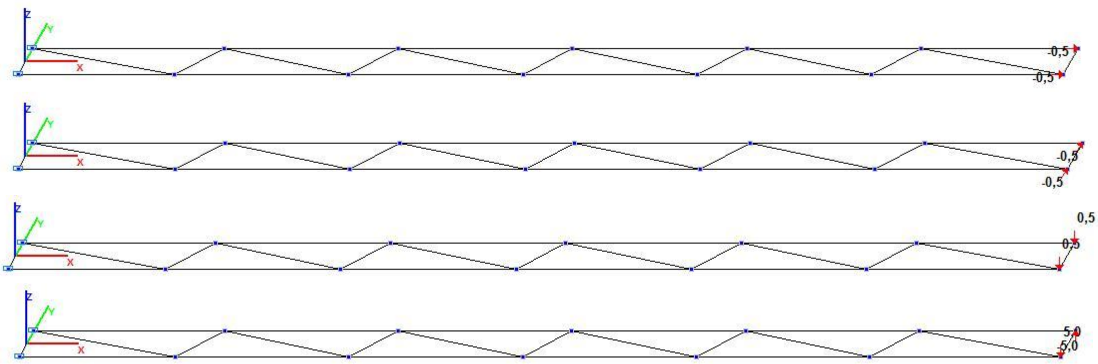
Model 10.
Design model with a trapezoidal finite element mesh



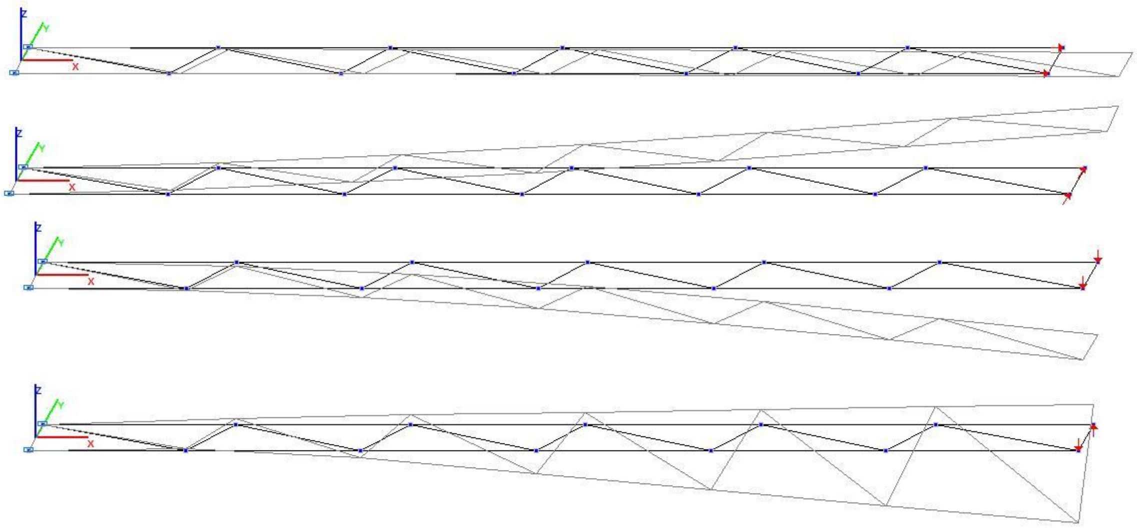
Model 10.
Deformed model with a trapezoidal finite element mesh



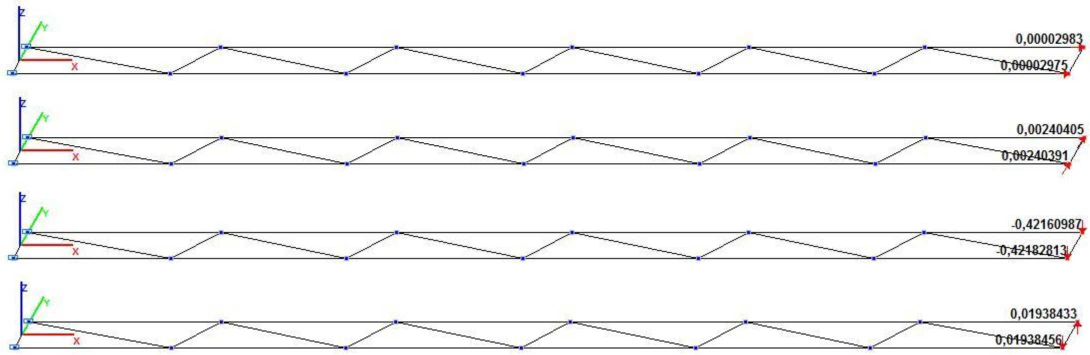
Model 10.
Values of the longitudinal displacement X from the action P_x,
transverse displacements Y, Z from the actions P_y, P_z
and transverse displacements Y, Z from the action M_x
of the free end of the rectilinear cantilever beam (m, m, m, m, m)



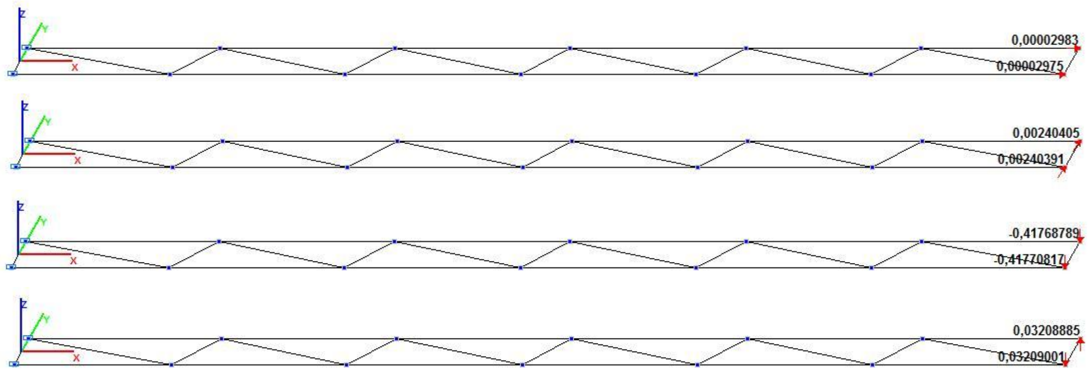
Models 1 and 2.
Design model with a parallelogram finite element mesh



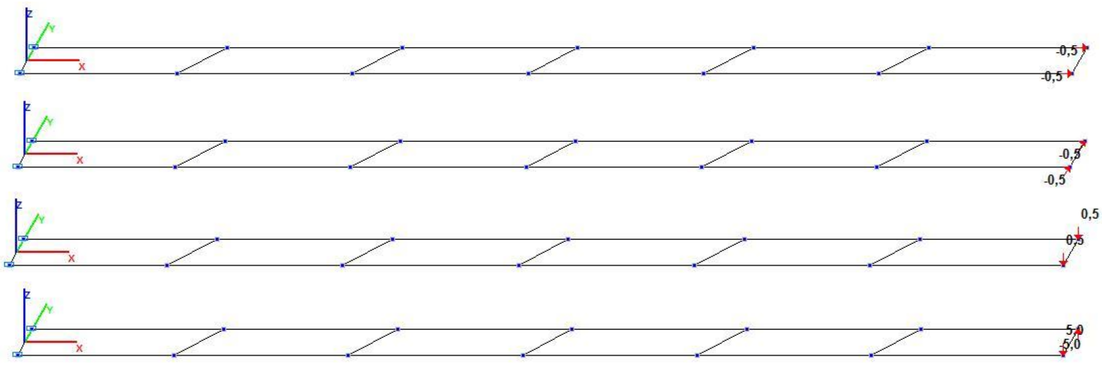
Models 1 and 2.
Deformed model with a parallelogram finite element mesh



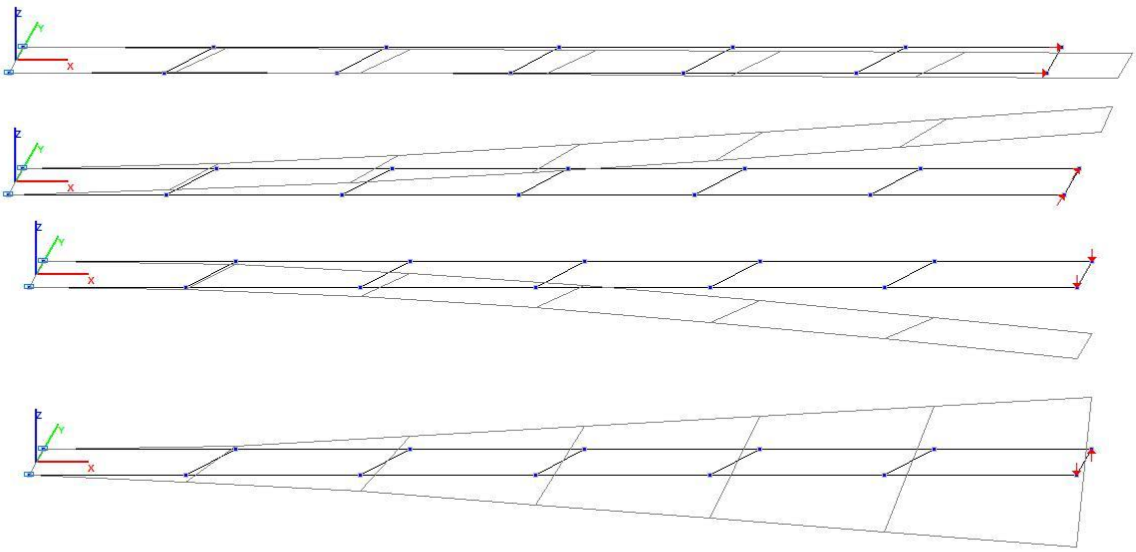
Model 1.
Values of the longitudinal displacement X, transverse displacements Y, Z and the torsional angle UX of the free end of the rectilinear cantilever beam (m, m, m, rad)



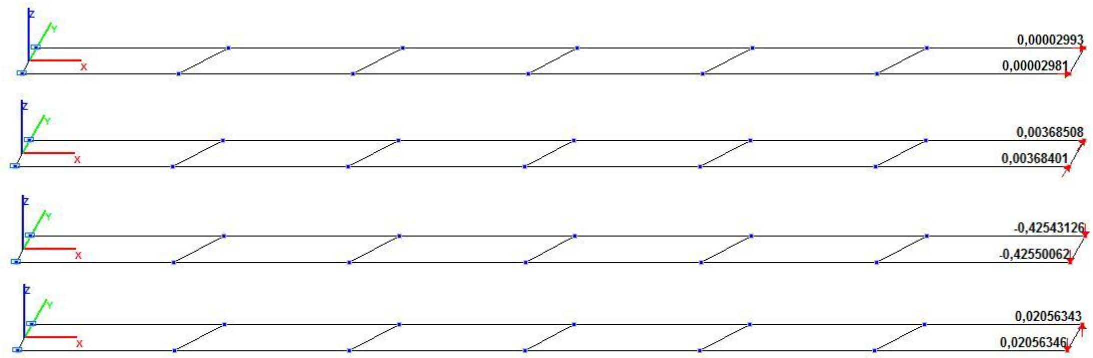
Model 2.
Values of the longitudinal displacement X, transverse displacements Y, Z and the torsional angle UX of the free end of the rectilinear cantilever beam (m, m, m, rad)



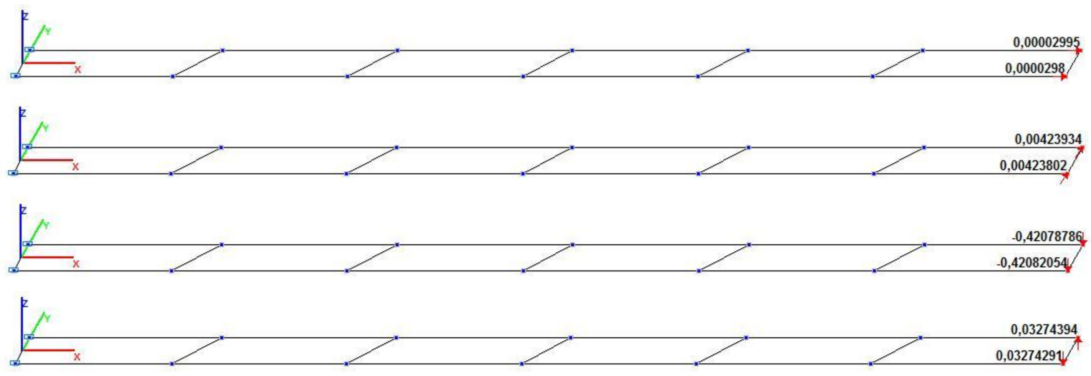
Models 3 and 4.
Design model with a parallelogram finite element mesh



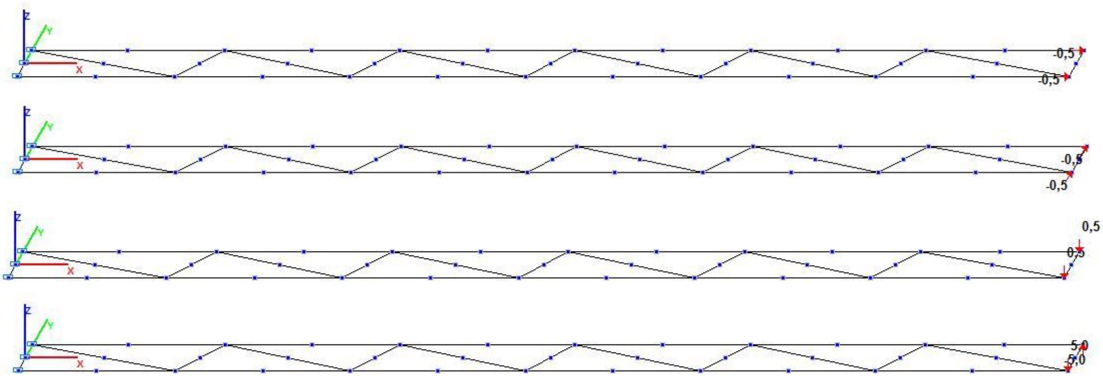
Models 3 and 4.
Deformed model with a parallelogram finite element mesh



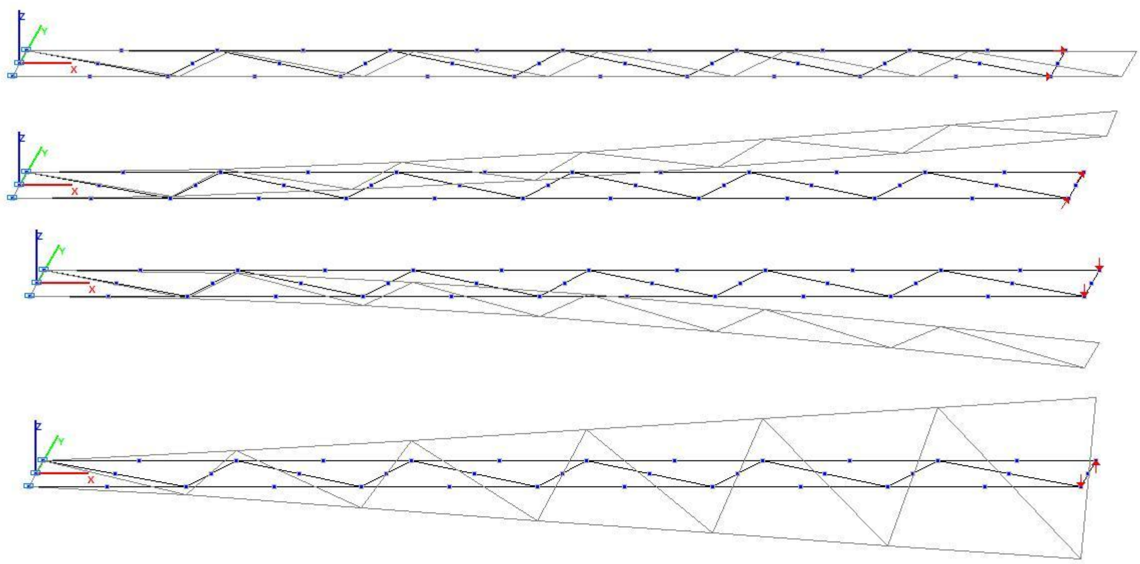
Model 3.
Values of the longitudinal displacement X, transverse displacements Y, Z and the torsional angle UX of the free end of the rectilinear cantilever beam (m, m, m, rad)



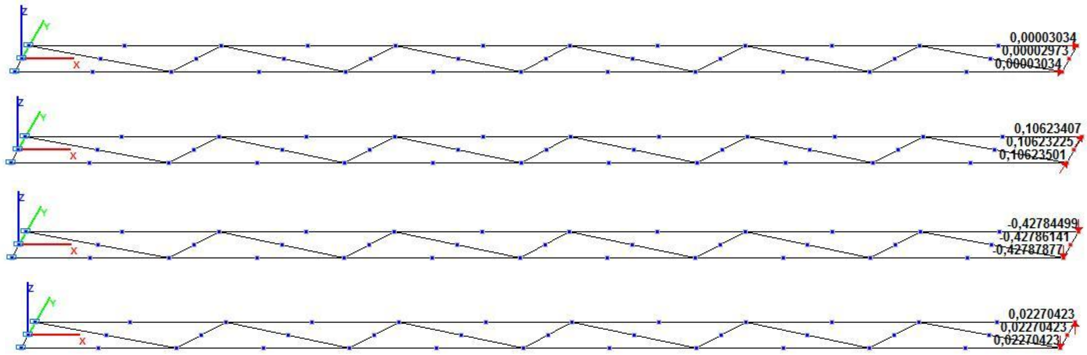
Model 4.
Values of the longitudinal displacement X, transverse displacements Y, Z and the torsional angle UX of the free end of the rectilinear cantilever beam (m, m, m, rad)



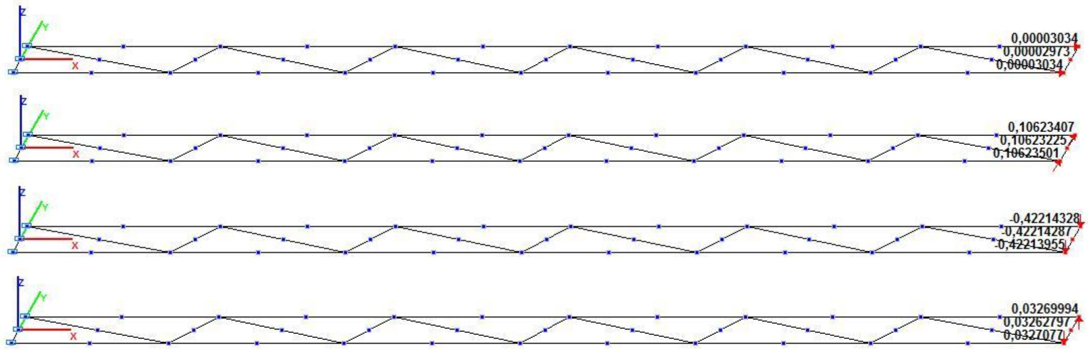
Models 5 and 6.
Design model with a parallelogram finite element mesh



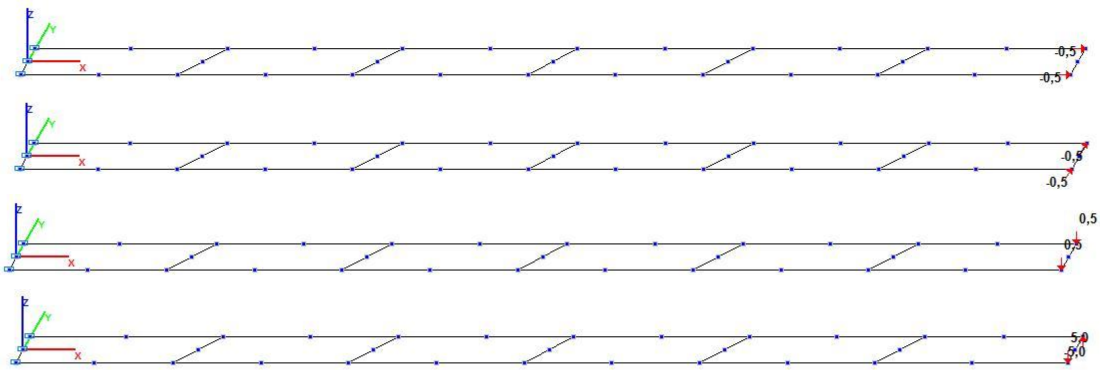
Models 5 and 6.
Deformed model with a parallelogram finite element mesh



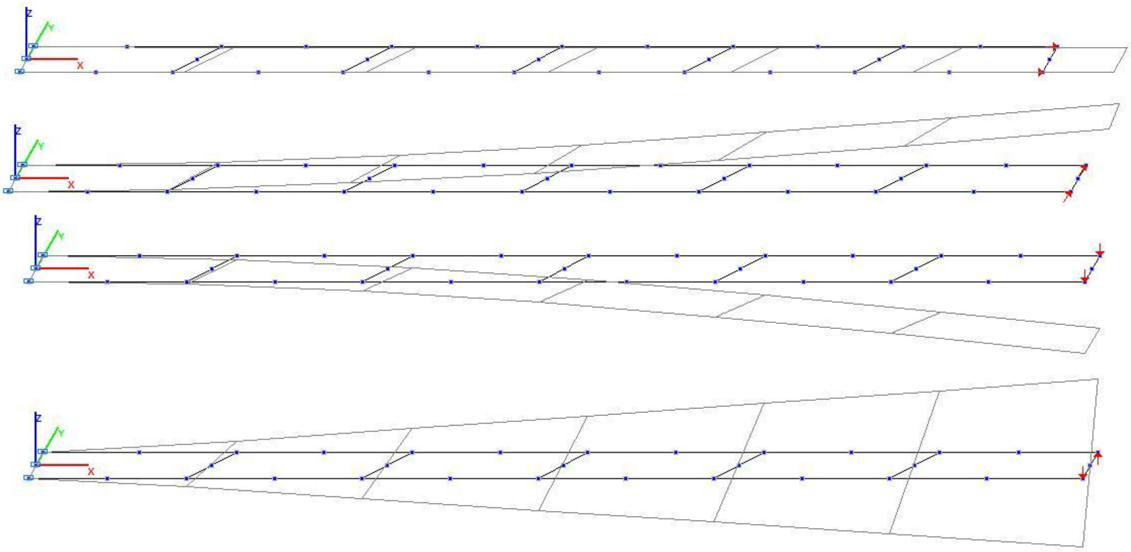
Model 5.
Values of the longitudinal displacement X, transverse displacements Y, Z and the torsional angle UX of the free end of the rectilinear cantilever beam (m, m, m, rad)



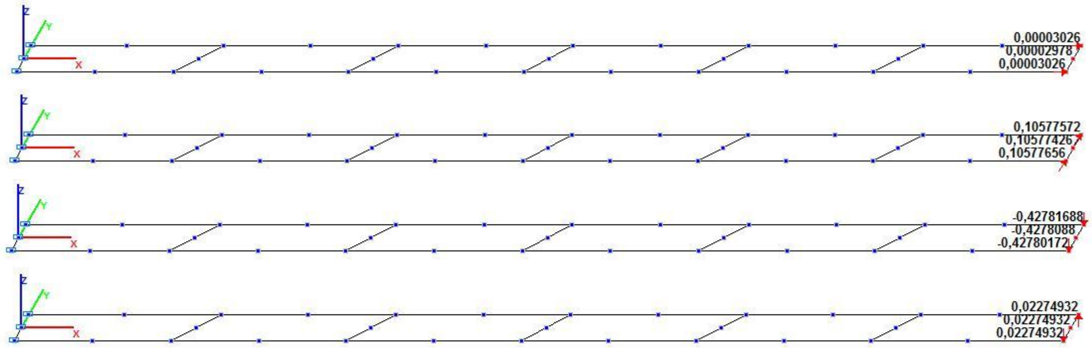
Model 6.
Values of the longitudinal displacement X, transverse displacements Y, Z and the torsional angle UX of the free end of the rectilinear cantilever beam (m, m, m, rad)



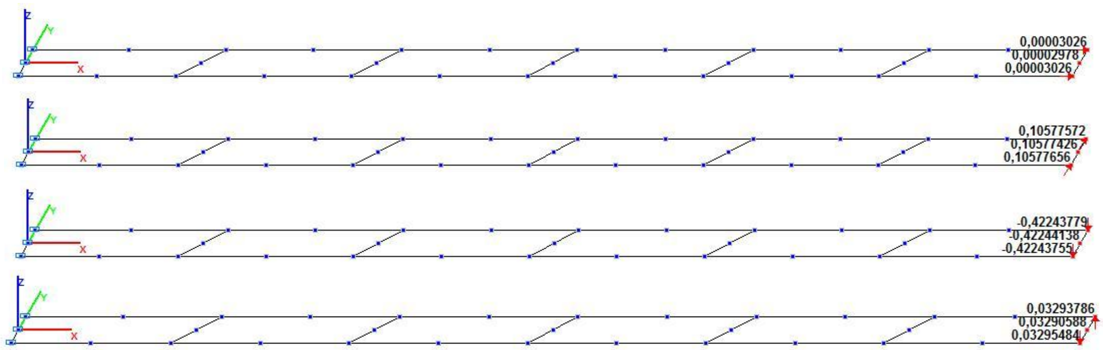
Models 7 and 8.
Design model with a parallelogram finite element mesh



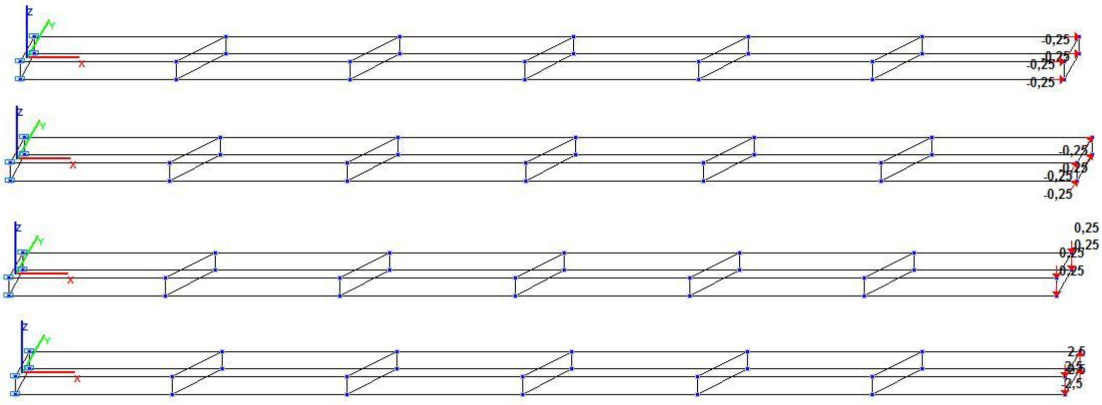
Models 7 and 8.
Deformed model with a parallelogram finite element mesh



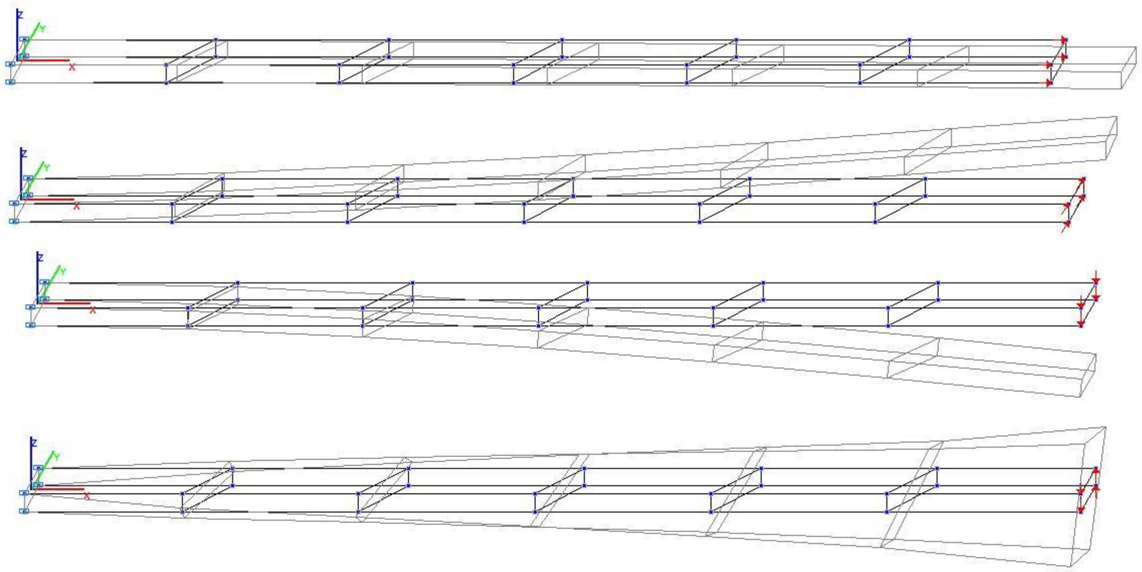
Model 7.
Values of the longitudinal displacement X, transverse displacements Y, Z and the torsional angle UX of the free end of the rectilinear cantilever beam (m, m, m, rad)



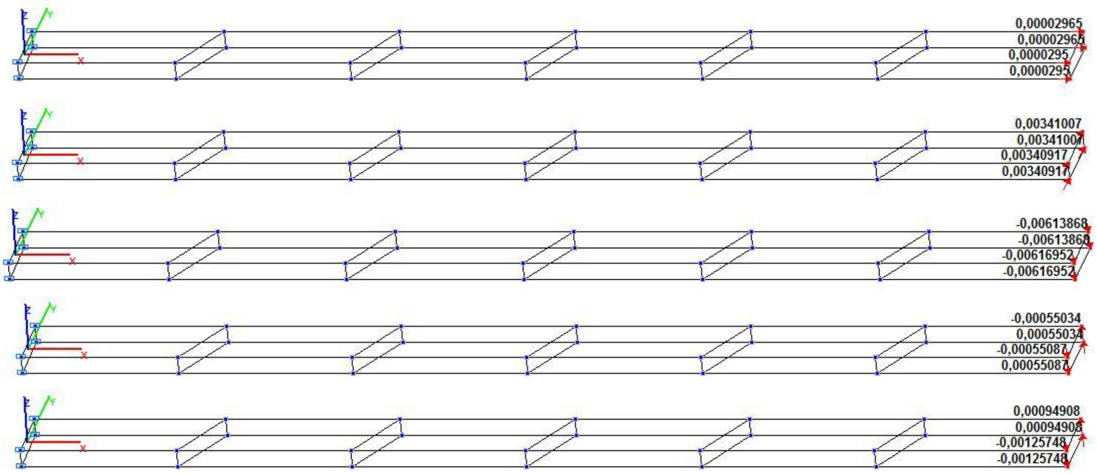
Model 8.
Values of the longitudinal displacement X, transverse displacements Y, Z and the torsional angle UX of the free end of the rectilinear cantilever beam (m, m, m, rad)



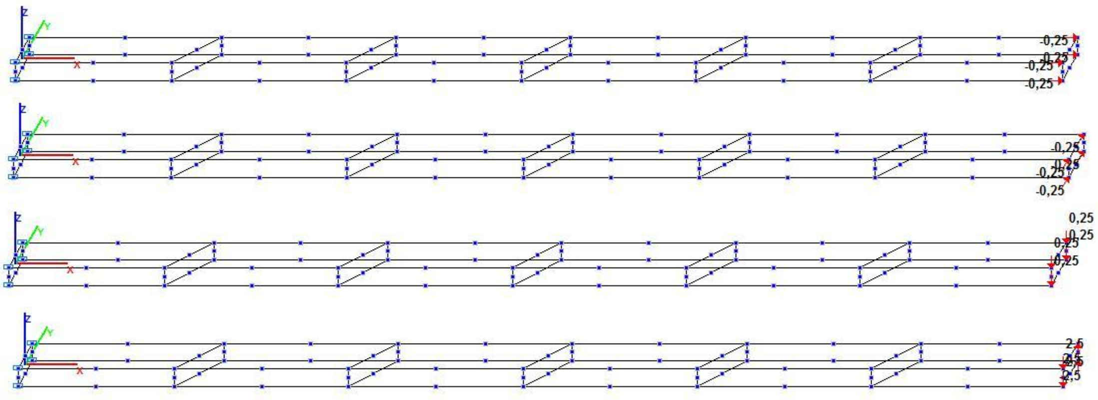
Model 9.
Design model with a parallelogram finite element mesh



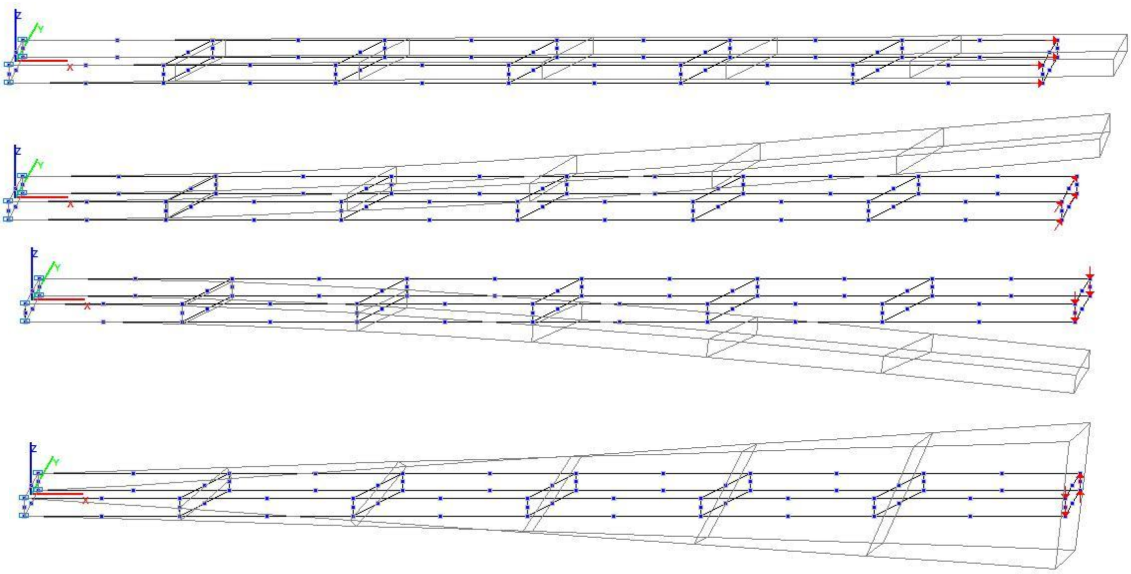
Model 9.
Deformed model with a parallelogram finite element mesh



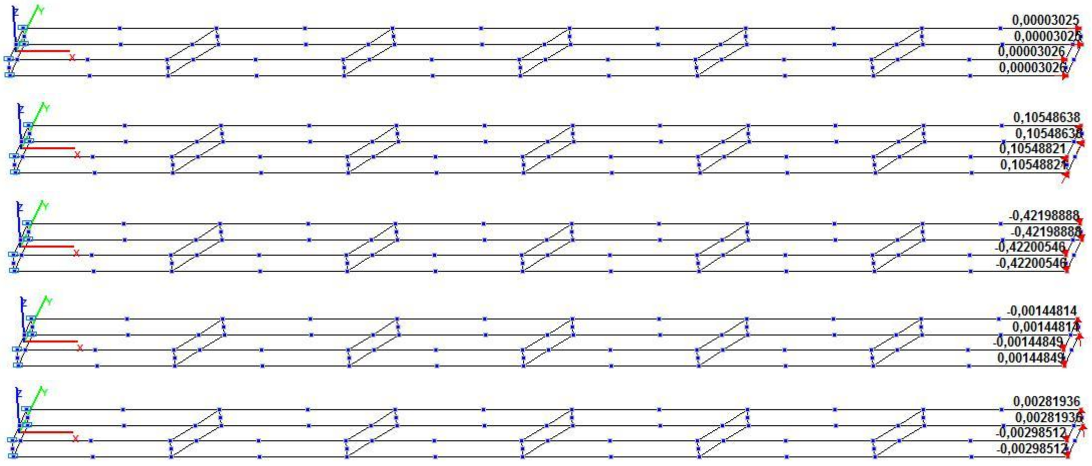
Model 9.
Values of the longitudinal displacement X from the action P_x,
transverse displacements Y, Z from the actions P_y, P_z
and transverse displacements Y, Z from the action M_x
of the free end of the rectilinear cantilever beam (m, m, m, m, m)



Model 10.
Design model with a parallelogram finite element mesh



Model 10.
Deformed model with a parallelogram finite element mesh



Model 10.
Values of the longitudinal displacement X from the action P_x,
transverse displacements Y, Z from the actions P_y, P_z
and transverse displacements Y, Z from the action M_x
of the free end of the rectilinear cantilever beam (m, m, m, m, m)

Comparison of solutions:

Design model with a regular finite element mesh

Model	Parameter	Theory	SCAD	Deviation, %
1 (Member type 42)	Longitudinal displacement X of the free end of the cantilever beam, m	0.00003000	0.00002972	0.93
	Transverse displacement Y of the free end of the cantilever beam, m	0.1080	0.0034	96.85
	Transverse displacement Z of the free end of the cantilever beam, m	0.4320	0.4212	2.50
	Torsional angle UX of the free end of the cantilever beam, rad	0.023400*	0.018560	20.68
2 (Member type 142)	Longitudinal displacement X of the free end of the cantilever beam, m	0.00003000	0.00002972	0.93
	Transverse displacement Y of the free end of the cantilever beam, m	0.1080	0.0034	96.85
	Transverse displacement Z of the free end of the cantilever beam, m	0.4320	0.4198	2.82
	Torsional angle UX of the free end of the cantilever beam, rad	0.034109	0.032096	5.90
3 (Member type 44)	Longitudinal displacement X of the free end of the cantilever beam, m	0.00003000	0.00002986	0.47
	Transverse displacement Y of the free end of the cantilever beam, m	0.1080	0.0101	90.65
	Transverse displacement Z of the free end of the cantilever beam, m	0.4320	0.4250	1.62
	Torsional angle UX of the free end of the cantilever beam, rad	0.023400*	0.019692	15.85
4 (Member type 144)	Longitudinal displacement X of the free end of the cantilever beam, m	0.00003000	0.00002987	0.43
	Transverse displacement Y of the free end of the cantilever beam, m	0.1080	0.0104	90.37
	Transverse displacement Z of the free end of the cantilever beam, m	0.4320	0.4235	1.97
	Torsional angle UX of the free end of the cantilever beam, rad	0.034109	0.032759	3.96
5 (Member type 45)	Longitudinal displacement X of the free end of the cantilever beam, m	0.00003000	0.00003032	1.07
	Transverse displacement Y of the free end of the cantilever beam, m	0.1080	0.1070	0.92
	Transverse displacement Z of the free end	0.4320	0.4276	1.02

Model	Parameter	Theory	SCAD	Deviation, %
	of the cantilever beam, m			
	Torsional angle UX of the free end of the cantilever beam, rad	0.023400*	0.022631	3.29
6 (Member type 145)	Longitudinal displacement X of the free end of the cantilever beam, m	0.00003000	0.00003032	1.07
	Transverse displacement Y of the free end of the cantilever beam, m	0.1080	0.1070	0.92
	Transverse displacement Z of the free end of the cantilever beam, m	0.4320	0.4259	1.41
	Torsional angle UX of the free end of the cantilever beam, rad	0.034109	0.032719	4.08
7 (Member type 50)	Longitudinal displacement X of the free end of the cantilever beam, m	0.00003000	0.00003026	0.87
	Transverse displacement Y of the free end of the cantilever beam, m	0.1080	0.1070	0.92
	Transverse displacement Z of the free end of the cantilever beam, m	0.4320	0.4276	1.02
	Torsional angle UX of the free end of the cantilever beam, rad	0.023400*	0.022813	2.51
8 (Member type 150)	Longitudinal displacement X of the free end of the cantilever beam, m	0.00003000	0.00003026	0.87
	Transverse displacement Y of the free end of the cantilever beam, m	0.1080	0.1070	0.92
	Transverse displacement Z of the free end of the cantilever beam, m	0.4320	0.4264	1.30
	Torsional angle UX of the free end of the cantilever beam, rad	0.034109	0.032959	3.37
9 (Member type 36)	Longitudinal displacement X of the free end of the cantilever beam, m	0.00003000	0.00002957	1.43
	Transverse displacement Y of the free end of the cantilever beam, m	0.1080	0.0100	90.74
	Transverse displacement Z of the free end of the cantilever beam, m	0.4320	0.0109	97.48
	Torsional angle UX of the free end of the cantilever beam, rad	0.034109	0.028974	15.05
10 (Member type 37)	Longitudinal displacement X of the free end of the cantilever beam, m	0.00003000	0.00003024	0.80
	Transverse displacement Y of the free end of the cantilever beam, m	0.1080	0.1072	0.74
	Transverse displacement Z of the free end of the cantilever beam, m	0.4320	0.4287	0.76

Model	Parameter	Theory	SCAD	Deviation, %
	Torsional angle UX of the free end of the cantilever beam, rad	0.034109	0.028974	15.05

Design model with a trapezoidal finite element mesh

Model	Parameter	Theory	SCAD	Deviation, %
1 (Member type 42)	Longitudinal displacement X of the free end of the cantilever beam, m	0.00003000	0.00002970	1.00
	Transverse displacement Y of the free end of the cantilever beam, m	0.1080	0.0016	98.52
	Transverse displacement Z of the free end of the cantilever beam, m	0.4320	0.4207	2.62
	Torsional angle UX of the free end of the cantilever beam, rad	0.023400*	0.018771	19.78
2 (Member type 142)	Longitudinal displacement X of the free end of the cantilever beam, m	0.00003000	0.00002970	1.00
	Transverse displacement Y of the free end of the cantilever beam, m	0.1080	0.0016	98.52
	Transverse displacement Z of the free end of the cantilever beam, m	0.4320	0.4117	4.70
	Torsional angle UX of the free end of the cantilever beam, rad	0.034109	0.031948	6.34
3 (Member type 44)	Longitudinal displacement X of the free end of the cantilever beam, m	0.00003000	0.00002980	0.67
	Transverse displacement Y of the free end of the cantilever beam, m	0.1080	0.0029	97.31
	Transverse displacement Z of the free end of the cantilever beam, m	0.4320	0.4254	1.53
	Torsional angle UX of the free end of the cantilever beam, rad	0.023400*	0.020509	12.35
4 (Member type 144)	Longitudinal displacement X of the free end of the cantilever beam, m	0.00003000	0.00002980	0.67
	Transverse displacement Y of the free end of the cantilever beam, m	0.1080	0.0030	97.22
	Transverse displacement Z of the free end of the cantilever beam, m	0.4320	0.4166	3.56
	Torsional angle UX of the free end of the cantilever beam, rad	0.034109	0.032724	4.06
5 (Member type 45)	Longitudinal displacement X of the free end of the cantilever beam, m	0.00003000	0.00003029	0.97
	Transverse displacement Y	0.1080	0.1059	1.94

Model	Parameter	Theory	SCAD	Deviation, %
	of the free end of the cantilever beam, m			
	Transverse displacement Z of the free end of the cantilever beam, m	0.4320	0.4272	1.11
	Torsional angle UX of the free end of the cantilever beam, rad	0.023400*	0.022536	3.69
6 (Member type 145)	Longitudinal displacement X of the free end of the cantilever beam, m	0.00003000	0.00003029	0.97
	Transverse displacement Y of the free end of the cantilever beam, m	0.1080	0.1059	1.94
	Transverse displacement Z of the free end of the cantilever beam, m	0.4320	0.4214	2.45
	Torsional angle UX of the free end of the cantilever beam, rad	0.034109	0.032659	4.25
7 (Member type 50)	Longitudinal displacement X of the free end of the cantilever beam, m	0.00003000	0.00003026	0.87
	Transverse displacement Y of the free end of the cantilever beam, m	0.1080	0.1057	2.13
	Transverse displacement Z of the free end of the cantilever beam, m	0.4320	0.4278	0.97
	Torsional angle UX of the free end of the cantilever beam, rad	0.023400*	0.022746	2.79
8 (Member type 150)	Longitudinal displacement X of the free end of the cantilever beam, m	0.00003000	0.00003026	0.87
	Transverse displacement Y of the free end of the cantilever beam, m	0.1080	0.1057	2.13
	Transverse displacement Z of the free end of the cantilever beam, m	0.4320	0.4226	2.18
	Torsional angle UX of the free end of the cantilever beam, rad	0.034109	0.032906	3.53
9 (Member type 36)	Longitudinal displacement X of the free end of the cantilever beam, m	0.00003000	0.00002950	1.67
	Transverse displacement Y of the free end of the cantilever beam, m	0.1080	0.0028	97.41
	Transverse displacement Z of the free end of the cantilever beam, m	0.4320	0.0045	98.96
	Torsional angle UX of the free end of the cantilever beam, rad	0.034109	0.016146	52.66
10 (Member type 37)	Longitudinal displacement X of the free end of the cantilever beam, m	0.00003000	0.00003095	3.17
	Transverse displacement Y of the free end	0.1080	0.0837	22.50

Model	Parameter	Theory	SCAD	Deviation, %
	of the cantilever beam, m			
	Transverse displacement Z of the free end of the cantilever beam, m	0.4320	0.3248	24.81
	Torsional angle UX of the free end of the cantilever beam, rad	0.034109	0.027779	18.56

Design model with a parallelogram finite element mesh

Model	Parameter	Theory	SCAD	Deviation, %
1 (Member type 42)	Longitudinal displacement X of the free end of the cantilever beam, m	0.00003000	0.00002975	0.83
	Transverse displacement Y of the free end of the cantilever beam, m	0.1080	0.0024	97.78
	Transverse displacement Z of the free end of the cantilever beam, m	0.4320	0.4216	2.41
	Torsional angle UX of the free end of the cantilever beam, rad	0.023400*	0.019384	17.16
2 (Member type 142)	Longitudinal displacement X of the free end of the cantilever beam, m	0.00003000	0.00002975	0.83
	Transverse displacement Y of the free end of the cantilever beam, m	0.1080	0.0024	97.78
	Transverse displacement Z of the free end of the cantilever beam, m	0.4320	0.4177	3.31
	Torsional angle UX of the free end of the cantilever beam, rad	0.034109	0.032089	5.92
3 (Member type 44)	Longitudinal displacement X of the free end of the cantilever beam, m	0.00003000	0.00002981	0.63
	Transverse displacement Y of the free end of the cantilever beam, m	0.1080	0.0037	96.57
	Transverse displacement Z of the free end of the cantilever beam, m	0.4320	0.4254	1.53
	Torsional angle UX of the free end of the cantilever beam, rad	0.023400*	0.020563	12.12
4 (Member type 144)	Longitudinal displacement X of the free end of the cantilever beam, m	0.00003000	0.00002980	0.67
	Transverse displacement Y of the free end of the cantilever beam, m	0.1080	0.0042	96.11
	Transverse displacement Z of the free end of the cantilever beam, m	0.4320	0.4208	2.59
	Torsional angle UX of the free end of the cantilever beam, rad	0.034109	0.032743	4.00

Model	Parameter	Theory	SCAD	Deviation, %
5 (Member type 45)	Longitudinal displacement X of the free end of the cantilever beam, m	0.00003000	0.00003034	1.13
	Transverse displacement Y of the free end of the cantilever beam, m	0.1080	0.1062	1.67
	Transverse displacement Z of the free end of the cantilever beam, m	0.4320	0.4278	0.97
	Torsional angle UX of the free end of the cantilever beam, rad	0.023400*	0.022704	2.97
6 (Member type 145)	Longitudinal displacement X of the free end of the cantilever beam, m	0.00003000	0.00003034	1.13
	Transverse displacement Y of the free end of the cantilever beam, m	0.1080	0.1062	1.67
	Transverse displacement Z of the free end of the cantilever beam, m	0.4320	0.4221	2.29
	Torsional angle UX of the free end of the cantilever beam, rad	0.034109	0.032700	4.13
7 (Member type 50)	Longitudinal displacement X of the free end of the cantilever beam, m	0.00003000	0.00003026	0.87
	Transverse displacement Y of the free end of the cantilever beam, m	0.1080	0.1058	2.04
	Transverse displacement Z of the free end of the cantilever beam, m	0.4320	0.4278	0.97
	Torsional angle UX of the free end of the cantilever beam, rad	0.023400*	0.022749	2.78
8 (Member type 150)	Longitudinal displacement X of the free end of the cantilever beam, m	0.00003000	0.00003026	0.87
	Transverse displacement Y of the free end of the cantilever beam, m	0.1080	0.1058	2.04
	Transverse displacement Z of the free end of the cantilever beam, m	0.4320	0.4224	2.22
	Torsional angle UX of the free end of the cantilever beam, rad	0.034109	0.032938	3.43
9 (Member type 36)	Longitudinal displacement X of the free end of the cantilever beam, m	0.00003000	0.00002950	1.67
	Transverse displacement Y of the free end of the cantilever beam, m	0.1080	0.0034	96.85
	Transverse displacement Z of the free end of the cantilever beam, m	0.4320	0.0061	98.59
	Torsional angle UX of the free end of the cantilever beam, rad	0.034109	0.011007	67.73
10	Longitudinal displacement X	0.00003000	0.00003026	0.87

Model	Parameter	Theory	SCAD	Deviation, %
(Member type 37)	of the free end of the cantilever beam, m			
	Transverse displacement Y of the free end of the cantilever beam, m	0.1080	0.1055	2.31
	Transverse displacement Z of the free end of the cantilever beam, m	0.4320	0.4220	2.31
	Torsional angle UX of the free end of the cantilever beam, rad	0.034109	0.028963	15.09

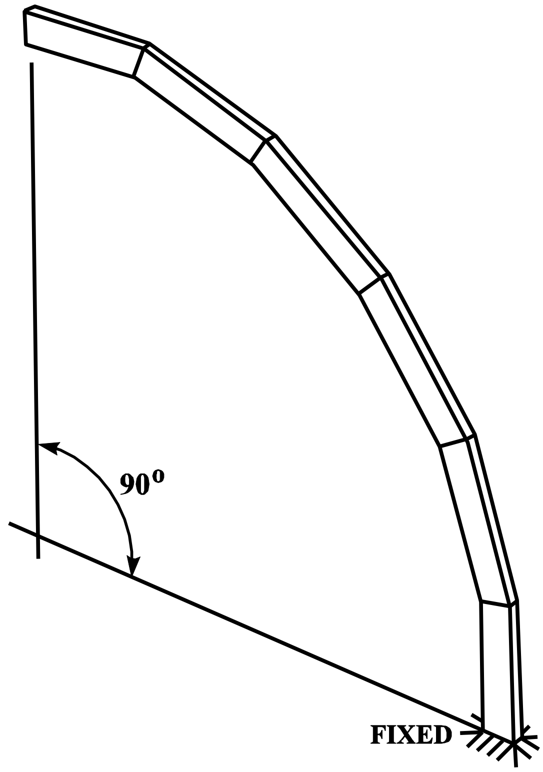
* The values of the torsional angles UX for thin plates (not allowing for shear) are determined at the free torsional inertia moment calculated with the value of the coefficient k_f, equal to 1/3 (h/b = ∞).

Notes: In the analytical solution the values of the longitudinal X and transverse displacements Y, Z and the torsional angle UX of the free end of the rectilinear cantilever beam from the respective actions are determined according to the following formulas:

$$X = \frac{P_x \cdot L}{E \cdot b \cdot h}; \quad Y = \frac{4 \cdot P_y \cdot L^3}{E \cdot b \cdot h^3}; \qquad Z = \frac{4 \cdot P_z \cdot L^3}{E \cdot b^3 \cdot h}; \qquad UX = \frac{2 \cdot (1+\nu) \cdot M_x \cdot L}{E \cdot k_f \cdot b^3 \cdot h},$$

where: $$k_f = \frac{1}{3} \cdot \left\{ 1 - \frac{192}{\pi^5} \cdot \frac{b}{h} \cdot \sum_{n=1}^{\infty} \left[sin^2\left(\frac{n \cdot \pi}{2}\right) \cdot \frac{1}{n^5} \cdot th\left(\frac{n \cdot \pi \cdot h}{2 \cdot b}\right) \right] \right\}.$$

Curvilinear Cantilever Beam with Concentrated Shear Forces at Its Free End



***Objective*:** Check of the obtained values of the transverse displacements of the free end of a curvilinear cantilever beam subjected to concentrated shear forces.

Initial data files:

File name	Description
Curved_cantilever_beam_Shell_42.SPR	Design model with the elements of type 42
Curved_cantilever_beam_Shell_142.SPR	Design model with the elements of type 142
Curved_cantilever_beam_Shell_44.SPR	Design model with the elements of type 44
Curved_cantilever_beam_Shell_144.SPR	Design model with the elements of type 144
Curved_cantilever_beam_Shell_45.SPR	Design model with the elements of type 45
Curved_cantilever_beam_Shell_145.SPR	Design model with the elements of type 145
Curved_cantilever_beam_Shell_50.SPR	Design model with the elements of type 50
Curved_cantilever_beam_Shell_150.SPR	Design model with the elements of type 150
Curved_cantilever_beam_ Solid _36.SPR	Design model with the elements of type 36
Curved_cantilever_beam_ Solid _37.SPR	Design model with the elements of type 37

***Problem formulation*:** The curvilinear isotropic cantilever beam of a rectangular cross-section is subjected to the concentrated shear forces P_y, P_z (bending in and out of the plane of the longitudinal axis of the beam) applied at its free end. Check the obtained values of the transverse displacements Y, Z of the free end of the curvilinear cantilever beam from the respective actions.

References: R. H. Macneal, R. L. Harder, A proposed standard set of problems to test finite element accuracy, North-Holland, Finite elements in analysis and design, 1, 1985, p. 3-20.

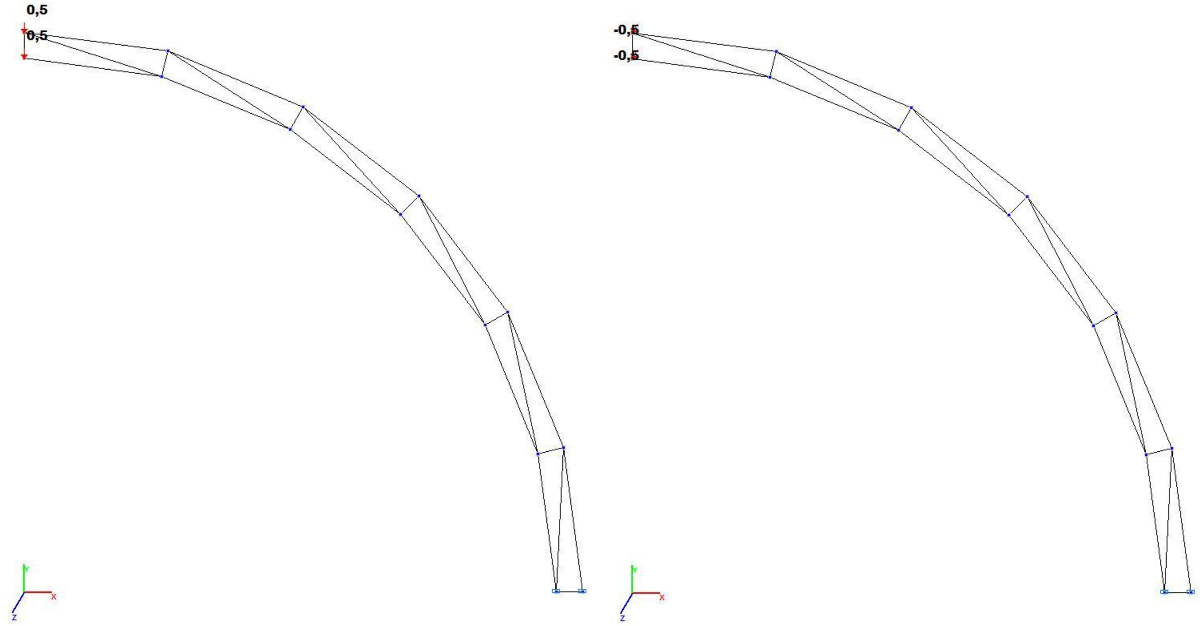
Initial data:

$E = 1.0 \cdot 10^7$ kPa - elastic modulus of the beam material;
$\nu = 0.25$ - Poisson’s ratio;
b = 0. 1 m - width of the beam;
h = 0. 2 m - height of the beam;
R = 4.22 m - radius of the arc of the longitudinal axis of the beam;
$\alpha = \pi/2$ rad - central angle of the arc of the longitudinal axis of the beam;
$P_y = 1.0$ kN - value of the shear force acting along the height of the beam (in the plane of the longitudinal axis);

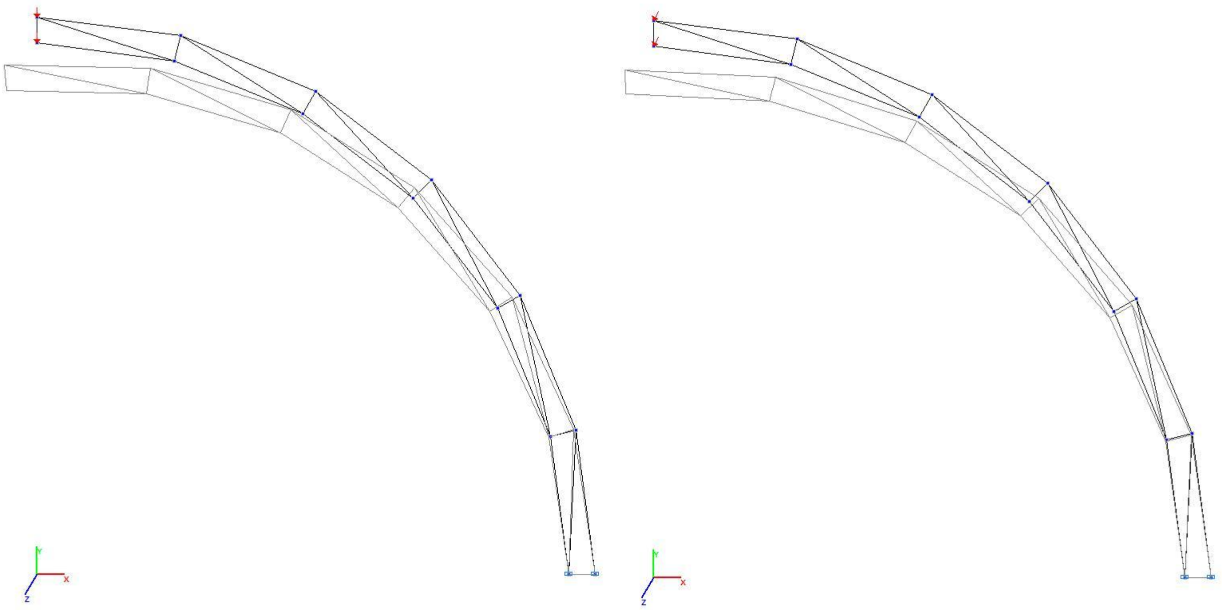
P_z = 1.0 kN - value of the shear force acting along the width of the beam (out of the plane of the longitudinal axis).

***Finite element model*:** Design model – general type system. Ten design models with a trapezoidal finite element mesh are considered:
Model 1 - 12 three-node shell elements of type 42. Boundary conditions are provided by imposing constraints on the nodes of the clamped end of the beam in the directions of the degrees of freedom X, Y, Z, UX, UY, UZ. The concentrated shear P_y, P_z forces are given in the form of two nodal forces (P_y = 2·0.5 kN, P_z = 2·0.5 kN). Number of nodes in the model – 14.
Model 2 - 12 three-node shell elements allowing for shear of type 142. Boundary conditions are provided by imposing constraints on the nodes of the clamped end of the beam in the directions of the degrees of freedom X, Y, Z, UX, UY, UZ. The concentrated shear P_y, P_z forces are given in the form of two nodal forces (P_y = 2·0.5 kN, P_z = 2·0.5 kN). Number of nodes in the model – 14.
Model 3 - 6 four-node shell elements of type 44. Boundary conditions are provided by imposing constraints on the nodes of the clamped end of the beam in the directions of the degrees of freedom X, Y, Z, UX, UY, UZ. The concentrated shear P_y, P_z forces are given in the form of two nodal forces (P_y = 2·0.5 kN, P_z = 2·0.5 kN).. Number of nodes in the model – 14.
Model 4 - 6 four-node shell elements allowing for shear of type 144. Boundary conditions are provided by imposing constraints on the nodes of the clamped end of the beam in the directions of the degrees of freedom X, Y, Z, UX, UY, UZ. The concentrated shear P_y, P_z forces are given in the form of two nodal forces (P_y = 2·0.5 kN, P_z = 2·0.5 kN). Number of nodes in the model – 14.
Model 5 - 12 six-node shell elements of type 45. Boundary conditions are provided by imposing constraints on the nodes of the clamped end of the beam in the directions of the degrees of freedom X, Y, Z, UX, UY, UZ. The concentrated shear P_y, P_z forces are given in the form of two nodal forces (P_y = 2·0.5 kN, P_z = 2·0.5 kN). Number of nodes in the model – 39.
Model 6 - 12 six-node shell elements allowing for shear of type 145. Boundary conditions are provided by imposing constraints on the nodes of the clamped end of the beam in the directions of the degrees of freedom X, Y, Z, UX, UY, UZ. The concentrated shear P_y, P_z forces are given in the form of two nodal forces (P_y = 2·0.5 kN, P_z = 2·0.5 kN). Number of nodes in the model – 39.
Model 7 - 6 eight-node shell elements of type 50. Boundary conditions are provided by imposing constraints on the nodes of the clamped end of the beam in the directions of the degrees of freedom X, Y, Z, UX, UY, UZ. The concentrated shear P_y, P_z forces are given in the form of two nodal forces (P_y = 2·0.5 kN, P_z = 2·0.5 kN). Number of nodes in the model – 33.
Model 8 - 6 eight-node shell elements allowing for shear of type 150. Boundary conditions are provided by imposing constraints on the nodes of the clamped end of the beam in the directions of the degrees of freedom X, Y, Z, UX, UY, UZ. The concentrated shear P_y, P_z forces are given in the form of two nodal forces (P_y = 2·0.5 kN, P_z = 2·0.5 kN). Number of nodes in the model – 33.
Model 9 - 6 eight-node isoparametric solid elements of type 36. Boundary conditions are provided by imposing constraints on the nodes of the clamped end of the beam in the directions of the degrees of freedom X, Y, Z, UX, UY, UZ. The concentrated shear P_y, P_z forces are given in the form of four nodal forces (P_y = 4·0.25 kN, P_z = 4·0.25 kN). Number of nodes in the model – 28.
Model 10 - 6 twenty-node isoparametric solid elements of type 37. Boundary conditions are provided by imposing constraints on the nodes of the clamped end of the beam in the directions of the degrees of freedom X, Y, Z, UX, UY, UZ. The concentrated shear P_y, P_z forces are given in the form of four nodal forces (P_y = 4·0.25 kN, P_z = 4·0.25 kN). Number of nodes in the model – 80.

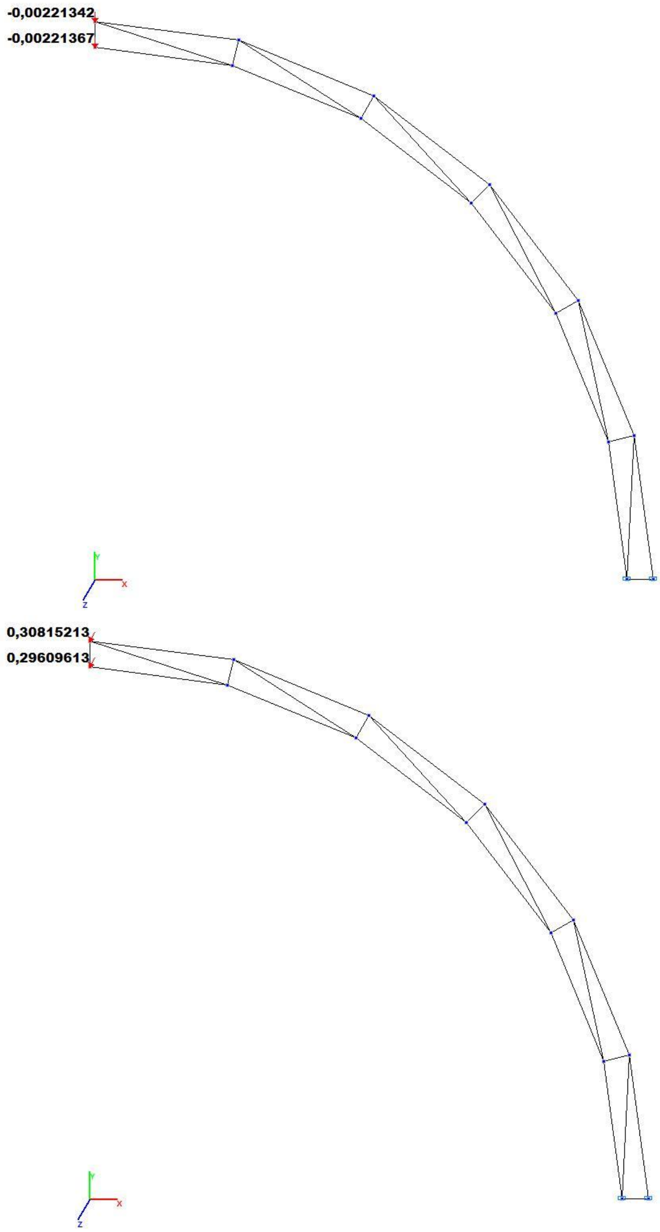
Results in SCAD



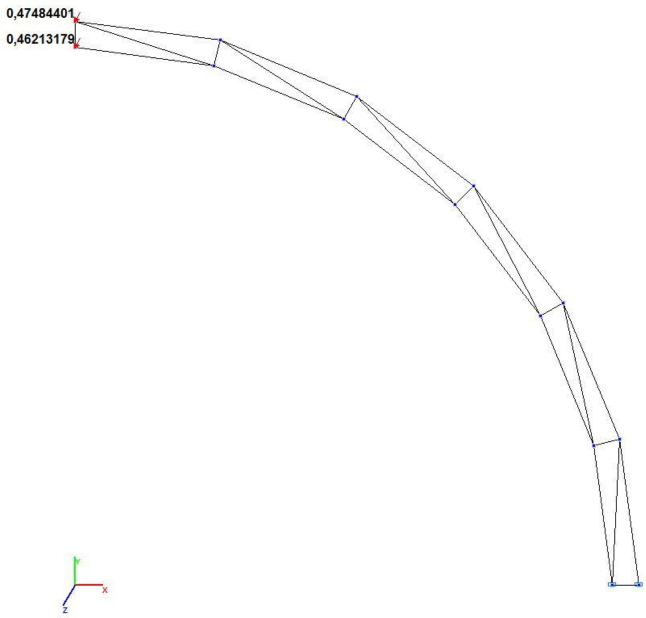
Models 1 and 2. Design model



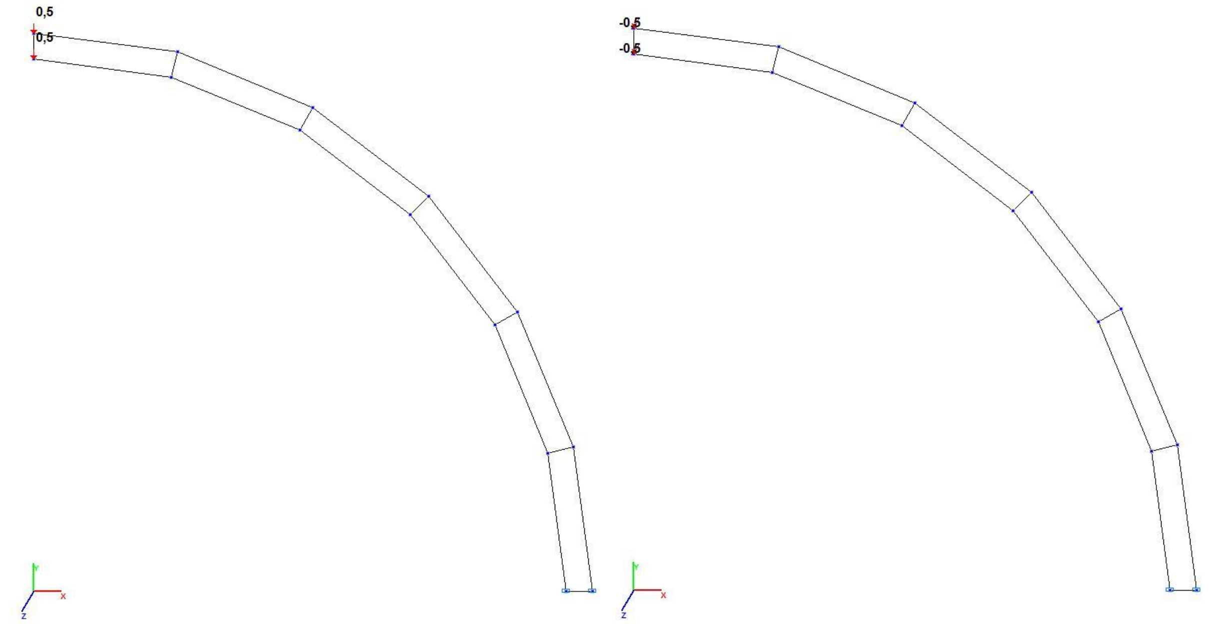
Models 1 and 2. Deformed model



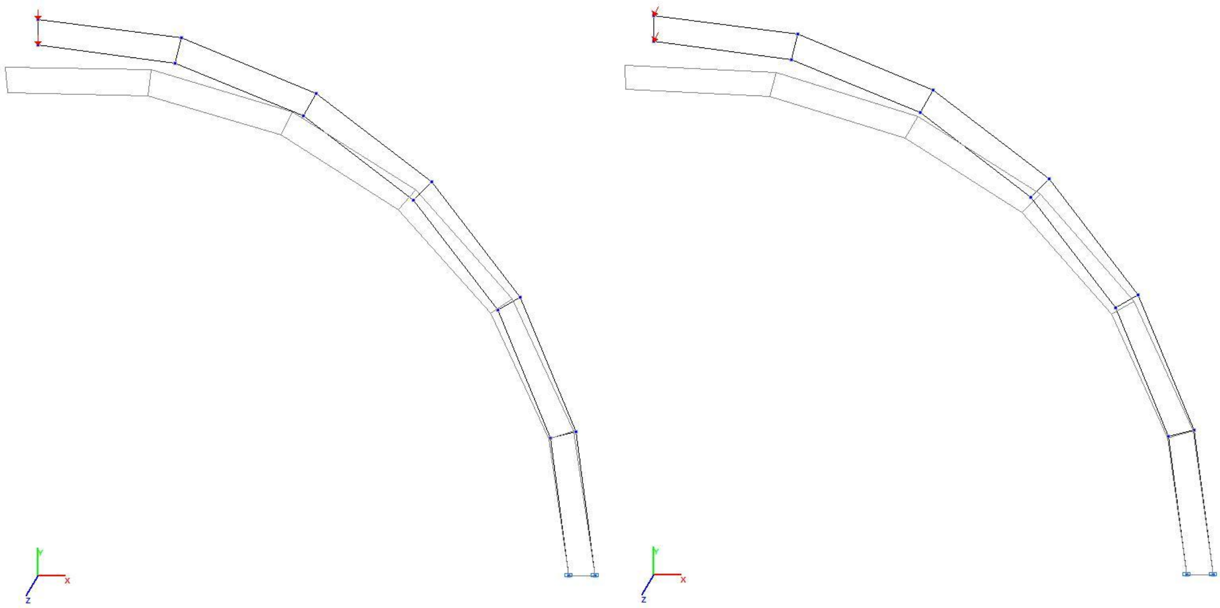
Model 1. Values of the transverse displacements Y, Z of the free end of the curvilinear cantilever beam (m, m)



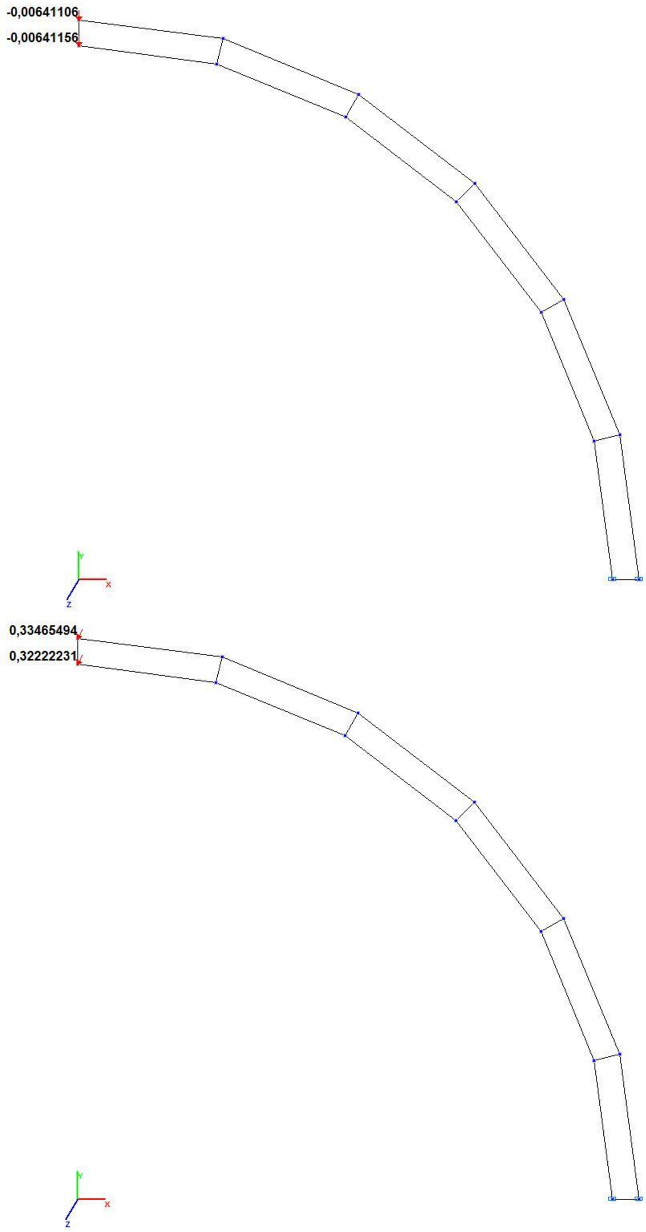
Model 2. Values of the transverse displacements Y, Z
of the free end of the curvilinear cantilever beam (m, m)



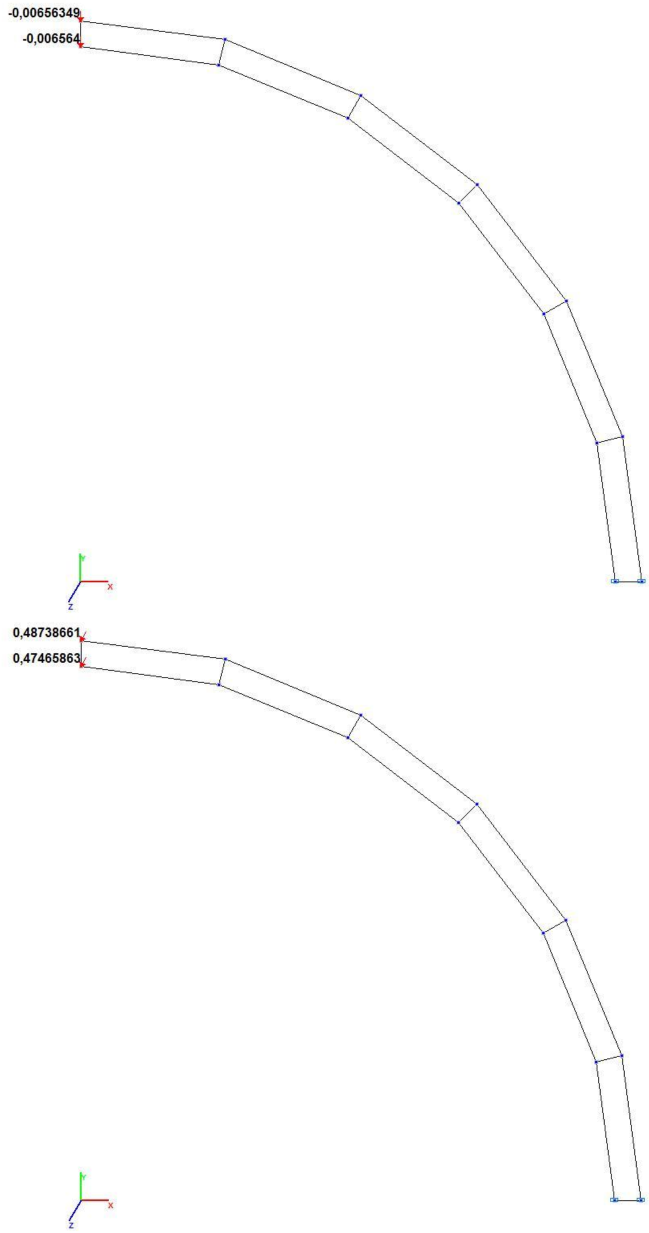
Models 3 and 4. Design model



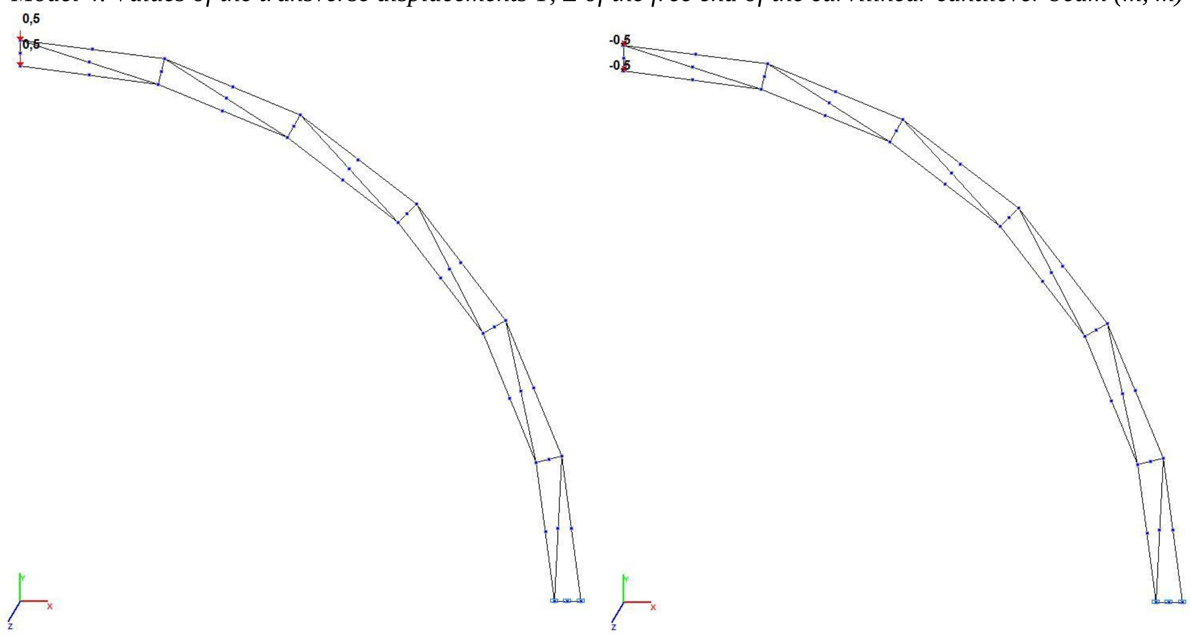
Models 3 and 4. Deformed model



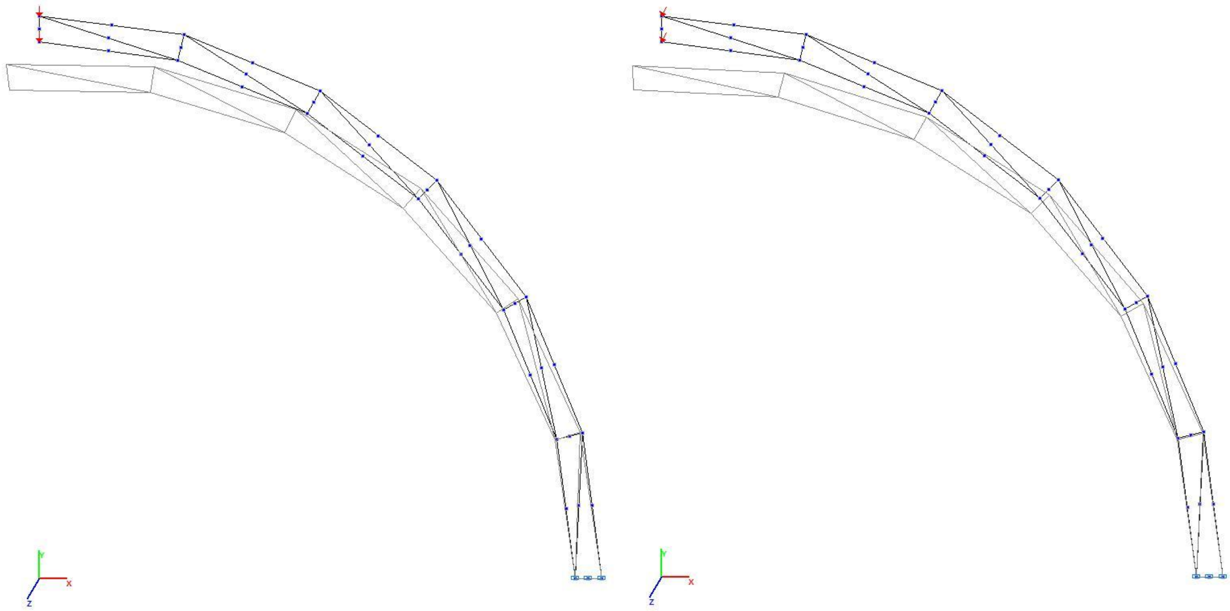
Model 3. Values of the transverse displacements Y, Z of the free end of the curvilinear cantilever beam (m, m)



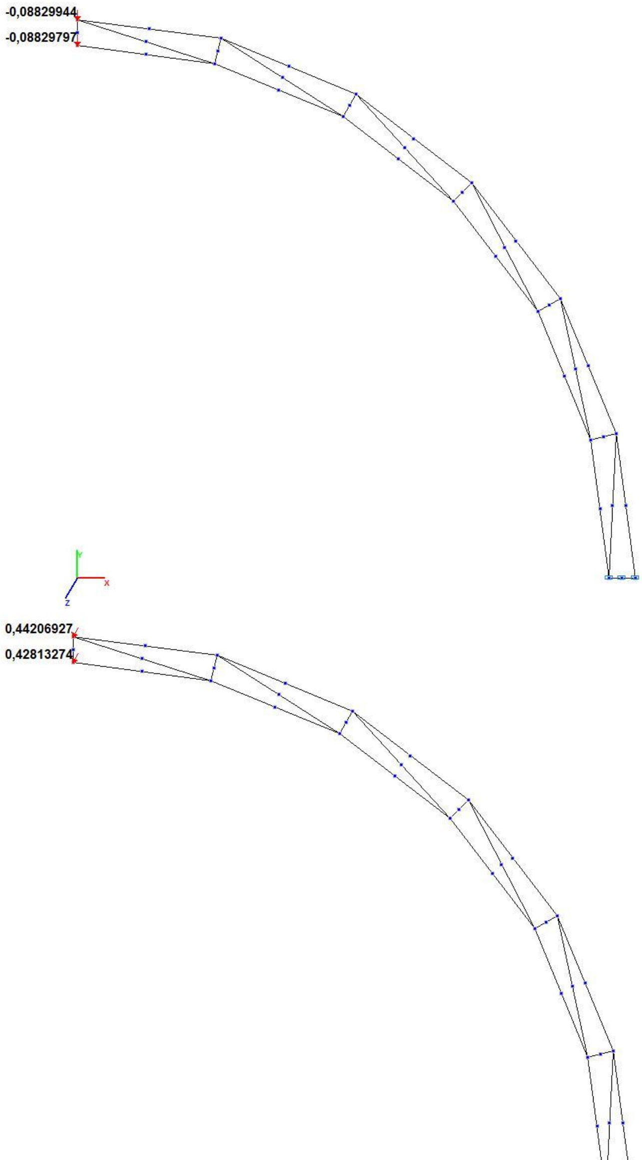
Model 4. Values of the transverse displacements Y, Z of the free end of the curvilinear cantilever beam (m, m)



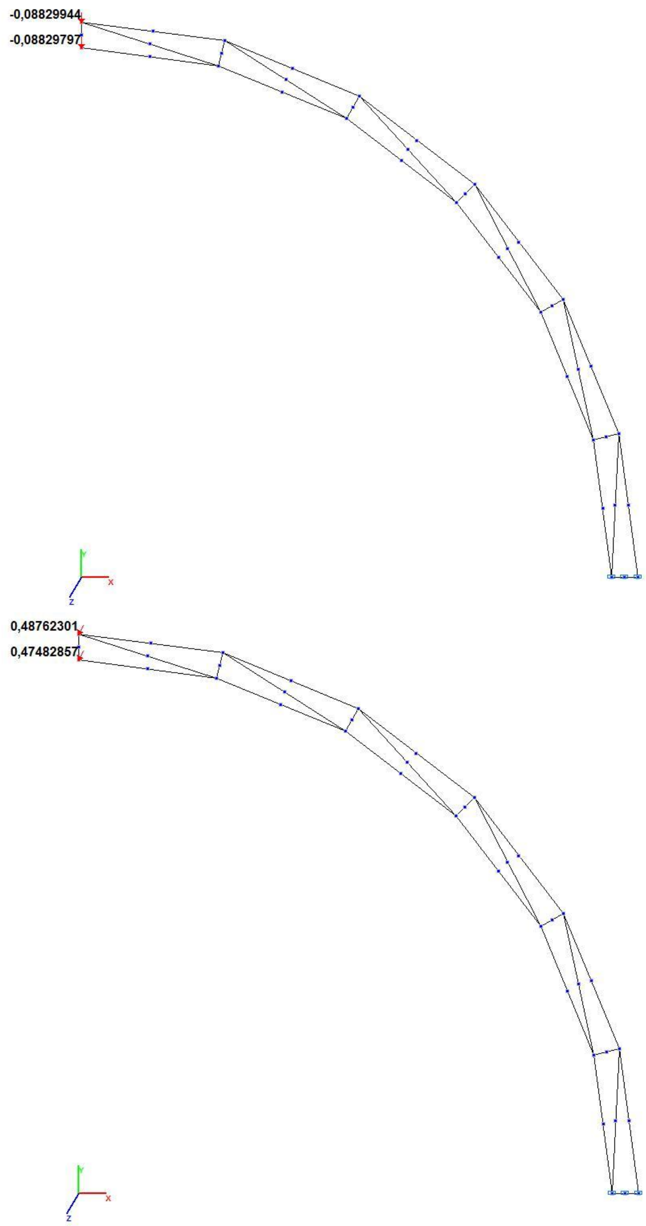
Models 5 and 6. Design model



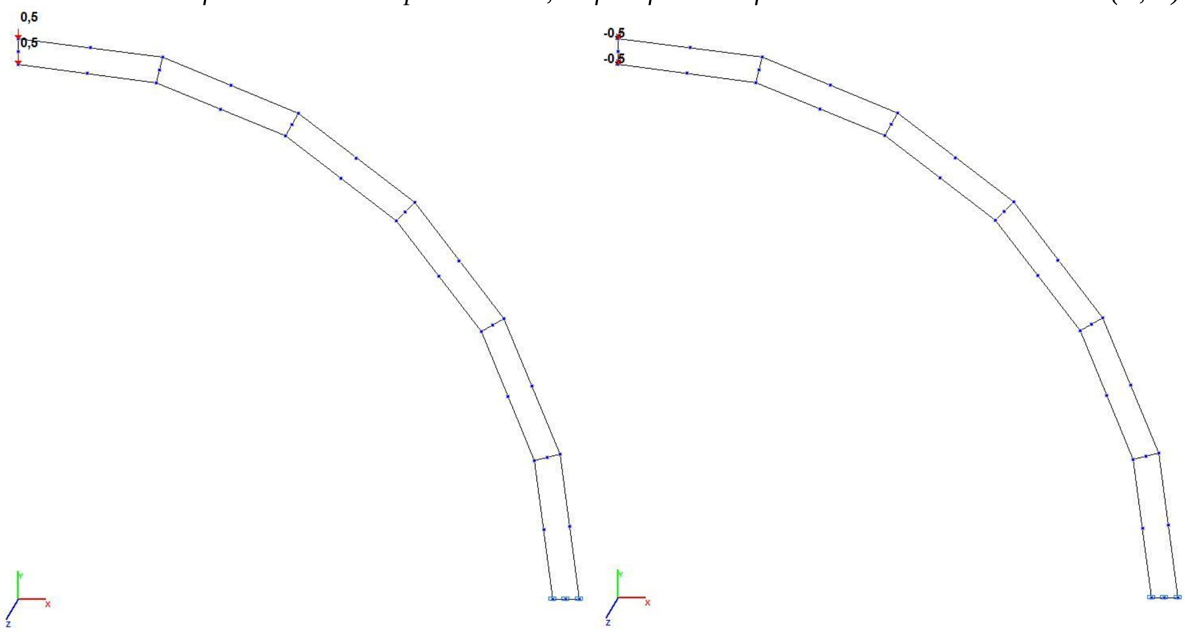
Models 5 and 6. Deformed model



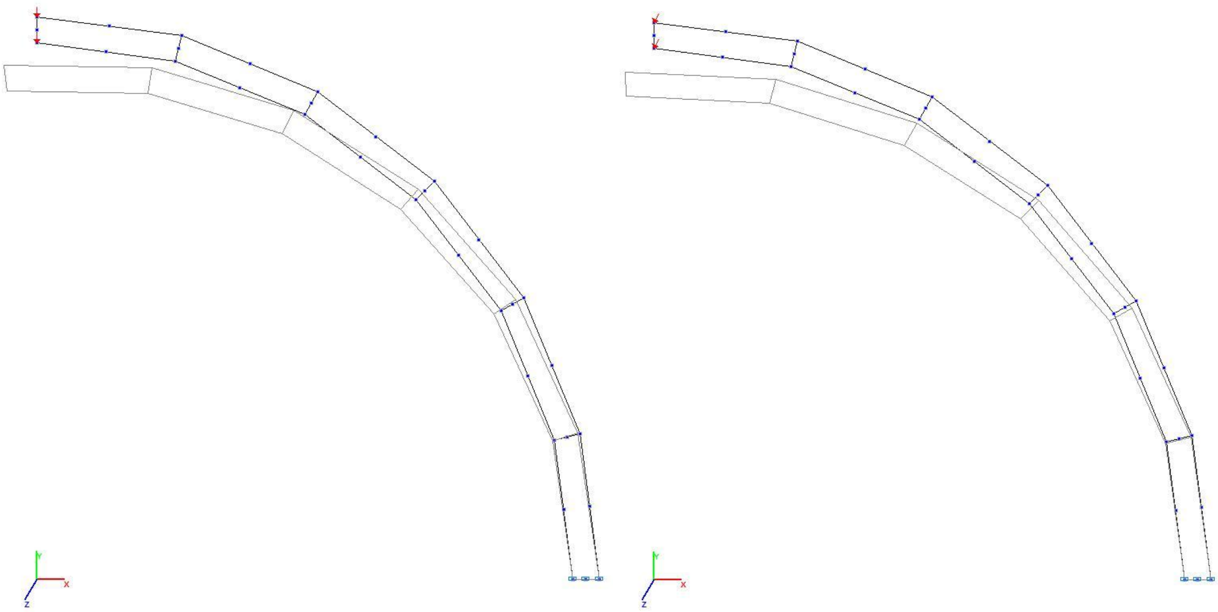
Model 5. Values of the transverse displacements Y, Z of the free end of the curvilinear cantilever beam (m, m)



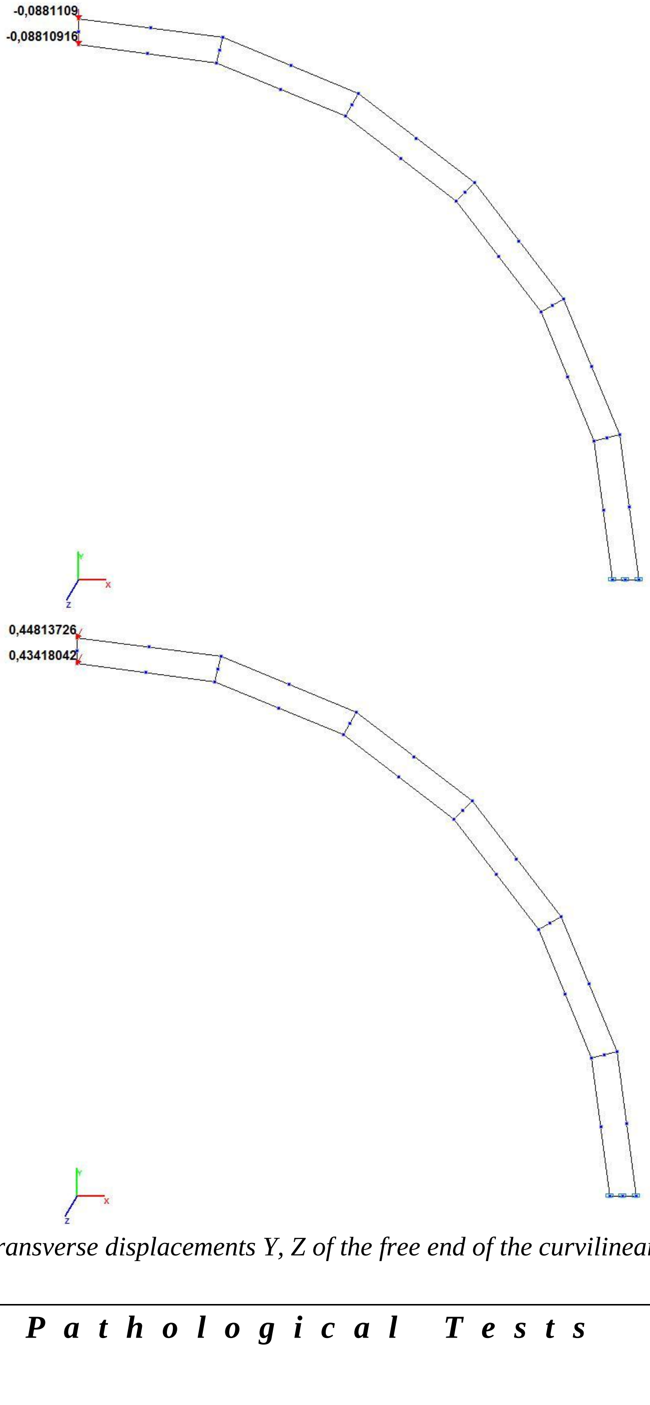
Model 6. Values of the transverse displacements Y, Z of the free end of the curvilinear cantilever beam (m, m)



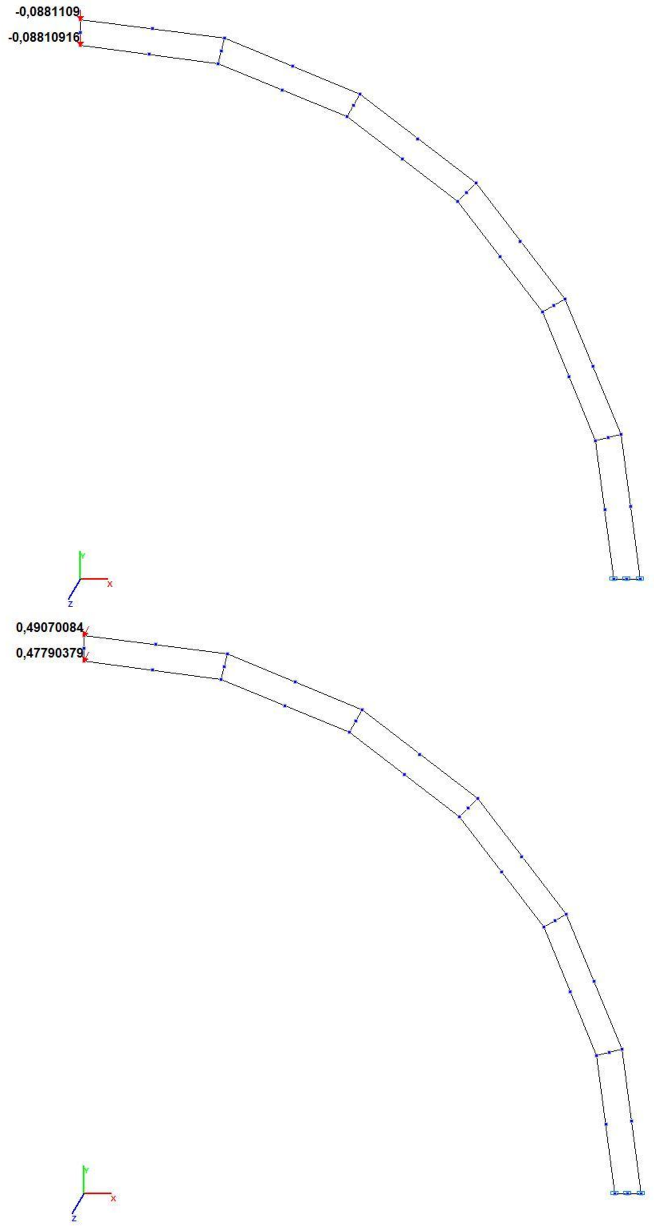
Models 7 and 8. Design model



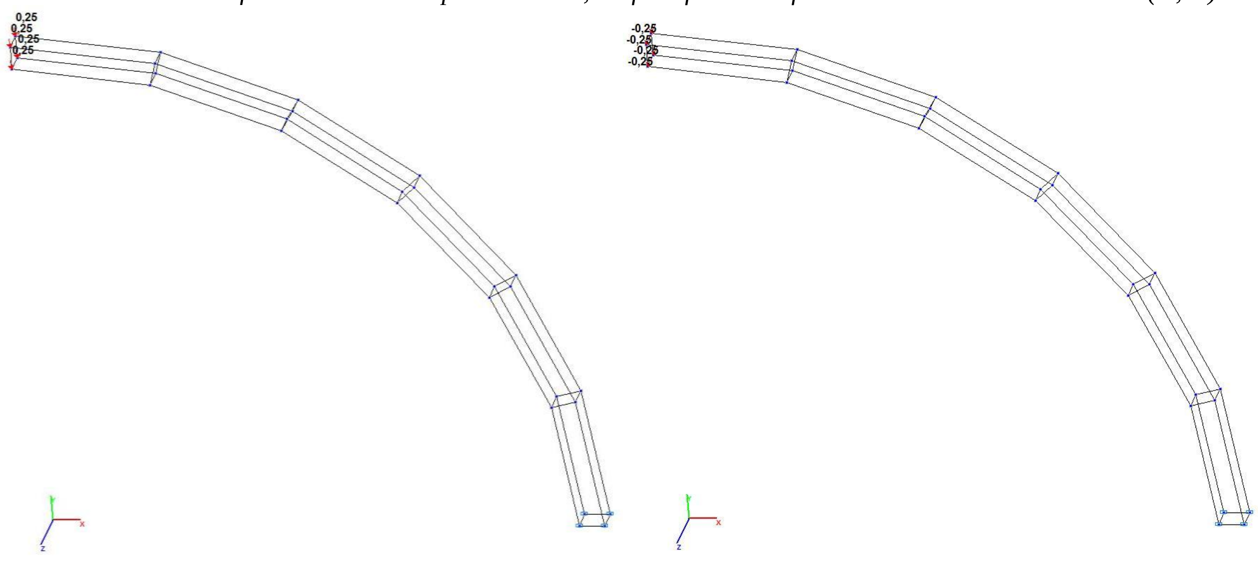
Models 7 and 8. Deformed model



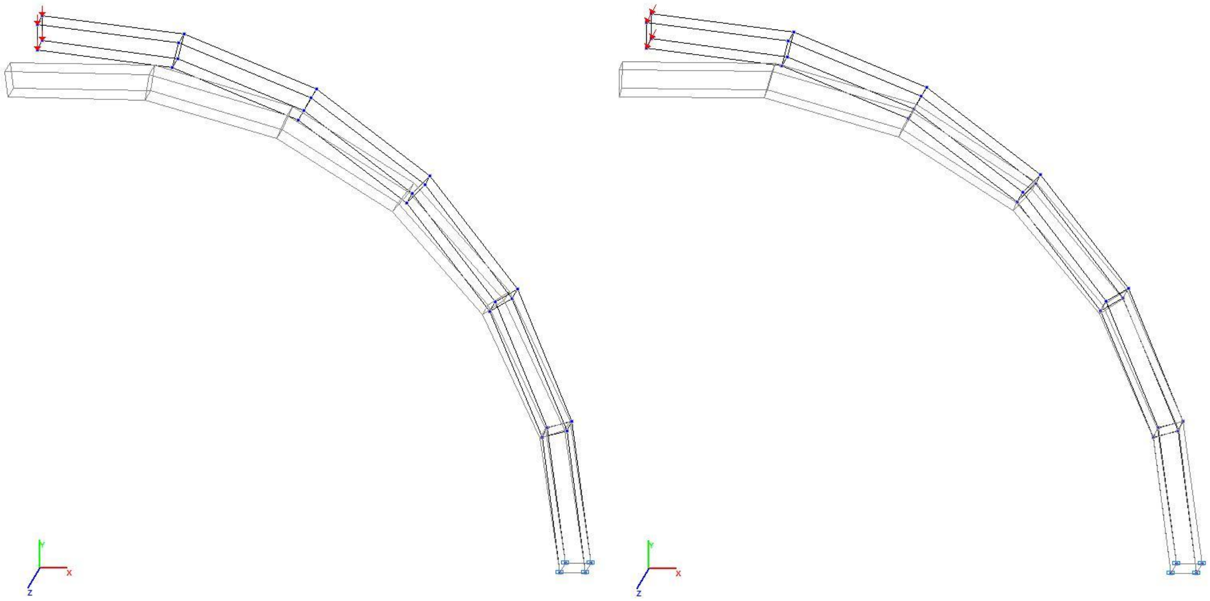
Model 7. Values of the transverse displacements Y, Z of the free end of the curvilinear cantilever beam (m, m)



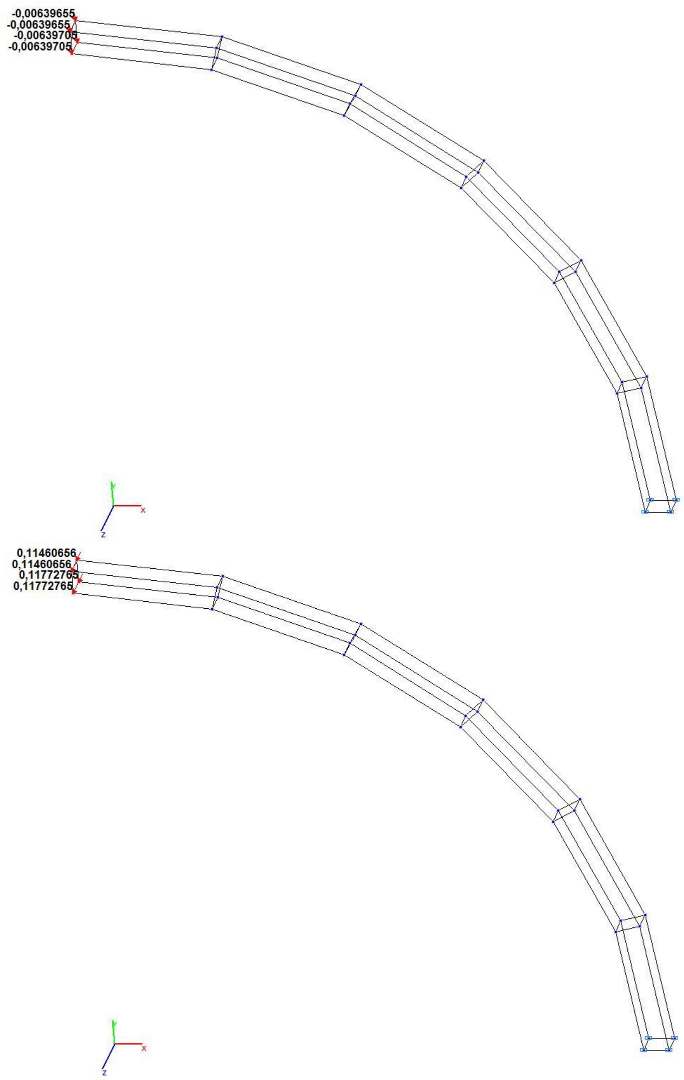
Model 8. Values of the transverse displacements Y, Z of the free end of the curvilinear cantilever beam (m, m)



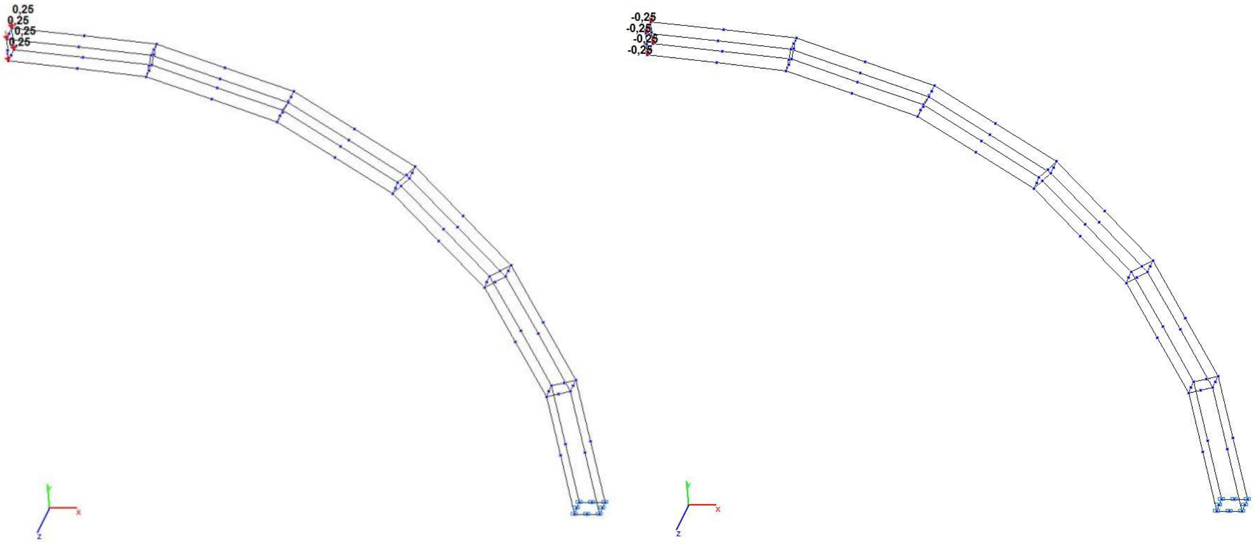
Model 9. Design model



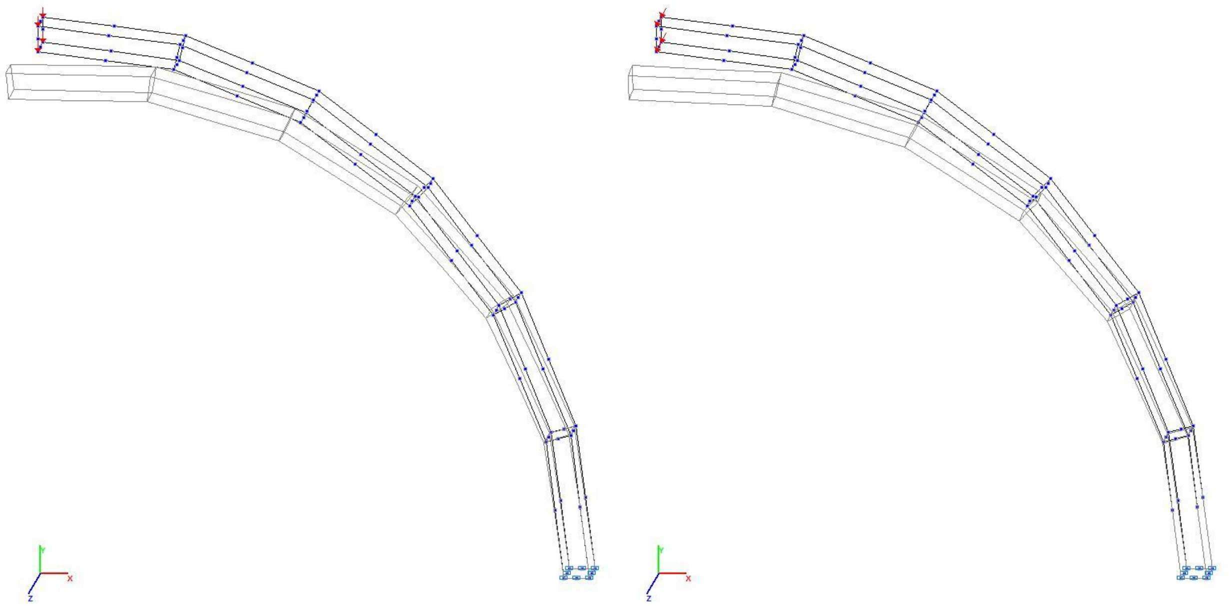
Model 9. Deformed model



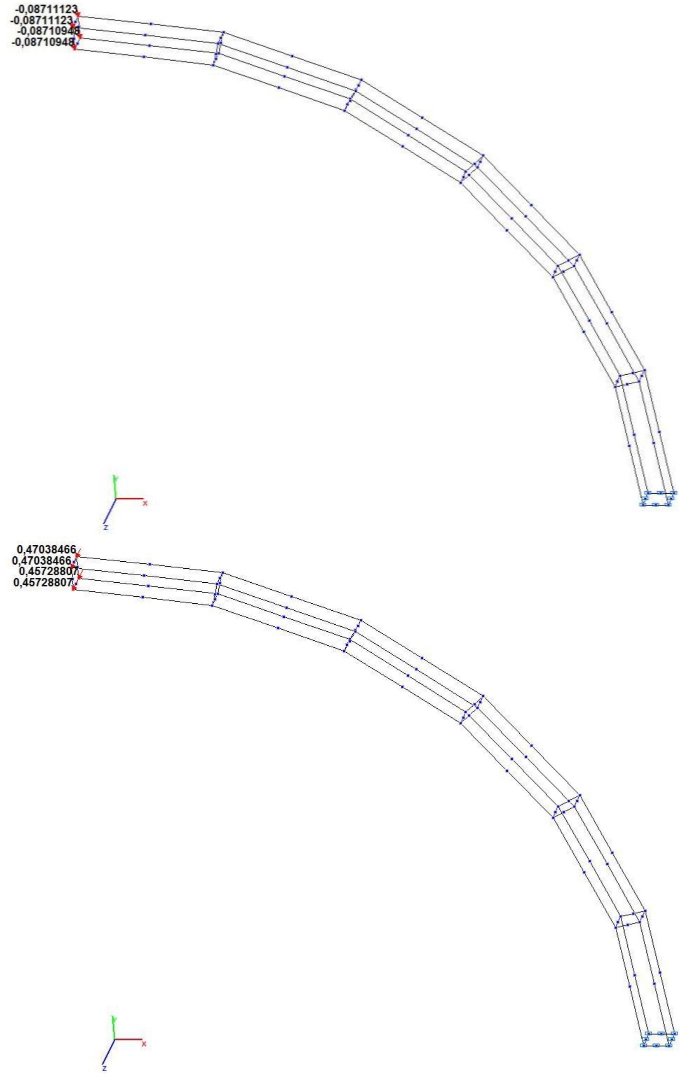
Model 9. Values of the transverse displacements Y, Z of the free end of the curvilinear cantilever beam (m, m)



Model 10. Design model



Model 10. Deformed model



Model 10. Values of the transverse displacements Y, Z of the free end of the curvilinear cantilever beam (m, m)

Comparison of solutions:

Model	Parameter	Theory	SCAD	Deviation, %
1 (Member type 42)	Transverse displacement Y of the free end of the cantilever beam, m	0.088536	0.002213	97.50
	Transverse displacement Z of the free end of the cantilever beam, m	0.454527*	0.308152	32.20
2 (Member type 142)	Transverse displacement Y of the free end of the cantilever beam, m	0.088536	0.02213	97.50
	Transverse displacement Z of the free end of the cantilever beam, m	0.500466	0.474844	5.12
3 (Member type 44)	Transverse displacement Y of the free end of the cantilever beam, m	0.088536	0.006411	92.76
	Transverse displacement Z of the free end of the cantilever beam, m	0.454527*	0.334655	26.37
4 (Member type 144)	Transverse displacement Y of the free end of the cantilever beam, m	0.088536	0.006563	92.59
	Transverse displacement Z	0.500466	0.487387	2.61

Model	Parameter	Theory	SCAD	Deviation, %
	of the free end of the cantilever beam, m			
5 (Member type 45)	Transverse displacement Y of the free end of the cantilever beam, m	0.088536	0.088299	0.27
	Transverse displacement Z of the free end of the cantilever beam, m	0.454527*	0.442069	2.74
6 (Member type 145)	Transverse displacement Y of the free end of the cantilever beam, m	0.088536	0.088299	0.27
	Transverse displacement Z of the free end of the cantilever beam, m	0.500466	0.487623	2.57
7 (Member type 50)	Transverse displacement Y of the free end of the cantilever beam, m	0.088536	0.088111	0.48
	Transverse displacement Z of the free end of the cantilever beam, m	0.454527*	0.448137	1.41
8 (Member type 150)	Transverse displacement Y of the free end of the cantilever beam, m	0.088536	0.088111	0.48
	Transverse displacement Z of the free end of the cantilever beam, m	0.500466	0.490701	1.95
9 (Member type 36)	Transverse displacement Y of the free end of the cantilever beam, m	0.088536	0.006397	92.77
	Transverse displacement Z of the free end of the cantilever beam, m	0.500466	0.114607	77.10
10 (Member type 37)	Transverse displacement Y of the free end of the cantilever beam, m	0.088536	0.087111	1.61
	Transverse displacement Z of the free end of the cantilever beam, m	0.500466	0.470384	6.01

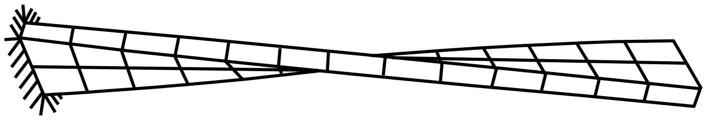
* The values of the transverse displacements Z for thin plates (not allowing for shear) are determined at the free torsional inertia moment calculated with the value of the coefficient k_f, equal to 1/3 (h/b = ∞).

Notes: In the analytical solution the values of the transverse displacements Y, Z of the free end of the curvilinear cantilever beam from the respective actions are determined according to the following formulas:

$$Y = \frac{3 \cdot \pi \cdot P_y \cdot R^3}{E \cdot b \cdot h^3}; \qquad Z = \frac{P_z \cdot R^3}{2 \cdot E \cdot b^3 \cdot h \cdot k_f} \cdot \left[6 \cdot \pi \cdot k_f + (3 \cdot \pi - 8) \cdot (1 + \nu)\right],$$

where: $$k_f = \frac{1}{3} \cdot \left\{1 - \frac{192}{\pi^5} \cdot \frac{b}{h} \cdot \sum_{n=1}^{\infty} \left[sin^2\left(\frac{n \cdot \pi}{2}\right) \cdot \frac{1}{n^5} \cdot th\left(\frac{n \cdot \pi \cdot h}{2 \cdot b}\right)\right]\right\}.$$

Twisted Cantilever Beam with Concentrated Shear Forces at Its Free End



Objective: Check of the obtained values of the transverse displacements of the free end of a twisted cantilever beam subjected to concentrated shear forces.

Initial data files:

File name	Description
Twisted_cantilever_beam_Shell_42.SPR	Design model with the elements of type 42
Twisted_cantilever_beam_Shell_142.SPR	Design model with the elements of type 142
Twisted_cantilever_beam_Shell_44.SPR	Design model with the elements of type 44
Twisted_cantilever_beam_Shell_144.SPR	Design model with the elements of type 144
Twisted_cantilever_beam_Shell_45.SPR	Design model with the elements of type 45
Twisted_cantilever_beam_Shell_145.SPR	Design model with the elements of type 145
Twisted_cantilever_beam_Shell_50.SPR	Design model with the elements of type 50
Twisted_cantilever_beam_Shell_150.SPR	Design model with the elements of type 150
Twisted_cantilever_beam_ Solid _36.SPR	Design model with the elements of type 36
Twisted_cantilever_beam_ Solid _37.SPR	Design model with the elements of type 37

***Problem formulation*:** The isotropic cantilever beam of a rectangular cross-section twisted along the longitudinal axis is subjected to the concentrated shear P_y, P_z forces (bending in and out of the plane of the beam height at the free end). Check the obtained values of the transverse displacements Y, Z of the free end of the twisted cantilever beam from the respective actions.

References: R. H. Macneal, R. L. Harder, A proposed standard set of problems to test finite element accuracy, North-Holland, Finite elements in analysis and design, 1, 1985, p. 3-20.

Initial data:

$E = 2.9 \cdot 10^7$ kPa	- elastic modulus of the beam material;
$\nu = 0.22$	- Poisson's ratio;
b = 0. 32 m	- width of the beam;
h = 1. 10 m	- height of the beam;
L = 12.0 m	- length of the longitudinal axis of the beam;
$\alpha = \pi/2$ rad	- twist angle of the longitudinal axis of the beam;
$P_y = 1.0$ kN	- value of the shear force acting along the height of the beam at the free end;
$P_z = 1.0$ kN	- value of the shear force acting along the width of the beam at the free end.

Finite element model: Design model – general type system. Ten design models with a regular finite element mesh 12x2 are considered:

Model 1 - 48 three-node shell elements of type 42. Boundary conditions are provided by imposing constraints on the nodes of the clamped end of the beam in the directions of the degrees of freedom X, Y, Z, UX, UY, UZ. The concentrated shear P_y, P_z forces are given in the form of two nodal forces ($P_y = 2 \cdot 0.5$ kN, $P_z = 2 \cdot 0.5$ kN). Number of nodes in the model – 39.

Model 2 - 48 three-node shell elements allowing for shear of type 142. Boundary conditions are provided by imposing constraints on the nodes of the clamped end of the beam in the directions of the degrees of freedom X, Y, Z, UX, UY, UZ. The concentrated shear P_y, P_z forces are given in the form of two nodal forces ($P_y = 2 \cdot 0.5$ kN, $P_z = 2 \cdot 0.5$ kN). Number of nodes in the model – 39.

Model 3 - 24 four-node shell elements of type 44. Boundary conditions are provided by imposing constraints on the nodes of the clamped end of the beam in the directions of the degrees of freedom X, Y, Z, UX, UY, UZ. The concentrated shear P_y, P_z forces are given in the form of two nodal forces ($P_y = 2 \cdot 0.5$ kN, $P_z = 2 \cdot 0.5$ kN). Number of nodes in the model – 39.

Model 4 - 24 four-node shell elements allowing for shear of type 144. Boundary conditions are provided by imposing constraints on the nodes of the clamped end of the beam in the directions of the degrees of freedom X, Y, Z, UX, UY, UZ. The concentrated shear P_y, P_z forces are given in the form of two nodal forces ($P_y = 2·0.5$ kN, $P_z = 2·0.5$ kN). Number of nodes in the model – 39.

Model 5 - 48 six-node shell elements of type 45. Boundary conditions are provided by imposing constraints on the nodes of the clamped end of the beam in the directions of the degrees of freedom X, Y, Z, UX, UY, UZ. The concentrated shear P_y, P_z forces are given in the form of two nodal forces ($P_y = 2·0.5$ kN, $P_z = 2·0.5$ kN). Number of nodes in the model – 125.

Model 6 - 48 six-node shell elements allowing for shear of type 145. Boundary conditions are provided by imposing constraints on the nodes of the clamped end of the beam in the directions of the degrees of freedom X, Y, Z, UX, UY, UZ. The concentrated shear P_y, P_z forces are given in the form of two nodal forces ($P_y = 2·0.5$ kN, $P_z = 2·0.5$ kN). Number of nodes in the model – 125.

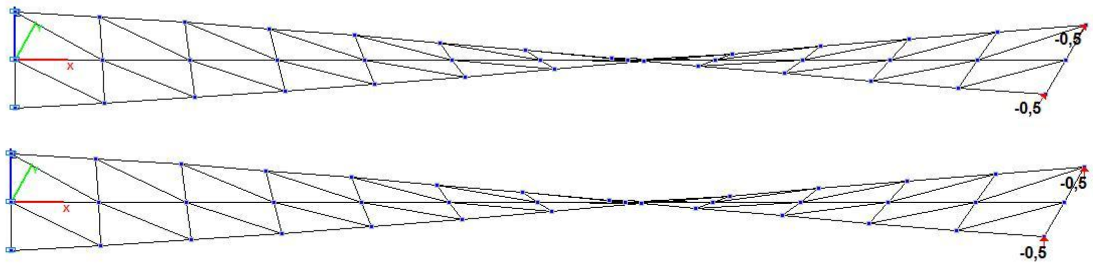
Model 7 - 24 eight-node shell elements of type 50. Boundary conditions are provided by imposing constraints on the nodes of the clamped end of the beam in the directions of the degrees of freedom X, Y, Z, UX, UY, UZ. The concentrated shear P_y, P_z forces are given in the form of two nodal forces ($P_y = 2·0.5$ kN, $P_z = 2·0.5$ kN). Number of nodes in the model – 101.

Model 8 - 24 eight-node shell elements allowing for shear of type 150. Boundary conditions are provided by imposing constraints on the nodes of the clamped end of the beam in the directions of the degrees of freedom X, Y, Z, UX, UY, UZ. The concentrated shear P_y, P_z forces are given in the form of two nodal forces ($P_y = 2·0.5$ kN, $P_z = 2·0.5$ kN). Number of nodes in the model – 101.

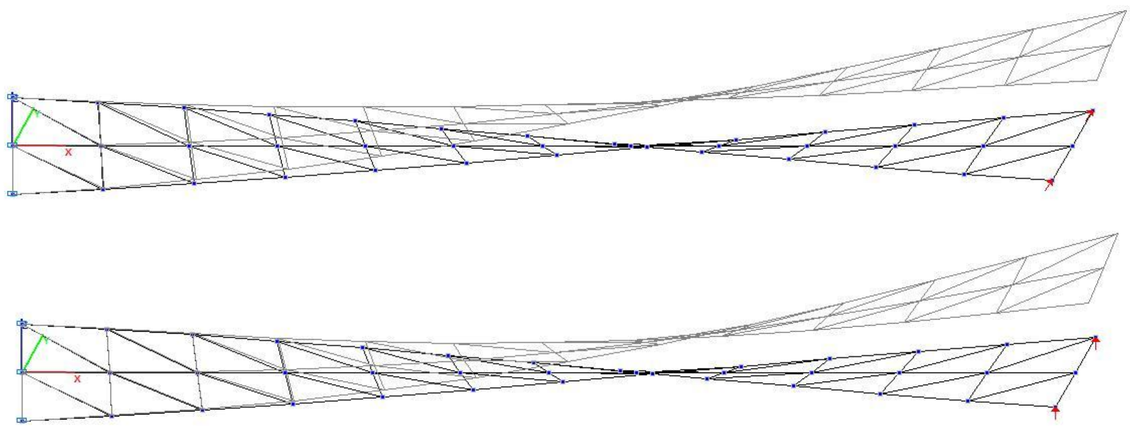
Model 9 - 24 eight-node isoparametric solid elements of type 36. Boundary conditions are provided by imposing constraints on the nodes of the clamped end of the beam in the directions of the degrees of freedom X, Y, Z, UX, UY, UZ. The concentrated shear P_y, P_z forces are given in the form of four nodal forces ($P_y = 4·0.25$ kN, $P_z = 4·0.25$ kN). Number of nodes in the model – 78.

Model 10 - 24 twenty-node isoparametric solid elements of type 37. Boundary conditions are provided by imposing constraints on the nodes of the clamped end of the beam in the directions of the degrees of freedom X, Y, Z, UX, UY, UZ. The concentrated shear P_y, P_z forces are given in the form of four nodal forces ($P_y = 4·0.25$ kN, $P_z = 4·0.25$ kN). Number of nodes in the model – 241.

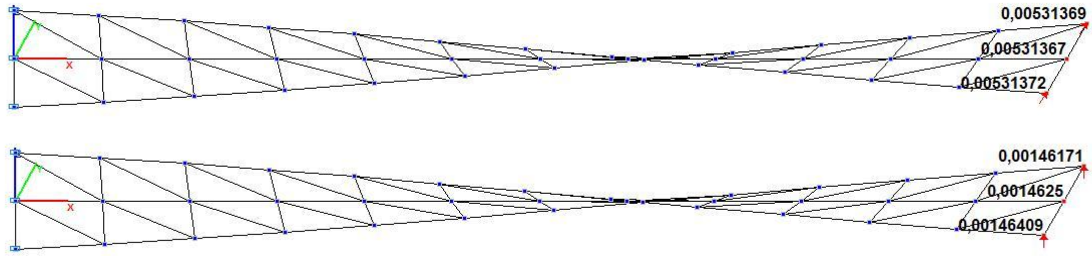
Results in SCAD



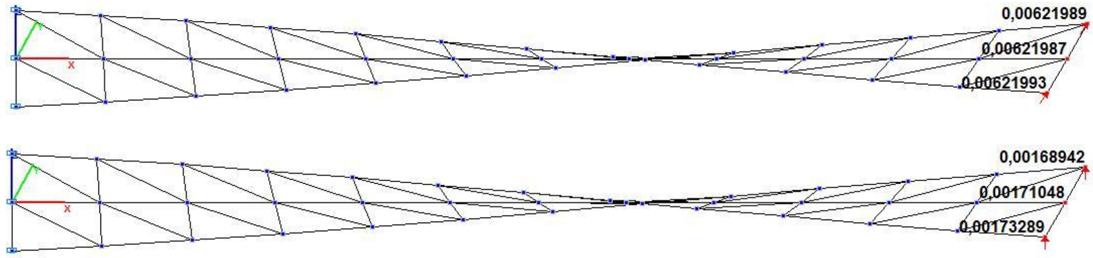
Models 1 and 2. Design model



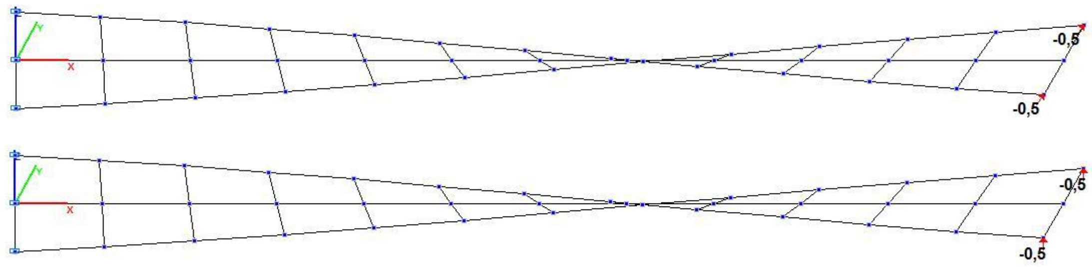
Models 1 and 2. Deformed model



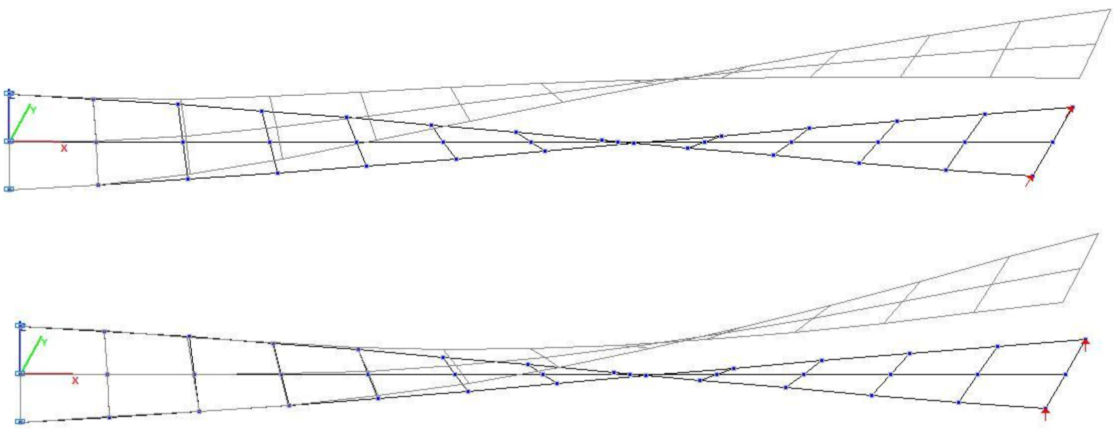
Model 1. Values of the transverse displacements Y, Z of the free end of the twisted cantilever beam (m, m)



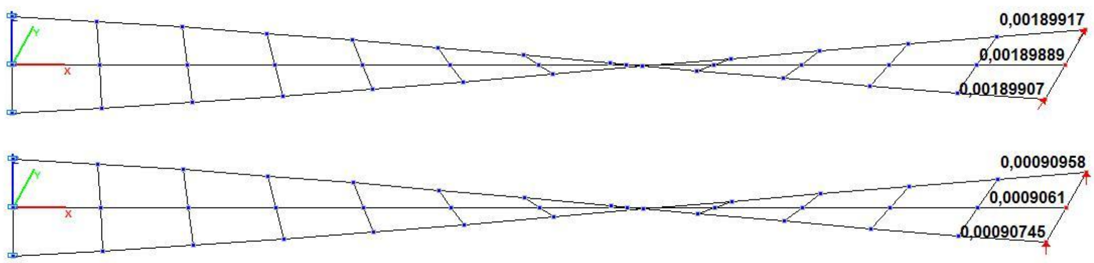
Model 2. Values of the transverse displacements Y, Z of the free end of the twisted cantilever beam (m, m)



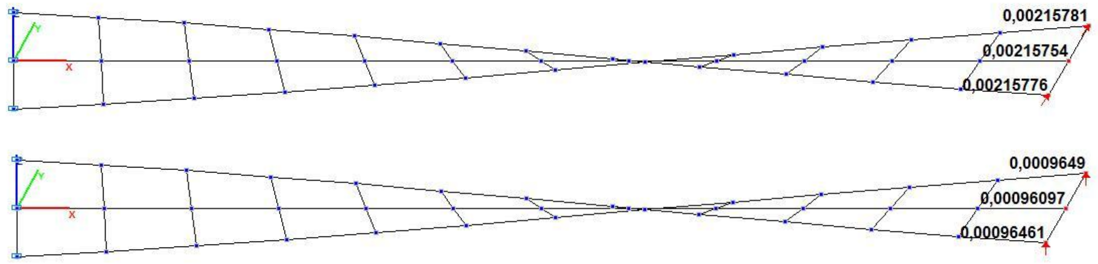
Models 3 and 4. Design model



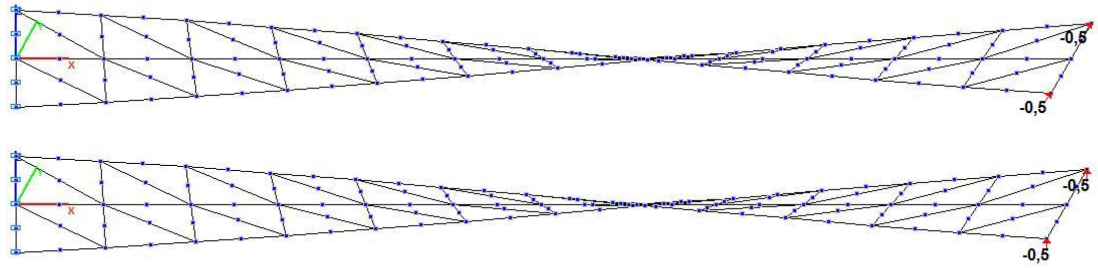
Models 3 and 4. Deformed model



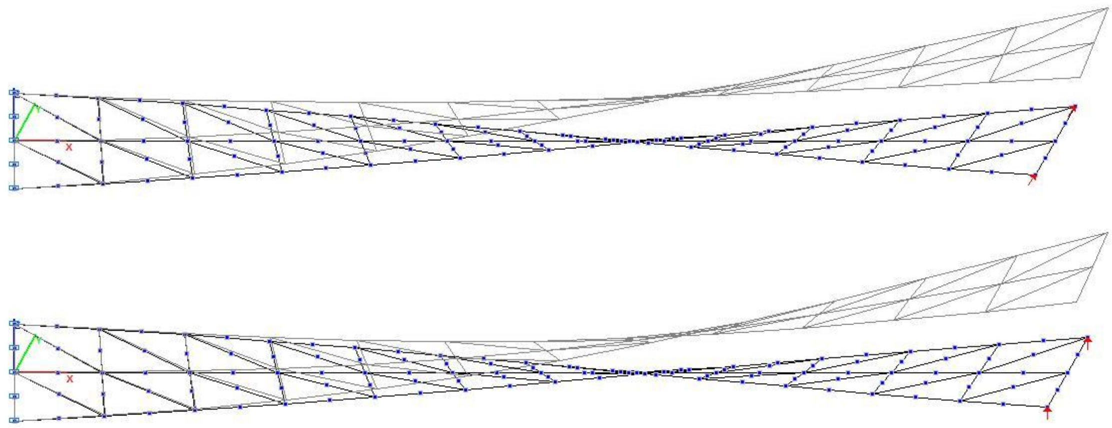
Model 3. Values of the transverse displacements Y, Z of the free end of the twisted cantilever beam (m, m)



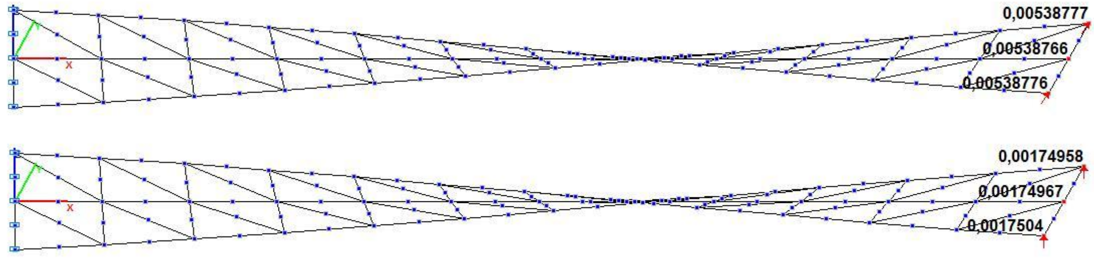
Model 4. Values of the transverse displacements Y, Z of the free end of the twisted cantilever beam (m, m)



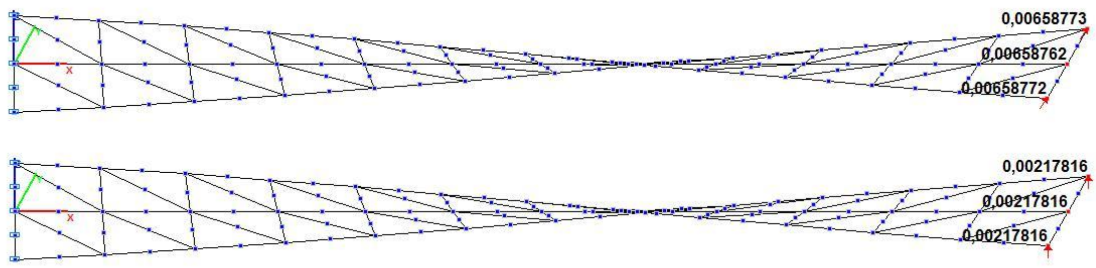
Models 5 and 6. Design model



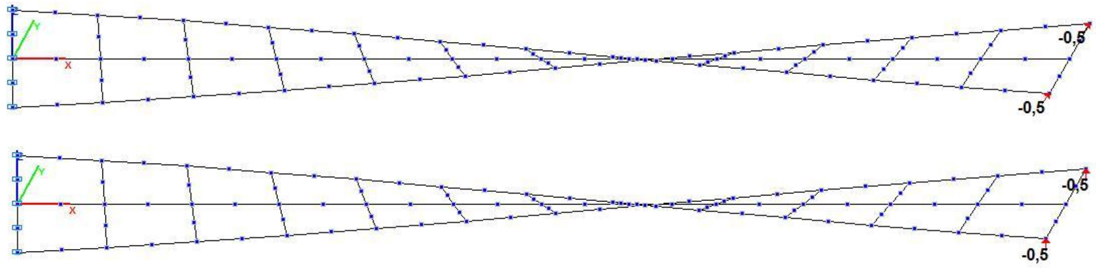
Models 5 and 6. Deformed model



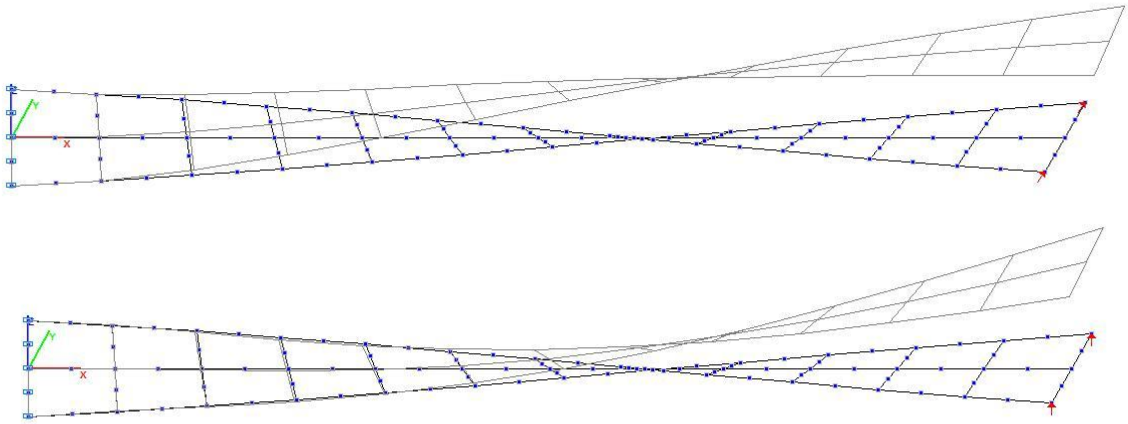
Model 5. Values of the transverse displacements Y, Z of the free end of the twisted cantilever beam (m, m)



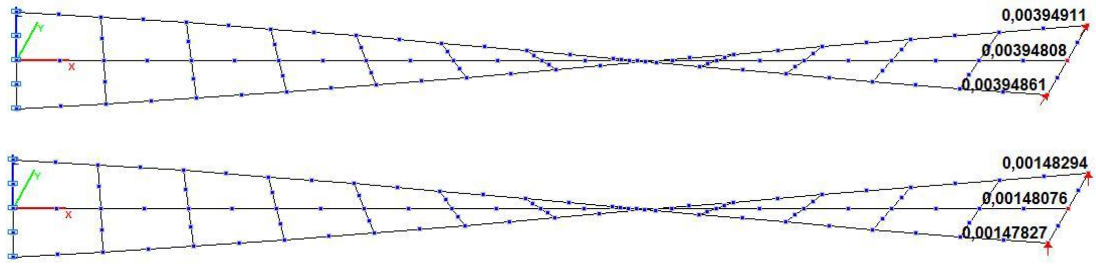
Model 6. Values of the transverse displacements Y, Z of the free end of the twisted cantilever beam (m, m)



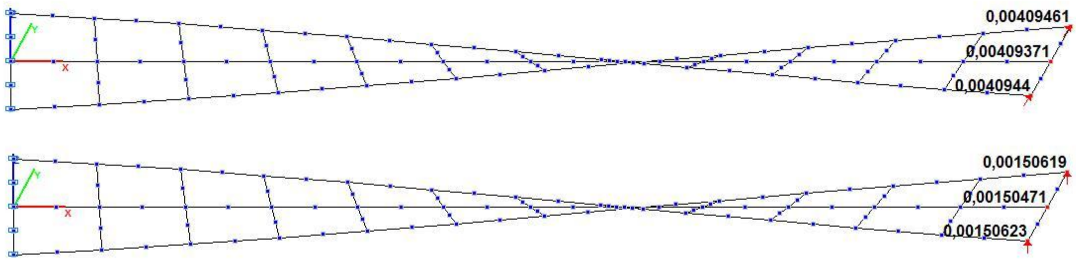
Models 7 and 8. Design model



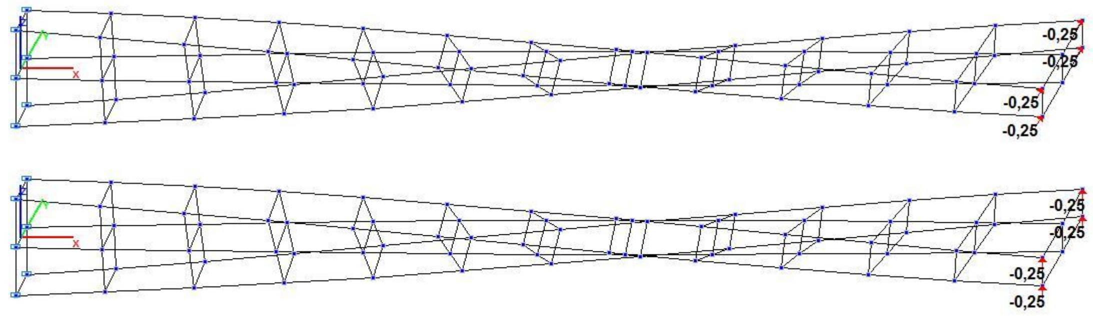
Models 7 and 8. Deformed model



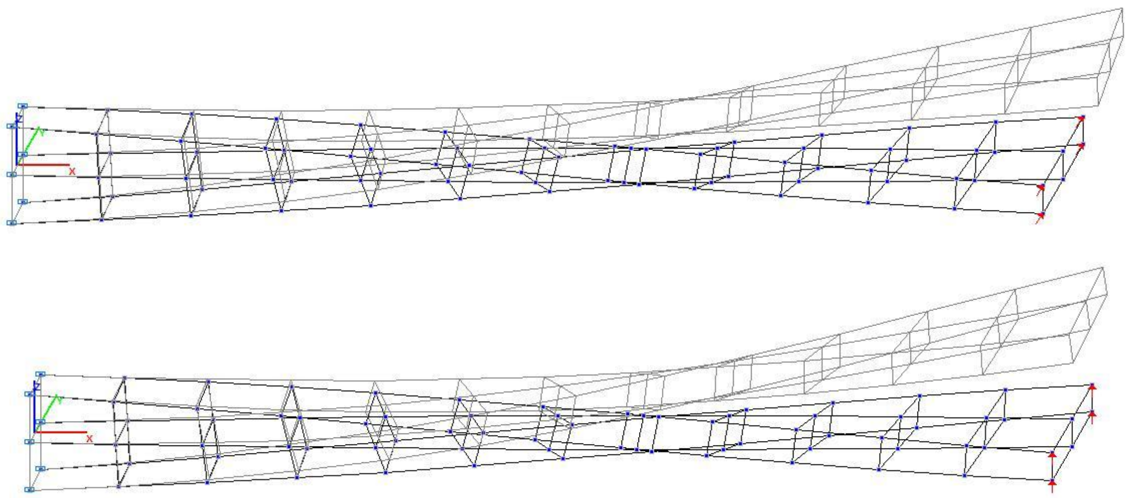
Model 7. Values of the transverse displacements Y, Z
of the free end of the twisted cantilever beam (m, m)



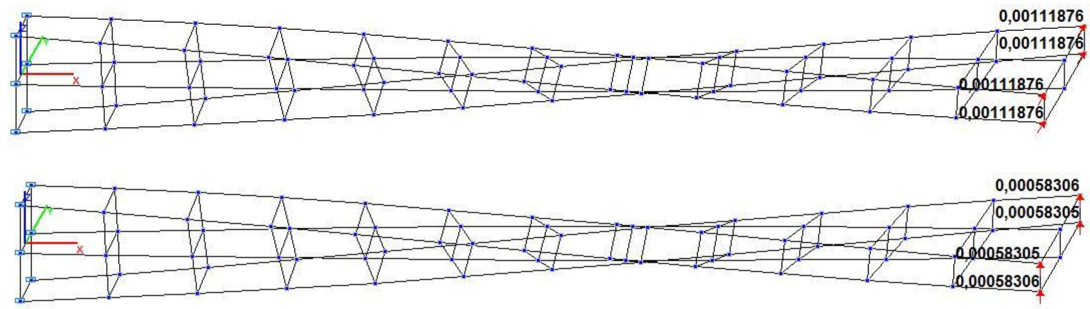
Model 8. Values of the transverse displacements Y, Z
of the free end of the twisted cantilever beam (m, m)



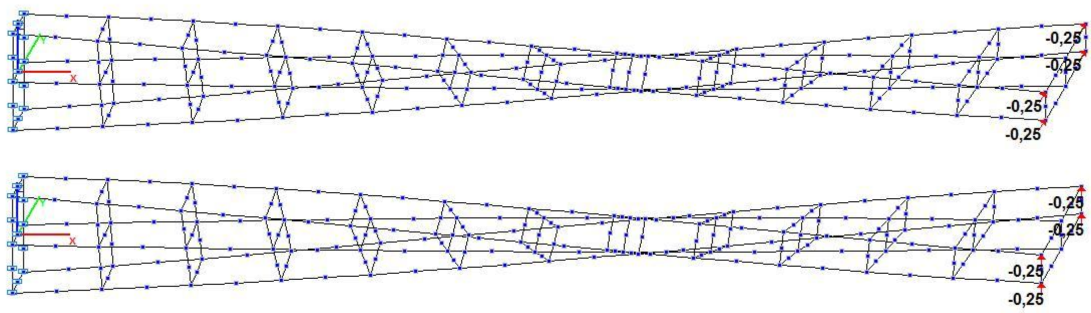
Model 9. Design model



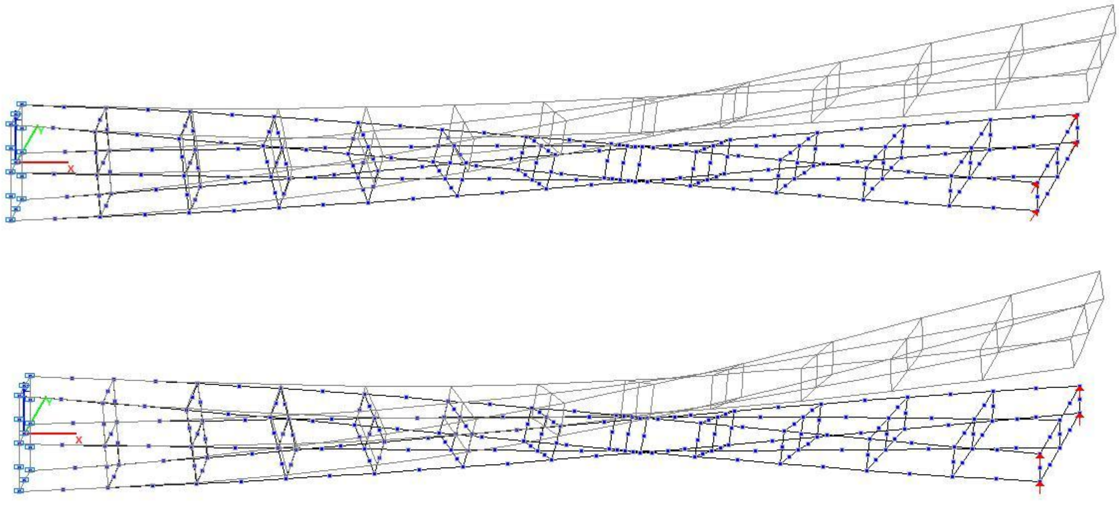
Model 9. Deformed model



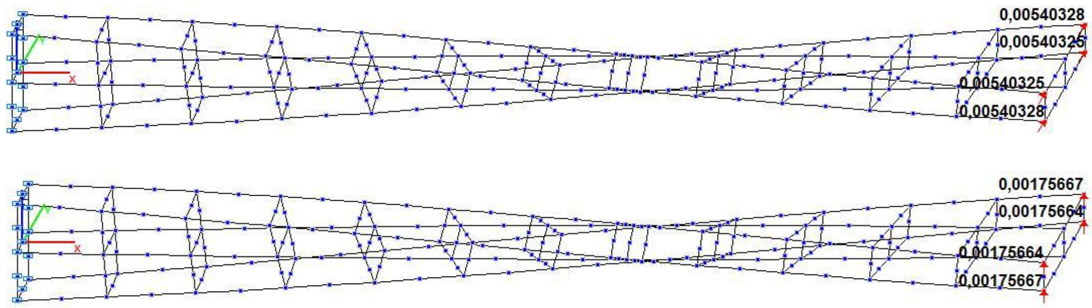
Model 9. Values of the transverse displacements Y, Z of the free end of the twisted cantilever beam (m, m)



Model 10. Design model



Model 10. Deformed model



Model 10. Values of the transverse displacements Y, Z
of the free end of the twisted cantilever beam (m, m)

Comparison of solutions:

Model	Parameter	Theory	SCAD	Deviation, %
1 (Member type 42)	Transverse displacement Y of the free end of the cantilever beam, m	0.005426	0.005314	2.06
	Transverse displacement Z of the free end of the cantilever beam, m	0.001746	0.001463	16.21
2 (Member type 142)	Transverse displacement Y of the free end of the cantilever beam, m	0.005426	0.006220	14.63
	Transverse displacement Z of the free end of the cantilever beam, m	0.001746	0.001710	2.06
3 (Member type 44)	Transverse displacement Y of the free end of the cantilever beam, m	0.005426	0.001899	65.00
	Transverse displacement Z of the free end of the cantilever beam, m	0.001746	0.000906	48.11
4 (Member type 144)	Transverse displacement Y of the free end of the cantilever beam, m	0.005426	0.002158	60.23
	Transverse displacement Z of the free end of the cantilever beam, m	0.001746	0.000961	44.96

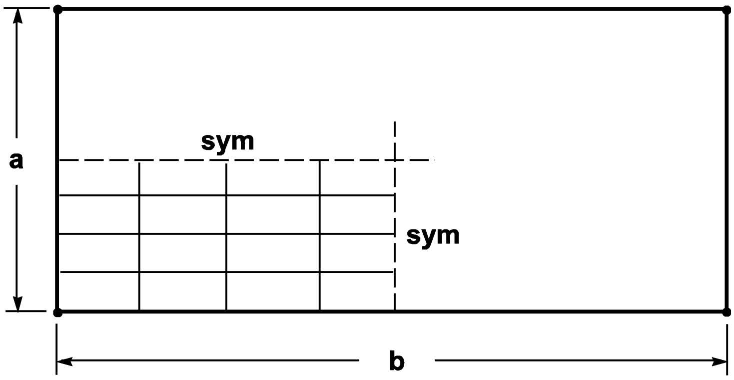
Model	Parameter	Theory	SCAD	Deviation, %
5 (Member type 45)	Transverse displacement Y of the free end of the cantilever beam, m	0.005426	0.005388	0.70
	Transverse displacement Z of the free end of the cantilever beam, m	0.001746	0.001750	0.23
6 (Member type 145)	Transverse displacement Y of the free end of the cantilever beam, m	0.005426	0.006588	21.42
	Transverse displacement Z of the free end of the cantilever beam, m	0.001746	0.002178	24.74
7 (Member type 50)	Transverse displacement Y of the free end of the cantilever beam, m	0.005426	0.003948	27.24
	Transverse displacement Z of the free end of the cantilever beam, m	0.001746	0.001481	15.18
8 (Member type 150)	Transverse displacement Y of the free end of the cantilever beam, m	0.005426	0.004094	24.55
	Transverse displacement Z of the free end of the cantilever beam, m	0.001746	0.001505	13.80
9 (Member type 36)	Transverse displacement Y of the free end of the cantilever beam, m	0.005426	0.001119	79.38
	Transverse displacement Z of the free end of the cantilever beam, m	0.001746	0.000583	66.61
10 (Member type 37)	Transverse displacement Y of the free end of the cantilever beam, m	0.005426	0.005403	0.42
	Transverse displacement Z of the free end of the cantilever beam, m	0.001746	0.001757	0.63

Notes: In the analytical solution the values of the transverse displacements Y, Z of the free end of the twisted cantilever beam from the respective actions are determined according to the following formulas:

$$Y = \frac{12 \cdot P_y \cdot L^3}{E \cdot b^3 \cdot h^3} \cdot \left[\left(\frac{1}{6} - \frac{1}{\pi^2} \right) \cdot h^2 + \left(\frac{1}{6} + \frac{1}{\pi^2} \right) \cdot b^2 \right];$$

$$Z = \frac{12 \cdot P_z \cdot L^3}{E \cdot b^3 \cdot h^3} \cdot \left[\left(\frac{1}{6} + \frac{1}{\pi^2} \right) \cdot h^2 + \left(\frac{1}{6} - \frac{1}{\pi^2} \right) \cdot b^2 \right].$$

Simply Supported Flat Square Plate Subjected to a Transverse Load Uniformly Distributed over the Entire Area and a Concentrated Shear Force Applied in the Center



Objective: Check of the obtained values of the transverse displacements in the center of a simply supported flat square plate subjected to a transverse load uniformly distributed over the entire area and a concentrated shear force applied in the center.

Initial data files:

File name	Description
Bending_of_square_flat_plate_Simply_supported_Shell_42_Mesh_2x2.SPR Bending_of_square_flat_plate_Simply_supported_Shell_42_Mesh_4x4.SPR Bending_of_square_flat_plate_Simply_supported_Shell_42_Mesh_8x8.SPR	Design model with the elements of type 42 for meshes 2x2, 4x4, 8x8
Bending_of_square_flat_plate_Simply_supported_Shell_44_Mesh_2x2.SPR Bending_of_square_flat_plate_Simply_supported_Shell_44_Mesh_4x4.SPR Bending_of_square_flat_plate_Simply_supported_Shell_44_Mesh_8x8.SPR	Design model with the elements of type 44 for meshes 2x2, 4x4, 8x8
Bending_of_square_flat_plate_Simply_supported_Shell_45_Mesh_2x2.SPR Bending_of_square_flat_plate_Simply_supported_Shell_45_Mesh_4x4.SPR Bending_of_square_flat_plate_Simply_supported_Shell_45_Mesh_8x8.SPR	Design model with the elements of type 45 for meshes 2x2, 4x4, 8x8
Bending_of_square_flat_plate_Simply_supported_Shell_50_Mesh_2x2.SPR Bending_of_square_flat_plate_Simply_supported_Shell_50_Mesh_4x4.SPR Bending_of_square_flat_plate_Simply_supported_Shell_50_Mesh_8x8.SPR	Design model with the elements of type 50 for meshes 2x2, 4x4, 8x8
Bending_of_square_flat_plate_Simply_supported_Solid_36_Mesh_2x2.SPR Bending_of_square_flat_plate_Simply_supported_Solid_36_Mesh_4x4.SPR Bending_of_square_flat_plate_Simply_supported_Solid_36_Mesh_8x8.SPR Bending_of_square_flat_plate_Simply_supported_Solid_36_Mesh_16x16.SPR Bending_of_square_flat_plate_Simply_supported_Solid_36_Mesh_32x32.SPR Bending_of_square_flat_plate_Simply_supported_Solid_36_Mesh_64x64.SPR Bending_of_square_flat_plate_Simply_supported_Solid_36_Mesh_128x128.SPR	Design model with the elements of type 36 for meshes 2x2, 4x4, 8x8, 16x16, 32x32, 64x64, 128x128
Bending_of_square_flat_plate_Simply_supported_Solid_37_Mesh_2x2.SPR Bending_of_square_flat_plate_Simply_supported_Solid_37_Mesh_4x4.SPR Bending_of_square_flat_plate_Simply_supported_Solid_37_Mesh_8x8.SPR Bending_of_square_flat_plate_Simply_supported_Solid_37_Mesh_16x16.SPR Bending_of_square_flat_plate_Simply_supported_Solid_37_Mesh_32x32.SPR Bending_of_square_flat_plate_Simply_supported_Solid_37_Mesh_64x64.SPR Bending_of_square_flat_plate_Simply_supported_Solid_37_Mesh_128x128.SPR	Design model with the elements of type 37 for meshes 2x2, 4x4, 8x8, 16x16, 32x32, 64x64, 128x128

Problem formulation: The simply supported flat square plate is subjected to the transverse load q uniformly distributed over the entire area and the concentrated shear force P applied in the center. Check the obtained values of the transverse displacements in the center of the simply supported flat square plate w_q and w_P from the respective actions.

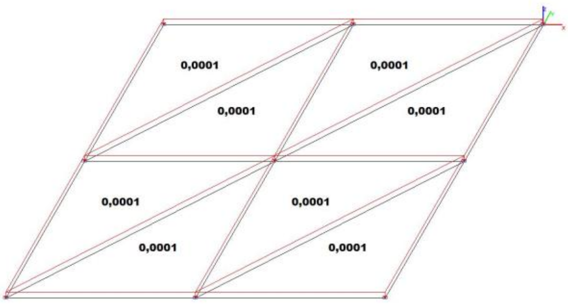
References: R. H. Macneal, R. L. Harder, A proposed standard set of problems to test finite element accuracy, North-Holland, Finite elements in analysis and design, 1, 1985, p. 3-20.
S. Timoshenko, S. Woinowsky-Krieger, Theory of plates and shells, New York, McGraw-Hill,1959, p. 120, 143, 202, 206.

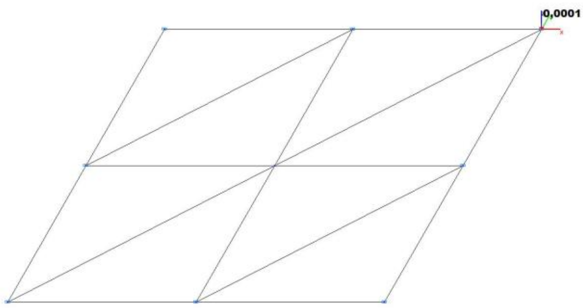
Initial data:

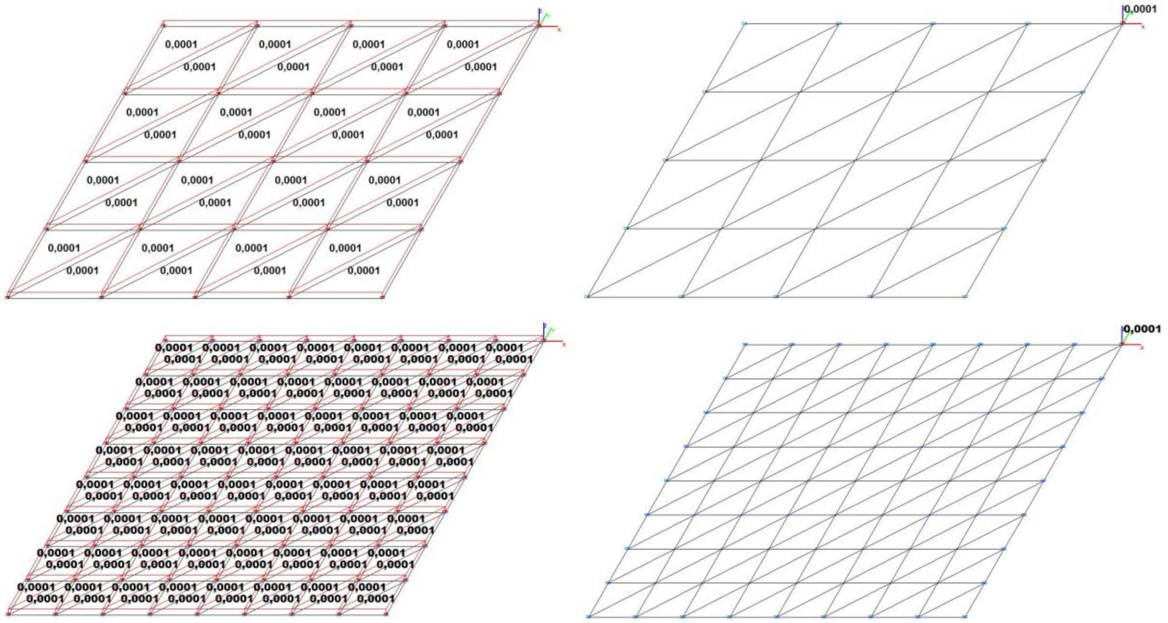
$E = 1.7472 \cdot 10^7$ kPa - elastic modulus of the plate material;
$\nu = 0.30$ - Poisson's ratio;
$a = 2.00$ m - width of the plate;
$b = 2.00$ m - length of the plate;
$h = 10^{-4}$ (10^{-2}) m - thickness of the plate;
$q = 1.0 \cdot 10^{-4}$ kN/m^2 - value of the transverse load uniformly distributed over the entire area of the plate;
$P = 4.0 \cdot 10^{-4}$ kN - value of the concentrated shear force in the center of the plate.

Finite element model: Design model – general type system. Six design models of a quarter of the plate according to the symmetry conditions are considered:
Model 1 – 8, 32, 128 three-node shell elements of type 42 with a regular mesh 2x2, 4x4, 8x8. The thickness of the plate – 10^{-4}m. Boundary conditions are provided by imposing constraints on the nodes of the support edges of the plate in the directions of the degrees of freedom X, Y, Z and constraints according to the symmetry conditions. Number of nodes in the model – 9, 25, 81.
Model 2 – 4, 16, 64 four-node shell elements of type 44 with a regular mesh 2x2, 4x4, 8x8. The thickness of the plate – 10^{-4} m. Boundary conditions are provided by imposing constraints on the nodes of the support edges of the plate in the directions of the degrees of freedom X, Y, Z and constraints according to the symmetry conditions. Number of nodes in the model – 9, 25, 81.
Model 3 – 8, 32, 128 six-node shell elements of type 45 with a regular mesh 2x2, 4x4, 8x8. The thickness of the plate – 10^{-4} m. Boundary conditions are provided by imposing constraints on the nodes of the support edges of the plate in the directions of the degrees of freedom X, Y, Z and constraints according to the symmetry conditions. Number of nodes in the model – 25, 81, 289.
Model 4 – 4, 16, 64 eight-node shell elements of type 50 with a regular mesh 2x2, 4x4, 8x8. The thickness of the plate – 10^{-4} m. Boundary conditions are provided by imposing constraints on the nodes of the support edges of the plate in the directions of the degrees of freedom X, Y, Z and constraints according to the symmetry conditions. Number of nodes in the model – 25, 81, 289.
Model 5 – 4, 16, 64, 256, 1024, 4096, 16384 eight-node isoparametric solid elements of type 36 with a regular mesh 2x2x1, 4x4x1, 8x8x1, 16x16x1, 32x32x1, 64x64x1, 128x128x1. The thickness of the plate – 10^{-2} m. Boundary conditions are provided by imposing constraints on the nodes of the support sides of the lower surface of the plate in the direction of the degree of freedom Z and constraints according to the symmetry conditions. Number of nodes in the model – 18, 50, 162, 578, 2178, 8450, 33282.
Model 6 – 4, 16, 64, 256, 1024, 4096, 16384 twenty-node isoparametric solid elements of type 37 with a regular mesh 2x2x1, 4x4x1, 8x8x1, 16x16x1, 32x32x1, 64x64x1, 128x128x1. The thickness of the plate – 10^{-2} m. Boundary conditions are provided by imposing constraints on the nodes of the support sides of the lower surface of the plate in the direction of the degree of freedom Z and constraints according to the symmetry conditions. Number of nodes in the model – 51, 155, 531, 1955, 7491, 29315, 115971.

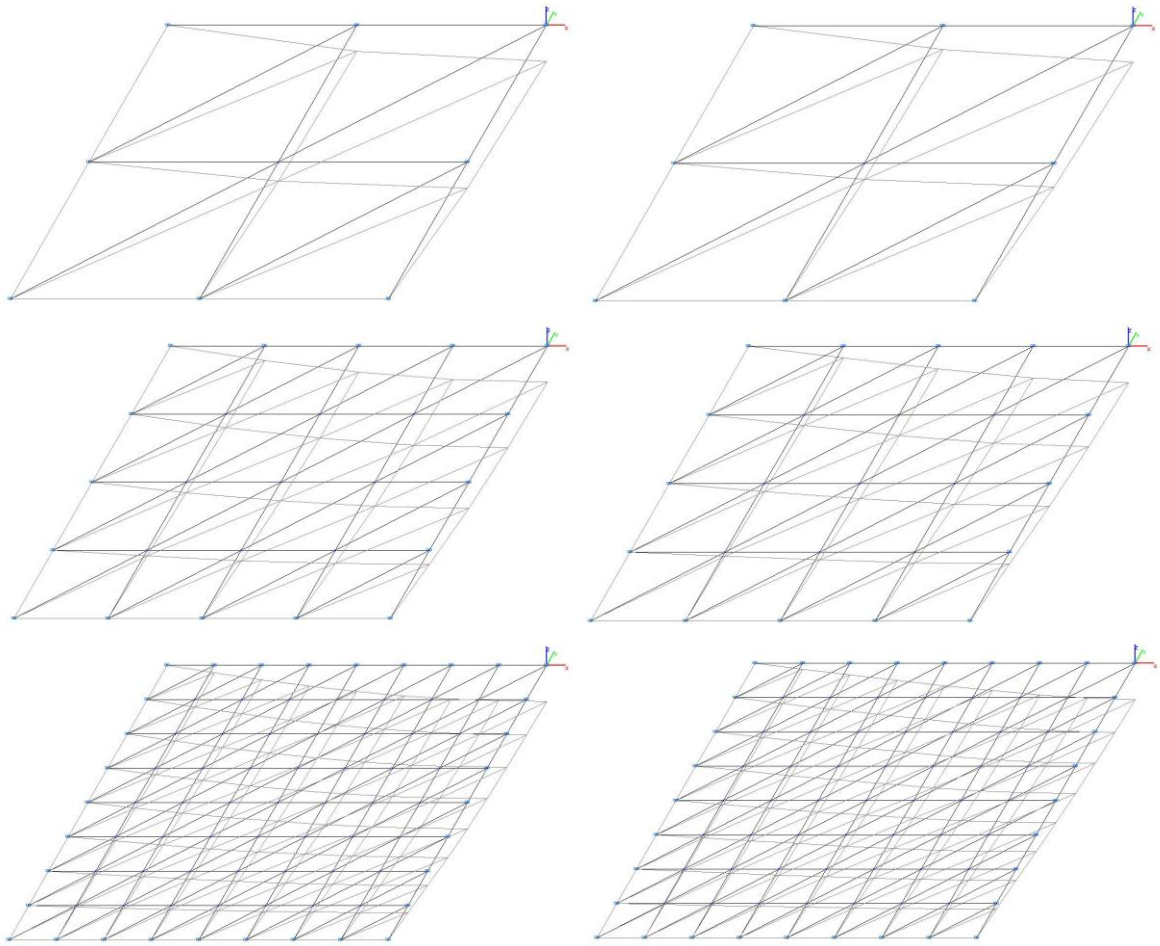
Results in SCAD



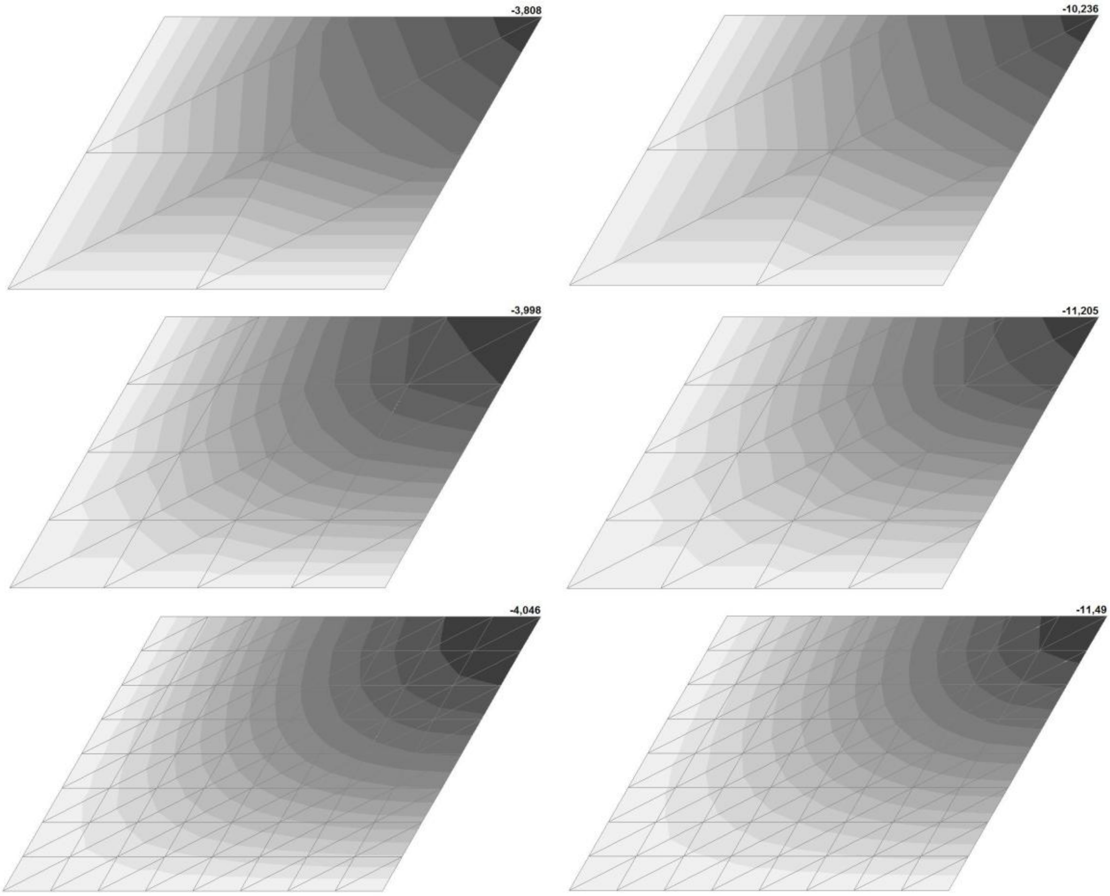




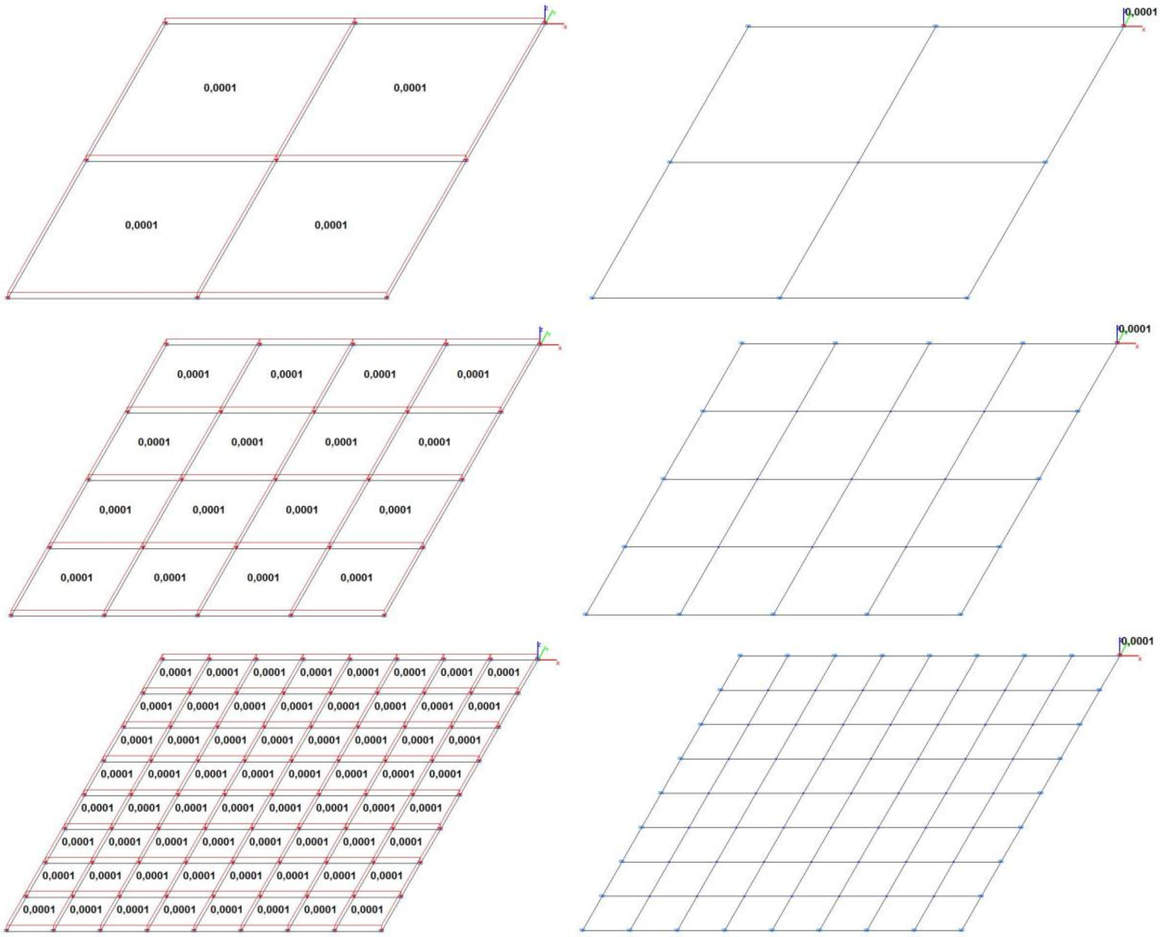
Model 1. Design model



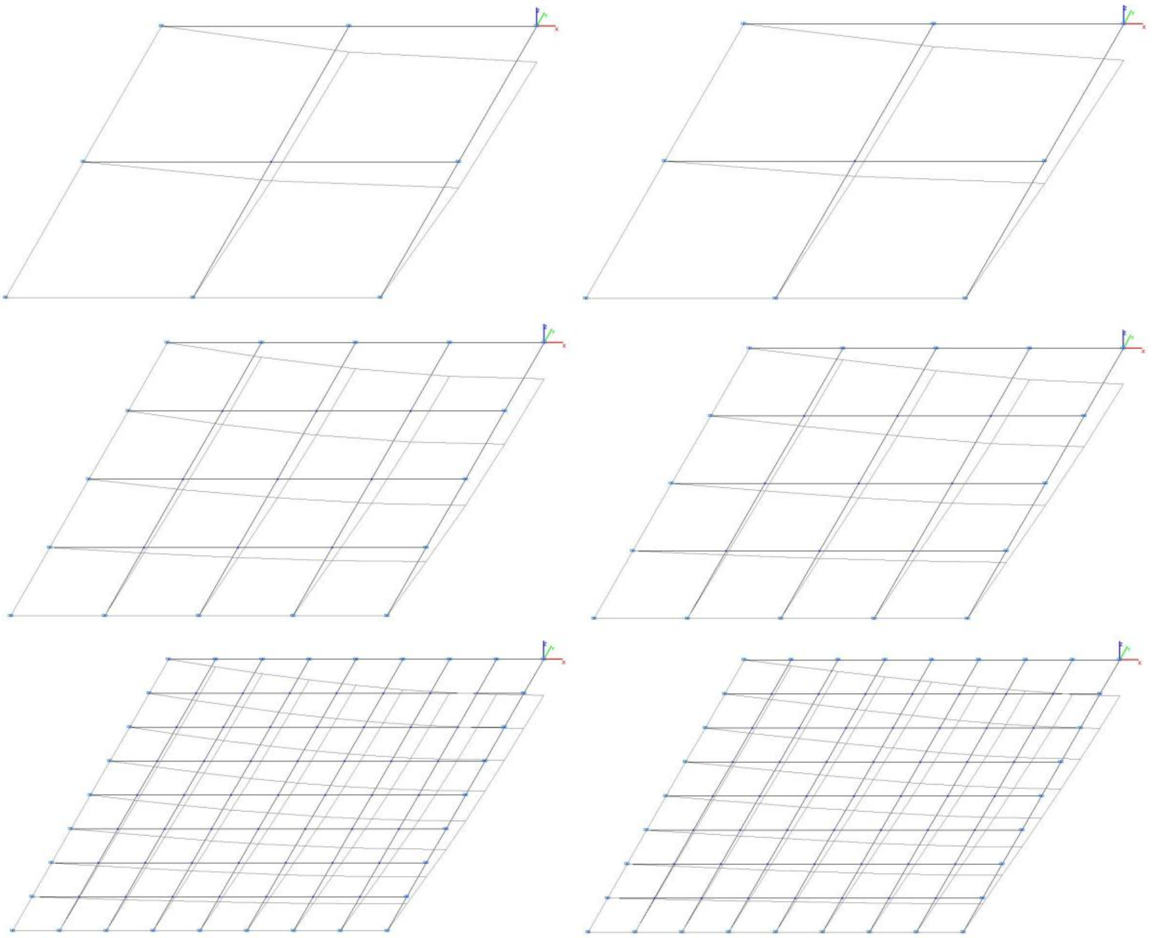
Model 1. Deformed model



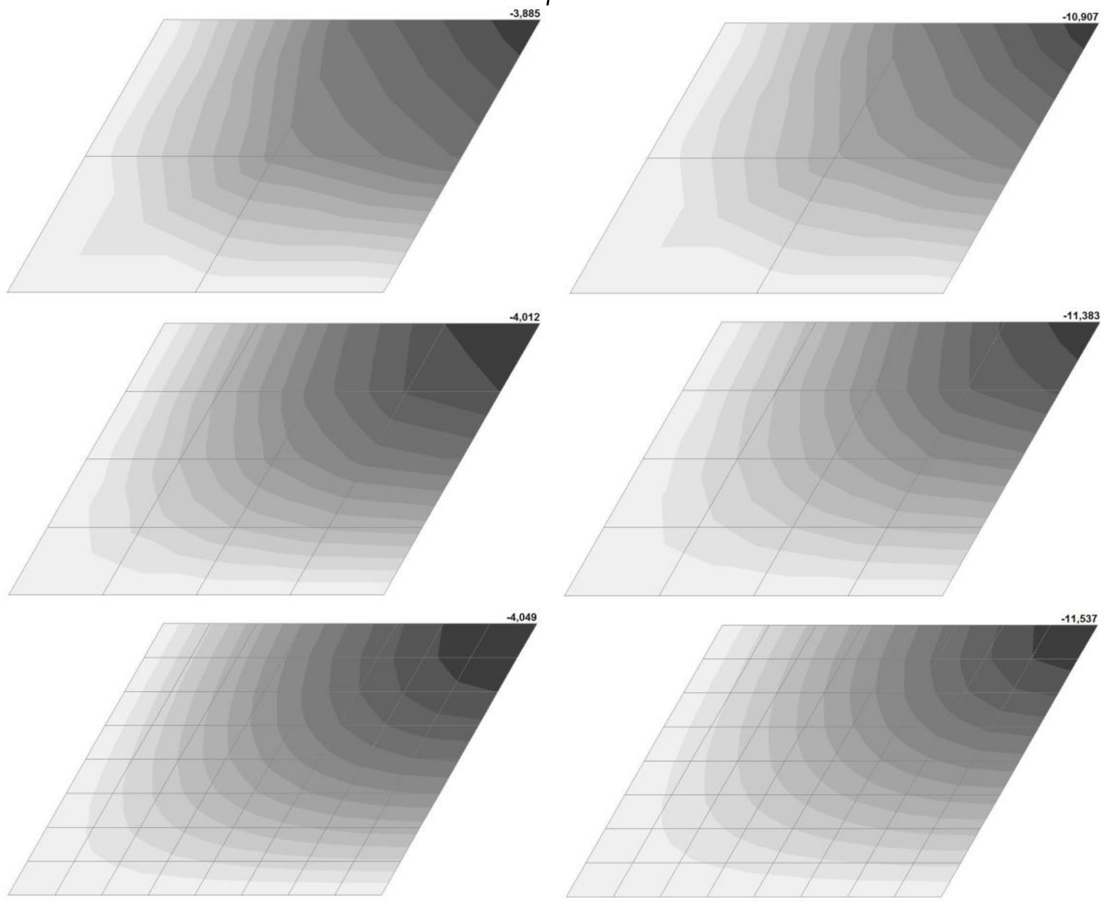
Model 1. Values of the transverse displacements in the center of the simply supported square plate w_q and w_P (m, m)



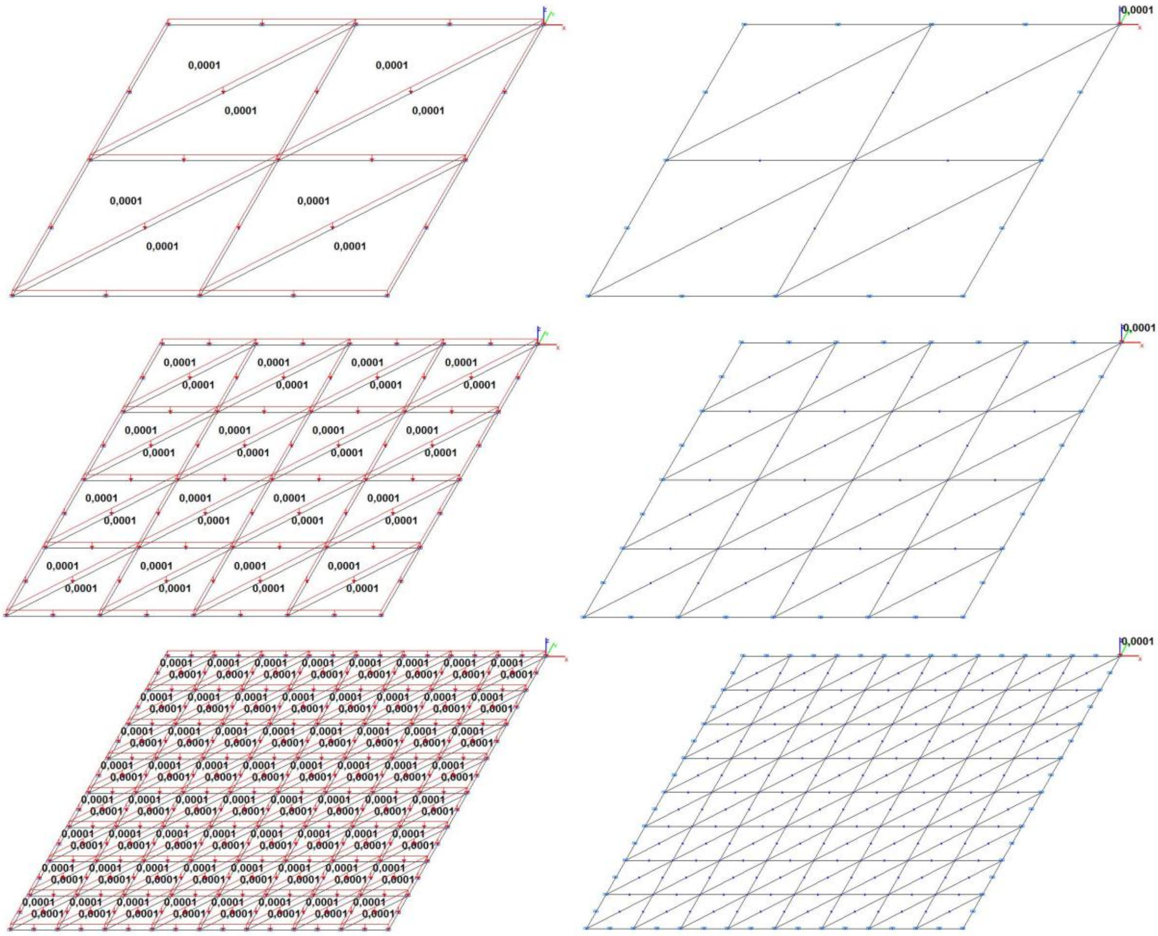
Model 2. Design model



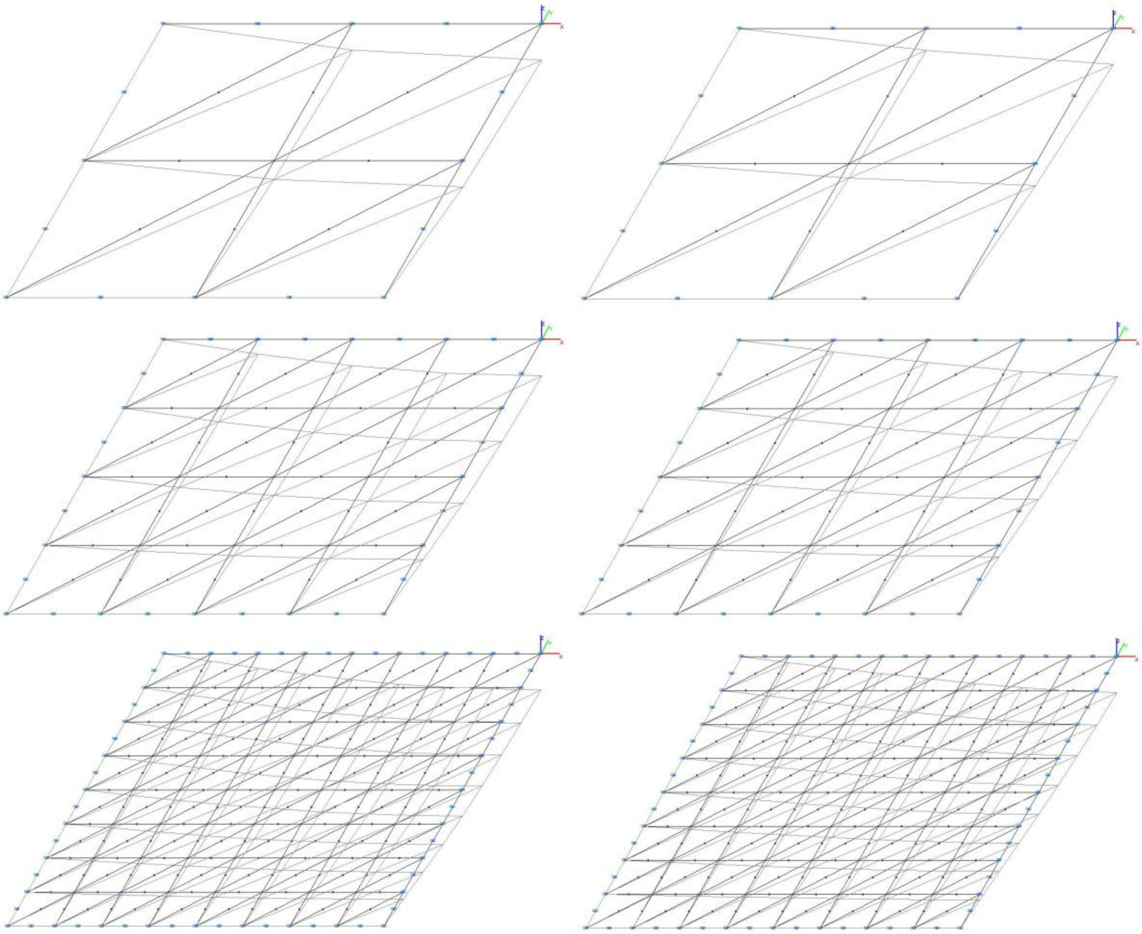
Model 2. Deformed model



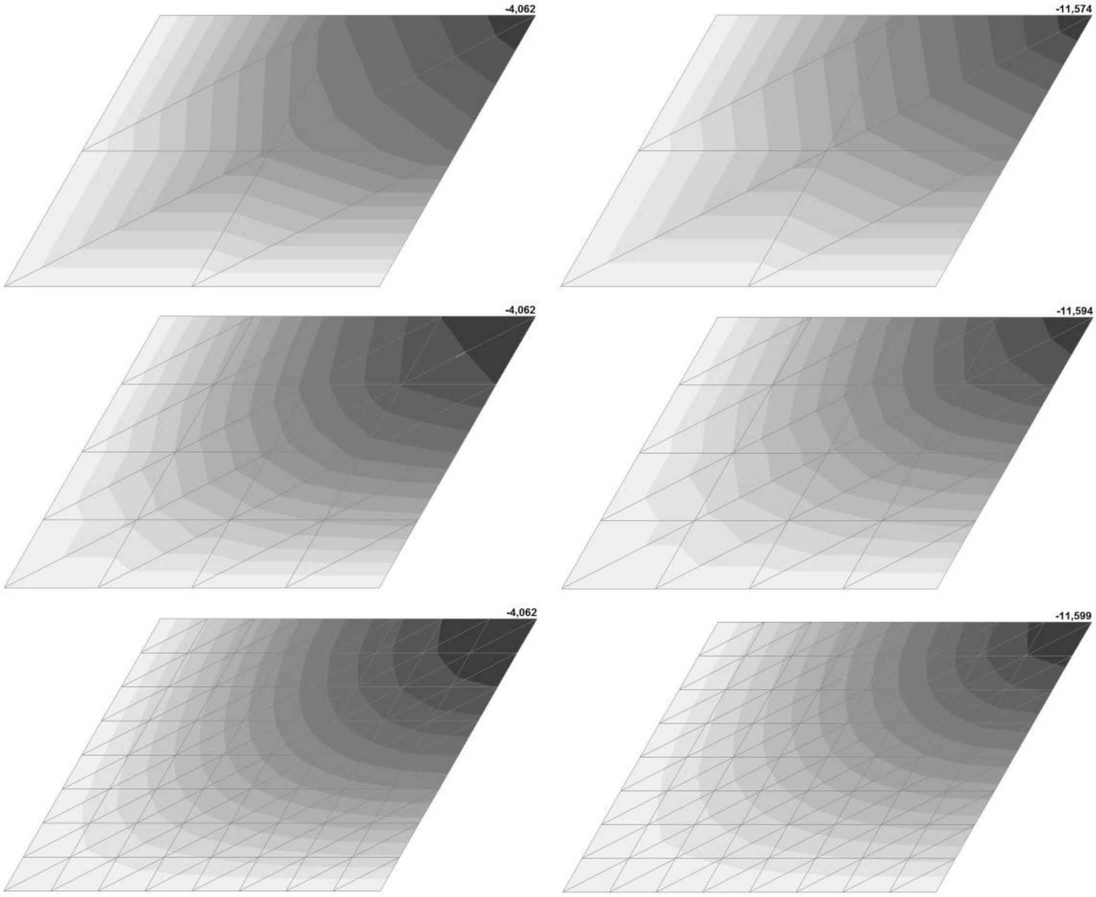
Model 2. Values of the transverse displacements in the center of the simply supported square plate w_q and w_P (m, m)



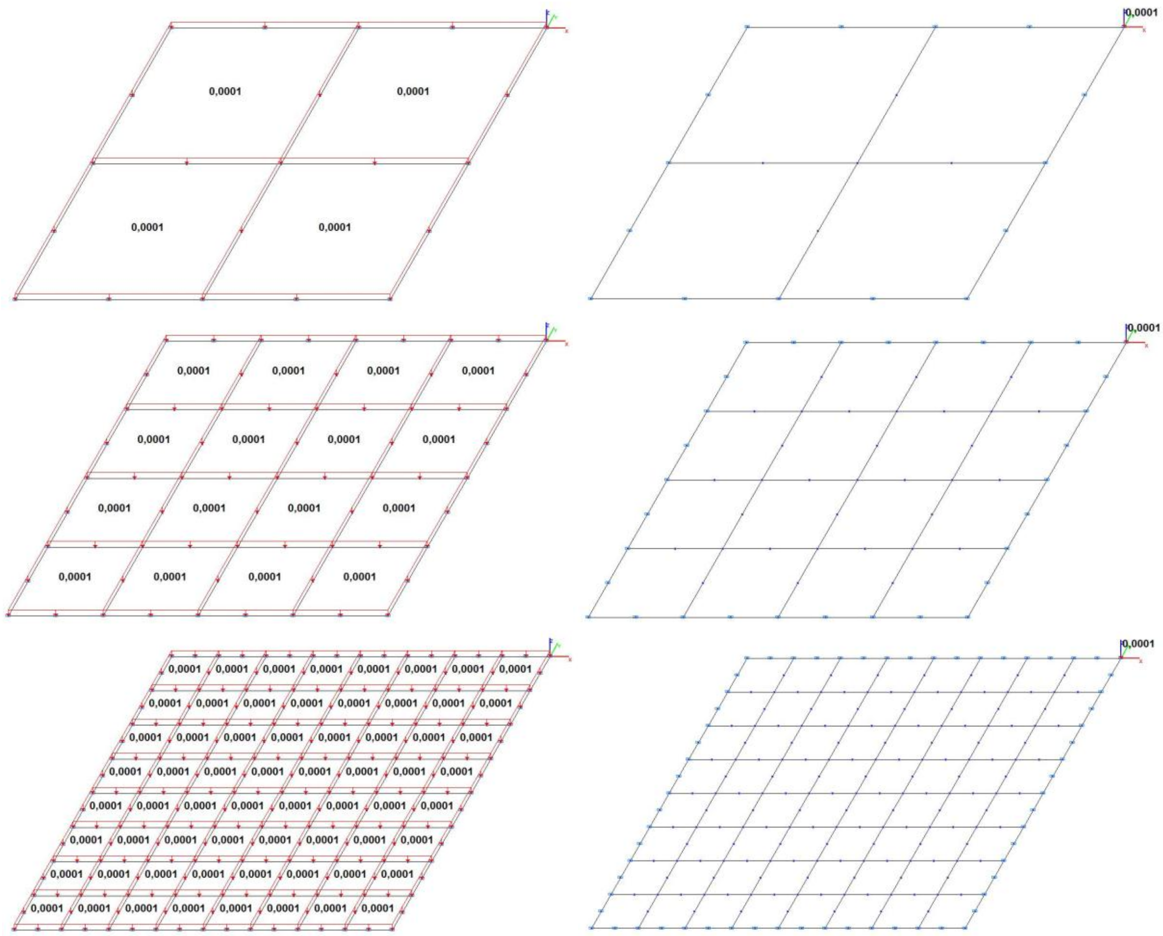
Model 3. Design model



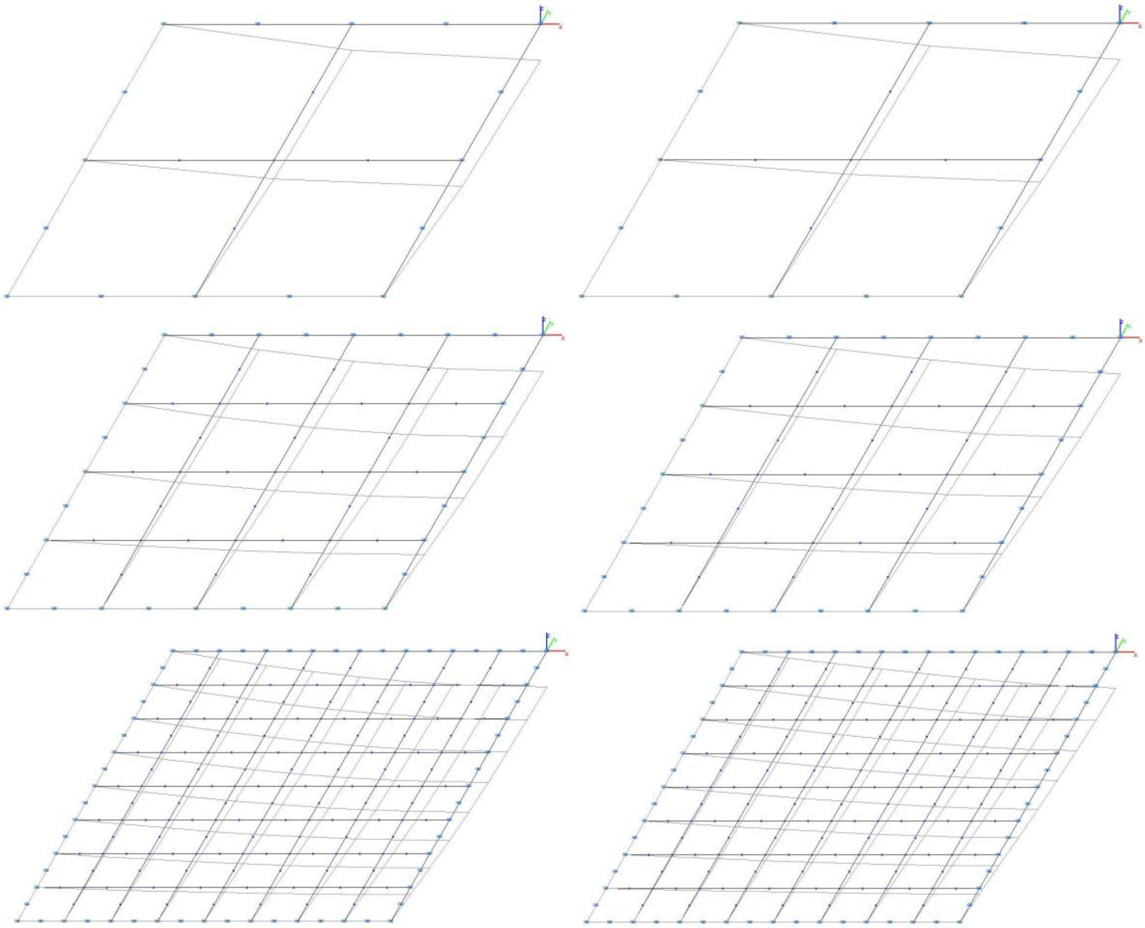
Model 3. Deformed model



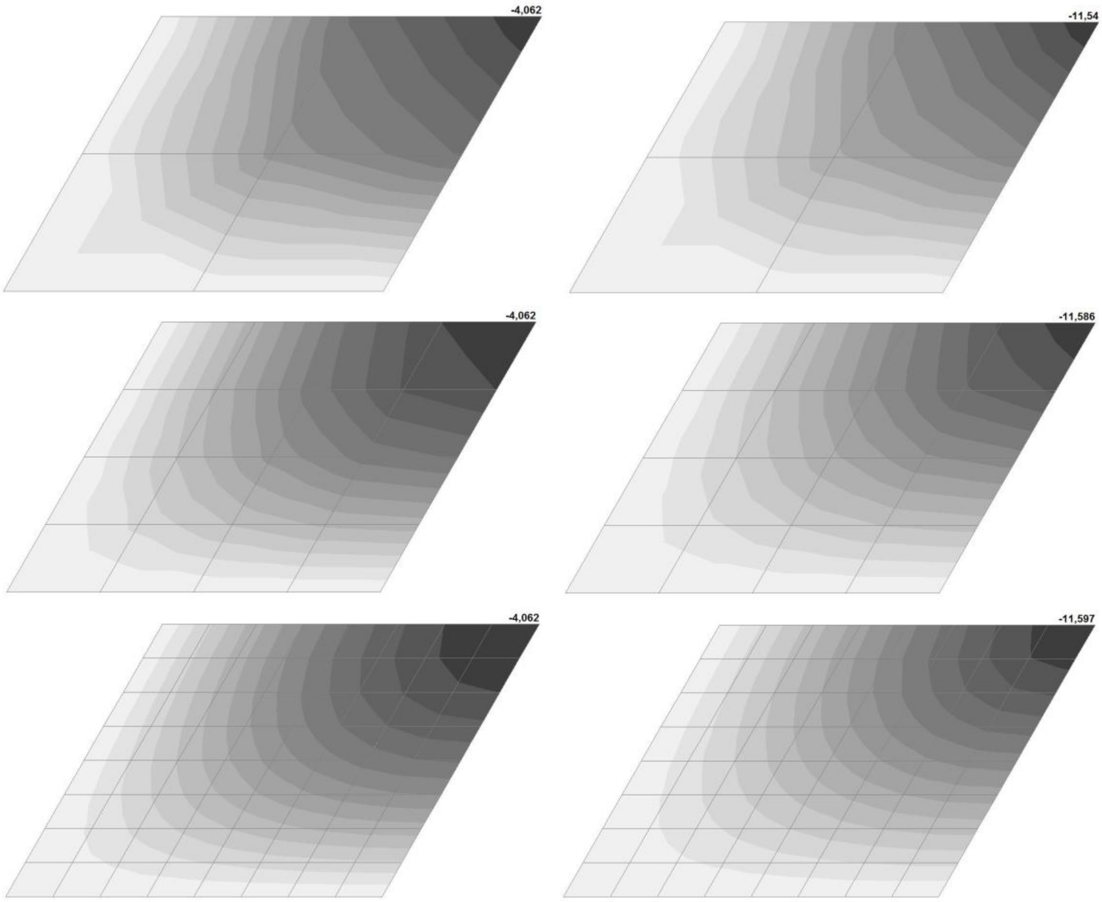
Model 3. Values of the transverse displacements in the center of the simply supported square plate w_q and w_P (m, m)



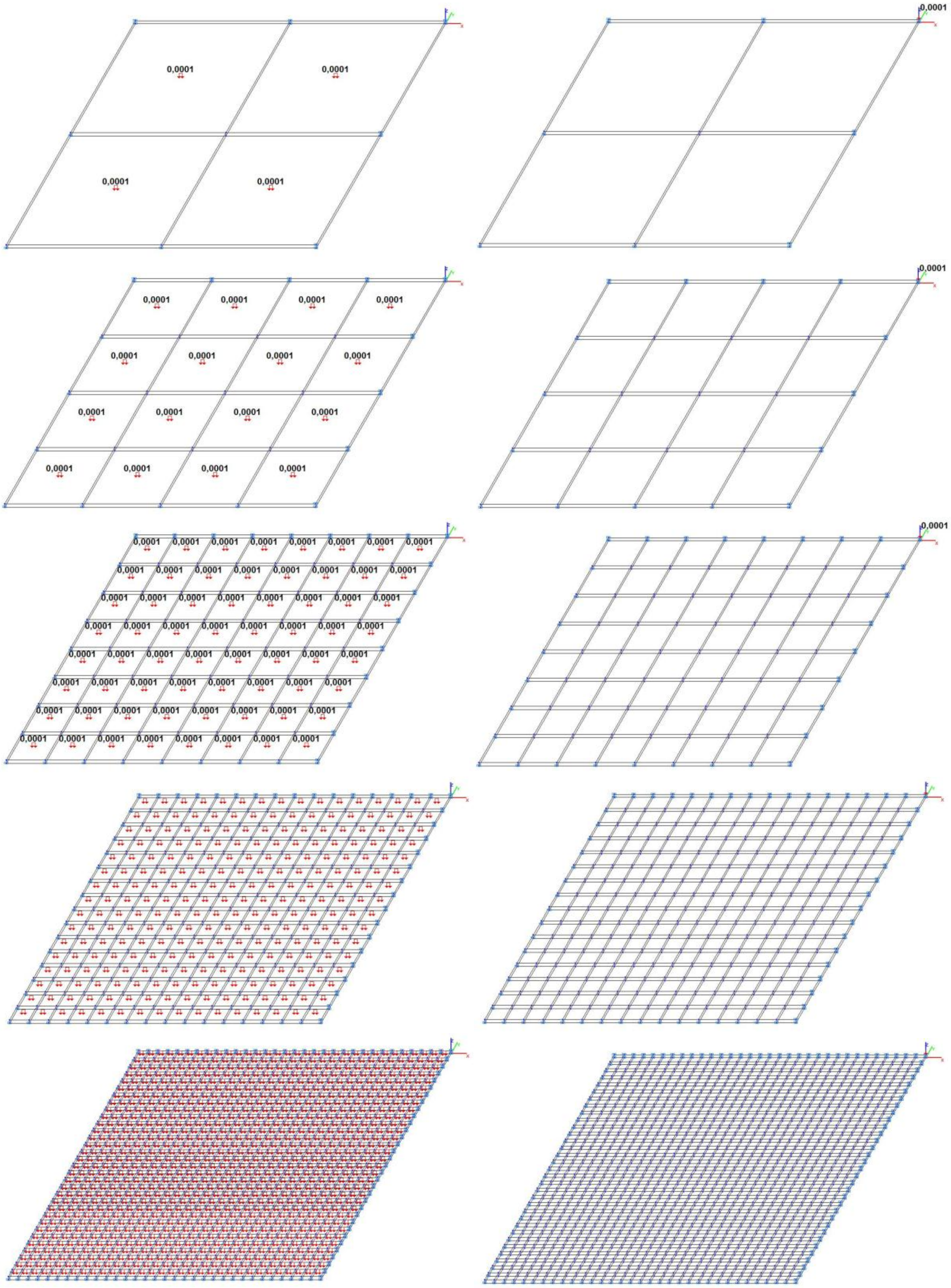
Model 4. Design model

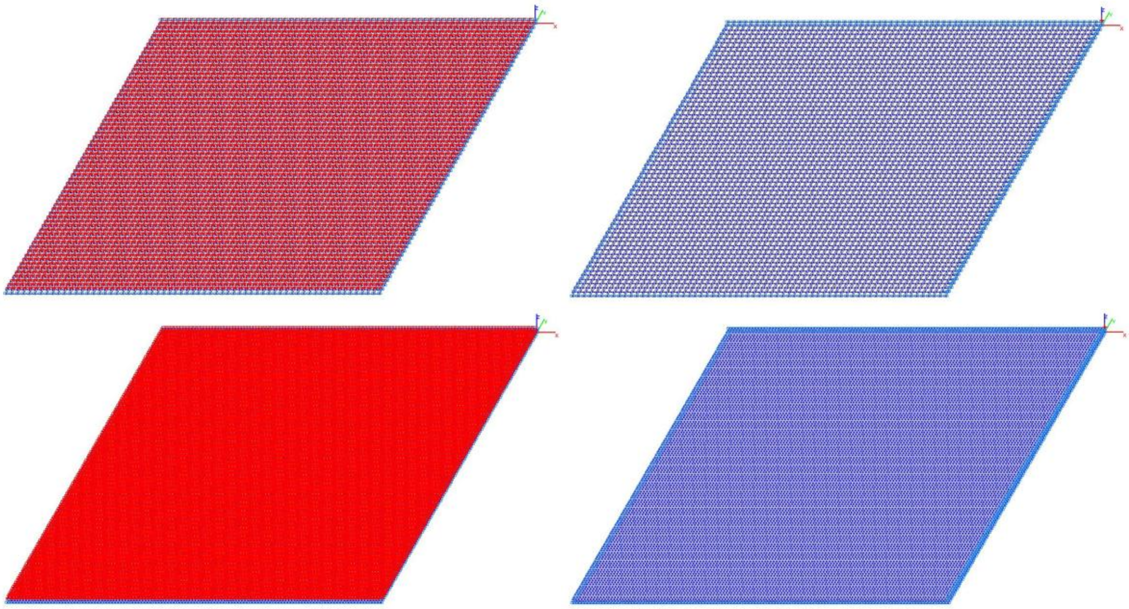


Model 4. Deformed model

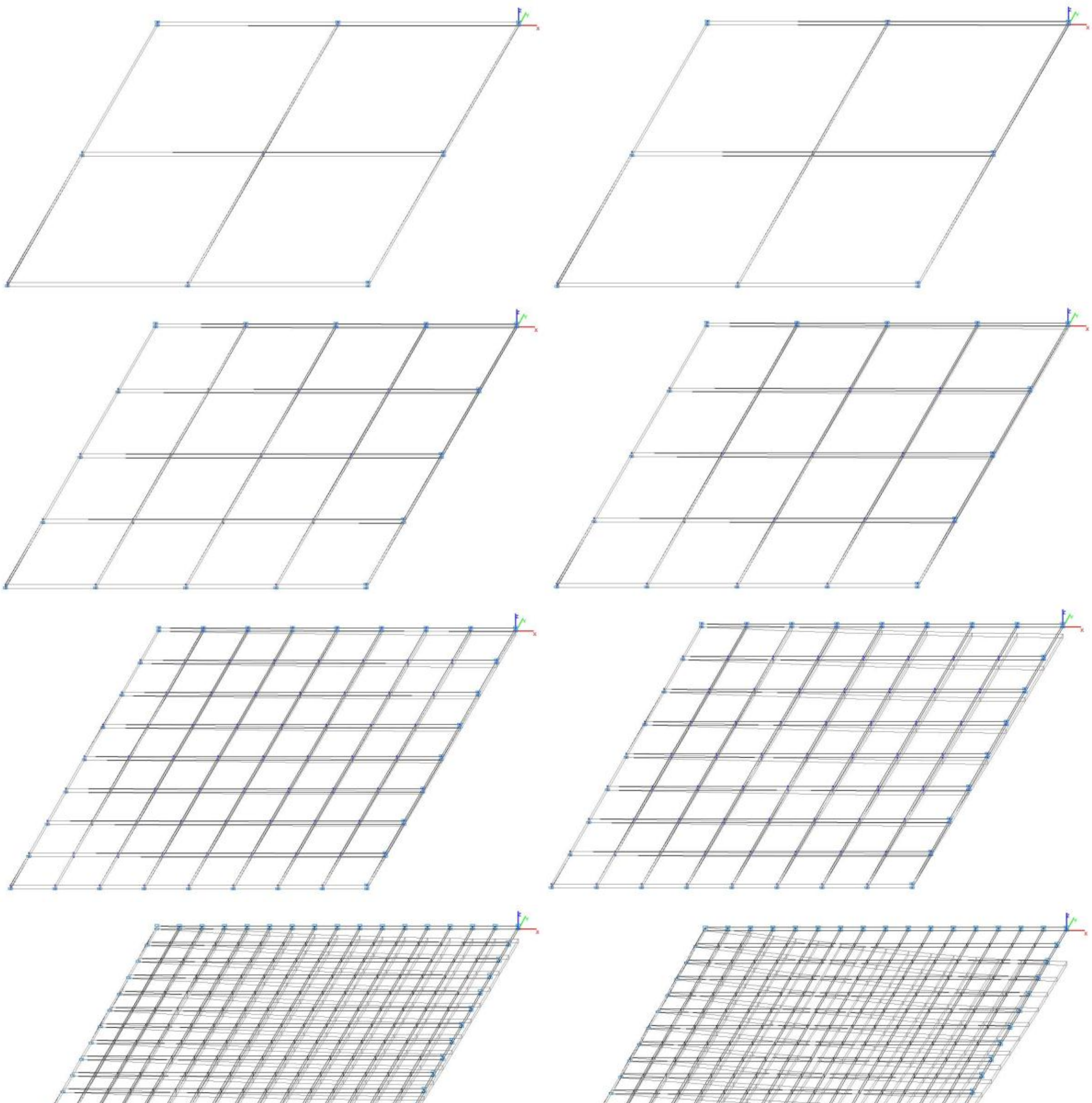


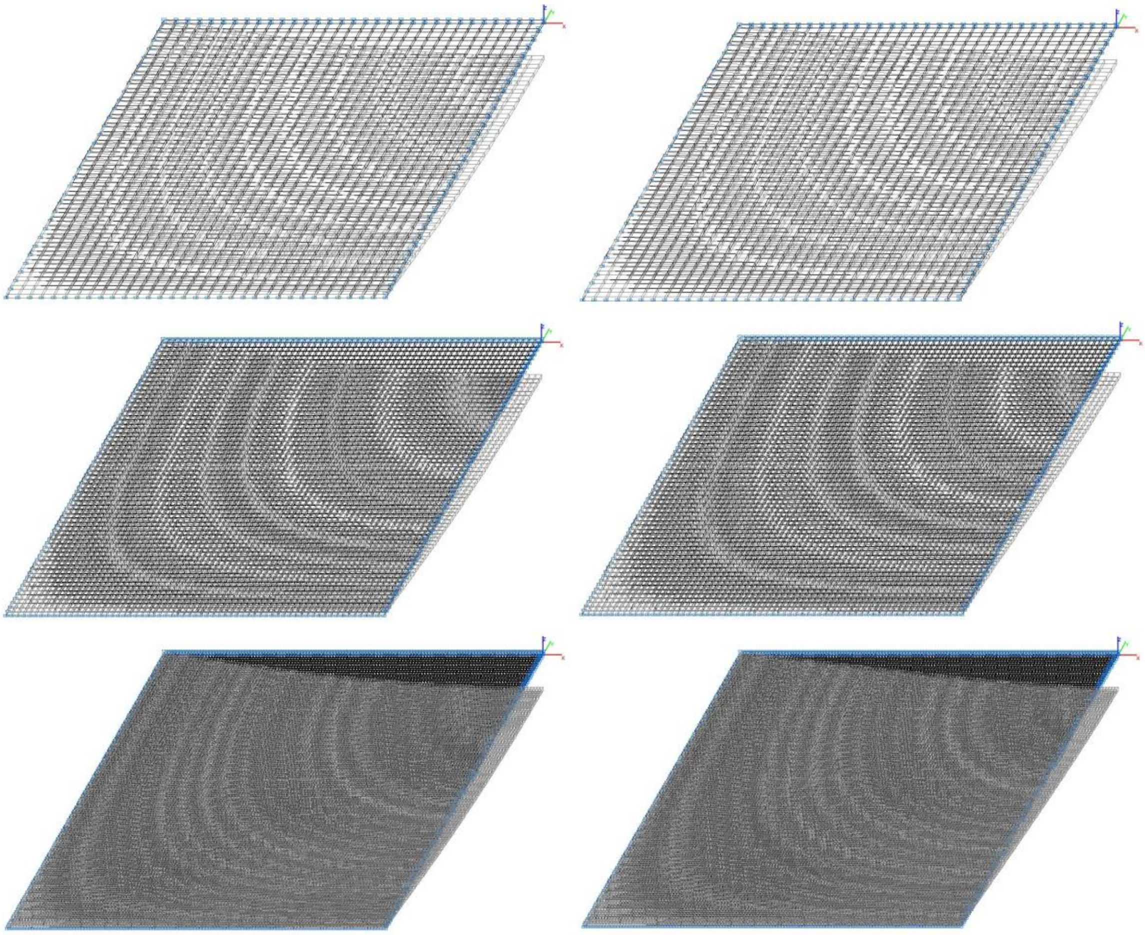
Model 4. Values of the transverse displacements in the center of the simply supported square plate w_q and w_P (m, m)


0,0001

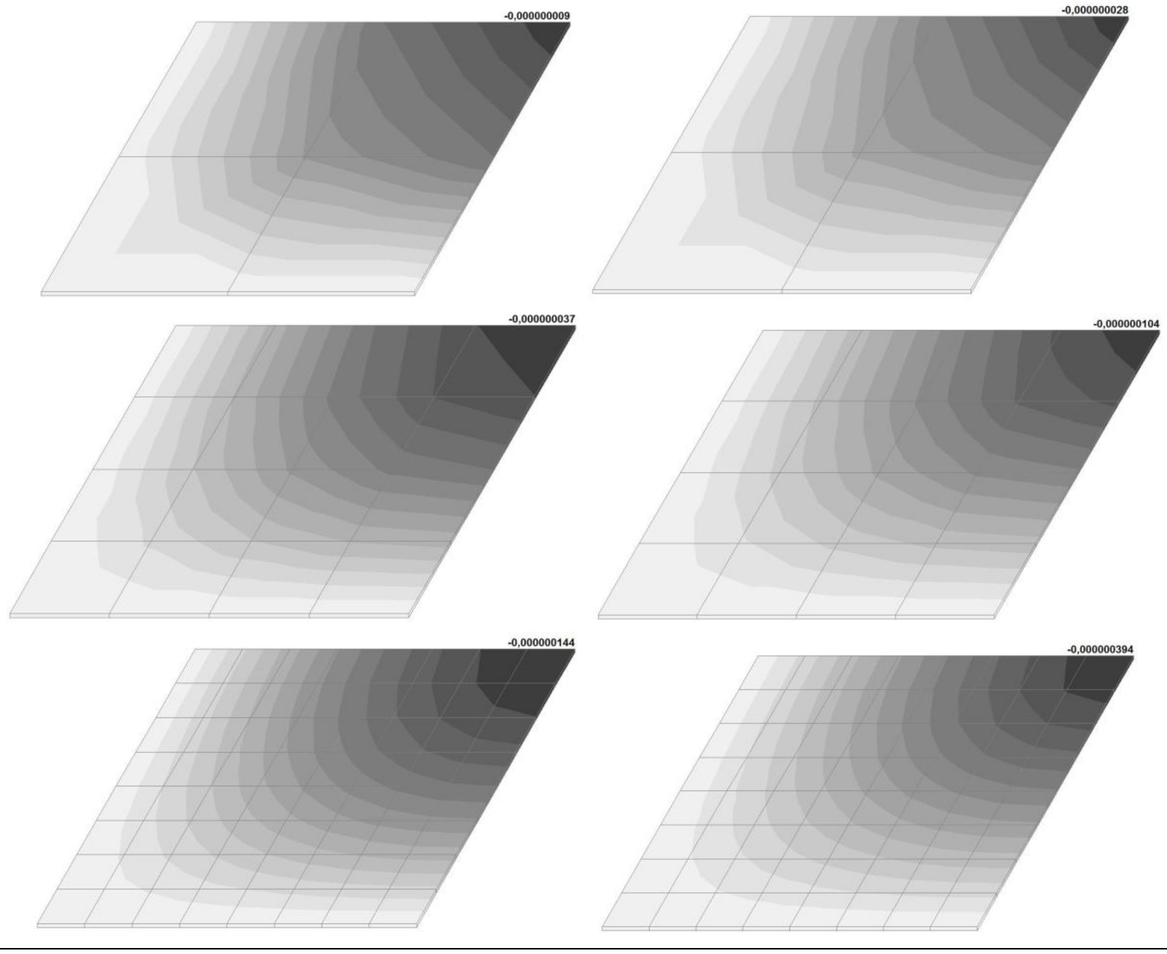


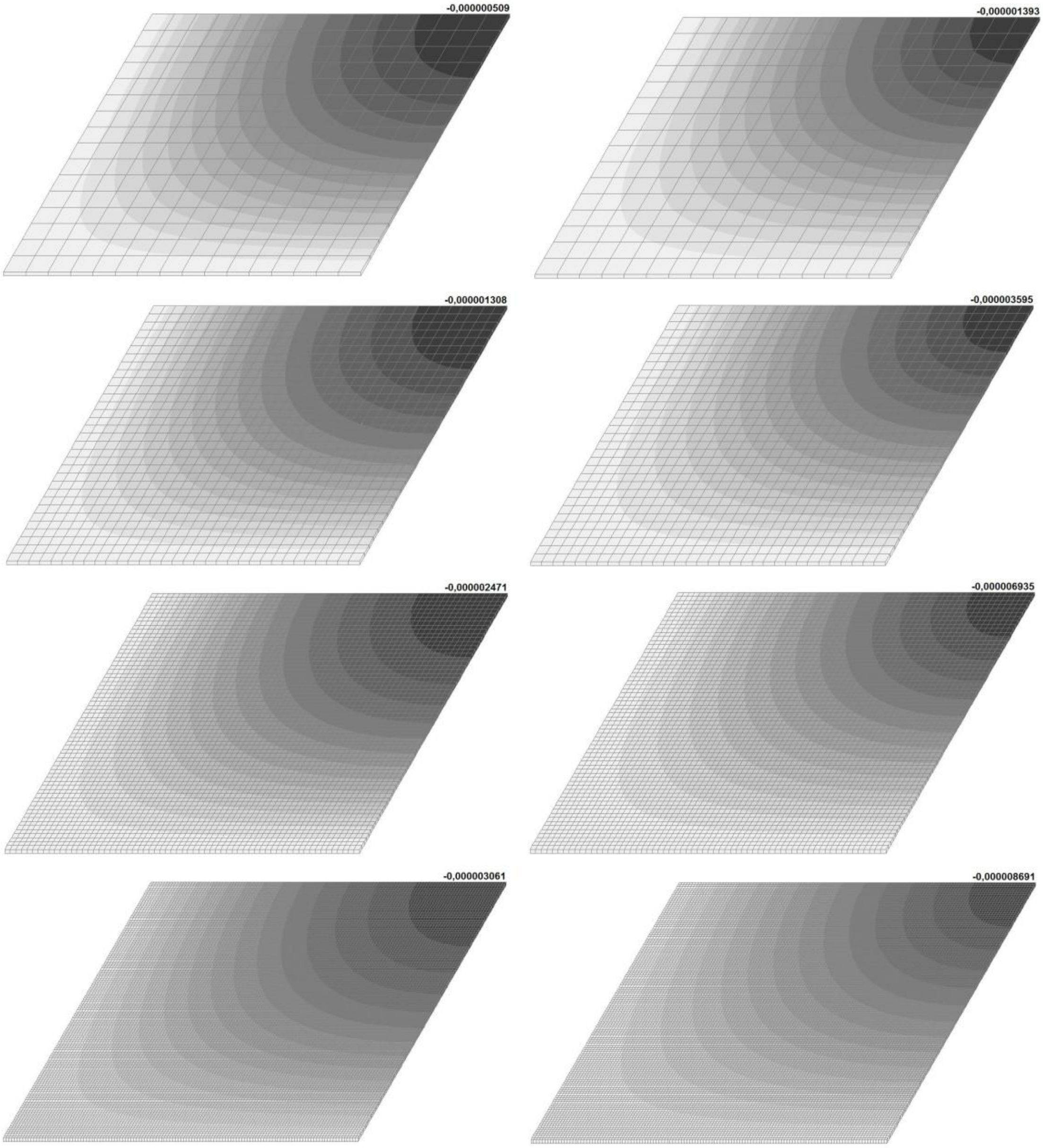
Model 5. Design model



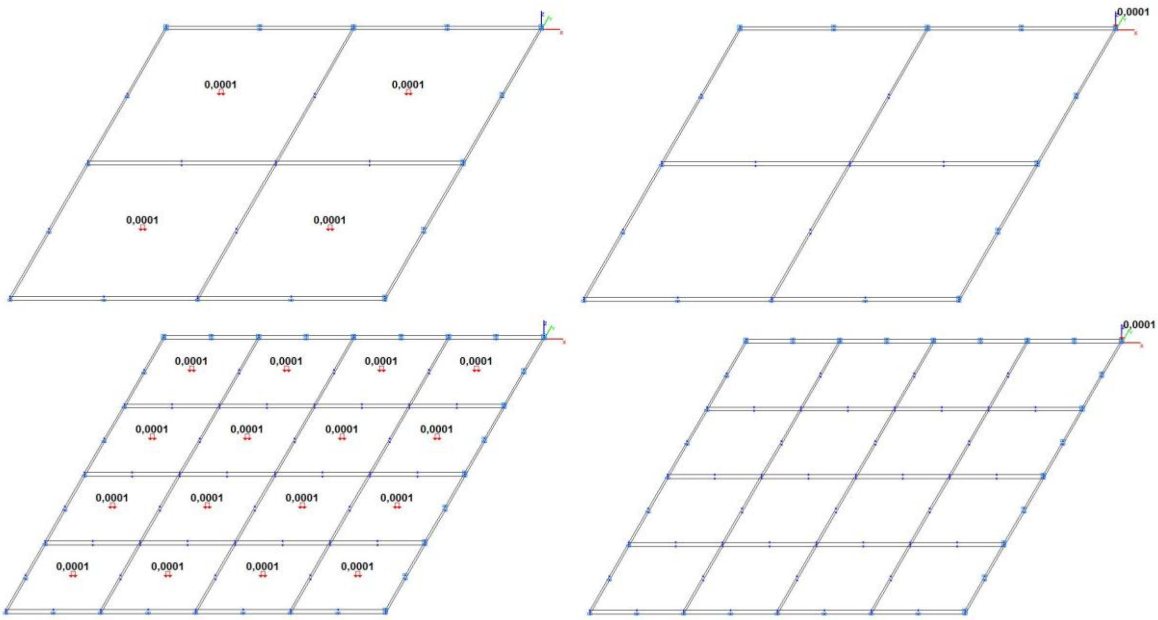


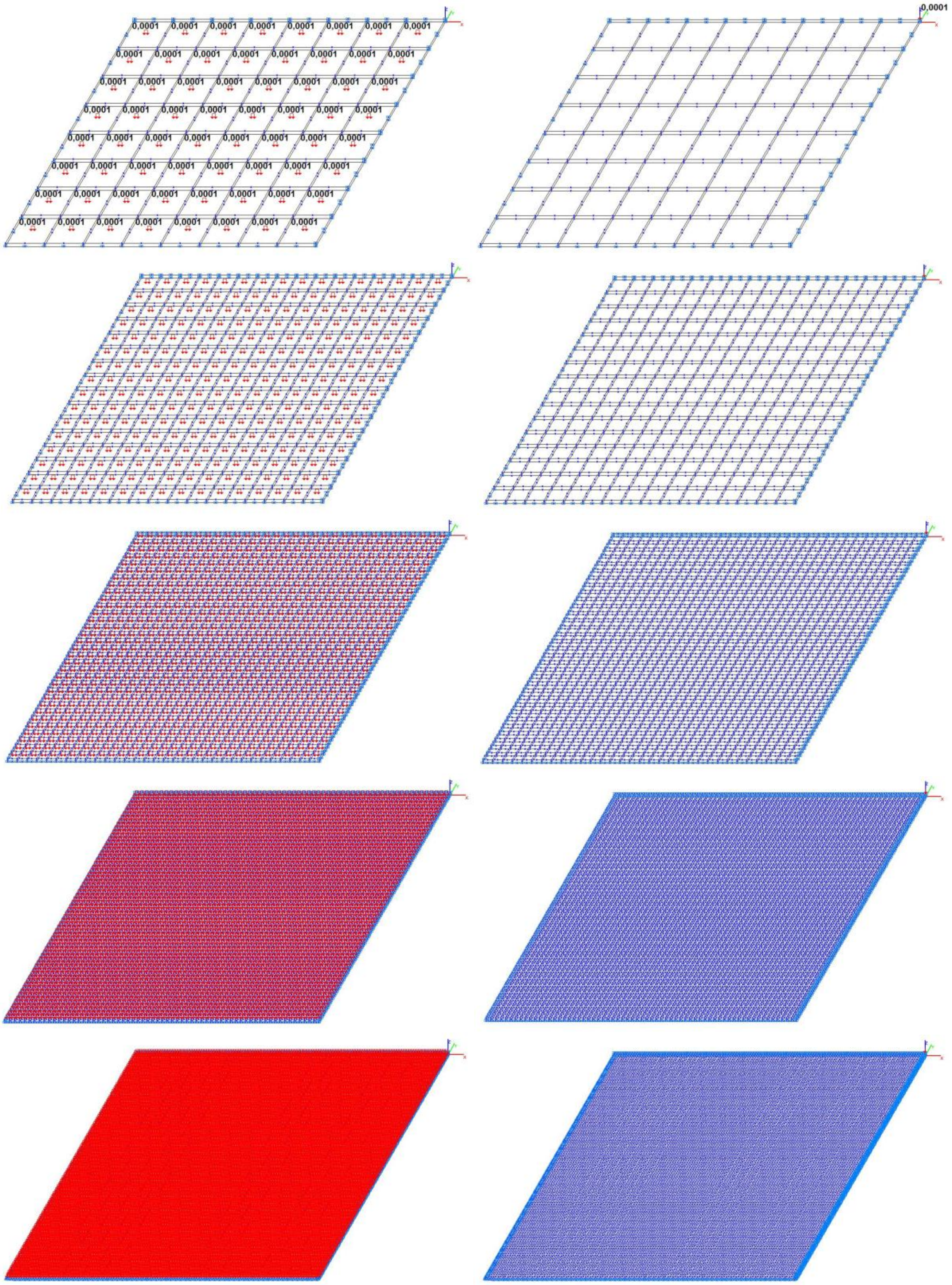
Model 5. Deformed model



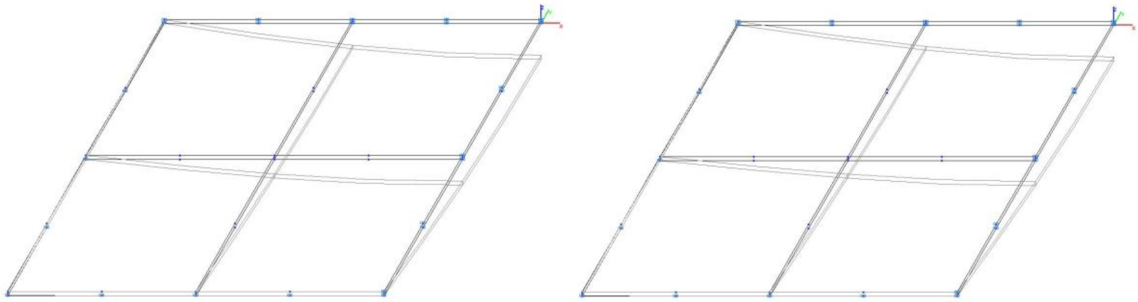


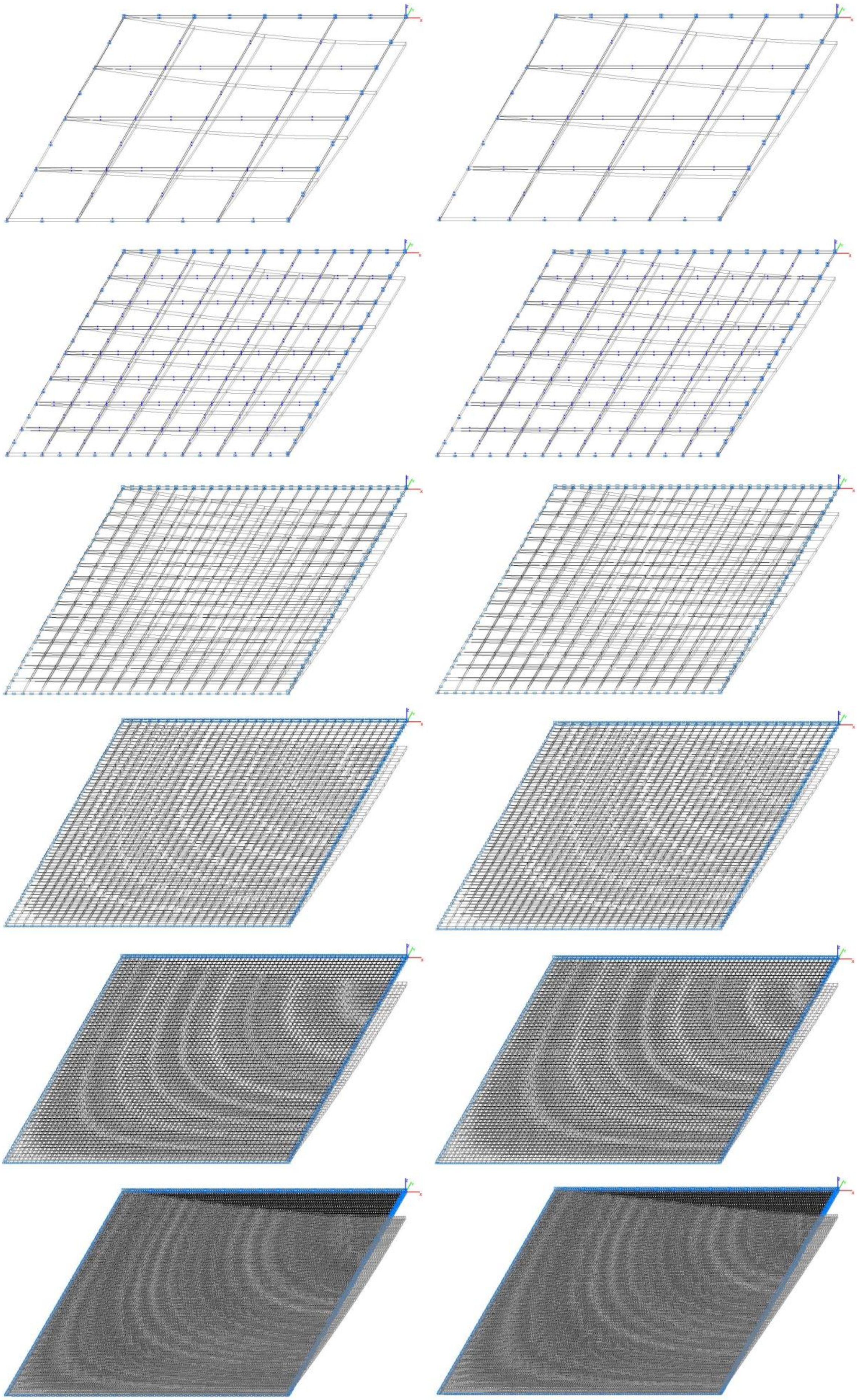
Model 5. Values of the transverse displacements in the center of the simply supported square plate w_q and w_P (m, m)



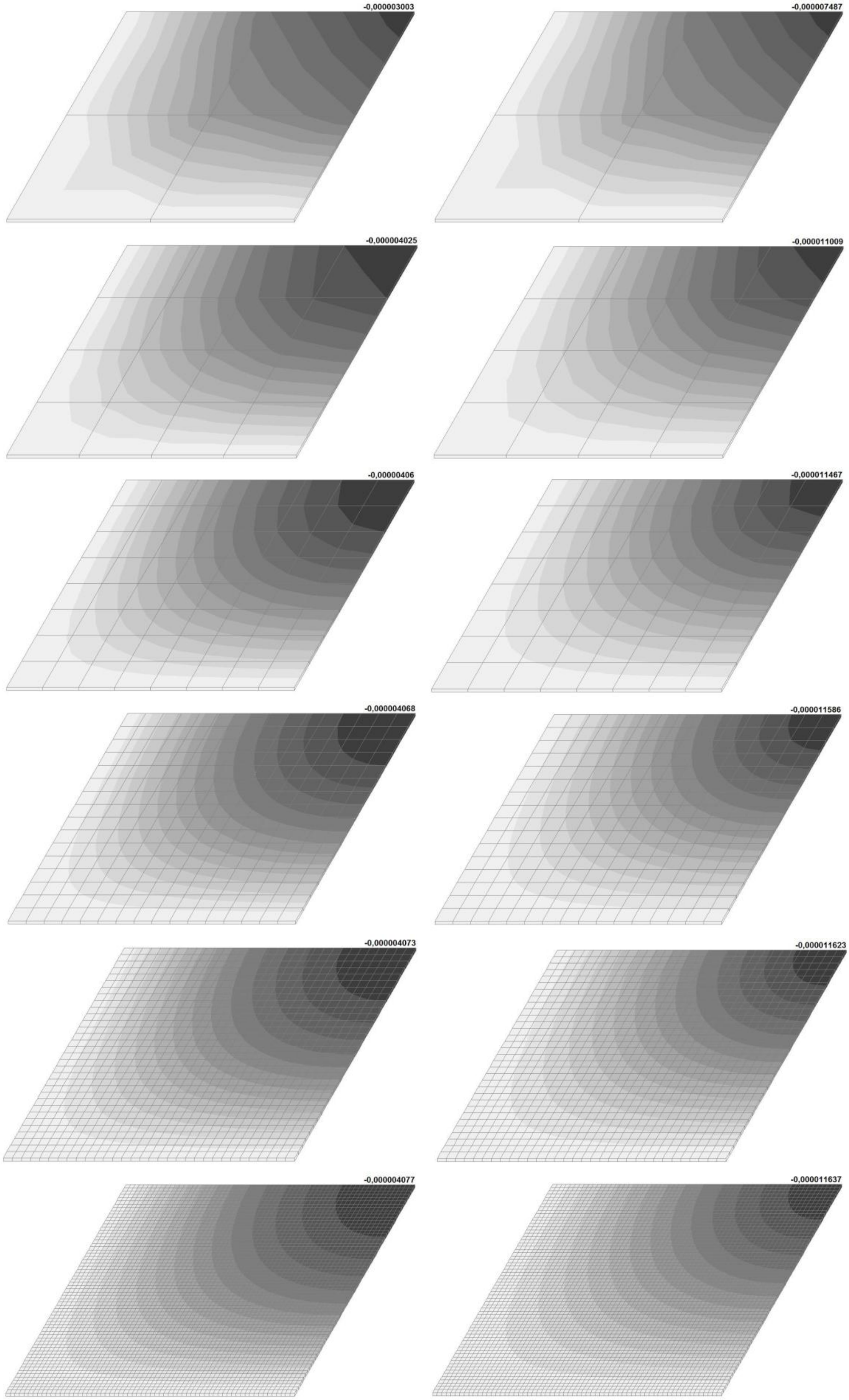


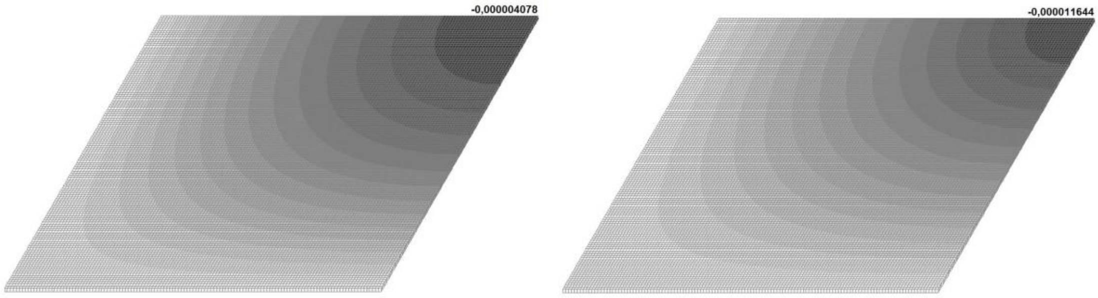
Model 6. Design model





Model 6. Deformed model


-0,000003003
-0,000007487
-0,000004025
-0,000011009
-0,00000406
-0,000011467
-0,000004068
-0,000011586
-0,000004073
-0,000011623
-0,000004077
-0,000011637



Model 6. Values of the transverse displacements in the center of the simply supported square plate w_q and w_P (m, m)

Comparison of solutions:

Transverse displacements in the center of the simply supported flat square plate w_q from the transverse load q uniformly distributed over the entire area

Model	Finite element mesh	Theory	SCAD	Deviation, %
1 (Member type 42)	2x2	4.062	3.808	6.25
	4x4		3.998	1.58
	8x8		4.046	0.39
2 (Member type 44)	2x2	4.062	3.885	4.36
	4x4		4.012	1.23
	8x8		4.049	0.32
3 (Member type 45)	2x2	4.062	4.062	0.00
	4x4		4.062	0.00
	8x8		4.062	0.00
4 (Member type 50)	2x2	4.062	4.062	0.00
	4x4		4.062	0.00
	8x8		4.062	0.00
5 (Member type 36)	2x2	$4.062 \cdot 10^{-6}$	$0.009 \cdot 10^{-6}$	99.78
	4x4		$0.037 \cdot 10^{-6}$	99.09
	8x8		$0.144 \cdot 10^{-6}$	96.45
	16x16		$0.509 \cdot 10^{-6}$	87.47
	32x32		$1.308 \cdot 10^{-6}$	67.80
	64x64		$2.471 \cdot 10^{-6}$	39.17
	128x128		$3.061 \cdot 10^{-6}$	24.64
6 (Member type 37)	2x2	$4.062 \cdot 10^{-6}$	$3.003 \cdot 10^{-6}$	26.07
	4x4		$4.025 \cdot 10^{-6}$	0.91
	8x8		$4.060 \cdot 10^{-6}$	0.05
	16x16		$4.068 \cdot 10^{-6}$	0.15
	32x32		$4.073 \cdot 10^{-6}$	0.27
	64x64		$4.077 \cdot 10^{-6}$	0.37
	128x128		$4.078 \cdot 10^{-6}$	0.39

Transverse displacements in the center of the simply supported flat square plate w_P from the concentrated shear force P applied in the center

Model	Finite element mesh	Theory	SCAD	Deviation, %
1 (Member type 42)	2x2	11.600	10.236	11.76
	4x4		11.205	3.41
	8x8		11.490	0.95
2 (Member type 44)	2x2	11.600	10.907	5.97
	4x4		11.383	1.87
	8x8		11.537	0.54
3 (Member type 45)	2x2	11.600	11.574	0.22
	4x4		11.594	0.05
	8x8		11.599	0.01

Model	Finite element mesh	Theory	SCAD	Deviation, %
4 (Member type 50)	2x2	11.600	11.540	0.52
	4x4		11.586	0.12
	8x8		11.597	0.02
5 (Member type 36)	2x2	$11.600 \cdot 10^{-6}$	$0.028 \cdot 10^{-6}$	99.76
	4x4		$0.104 \cdot 10^{-6}$	99.10
	8x8		$0.394 \cdot 10^{-6}$	96.60
	16x16		$1.393 \cdot 10^{-6}$	87.98
	32x32		$3.595 \cdot 10^{-6}$	69.01
	64x64		$6.935 \cdot 10^{-6}$	40.21
	128x128		$8.691 \cdot 10^{-6}$	25.08
6 (Member type 37)	2x2	$11.600 \cdot 10^{-6}$	$7.487 \cdot 10^{-6}$	35.46
	4x4		$11.009 \cdot 10^{-6}$	5.09
	8x8		$11.467 \cdot 10^{-6}$	1.15
	16x16		$11.586 \cdot 10^{-6}$	0.12
	32x32		$11.623 \cdot 10^{-6}$	0.20
	64x64		$11.637 \cdot 10^{-6}$	0.32
	128x128		$11.644 \cdot 10^{-6}$	0.38

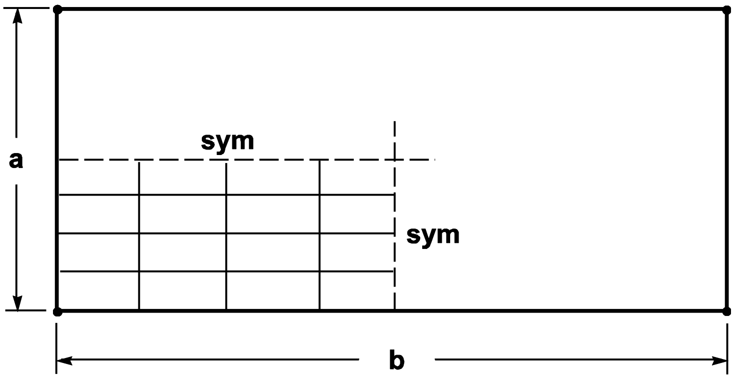
Notes: In the analytical solution the values of the transverse displacements in the center of the simply supported flat square plate w_q and w_P from the respective actions are determined according to the following formulas:

$$w_q = \frac{4 \cdot q \cdot a^4}{\pi^5 \cdot D} \cdot \sum_{m=1}^{\infty} \left\{ \frac{1}{m^5} \cdot \left[1 - \frac{\frac{m \cdot \pi \cdot b}{2 \cdot a} \cdot th\left(\frac{m \cdot \pi \cdot b}{2 \cdot a} \right) + 2}{2 \cdot ch\left(\frac{m \cdot \pi \cdot b}{2 \cdot a} \right)} \right] \cdot sin\left(\frac{m \cdot \pi}{2} \right) \right\};$$

$$w_P = \frac{P \cdot a^2}{2 \cdot \pi^3 \cdot D} \cdot \sum_{m=1}^{\infty} \left\{ \frac{1}{m^3} \cdot \left[th\left(\frac{m \cdot \pi \cdot b}{2 \cdot a} \right) - \frac{\frac{m \cdot \pi \cdot b}{2 \cdot a}}{ch^2\left(\frac{m \cdot \pi \cdot b}{2 \cdot a} \right)} \right] \cdot sin^2\left(\frac{m \cdot \pi}{2} \right) \right\}, \text{ where:}$$

$$D = \frac{E \cdot h^3}{12 \cdot (1 - \nu^2)}.$$

Flat Square Plate Clamped along the Outer Edges and Subjected to a Transverse Load Uniformly Distributed over the Entire Area and a Concentrated Shear Force Applied in the Center



Objective: Check of the obtained values of the transverse displacements in the center of a flat square plate clamped along the outer edges and subjected to a transverse load uniformly distributed over the entire area and a concentrated shear force applied in the center.

Initial data files:

File name	Description
Bending_of_square_flat_plate_Clamped_supported_Shell_42_Mesh_2x2.SPR Bending_of_square_flat_plate_Clamped_supported_Shell_42_Mesh_4x4.SPR Bending_of_square_flat_plate_Clamped_supported_Shell_42_Mesh_8x8.SPR	Design model with the elements of type 42 for meshes 2x2, 4x4, 8x8
Bending_of_square_flat_plate_Clamped_supported_Shell_44_Mesh_2x2.SPR Bending_of_square_flat_plate_Clamped_supported_Shell_44_Mesh_4x4.SPR Bending_of_square_flat_plate_Clamped_supported_Shell_44_Mesh_8x8.SPR	Design model with the elements of type 44 for meshes 2x2, 4x4, 8x8
Bending_of_square_flat_plate_Simply_supported_Shell_45_Mesh_2x2.SPR Bending_of_square_flat_plate_Clamped_supported_Shell_45_Mesh_4x4.SPR Bending_of_square_flat_plate_Clamped_supported_Shell_45_Mesh_8x8.SPR	Design model with the elements of type 45 for meshes 2x2, 4x4, 8x8
Bending_of_square_flat_plate_Clamped_supported_Shell_50_Mesh_2x2.SPR Bending_of_square_flat_plate_Clamped_supported_Shell_50_Mesh_4x4.SPR Bending_of_square_flat_plate_Clamped_supported_Shell_50_Mesh_8x8.SPR	Design model with the elements of type 50 for meshes 2x2, 4x4, 8x8
Bending_of_square_flat_plate_Clamped_supported_Solid_36_Mesh_2x2.SPR Bending_of_square_flat_plate_Clamped_supported_Solid_36_Mesh_4x4.SPR Bending_of_square_flat_plate_Clamped_supported_Solid_36_Mesh_8x8.SPR Bending_of_square_flat_plate_Clamped_supported_Solid_36_Mesh_16x16.SPR Bending_of_square_flat_plate_Clamped_supported_Solid_36_Mesh_32x32.SPR Bending_of_square_flat_plate_Clamped_supported_Solid_36_Mesh_64x64.SPR Bending_of_square_flat_plate_Clamped_supported_Solid_36_Mesh_128x128.SPR	Design model with the elements of type 36 for meshes 2x2, 4x4, 8x8, 16x16, 32x32, 64x64, 128x128
Bending_of_square_flat_plate_Clamped_supported_Solid_37_Mesh_2x2.SPR Bending_of_square_flat_plate_Clamped_supported_Solid_37_Mesh_4x4.SPR Bending_of_square_flat_plate_Clamped_supported_Solid_37_Mesh_8x8.SPR Bending_of_square_flat_plate_Clamped_supported_Solid_37_Mesh_16x16.SPR Bending_of_square_flat_plate_Clamped_supported_Solid_37_Mesh_32x32.SPR Bending_of_square_flat_plate_Clamped_supported_Solid_37_Mesh_64x64.SPR Bending_of_square_flat_plate_Clamped_supported_Solid_37_Mesh_128x128.SPR	Design model with the elements of type 37 for meshes 2x2, 4x4, 8x8, 16x16, 32x32, 64x64, 128x128

Problem formulation: The flat square plate clamped along the outer edges is subjected to the transverse load q uniformly distributed over the entire area and the concentrated shear force P applied in the center. Check the obtained values of the transverse displacements in the center of the flat square plate clamped along the outer edges w_q and w_P from the respective actions.

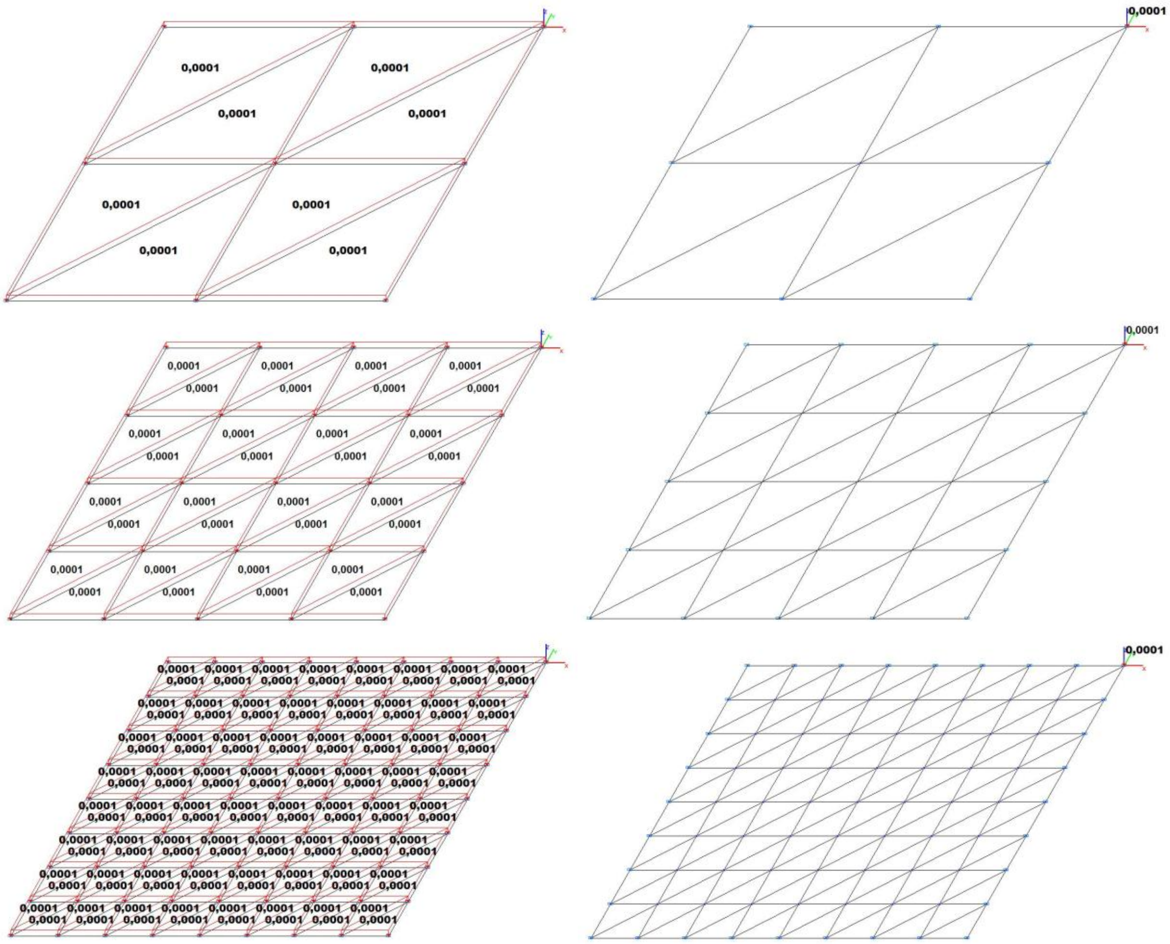
References: R. H. Macneal, R. L. Harder, A proposed standard set of problems to test finite element accuracy, North-Holland, Finite elements in analysis and design, 1, 1985, p. 3-20.
S. Timoshenko, S. Woinowsky-Krieger, Theory of plates and shells, New York, McGraw-Hill,1959, p. 120, 143, 202, 206.

Initial data:

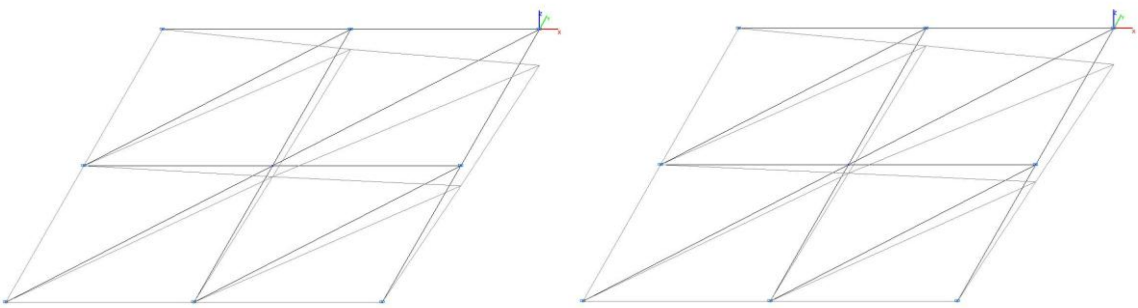
$E = 1.7472 \cdot 10^7$ kPa	- elastic modulus of the plate material;
$\nu = 0.30$	- Poisson's ratio;
a = 2.00 m	- width of the plate;
b = 2.00 m	- length of the plate;
$h = 10^{-4}$ (10^{-2}) m	- thickness of the plate;
$q = 1.0 \cdot 10^{-4}$ kN/m^2	- value of the transverse load uniformly distributed over the entire area of the plate;
$P = 4.0 \cdot 10^{-4}$ kN	- value of the concentrated shear force in the center of the plate.

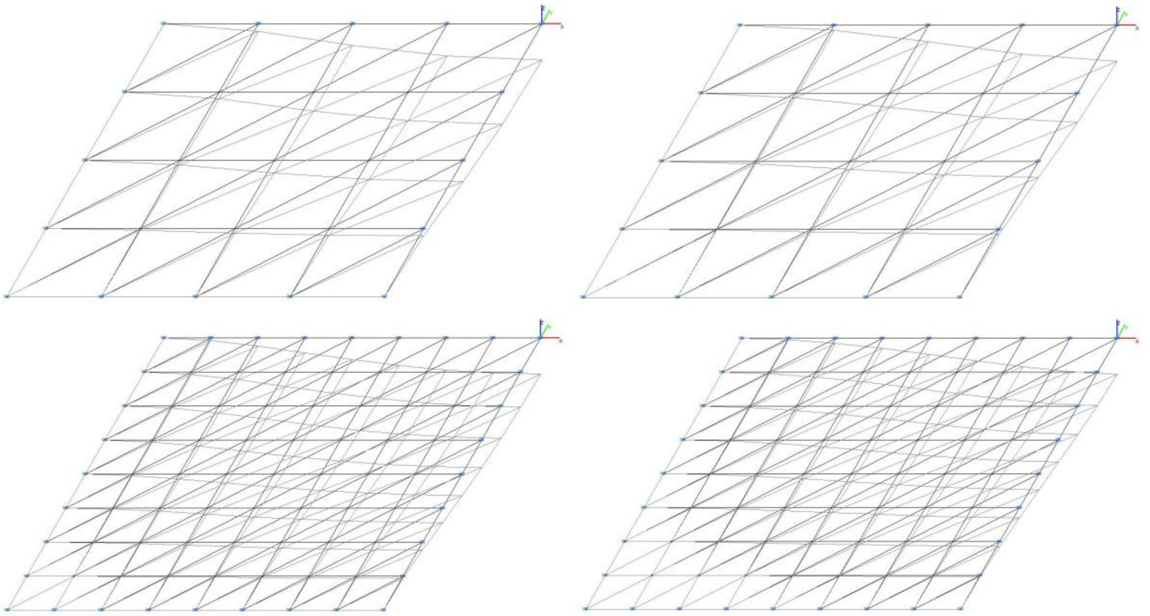
Finite element model: Design model – general type system. Six design models of a quarter of the plate according to the symmetry conditions are considered:
Model 1 – 8, 32, 128 three-node shell elements of type 42 with a regular mesh 2x2, 4x4, 8x8. The thickness of the plate – 10^{-4} m. Boundary conditions are provided by imposing constraints on the nodes of the clamped edges of the plate in the directions of the degrees of freedom X, Y, Z, UX, UY, UZ and constraints according to the symmetry conditions. Number of nodes in the model – 9, 25, 81.
Model 2 – 4, 16, 64 four-node shell elements of type 44 with a regular mesh 2x2, 4x4, 8x8. The thickness of the plate – 10^{-4} m. Boundary conditions are provided by imposing constraints on the nodes of the clamped edges of the plate in the directions of the degrees of freedom X, Y, Z, UX, UY, UZ and constraints according to the symmetry conditions. Number of nodes in the model – 9, 25, 81.
Model 3 – 8, 32, 128 six-node shell elements of type 45 with a regular mesh 2x2, 4x4, 8x8. The thickness of the plate – 10^{-4} m. Boundary conditions are provided by imposing constraints on the nodes of the clamped edges of the plate in the directions of the degrees of freedom X, Y, Z, UX, UY, UZ and constraints according to the symmetry conditions. Number of nodes in the model – 25, 81, 289.
Model 4 – 4, 16, 64 eight-node shell elements of type 50 with a regular mesh 2x2, 4x4, 8x8. The thickness of the plate – 10^{-4} m. Boundary conditions are provided by imposing constraints on the nodes of the clamped edges of the plate in the directions of the degrees of freedom X, Y, Z, UX, UY, UZ and constraints according to the symmetry conditions. Number of nodes in the model – 25, 81, 289.
Model 5 – 4, 16, 64, 256, 1024, 4096, 16384 eight-node isoparametric solid elements of type 36 with a regular mesh 2x2x1, 4x4x1, 8x8x1, 16x16x1, 32x32x1, 64x64x1, 128x128x1. The thickness of the plate – 10^{-2} m. Boundary conditions are provided by imposing constraints on the nodes of the clamped sides of the lower surface of the plate in the directions of the degrees of freedom X, Y, Z, on the nodes of the clamped sides of the upper surface of the plate parallel to the Y axis of the global coordinate system in the direction of the degree of freedom X, on the nodes of the clamped sides of the upper surface of the plate parallel to the X axis of the global coordinate system in the direction of the degree of freedom Y and constraints according to the symmetry conditions. Number of nodes in the model – 18, 50, 162, 578, 2178, 8450, 33282.
Model 6 – 4, 16, 64, 256, 1024, 4096, 16384 twenty-node isoparametric solid elements of type 37 with a regular mesh 2x2x1, 4x4x1, 8x8x1, 16x16x1, 32x32x1, 64x64x1, 128x128x1. The thickness of the plate – 10^{-2} m. Boundary conditions are provided by imposing constraints on the nodes of the clamped sides of the lower surface of the plate in the directions of the degrees of freedom X, Y, Z, on the nodes of the clamped sides of the upper surface of the plate parallel to the Y axis of the global coordinate system in the direction of the degree of freedom X, on the nodes of the clamped sides of the upper surface of the plate parallel to the X axis of the global coordinate system in the direction of the degree of freedom Y and constraints according to the symmetry conditions. Number of nodes in the model – 51, 155, 531, 1955, 7491, 29315, 115971.

Results in SCAD

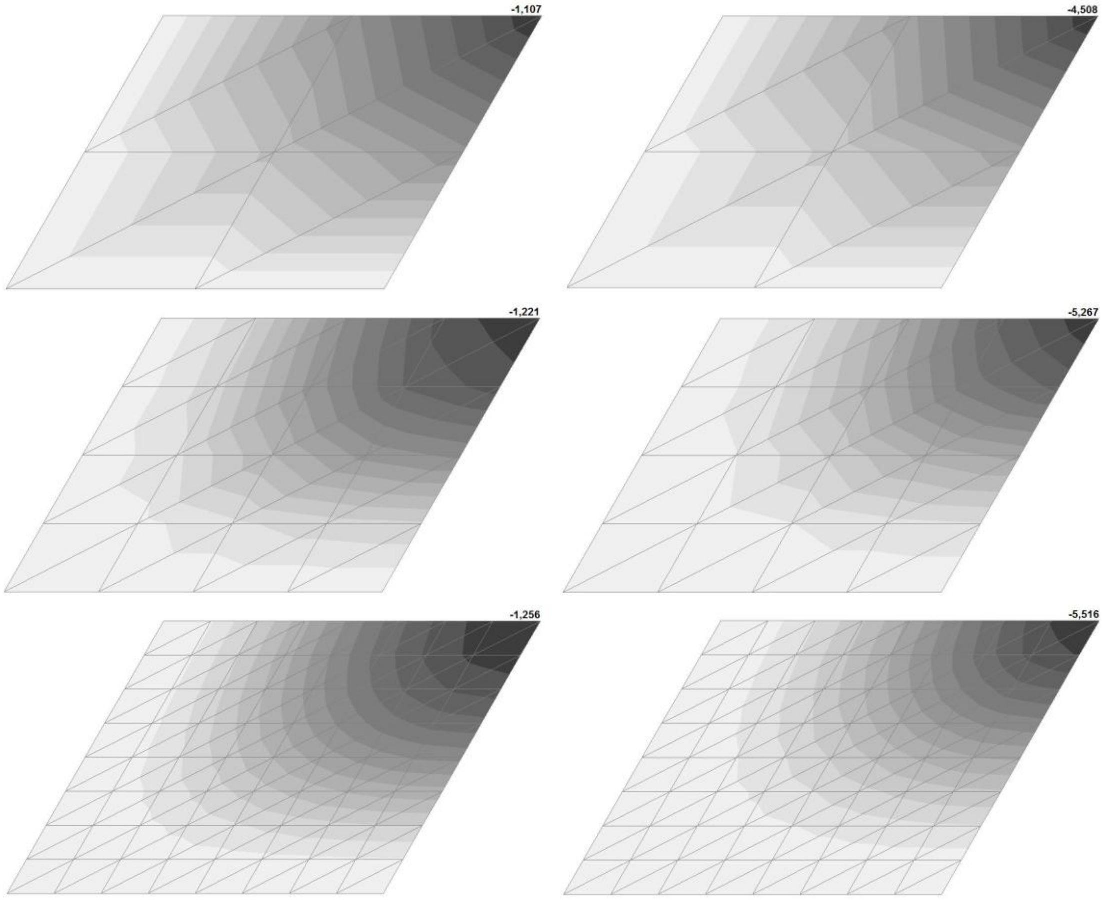


Model 1.Design model

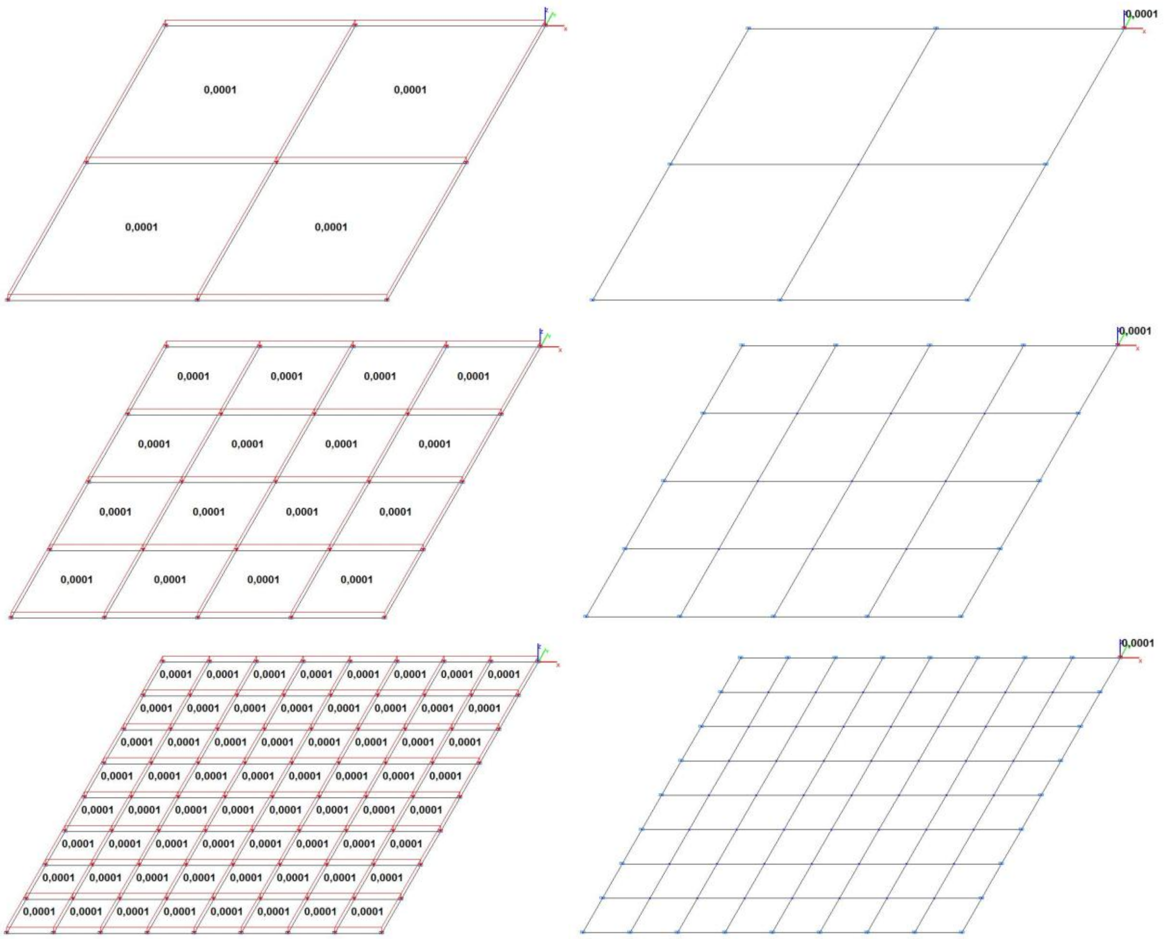




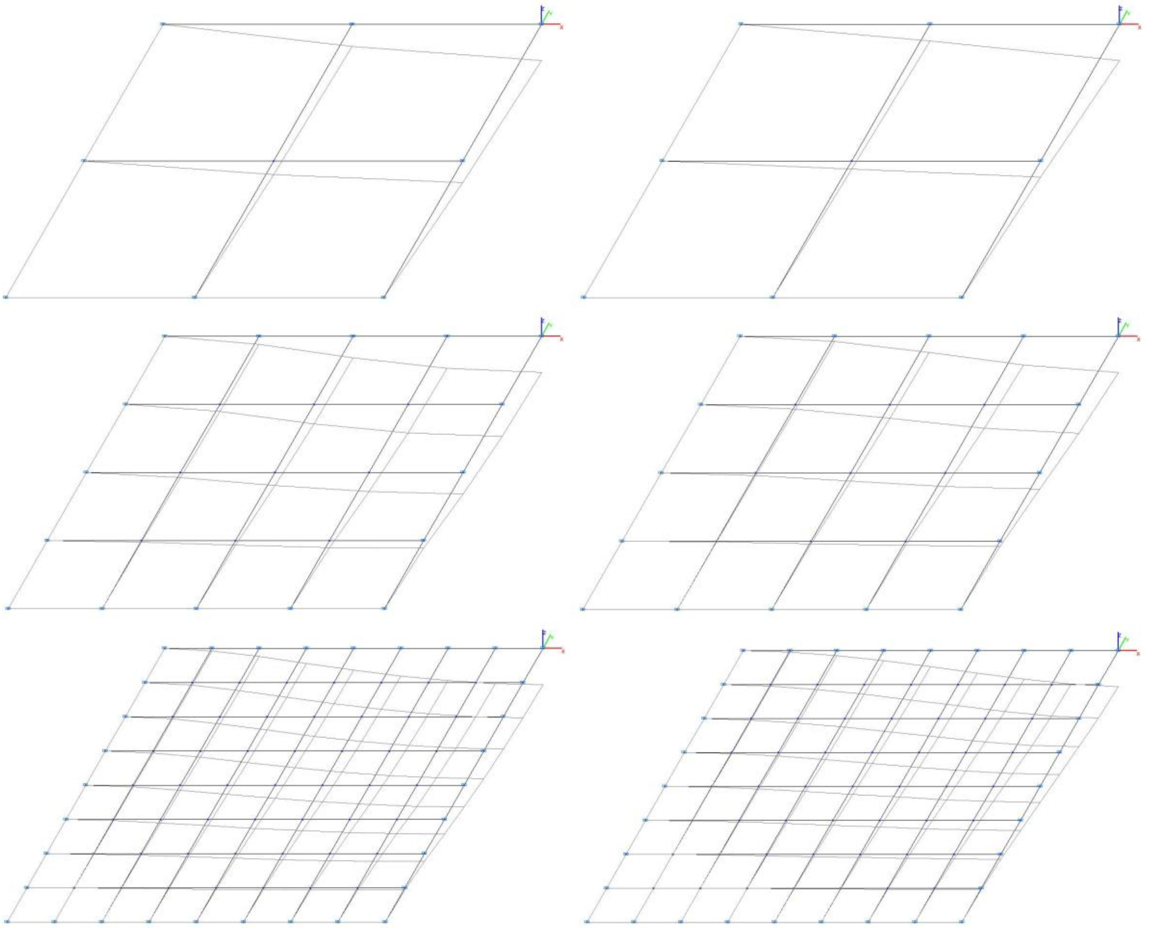
Model 1. Deformed model



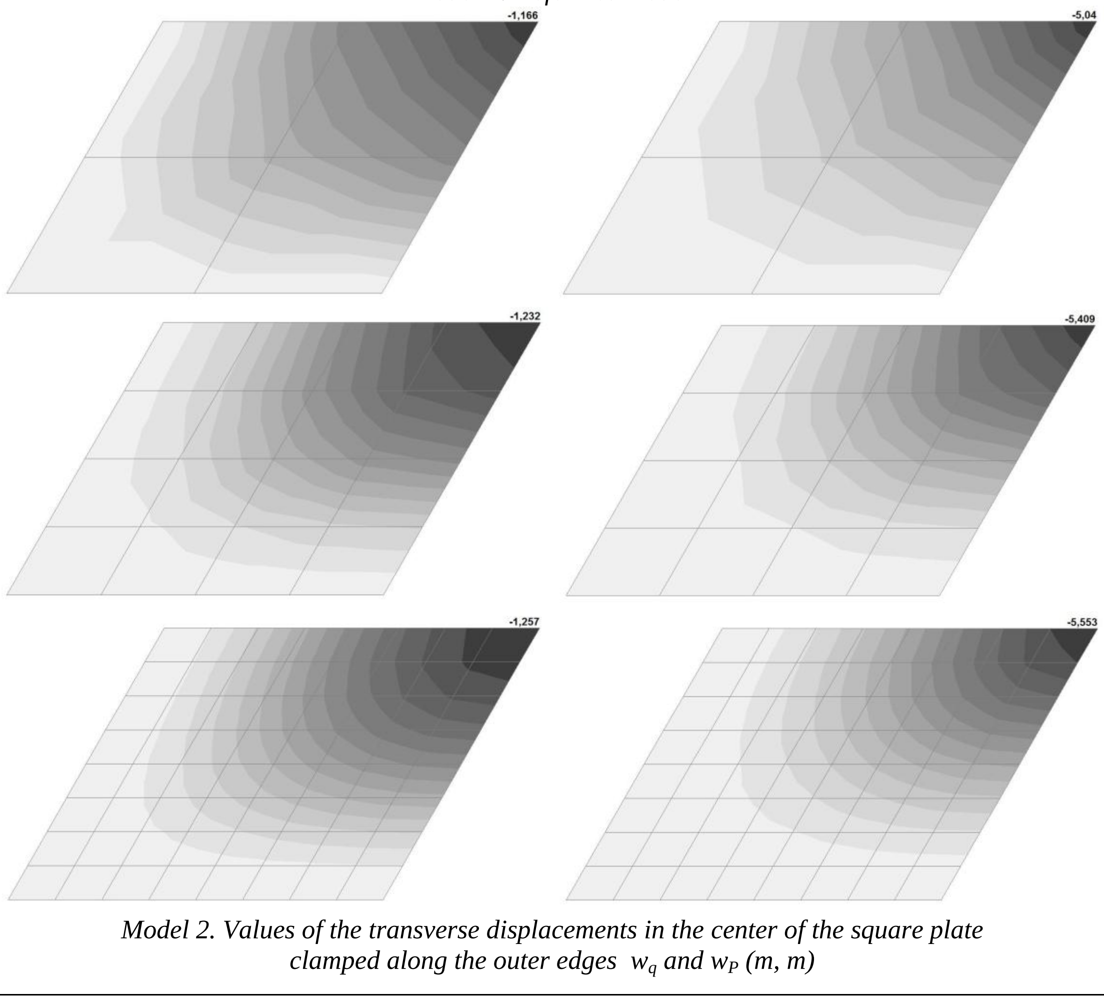
Model 1. Values of the transverse displacements in the center of the square plate clamped along the outer edges w_q and w_P (m, m)



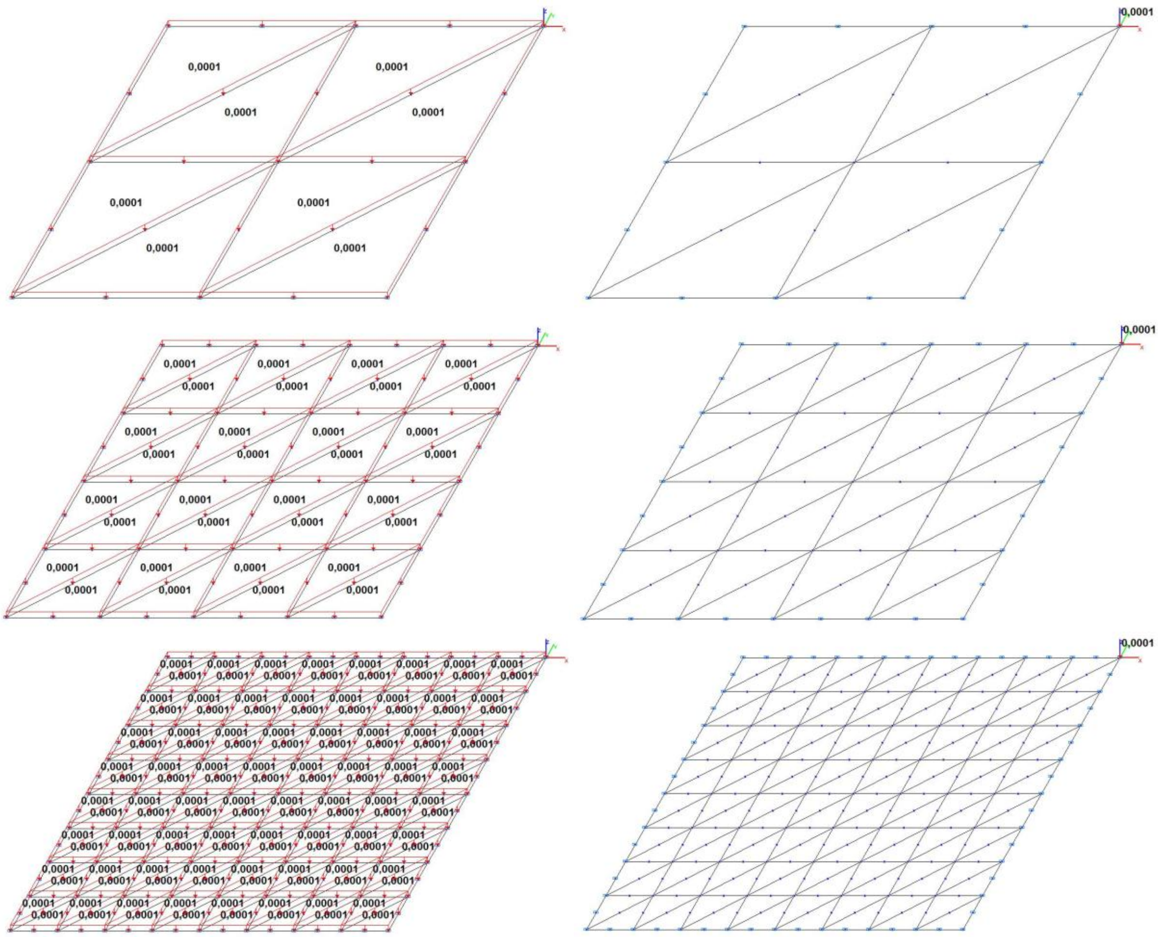
Model 2. Design model



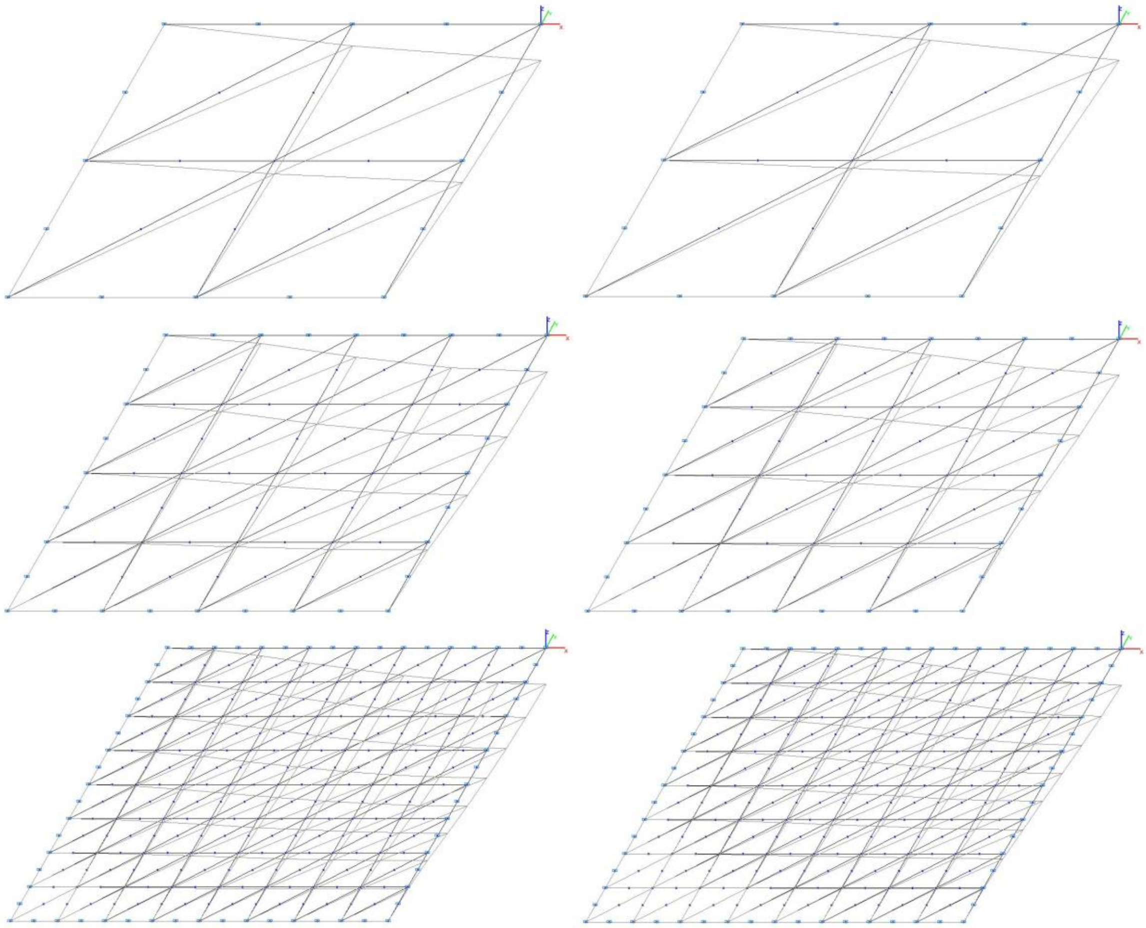
Model 2. Deformed model



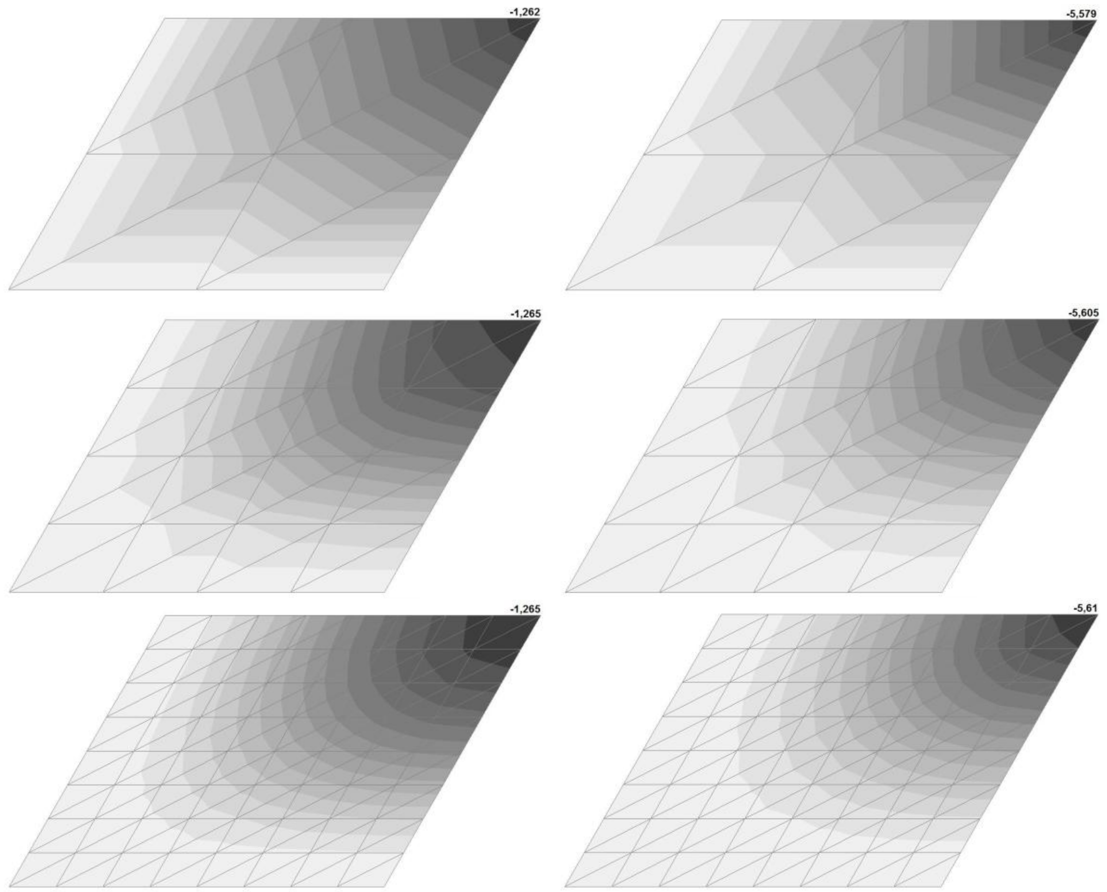
Model 2. Values of the transverse displacements in the center of the square plate clamped along the outer edges w_q and w_P (m, m)



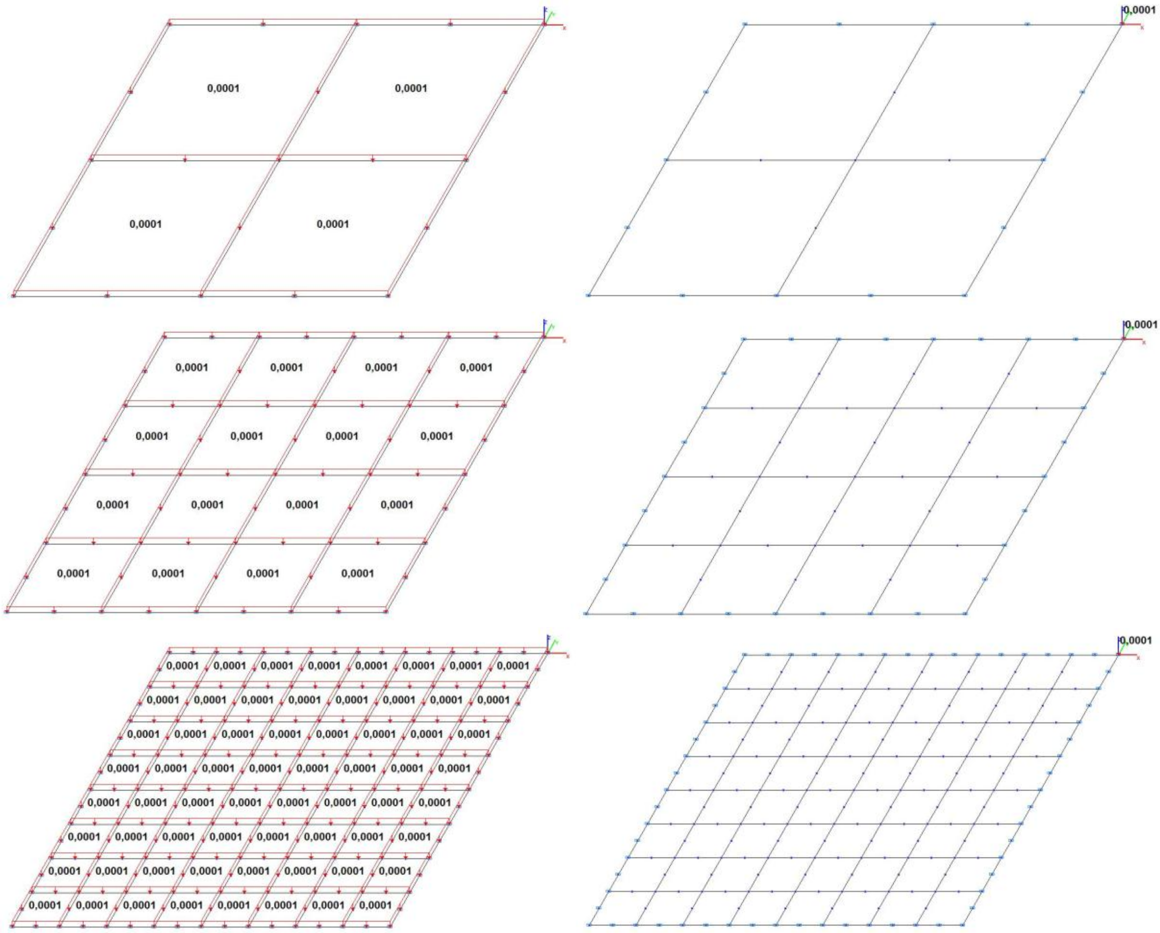
Model 3. Design model



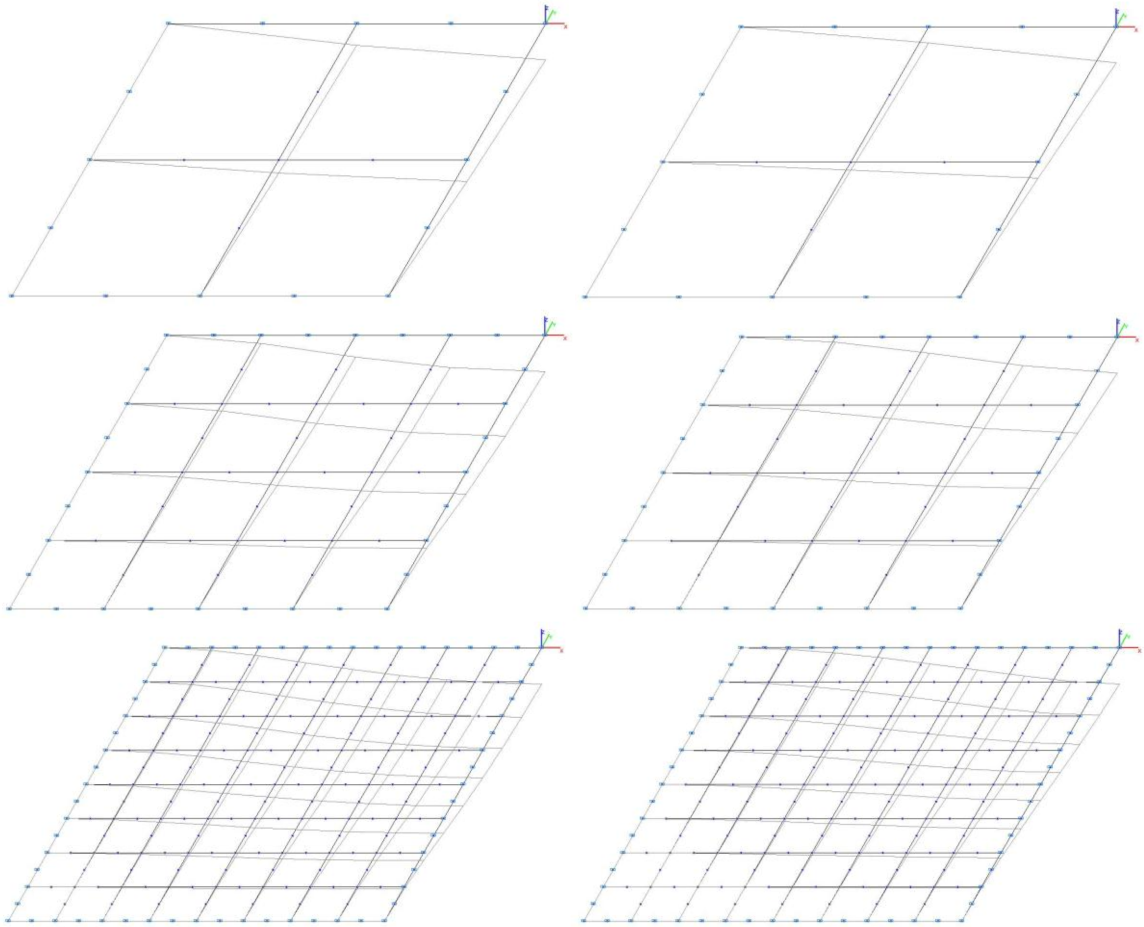
Model 3. Deformed model



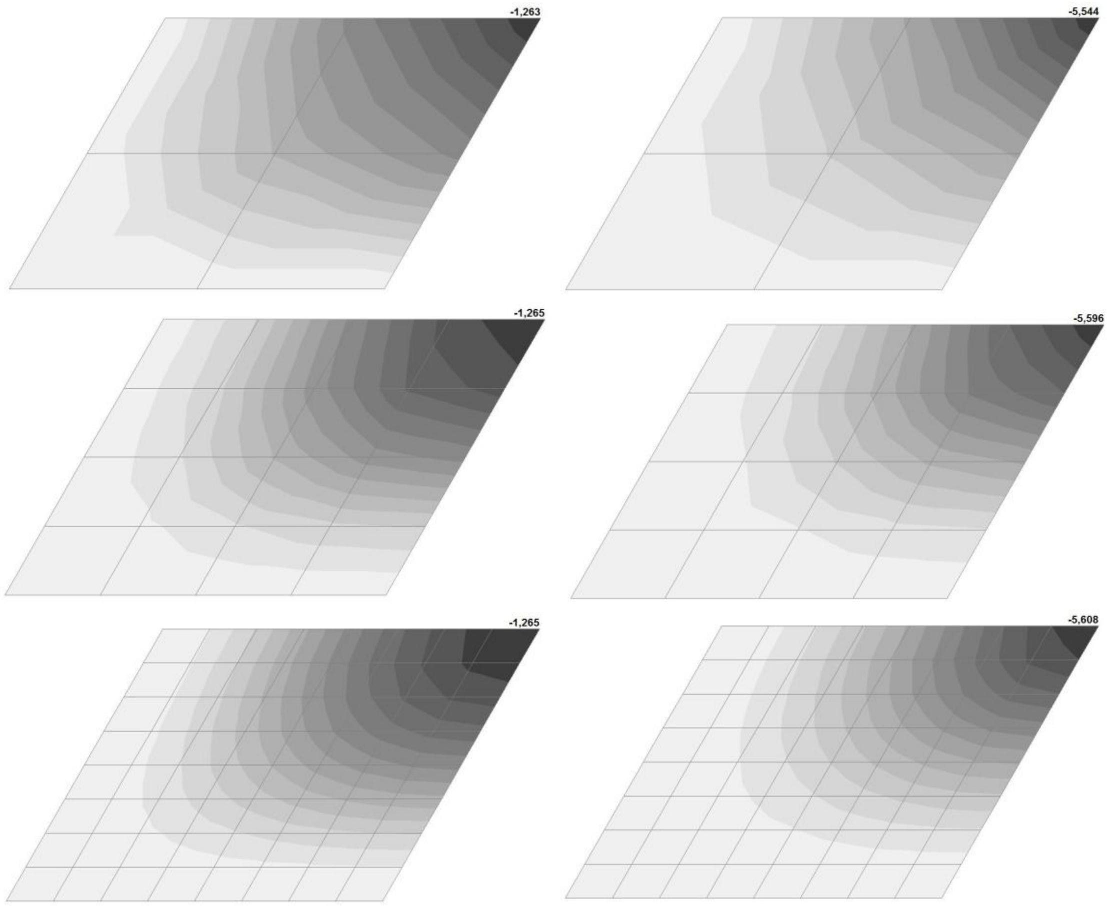
Model 3. Values of the transverse displacements in the center of the square plate clamped along the outer edges w_q and w_P (m, m)



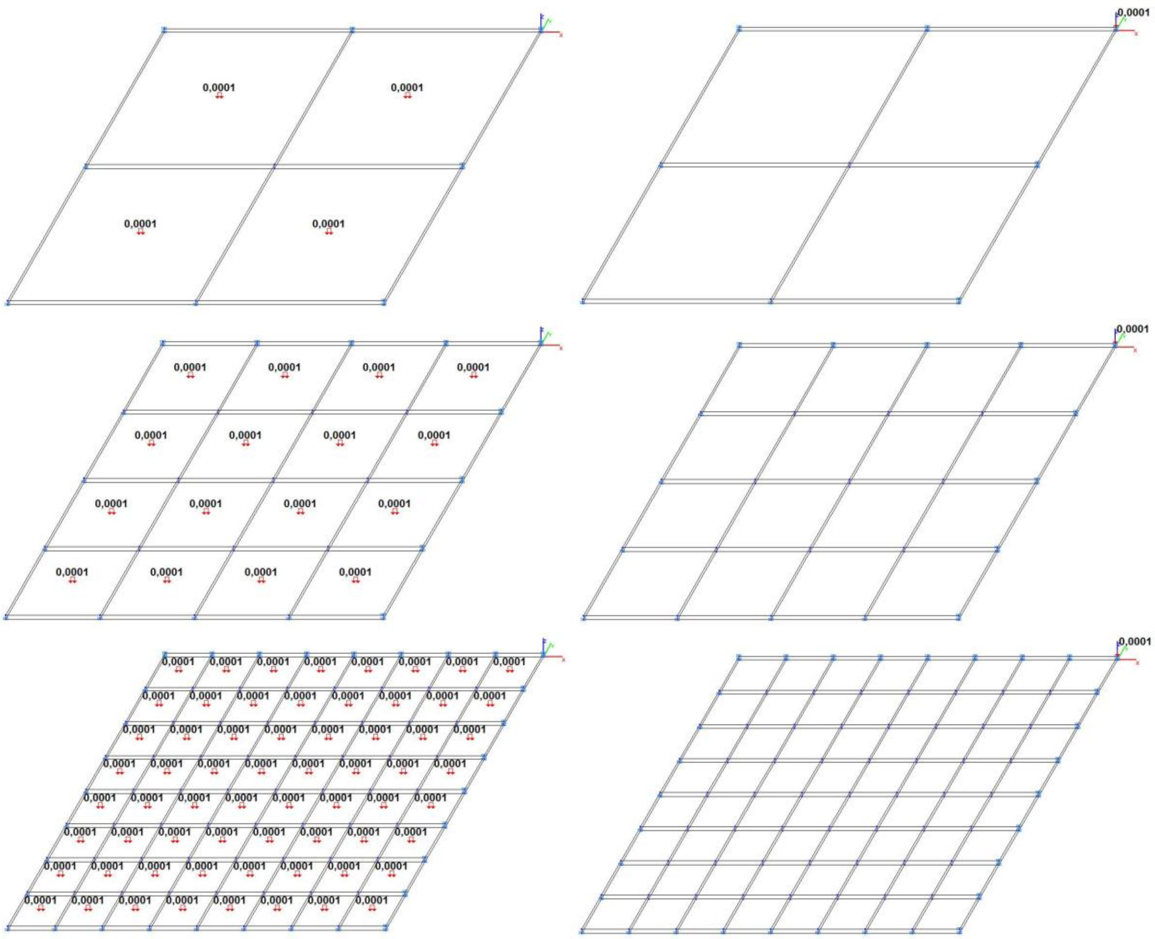
Model 4. Design model

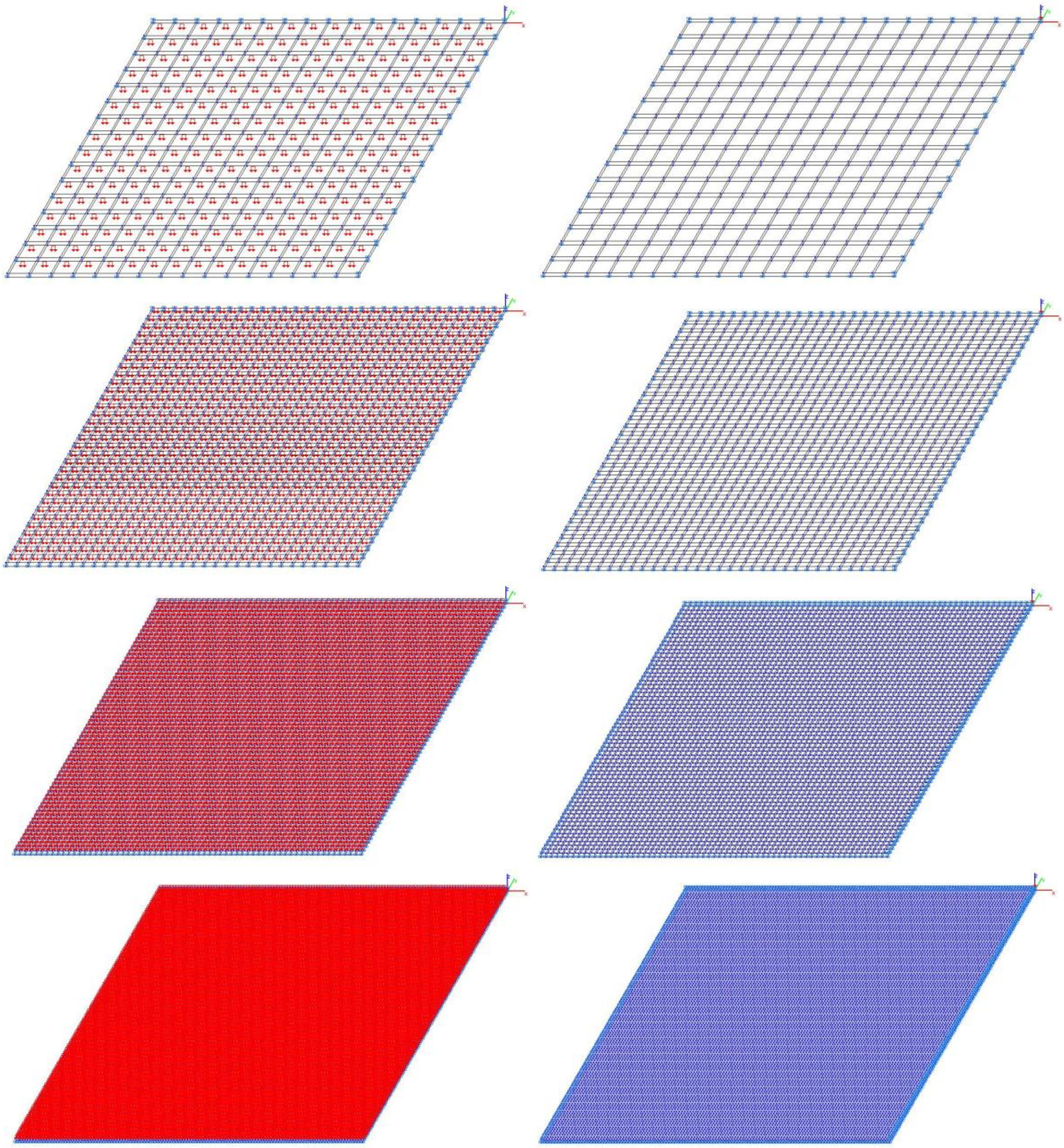


Model 4. Deformed model

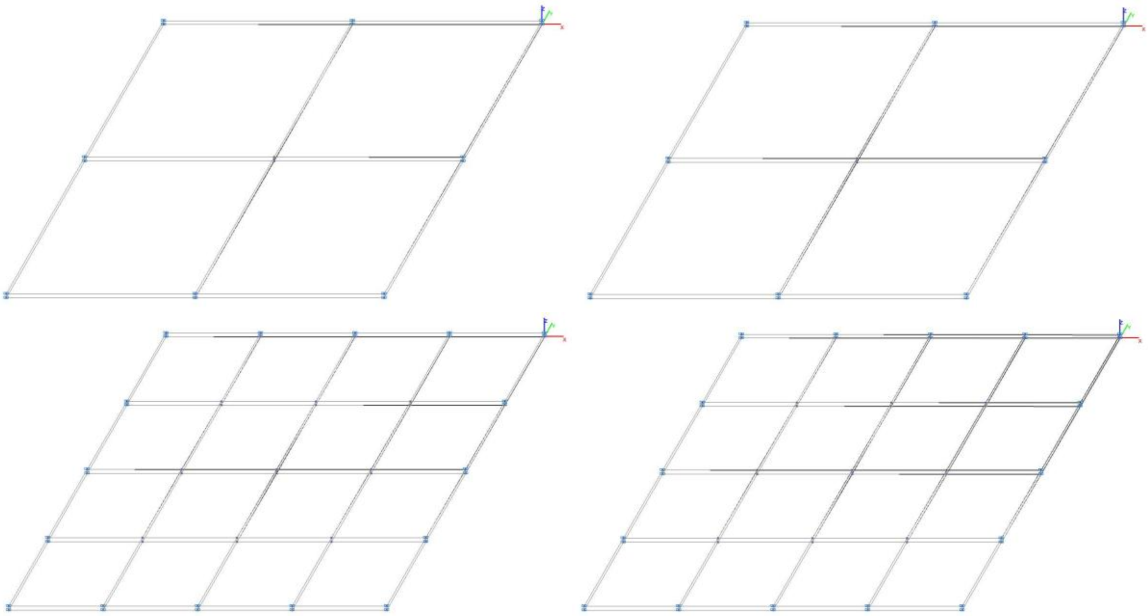


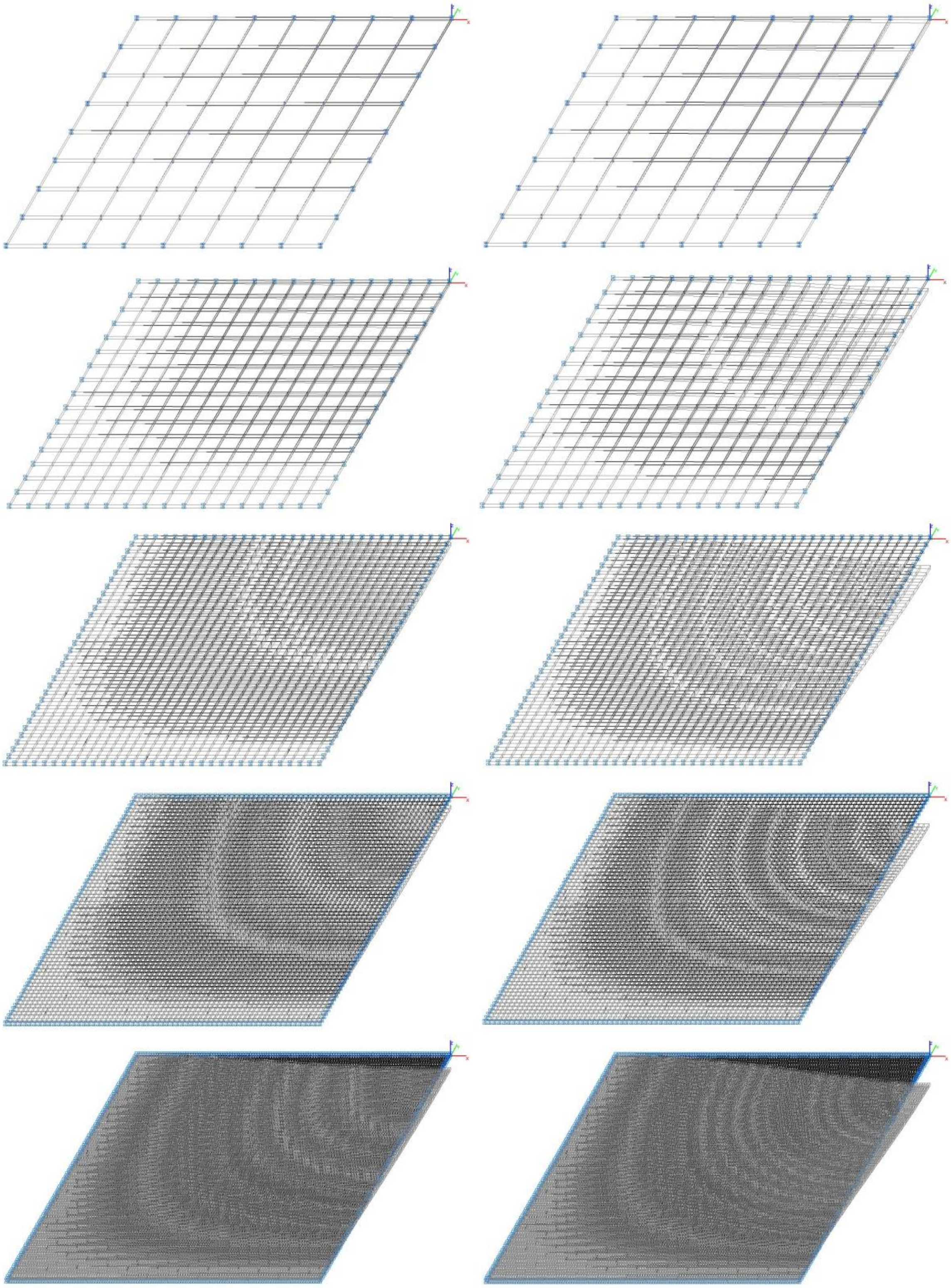
Model 4. Values of the transverse displacements in the center of the square plate clamped along the outer edges w_q *and* w_P *(m, m)*



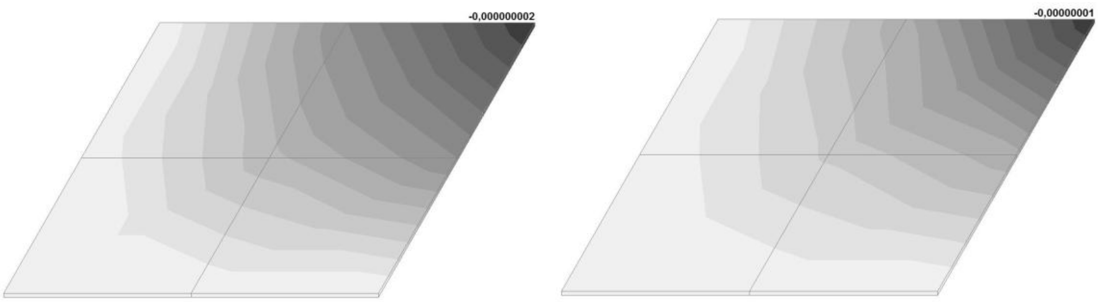


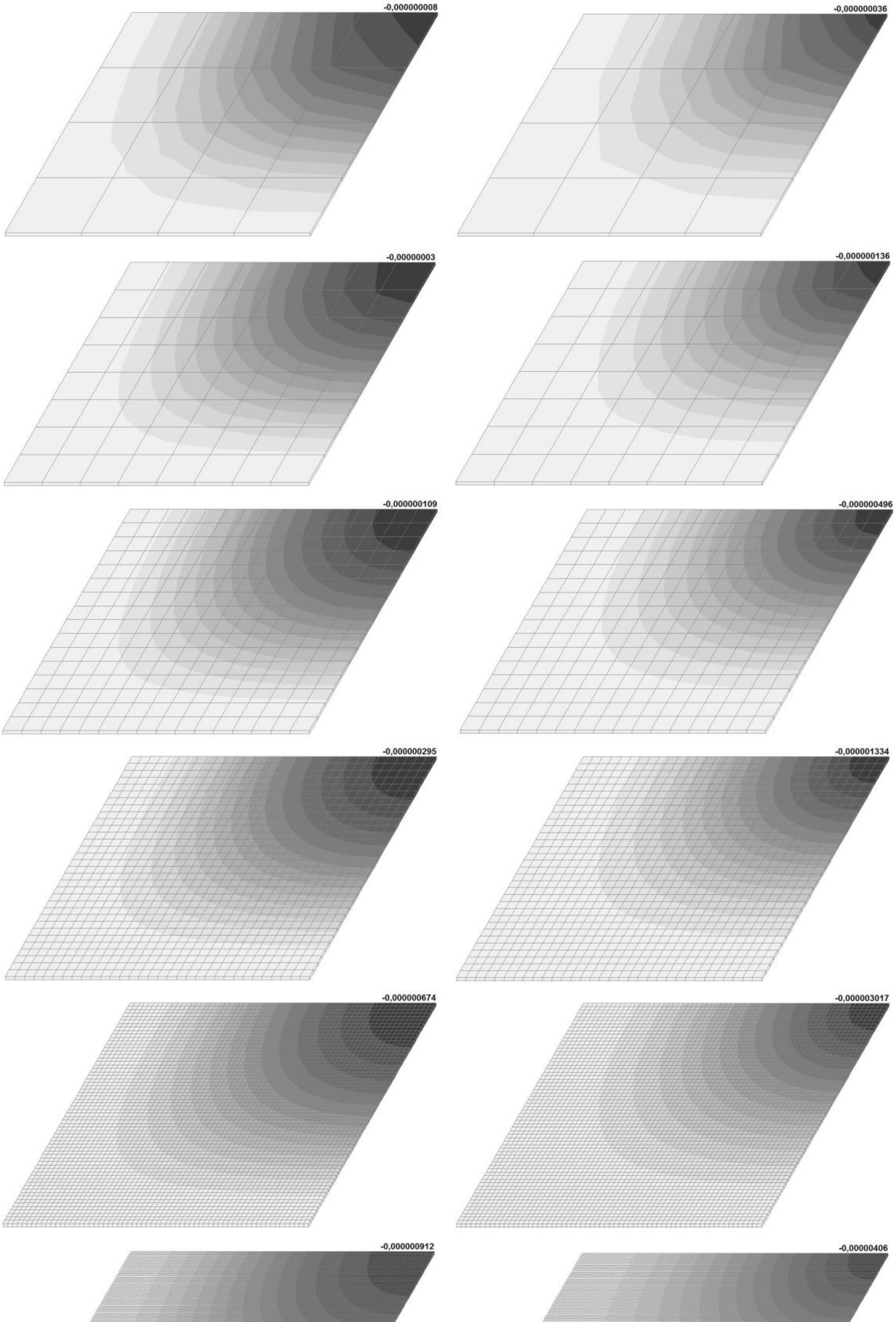
Model 5. Design model



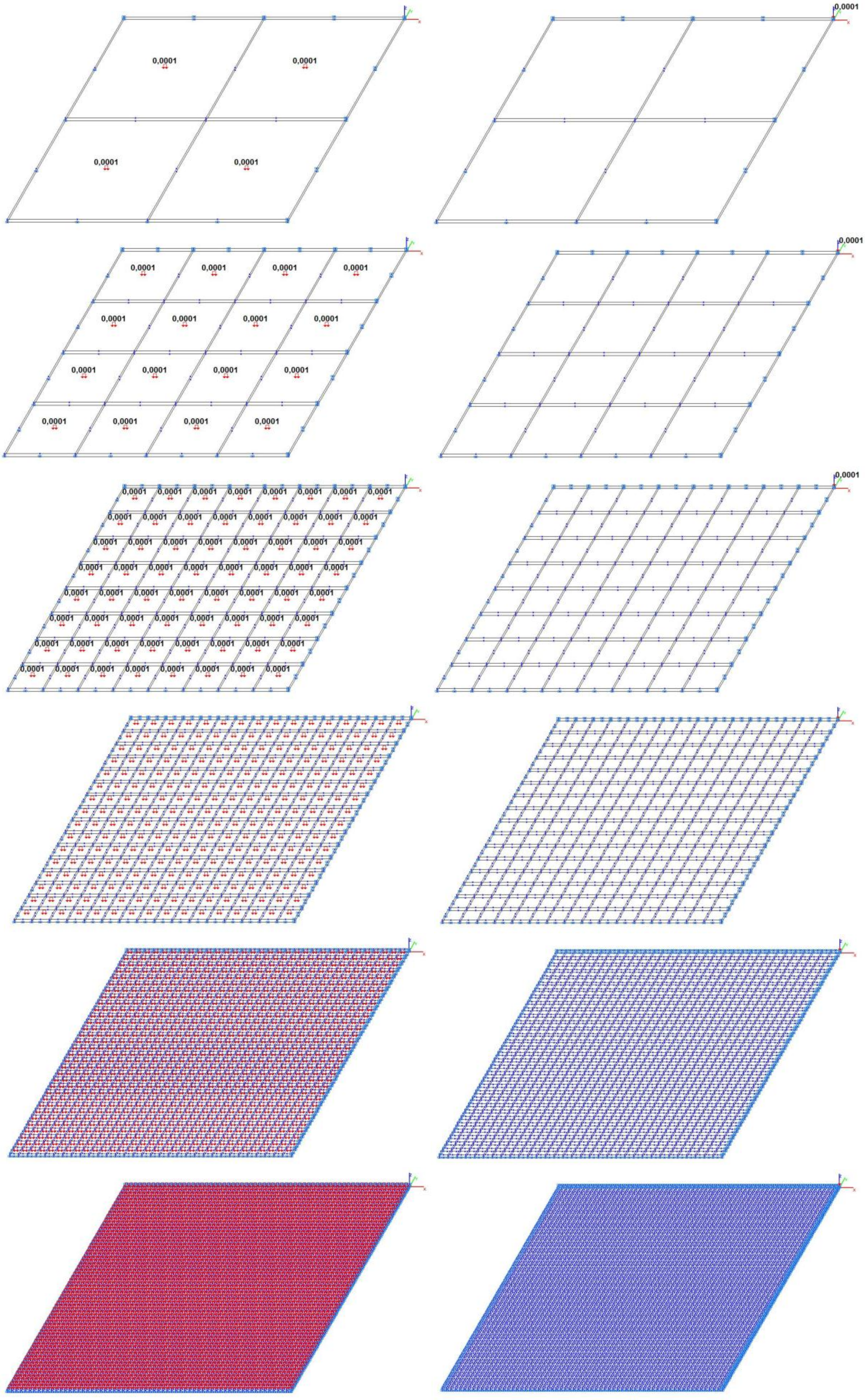


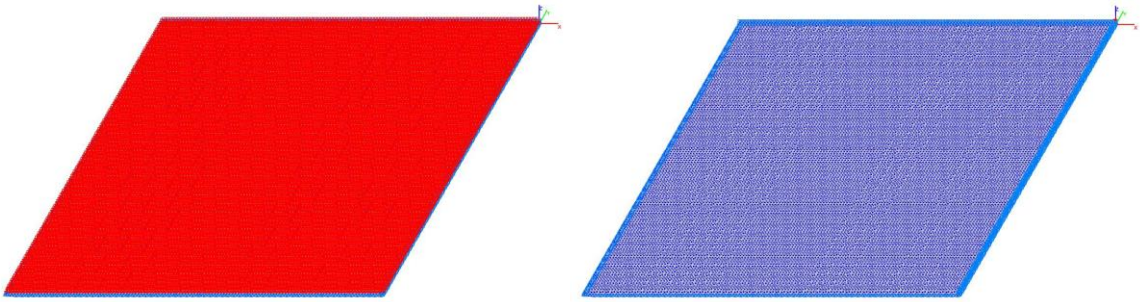
Model 5. Deformed model



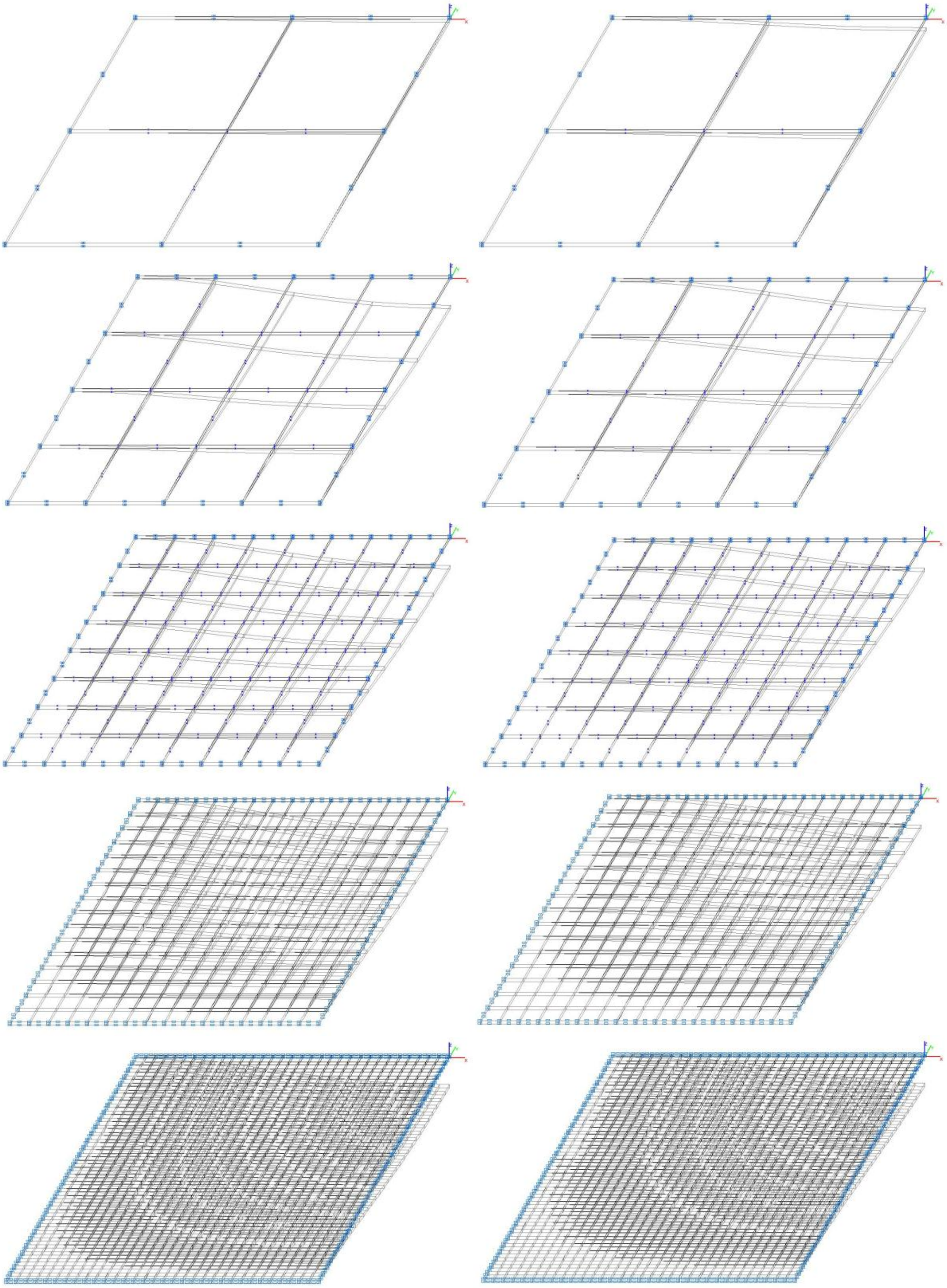


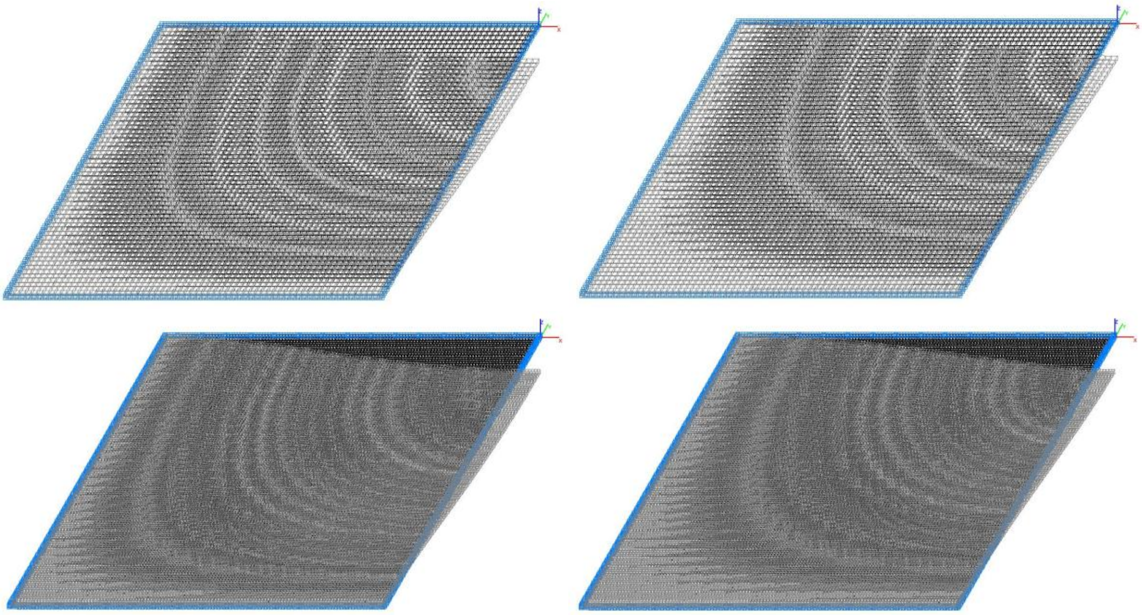
Model 5. Values of the transverse displacements in the center of the square plate clamped along the outer edges w_q and w_P (m, m)


0,0001
0,0001
0,0001
0,0001
0,0001

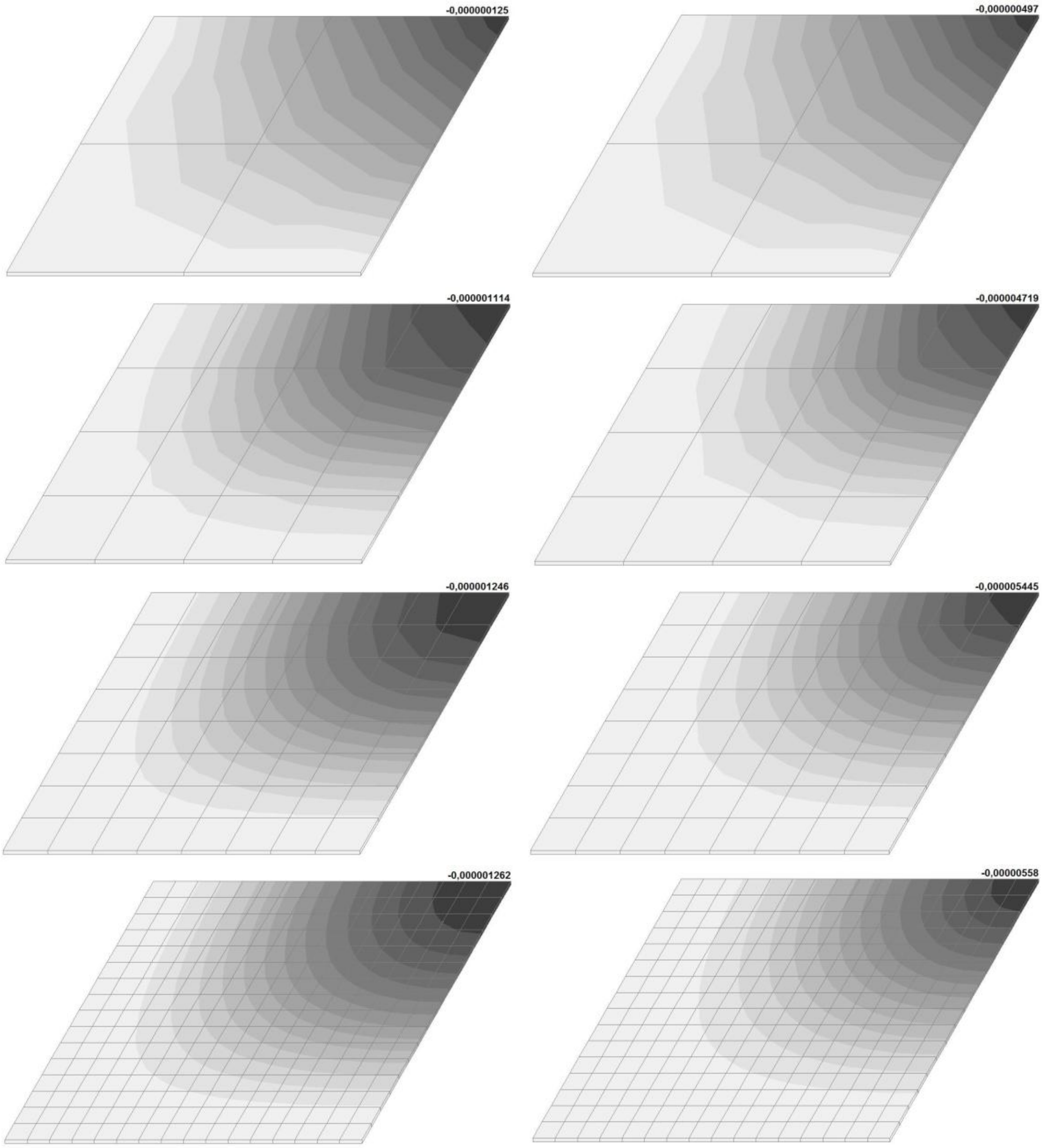


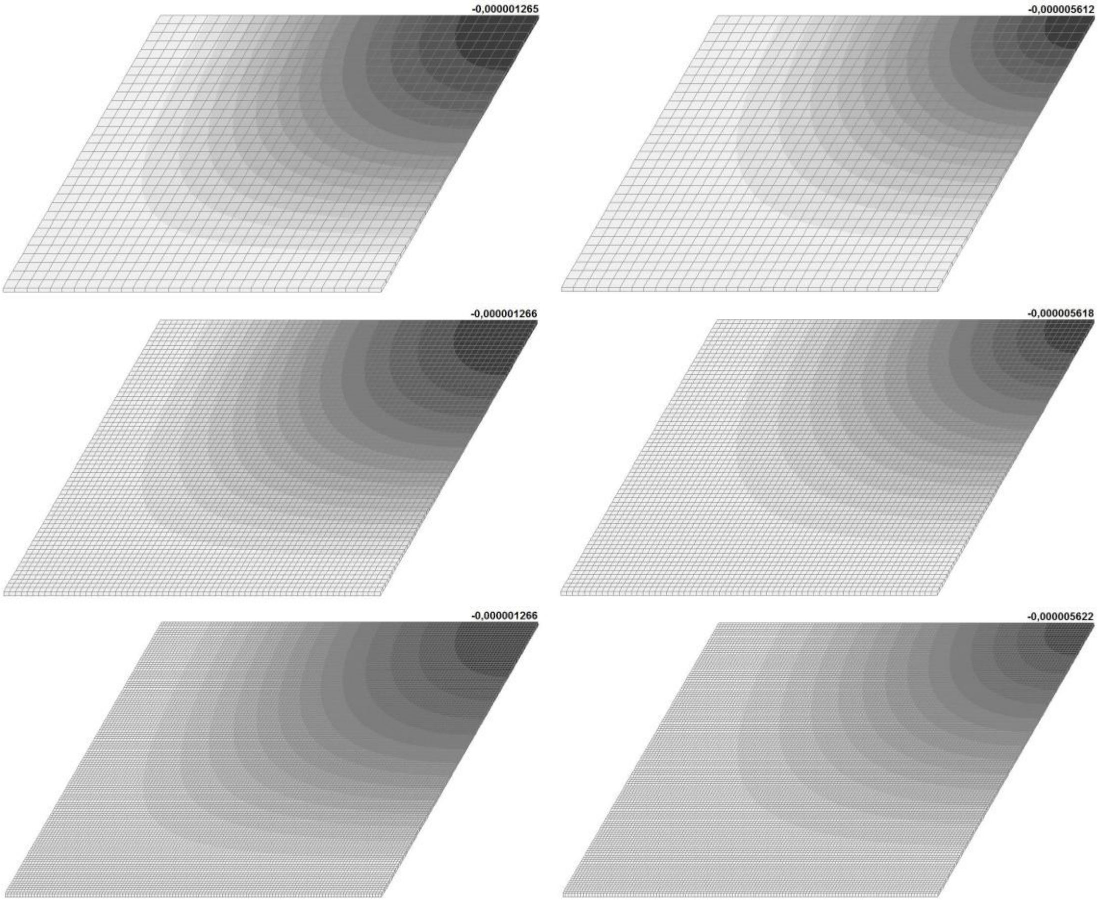
Model 6. Design model





Model 6. Deformed model





Model 6. Values of the transverse displacements in the center of the square plate clamped along the outer edges w_q and w_P (m, m)

Comparison of solutions:

Transverse displacements in the center of the flat square plate clamped along the outer edges w_q from the transverse load q uniformly distributed over the entire area

Model	Finite element mesh	Theory	SCAD	Deviation, %
1 (Member type 42)	2x2	1.265	1.107	12.49
	4x4		1.221	3.48
	8x8		1.256	0.71
2 (Member type 44)	2x2	1.265	1.166	7.83
	4x4		1.232	2.61
	8x8		1.257	0.63
3 (Member type 45)	2x2	1.265	1.262	0.24
	4x4		1.265	0.00
	8x8		1.265	0.00
4 (Member type 50)	2x2	1.265	1.263	0.16
	4x4		1.265	0.00
	8x8		1.265	0.00
5 (Member type 36)	2x2	$1.265\cdot10^{-6}$	$0.002\cdot10^{-6}$	99.84
	4x4		$0.008\cdot10^{-6}$	99.37
	8x8		$0.030\cdot10^{-6}$	97.63
	16x16		$0.109\cdot10^{-6}$	91.38
	32x32		$0.295\cdot10^{-6}$	76.68
	64x64		$0.674\cdot10^{-6}$	46.72
	128x128		$0.912\cdot10^{-6}$	27.91
6 (Member type 37)	2x2	$1.265\cdot10^{-6}$	$0.125\cdot10^{-6}$	90.12
	4x4		$1.114\cdot10^{-6}$	11.94
	8x8		$1.246\cdot10^{-6}$	1.50

Model	Finite element mesh	Theory	SCAD	Deviation, %
	16x16		$1.262\cdot10^{-6}$	0.24
	32x32		$1.265\cdot10^{-6}$	0.00
	64x64		$1.266\cdot10^{-6}$	0.08
	128x128		$1.266\cdot10^{-6}$	0.08

Transverse displacements in the center of the flat square plate clamped along the outer edges w_P from the concentrated shear force P applied in the center

Model	Finite element mesh	Theory	SCAD	Deviation, %
1 (Member type 42)	2x2	5.612	4.508	19.67
	4x4		5.267	6.15
	8x8		5.516	1.71
2 (Member type 44)	2x2	5.612	5.040	10.19
	4x4		5.409	3.62
	8x8		5.553	1.05
3 (Member type 45)	2x2	5.612	5.579	0.59
	4x4		5.605	0.12
	8x8		5.610	0.04
4 (Member type 50)	2x2	5.612	5.554	1.03
	4x4		5.596	0.29
	8x8		5.608	0.07
5 (Member type 36)	2x2	$5.612\cdot10^{-6}$	$0.010\cdot10^{-6}$	99.82
	4x4		$0.036\cdot10^{-6}$	99.36
	8x8		$0.136\cdot10^{-6}$	97.58
	16x16		$0.496\cdot10^{-6}$	91.16
	32x32		$1.334\cdot10^{-6}$	76.23
	64x64		$3.017\cdot10^{-6}$	46.24
	128x128		$4.060\cdot10^{-6}$	27.66
6 (Member type 37)	2x2	$5.612\cdot10^{-6}$	$0.497\cdot10^{-6}$	91.14
	4x4		$4.719\cdot10^{-6}$	15.91
	8x8		$5.445\cdot10^{-6}$	2.98
	16x16		$5.580\cdot10^{-6}$	0.57
	32x32		$5.612\cdot10^{-6}$	0.00
	64x64		$5.618\cdot10^{-6}$	0.11
	128x128		$5.622\cdot10^{-6}$	0.18

Notes: In the analytical solution the values of the transverse displacements in the center of the flat square plate clamped along the outer edges w_q and w_P from the respective actions are determined according to the following formulas:

$$w_q = \frac{4\cdot q\cdot a^4}{\pi^5\cdot D}\cdot\sum_{m=1}^{M}\left\{\frac{1}{m^5}\cdot\left[1-\frac{\frac{m\cdot\pi\cdot b}{2\cdot a}\cdot th\left(\frac{m\cdot\pi\cdot b}{2\cdot a}\right)+2}{2\cdot ch\left(\frac{m\cdot\pi\cdot b}{2\cdot a}\right)}\right]\cdot\sin\left(\frac{m\cdot\pi}{2}\right)\right\}+$$

$$\frac{a^2}{2\cdot\pi^2\cdot D}\cdot\sum_{m=1}^{M}\left\{E_m\cdot\frac{1}{m^2}\cdot\frac{\frac{m\cdot\pi\cdot b}{2\cdot a}\cdot sh\left(\frac{m\cdot\pi\cdot b}{2\cdot a}\right)}{ch^2\left(\frac{m\cdot\pi\cdot b}{2\cdot a}\right)}\cdot\sin\left(\frac{m\cdot\pi}{2}\right)\right\}+$$

$$\frac{b^2}{2\cdot\pi^2\cdot D}\cdot\sum_{m=1}^{M}\left\{F_m\cdot\frac{1}{m^2}\cdot\frac{\frac{m\cdot\pi\cdot a}{2\cdot b}\cdot sh\left(\frac{m\cdot\pi\cdot a}{2\cdot b}\right)}{ch^2\left(\frac{m\cdot\pi\cdot a}{2\cdot b}\right)}\cdot\sin\left(\frac{m\cdot\pi}{2}\right)\right\}$$

The values of the coefficients E_m and F_m are determined by solving the system of 2·M equations:

$$\left.\begin{aligned}
&\frac{4\cdot q\cdot a^2}{\pi^3}\cdot\frac{1}{i^4}\cdot\left(\frac{\frac{i\cdot\pi\cdot b}{2\cdot a}}{ch^2\left(\frac{i\cdot\pi\cdot b}{2\cdot a}\right)}-th\left(\frac{i\cdot\pi\cdot b}{2\cdot a}\right)\right)-\frac{E_i}{i}\cdot\left(\frac{\frac{i\cdot\pi\cdot b}{2\cdot a}}{ch^2\left(\frac{i\cdot\pi\cdot b}{2\cdot a}\right)}+th\left(\frac{i\cdot\pi\cdot b}{2\cdot a}\right)\right)-\frac{8\cdot i\cdot a}{\pi\cdot b}\cdot\sum_{m=1}^{M}\left[F_m\cdot\frac{1}{m^3}\cdot\frac{1}{\left(\frac{a^2}{b^2}+\frac{i^2}{m^2}\right)^2}\cdot sin^2\left(\frac{m\cdot\pi}{2}\right)\right]\\
&\frac{4\cdot q\cdot b^2}{\pi^3}\cdot\frac{1}{i^4}\cdot\left(\frac{\frac{i\cdot\pi\cdot a}{2\cdot b}}{ch^2\left(\frac{i\cdot\pi\cdot a}{2\cdot b}\right)}-th\left(\frac{i\cdot\pi\cdot a}{2\cdot b}\right)\right)-\frac{F_i}{i}\cdot\left(\frac{\frac{i\cdot\pi\cdot a}{2\cdot b}}{ch^2\left(\frac{i\cdot\pi\cdot a}{2\cdot b}\right)}+th\left(\frac{i\cdot\pi\cdot a}{2\cdot b}\right)\right)-\frac{8\cdot i\cdot b}{\pi\cdot a}\cdot\sum_{m=1}^{M}\left[E_m\cdot\frac{1}{m^3}\cdot\frac{1}{\left(\frac{b^2}{a^2}+\frac{i^2}{m^2}\right)^2}\cdot sin^2\left(\frac{m\cdot\pi}{2}\right)\right]
\end{aligned}\right\};$$

$$\begin{aligned}
w_P=&\frac{P\cdot a^2}{2\cdot\pi^3\cdot D}\cdot\sum_{m=1}^{M}\left\{\frac{1}{m^3}\cdot\left[th\left(\frac{m\cdot\pi\cdot b}{2\cdot a}\right)-\frac{\frac{m\cdot\pi\cdot b}{2\cdot a}}{ch^2\left(\frac{m\cdot\pi\cdot b}{2\cdot a}\right)}\right]\cdot\sin^2\left(\frac{m\cdot\pi}{2}\right)\right\}+\\
&\frac{a^2}{2\cdot\pi^2\cdot D}\cdot\sum_{m=1}^{M}\left\{E_m\cdot\frac{1}{m^2}\cdot\frac{\frac{m\cdot\pi\cdot b}{2\cdot a}\cdot sh\left(\frac{m\cdot\pi\cdot b}{2\cdot a}\right)}{ch^2\left(\frac{m\cdot\pi\cdot b}{2\cdot a}\right)}\cdot\sin\left(\frac{m\cdot\pi}{2}\right)\right\}+\\
&\frac{b^2}{2\cdot\pi^2\cdot D}\cdot\sum_{m=1}^{M}\left\{F_m\cdot\frac{1}{m^2}\cdot\frac{\frac{m\cdot\pi\cdot a}{2\cdot b}\cdot sh\left(\frac{m\cdot\pi\cdot a}{2\cdot b}\right)}{ch^2\left(\frac{m\cdot\pi\cdot a}{2\cdot b}\right)}\cdot\sin\left(\frac{m\cdot\pi}{2}\right)\right\}
\end{aligned}$$

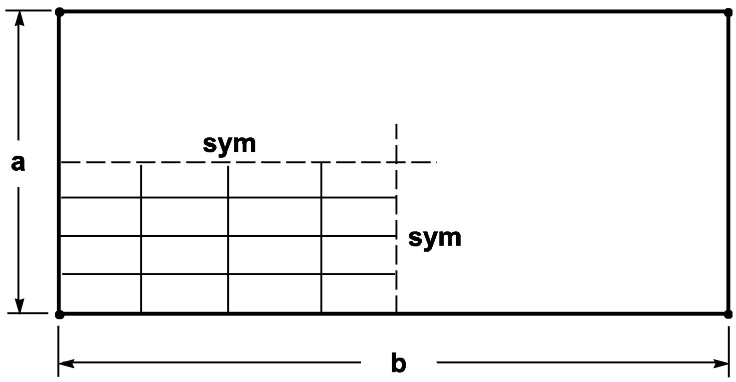
The values of the coefficients E_m and F_m are determined by solving the system of 2·M equations:

$$\left.\begin{aligned}
&-\frac{P}{\pi}\cdot\frac{1}{i^2}\cdot\frac{\frac{i\cdot\pi\cdot b}{2\cdot a}\cdot sh\left(\frac{i\cdot\pi\cdot b}{2\cdot a}\right)}{ch^2\left(\frac{i\cdot\pi\cdot b}{2\cdot a}\right)}\cdot sin\left(\frac{i\cdot\pi}{2}\right)-\frac{E_i}{i}\cdot\left(\frac{\frac{i\cdot\pi\cdot b}{2\cdot a}}{ch^2\left(\frac{i\cdot\pi\cdot b}{2\cdot a}\right)}+th\left(\frac{i\cdot\pi\cdot b}{2\cdot a}\right)\right)-\frac{8\cdot i\cdot a}{\pi\cdot b}\cdot\sum_{m=1}^{M}\left[F_m\cdot\frac{1}{m^3}\cdot\frac{1}{\left(\frac{a^2}{b^2}+\frac{i^2}{m^2}\right)^2}\cdot sin^2\left(\frac{m\cdot\pi}{2}\right)\right]\\
&-\frac{P}{\pi}\cdot\frac{1}{i^2}\cdot\frac{\frac{i\cdot\pi\cdot a}{2\cdot b}\cdot sh\left(\frac{i\cdot\pi\cdot a}{2\cdot b}\right)}{ch^2\left(\frac{i\cdot\pi\cdot a}{2\cdot b}\right)}\cdot sin\left(\frac{i\cdot\pi}{2}\right)-\frac{F_i}{i}\cdot\left(\frac{\frac{i\cdot\pi\cdot a}{2\cdot b}}{ch^2\left(\frac{i\cdot\pi\cdot a}{2\cdot b}\right)}+th\left(\frac{i\cdot\pi\cdot a}{2\cdot b}\right)\right)-\frac{8\cdot i\cdot b}{\pi\cdot a}\cdot\sum_{m=1}^{M}\left[E_m\cdot\frac{1}{m^3}\cdot\frac{1}{\left(\frac{b^2}{a^2}+\frac{i^2}{m^2}\right)^2}\cdot sin^2\left(\frac{m\cdot\pi}{2}\right)\right]
\end{aligned}\right\};$$

$$D=\frac{E\cdot h^3}{12\cdot\left(1-\nu^2\right)}.$$

The accuracy of the solution has decreased for the coarse meshes (64x64, 128x128) due to the accumulation of computational errors.

Simply Supported Flat Rectangular Plate Subjected to a Transverse Load Uniformly Distributed over the Entire Area and a Concentrated Shear Force Applied in the Center



Objective: Check of the obtained values of the transverse displacements in the center of a simply supported flat rectangular plate subjected to a transverse load uniformly distributed over the entire area and a concentrated shear force applied in the center.

Initial data files:

File name	Description
Bending_of_rectangular_flat_plate_Simply_supported_Shell_42_Mesh_2x2.SPR Bending_of_rectangular_flat_plate_Simply_supported_Shell_42_Mesh_4x4.SPR Bending_of_rectangular_flat_plate_Simply_supported_Shell_42_Mesh_8x8.SPR	Design model with the elements of type 42 for meshes 2x2, 4x4, 8x8
Bending_of_rectangular_flat_plate_Simply_supported_Shell_44_Mesh_2x2.SPR Bending_of_rectangular_flat_plate_Simply_supported_Shell_44_Mesh_4x4.SPR Bending_of_rectangular_flat_plate_Simply_supported_Shell_44_Mesh_8x8.SPR	Design model with the elements of type 44 for meshes 2x2, 4x4, 8x8
Bending_of_rectangular_flat_plate_Simply_supported_Shell_45_Mesh_2x2.SPR Bending_of_rectangular_flat_plate_Simply_supported_Shell_45_Mesh_4x4.SPR Bending_of_rectangular_flat_plate_Simply_supported_Shell_45_Mesh_8x8.SPR	Design model with the elements of type 45 for meshes 2x2, 4x4, 8x8
Bending_of_rectangular_flat_plate_Simply_supported_Shell_50_Mesh_2x2.SPR Bending_of_rectangular_flat_plate_Simply_supported_Shell_50_Mesh_4x4.SPR Bending_of_rectangular_flat_plate_Simply_supported_Shell_50_Mesh_8x8.SPR	Design model with the elements of type 50 for meshes 2x2, 4x4, 8x8
Bending_of_rectangular_flat_plate_Simply_supported_Solid_36_Mesh_2x2.SPR Bending_of_rectangular_flat_plate_Simply_supported_Solid_36_Mesh_4x4.SPR Bending_of_rectangular_flat_plate_Simply_supported_Solid_36_Mesh_8x8.SPR Bending_of_rectangular_flat_plate_Simply_supported_Solid_36_Mesh_16x16.SPR Bending_of_rectangular_flat_plate_Simply_supported_Solid_36_Mesh_32x32.SPR Bending_of_rectangular_flat_plate_Simply_supported_Solid_36_Mesh_64x64.SPR Bending_of_rectangular_flat_plate_Simply_supported_Solid_36_Mesh_128x128.SPR	Design model with the elements of type 36 for meshes 2x2, 4x4, 8x8, 16x16, 32x32, 64x64, 128x128
Bending_of_rectangular_flat_plate_Simply_supported_Solid_37_Mesh_2x2.SPR Bending_of_rectangular_flat_plate_Simply_supported_Solid_37_Mesh_4x4.SPR Bending_of_rectangular_flat_plate_Simply_supported_Solid_37_Mesh_8x8.SPR Bending_of_rectangular_flat_plate_Simply_supported_Solid_37_Mesh_16x16.SPR Bending_of_rectangular_flat_plate_Simply_supported_Solid_37_Mesh_32x32.SPR Bending_of_rectangular_flat_plate_Simply_supported_Solid_37_Mesh_64x64.SPR Bending_of_rectangular_flat_plate_Simply_supported_Solid_37_Mesh_128x128.SPR	Design model with the elements of type 37 for meshes 2x2, 4x4, 8x8, 16x16, 32x32, 64x64, 128x128

Problem formulation: The simply supported flat rectangular plate is subjected to the transverse load q uniformly distributed over the entire area and the concentrated shear force P applied in the center. Check the obtained values of the transverse displacements in the center of the simply supported flat rectangular plate w_q and w_P from the respective actions.

References: R. H. Macneal, R. L. Harder, A proposed standard set of problems to test finite element accuracy, North-Holland, Finite elements in analysis and design, 1, 1985, p. 3-20.
S. Timoshenko, S. Woinowsky-Krieger, Theory of plates and shells, New York, McGraw-Hill,1959, p. 120, 143, 202, 206.

Initial data:

$E = 1.7472 \cdot 10^7$ kPa	- elastic modulus of the plate material;
$\nu = 0.30$	- Poisson's ratio;
a = 2.00 m	- width of the plate;
b = 10.00 m	- length of the plate;
$h = 10^{-4}$ (10^{-2}) m	- thickness of the plate;
$q = 1.0 \cdot 10^{-4}$ kN/m^2	- value of the transverse load uniformly distributed over the entire area of the plate;
$P = 4.0 \cdot 10^{-4}$ kN	- value of the concentrated shear force in the center of the plate.

***Finite element model*:** Design model – general type system. Six design models of a quarter of the plate according to the symmetry conditions are considered:

Model 1 – 8, 32, 128 three-node shell elements of type 42 with a regular mesh 2x2, 4x4, 8x8. The thickness of the plate – 10^{-4} m. Boundary conditions are provided by imposing constraints on the nodes of the support edges of the plate in the directions of the degrees of freedom X, Y, Z and constraints according to the symmetry conditions. Number of nodes in the model – 9, 25, 81.

Model 2 – 4, 16, 64 four-node shell elements of type 44 with a regular mesh 2x2, 4x4, 8x8. The thickness of the plate – 10^{-4} m. Boundary conditions are provided by imposing constraints on the nodes of the support edges of the plate in the directions of the degrees of freedom X, Y, Z and constraints according to the symmetry conditions. Number of nodes in the model – 9, 25, 81.

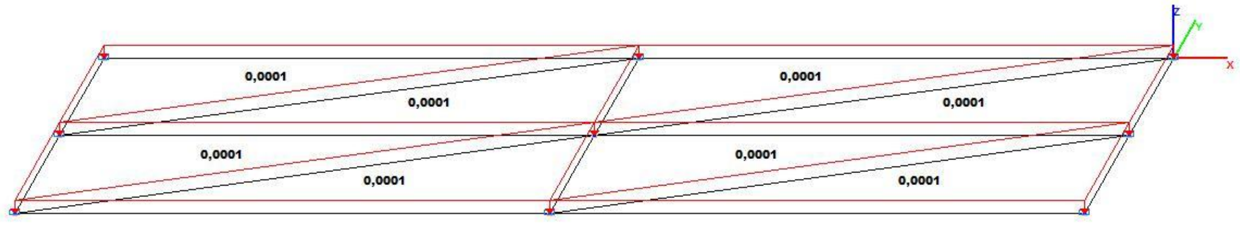
Model 3 – 8, 32, 128 six-node shell elements of type 45 with a regular mesh 2x2, 4x4, 8x8. The thickness of the plate – 10^{-4} m. Boundary conditions are provided by imposing constraints on the nodes of the support edges of the plate in the directions of the degrees of freedom X, Y, Z and constraints according to the symmetry conditions. Number of nodes in the model – 25, 81, 289.

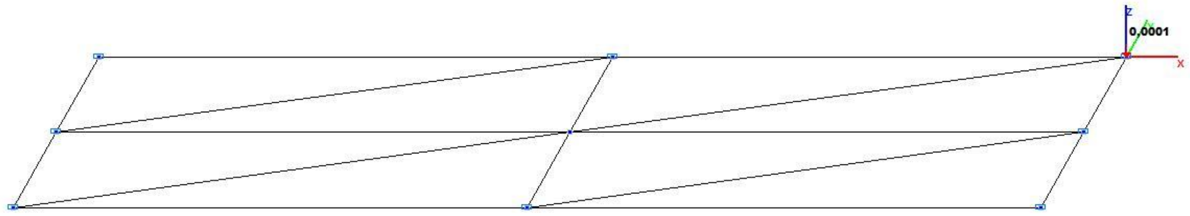
Model 4 – 4, 16, 64 eight-node shell elements of type 50 with a regular mesh 2x2, 4x4, 8x8. The thickness of the plate – 10^{-4} m. Boundary conditions are provided by imposing constraints on the nodes of the support edges of the plate in the directions of the degrees of freedom X, Y, Z and constraints according to the symmetry conditions. Number of nodes in the model – 25, 81, 289.

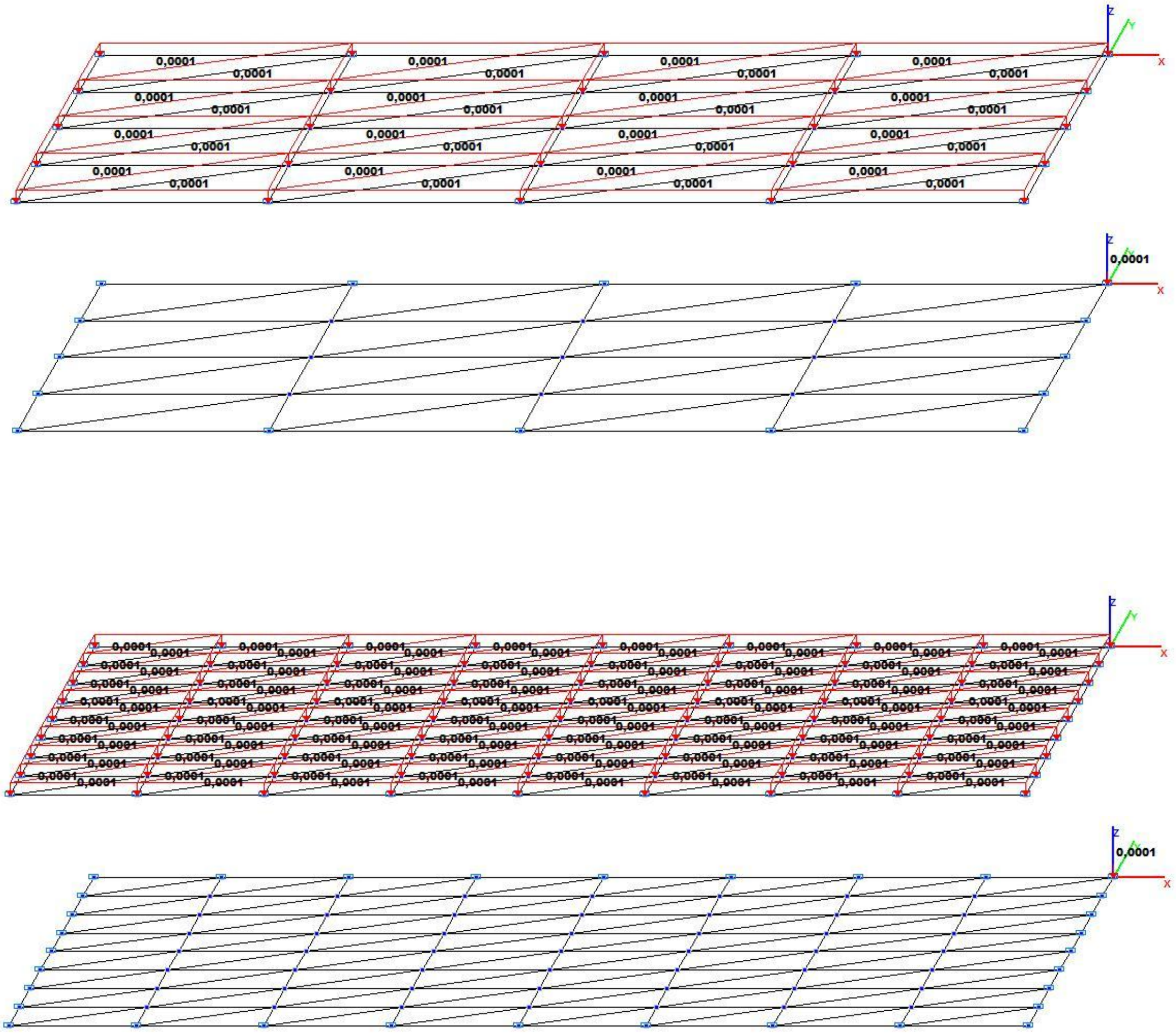
Model 5 – 4, 16, 64, 256, 1024, 4096, 16384 eight-node isoparametric solid elements of type 36 with a regular mesh 2x2x1, 4x4x1, 8x8x1, 16x16x1, 32x32x1, 64x64x1, 128x128x1. The thickness of the plate – 10^{-2} m. Boundary conditions are provided by imposing constraints on the nodes of the support sides of the lower surface of the plate in the direction of the degree of freedom Z and constraints according to the symmetry conditions. Number of nodes in the model – 18, 50, 162, 578, 2178, 8450, 33282.

Model 6 – 4, 16, 64, 256, 1024, 4096, 16384 twenty-node isoparametric solid elements of type 37 with a regular mesh 2x2x1, 4x4x1, 8x8x1, 16x16x1, 32x32x1, 64x64x1, 128x128x1. The thickness of the plate – 10^{-2} m. Boundary conditions are provided by imposing constraints on the nodes of the support sides of the lower surface of the plate in the direction of the degree of freedom Z and constraints according to the symmetry conditions. Number of nodes in the model – 51, 155, 531, 1955, 7491, 29315, 115971.

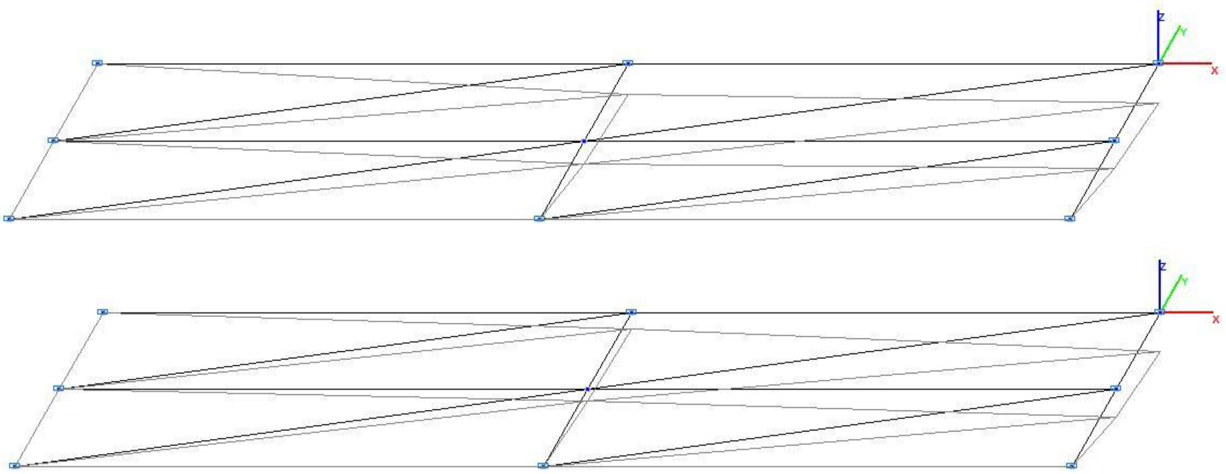
Results in SCAD

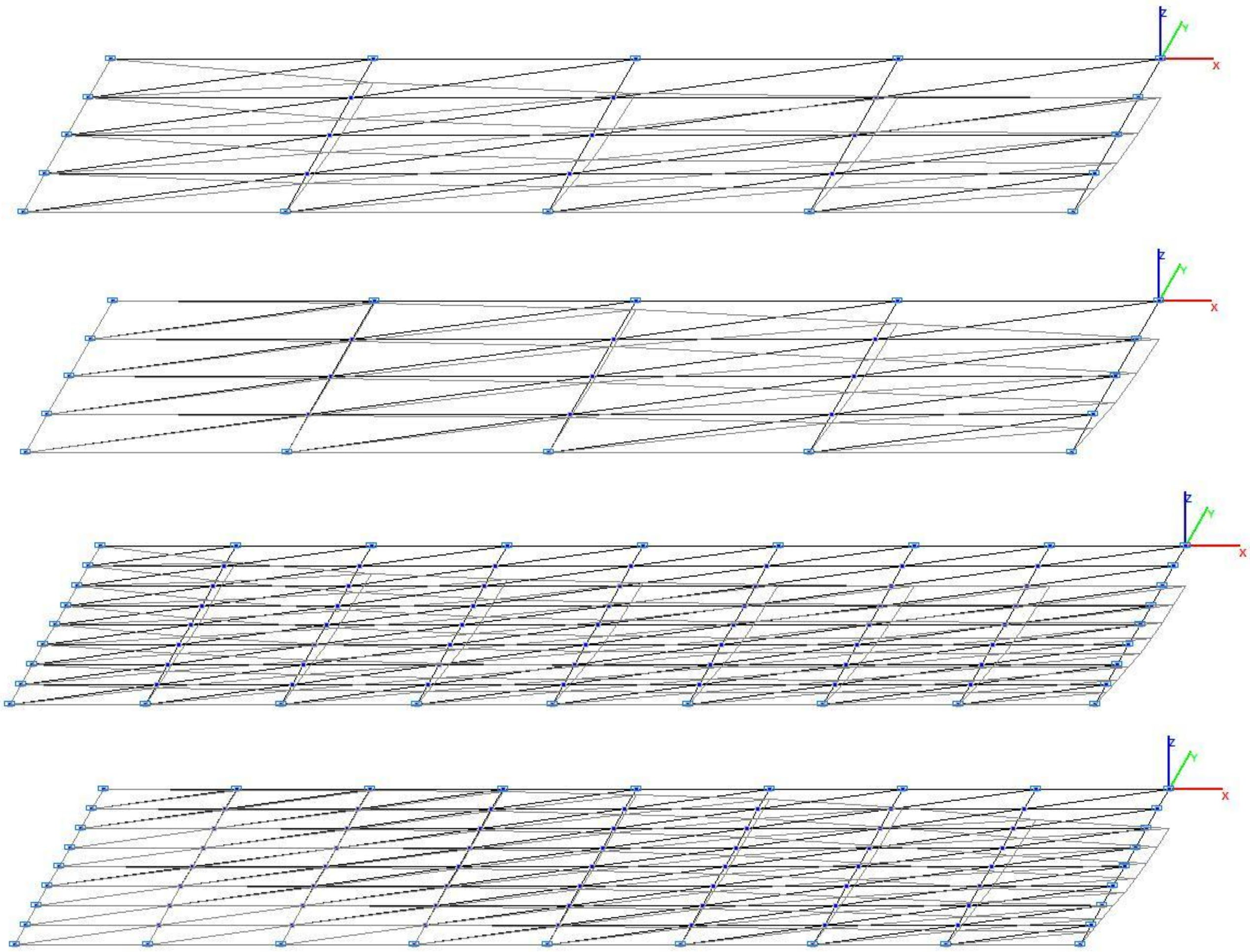




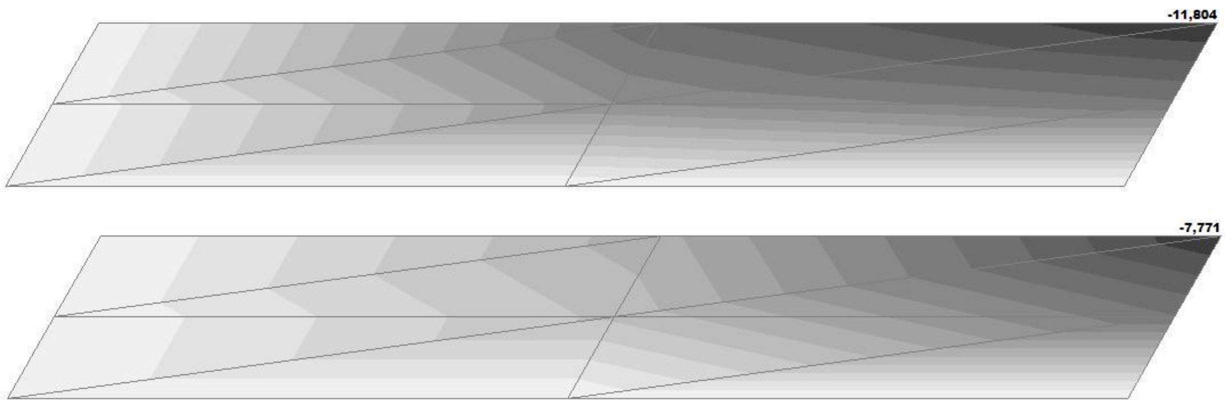


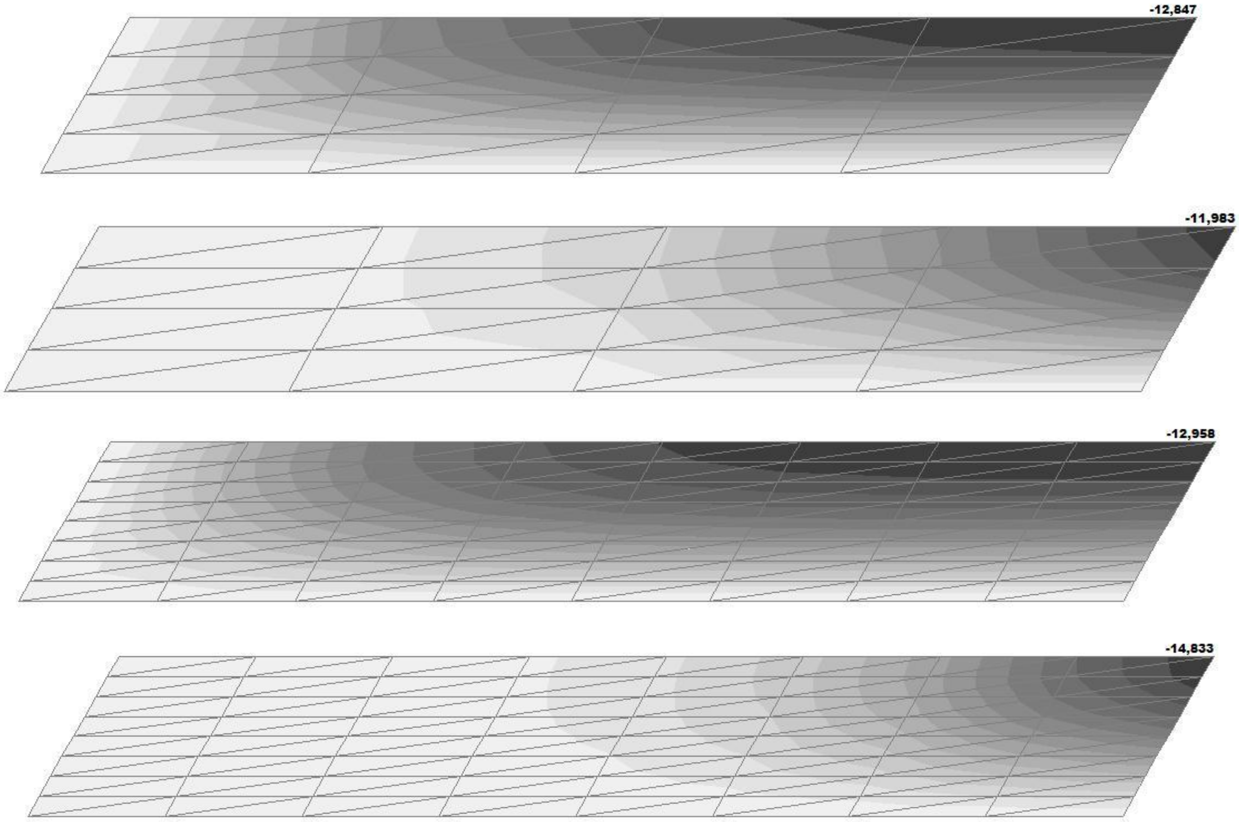
Model 1. Design model



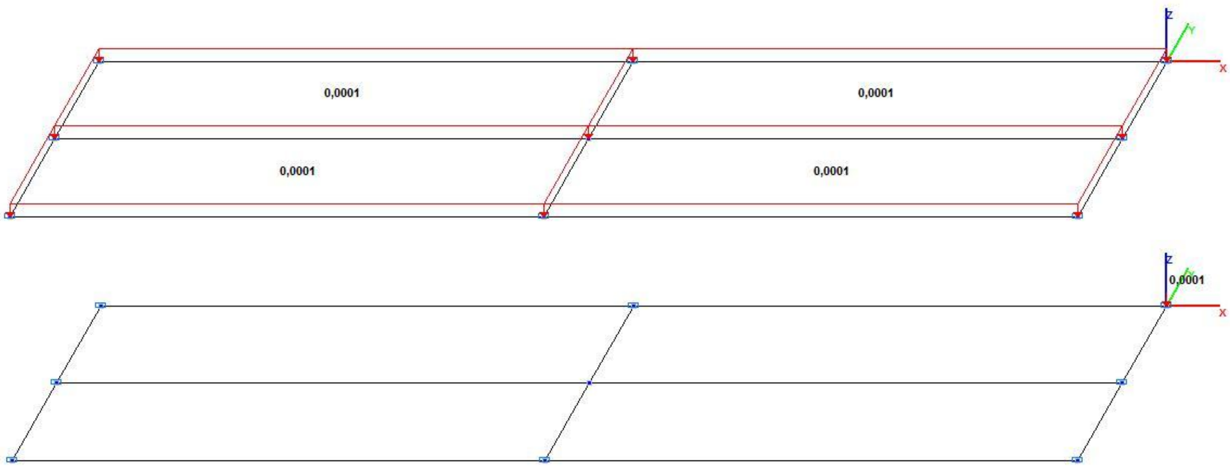


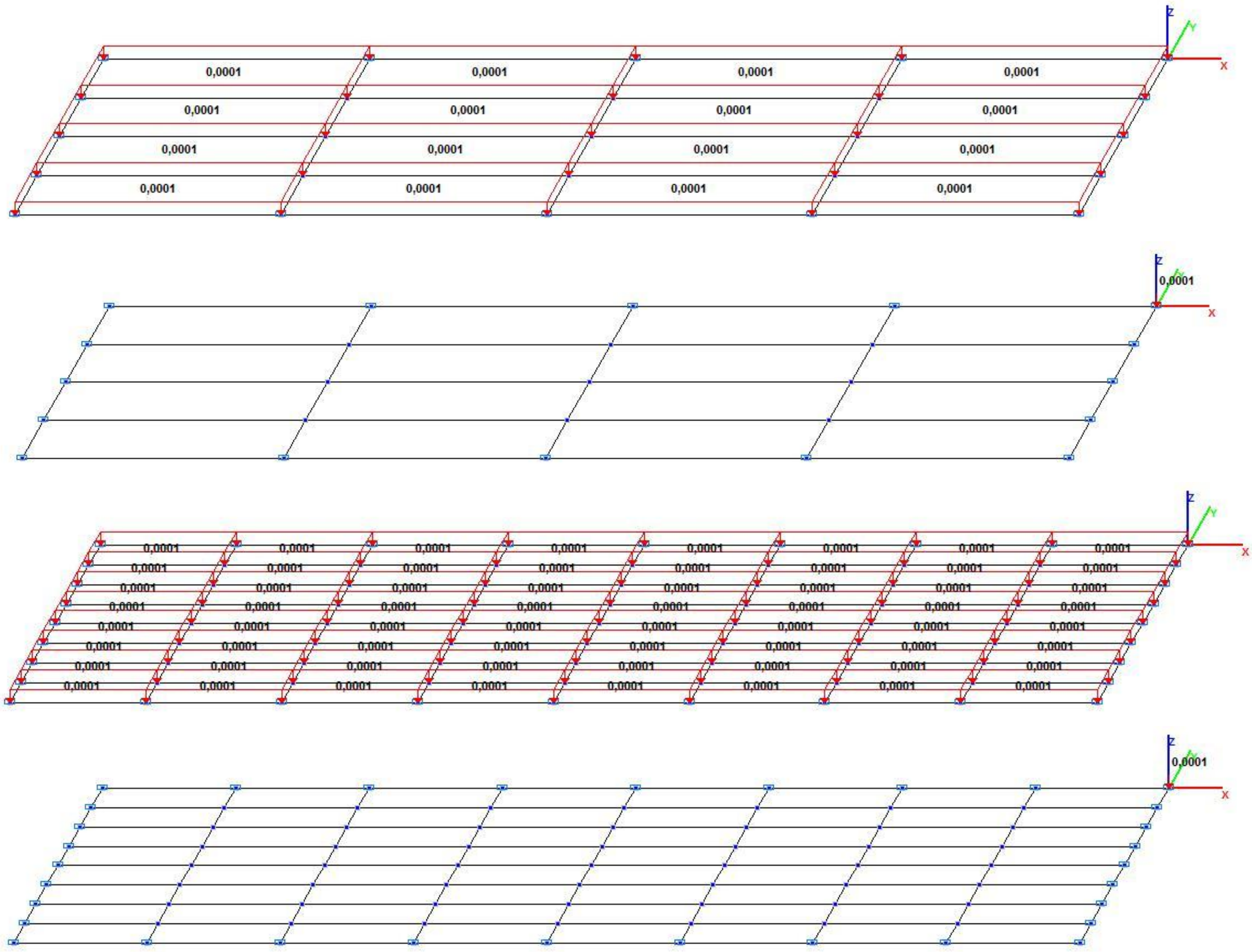
Model 1. Deformed model



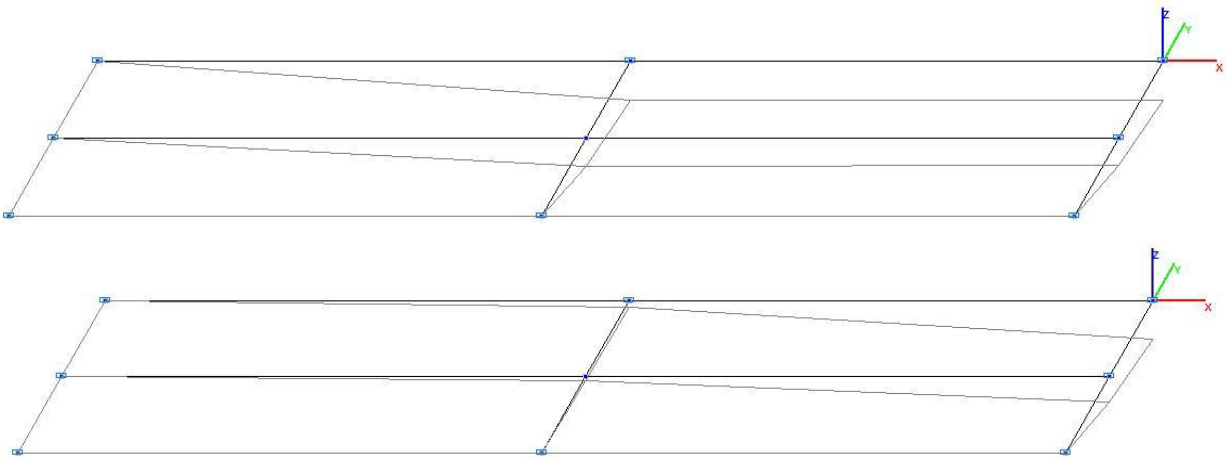


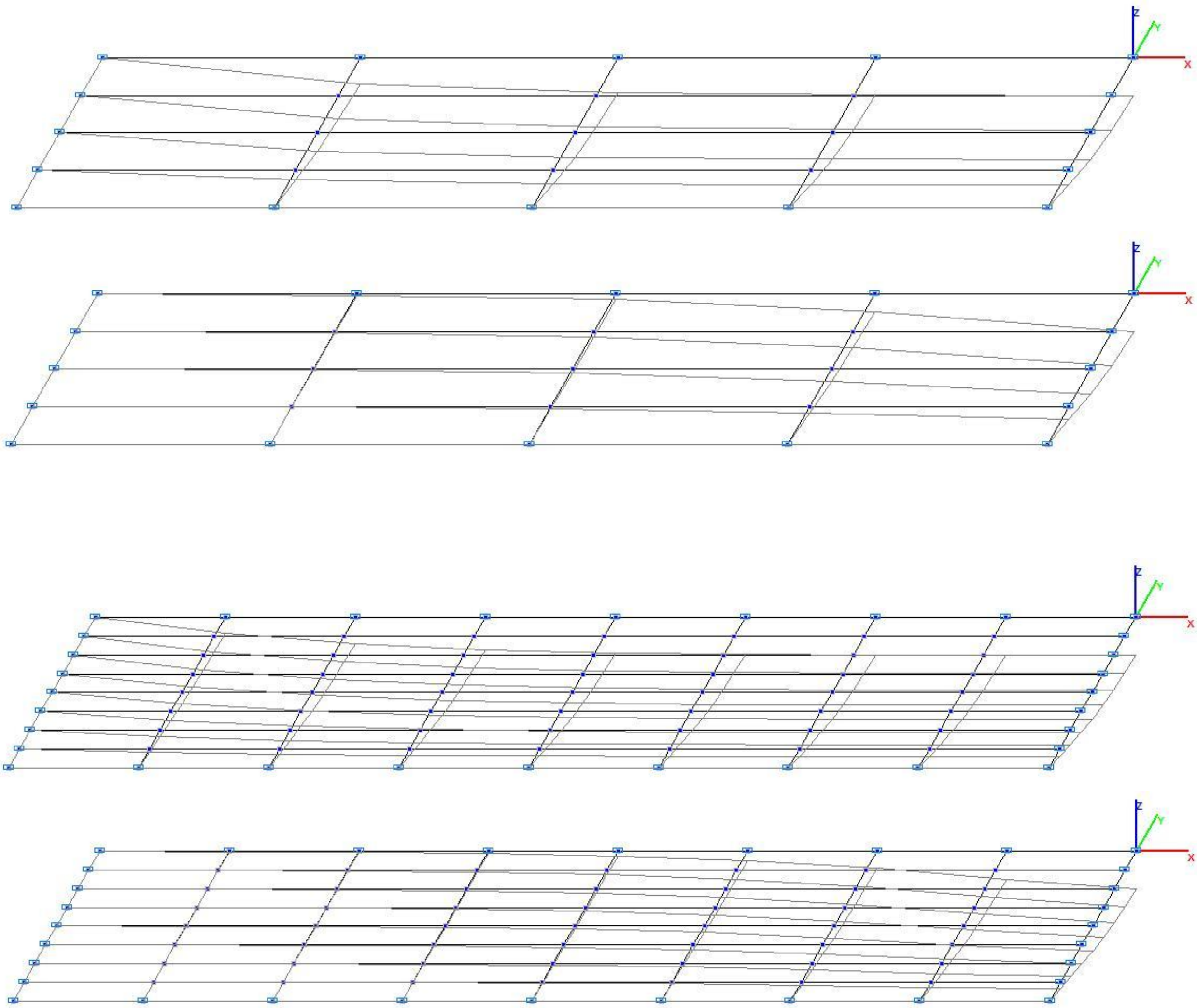
Model 1. Values of the transverse displacements in the center of the simply supported rectangular plate w_q and w_P (m, m)



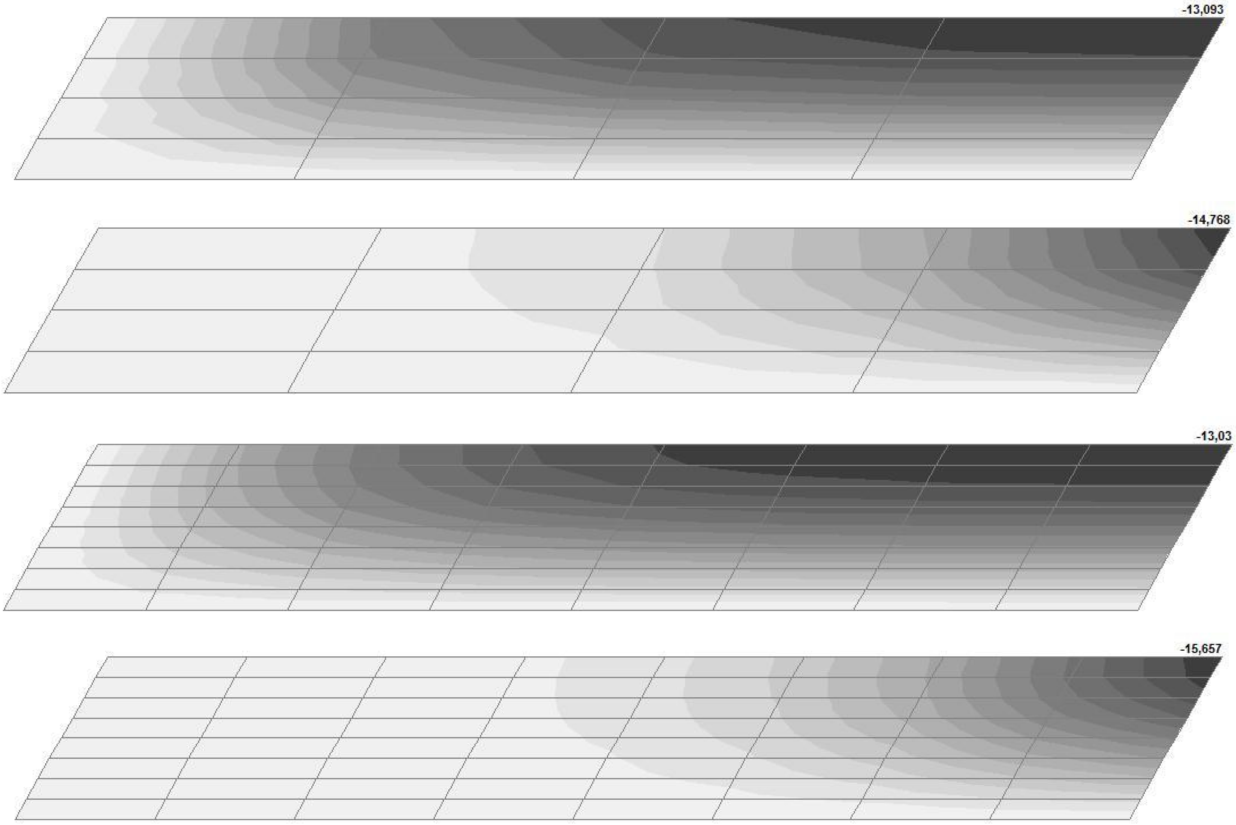


Model 2. Design model

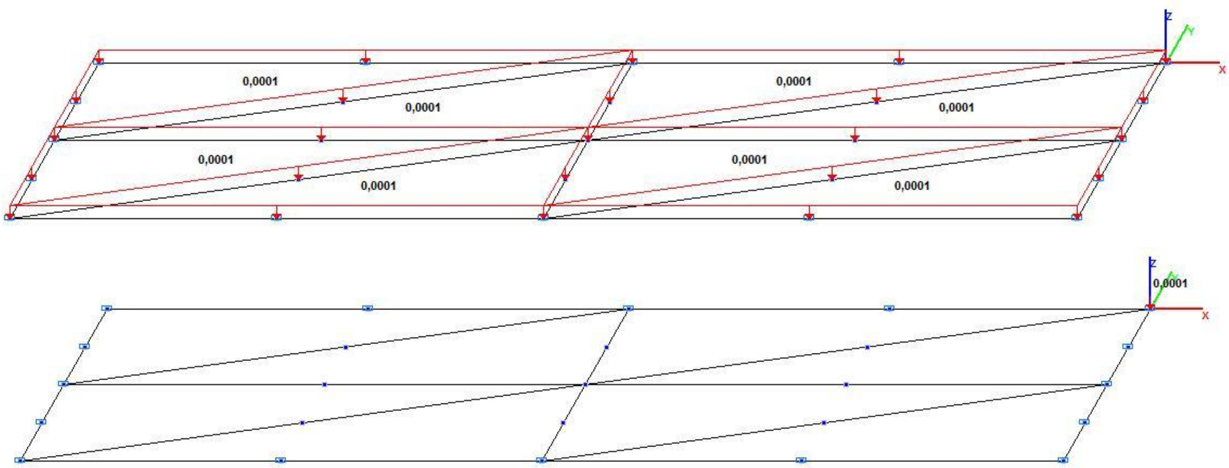


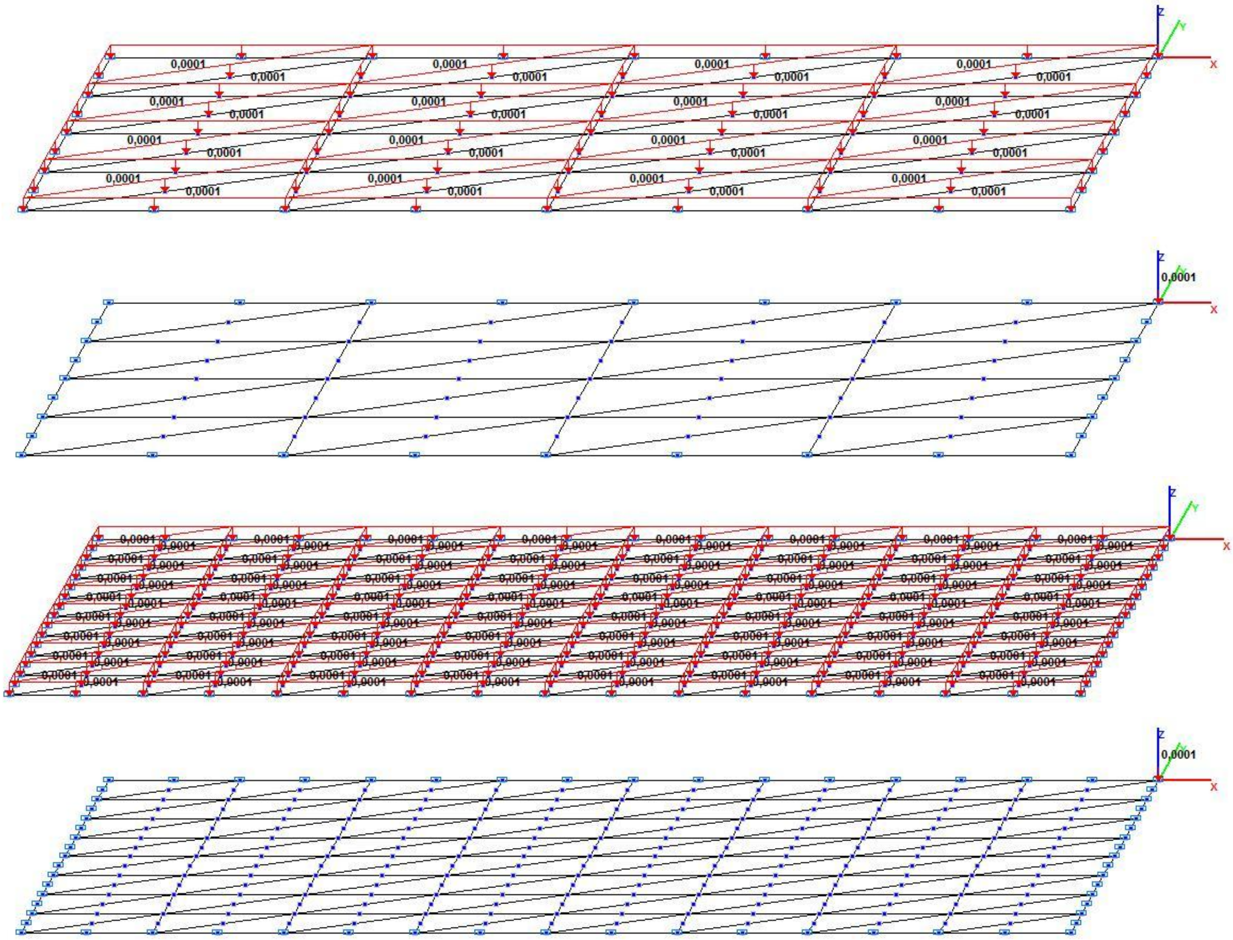


Model 2. Deformed model

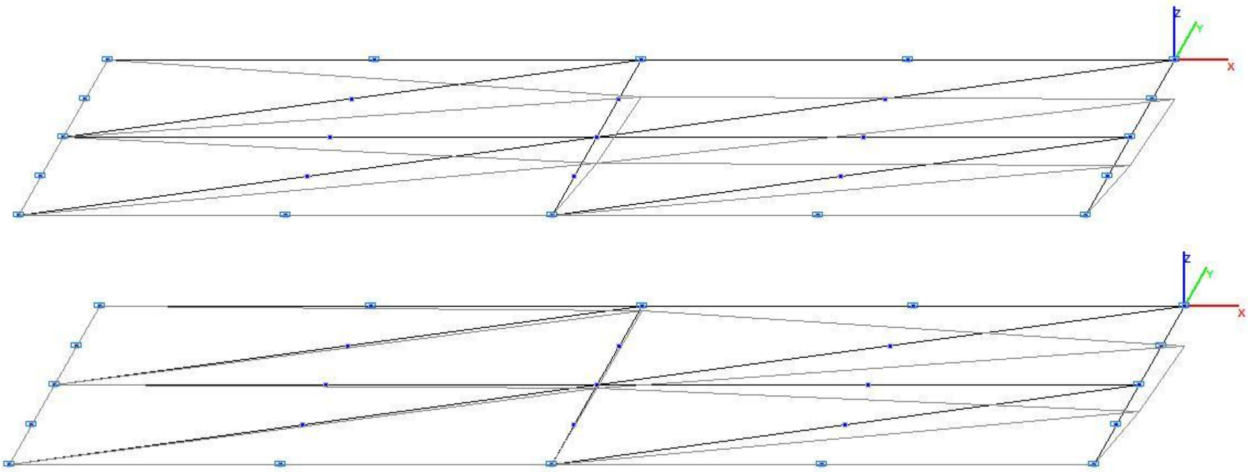


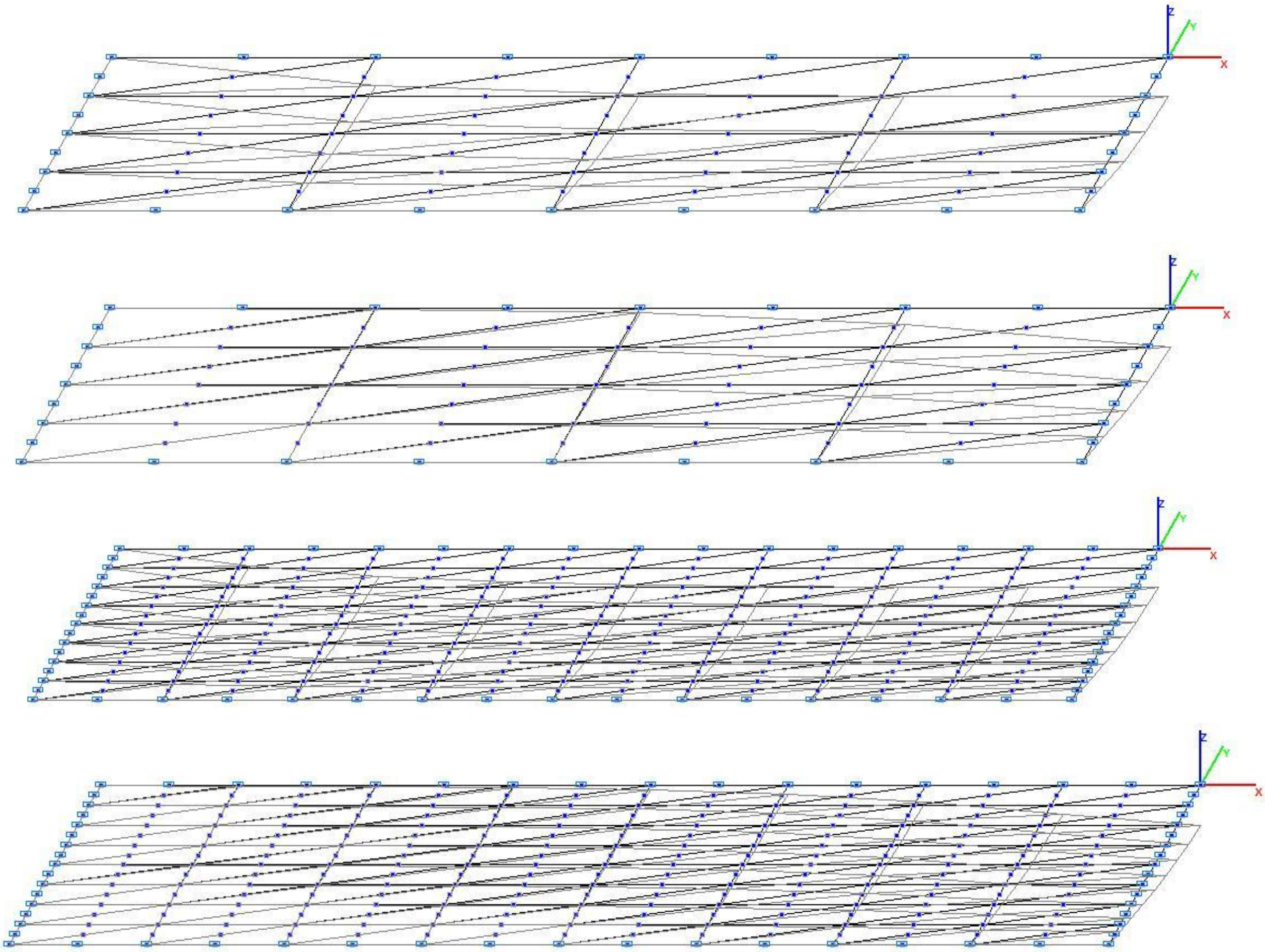
Model 2.
Values of the transverse displacements in the center of the simply supported rectangular plate w_q and w_P (m, m)



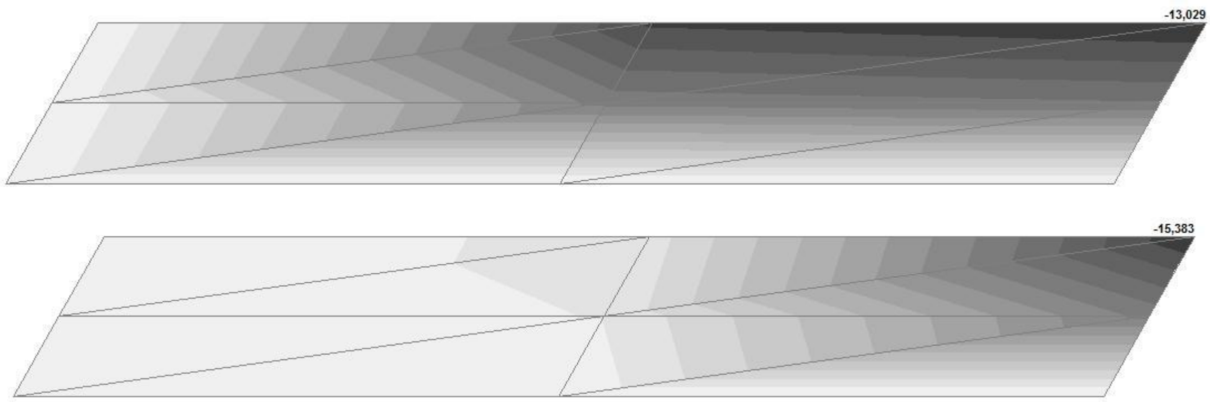


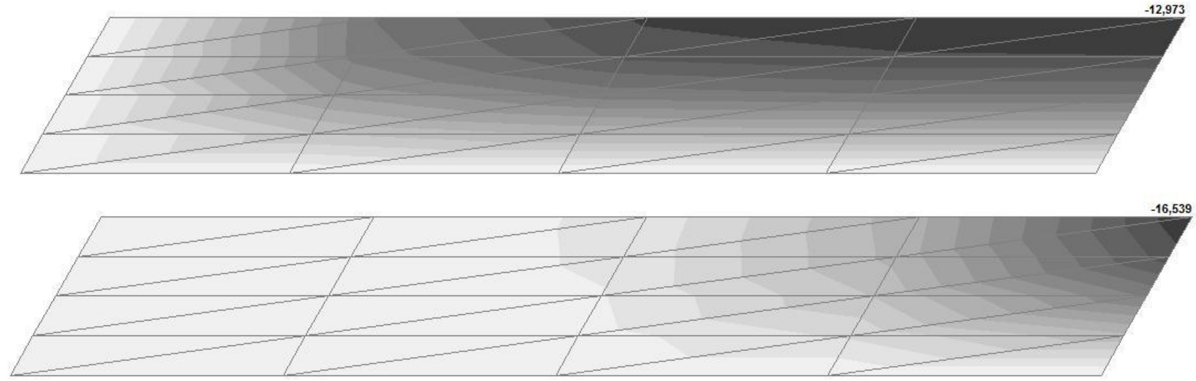
Model 3. Design model

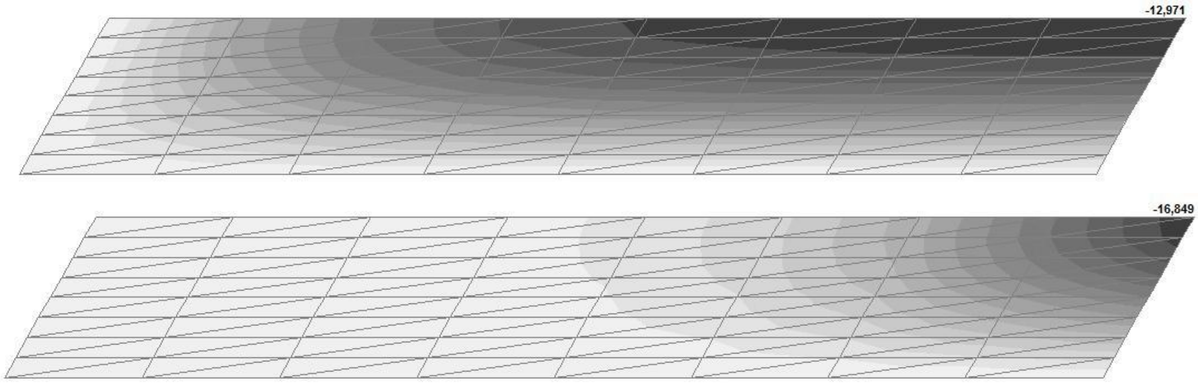




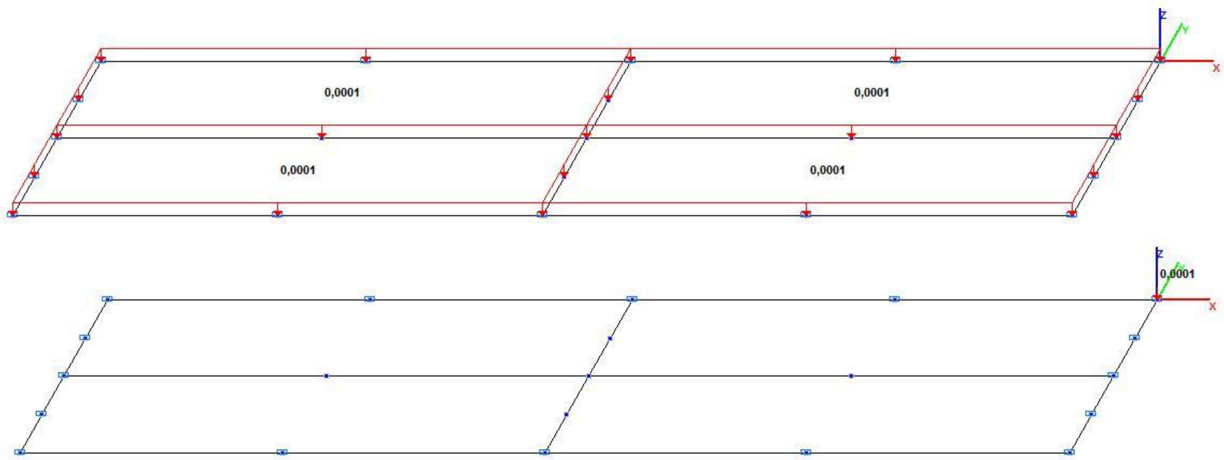
Model 3. Deformed model

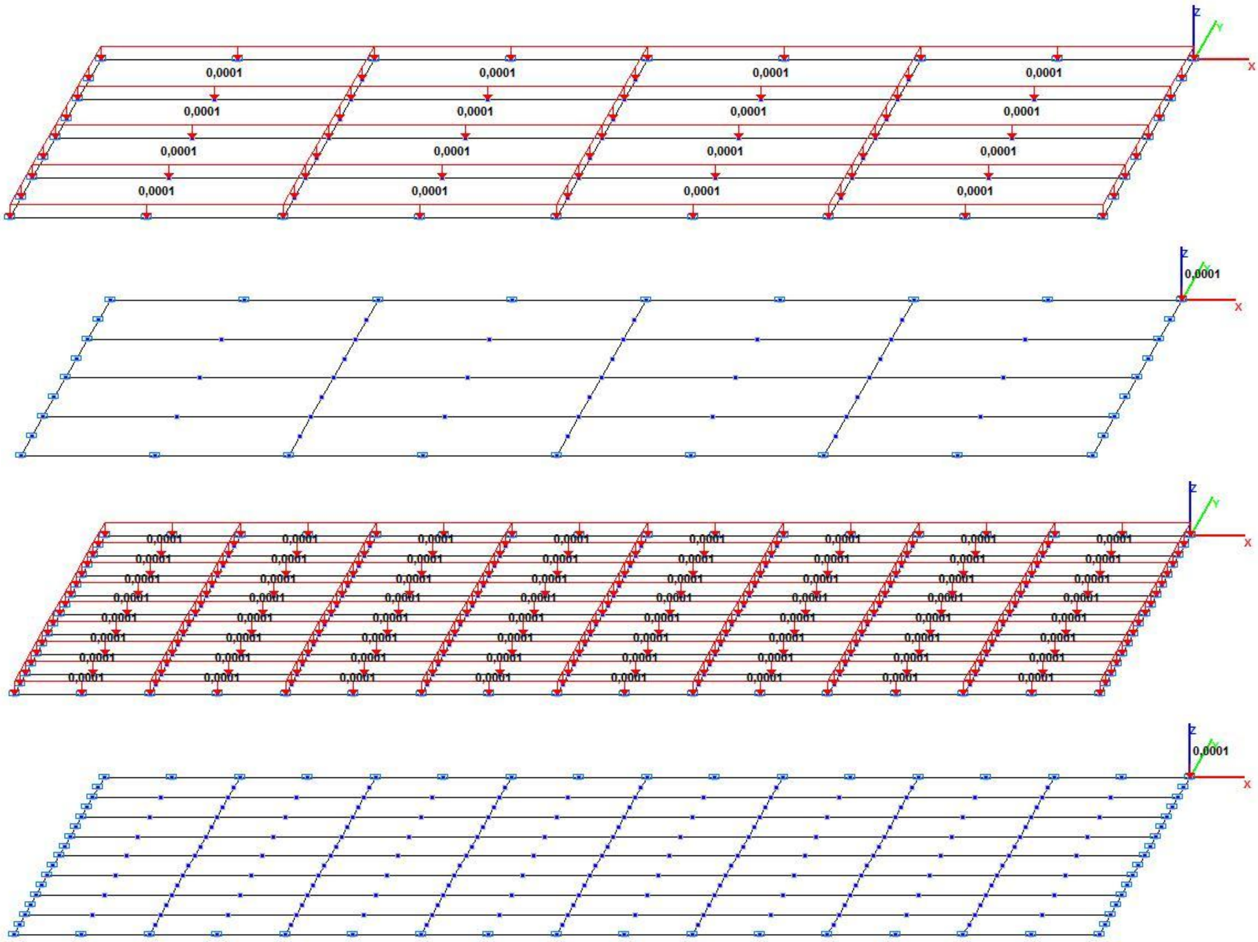




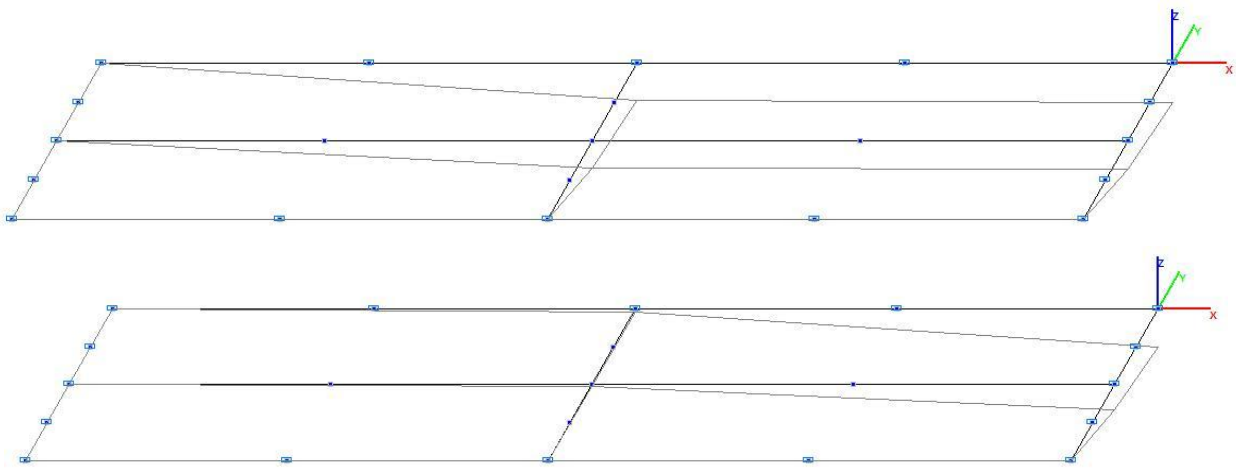


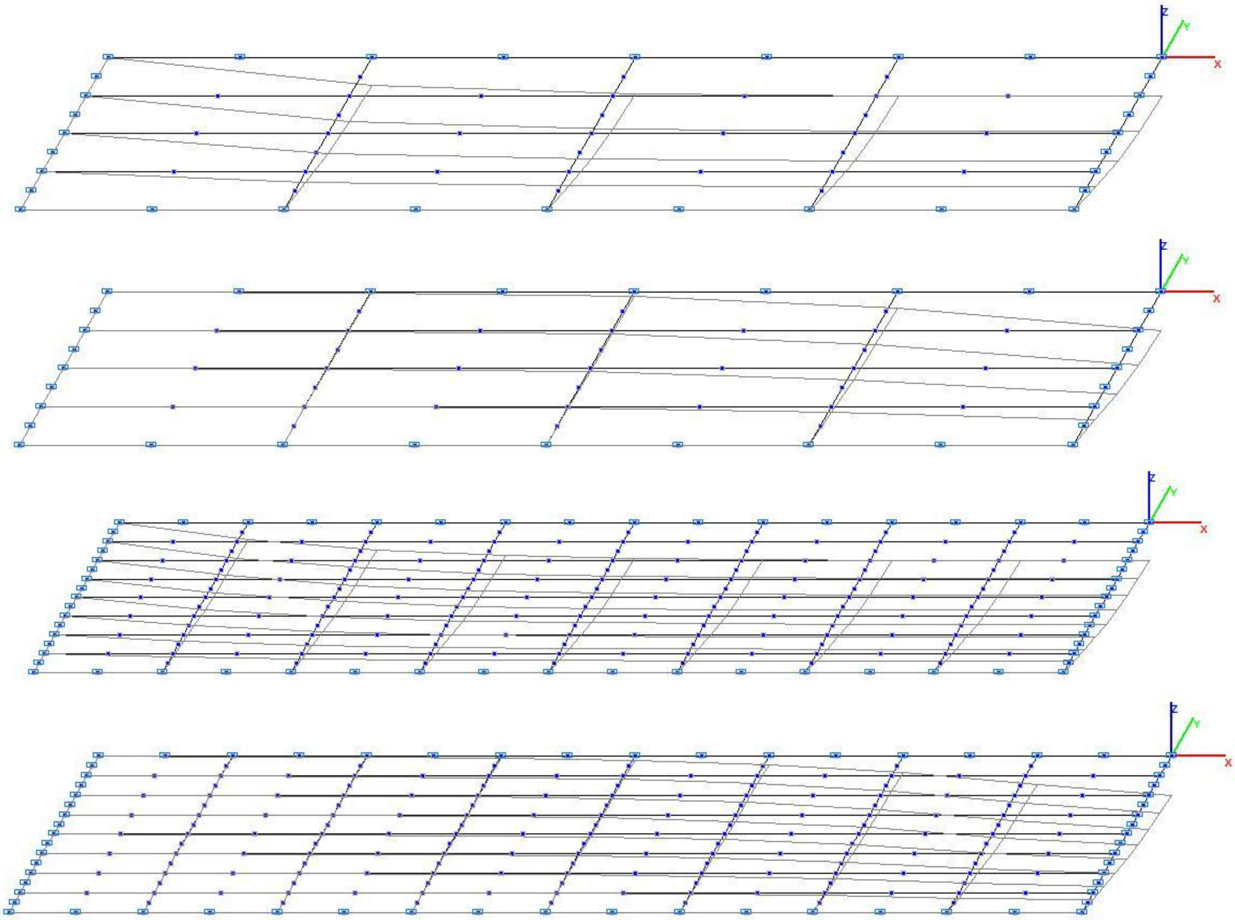
Model 3.
Values of the transverse displacements in the center of the simply supported rectangular plate w_q and w_P (m, m)



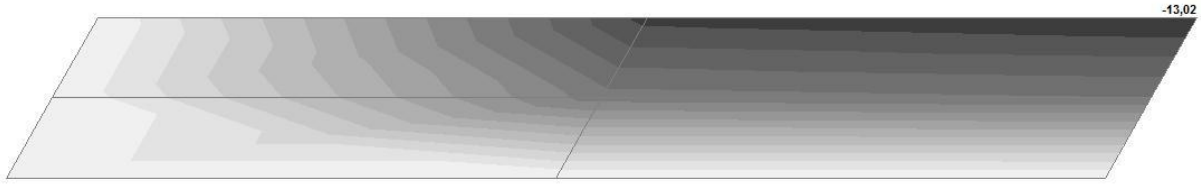


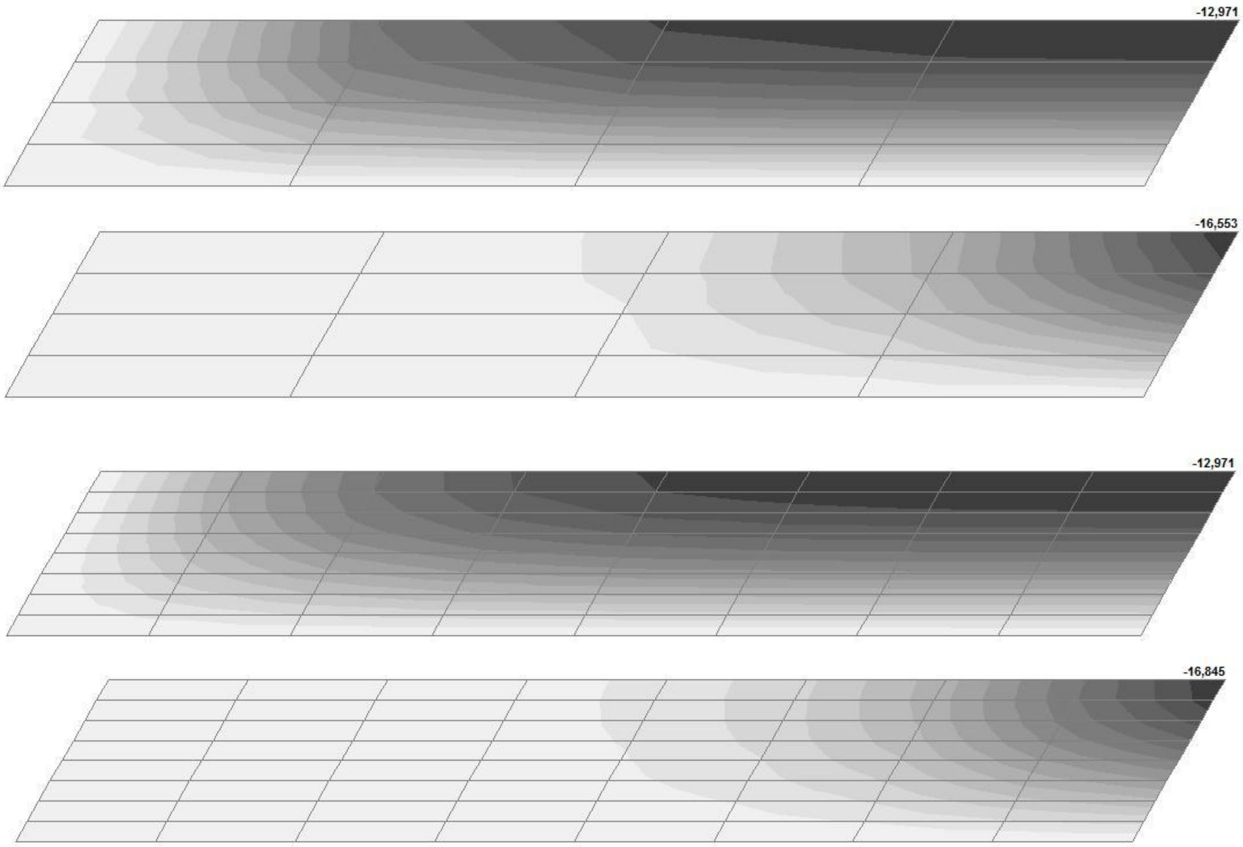
Model 4. Design model



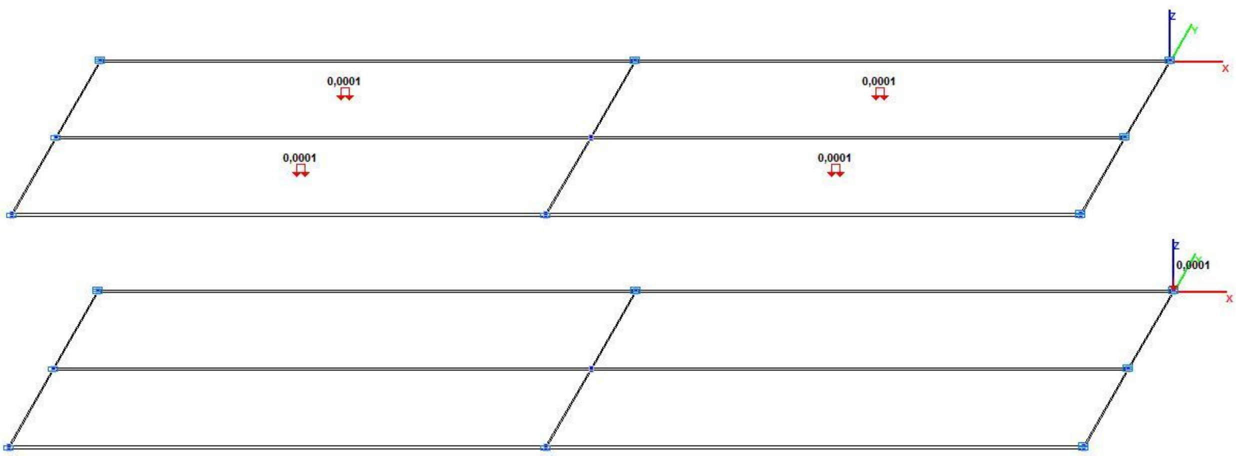


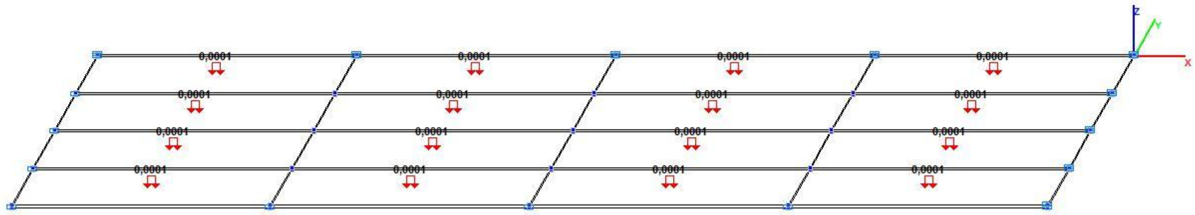
Model 4. Deformed model

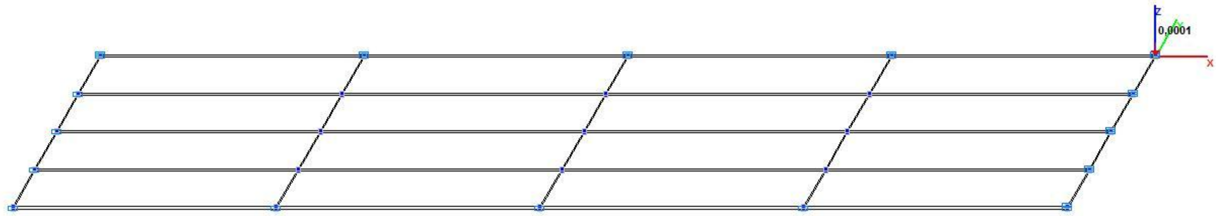


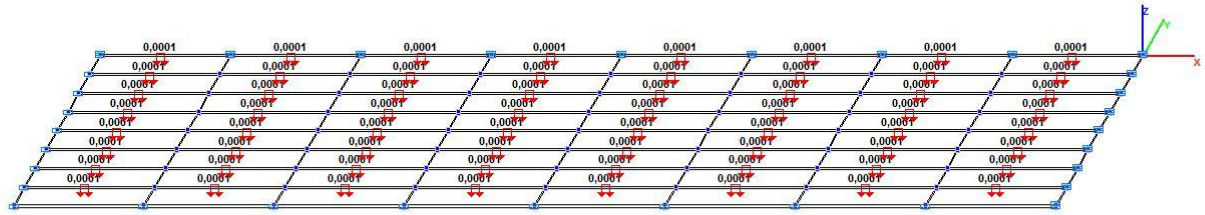


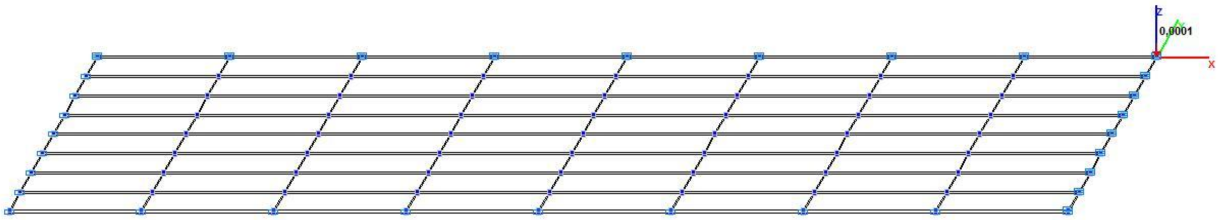
Model 4. Values of the transverse displacements in the center of the simply supported rectangular plate w_q and w_P (m, m)

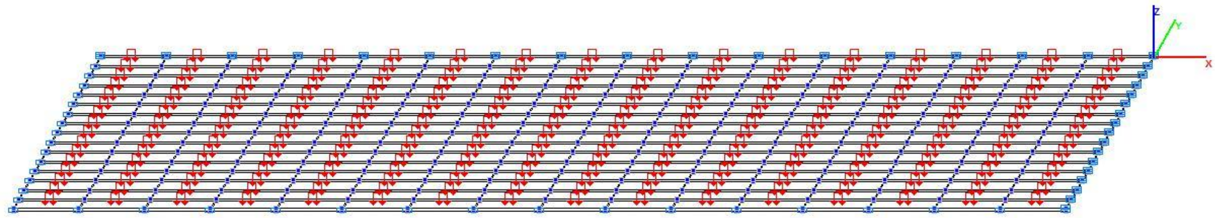


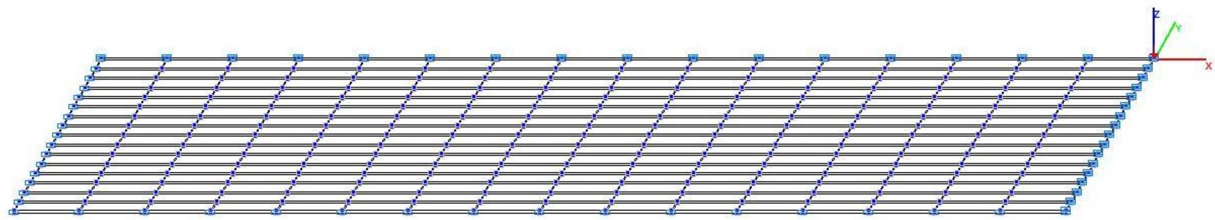

0,0001
z
y
x

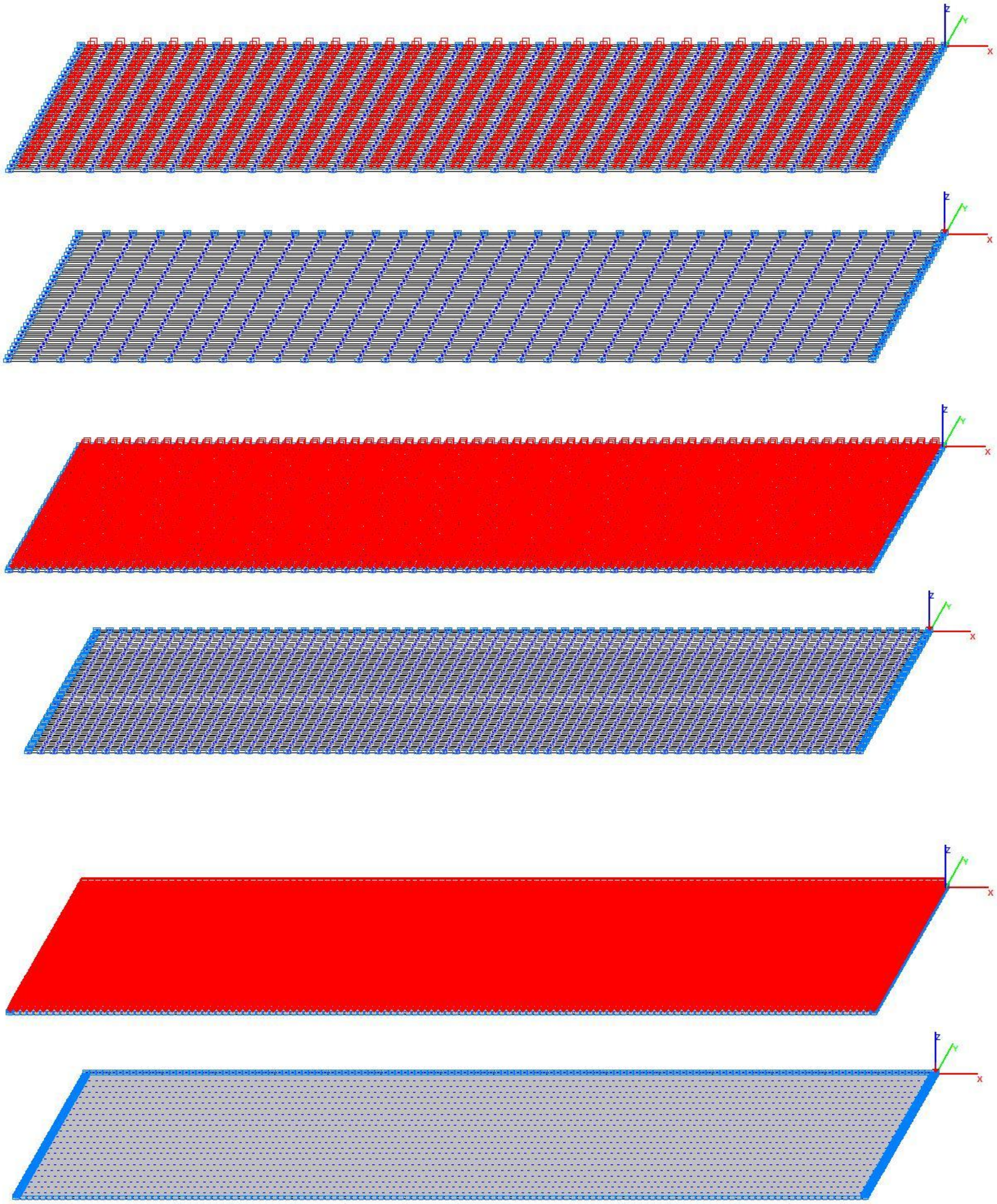

0,0001
z
x


0,0001
z
y
x

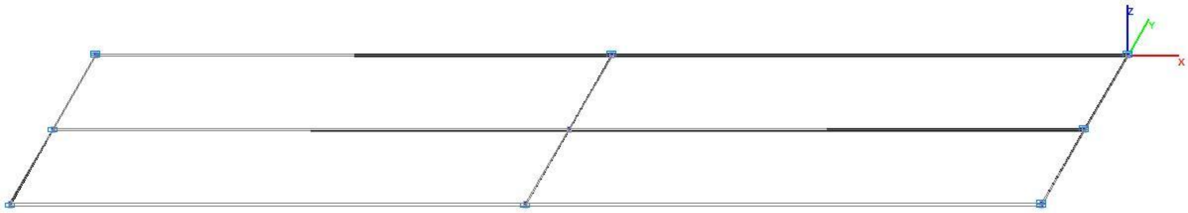

0,0001
z
x

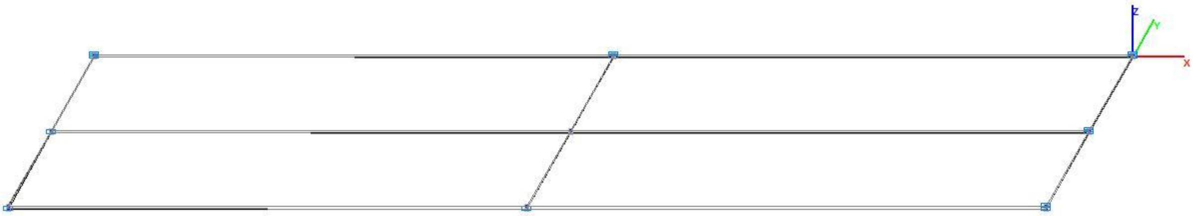

z
y
x

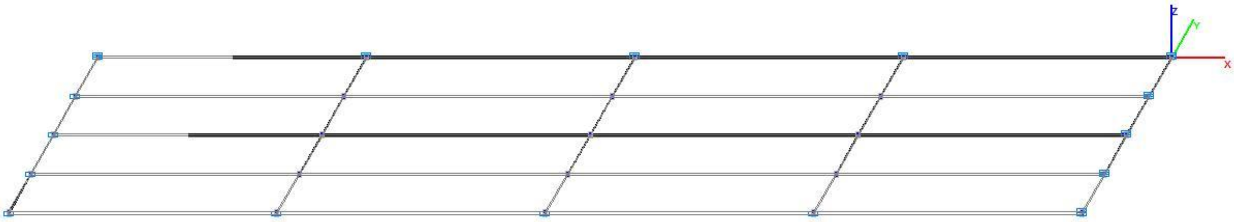

z
y
x

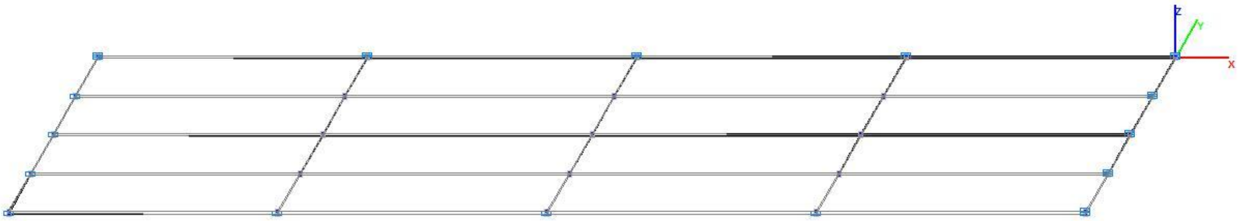


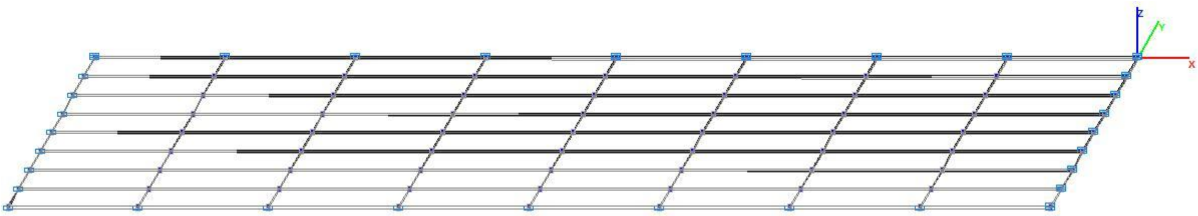
Model 5. Design model

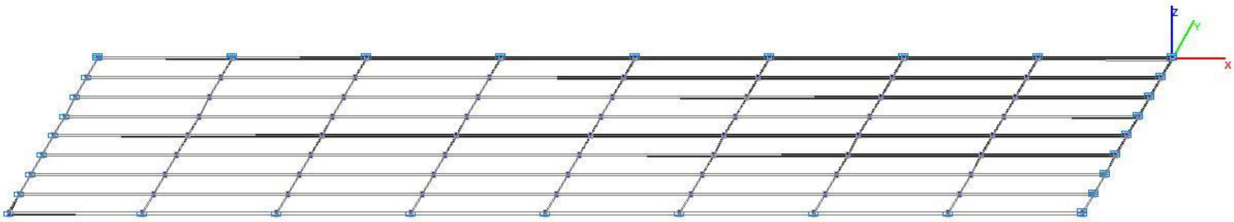

z
y
x

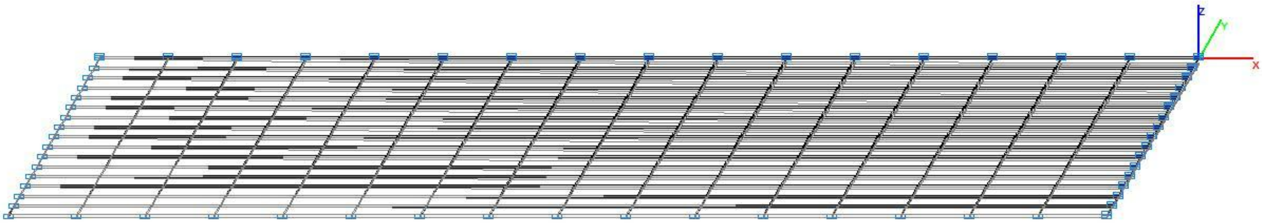

z
y
x

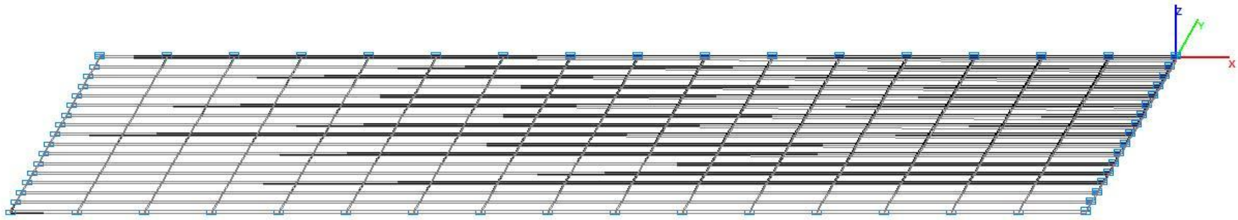

z
y
x

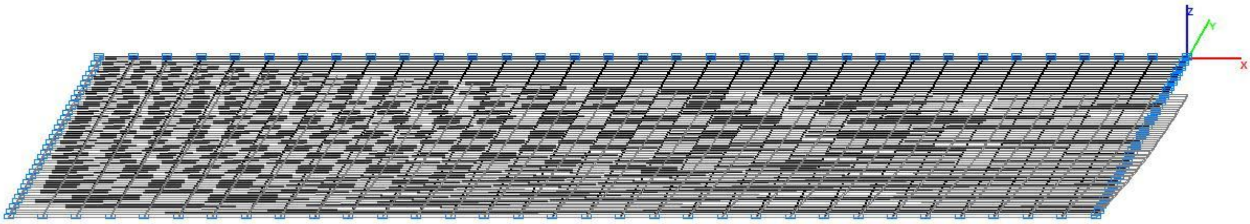

z
y
x

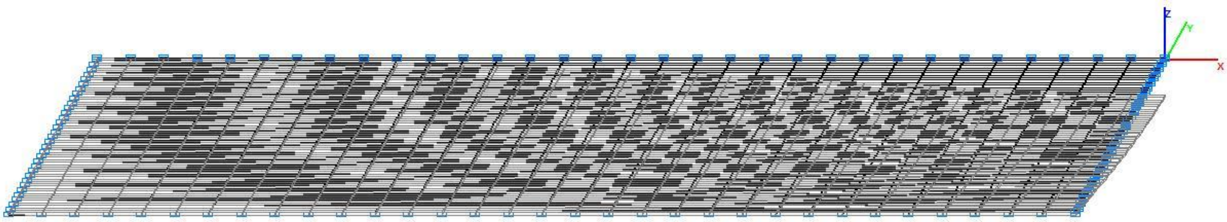

z
y
x

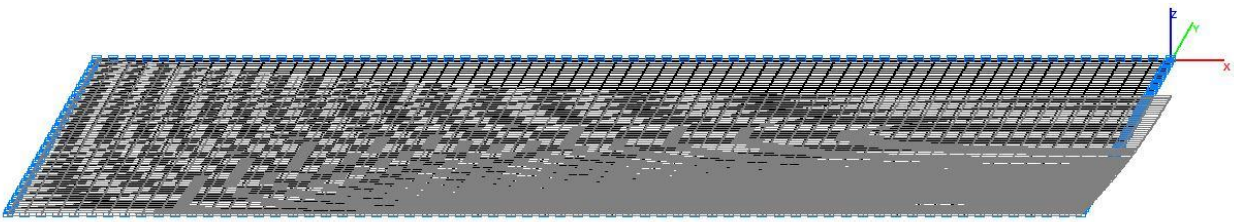

z
y
x

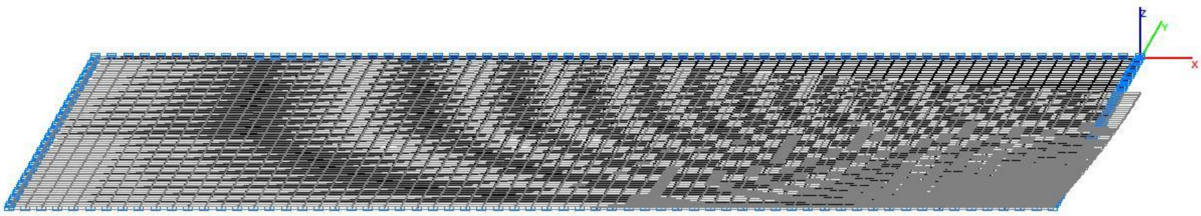


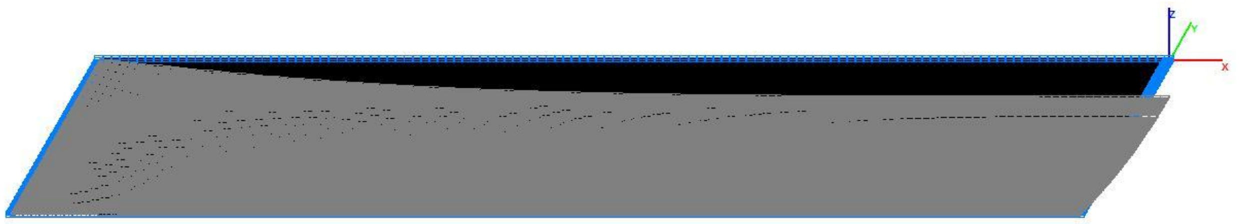


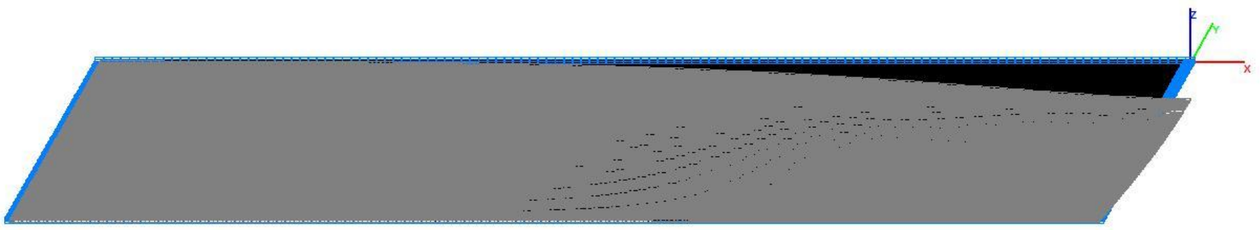




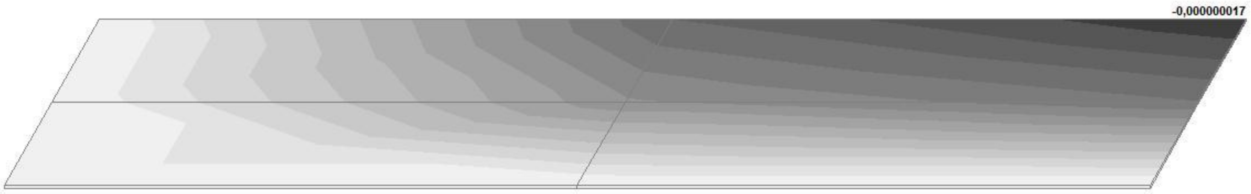


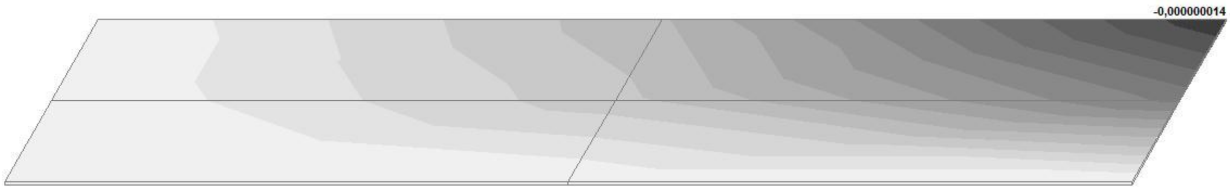


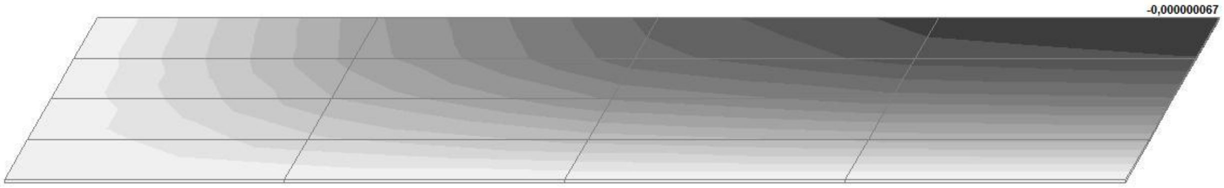


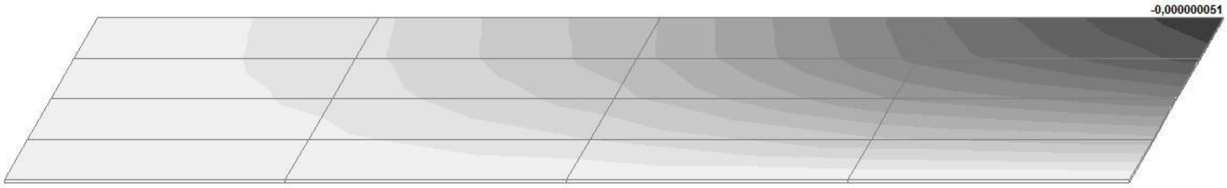


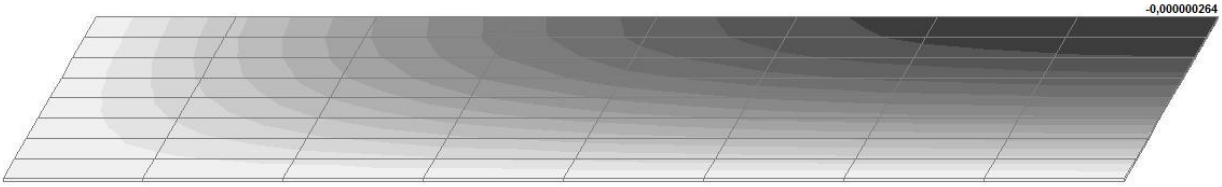
Model 5. Deformed model

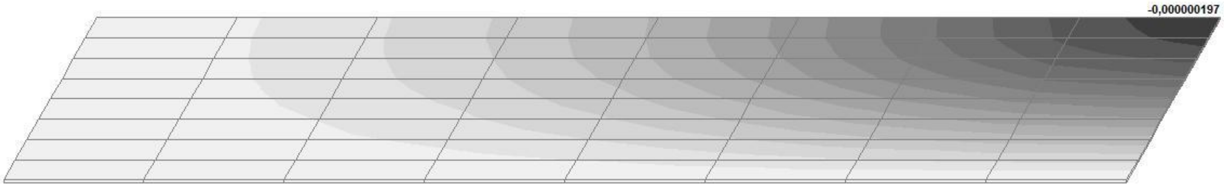


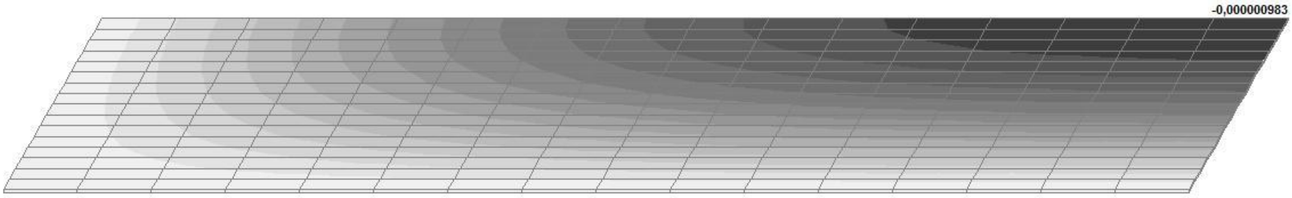


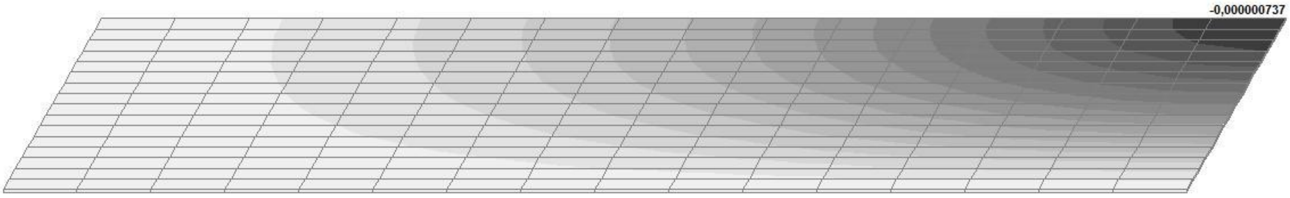


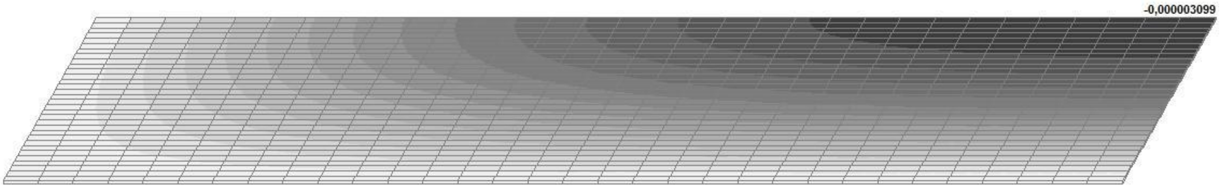


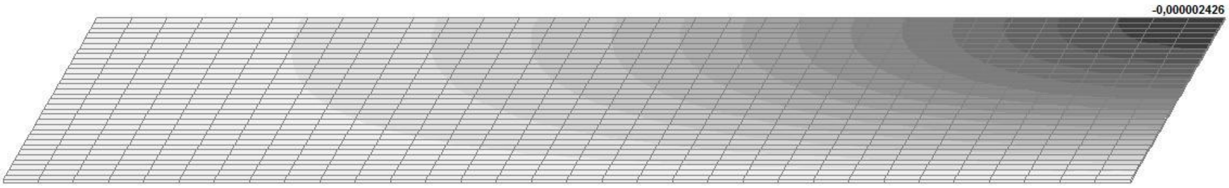

-0,000000264

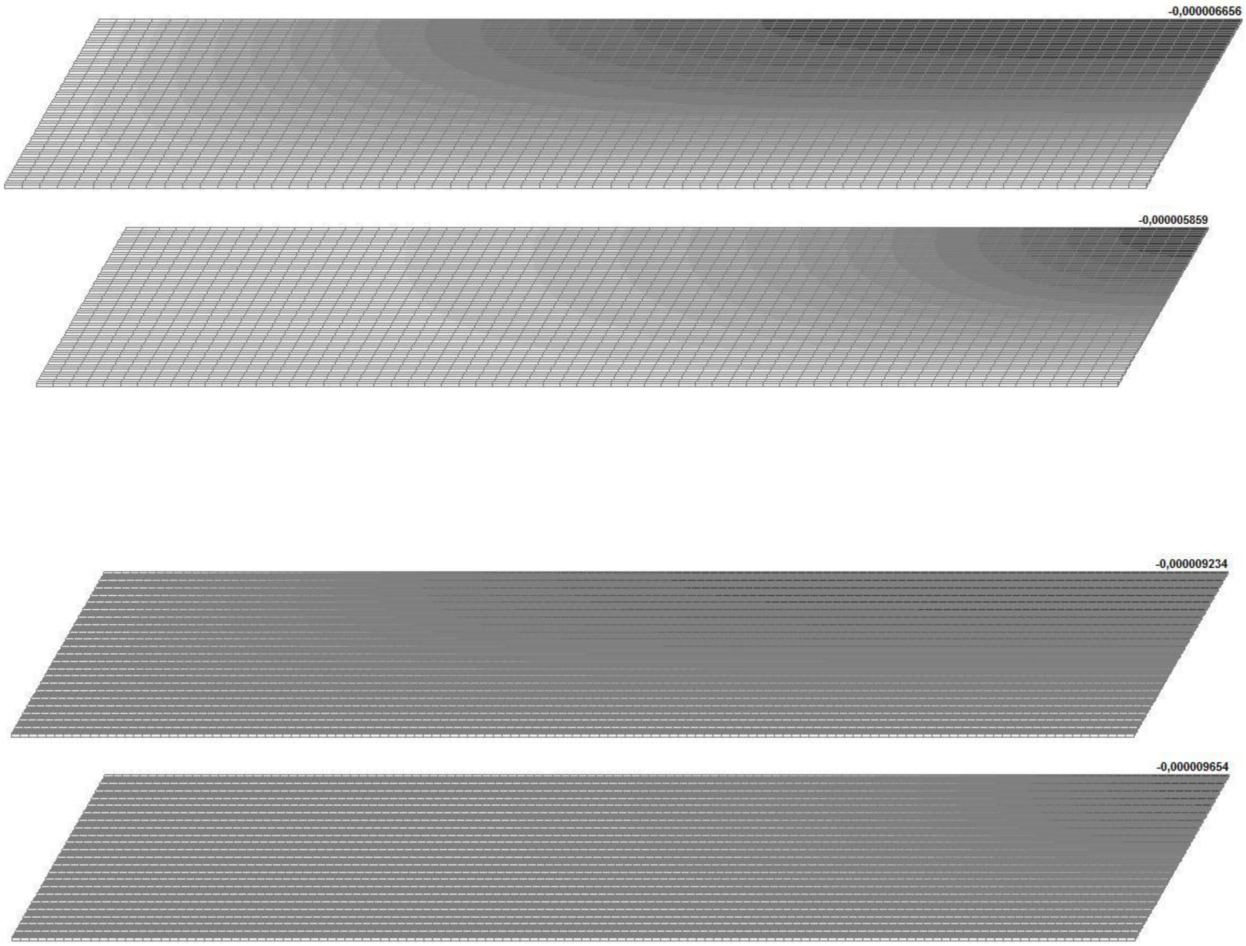

-0,000000197


-0,000000983

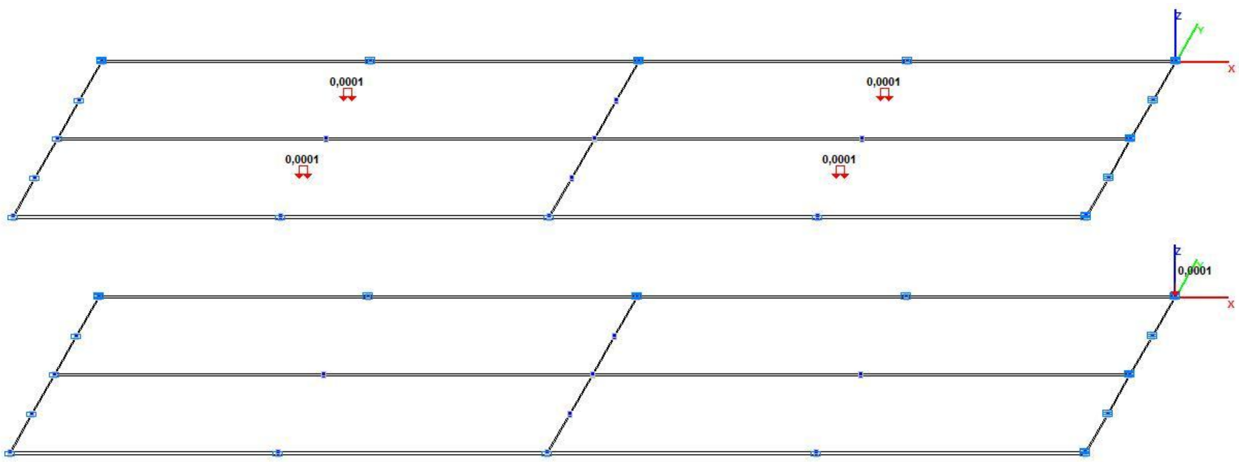

-0,000000737

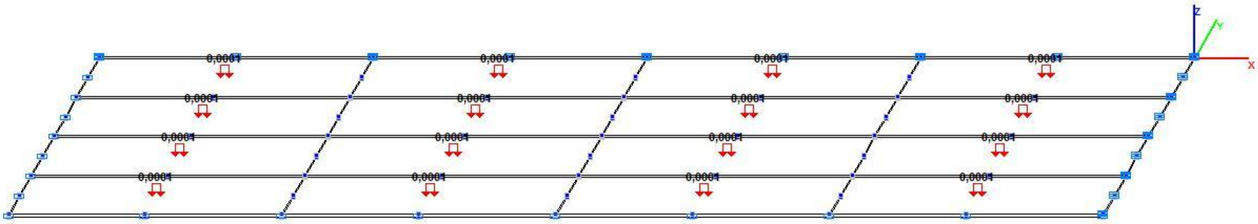

-0,000003099

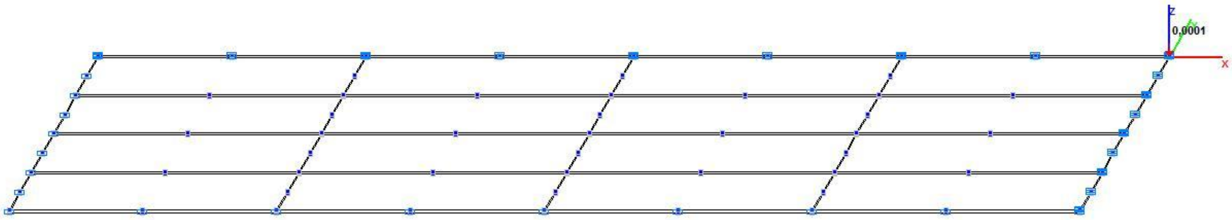

-0,000002426

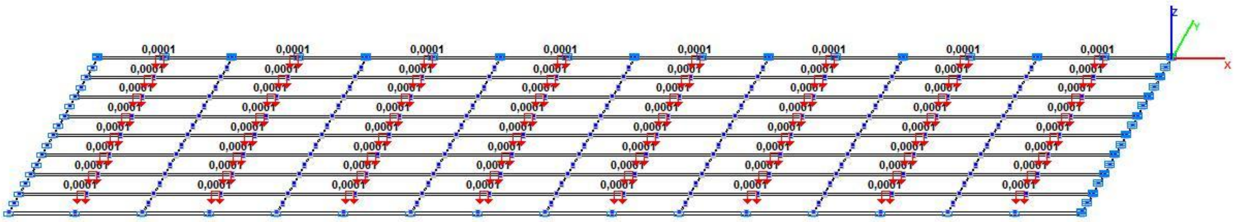


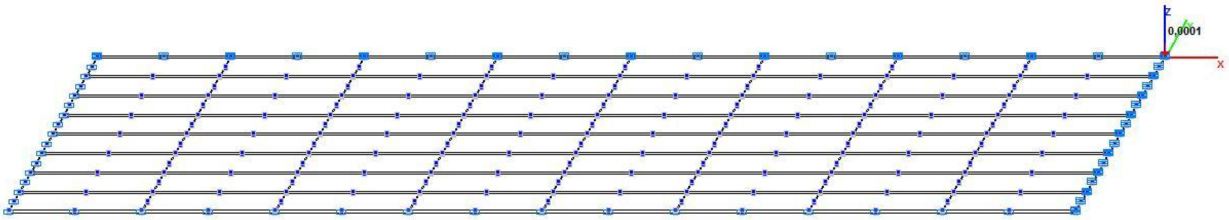
Model 5. Values of the transverse displacements in the center of the simply supported rectangular plate w_q and w_P (m, m)

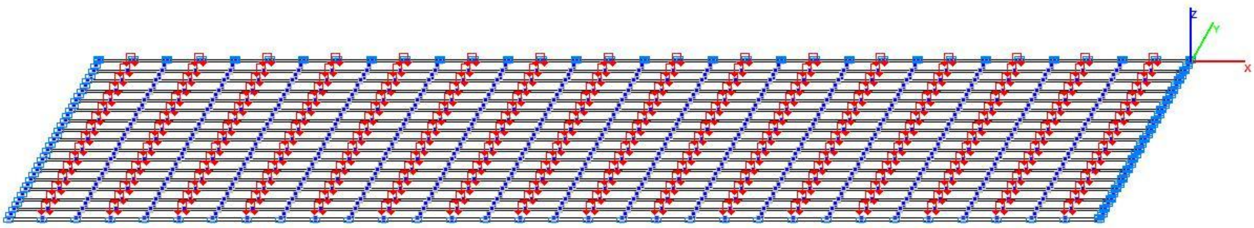


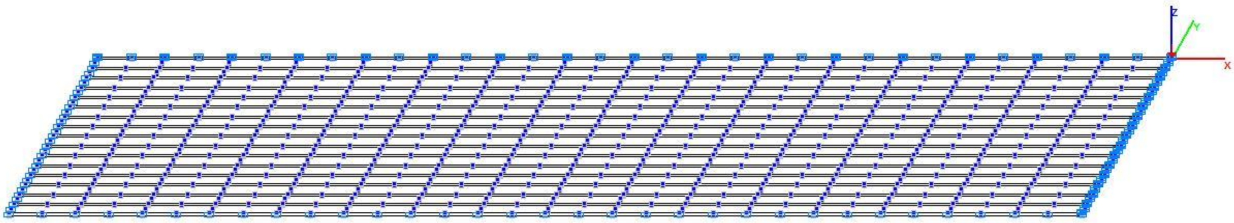


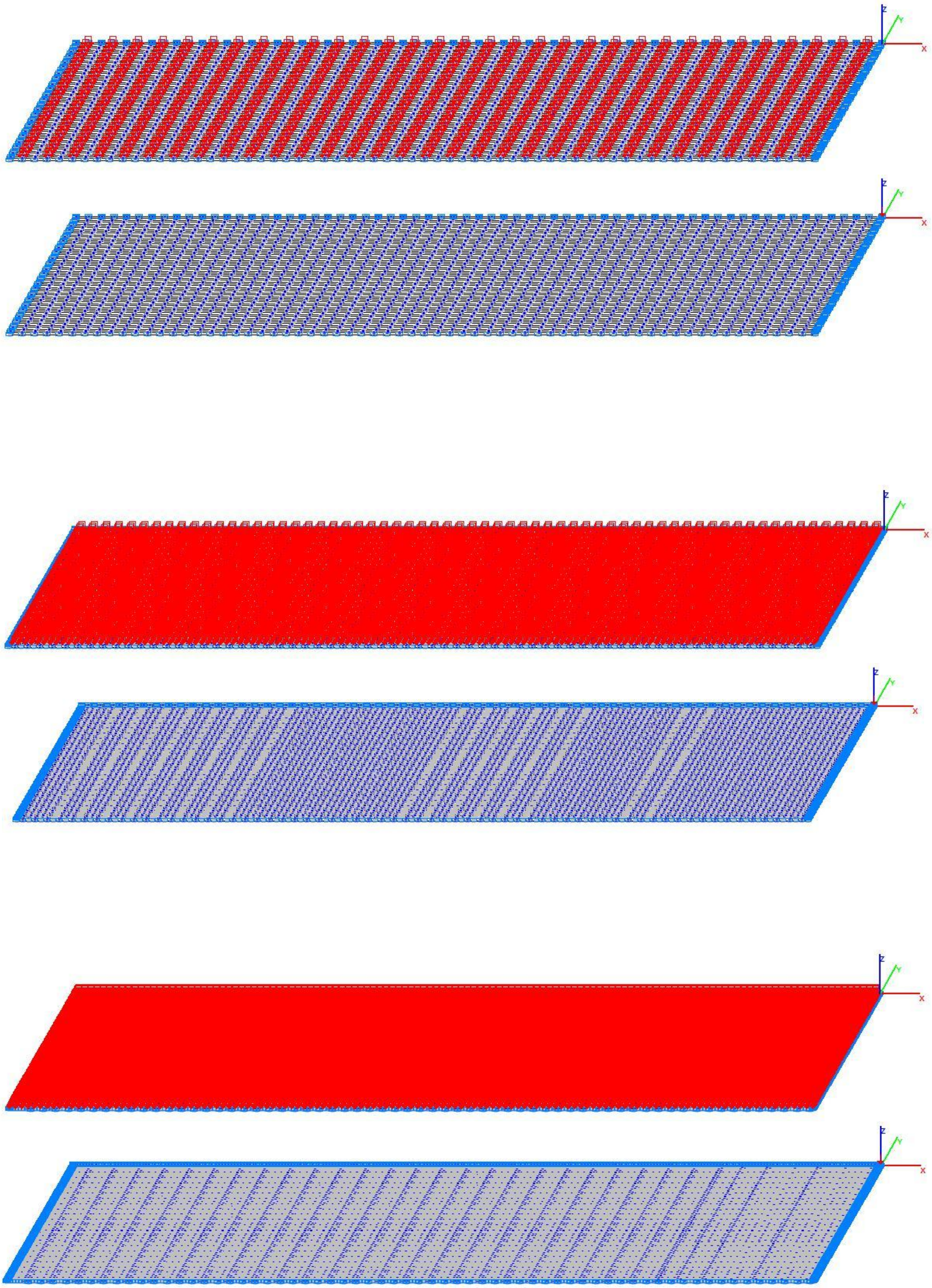




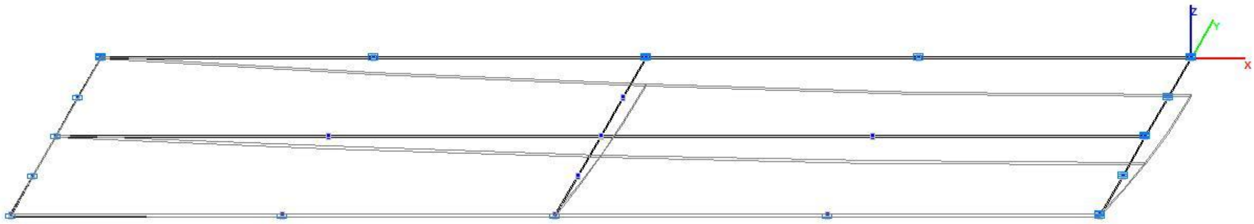


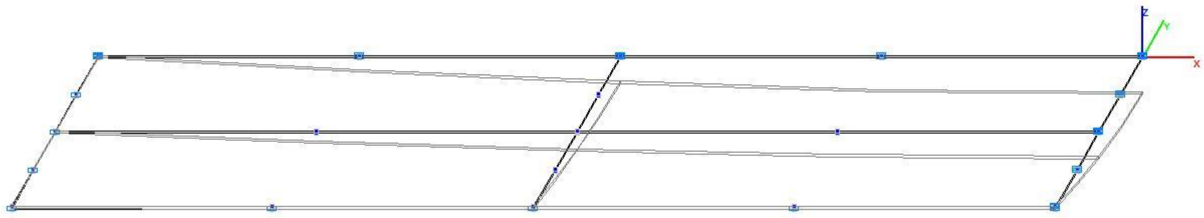


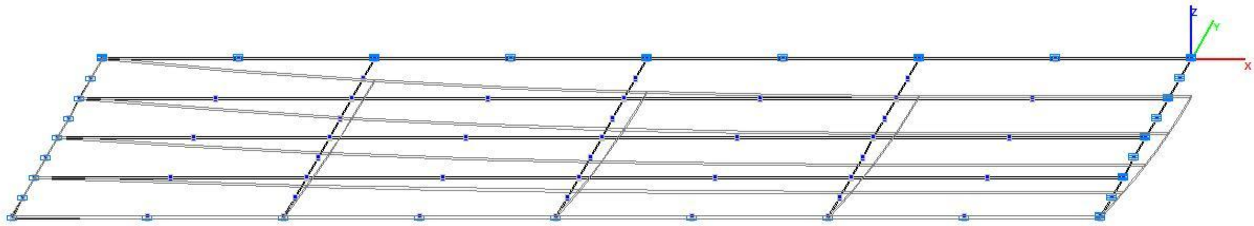


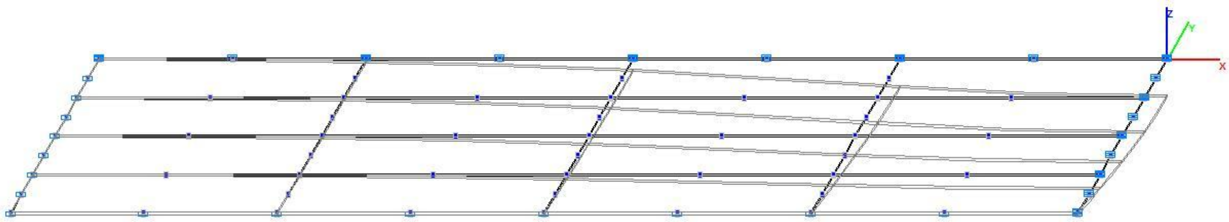


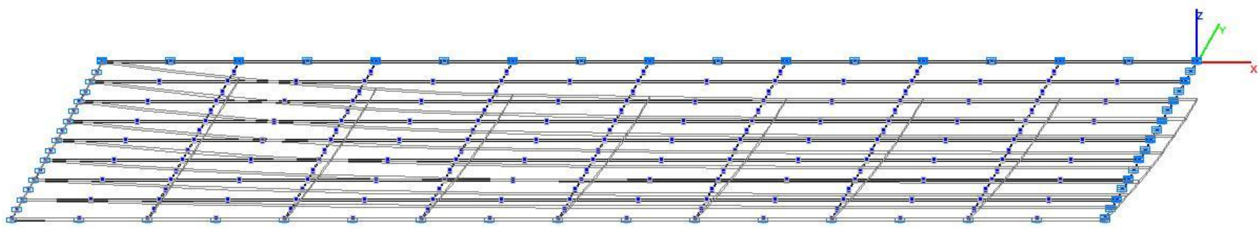
Model 6. Design model

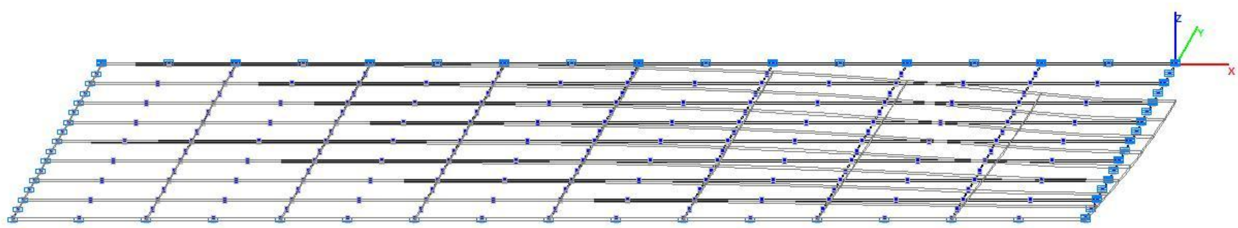

Z
Y
X

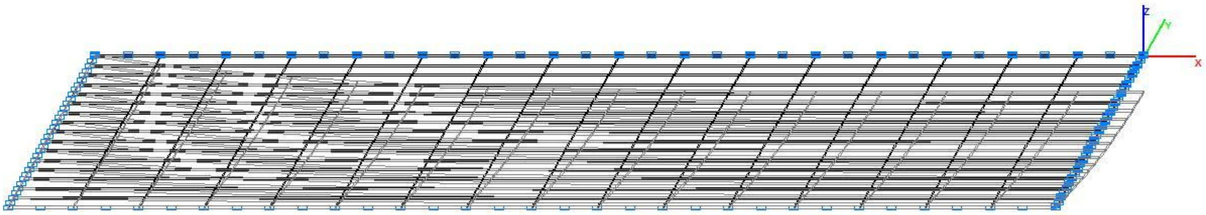

Z
Y
X

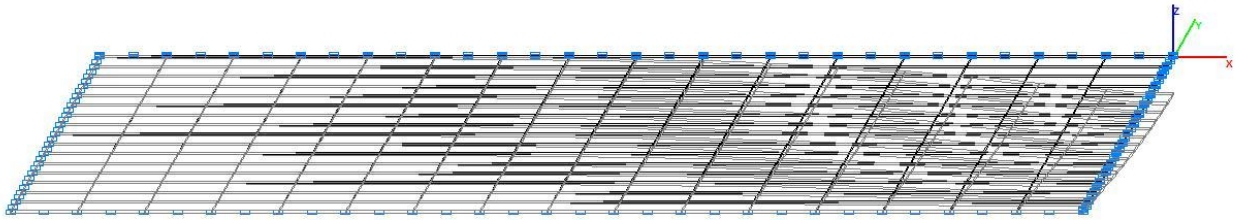

Z
Y
X

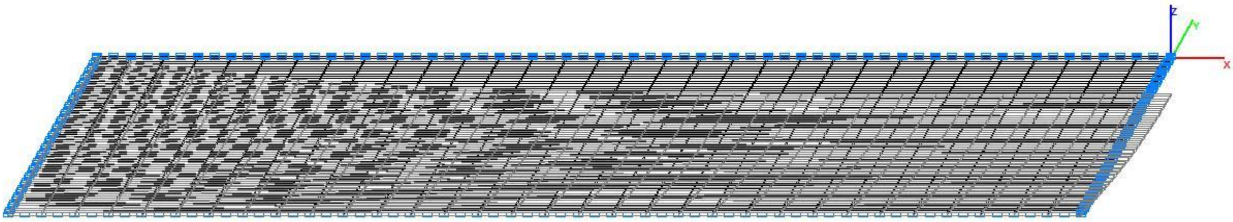

Z
Y
X

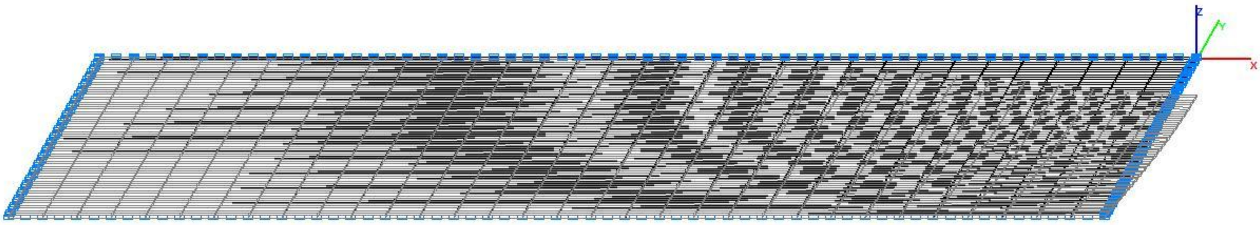

Z
Y
X

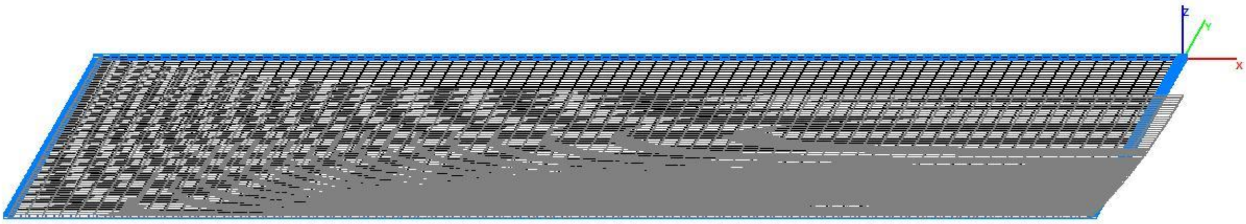

Z
Y
X

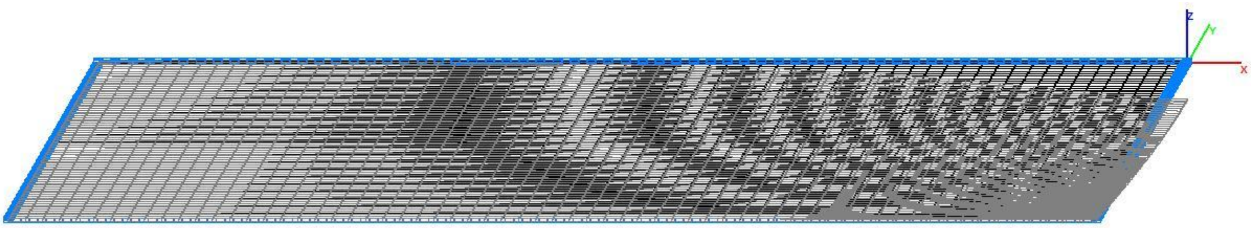




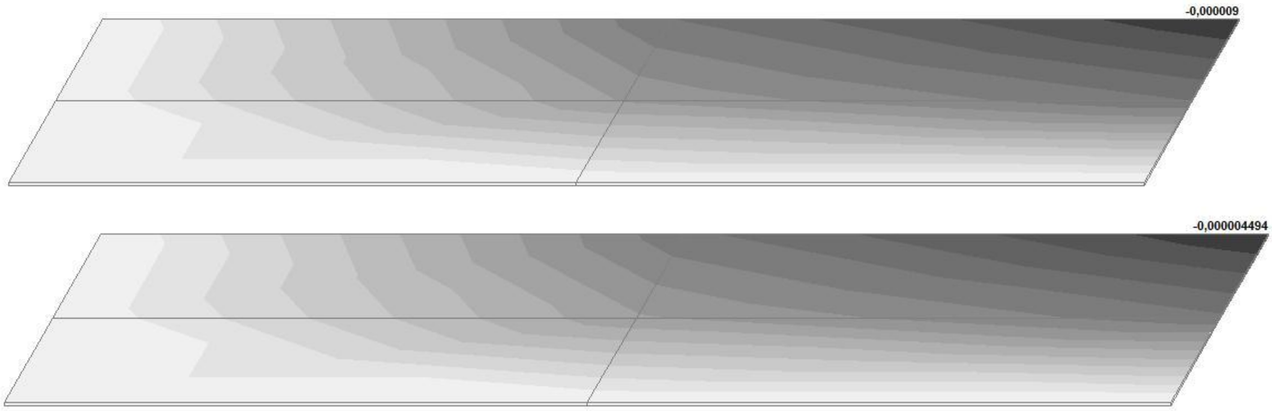


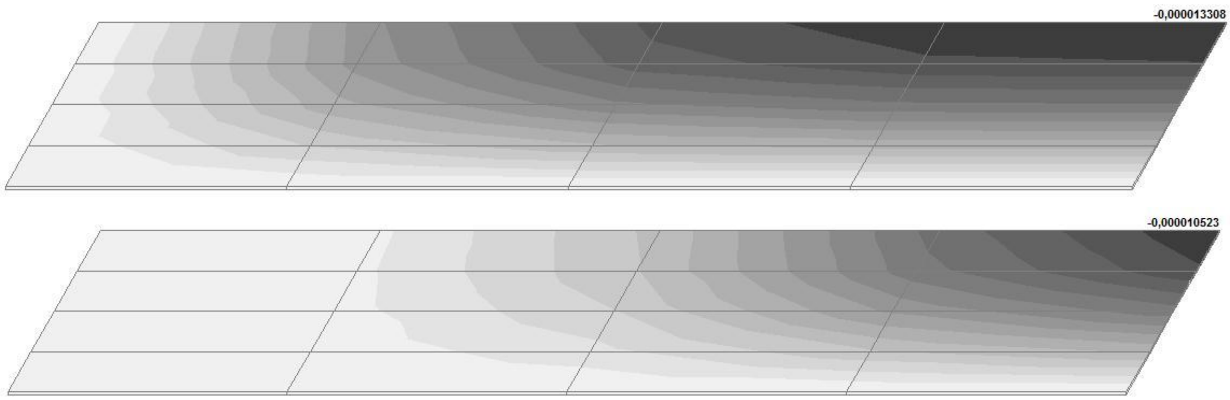


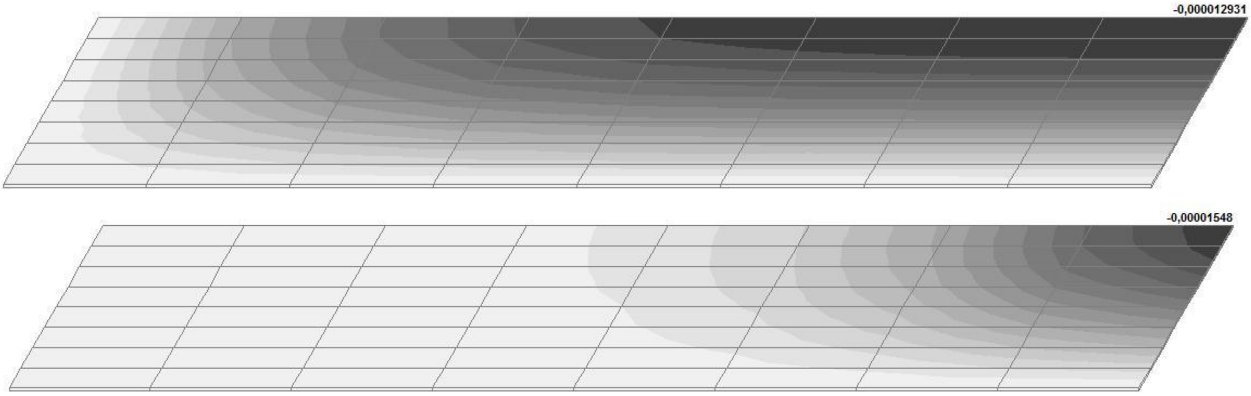


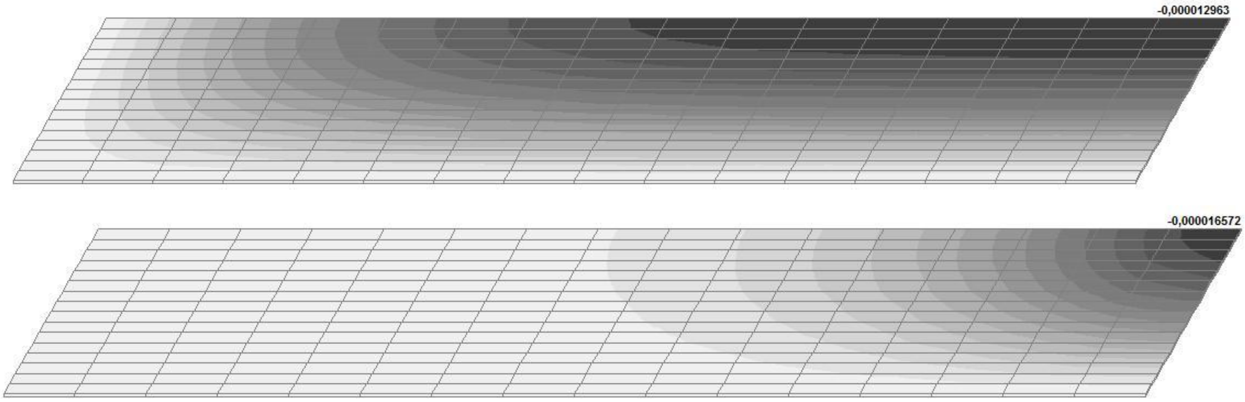


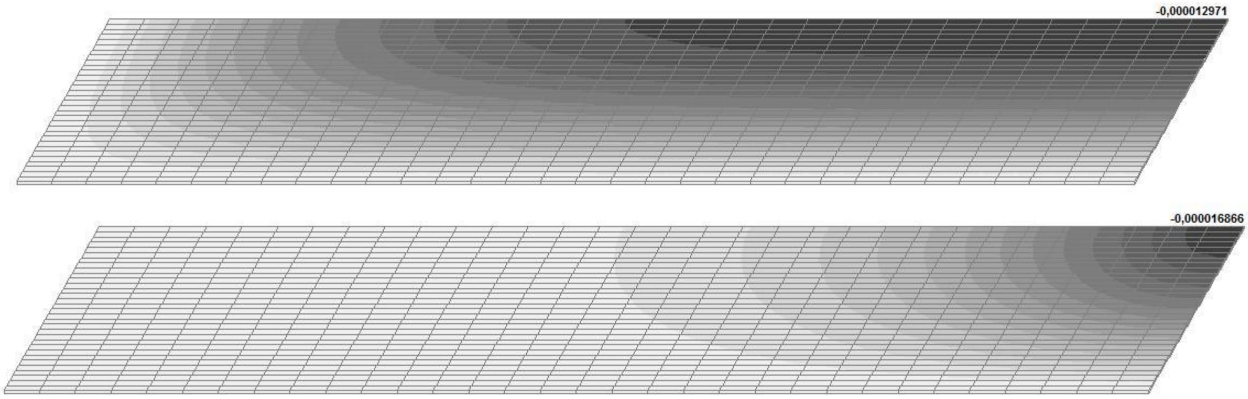
Model 6. Deformed model

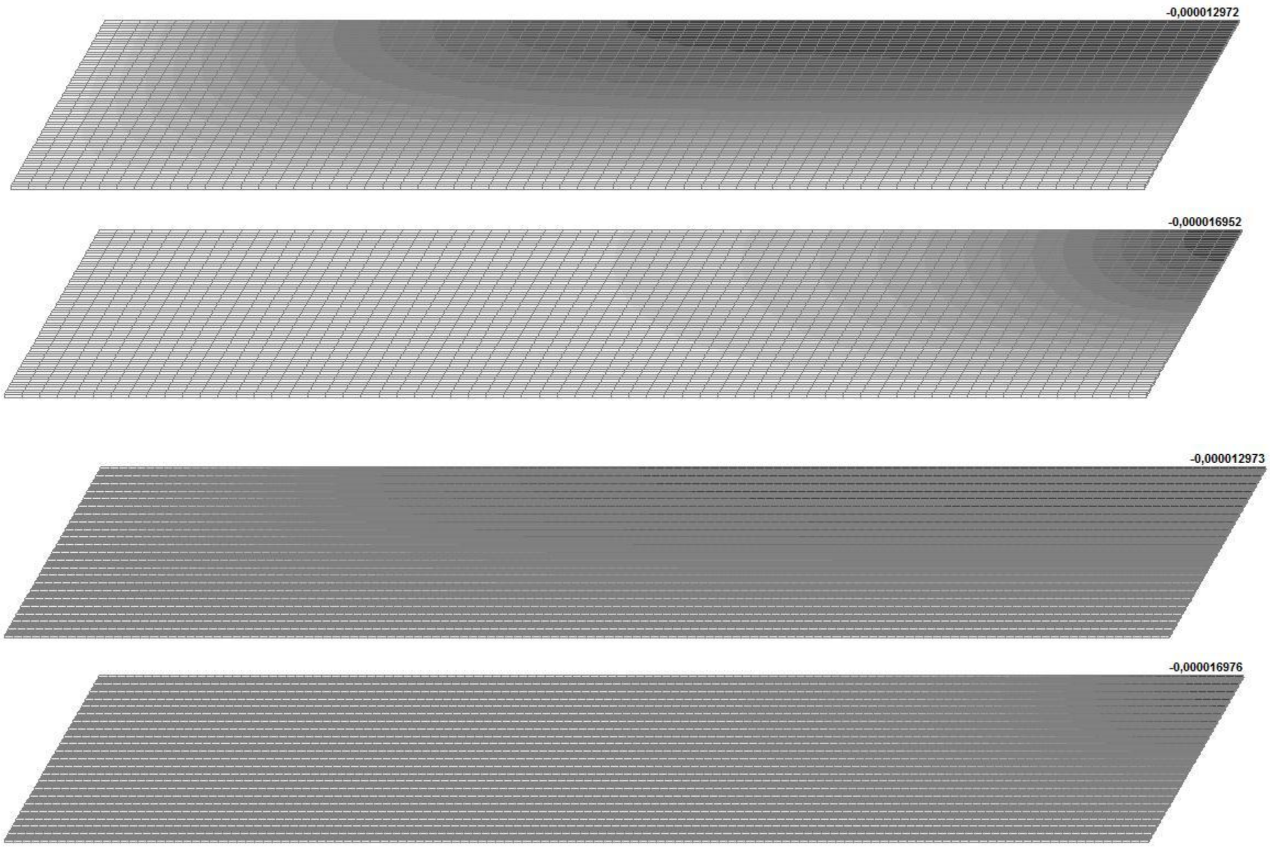





-0,000012931
-0,00001548


-0,000012963
-0,000016572


-0,000012971
-0,000016866



Model 6. Values of the transverse displacements in the center of the simply supported rectangular plate w_q and w_P (m, m)

Comparison of solutions:

Transverse displacements in the center of the simply supported flat rectangular plate w_q from the transverse load q uniformly distributed over the entire area

Model	Finite element mesh	Theory	SCAD	Deviation, %
1 (Member type 42)	2x2	12.971	11.804	9.00
	4x4		12.847	0.96
	8x8		12.958	0.10
2 (Member type 44)	2x2	12.971	12.528	3.42
	4x4		13.093	0.94
	8x8		13.030	0.45
3 (Member type 45)	2x2	12.971	13.029	0.45
	4x4		12.973	0.02
	8x8		12.971	0.00
4 (Member type 50)	2x2	12.971	13.020	0.38
	4x4		12.971	0.00
	8x8		12.971	0.00
5 (Member type 36)	2x2	$12.971 \cdot 10^{-6}$	$0.017 \cdot 10^{-6}$	99.87
	4x4		$0.067 \cdot 10^{-6}$	99.48
	8x8		$0.264 \cdot 10^{-6}$	97.96
	16x16		$0.983 \cdot 10^{-6}$	92.42
	32x32		$3.099 \cdot 10^{-6}$	76.11
	64x64		$6.656 \cdot 10^{-6}$	48.69
	128x128		$9.234 \cdot 10^{-6}$	28.81
6 (Member type 37)	2x2	$12.971 \cdot 10^{-6}$	$9.000 \cdot 10^{-6}$	30.61
	4x4		$13.308 \cdot 10^{-6}$	2.60
	8x8		$12.931 \cdot 10^{-6}$	0.31
	16x16		$12.963 \cdot 10^{-6}$	0.06
	32x32		$12.971 \cdot 10^{-6}$	0.00
	64x64		$12.972 \cdot 10^{-6}$	0.01
	128x128		$12.973 \cdot 10^{-6}$	0.02

Transverse displacements in the center of the simply supported flat rectangular plate w_P from the concentrated shear force P applied in the center

Model	Finite element mesh	Theory	SCAD	Deviation, %
1 (Member type 42)	2x2	16.960	7.771	54.18
	4x4		11.983	29.34
	8x8		14.833	12.54
2 (Member type 44)	2x2	16.960	12.674	25.27
	4x4		14.768	12.92
	8x8		15.657	7.68
3 (Member type 45)	2x2	16.960	15.383	9.30
	4x4		16.539	2.48
	8x8		16.849	0.65
4 (Member type 50)	2x2	16.960	15.862	6.47
	4x4		16.553	2.40
	8x8		16.845	0.68
5 (Member type 36)	2x2	$16.960\cdot10^{-6}$	$0.014\cdot10^{-6}$	99.92
	4x4		$0.051\cdot10^{-6}$	99.70
	8x8		$0.197\cdot10^{-6}$	98.84
	16x16		$0.737\cdot10^{-6}$	95.65
	32x32		$2.426\cdot10^{-6}$	85.70
	64x64		$5.859\cdot10^{-6}$	65.45
	128x128		$9.654\cdot10^{-6}$	43.08
6 (Member type 37)	2x2	$16.960\cdot10^{-6}$	$4.494\cdot10^{-6}$	73.50
	4x4		$10.523\cdot10^{-6}$	37.95
	8x8		$15.480\cdot10^{-6}$	8.73
	16x16		$16.572\cdot10^{-6}$	2.29
	32x32		$16.866\cdot10^{-6}$	0.55
	64x64		$16.952\cdot10^{-6}$	0.05
	128x128		$16.976\cdot10^{-6}$	0.09

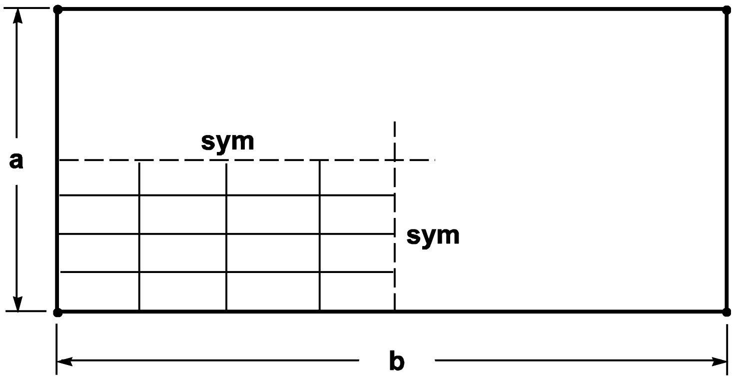
Notes: In the analytical solution the values of the transverse displacements in the center of the simply supported flat rectangular plate w_q and w_P from the respective actions are determined according to the following formulas:

$$w_q = \frac{4\cdot q\cdot a^4}{\pi^5\cdot D}\cdot\sum_{m=1}^{\infty}\left\{\frac{1}{m^5}\cdot\left[1-\frac{\frac{m\cdot\pi\cdot b}{2\cdot a}\cdot th\left(\frac{m\cdot\pi\cdot b}{2\cdot a}\right)+2}{2\cdot ch\left(\frac{m\cdot\pi\cdot b}{2\cdot a}\right)}\right]\cdot sin\left(\frac{m\cdot\pi}{2}\right)\right\};$$

$$w_P = \frac{P\cdot a^2}{2\cdot\pi^3\cdot D}\cdot\sum_{m=1}^{\infty}\left\{\frac{1}{m^3}\cdot\left[th\left(\frac{m\cdot\pi\cdot b}{2\cdot a}\right)-\frac{\frac{m\cdot\pi\cdot b}{2\cdot a}}{ch^2\left(\frac{m\cdot\pi\cdot b}{2\cdot a}\right)}\right]\cdot sin^2\left(\frac{m\cdot\pi}{2}\right)\right\},\text{ where:}$$

$$D = \frac{E\cdot h^3}{12\cdot\left(1-\nu^2\right)}.$$

Flat Rectangular Plate Clamped along the Outer Edges and Subjected to a Transverse Load Uniformly Distributed over the Entire Area and a Concentrated Shear Force Applied in the Center



Objective: Check of the obtained values of the transverse displacements in the center of a flat rectangular plate clamped along the outer edges and subjected to a transverse load uniformly distributed over the entire area and a concentrated shear force applied in the center.

Initial data files:

File name	Description
Bending_of_rectangular_flat_plate_Clamped_supported_Shell_42_Mesh_2x2.SPR Bending_of_rectangular_flat_plate_Clamped_supported_Shell_42_Mesh_4x4.SPR Bending_of_rectangular_flat_plate_Clamped_supported_Shell_42_Mesh_8x8.SPR	Design model with the elements of type 42 for meshes 2x2, 4x4, 8x8
Bending_of_rectangular_flat_plate_Clamped_supported_Shell_44_Mesh_2x2.SPR Bending_of_rectangular_flat_plate_Clamped_supported_Shell_44_Mesh_4x4.SPR Bending_of_rectangular_flat_plate_Clamped_supported_Shell_44_Mesh_8x8.SPR	Design model with the elements of type 44 for meshes 2x2, 4x4, 8x8
Bending_of_rectangular_flat_plate_Clamped_supported_Shell_45_Mesh_2x2.SPR Bending_of_rectangular_flat_plate_Clamped_supported_Shell_45_Mesh_4x4.SPR Bending_of_rectangular_flat_plate_Clamped_supported_Shell_45_Mesh_8x8.SPR	Design model with the elements of type 45 for meshes 2x2, 4x4, 8x8
Bending_of_rectangular_flat_plate_Clamped_supported_Shell_50_Mesh_2x2.SPR Bending_of_rectangular_flat_plate_Clamped_supported_Shell_50_Mesh_4x4.SPR Bending_of_rectangular_flat_plate_Clamped_supported_Shell_50_Mesh_8x8.SPR	Design model with the elements of type 50 for meshes 2x2, 4x4, 8x8
Bending_of_rectangular_flat_plate_Clamped_supported_Solid_36_Mesh_2x2.SPR Bending_of_rectangular_flat_plate_Clamped_supported_Solid_36_Mesh_4x4.SPR Bending_of_rectangular_flat_plate_Clamped_supported_Solid_36_Mesh_8x8.SPR Bending_of_rectangular_flat_plate_Clamped_supported_Solid_36_Mesh_16x16.SPR Bending_of_rectangular_flat_plate_Clamped_supported_Solid_36_Mesh_32x32.SPR Bending_of_rectangular_flat_plate_Clamped_supported_Solid_36_Mesh_64x64.SPR Bending_of_rectangular_flat_plate_Clamped_supported_Solid_36_Mesh_128x128.SPR	Design model with the elements of type 36 for meshes 2x2, 4x4, 8x8, 16x16, 32x32, 64x64, 128x128
Bending_of_rectangular_flat_plate_Clamped_supported_Solid_37_Mesh_2x2.SPR Bending_of_rectangular_flat_plate_Clamped_supported_Solid_37_Mesh_4x4.SPR Bending_of_rectangular_flat_plate_Clamped_supported_Solid_37_Mesh_8x8.SPR Bending_of_rectangular_flat_plate_Clamped_supported_Solid_37_Mesh_16x16.SPR Bending_of_rectangular_flat_plate_Clamped_supported_Solid_37_Mesh_32x32.SPR Bending_of_rectangular_flat_plate_Clamped_supported_Solid_37_Mesh_64x64.SPR Bending_of_rectangular_flat_plate_Clamped_supported_Solid_37_Mesh_128x128.SPR	Design model with the elements of type 37 for meshes 2x2, 4x4, 8x8, 16x16, 32x32, 64x64, 128x128

Problem formulation: The flat rectangular plate clamped along the outer edges is subjected to the transverse load q uniformly distributed over the entire area and the concentrated shear force P applied in the center. Check the obtained values of the transverse displacements in the center of the flat rectangular plate clamped along the outer edges w_q and w_P from the respective actions.

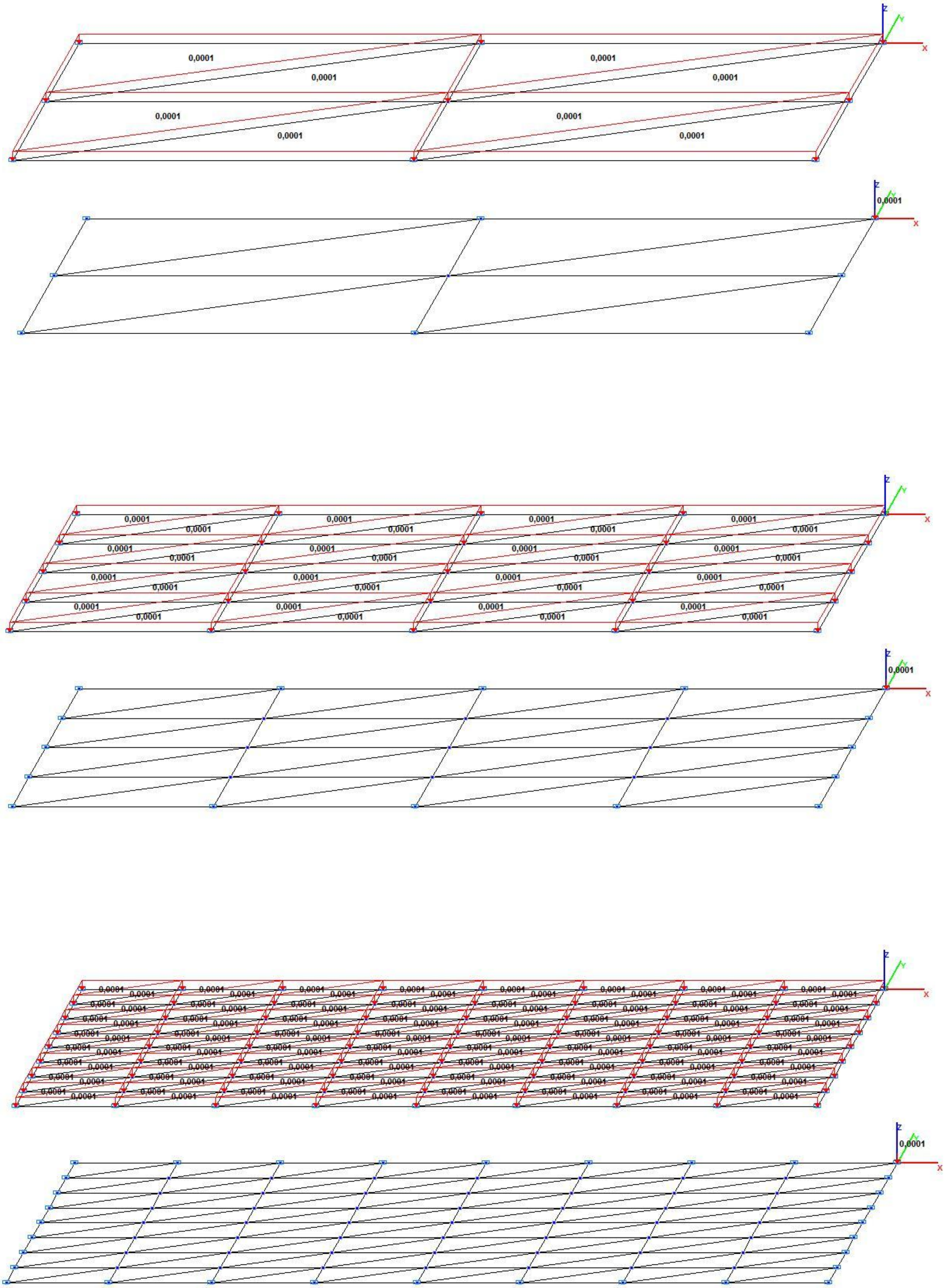
References: R. H. Macneal, R. L. Harder, A proposed standard set of problems to test finite element accuracy, North-Holland, Finite elements in analysis and design, 1, 1985, p. 3-20.
S. Timoshenko, S. Woinowsky-Krieger, Theory of plates and shells, New York, McGraw-Hill,1959, p. 120, 143, 202, 206.

Initial data:

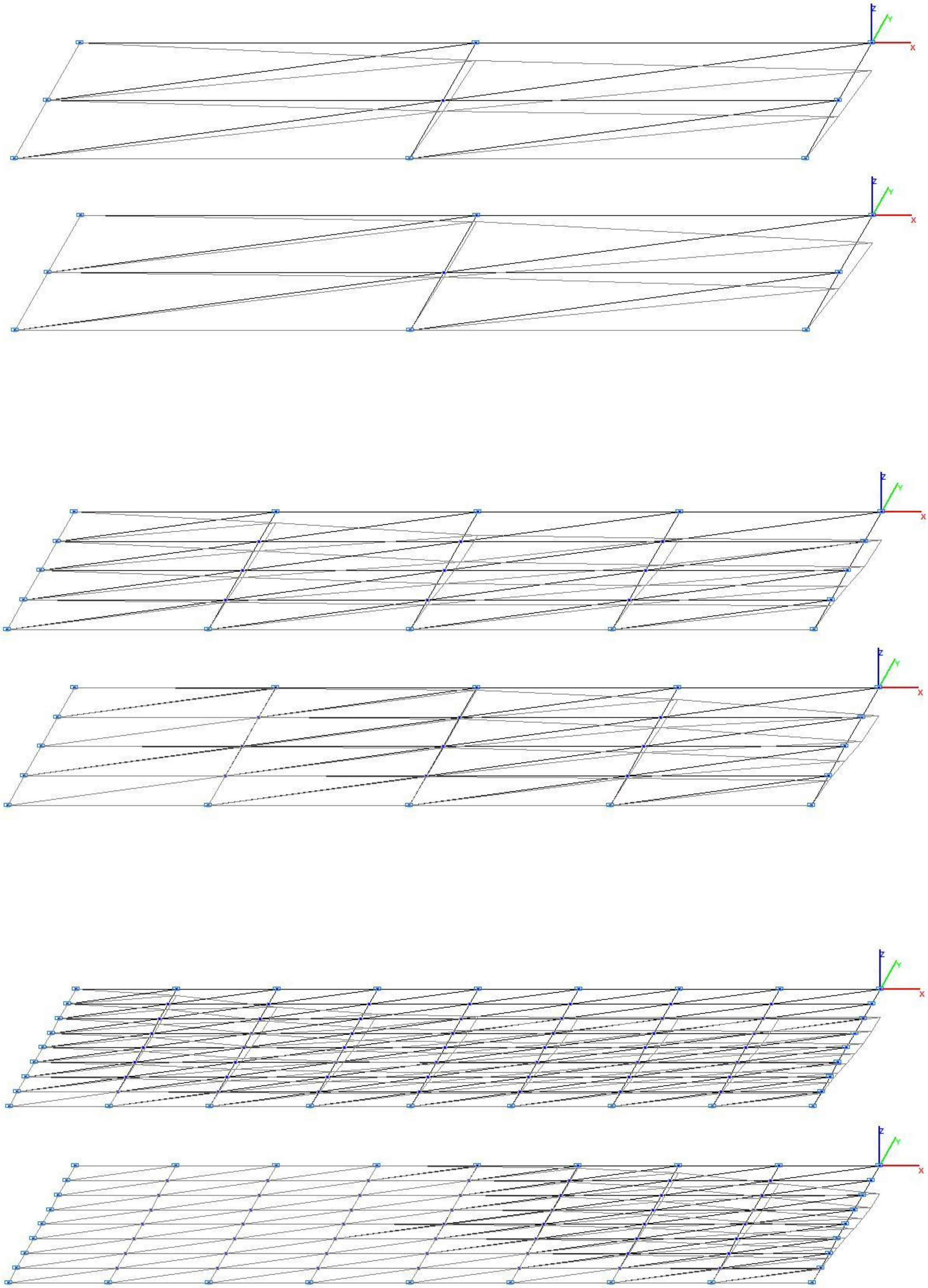
$E = 1.7472 \cdot 10^7$ kPa - elastic modulus of the plate material;
$\nu = 0.30$ - Poisson's ratio;
$a = 2.00$ m - width of the plate;
$b = 10.00$ m - length of the plate;
$h = 10^{-4}$ (10^{-2}) m - thickness of the plate;
$q = 1.0 \cdot 10^{-4}$ kN/m^2 - value of the transverse load uniformly distributed over the entire area of the plate;
$P = 4.0 \cdot 10^{-4}$ kN - value of the concentrated shear force in the center of the plate.

Finite element model: Design model – general type system. Six design models of a quarter of the plate according to the symmetry conditions are considered:
Model 1 – 8, 32, 128 three-node shell elements of type 42 with a regular mesh 2x2, 4x4, 8x8. The thickness of the plate – 10^{-4} m. Boundary conditions are provided by imposing constraints on the nodes of the clamped edges of the plate in the directions of the degrees of freedom X, Y, Z, UX, UY, UZ and constraints according to the symmetry conditions. Number of nodes in the model – 9, 25, 81.
Model 2 – 4, 16, 64 four-node shell elements of type 44 with a regular mesh 2x2, 4x4, 8x8. The thickness of the plate – 10^{-4} m. Boundary conditions are provided by imposing constraints on the nodes of the clamped edges of the plate in the directions of the degrees of freedom X, Y, Z, UX, UY, UZ and constraints according to the symmetry conditions. Number of nodes in the model – 9, 25, 81.
Model 3 – 8, 32, 128 six-node shell elements of type 45 with a regular mesh 2x2, 4x4, 8x8. The thickness of the plate – 10^{-4} m. Boundary conditions are provided by imposing constraints on the nodes of the clamped edges of the plate in the directions of the degrees of freedom X, Y, Z, UX, UY, UZ and constraints according to the symmetry conditions. Number of nodes in the model – 25, 81, 289.
Model 4 – 4, 16, 64 eight-node shell elements of type 50 with a regular mesh 2x2, 4x4, 8x8. The thickness of the plate – 10^{-4} m. Boundary conditions are provided by imposing constraints on the nodes of the clamped edges of the plate in the directions of the degrees of freedom X, Y, Z, UX, UY, UZ and constraints according to the symmetry conditions. Number of nodes in the model – 25, 81, 289.
Model 5 – 4, 16, 64, 256, 1024, 4096, 16384 в eight-node isoparametric solid elements of type 36 with a regular mesh 2x2x1, 4x4x1, 8x8x1, 16x16x1, 32x32x1, 64x64x1, 128x128x1. The thickness of the plate – 10^{-2} m. Boundary conditions are provided by imposing constraints on the nodes of the clamped sides of the lower surface of the plate in the directions of the degrees of freedom X, Y, Z, on the nodes of the clamped sides of the upper surface of the plate parallel to the Y axis of the global coordinate system in the direction of the degree of freedom X, on the nodes of the clamped sides of the upper surface of the plate parallel to the X axis of the global coordinate system in the direction of the degree of freedom Y and constraints according to the symmetry conditions. Number of nodes in the model – 18, 50, 162, 578, 2178, 8450, 33282.
Model 6 – 4, 16, 64, 256, 1024, 4096, 16384 twenty-node isoparametric solid elements of type 37 with a regular mesh 2x2x1, 4x4x1, 8x8x1, 16x16x1, 32x32x1, 64x64x1, 128x128x1. The thickness of the plate – 10^{-2} m. Boundary conditions are provided by imposing constraints on the nodes of the clamped sides of the lower surface of the plate in the directions of the degrees of freedom X, Y, Z, on the nodes of the clamped sides of the upper surface of the plate parallel to the Y axis of the global coordinate system in the direction of the degree of freedom X, on the nodes of the clamped sides of the upper surface of the plate parallel to the X axis of the global coordinate system in the direction of the degree of freedom Y and constraints according to the symmetry conditions. Number of nodes in the model – 51, 155, 531, 1955, 7491, 29315, 115971.

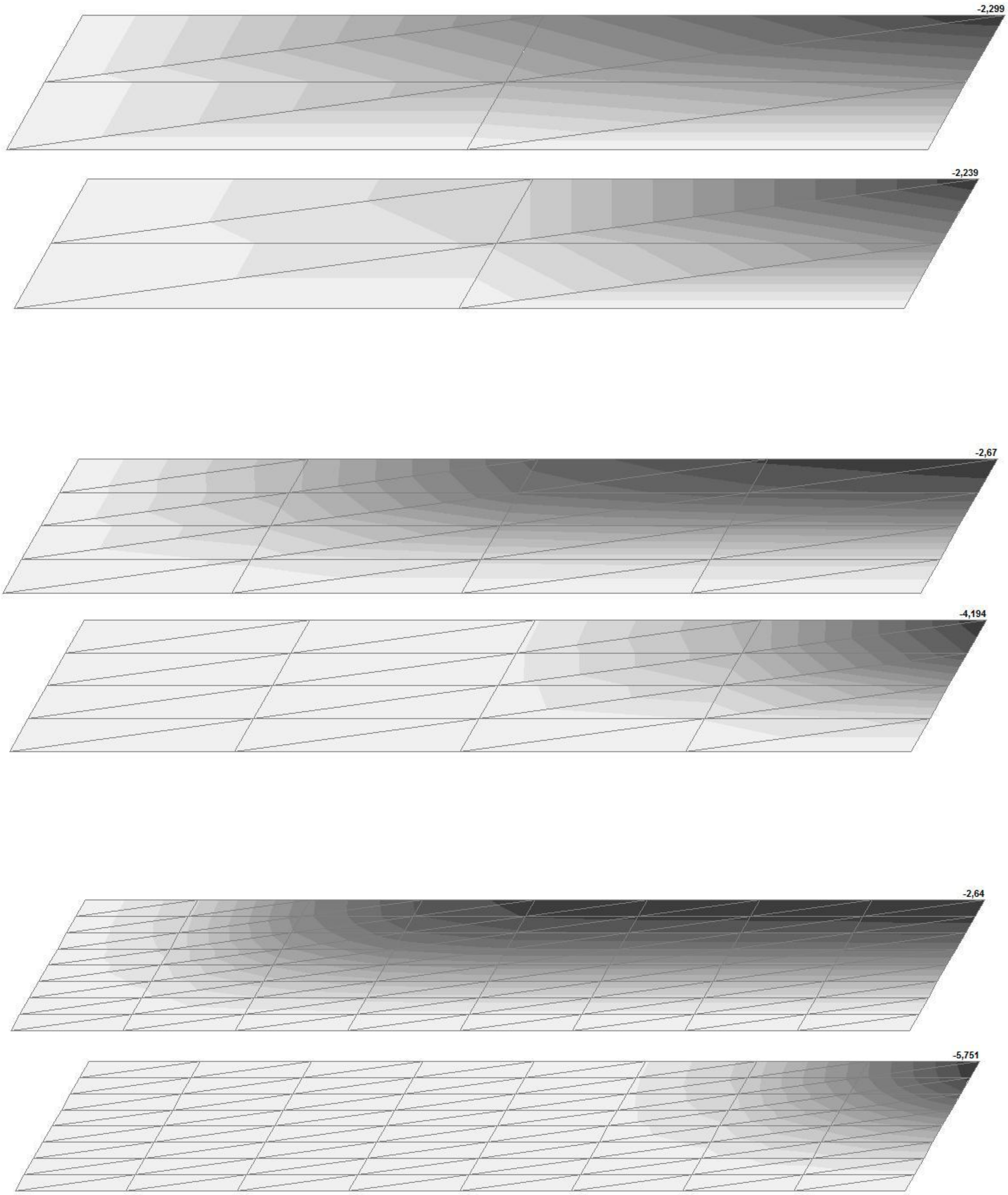
Results in SCAD



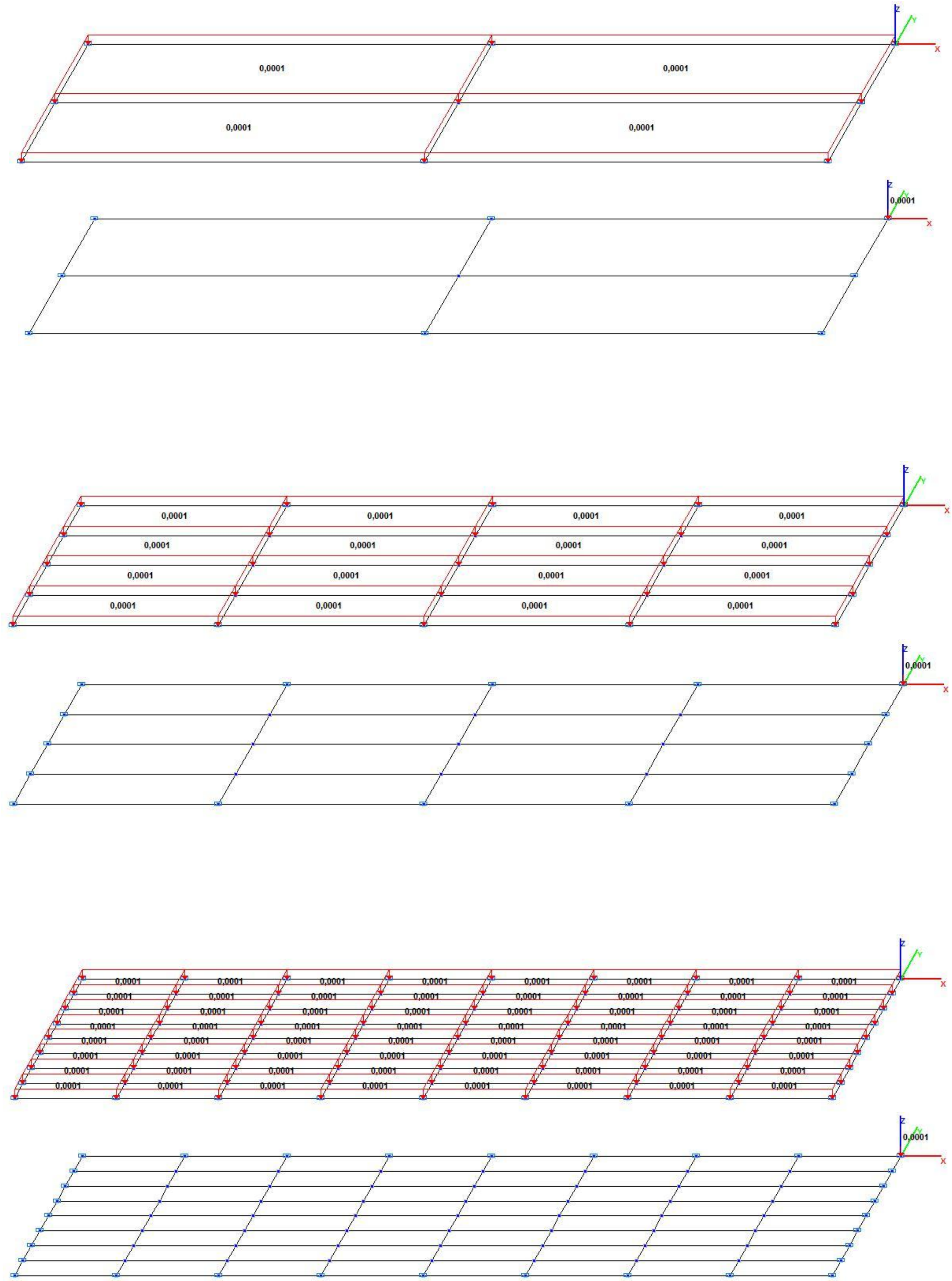
Model 1. Design model



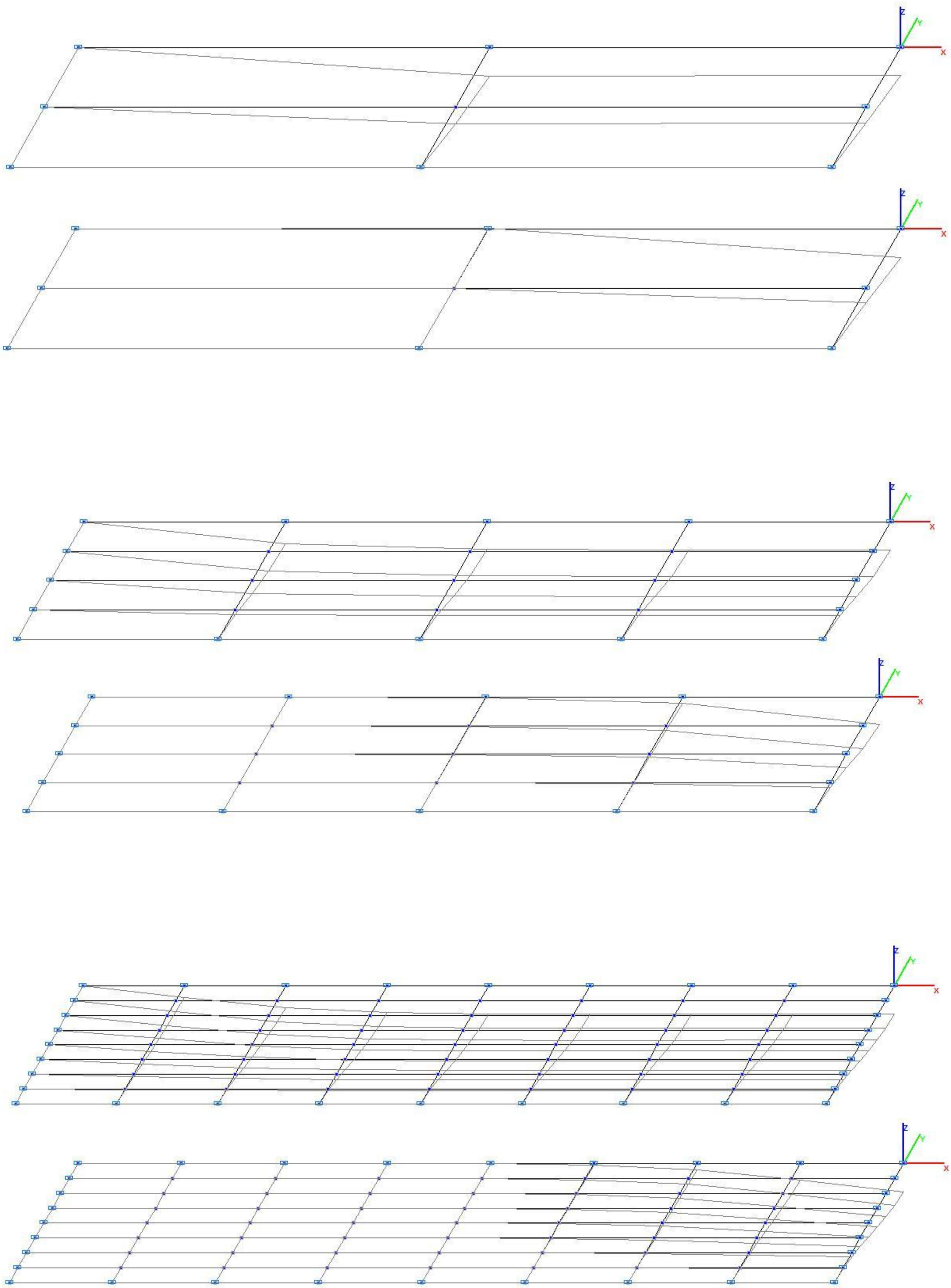
Model 1. Deformed model



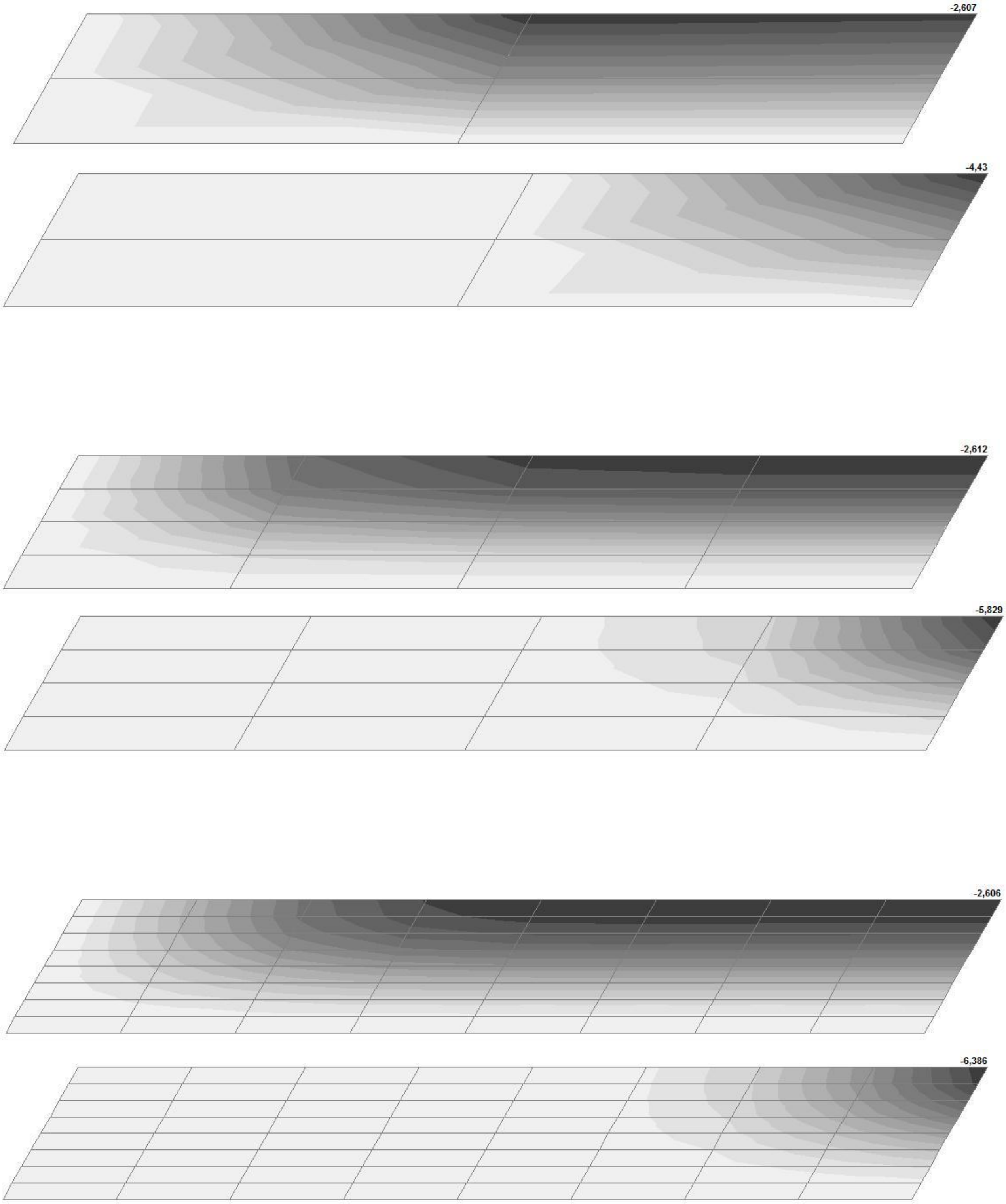
Model 1. Values of the transverse displacements in the center of the rectangular plate clamped along the outer edges w_q *and* w_P *(m, m)*



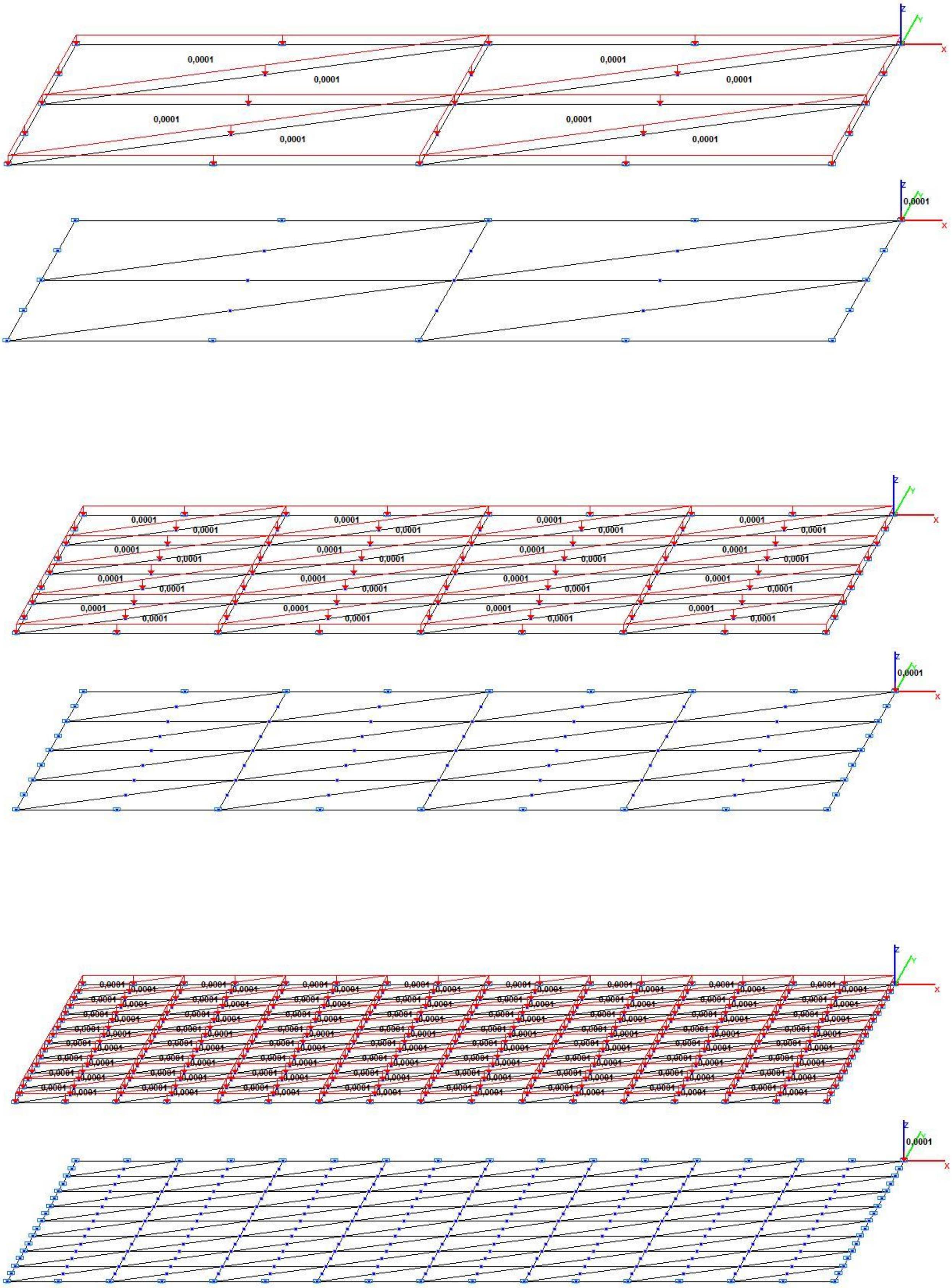
Model 2. Design model



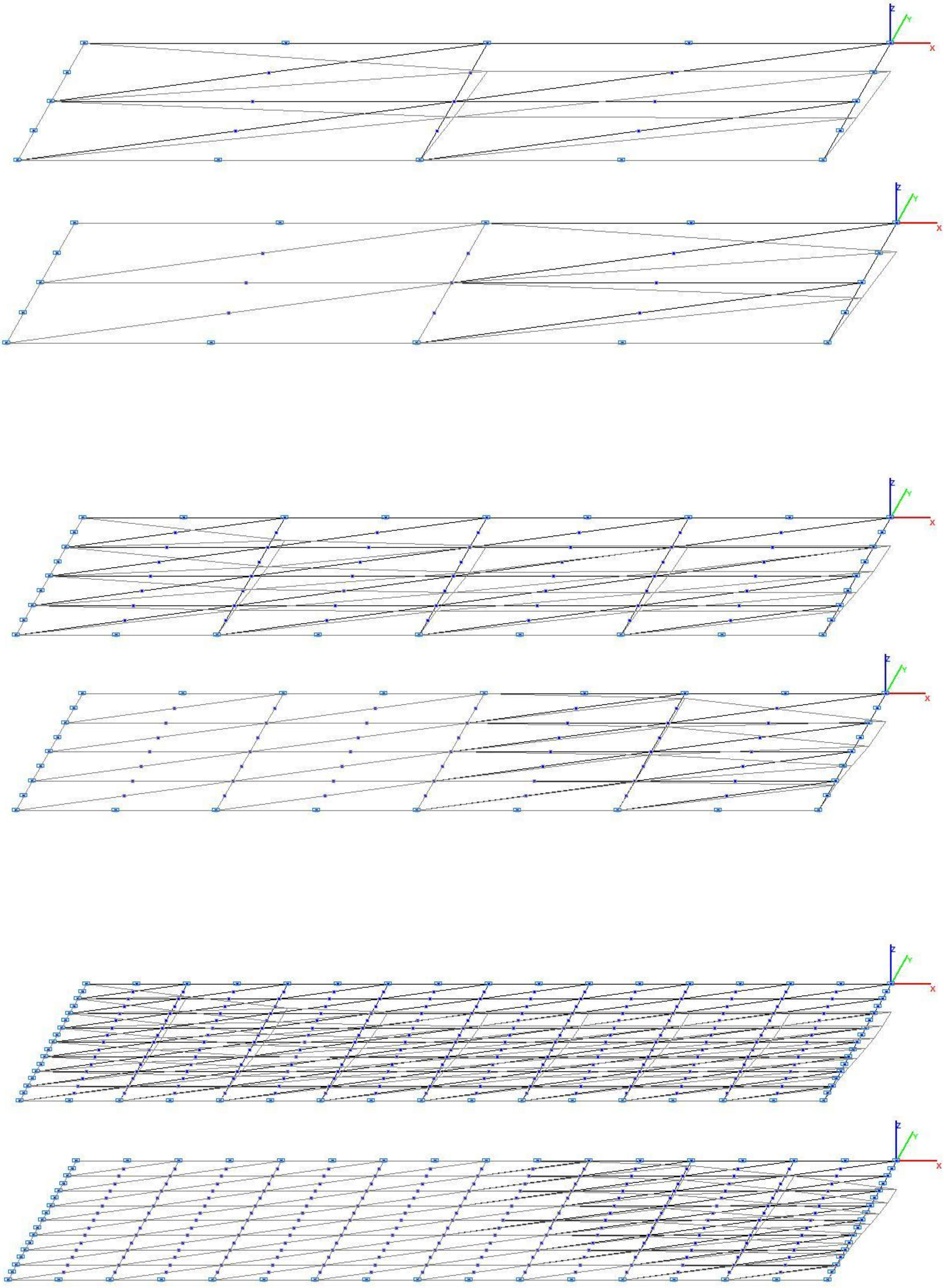
Model 2. Deformed model



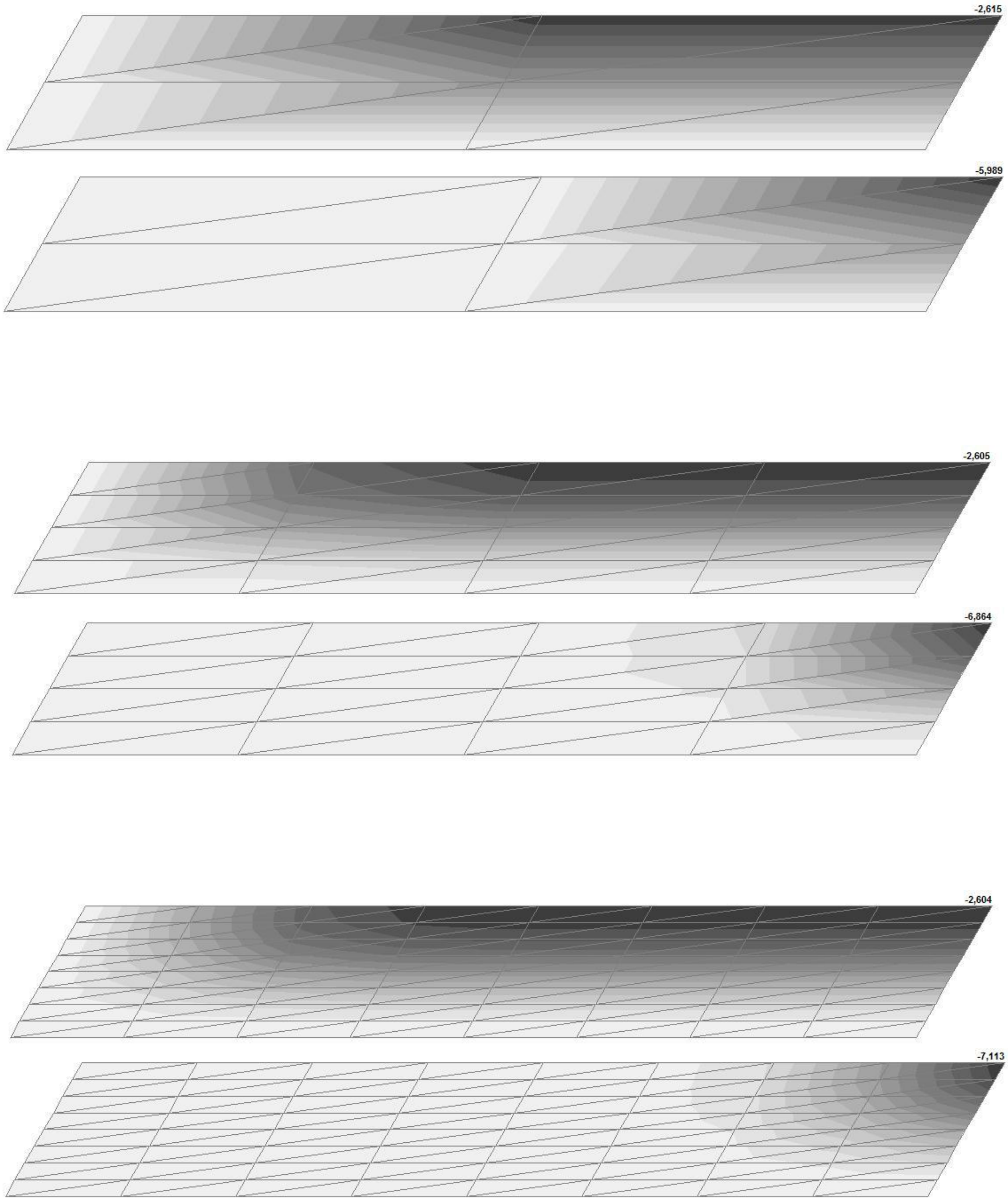
Model 2. Values of the transverse displacements in the center of the rectangular plate clamped along the outer edges w_q and w_P (m, m)



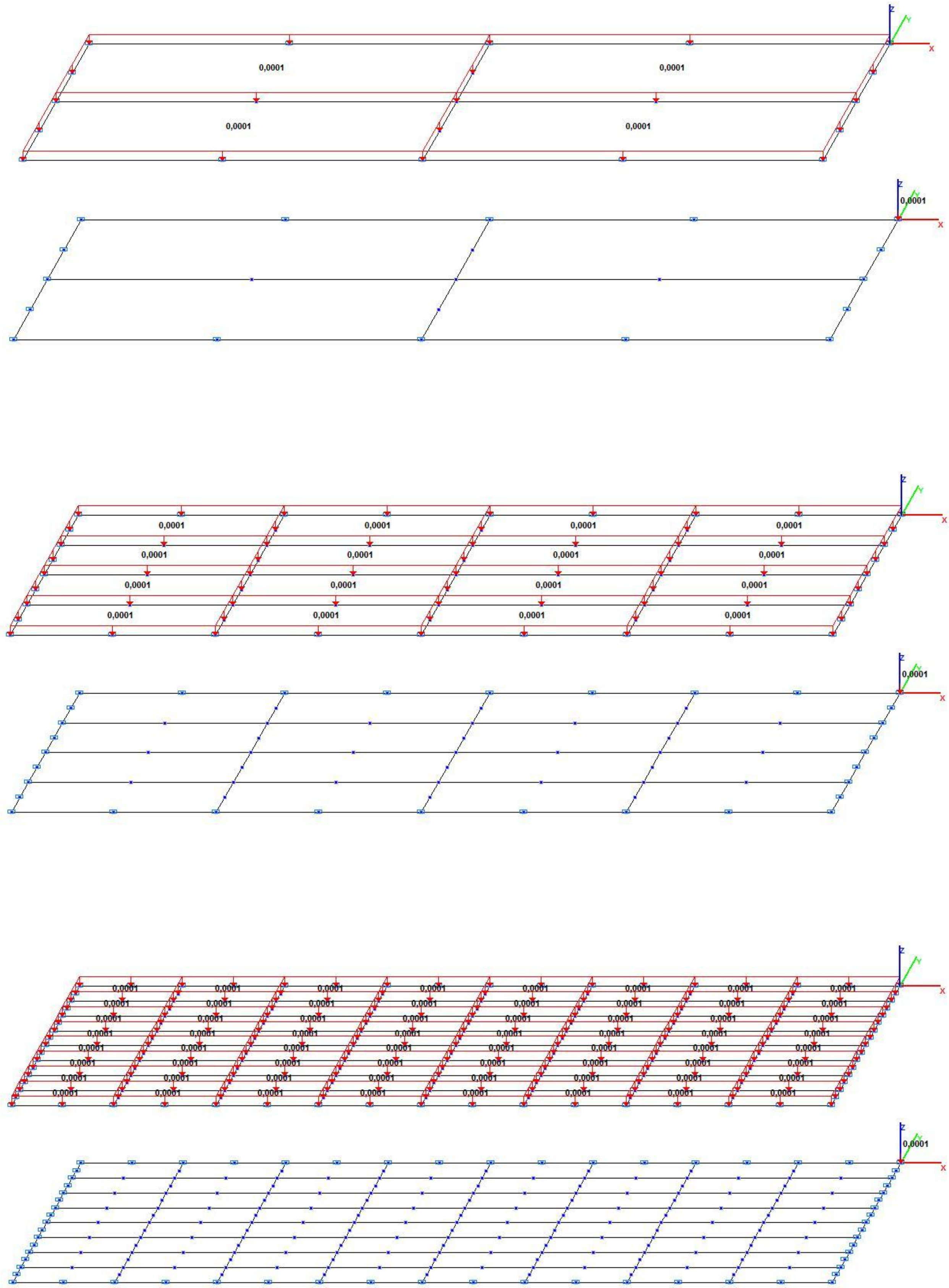
Model 3. Design model



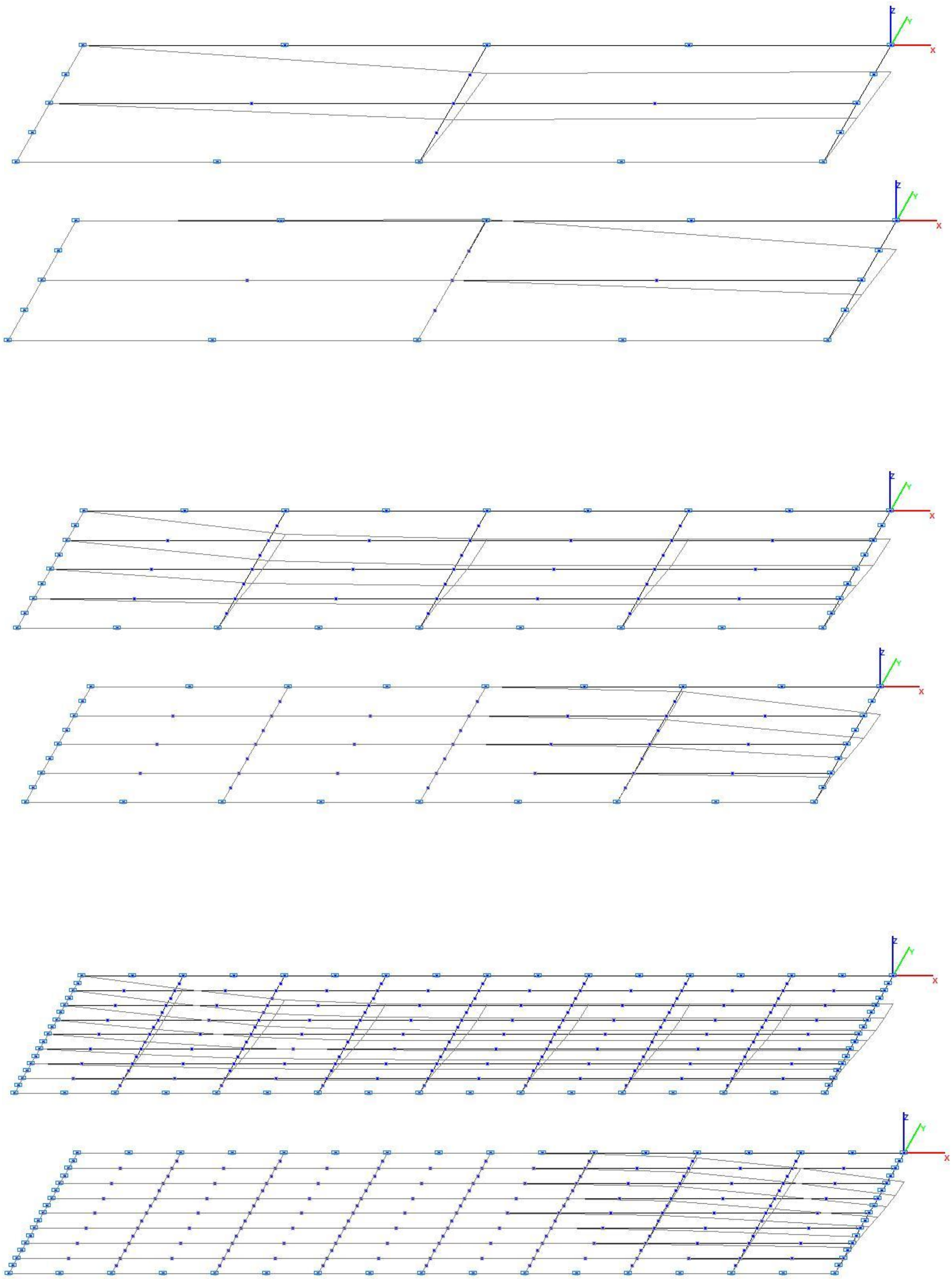
Model 3. Deformed model



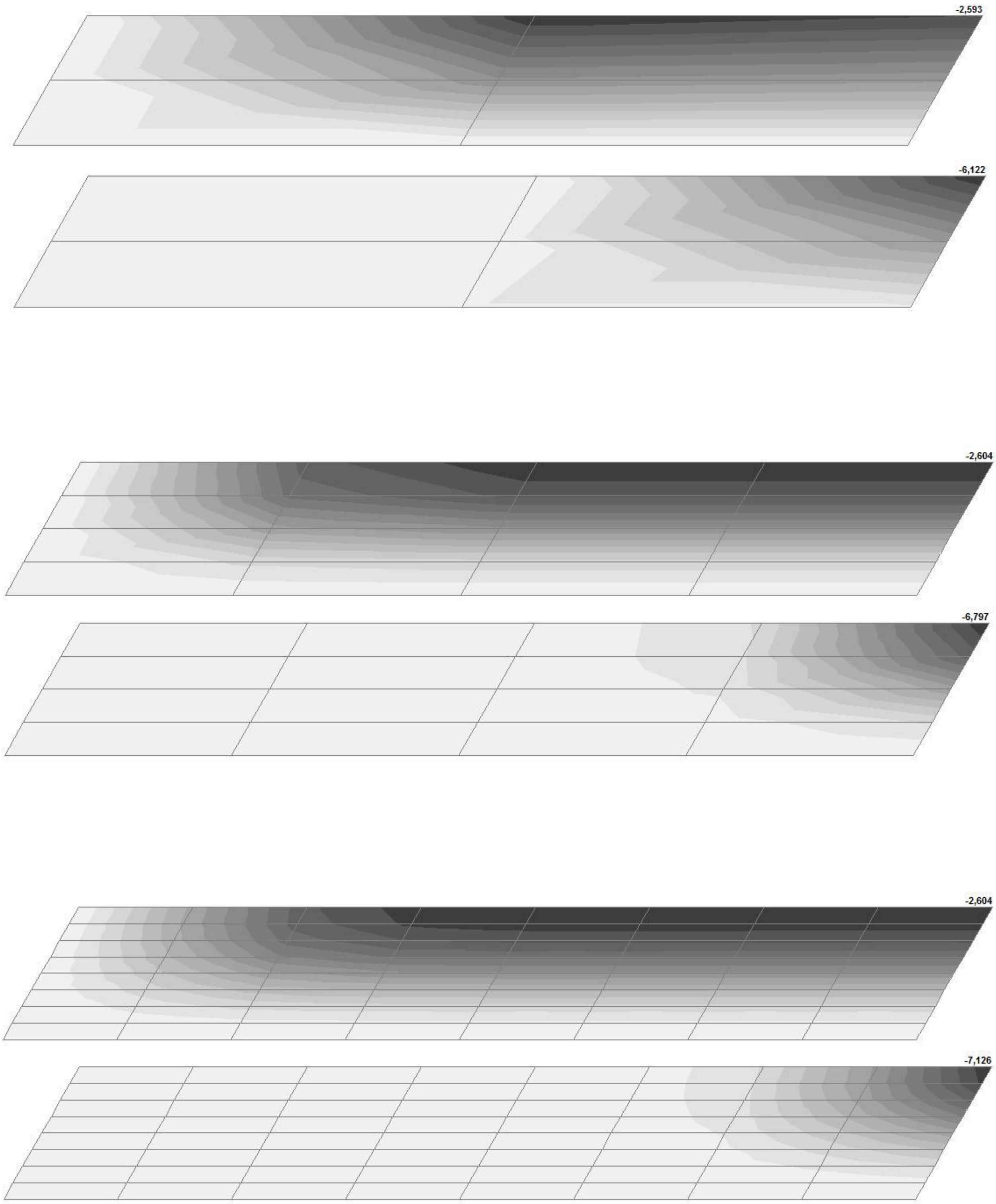
Model 3. Values of the transverse displacements in the center of the rectangular plate clamped along the outer edges w_q and w_P (m, m)



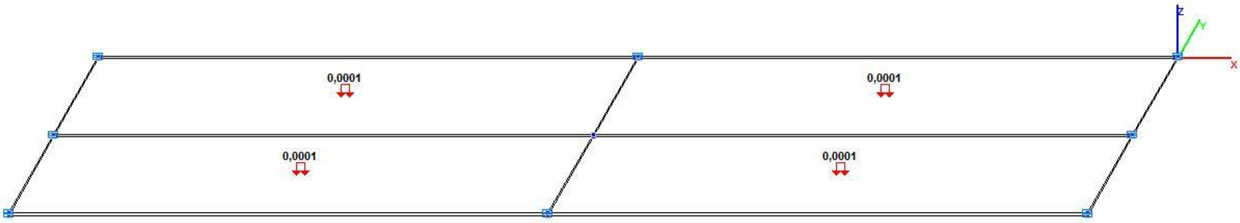
Model 4. Design model

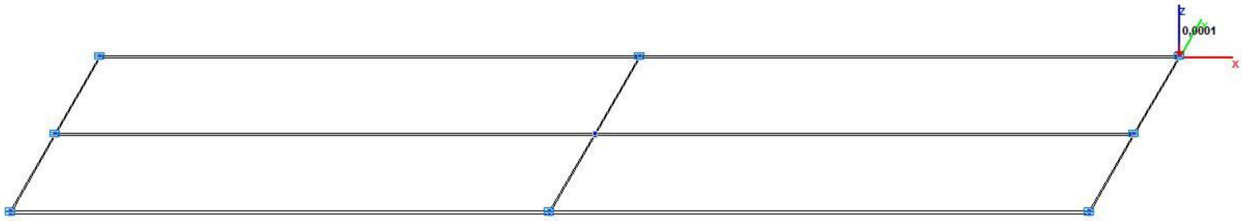


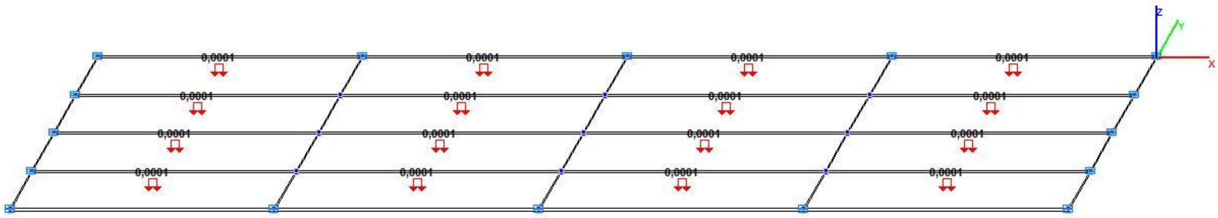
Model 4. Deformed model

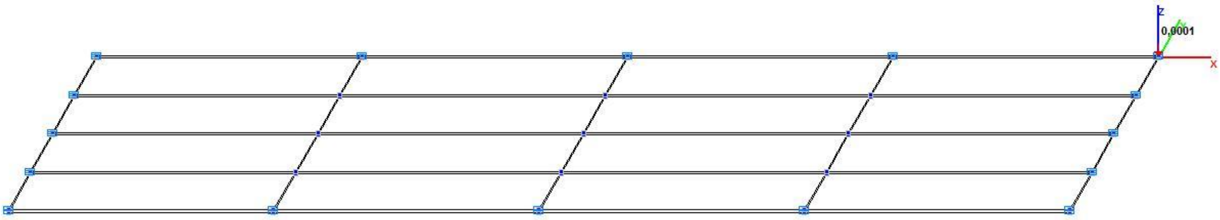


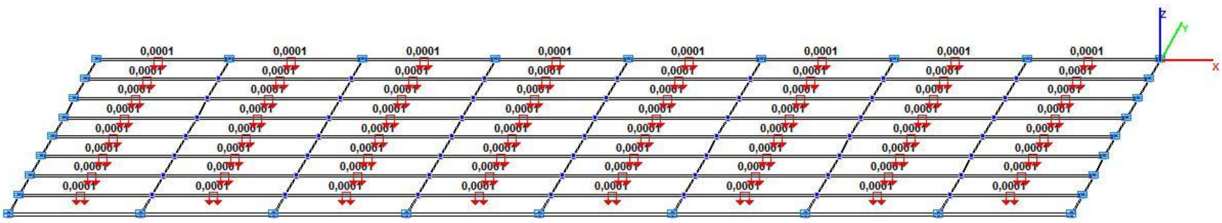
Model 4. Values of the transverse displacements in the center of the rectangular plate clamped along the outer edges w_q *and* w_P *(m, m)*

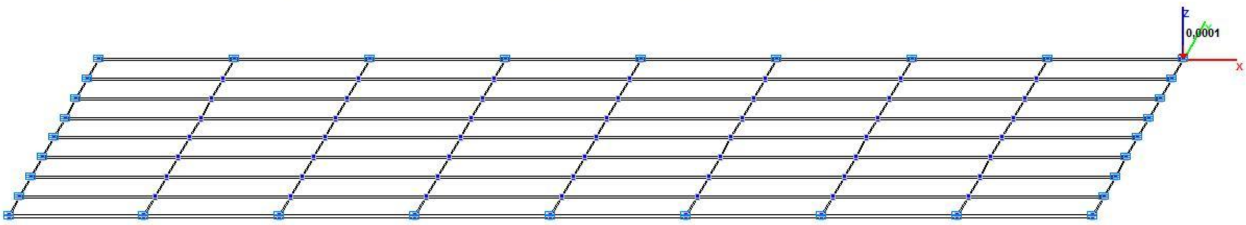

0,0001
z
y
x

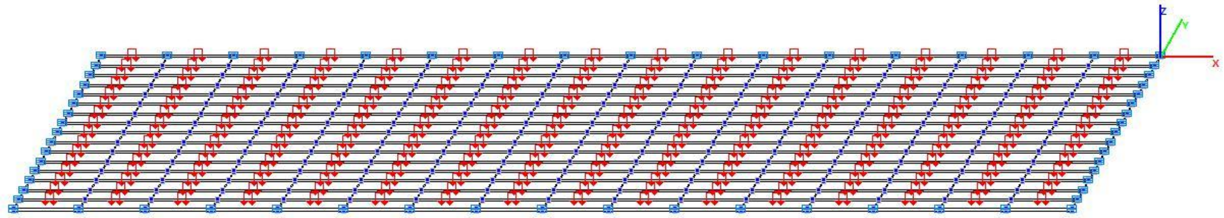

0,0001
z
x

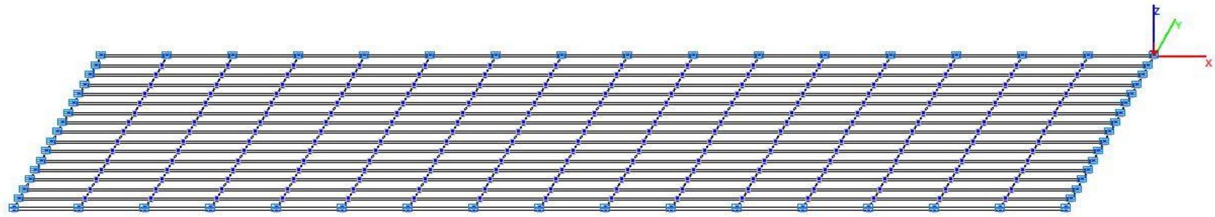

0,0001
z
y
x

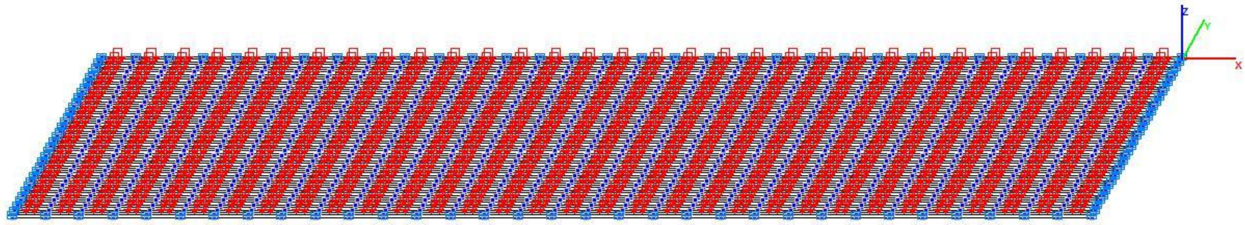

0,0001
z
x

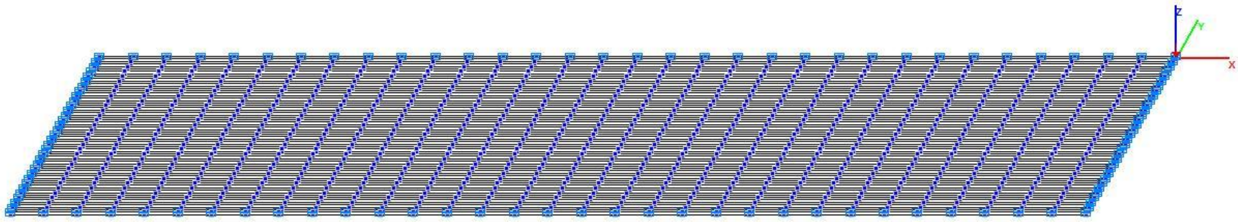

0,0001
z
y
x

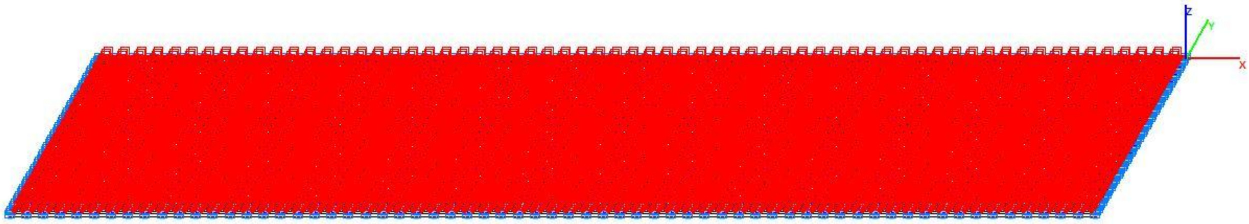

0,0001
z
x

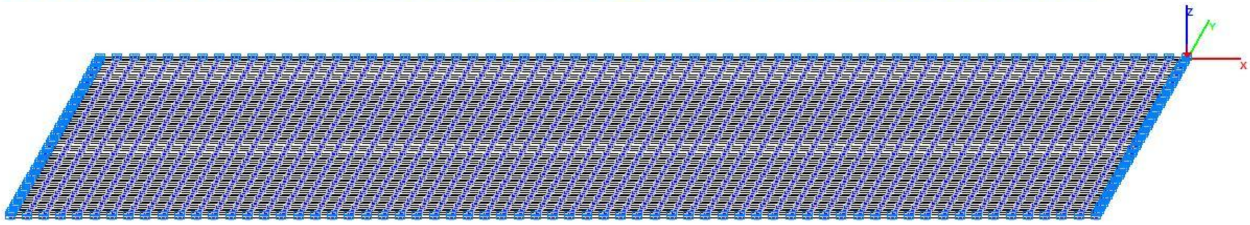


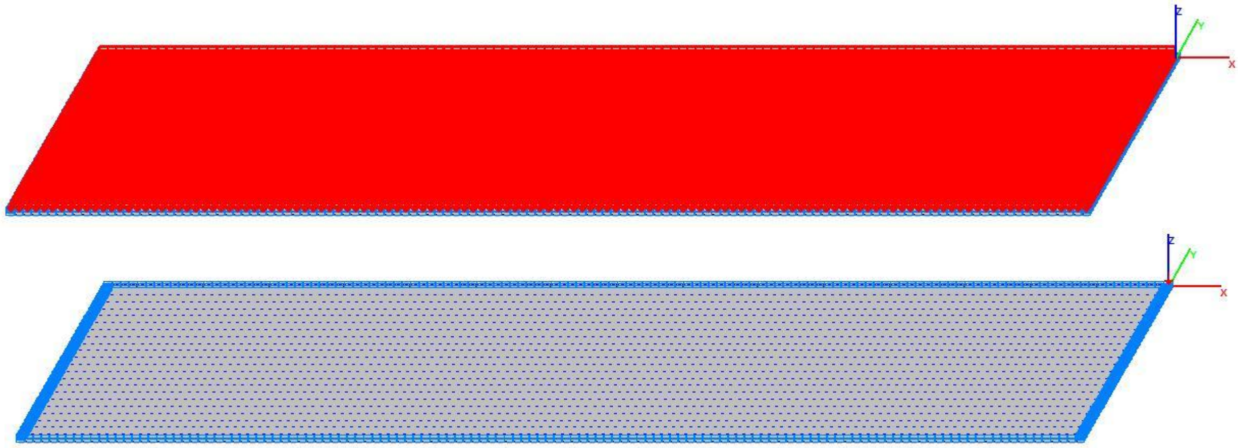




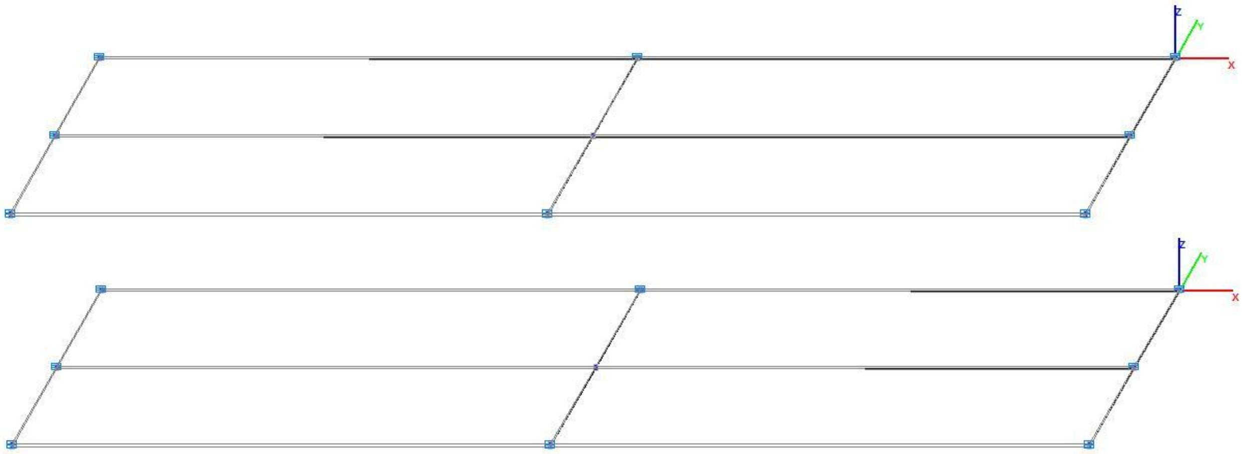


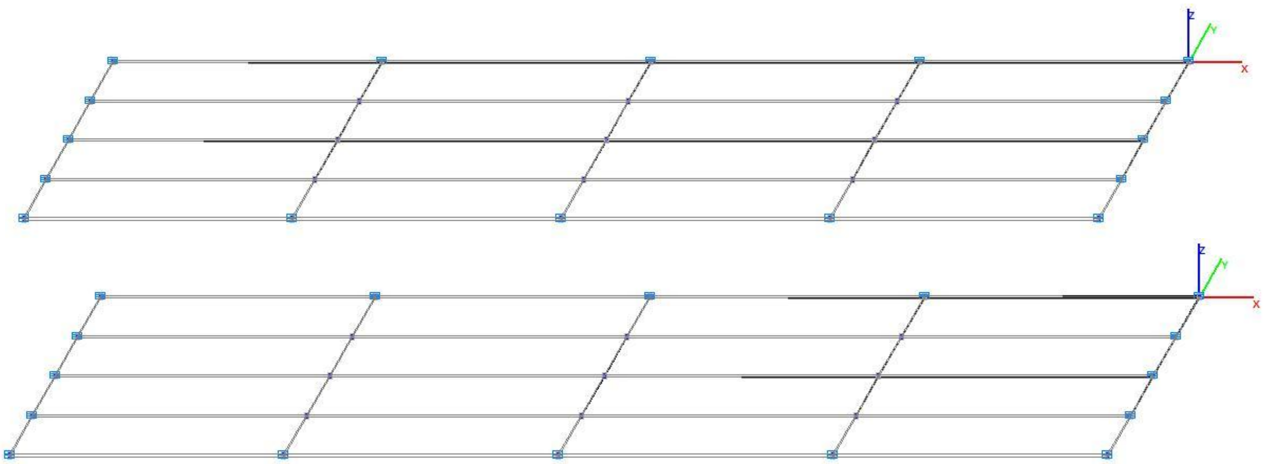


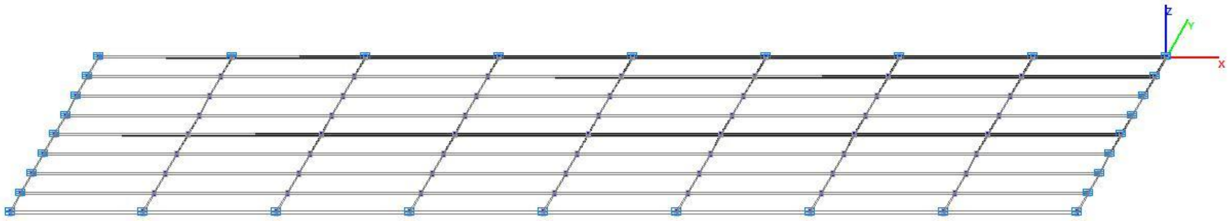


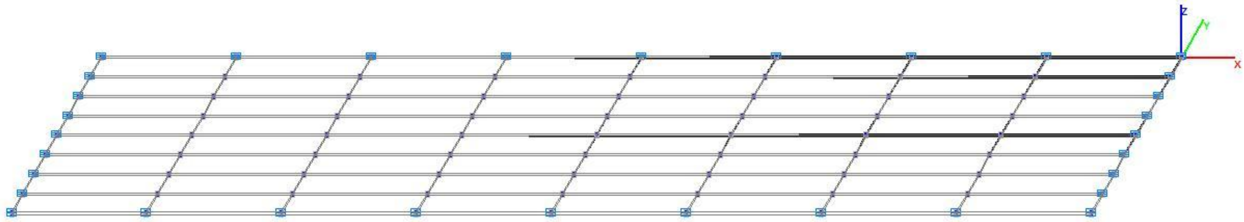


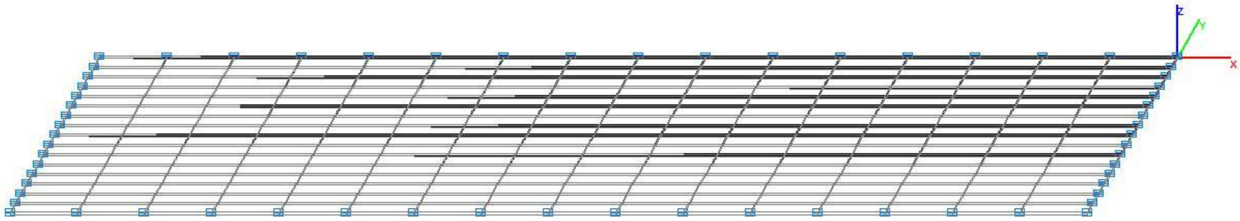
Model 5. Design model

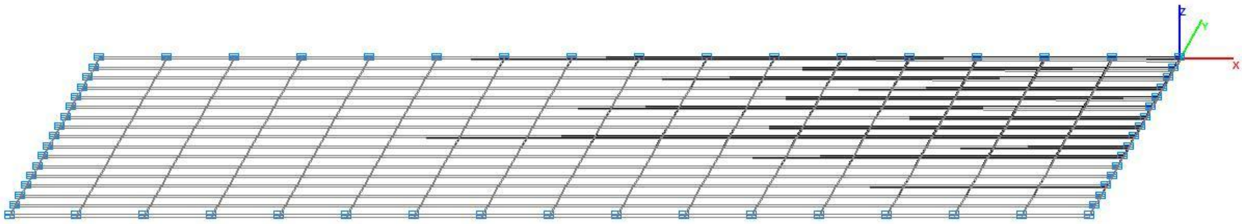


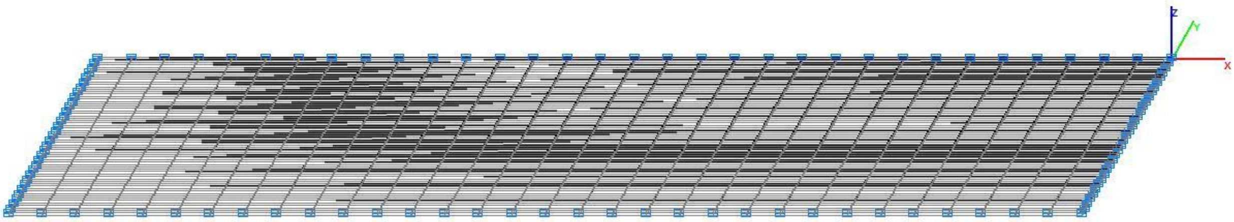


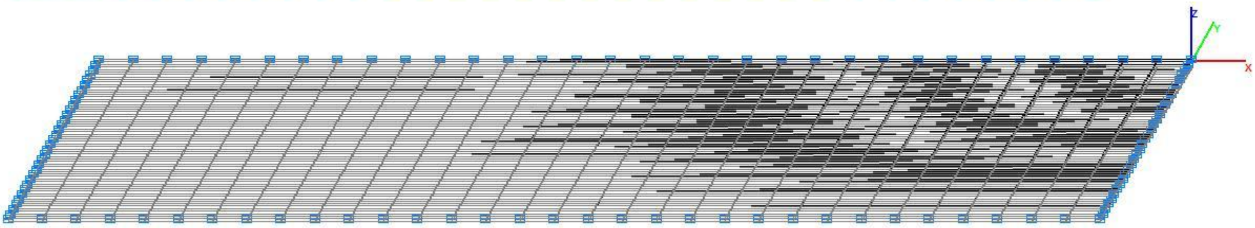


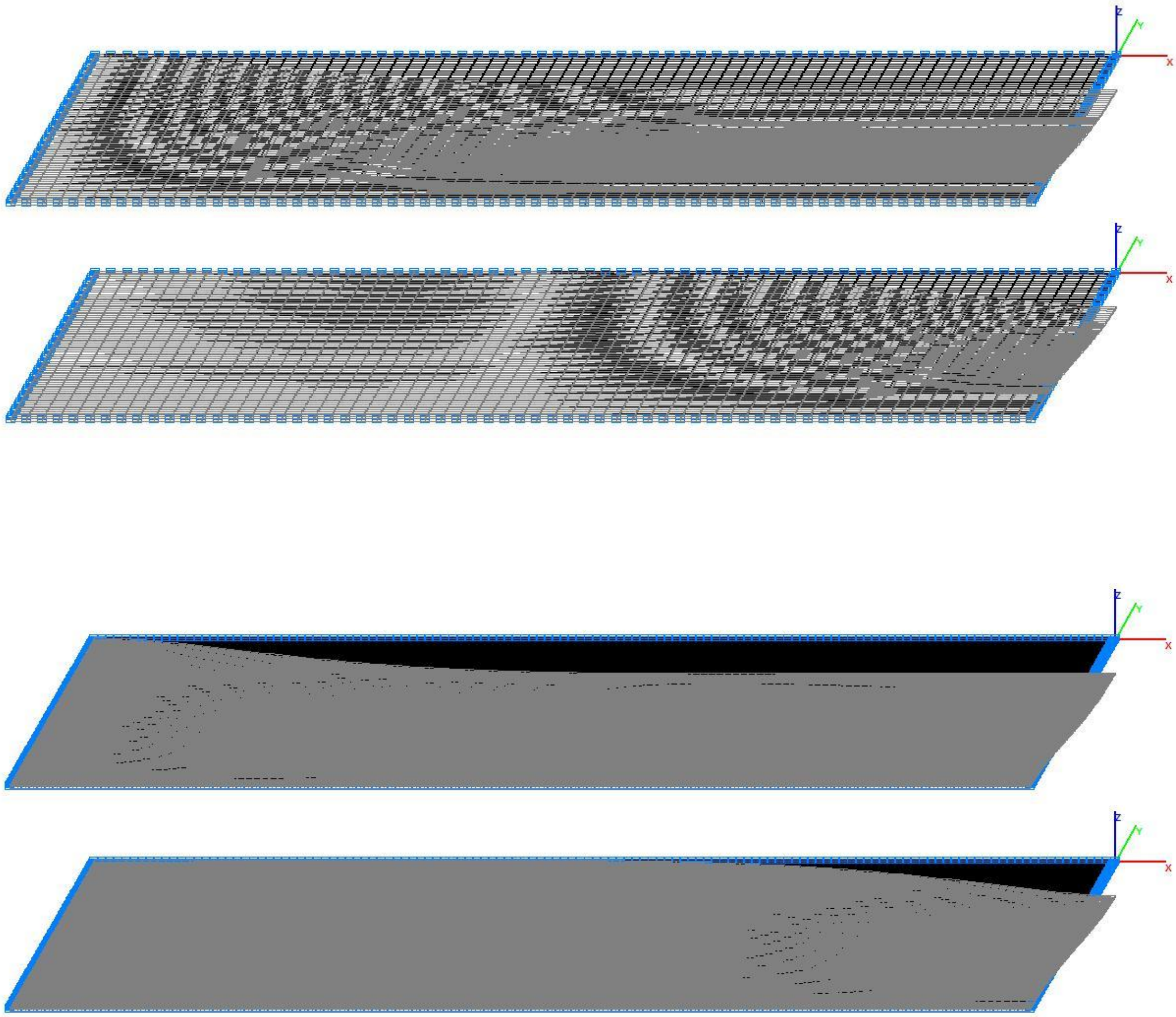




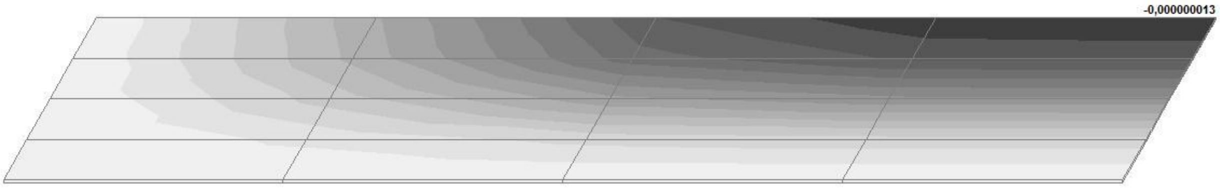


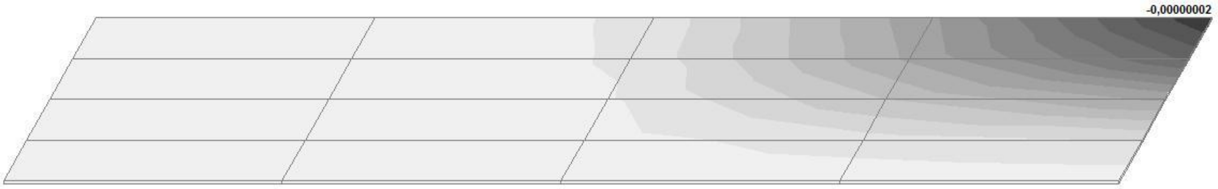


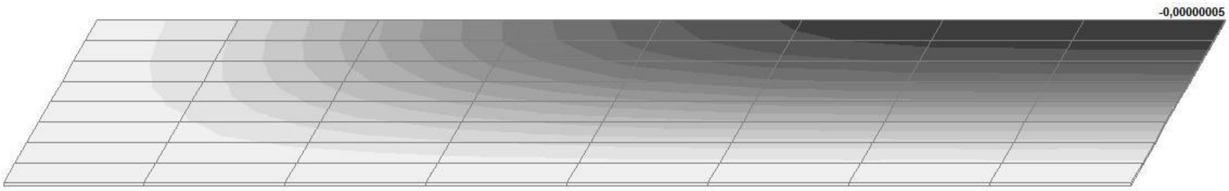


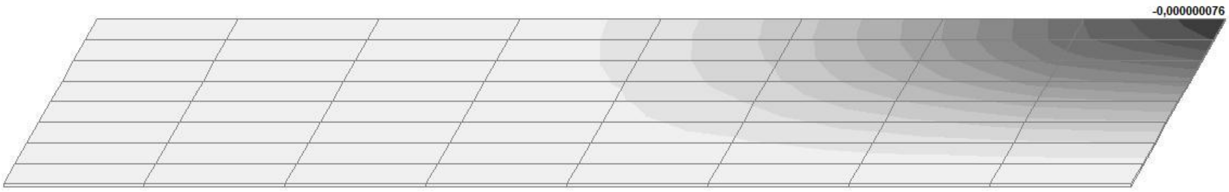


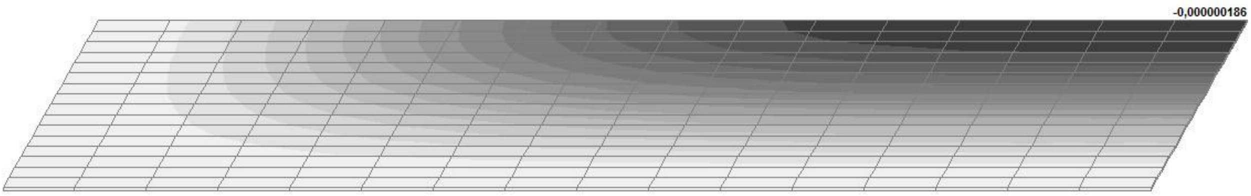
Model 5. Deformed model

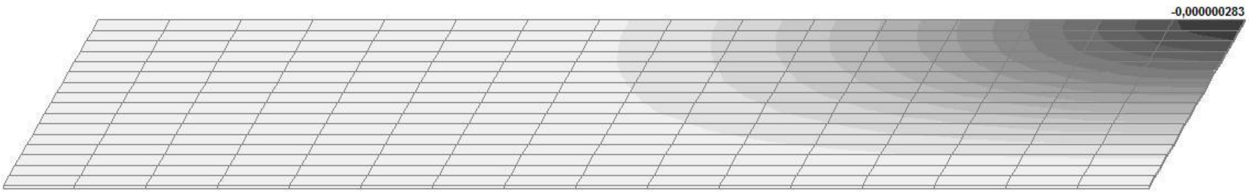

-0,000000013

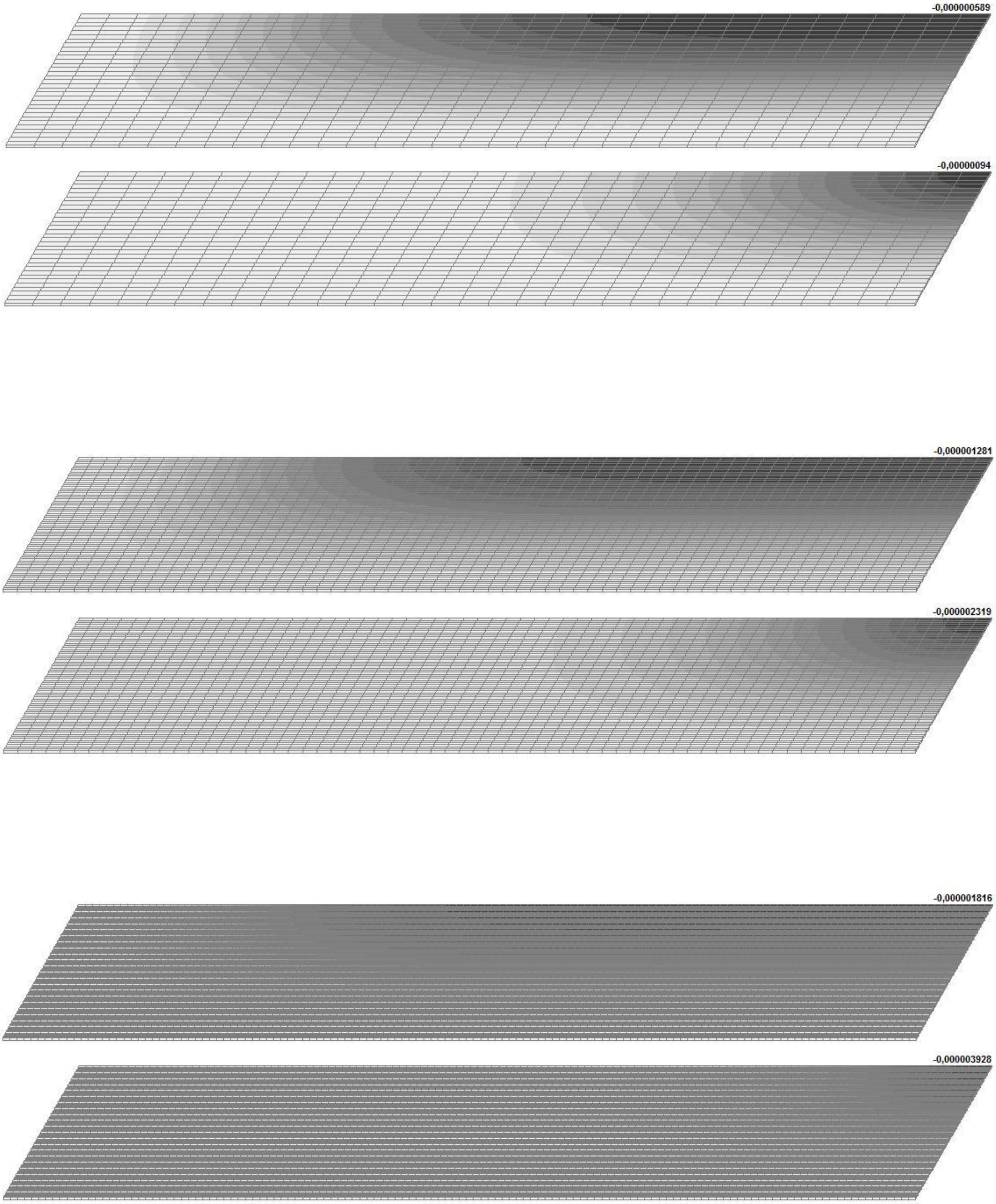

-0,00000002


-0,00000005

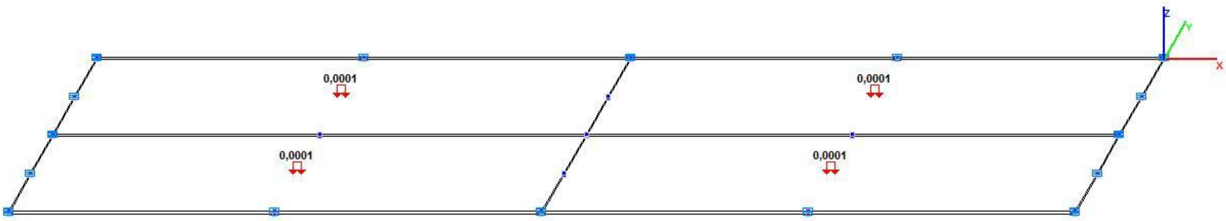

-0,000000076

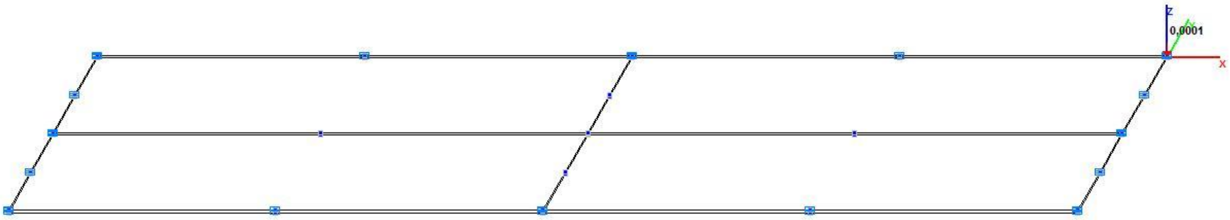

-0,000000186

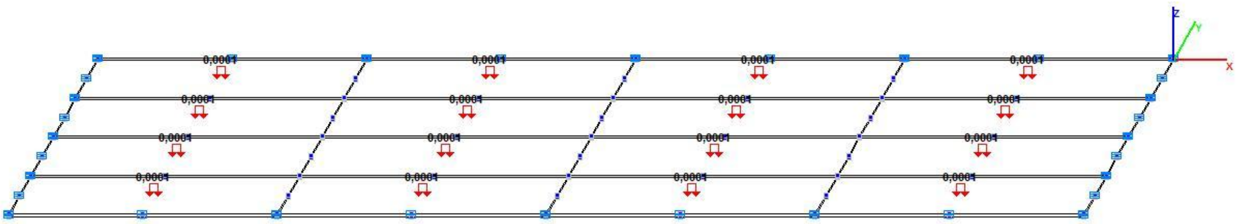

-0,000000283

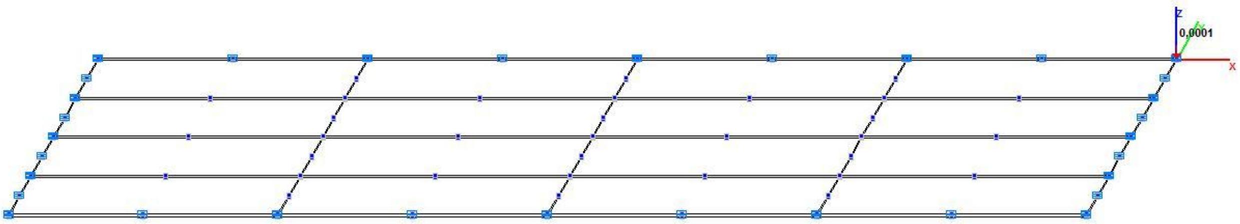


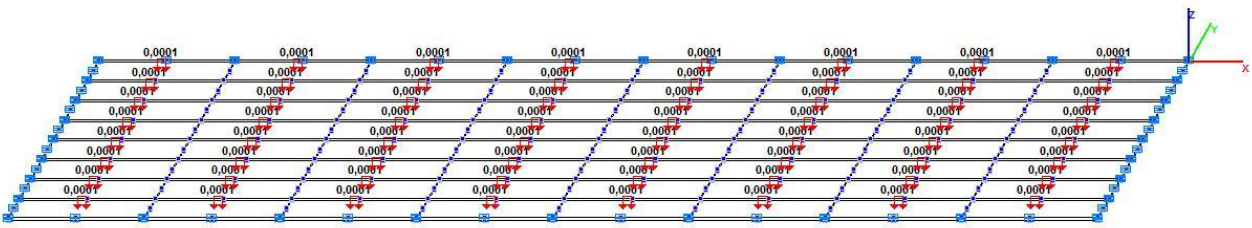
Model 5. Values of the transverse displacements in the center of the rectangular plate clamped along the outer edges w_q *and* w_P *(m, m)*

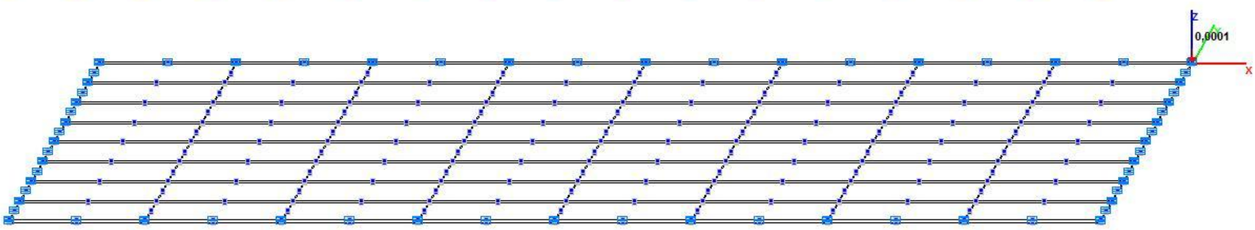

0,0001

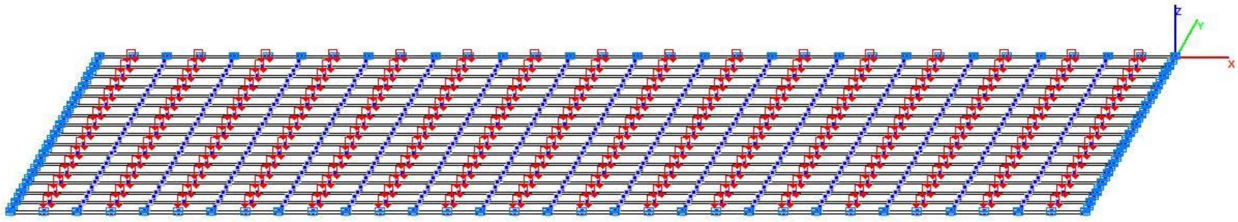

0,0001

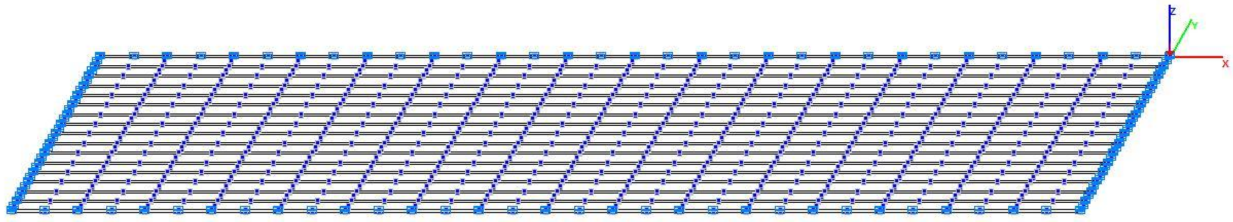

0,0001

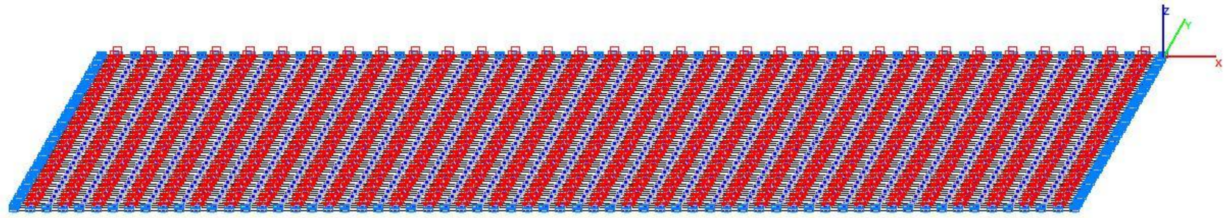

0,0001

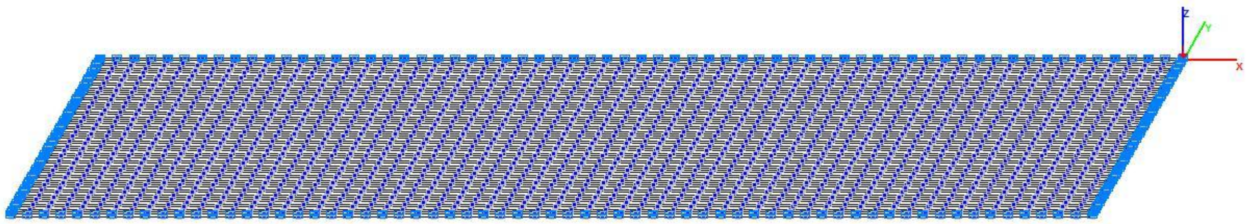

0,0001

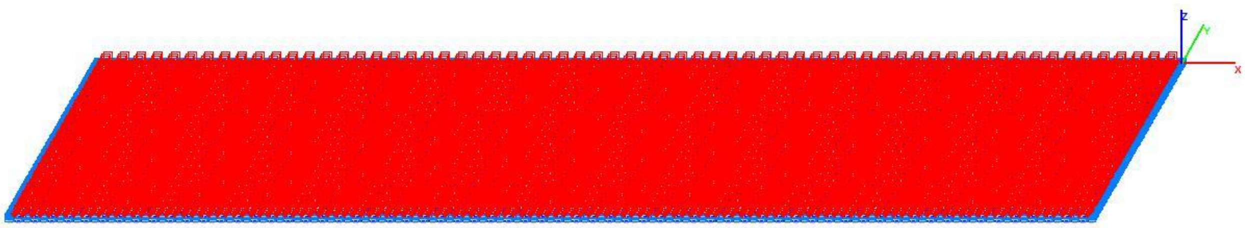

0,0001

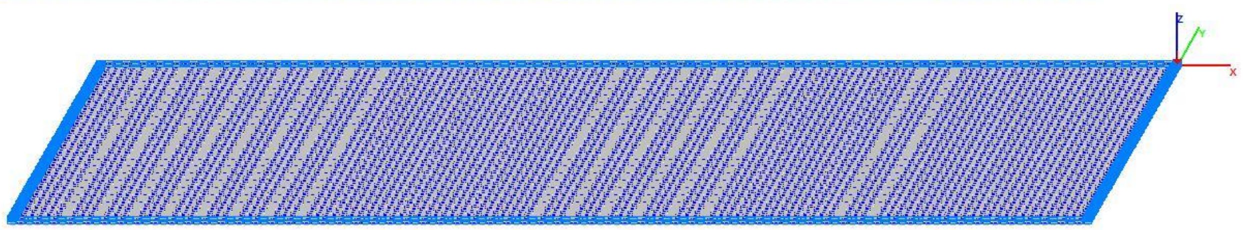


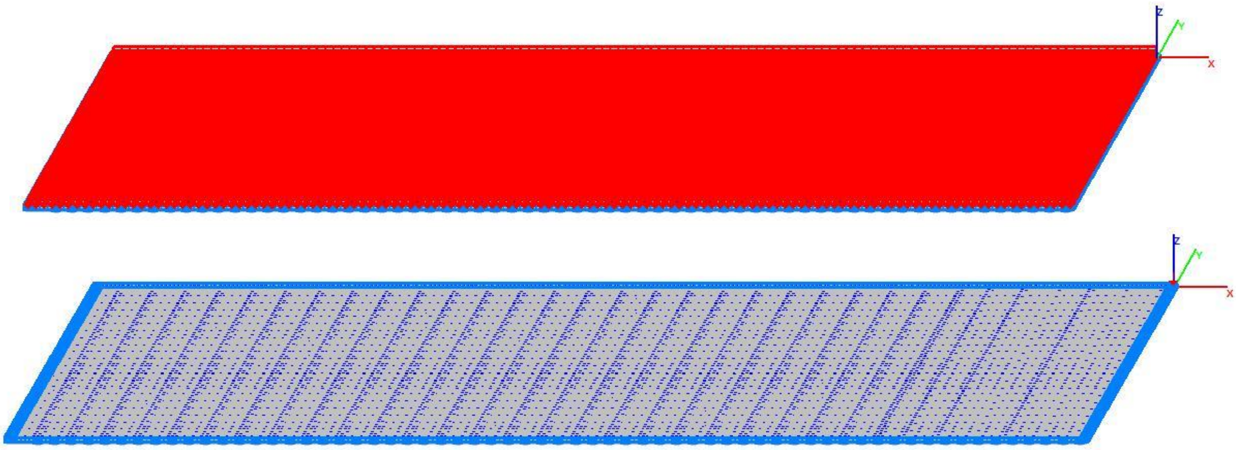




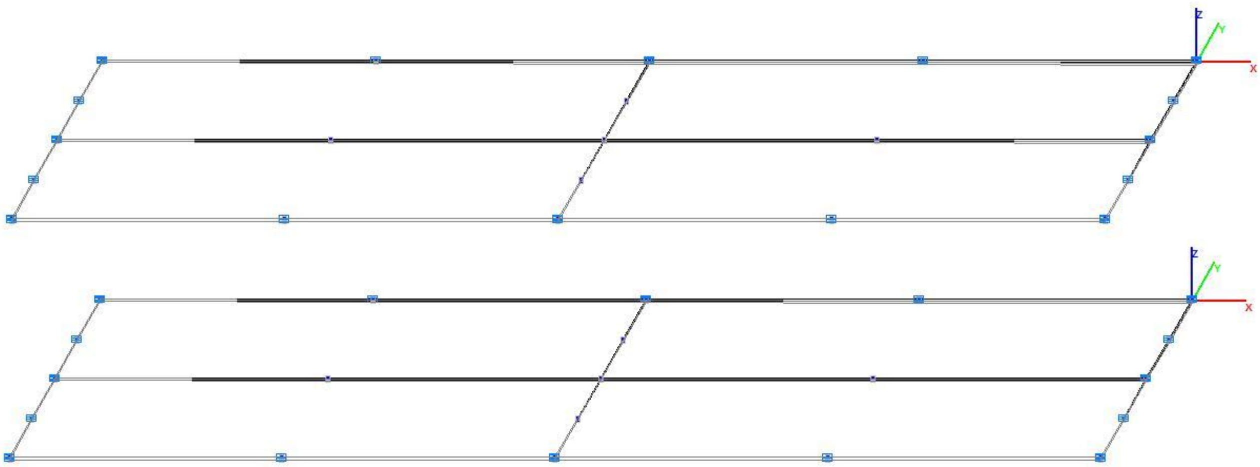


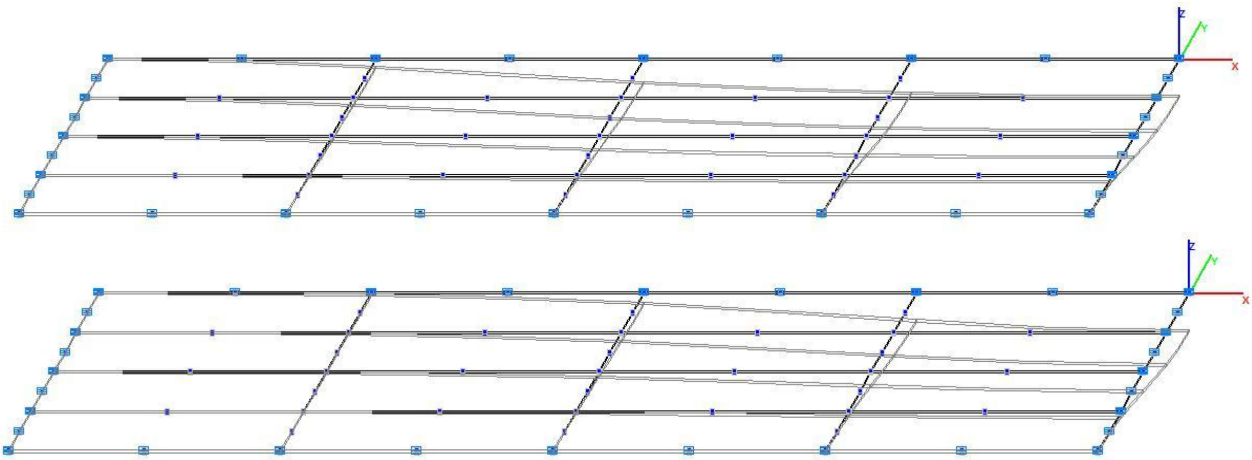


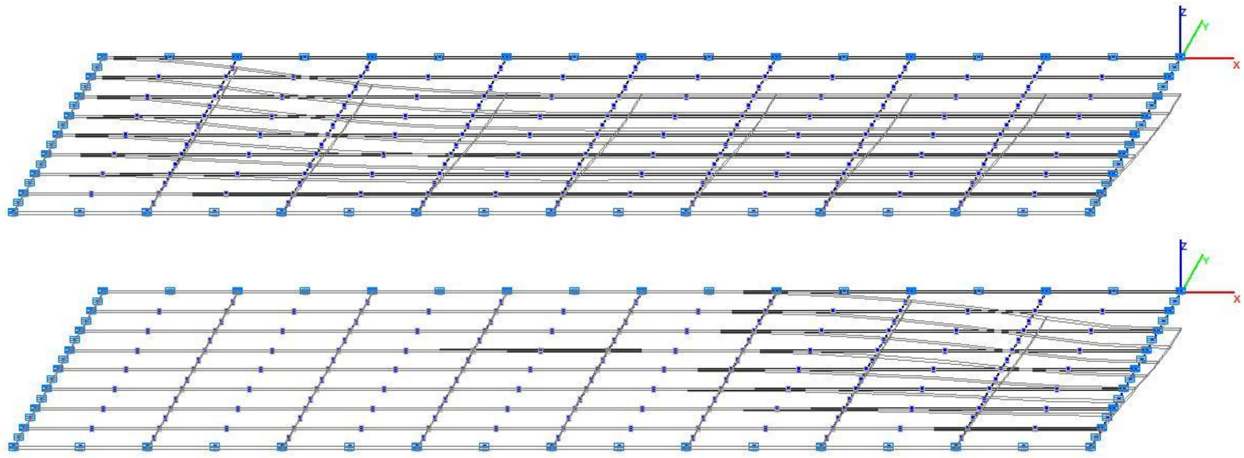


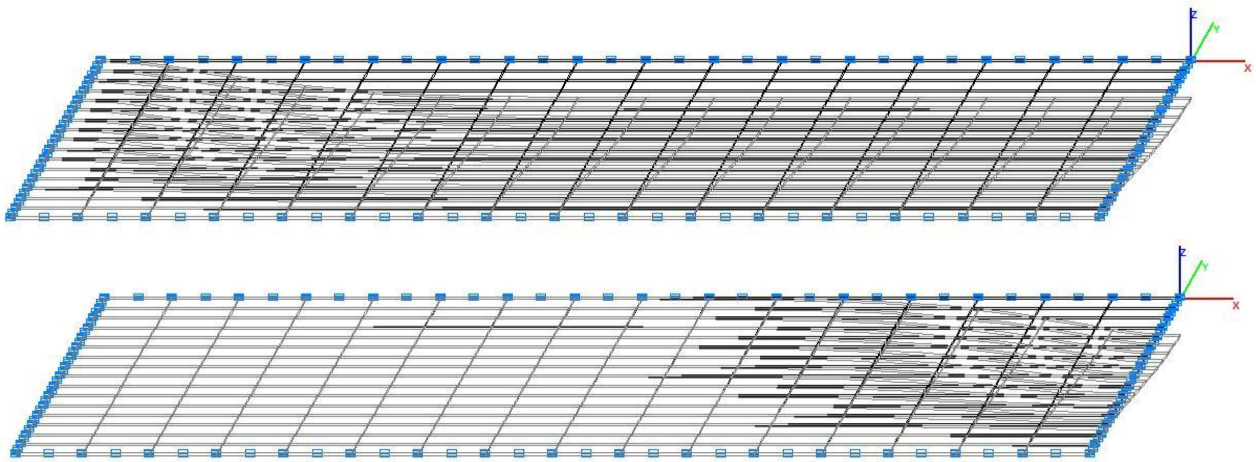


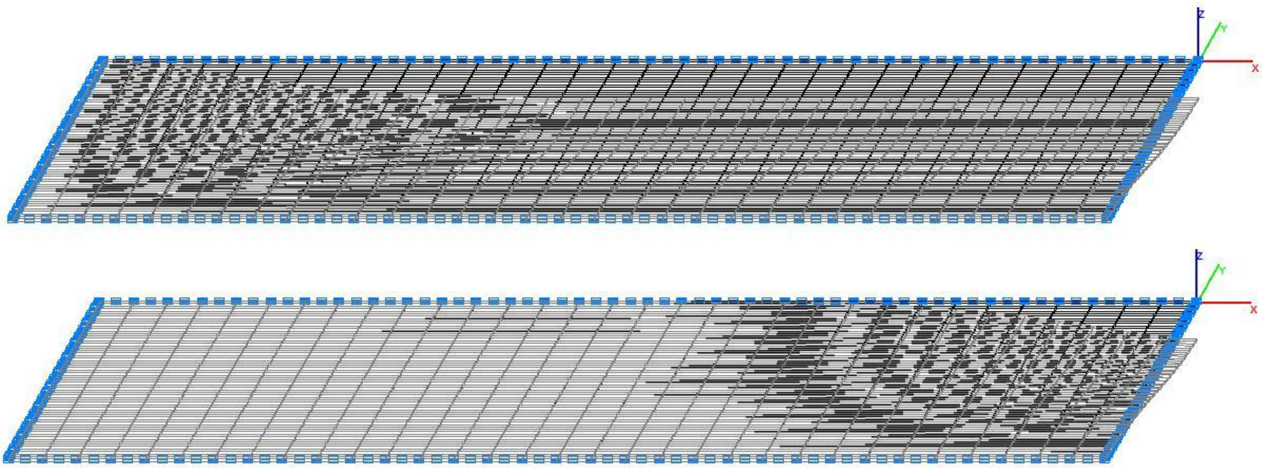
Model 6. Design model

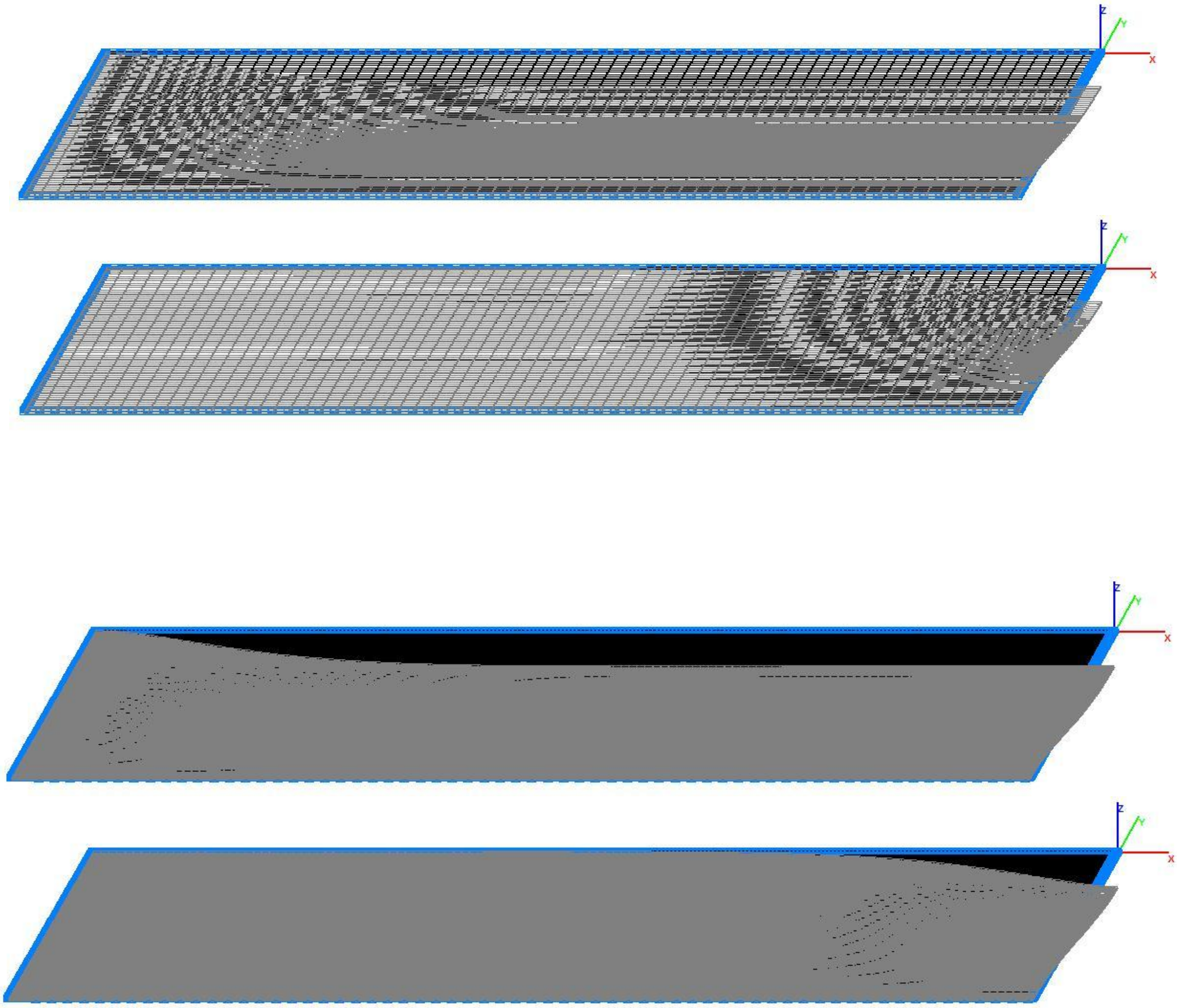




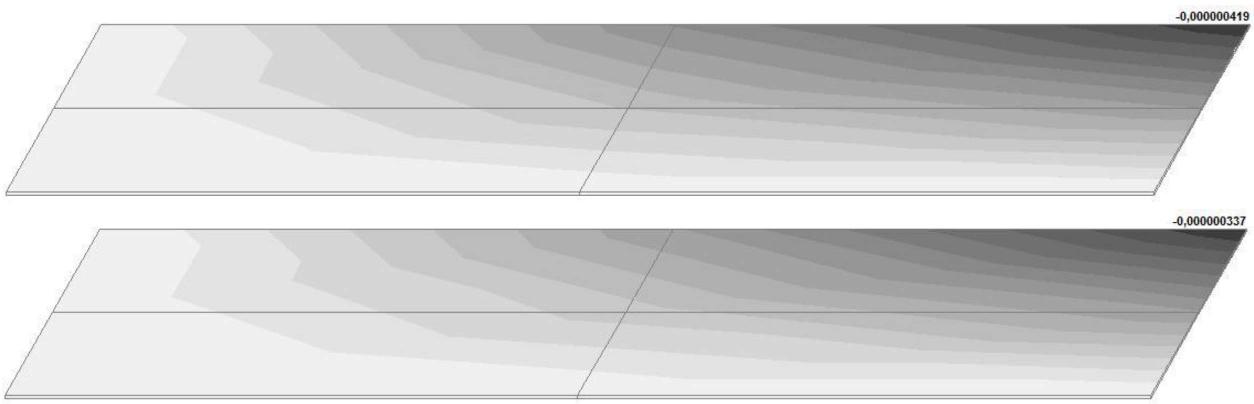


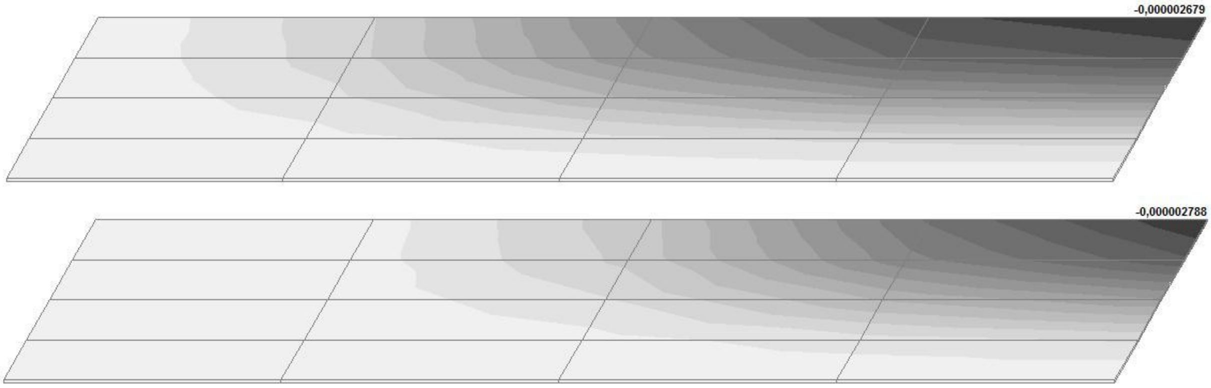




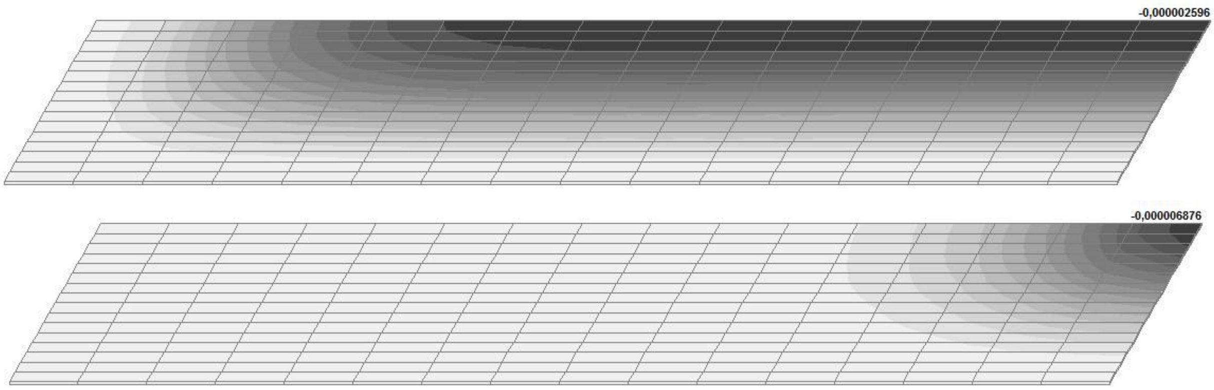


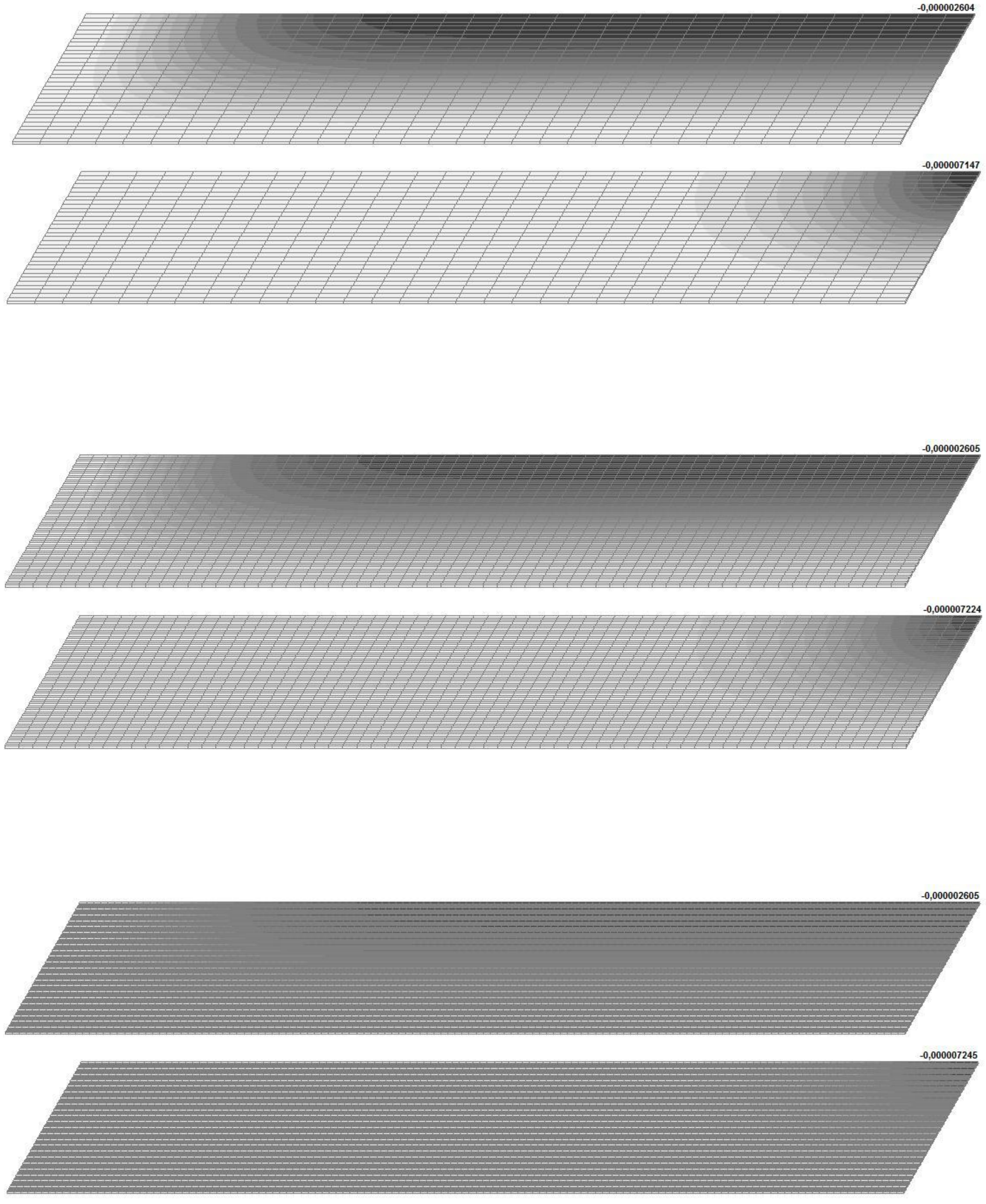
Model 6. Deformed model




-0,000002679
-0,000002788


-0,00000256
-0,000005735


-0,000002596
-0,000006876



Model 6. Values of the transverse displacements in the center of the rectangular plate clamped along the outer edges w_q *and* w_P *(m, m)*

Comparison of solutions:

Transverse displacements in the center of the flat rectangular plate clamped along the outer edges w_q from the transverse load q uniformly distributed over the entire area

Model	Finite element mesh	Theory	SCAD	Deviation, %
1 (Member type 42)	2x2	2.605	2.299	11.75
	4x4		2.670	2.50
	8x8		2.640	1.34
2 (Member type 44)	2x2	2.605	2.607	0.08
	4x4		2.612	0.27
	8x8		2.606	0.04
3 (Member type 45)	2x2	2.605	2.615	0.38
	4x4		2.605	0.00
	8x8		2.604	0.04
4 (Member type 50)	2x2	2.605	2.593	0.46
	4x4		2.604	0.04
	8x8		2.604	0.04
5 (Member type 36)	2x2	$2.605 \cdot 10^{-6}$	$0.003 \cdot 10^{-6}$	99.88
	4x4		$0.013 \cdot 10^{-6}$	99.50
	8x8		$0.050 \cdot 10^{-6}$	98.08
	16x16		$0.186 \cdot 10^{-6}$	92.86
	32x32		$0.589 \cdot 10^{-6}$	77.39
	64x64		$1.281 \cdot 10^{-6}$	50.83
	128x128		$1.816 \cdot 10^{-6}$	30.29
6 (Member type 37)	2x2	$2.605 \cdot 10^{-6}$	$0.419 \cdot 10^{-6}$	83.92
	4x4		$2.679 \cdot 10^{-6}$	2.84
	8x8		$2.560 \cdot 10^{-6}$	1.73
	16x16		$2.596 \cdot 10^{-6}$	0.35
	32x32		$2.604 \cdot 10^{-6}$	0.04
	64x64		$2.605 \cdot 10^{-6}$	0.00
	128x128		$2.605 \cdot 10^{-6}$	0.00

Transverse displacements in the center of the flat rectangular plate clamped along the outer edges w_P from the concentrated shear force P applied in the center

Model	Finite element mesh	Theory	SCAD	Deviation, %
1 (Member type 42)	2x2	7.260	2.239	69.16
	4x4		4.194	42.23
	8x8		5.751	20.79
2 (Member type 44)	2x2	7.260	4.430	38.98
	4x4		5.829	19.71
	8x8		6.386	12.04
3 (Member type 45)	2x2	7.260	5.989	17.51
	4x4		6.864	5.45
	8x8		7.113	2.02
4 (Member type 50)	2x2	7.260	6.122	15.67
	4x4		6.797	6.38
	8x8		7.126	1.85
5 (Member type 36)	2x2	$7.260 \cdot 10^{-6}$	$0.005 \cdot 10^{-6}$	99.93
	4x4		$0.020 \cdot 10^{-6}$	99.72
	8x8		$0.076 \cdot 10^{-6}$	98.95
	16x16		$0.283 \cdot 10^{-6}$	96.10
	32x32		$0.940 \cdot 10^{-6}$	87.05
	64x64		$2.319 \cdot 10^{-6}$	68.06
	128x128		$3.928 \cdot 10^{-6}$	45.90
6 (Member type 37)	2x2	$7.260 \cdot 10^{-6}$	$0.337 \cdot 10^{-6}$	95.36
	4x4		$2.788 \cdot 10^{-6}$	61.60
	8x8		$5.735 \cdot 10^{-6}$	21.01
	16x16		$6.876 \cdot 10^{-6}$	5.29
	32x32		$7.147 \cdot 10^{-6}$	1.56
	64x64		$7.224 \cdot 10^{-6}$	0.50

Model	Finite element mesh	Theory	SCAD	Deviation, %
	128x128		$7.245 \cdot 10^{-6}$	0.21

Notes: In the analytical solution the values of the transverse displacements in the center of the flat rectangular plate clamped along the outer edges w_q and w_P from the respective actions are determined according to the following formulas:

$$
\begin{aligned}
w_q = \frac{4 \cdot q \cdot a^4}{\pi^5 \cdot D} \cdot \sum_{m=1}^{M} \left\{ \frac{1}{m^5} \cdot \left[1 - \frac{\frac{m \cdot \pi \cdot b}{2 \cdot a} \cdot th\left(\frac{m \cdot \pi \cdot b}{2 \cdot a} \right) + 2}{2 \cdot ch\left(\frac{m \cdot \pi \cdot b}{2 \cdot a} \right)} \right] \cdot sin\left(\frac{m \cdot \pi}{2} \right) \right\} + \\
+ \frac{a^2}{2 \cdot \pi^2 \cdot D} \cdot \sum_{m=1}^{M} \left\{ E_m \cdot \frac{1}{m^2} \cdot \frac{\frac{m \cdot \pi \cdot b}{2 \cdot a} \cdot sh\left(\frac{m \cdot \pi \cdot b}{2 \cdot a} \right)}{ch^2\left(\frac{m \cdot \pi \cdot b}{2 \cdot a} \right)} \cdot sin\left(\frac{m \cdot \pi}{2} \right) \right\} + \\
+ \frac{b^2}{2 \cdot \pi^2 \cdot D} \cdot \sum_{m=1}^{M} \left\{ F_m \cdot \frac{1}{m^2} \cdot \frac{\frac{m \cdot \pi \cdot a}{2 \cdot b} \cdot sh\left(\frac{m \cdot \pi \cdot a}{2 \cdot b} \right)}{ch^2\left(\frac{m \cdot \pi \cdot a}{2 \cdot b} \right)} \cdot sin\left(\frac{m \cdot \pi}{2} \right) \right\} +
\end{aligned}
.
$$

The values of the coefficients E_m and F_m are determined by solving the system of 2·M equations:

$$
\left\{
\begin{aligned}
\frac{4 \cdot q \cdot a^2}{\pi^3} \cdot \frac{1}{i^4} \cdot \left(\frac{\frac{i \cdot \pi \cdot b}{2 \cdot a}}{ch^2\left(\frac{i \cdot \pi \cdot b}{2 \cdot a} \right)} - th\left(\frac{i \cdot \pi \cdot b}{2 \cdot a} \right) \right) - \frac{E_i}{i} \cdot \left(\frac{\frac{i \cdot \pi \cdot b}{2 \cdot a}}{ch^2\left(\frac{i \cdot \pi \cdot b}{2 \cdot a} \right)} + th\left(\frac{i \cdot \pi \cdot b}{2 \cdot a} \right) \right) - \frac{8 \cdot i \cdot a}{\pi \cdot b} \cdot \sum_{m=1}^{M} \left[F_m \cdot \frac{1}{m^3} \cdot \frac{1}{\left(\frac{a^2}{b^2} + \frac{i^2}{m^2} \right)^2} \cdot sin^2\left(\frac{m \cdot \pi}{2} \right) \right] \\
\frac{4 \cdot q \cdot b^2}{\pi^3} \cdot \frac{1}{i^4} \cdot \left(\frac{\frac{i \cdot \pi \cdot a}{2 \cdot b}}{ch^2\left(\frac{i \cdot \pi \cdot a}{2 \cdot b} \right)} - th\left(\frac{i \cdot \pi \cdot a}{2 \cdot b} \right) \right) - \frac{F_i}{i} \cdot \left(\frac{\frac{i \cdot \pi \cdot a}{2 \cdot b}}{ch^2\left(\frac{i \cdot \pi \cdot a}{2 \cdot b} \right)} + th\left(\frac{i \cdot \pi \cdot a}{2 \cdot b} \right) \right) - \frac{8 \cdot i \cdot b}{\pi \cdot a} \cdot \sum_{m=1}^{M} \left[E_m \cdot \frac{1}{m^3} \cdot \frac{1}{\left(\frac{b^2}{a^2} + \frac{i^2}{m^2} \right)^2} \cdot sin^2\left(\frac{m \cdot \pi}{2} \right) \right]
\end{aligned}
\right\};
$$

$$
\begin{aligned}
w_P = \frac{P \cdot a^2}{2 \cdot \pi^3 \cdot D} \cdot \sum_{m=1}^{M} \left\{ \frac{1}{m^3} \cdot \left[th\left(\frac{m \cdot \pi \cdot b}{2 \cdot a} \right) - \frac{\frac{m \cdot \pi \cdot b}{2 \cdot a}}{ch^2\left(\frac{m \cdot \pi \cdot b}{2 \cdot a} \right)} \right] \cdot sin^2\left(\frac{m \cdot \pi}{2} \right) \right\} + \\
+ \frac{a^2}{2 \cdot \pi^2 \cdot D} \cdot \sum_{m=1}^{M} \left\{ E_m \cdot \frac{1}{m^2} \cdot \frac{\frac{m \cdot \pi \cdot b}{2 \cdot a} \cdot sh\left(\frac{m \cdot \pi \cdot b}{2 \cdot a} \right)}{ch^2\left(\frac{m \cdot \pi \cdot b}{2 \cdot a} \right)} \cdot sin\left(\frac{m \cdot \pi}{2} \right) \right\} + \\
+ \frac{b^2}{2 \cdot \pi^2 \cdot D} \cdot \sum_{m=1}^{M} \left\{ F_m \cdot \frac{1}{m^2} \cdot \frac{\frac{m \cdot \pi \cdot a}{2 \cdot b} \cdot sh\left(\frac{m \cdot \pi \cdot a}{2 \cdot b} \right)}{ch^2\left(\frac{m \cdot \pi \cdot a}{2 \cdot b} \right)} \cdot sin\left(\frac{m \cdot \pi}{2} \right) \right\} +
\end{aligned}
.
$$

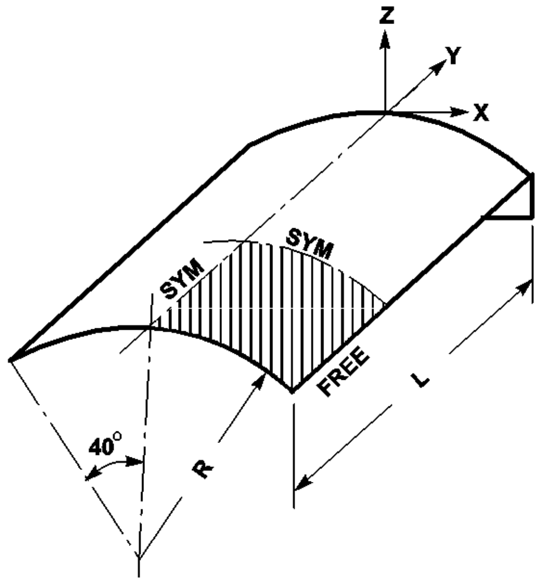
The values of the coefficients E_m and F_m are determined by solving the system of 2·M equations:

$$-\frac{P}{\pi}\cdot\frac{1}{i^2}\cdot\frac{\frac{i\cdot\pi\cdot b}{2\cdot a}\cdot sh\left(\frac{i\cdot\pi\cdot b}{2\cdot a}\right)}{ch^2\left(\frac{i\cdot\pi\cdot b}{2\cdot a}\right)}\cdot sin\left(\frac{i\cdot\pi}{2}\right)-\frac{E_i}{i}\cdot\left(\frac{\frac{i\cdot\pi\cdot b}{2\cdot a}}{ch^2\left(\frac{i\cdot\pi\cdot b}{2\cdot a}\right)}+th\left(\frac{i\cdot\pi\cdot b}{2\cdot a}\right)\right)-\frac{8\cdot i\cdot a}{\pi\cdot b}\cdot\sum_{m=1}^{M}\left[F_m\cdot\frac{1}{m^3}\cdot\frac{1}{\left(\frac{a^2}{b^2}+\frac{i^2}{m^2}\right)^2}\cdot sin^2\left(\frac{m\cdot\pi}{2}\right)\right]$$

$$-\frac{P}{\pi}\cdot\frac{1}{i^2}\cdot\frac{\frac{i\cdot\pi\cdot a}{2\cdot b}\cdot sh\left(\frac{i\cdot\pi\cdot a}{2\cdot b}\right)}{ch^2\left(\frac{i\cdot\pi\cdot a}{2\cdot b}\right)}\cdot sin\left(\frac{i\cdot\pi}{2}\right)-\frac{F_i}{i}\cdot\left(\frac{\frac{i\cdot\pi\cdot a}{2\cdot b}}{ch^2\left(\frac{i\cdot\pi\cdot a}{2\cdot b}\right)}+th\left(\frac{i\cdot\pi\cdot a}{2\cdot b}\right)\right)-\frac{8\cdot i\cdot b}{\pi\cdot a}\cdot\sum_{m=1}^{M}\left[E_m\cdot\frac{1}{m^3}\cdot\frac{1}{\left(\frac{b^2}{a^2}+\frac{i^2}{m^2}\right)^2}\cdot sin^2\left(\frac{m\cdot\pi}{2}\right)\right]\Bigg\};$$

$$D=\frac{E\cdot h^3}{12\cdot\left(1-\nu^2\right)}.$$

Open Cylindrical Shell Rectangular in Plan and Simply Supported along the Curvilinear Edges Subjected to a Transverse Load Uniformly Distributed over the Entire Area



Objective: Check of the obtained values of the transverse displacements in the middle of the free rectilinear edges of an open cylindrical shell rectangular in plan and simply supported along the curvilinear edges subjected to a transverse load uniformly distributed over the entire area.

Initial data files:

File name	Description
Scordelis-Lo_roof _Shell_42_Mesh_2x2.SPR Scordelis-Lo_roof _Shell_42_Mesh_4x4.SPR Scordelis-Lo_roof _Shell_42_Mesh_8x8.SPR	Design model with the elements of type 42 for meshes 2x2, 4x4, 8x8
Scordelis-Lo_roof _Shell_44_Mesh_2x2.SPR Scordelis-Lo_roof _Shell_44_Mesh_4x4.SPR Scordelis-Lo_roof _Shell_44_Mesh_8x8.SPR	Design model with the elements of type 44 for meshes 2x2, 4x4, 8x8
Scordelis-Lo_roof _Shell_45_Mesh_2x2.SPR Scordelis-Lo_roof _Shell_45_Mesh_4x4.SPR Scordelis-Lo_roof _Shell_45_Mesh_8x8.SPR	Design model with the elements of type 45 for meshes 2x2, 4x4, 8x8
Scordelis-Lo_roof _Shell_50_Mesh_2x2.SPR Scordelis-Lo_roof _Shell_50_Mesh_4x4.SPR Scordelis-Lo_roof _Shell_50_Mesh_8x8.SPR	Design model with the elements of type 50 for meshes 2x2, 4x4, 8x8
Scordelis-Lo_roof _ Solid _36.SPR _Mesh_2x2.SPR Scordelis-Lo_roof _ Solid _36.SPR _Mesh_4x4.SPR Scordelis-Lo_roof _ Solid _36.SPR _Mesh_8x8.SPR Scordelis-Lo_roof _ Solid _36.SPR _Mesh_16x16.SPR Scordelis-Lo_roof _ Solid _36.SPR _Mesh_32x32.SPR Scordelis-Lo_roof _ Solid _36.SPR _Mesh_64x64.SPR Scordelis-Lo_roof _ Solid _36.SPR _Mesh_128x128.SPR	Design model with the elements of type 36 for meshes 2x2, 4x4, 8x8, 16x16, 32x32, 64x64, 128x128
Scordelis-Lo_roof _ Solid _37.SPR _Mesh_2x2.SPR Scordelis-Lo_roof _ Solid _37.SPR _Mesh_4x4.SPR Scordelis-Lo_roof _ Solid _37.SPR _Mesh_8x8.SPR Scordelis-Lo_roof _ Solid _37.SPR _Mesh_16x16.SPR Scordelis-Lo_roof _ Solid _37.SPR _Mesh_32x32.SPR Scordelis-Lo_roof _ Solid _37.SPR _Mesh_64x64.SPR Scordelis-Lo_roof _ Solid _37.SPR _Mesh_128x128.SPR	Design model with the elements of type 37 for meshes 2x2, 4x4, 8x8, 16x16, 32x32, 64x64, 128x128

Problem formulation: The open cylindrical shell rectangular in plan and simply supported along the curvilinear edges by ideal end diaphragms rigid in their plane and compliant out of their plane is subjected to the transverse load q uniformly distributed over the entire area. Check the obtained values of the transverse displacements in the middle of the free rectilinear edges of the open cylindrical shell w_q.

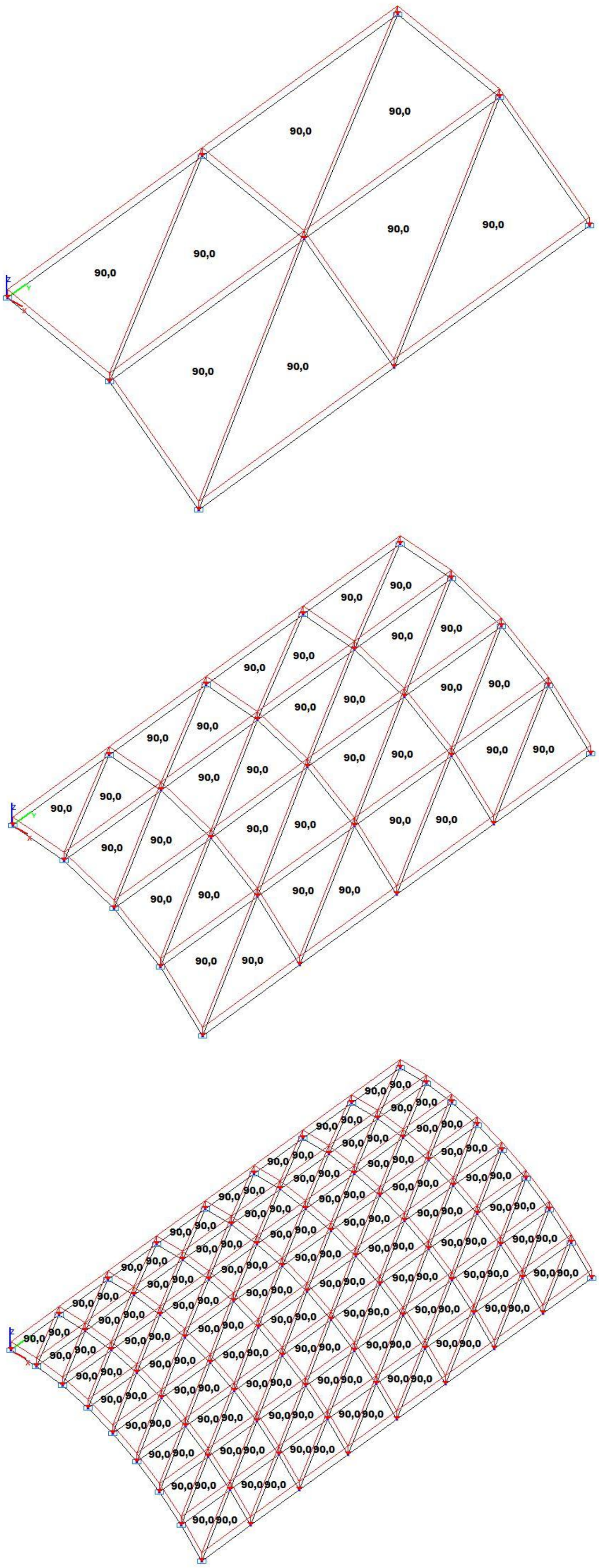
References: R. H. Macneal, R. L. Harder, A proposed standard set of problems to test finite element accuracy, North-Holland, Finite elements in analysis and design, 1, 1985, p. 3-20.
A. C. Scordelis, K. S. Lo, Computer analysis of cylindrical shells, Journal of the American concrete institute, Title No 61-33, May 1964, p. 539-561.
Design of cylindrical concrete shell roofs, New York, Manual No 31 American society of civil engineers, 1952.

Initial data:

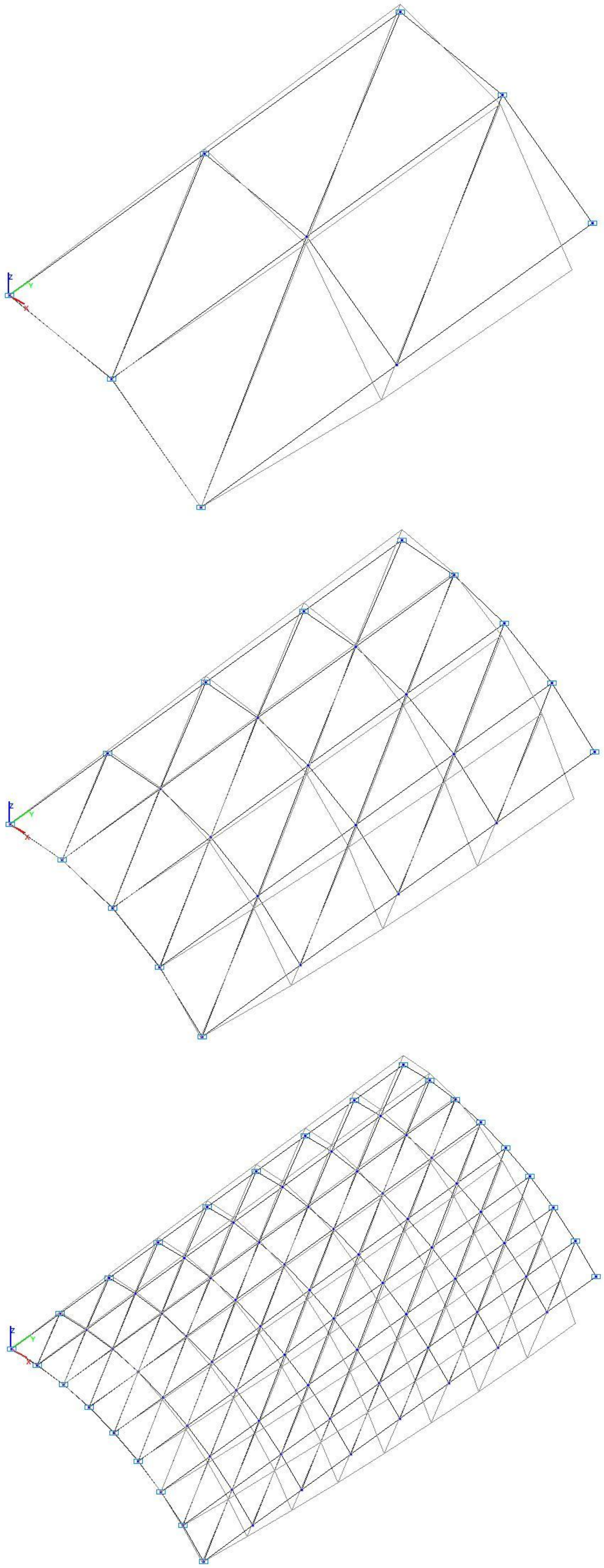
$E = 4.32 \cdot 10^8$ kPa	- elastic modulus of the material of the cylindrical shell;
$\nu = 0.00$	- Poisson's ratio;
L = 50.00 m	- length of the generatrix of the cylindrical shell;
R = 25.00 m	- radius of the midsurface of the cylindrical shell;
$2 \cdot \theta = 2 \cdot 40°$	- central angle of the arc of the director of the cylindrical shell;
h = 0.25 m	- thickness of the cylindrical shell;
q = 90.0 kN/m^2	- value of the transverse load uniformly distributed over the entire area of the cylindrical shell.

Finite element model: Design model – general type system. Six design models of a quarter of the cylindrical shell according to the symmetry conditions are considered:
Model 1 – 8, 32, 128 three-node shell elements of type 42 with a regular mesh 2x2, 4x4, 8x8. Boundary conditions are provided by imposing constraints on the nodes of the support curvilinear edges of the cylindrical shell in the directions of the degrees of freedom X, Z and constraints according to the symmetry conditions. Number of nodes in the model – 9, 25, 81.
Model 2 – 4, 16, 64 four-node shell elements of type 44 with a regular mesh 2x2, 4x4, 8x8. Boundary conditions are provided by imposing constraints on the nodes of the support curvilinear edges of the cylindrical shell in the directions of the degrees of freedom X, Z and constraints according to the symmetry conditions. Number of nodes in the model – 9, 25, 81.
Model 3 – 8, 32, 128 six-node shell elements of type 45 with a regular mesh 2x2, 4x4, 8x8. Boundary conditions are provided by imposing constraints on the nodes of the support curvilinear edges of the cylindrical shell in the directions of the degrees of freedom X, Z and constraints according to the symmetry conditions. Number of nodes in the model – 25, 81, 289.
Model 4 – 4, 16, 64 eight-node shell elements of type 50 with a regular mesh 2x2, 4x4, 8x8. Boundary conditions are provided by imposing constraints on the nodes of the support curvilinear edges of the cylindrical shell in the directions of the degrees of freedom X, Z and constraints according to the symmetry conditions. Number of nodes in the model – 25, 81, 289.
Model 5 – 4, 16, 64, 256, 1024, 4096, 16384 eight-node isoparametric solid elements of type 36 with a regular mesh 2x2x1, 4x4x1, 8x8x1, 16x16x1, 32x32x1, 64x64x1, 128x128x1. Boundary conditions are provided by imposing constraints on the nodes of the support curvilinear sides of the cylindrical shell in the directions of the degrees of freedom X, Z and constraints according to the symmetry conditions. Number of nodes in the model – 18, 50, 162, 578, 2178, 8450, 33282.
Model 6 – 4, 16, 64, 256, 1024, 4096, 16384 twenty-node isoparametric solid elements of type 37 with a regular mesh 2x2x1, 4x4x1, 8x8x1, 16x16x1, 32x32x1, 64x64x1, 128x128x1. Boundary conditions are provided by imposing constraints on the nodes of the support curvilinear sides of the cylindrical shell in the directions of the degrees of freedom X, Z and constraints according to the symmetry conditions. Number of nodes in the model – 51, 155, 531, 1955, 7491, 29315, 115971.

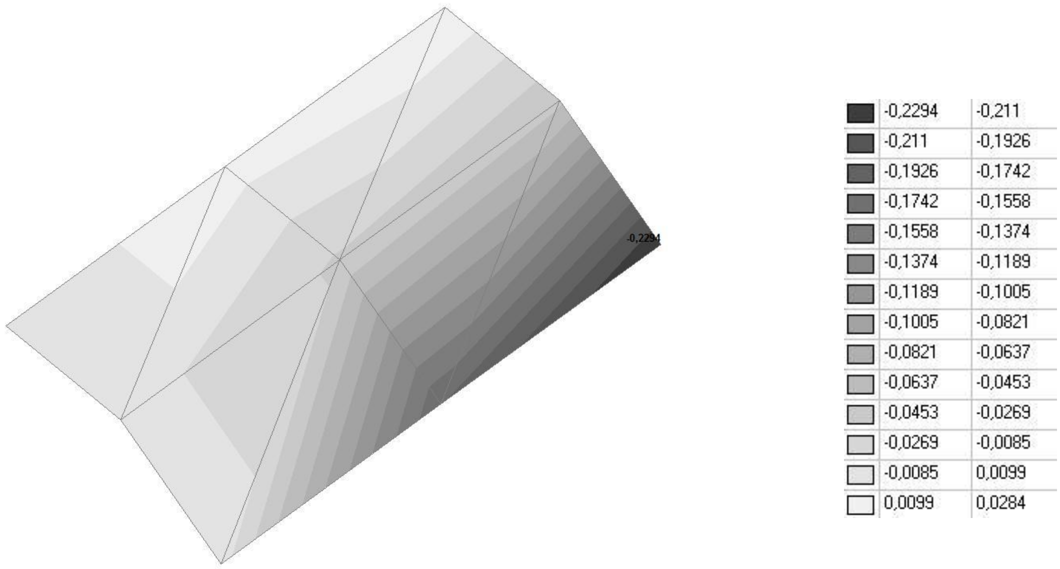
Results in SCAD

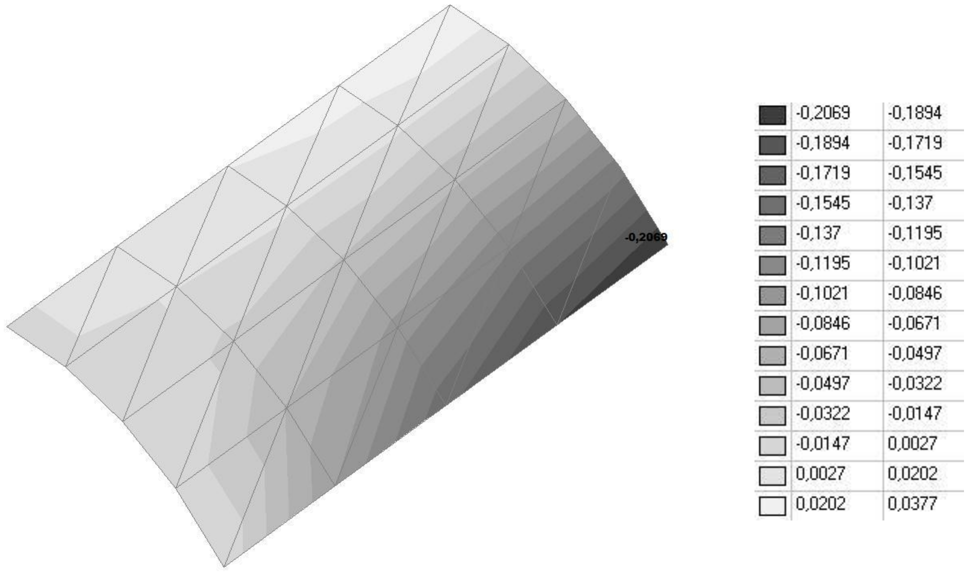


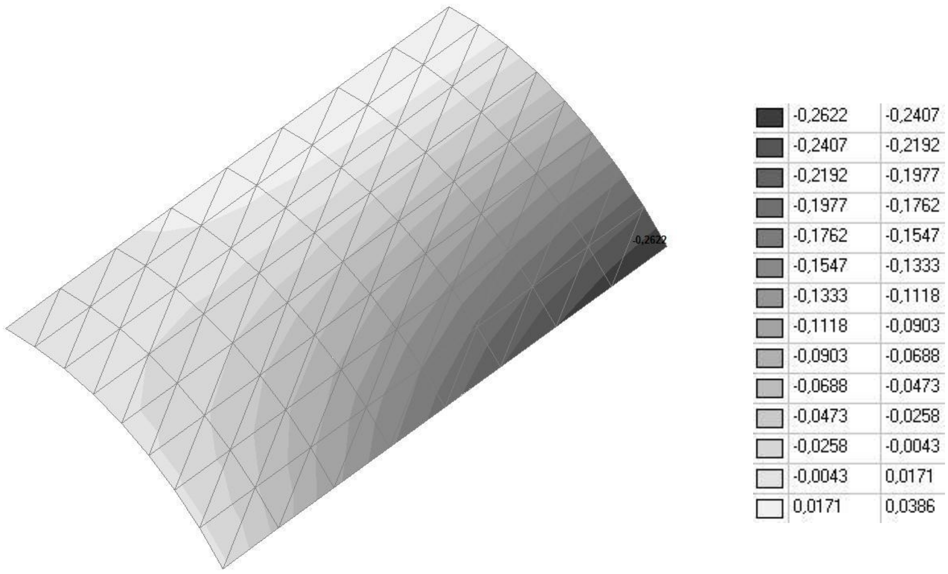
Model 1. Design model



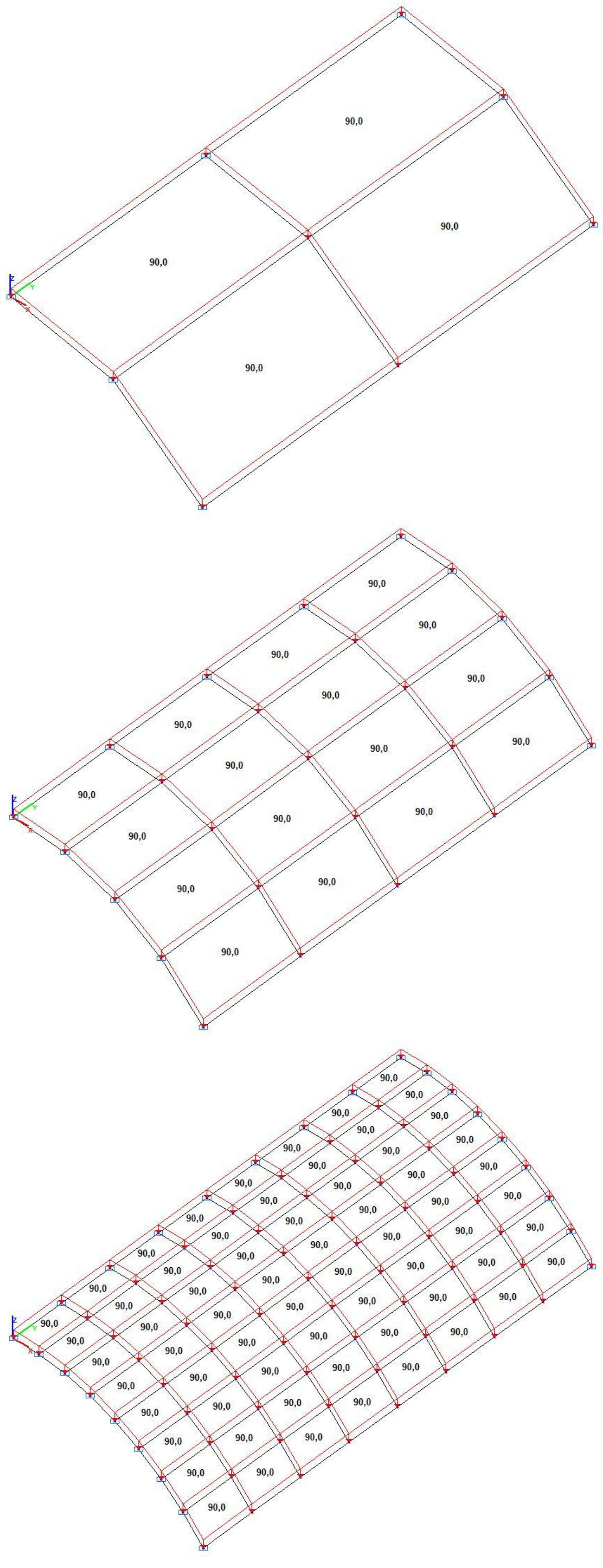
Model 1. Deformed model



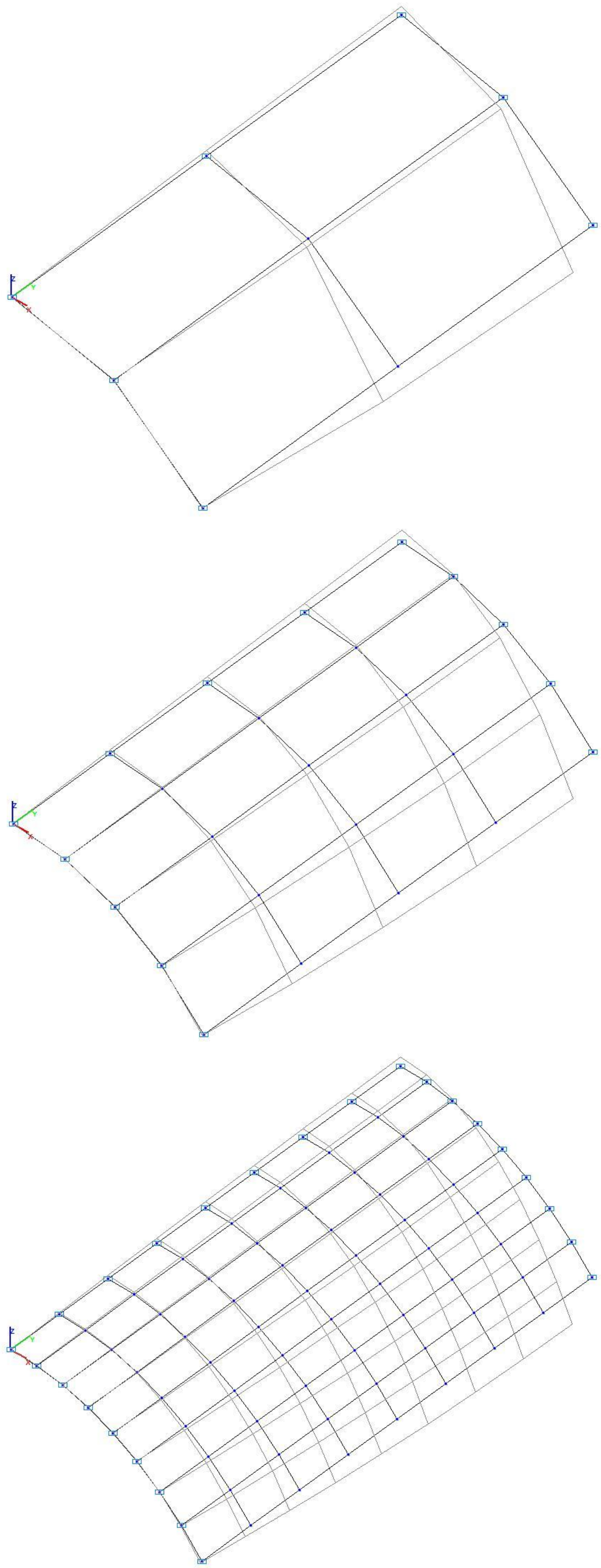




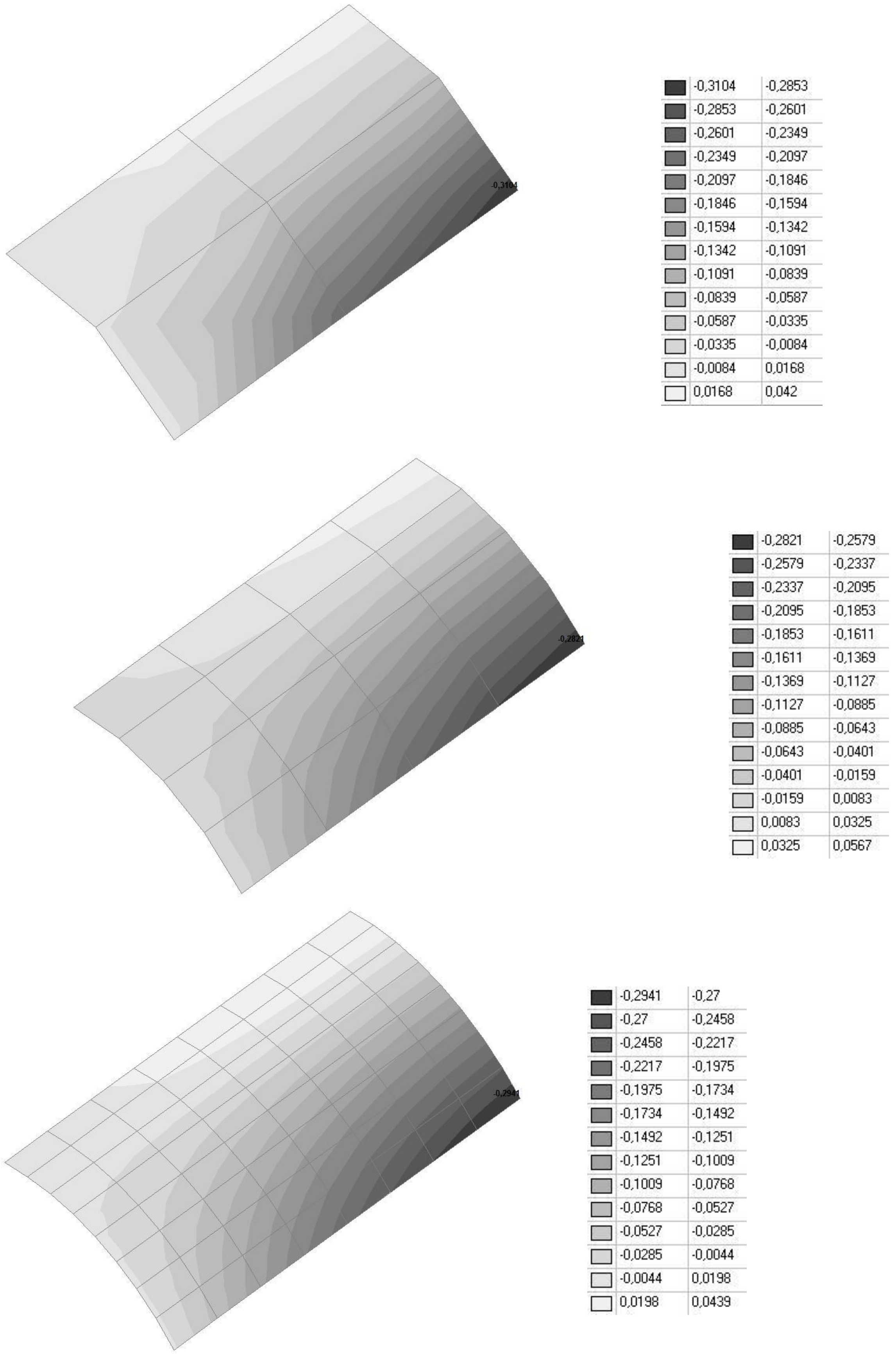
Model 1. Values of the transverse displacements in the middle of the free rectilinear edges of the open cylindrical shell w_q (m)



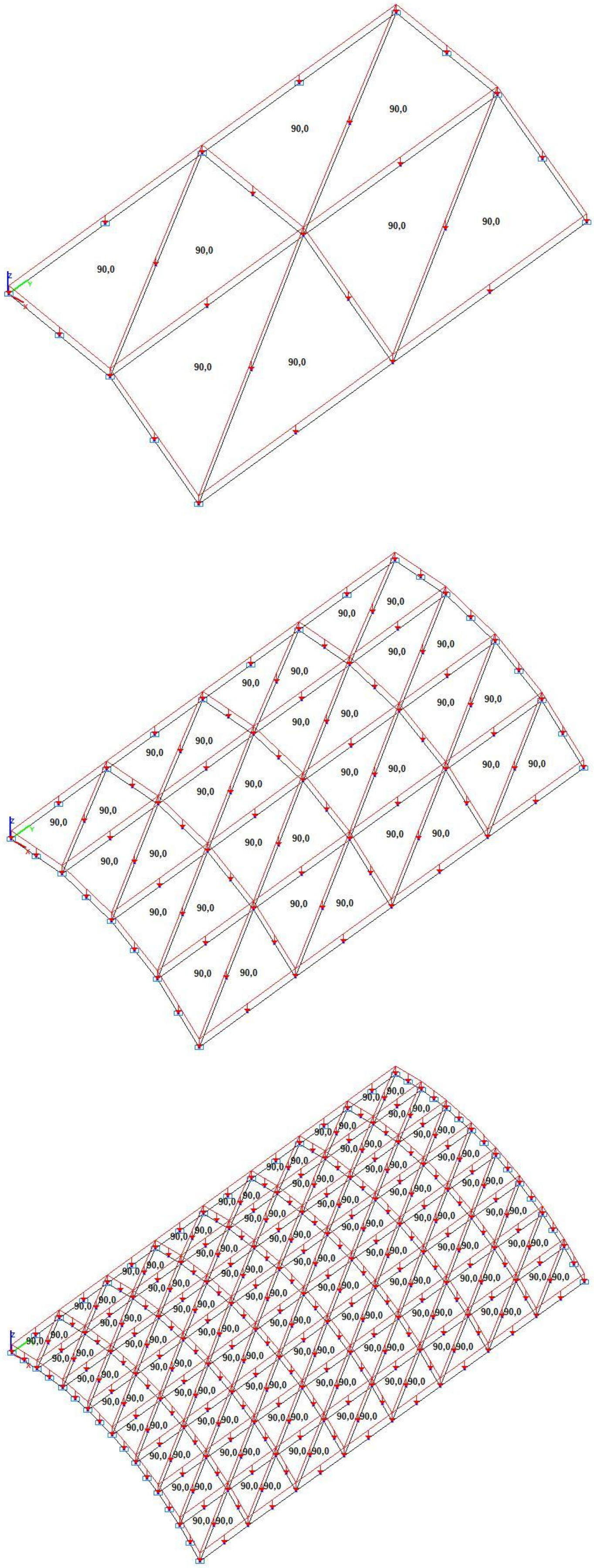
Model 2. Design model



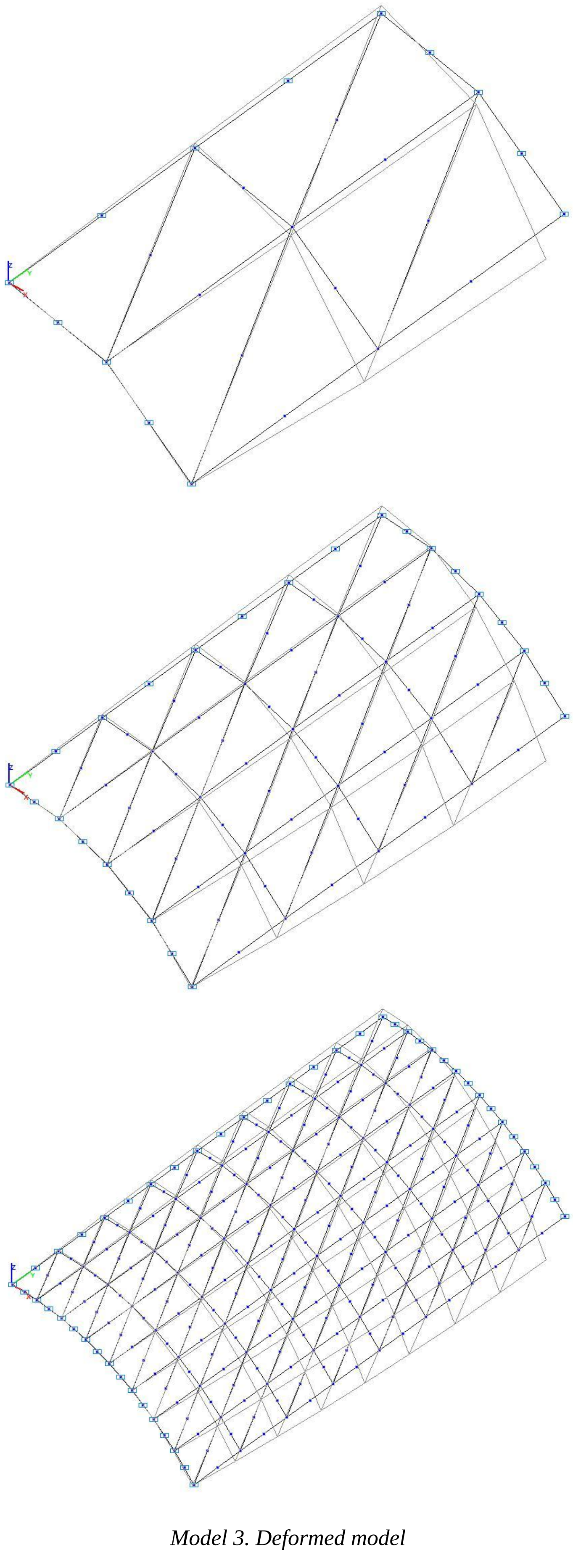
Model 2. Deformed model



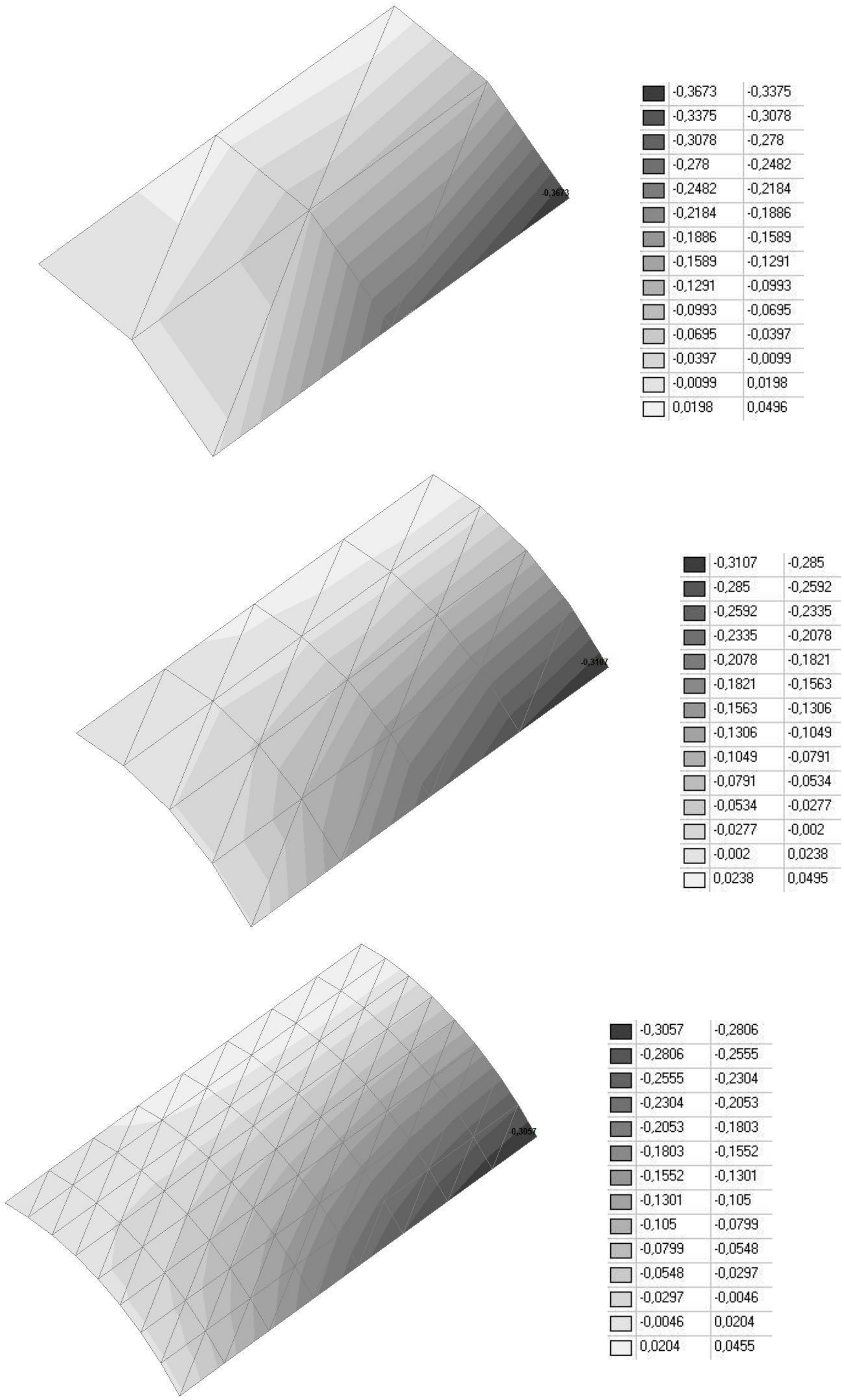
Model 2. Values of the transverse displacements in the middle of the free rectilinear edges of the open cylindrical shell w_q (m)



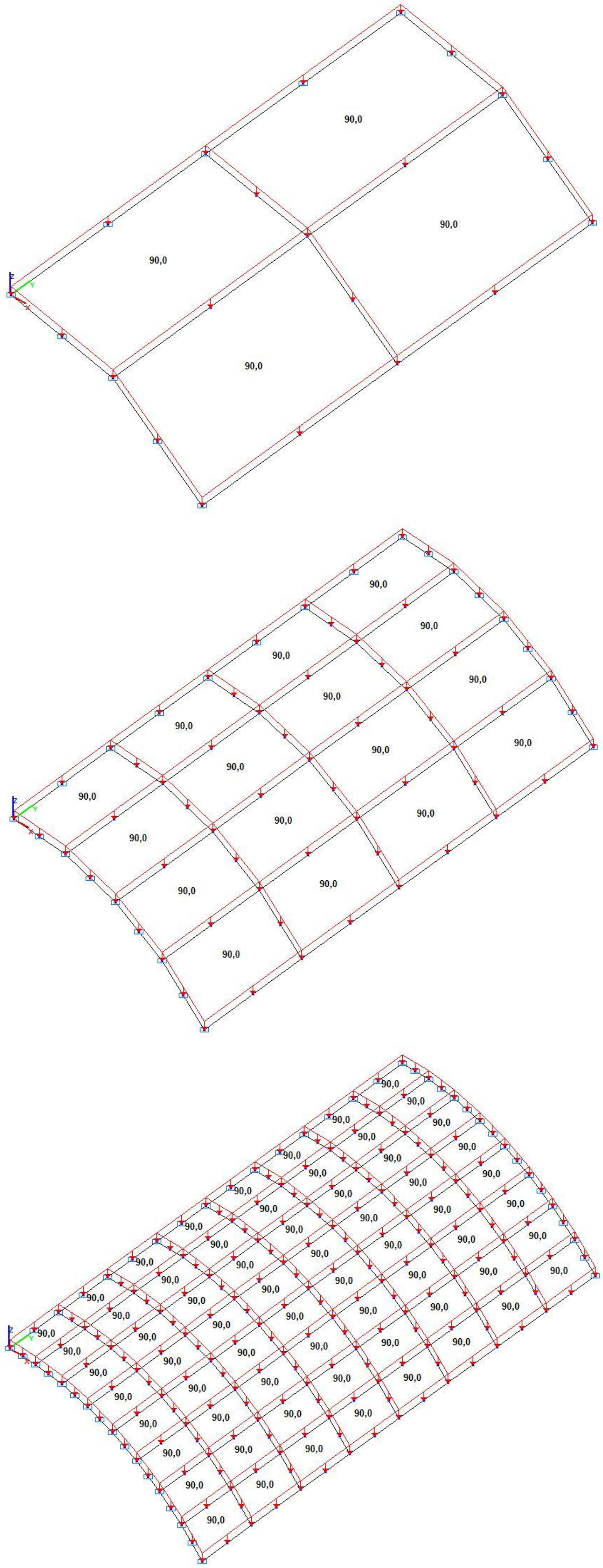
Model 3. Design model



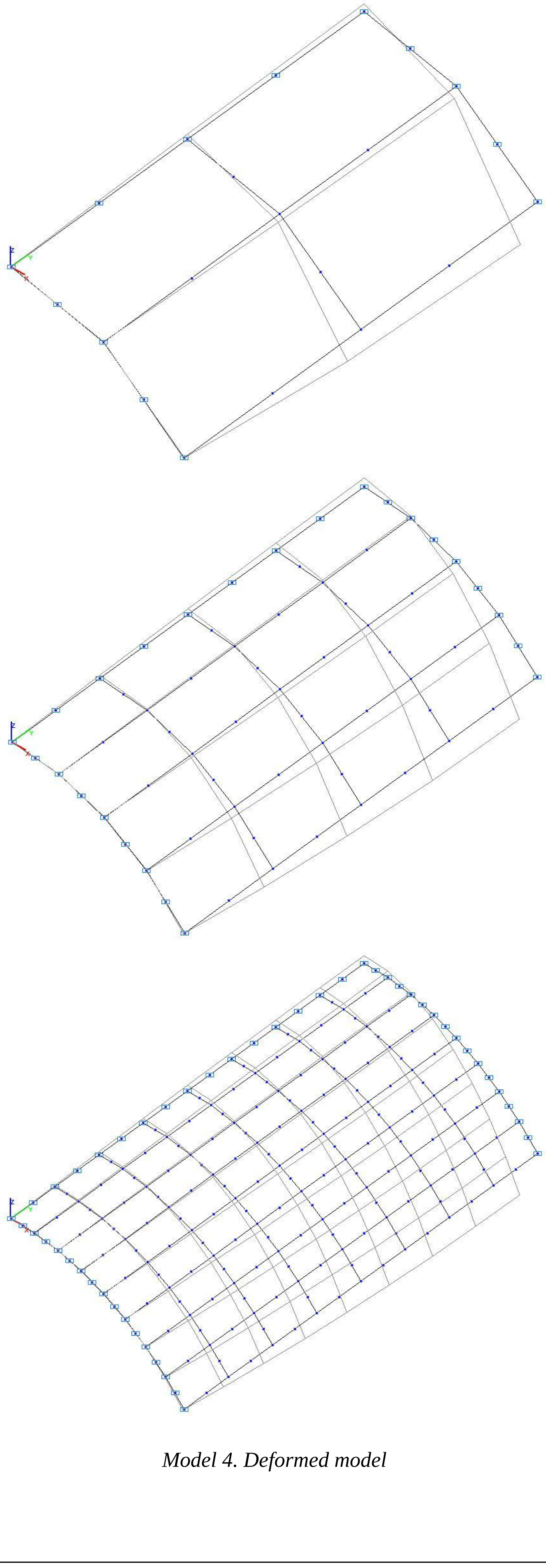
Model 3. Deformed model



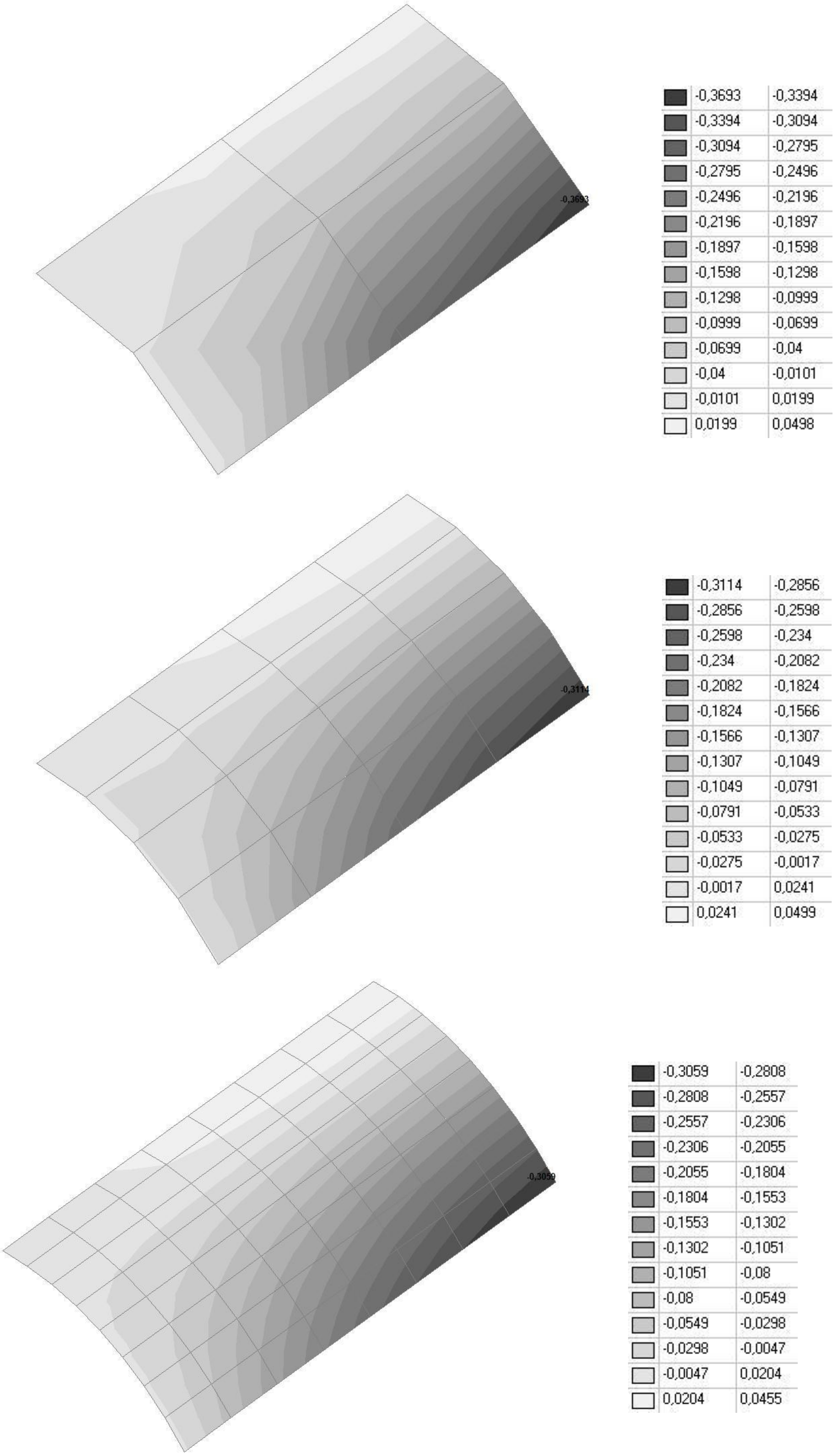
Model 3. Values of the transverse displacements in the middle of the free rectilinear edges of the open cylindrical shell w_q (m)



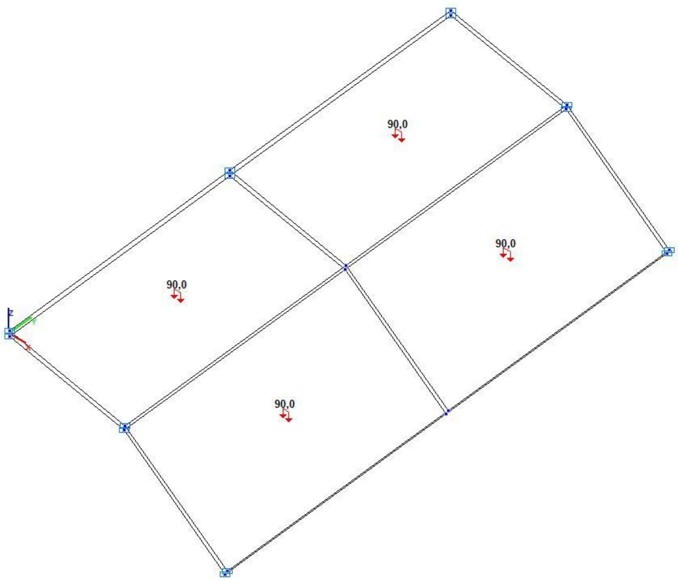
Model 4. Design mode



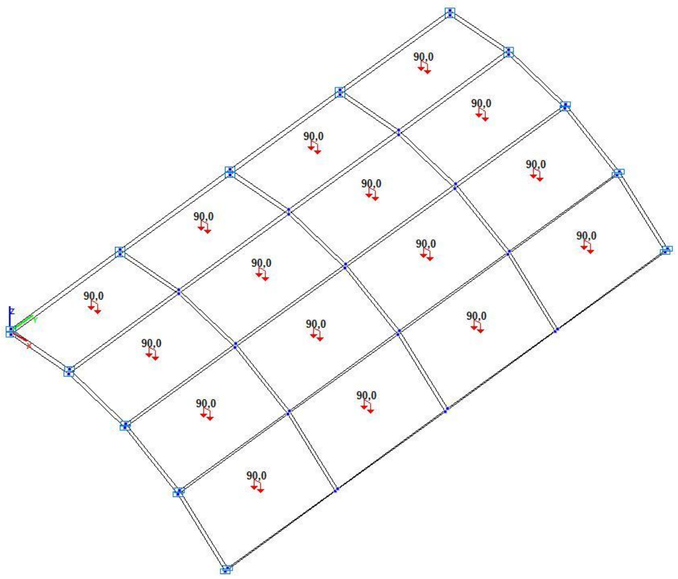
Model 4. Deformed model



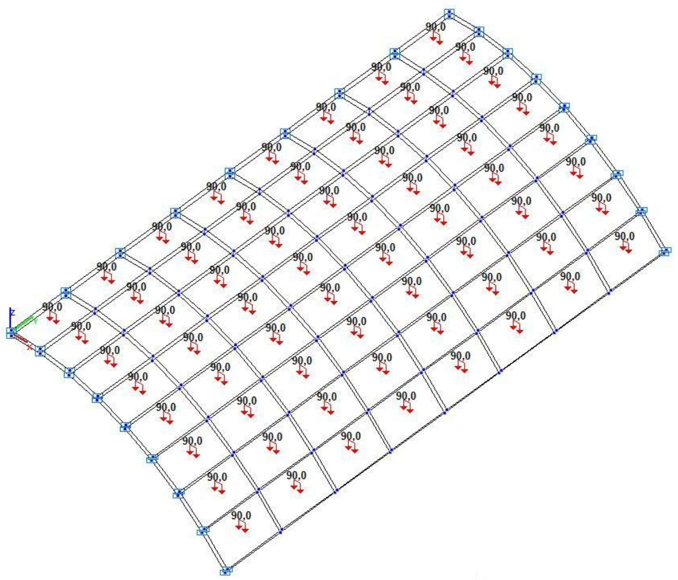
Model 4. Values of the transverse displacements in the middle of the free rectilinear edges of the open cylindrical shell w_q (m)

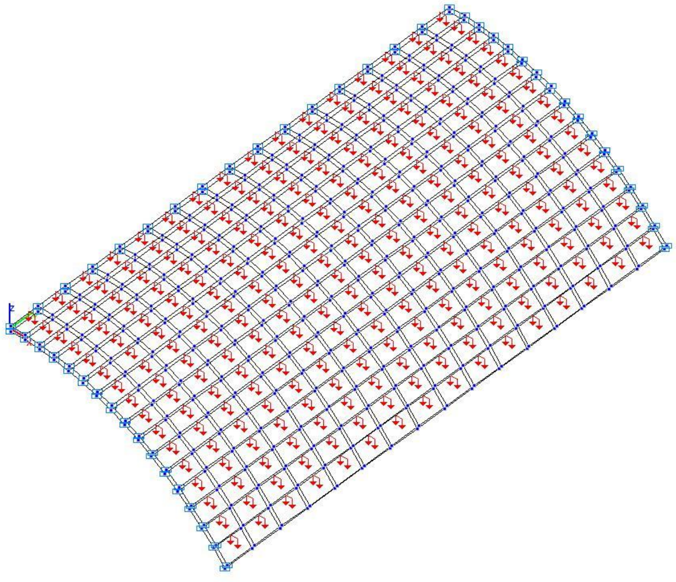


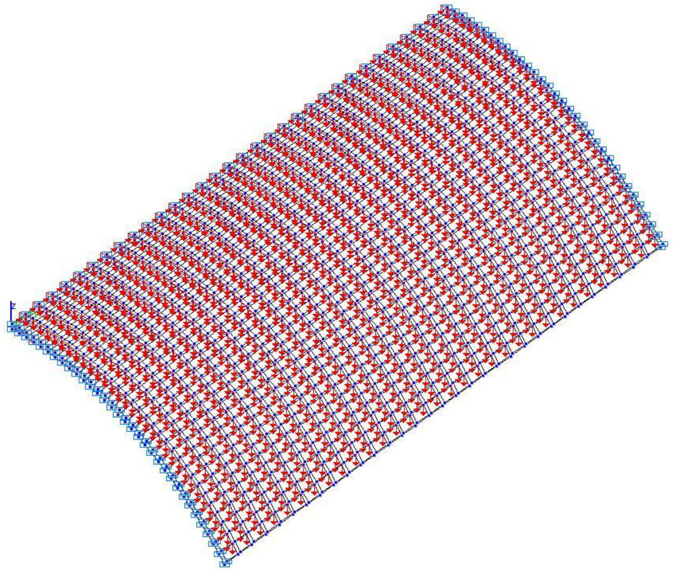
90,0
90,0
90,0
90,0

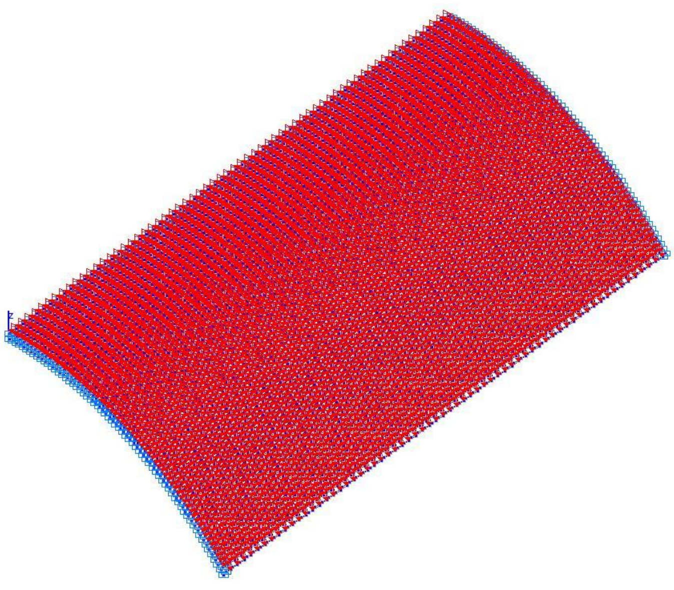


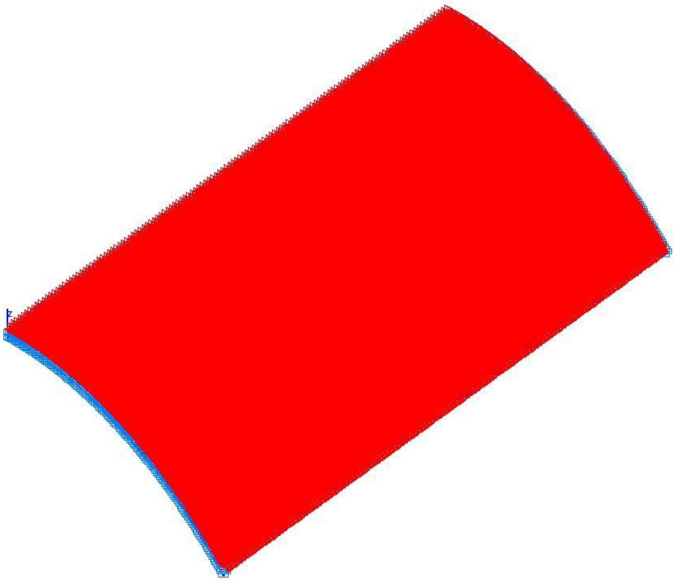
90,0
90,0
90,0
90,0
90,0
90,0
90,0
90,0
90,0
90,0
90,0
90,0
90,0
90,0
90,0
90,0



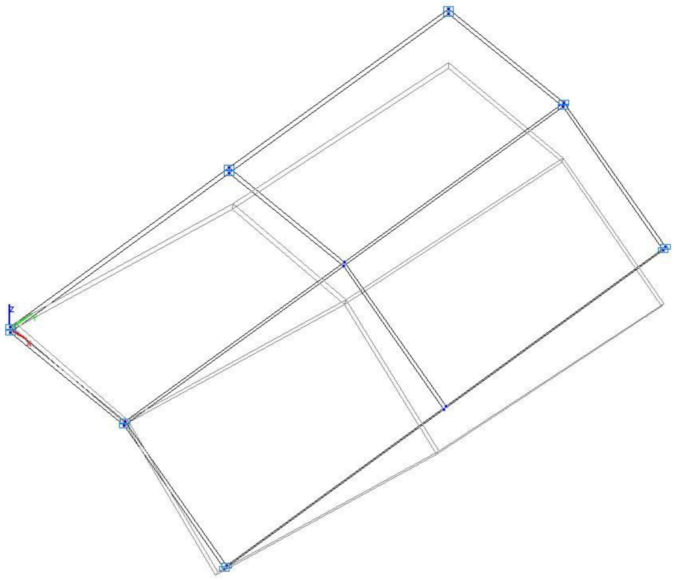


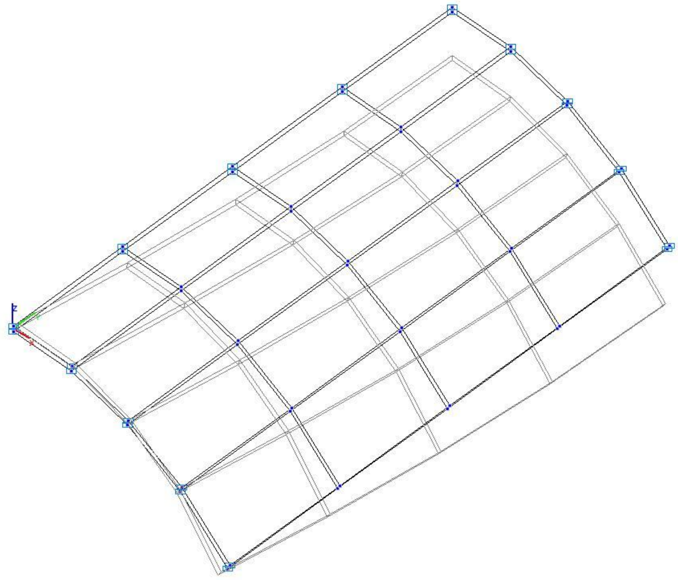


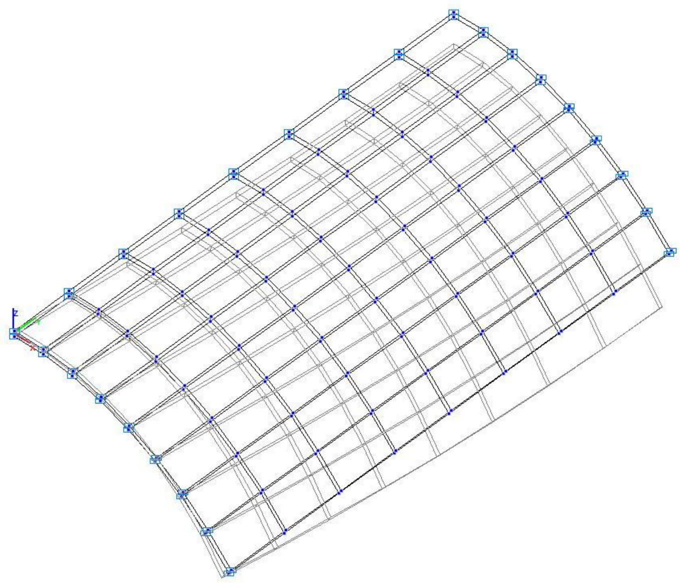


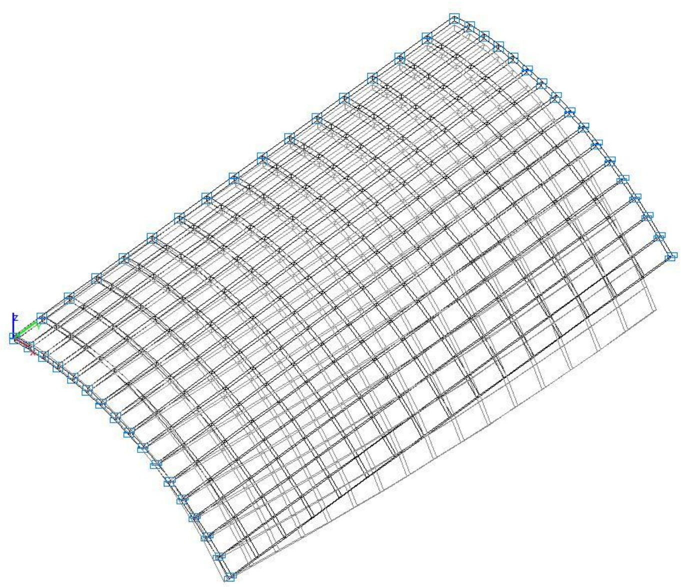


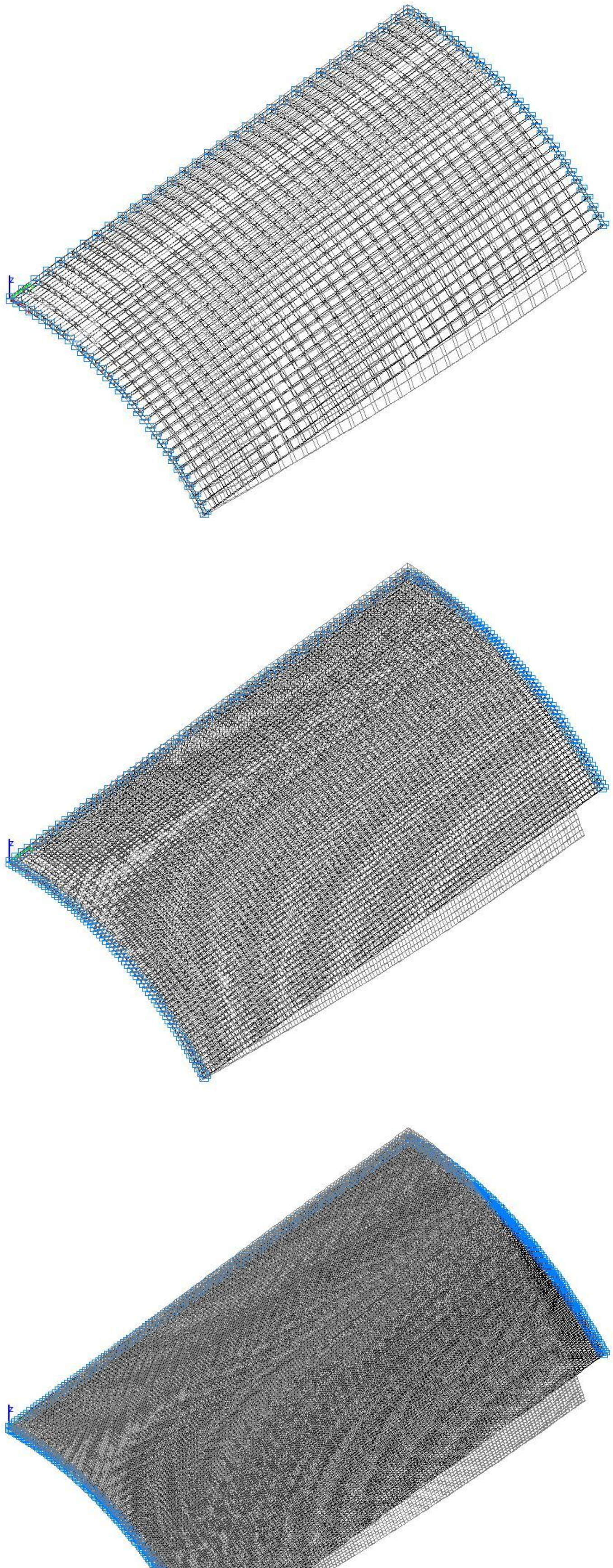
Model 5. Design model



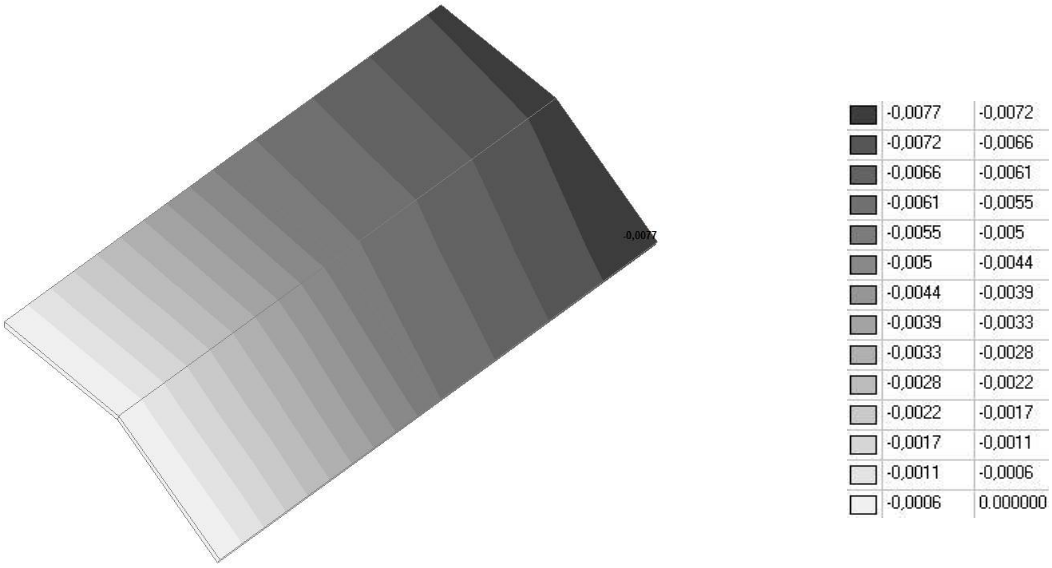


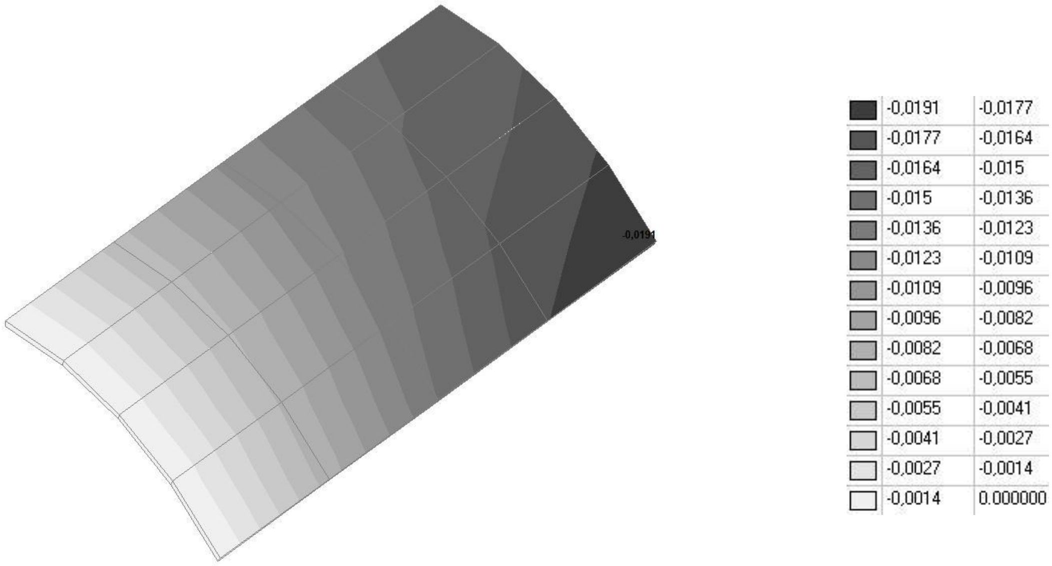


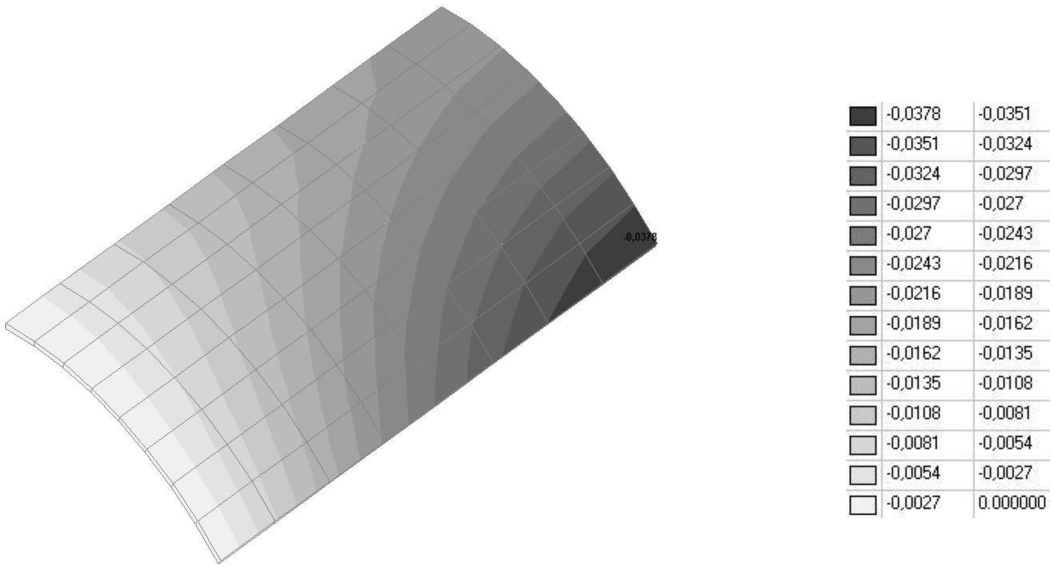


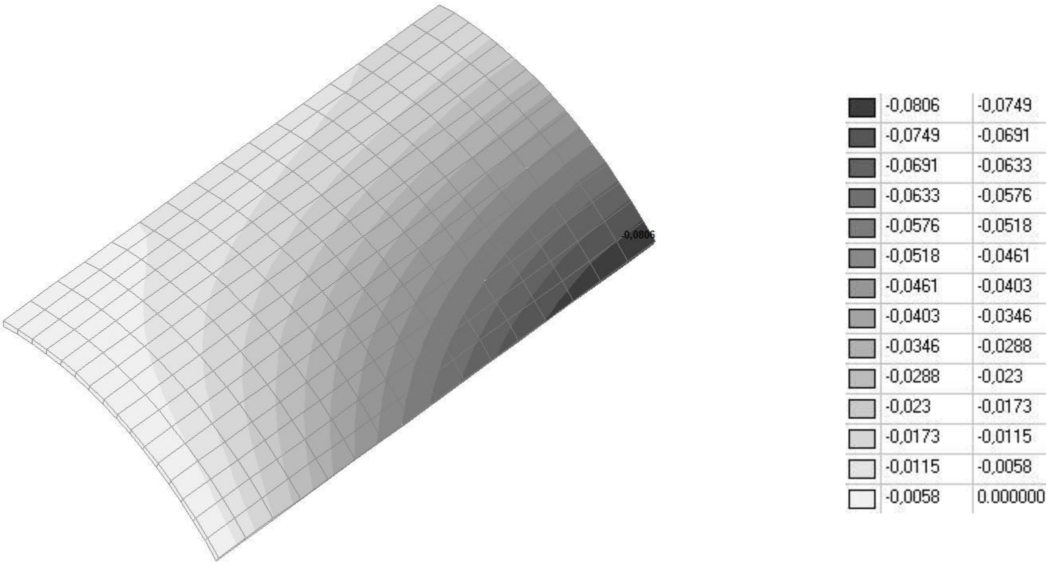


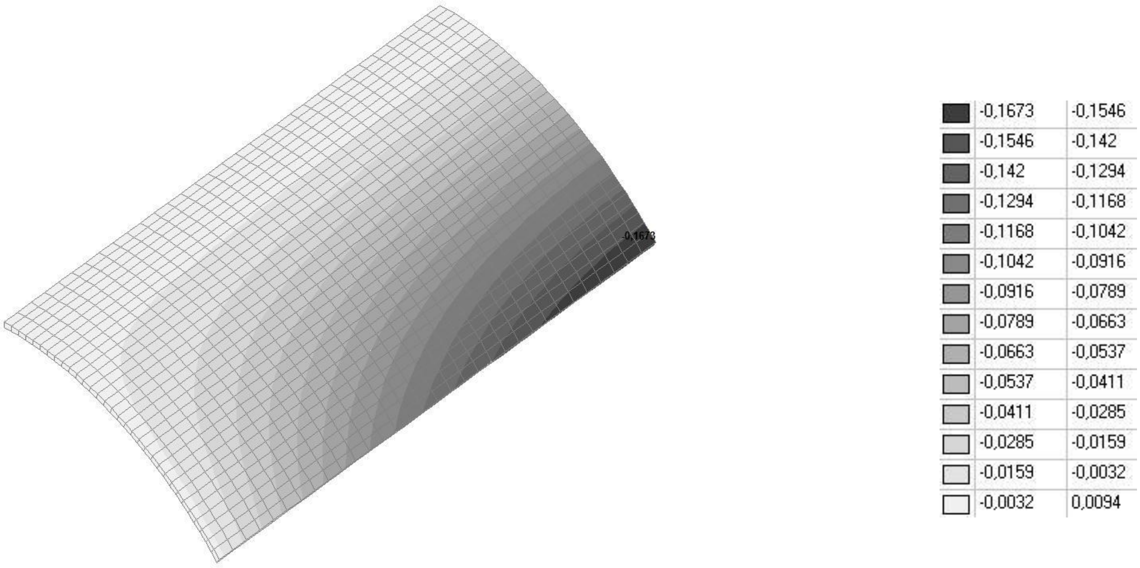
Model 5. Deformed model

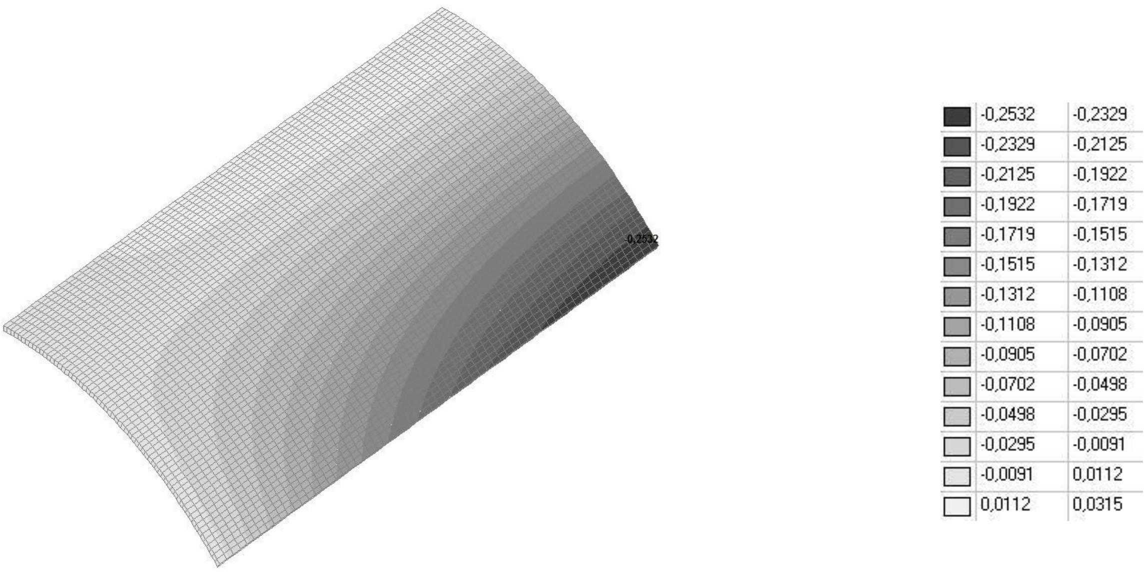

-0,0077
-0,0077 -0,0072
-0,0072 -0,0066
-0,0066 -0,0061
-0,0061 -0,0055
-0,0055 -0,005
-0,005 -0,0044
-0,0044 -0,0039
-0,0039 -0,0033
-0,0033 -0,0028
-0,0028 -0,0022
-0,0022 -0,0017
-0,0017 -0,0011
-0,0011 -0,0006
-0,0006 0.000000

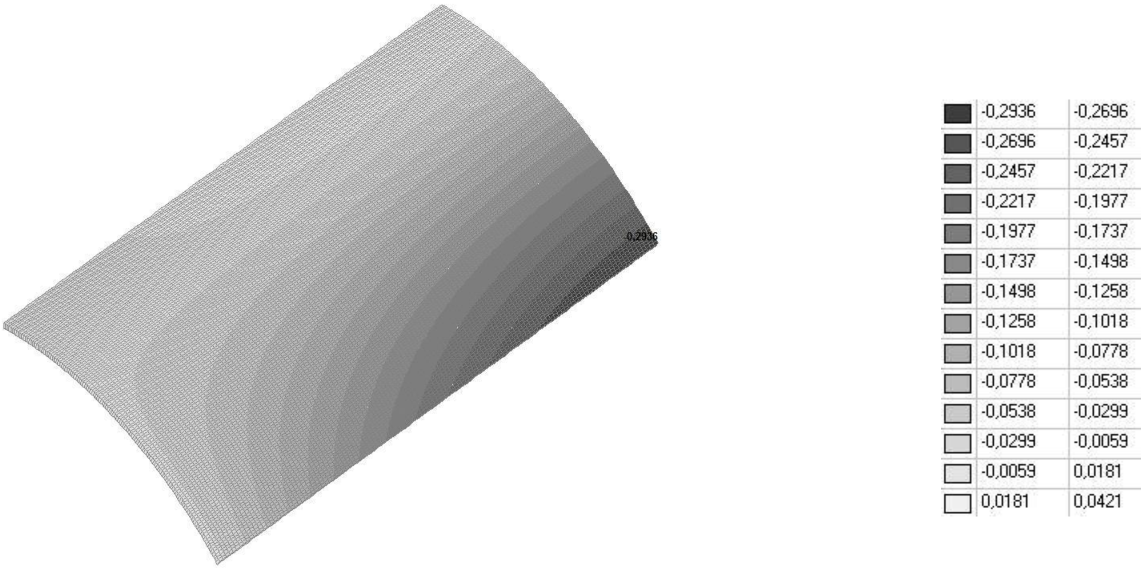

-0,0191
-0,0191 -0,0177
-0,0177 -0,0164
-0,0164 -0,015
-0,015 -0,0136
-0,0136 -0,0123
-0,0123 -0,0109
-0,0109 -0,0096
-0,0096 -0,0082
-0,0082 -0,0068
-0,0068 -0,0055
-0,0055 -0,0041
-0,0041 -0,0027
-0,0027 -0,0014
-0,0014 0.000000


-0,0378
-0,0378 -0,0351
-0,0351 -0,0324
-0,0324 -0,0297
-0,0297 -0,027
-0,027 -0,0243
-0,0243 -0,0216
-0,0216 -0,0189
-0,0189 -0,0162
-0,0162 -0,0135
-0,0135 -0,0108
-0,0108 -0,0081
-0,0081 -0,0054
-0,0054 -0,0027
-0,0027 0.000000

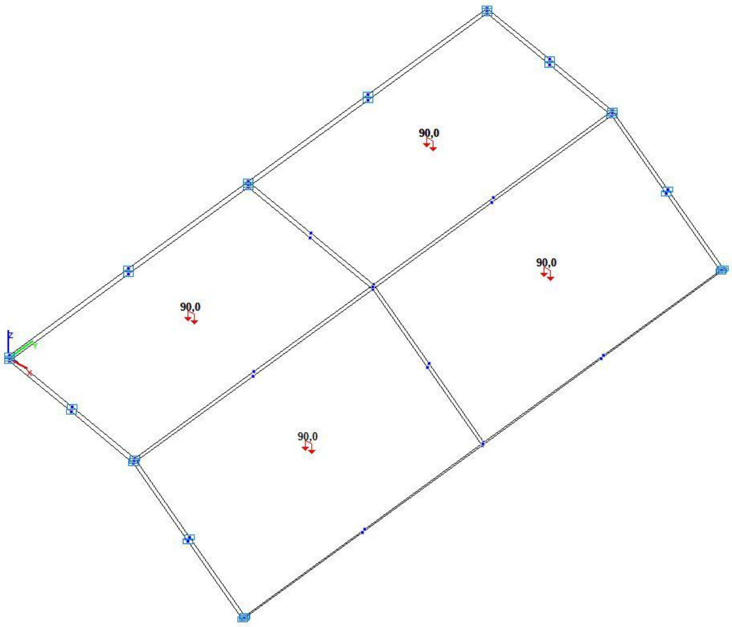

-0,0806 -0,0749
-0,0749 -0,0691
-0,0691 -0,0633
-0,0633 -0,0576
-0,0576 -0,0518
-0,0518 -0,0461
-0,0461 -0,0403
-0,0403 -0,0346
-0,0346 -0,0288
-0,0288 -0,023
-0,023 -0,0173
-0,0173 -0,0115
-0,0115 -0,0058
-0,0058 0.000000

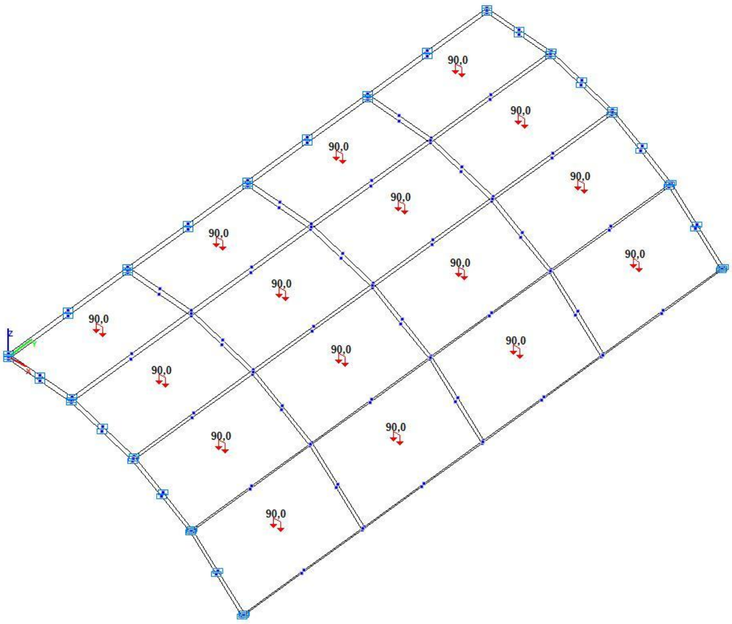

-0,1673 -0,1546
-0,1546 -0,142
-0,142 -0,1294
-0,1294 -0,1168
-0,1168 -0,1042
-0,1042 -0,0916
-0,0916 -0,0789
-0,0789 -0,0663
-0,0663 -0,0537
-0,0537 -0,0411
-0,0411 -0,0285
-0,0285 -0,0159
-0,0159 -0,0032
-0,0032 0,0094

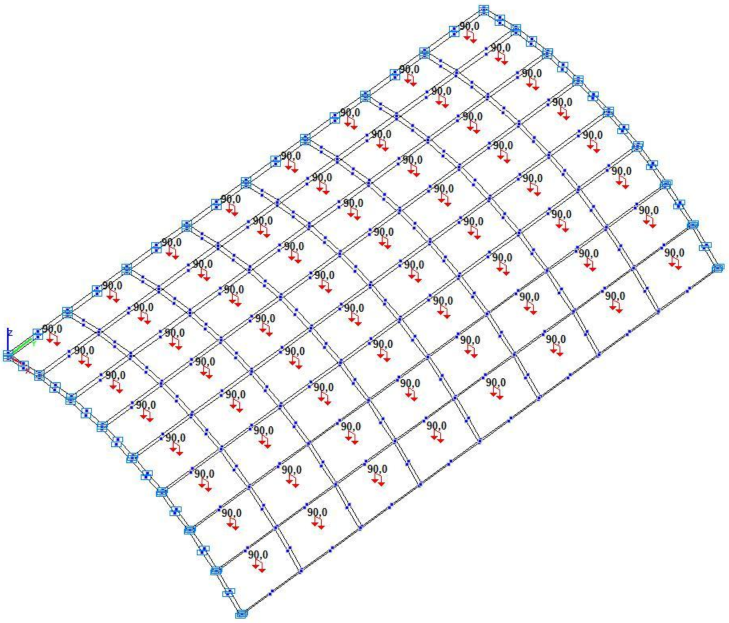

-0,2532 -0,2329
-0,2329 -0,2125
-0,2125 -0,1922
-0,1922 -0,1719
-0,1719 -0,1515
-0,1515 -0,1312
-0,1312 -0,1108
-0,1108 -0,0905
-0,0905 -0,0702
-0,0702 -0,0498
-0,0498 -0,0295
-0,0295 -0,0091
-0,0091 0,0112
0,0112 0,0315

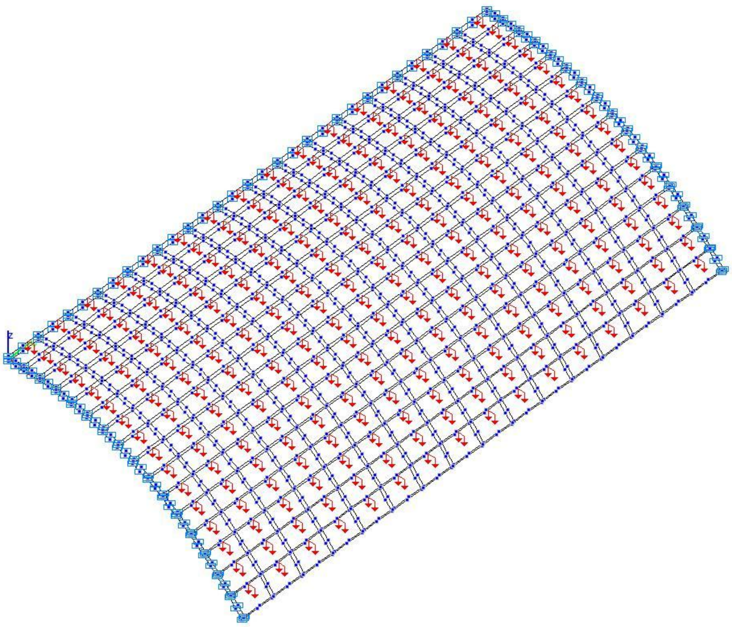


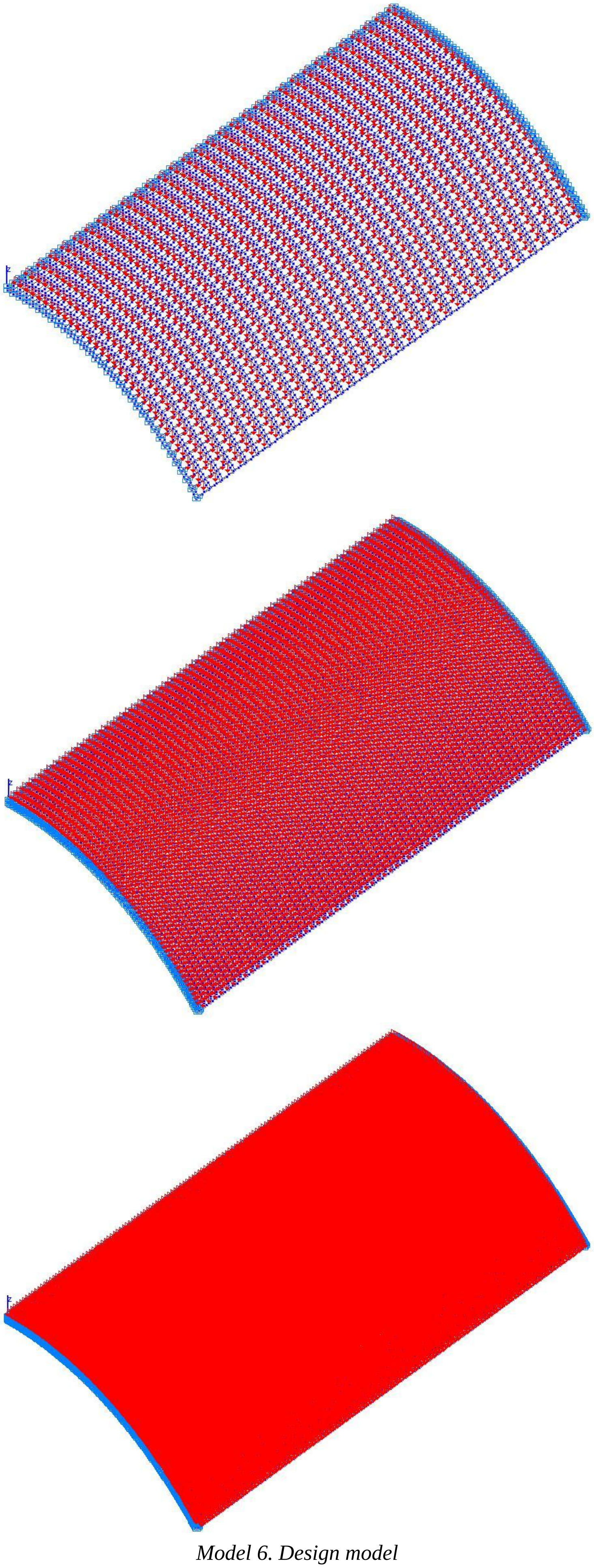
Model 5. Values of the transverse displacements in the middle of the free rectilinear edges of the open cylindrical shell w_q (m)



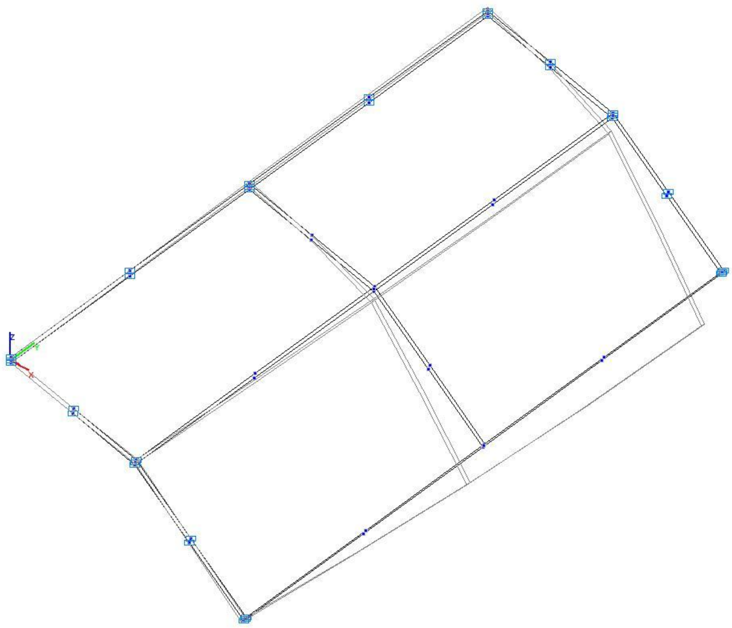

90,0

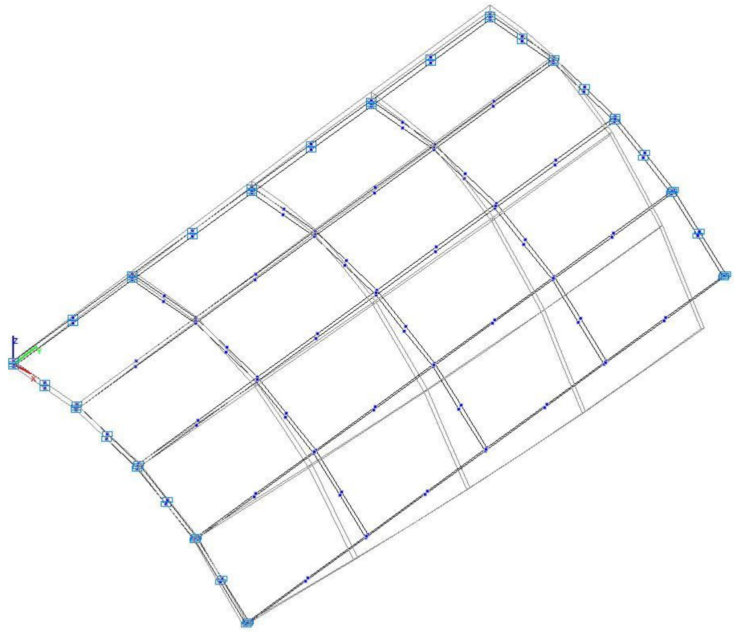

90,0

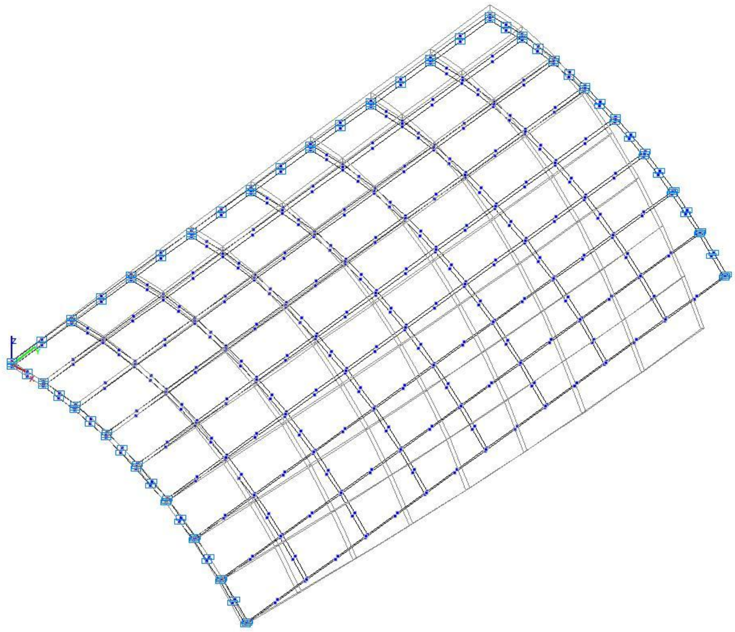


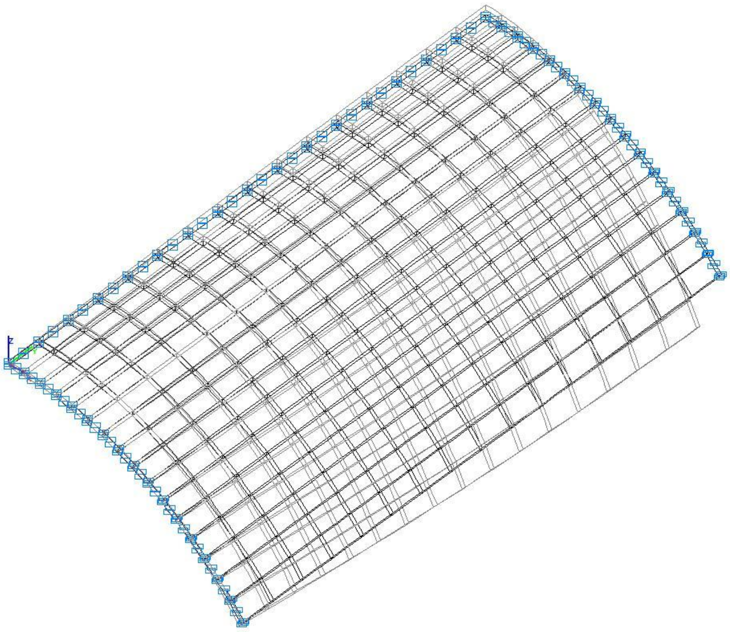


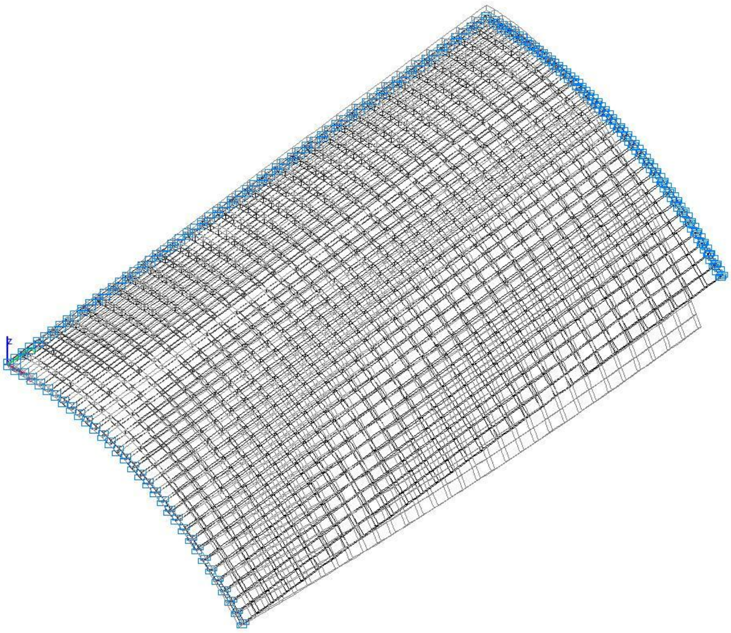
Model 6. Design model

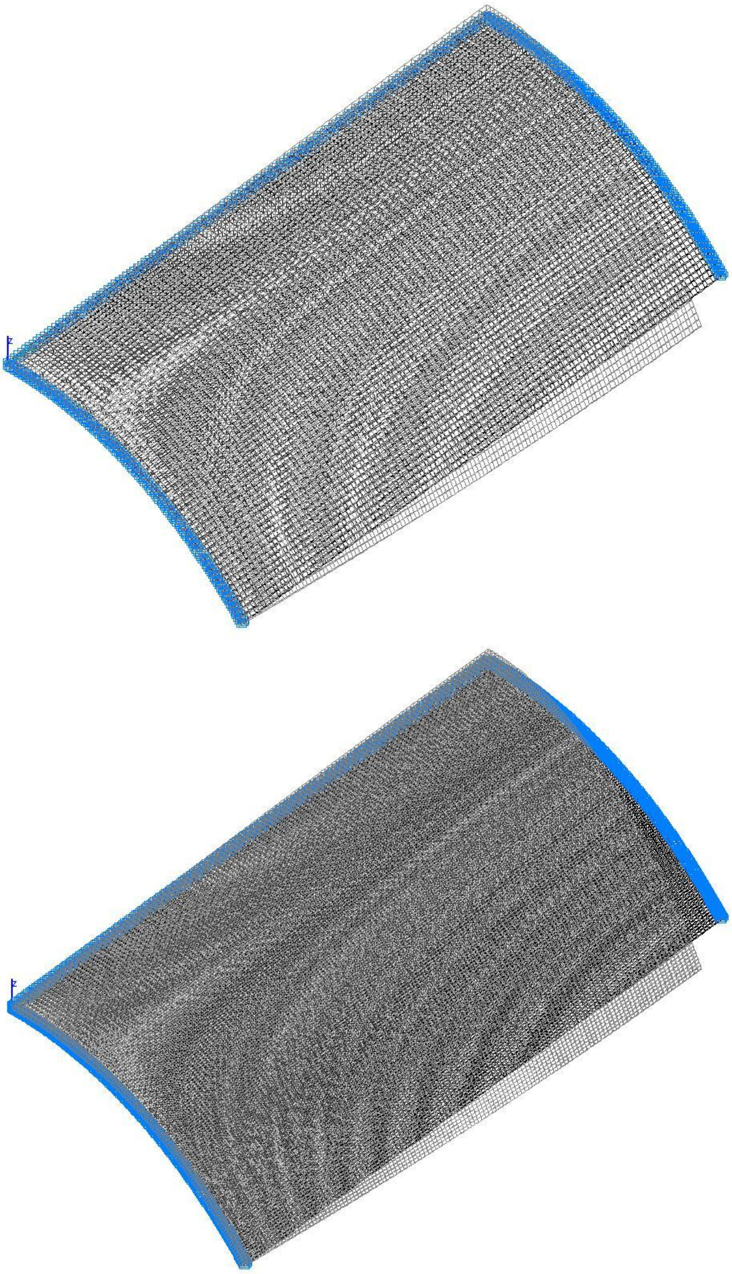




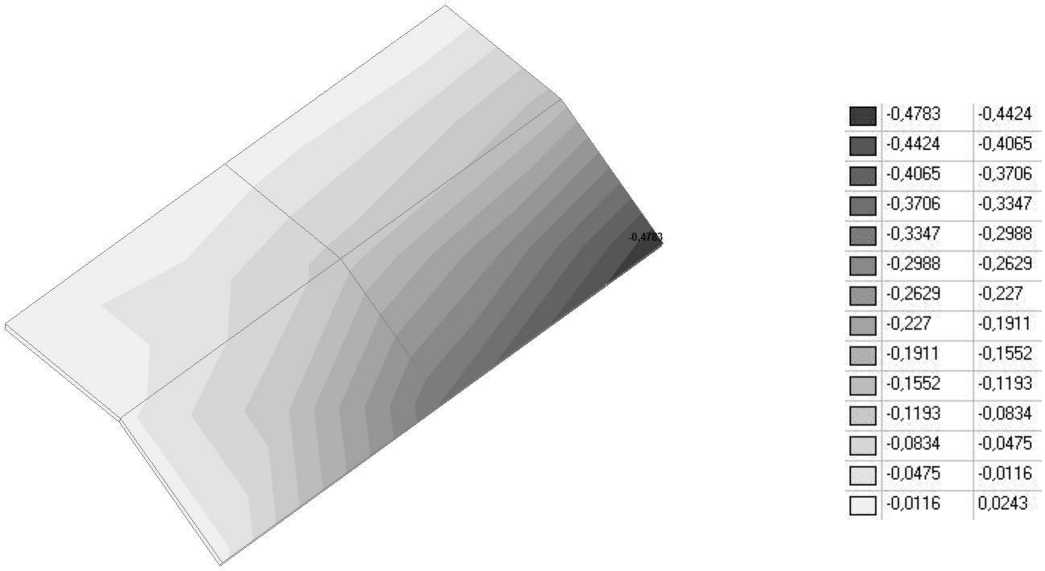


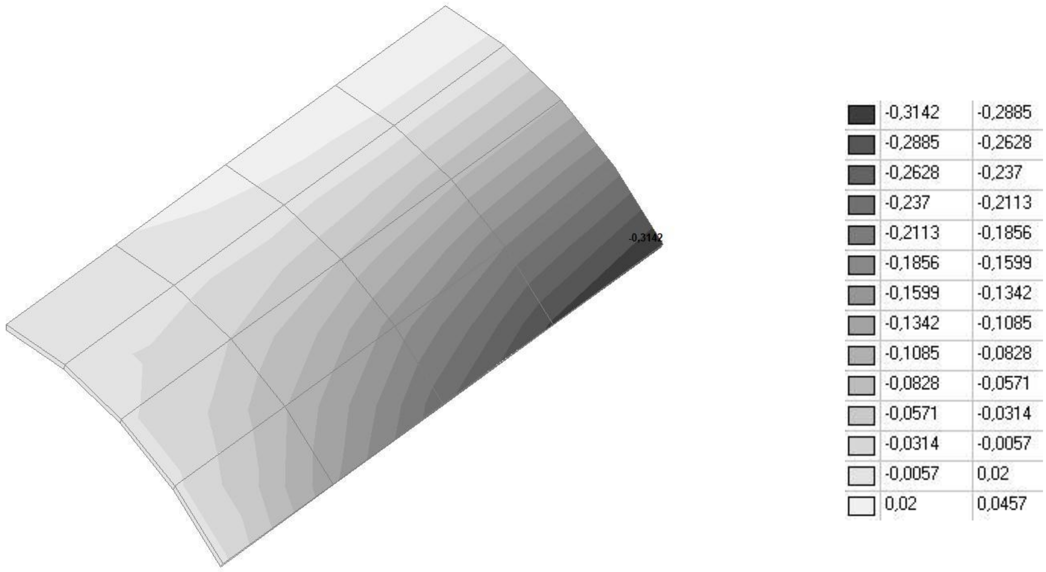


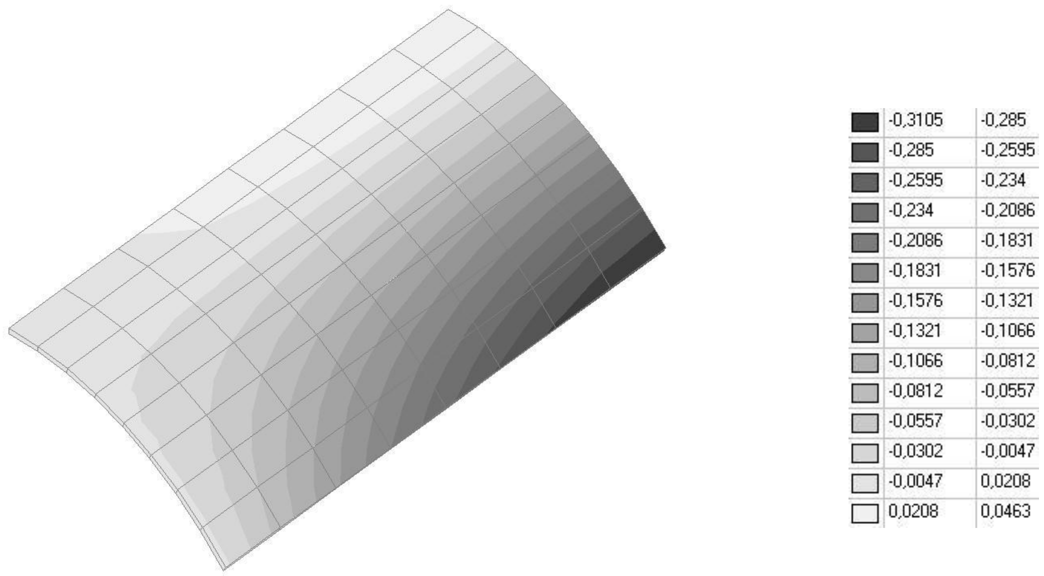


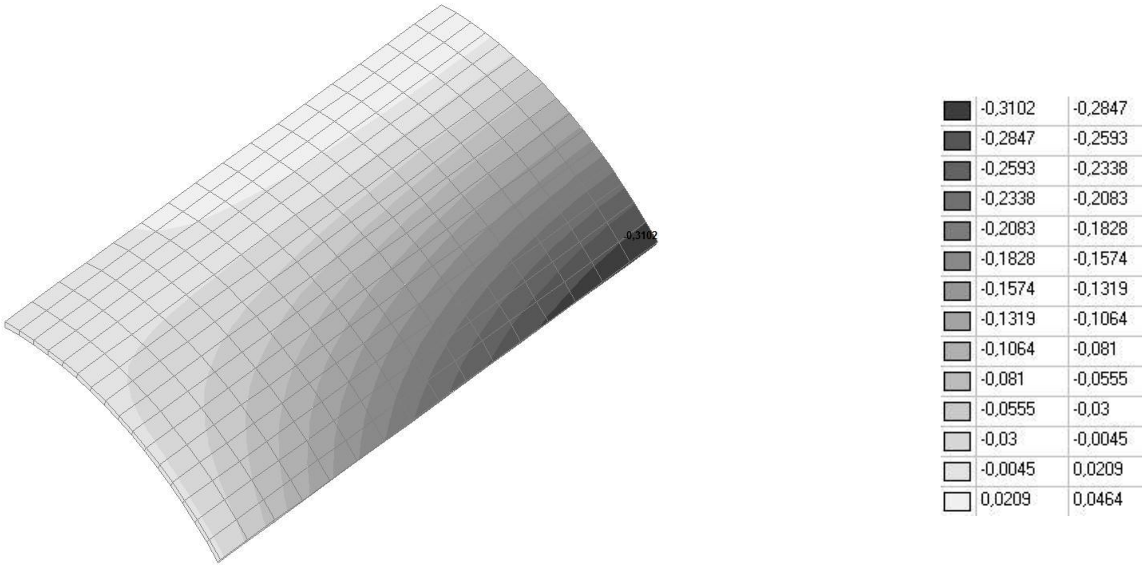


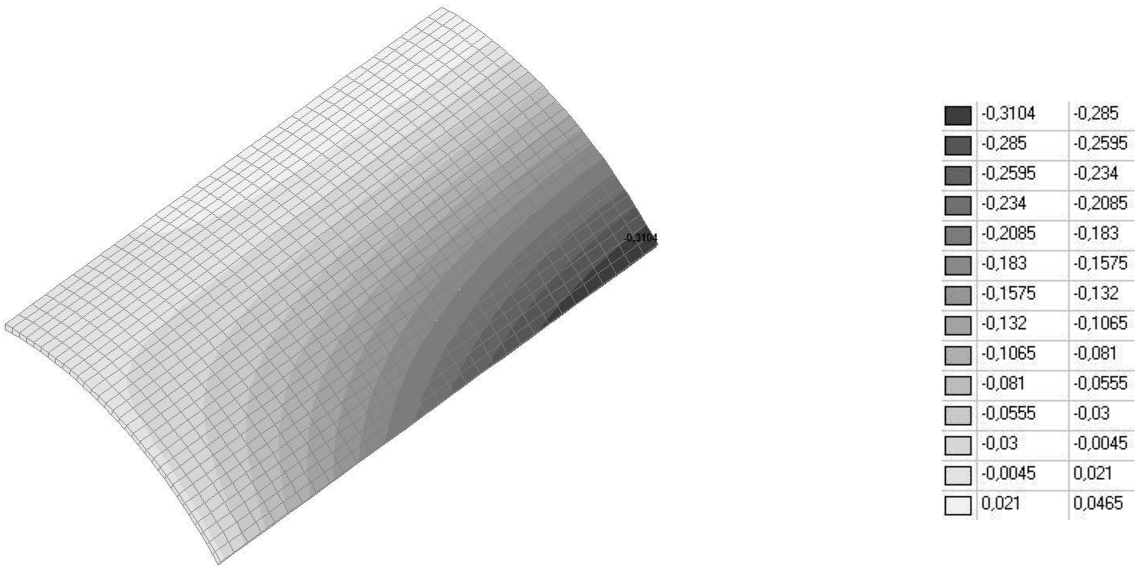
Model 6. Deformed model

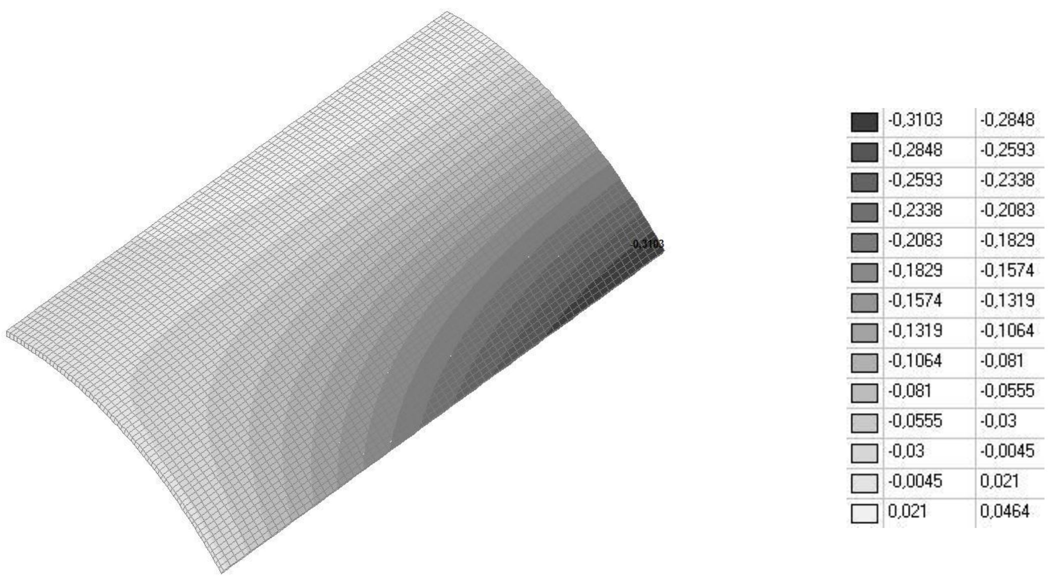

-0,4783
-0,4783 -0,4424
-0,4424 -0,4065
-0,4065 -0,3706
-0,3706 -0,3347
-0,3347 -0,2988
-0,2988 -0,2629
-0,2629 -0,227
-0,227 -0,1911
-0,1911 -0,1552
-0,1552 -0,1193
-0,1193 -0,0834
-0,0834 -0,0475
-0,0475 -0,0116
-0,0116 0,0243

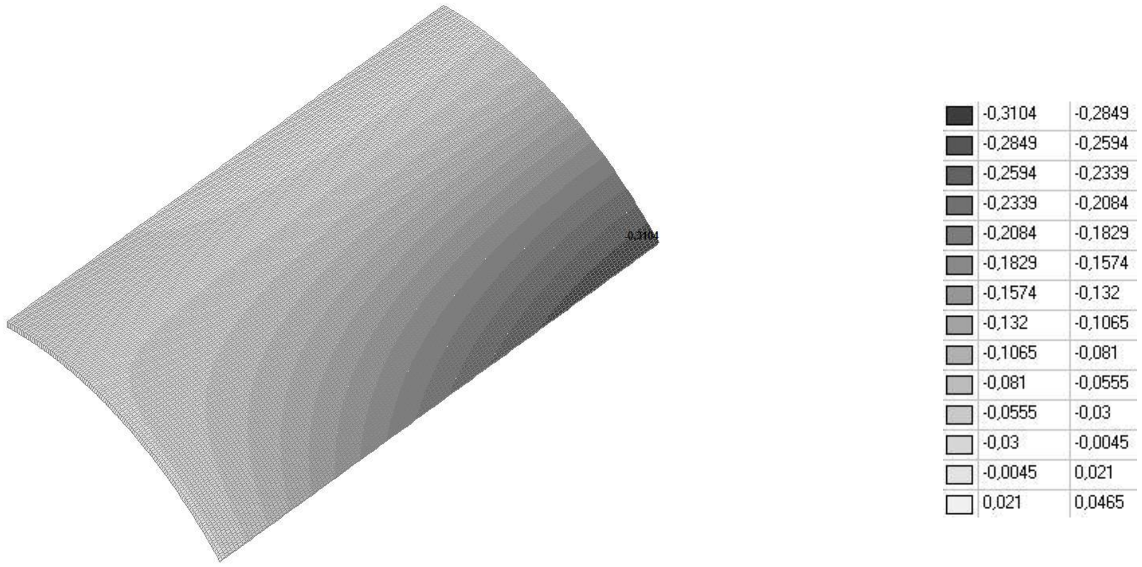

-0,3142
-0,3142 -0,2885
-0,2885 -0,2628
-0,2628 -0,237
-0,237 -0,2113
-0,2113 -0,1856
-0,1856 -0,1599
-0,1599 -0,1342
-0,1342 -0,1085
-0,1085 -0,0828
-0,0828 -0,0571
-0,0571 -0,0314
-0,0314 -0,0057
-0,0057 0,02
0,02 0,0457


-0,3105 -0,285
-0,285 -0,2595
-0,2595 -0,234
-0,234 -0,2086
-0,2086 -0,1831
-0,1831 -0,1576
-0,1576 -0,1321
-0,1321 -0,1066
-0,1066 -0,0812
-0,0812 -0,0557
-0,0557 -0,0302
-0,0302 -0,0047
-0,0047 0,0208
0,0208 0,0463


-0,3102
-0,3102 -0,2847
-0,2847 -0,2593
-0,2593 -0,2338
-0,2338 -0,2083
-0,2083 -0,1828
-0,1828 -0,1574
-0,1574 -0,1319
-0,1319 -0,1064
-0,1064 -0,081
-0,081 -0,0555
-0,0555 -0,03
-0,03 -0,0045
-0,0045 0,0209
0,0209 0,0464


-0,3104
-0,3104 -0,285
-0,285 -0,2595
-0,2595 -0,234
-0,234 -0,2085
-0,2085 -0,183
-0,183 -0,1575
-0,1575 -0,132
-0,132 -0,1065
-0,1065 -0,081
-0,081 -0,0555
-0,0555 -0,03
-0,03 -0,0045
-0,0045 0,021
0,021 0,0465


-0,3103
-0,3103 -0,2848
-0,2848 -0,2593
-0,2593 -0,2338
-0,2338 -0,2083
-0,2083 -0,1829
-0,1829 -0,1574
-0,1574 -0,1319
-0,1319 -0,1064
-0,1064 -0,081
-0,081 -0,0555
-0,0555 -0,03
-0,03 -0,0045
-0,0045 0,021
0,021 0,0464



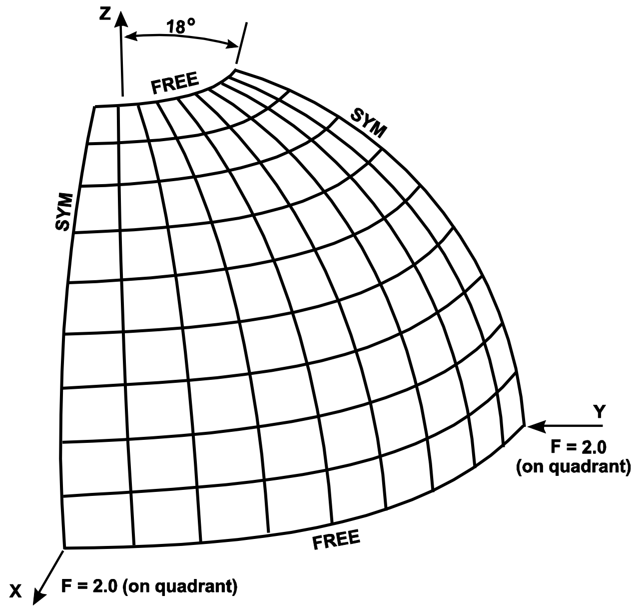
Model 6. Values of the transverse displacements in the middle of the free rectilinear edges of the open cylindrical shell w_q (m)

Comparison of solutions:

Transverse displacements in the middle of the free rectilinear edges of the open cylindrical shell w_q from the transverse load q uniformly distributed over the entire area

Model	Finite element mesh	Theory	SCAD	Deviation, %
1 (Member type 42)	2x2	0.3086	0.2294	25.66
	4x4		0.2069	32.95
	8x8		0.2622	15.04
2 (Member type 44)	2x2	0.3086	0.3104	0.58
	4x4		0.2821	8.59
	8x8		0.2941	4.70
3 (Member type 45)	2x2	0.3086	0.3673	19.02
	4x4		0.3107	0.68
	8x8		0.3057	0.94
4 (Member type 50)	2x2	0.3086	0.3693	19.67
	4x4		0.3114	0.91
	8x8		0.3059	0.87
5 (Member type 36)	2x2	0.3086	0.0077	97.50
	4x4		0.0191	93.81
	8x8		0.0378	87.75
	16x16		0.0806	73.88
	32x32		0.1673	45.79
	64x64		0.2532	17.95
	128x128		0.2936	4.86
6 (Member type 37)	2x2	0.3086	0.4783	54.99
	4x4		0.3142	1.81
	8x8		0.3105	0.62
	16x16		0.3102	0.52
	32x32		0.3104	0.58
	64x64		0.3103	0.55
	128x128		0.3104	0.58

Free Hemispherical Shell with a Circular Pole Hole Subjected to Two Orthogonal Pairs of Mutually Balanced Radial Tensile and Compressive Forces Applied at the Equator



Objective: Check of the obtained values of the transverse displacements of a free hemispherical shell with a circular pole hole in the direction of action of two orthogonal pairs of mutually balanced radial tensile and compressive forces applied at the equator.

Initial data files:

File name	Description
Quadrant_of_a_spherical_shell _Shell_42_Mesh_2x2.SPR Quadrant_of_a_spherical_shell _Shell_42_Mesh_4x4.SPR Quadrant_of_a_spherical_shell _Shell_42_Mesh_8x8.SPR Quadrant_of_a_spherical_shell _Shell_42_Mesh_16x16.SPR Quadrant_of_a_spherical_shell _Shell_42_Mesh_32x32.SPR	Design model with the elements of type 42 for meshes 2x2, 4x4, 8x8, 16x16, 32x32
Quadrant_of_a_spherical_shell _Shell_44_Mesh_2x2.SPR Quadrant_of_a_spherical_shell _Shell_44_Mesh_4x4.SPR Quadrant_of_a_spherical_shell _Shell_44_Mesh_8x8.SPR Quadrant_of_a_spherical_shell _Shell_44_Mesh_16x16.SPR Quadrant_of_a_spherical_shell _Shell_44_Mesh_32x32.SPR	Design model with the elements of type 44 for meshes 2x2, 4x4, 8x8, 16x16, 32x32
Quadrant_of_a_spherical_shell _Shell_45_Mesh_2x2.SPR Quadrant_of_a_spherical_shell _Shell_45_Mesh_4x4.SPR Quadrant_of_a_spherical_shell _Shell_45_Mesh_8x8.SPR Quadrant_of_a_spherical_shell _Shell_45_Mesh_16x16.SPR Quadrant_of_a_spherical_shell _Shell_45_Mesh_32x32.SPR	Design model with the elements of type 45 for meshes 2x2, 4x4, 8x8, 16x16, 32x32
Quadrant_of_a_spherical_shell _Shell_50_Mesh_2x2.SPR Quadrant_of_a_spherical_shell _Shell_50_Mesh_4x4.SPR Quadrant_of_a_spherical_shell _Shell_50_Mesh_8x8.SPR Quadrant_of_a_spherical_shell _Shell_50_Mesh_16x16.SPR Quadrant_of_a_spherical_shell _Shell_50_Mesh_32x32.SPR	Design model with the elements of type 50 for meshes 2x2, 4x4, 8x8, 16x16, 32x32
Quadrant_of_a_spherical_shell _ Solid _36.SPR _Mesh_2x2.SPR Quadrant_of_a_spherical_shell _ Solid _36.SPR _Mesh_4x4.SPR Quadrant_of_a_spherical_shell _ Solid _36.SPR _Mesh_8x8.SPR Quadrant_of_a_spherical_shell _ Solid _36.SPR _Mesh_16x16.SPR Quadrant_of_a_spherical_shell _ Solid _36.SPR _Mesh_32x32.SPR Quadrant_of_a_spherical_shell _ Solid _36.SPR _Mesh_64x64.SPR Quadrant_of_a_spherical_shell _ Solid _36.SPR _Mesh_128x128.SPR Quadrant_of_a_spherical_shell _ Solid _36.SPR _Mesh_256x256.SPR Quadrant_of_a_spherical_shell _ Solid _36.SPR _Mesh_512x512.SPR	Design model with the elements of type 36 for meshes 2x2, 4x4, 8x8, 16x16, 32x32, 64x64, 128x128, 256x256, 512x512

File name	Description
Quadrant_of_a_spherical_shell _ Solid _37.SPR _Mesh_2x2.SPR Quadrant_of_a_spherical_shell _ Solid _37.SPR _Mesh_4x4.SPR Quadrant_of_a_spherical_shell _ Solid _37.SPR _Mesh_8x8.SPR Quadrant_of_a_spherical_shell _ Solid _37.SPR _Mesh_16x16.SPR Quadrant_of_a_spherical_shell _ Solid _37.SPR _Mesh_32x32.SPR Quadrant_of_a_spherical_shell _ Solid _37.SPR _Mesh_64x64.SPR Quadrant_of_a_spherical_shell _ Solid _37.SPR _Mesh_128x128.SPR	Design model with the elements of type 37 for meshes 2x2, 4x4, 8x8, 16x16, 32x32, 64x64, 128x128

Problem formulation: The free hemispherical shell with a circular pole hole is subjected to two orthogonal pairs of mutually balanced radial tensile and compressive forces F applied at the equator. Check the obtained values of the transverse displacements of the free hemispherical shell w_{FX} and w_{FY} in the direction of the action of forces applied at the equator.

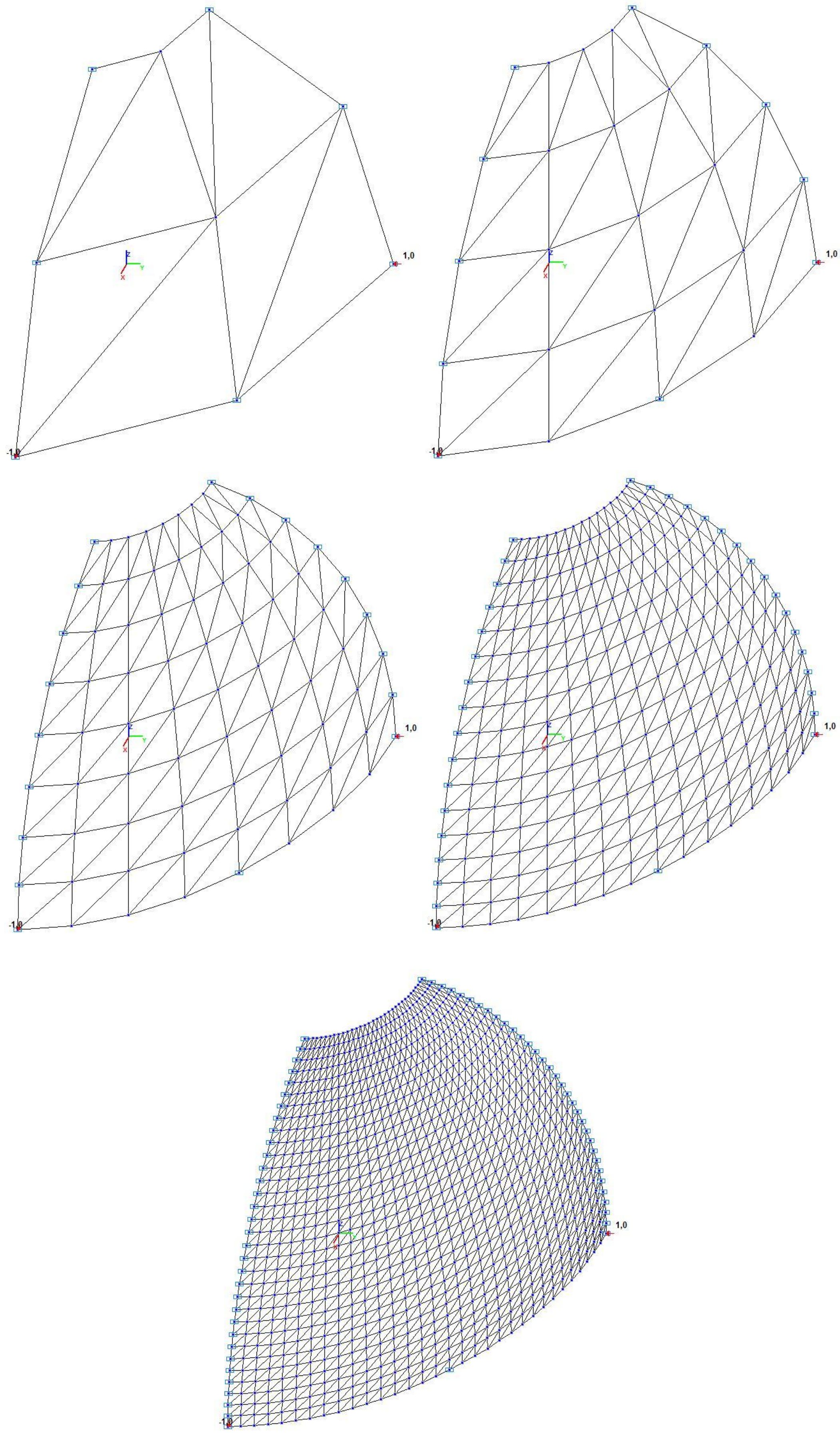
References: R. H. Macneal, R. L. Harder, A proposed standard set of problems to test finite element accuracy, North-Holland, Finite elements in analysis and design, 1, 1985, p. 3-20.
L. S. D. Morley, A. J. Morris, Conflict between finite elements and shell theory, London, Royal aircraft establishment report, 1978.

Initial data:

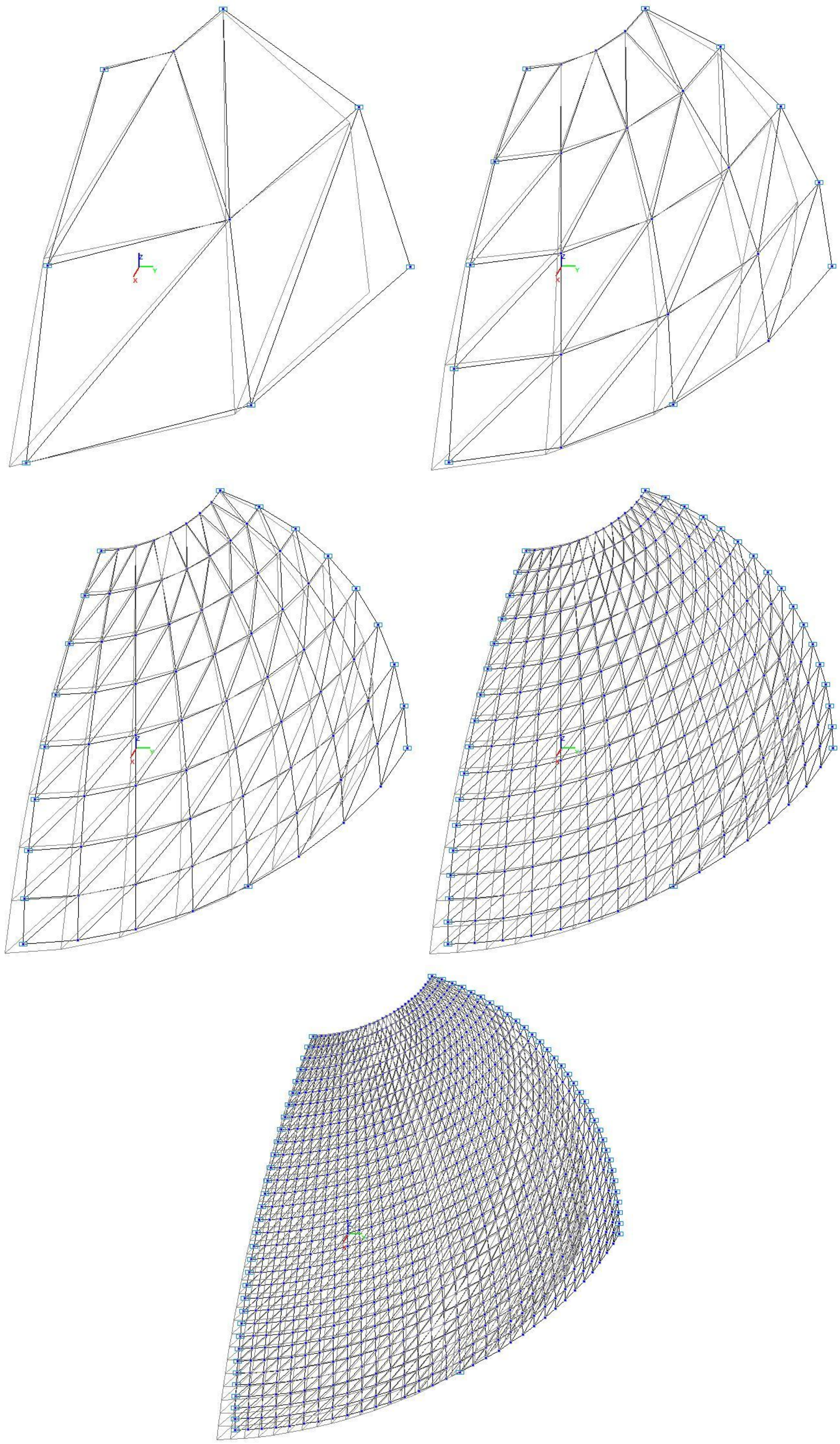
$E = 6.825 \cdot 10^7$ kPa	- elastic modulus of the material of the hemispherical shell;
$\nu = 0.30$	- Poisson's ratio;
R = 10.00 m	- radius of the midsurface of the hemispherical shell;
$2 \cdot \theta = 2 \cdot 18°$	- central angle of the surface of the circular hole of the hemispherical shell;
h = 0.04 m	- thickness of the hemispherical shell;
$F_X = + 2.0$ kN	- values of the concentrated radial tensile forces applied at the equator of the hemispherical shell;
$F_Y = – 2.0$ kN	- values of the concentrated radial compressive forces applied at the equator of the hemispherical shell.

Finite element model: Design model – general type system. Six design models of a quarter of the hemispherical shell according to the symmetry conditions are considered:
Model 1 – 8, 32, 128, 512, 2048 three-node shell elements of type 42 with a regular mesh 2x2, 4x4, 8x8, 16x16, 32x32. Boundary conditions and the dimensional stability are provided by imposing constraints according to the symmetry conditions. Number of nodes in the model – 9, 25, 81, 289, 1089.
Model 2 – 4, 16, 64, 256, 1024 four-node shell elements of type 44 with a regular mesh 2x2, 4x4, 8x8, 16x16, 32x32. Boundary conditions and the dimensional stability are provided by imposing constraints according to the symmetry conditions. Number of nodes in the model – 9, 25, 81, 289,1089.
Model 3 – 8, 32, 128, 512, 2048 six-node shell elements of type 45 with a regular mesh 2x2, 4x4, 8x8, 16x16, 32x32. Boundary conditions and the dimensional stability are provided by imposing constraints according to the symmetry conditions. Number of nodes in the model – 25, 81, 289, 1089, 4225.
Model 4 – 4, 16, 64, 256, 1024 eight- node shell elements of type 50 with a regular mesh 2x2, 4x4, 8x8, 16x16, 32x32. Boundary conditions and the dimensional stability are provided by imposing constraints according to the symmetry conditions. Number of nodes in the model – 21, 65, 225, 833, 3201.
Model 5 – 4, 16, 64, 256, 1024, 4096, 16384, 65536, 262144 eight-node isoparametric solid elements of type 36 with a regular mesh 2x2x1, 4x4x1, 8x8x1, 16x16x1, 32x32x1, 64x64x1, 128x128x1, 256x256x1, 512x512x1. Boundary conditions and the dimensional stability are provided by imposing constraints according to the symmetry conditions. Number of nodes in the model – 18, 50, 162, 578, 2178, 8450, 33282, 132149, 526338.
Model 6 – 4, 16, 64, 256, 1024, 4096, 16384 twenty-node isoparametric solid elements of type 37 with a regular mesh 2x2x1, 4x4x1, 8x8x1, 16x16x1, 32x32x1, 64x64x1, 128x128x1. Boundary conditions and the dimensional stability are provided by imposing constraints according to the symmetry conditions. Number of nodes in the model – 51, 155, 531, 1955, 7491, 29315, 115971.

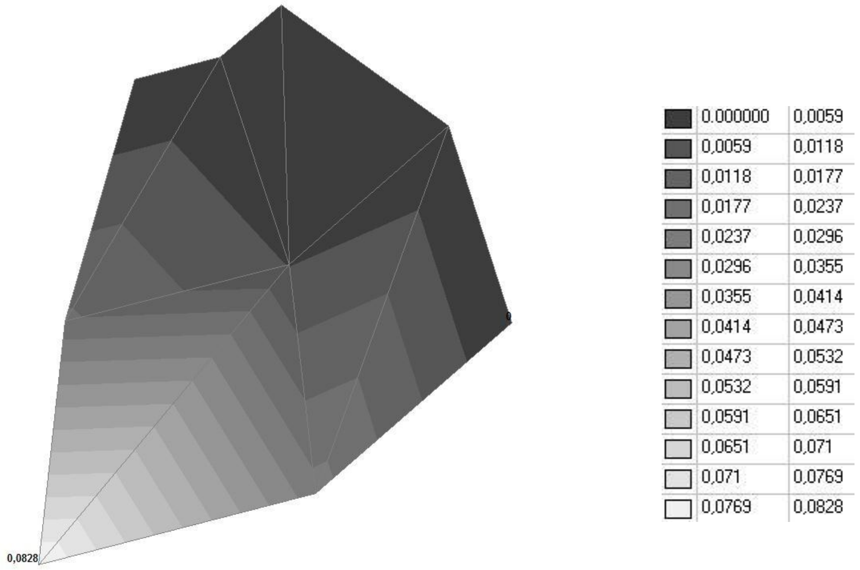
Results in SCAD

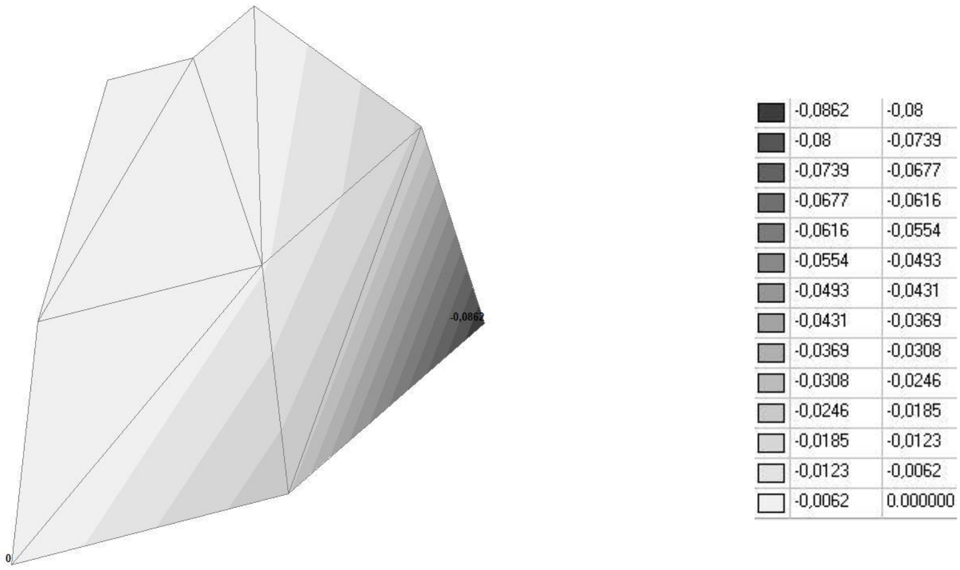


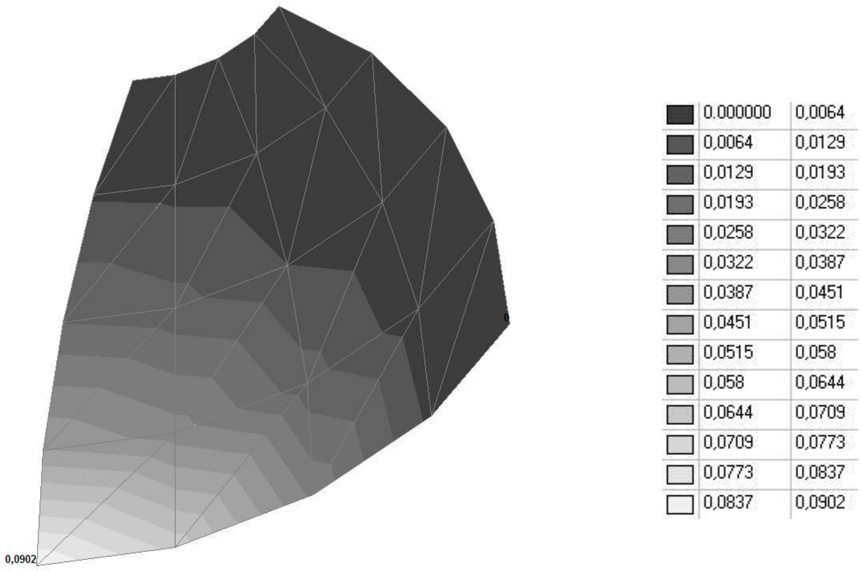
Model 1. Design model

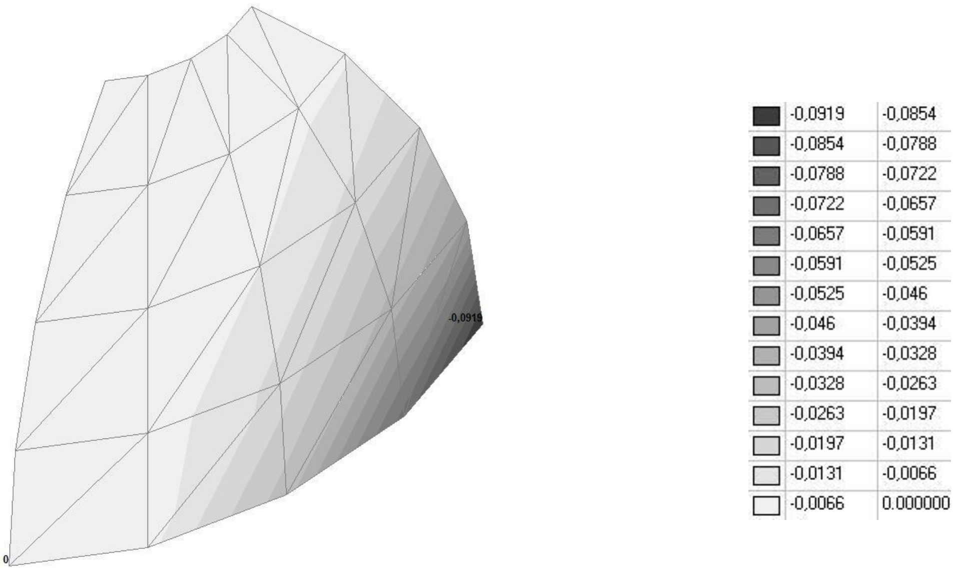


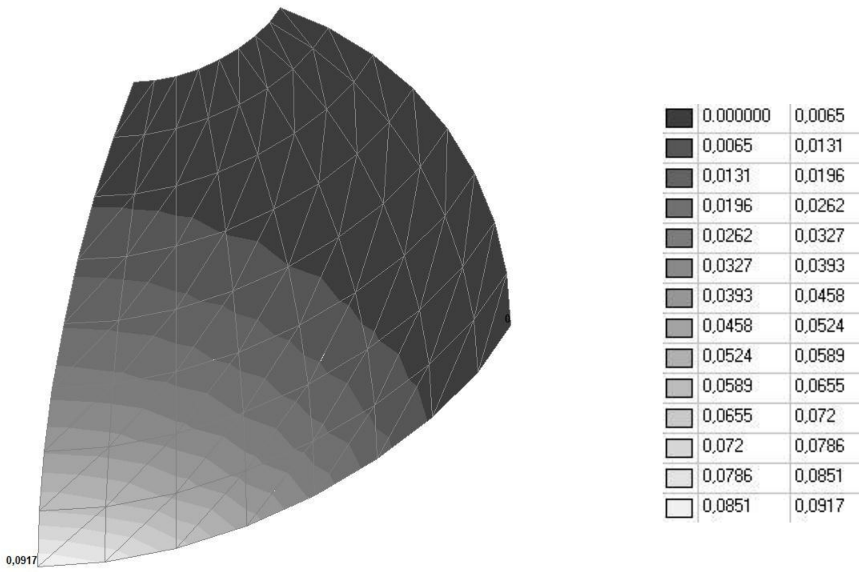
Model 1. Deformed model

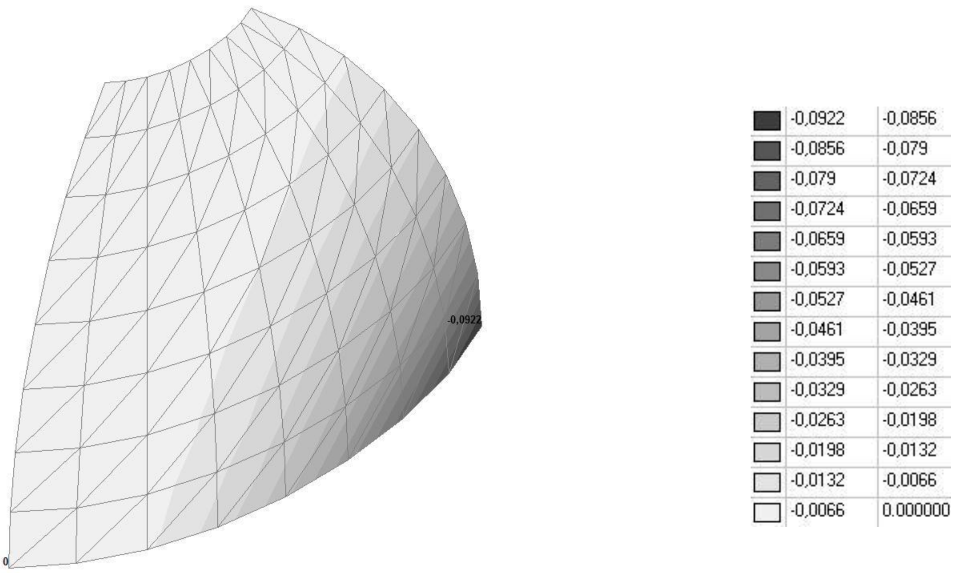

0,0828
0
0.000000 0,0059
0,0059 0,0118
0,0118 0,0177
0,0177 0,0237
0,0237 0,0296
0,0296 0,0355
0,0355 0,0414
0,0414 0,0473
0,0473 0,0532
0,0532 0,0591
0,0591 0,0651
0,0651 0,071
0,071 0,0769
0,0769 0,0828

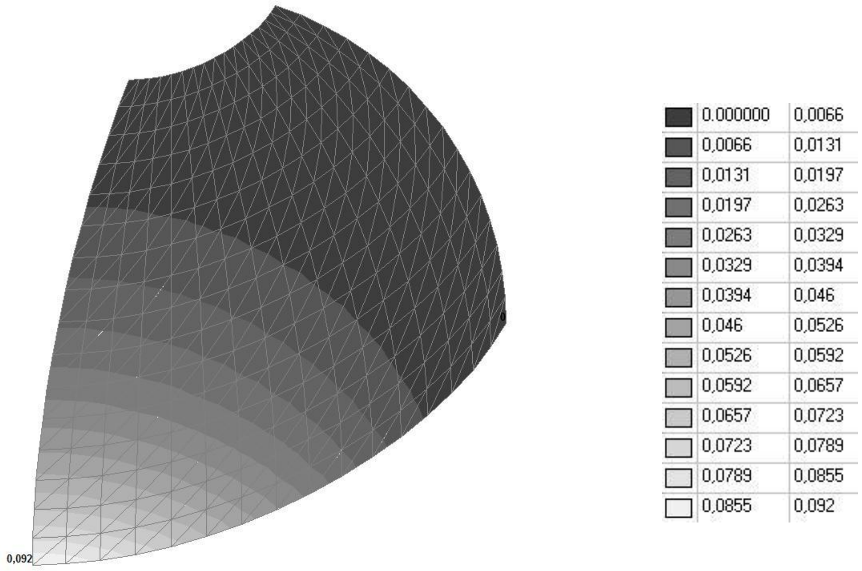

0
-0,0862
-0,0862 -0,08
-0,08 -0,0739
-0,0739 -0,0677
-0,0677 -0,0616
-0,0616 -0,0554
-0,0554 -0,0493
-0,0493 -0,0431
-0,0431 -0,0369
-0,0369 -0,0308
-0,0308 -0,0246
-0,0246 -0,0185
-0,0185 -0,0123
-0,0123 -0,0062
-0,0062 0.000000

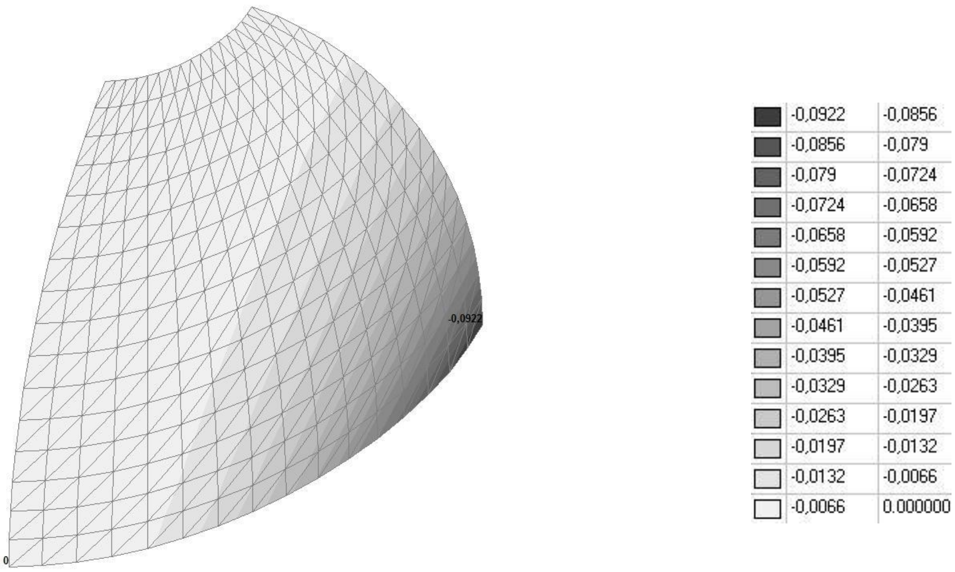

0,0902
0
0.000000 0,0064
0,0064 0,0129
0,0129 0,0193
0,0193 0,0258
0,0258 0,0322
0,0322 0,0387
0,0387 0,0451
0,0451 0,0515
0,0515 0,058
0,058 0,0644
0,0644 0,0709
0,0709 0,0773
0,0773 0,0837
0,0837 0,0902

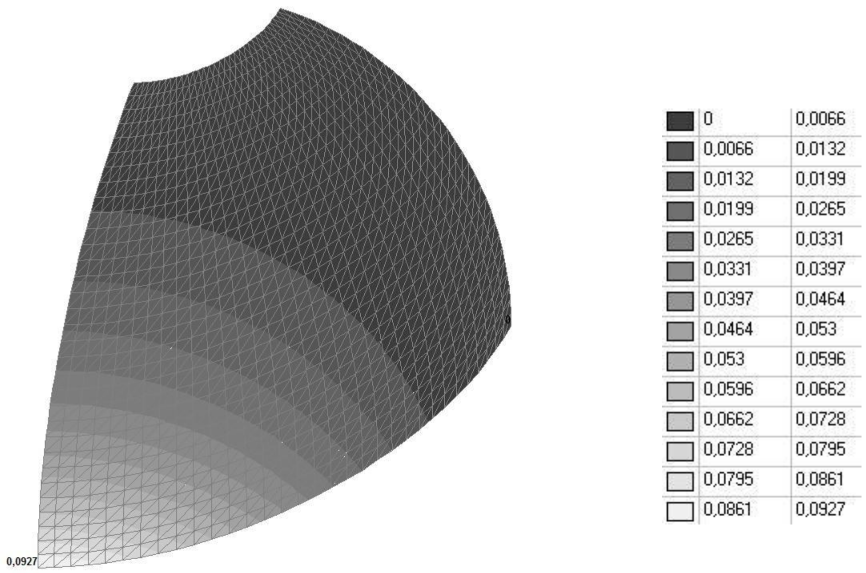

-0,0919
0
-0,0919 -0,0854
-0,0854 -0,0788
-0,0788 -0,0722
-0,0722 -0,0657
-0,0657 -0,0591
-0,0591 -0,0525
-0,0525 -0,046
-0,046 -0,0394
-0,0394 -0,0328
-0,0328 -0,0263
-0,0263 -0,0197
-0,0197 -0,0131
-0,0131 -0,0066
-0,0066 0.000000

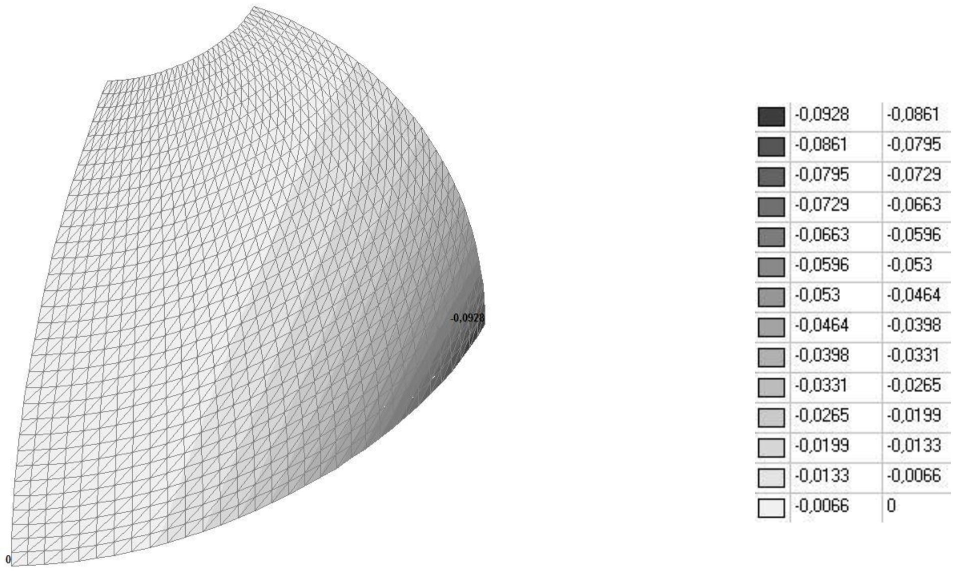

0,0917
0
0.000000 0,0065
0,0065 0,0131
0,0131 0,0196
0,0196 0,0262
0,0262 0,0327
0,0327 0,0393
0,0393 0,0458
0,0458 0,0524
0,0524 0,0589
0,0589 0,0655
0,0655 0,072
0,072 0,0786
0,0786 0,0851
0,0851 0,0917


-0,0922
0
-0,0922 -0,0856
-0,0856 -0,079
-0,079 -0,0724
-0,0724 -0,0659
-0,0659 -0,0593
-0,0593 -0,0527
-0,0527 -0,0461
-0,0461 -0,0395
-0,0395 -0,0329
-0,0329 -0,0263
-0,0263 -0,0198
-0,0198 -0,0132
-0,0132 -0,0066
-0,0066 0.000000

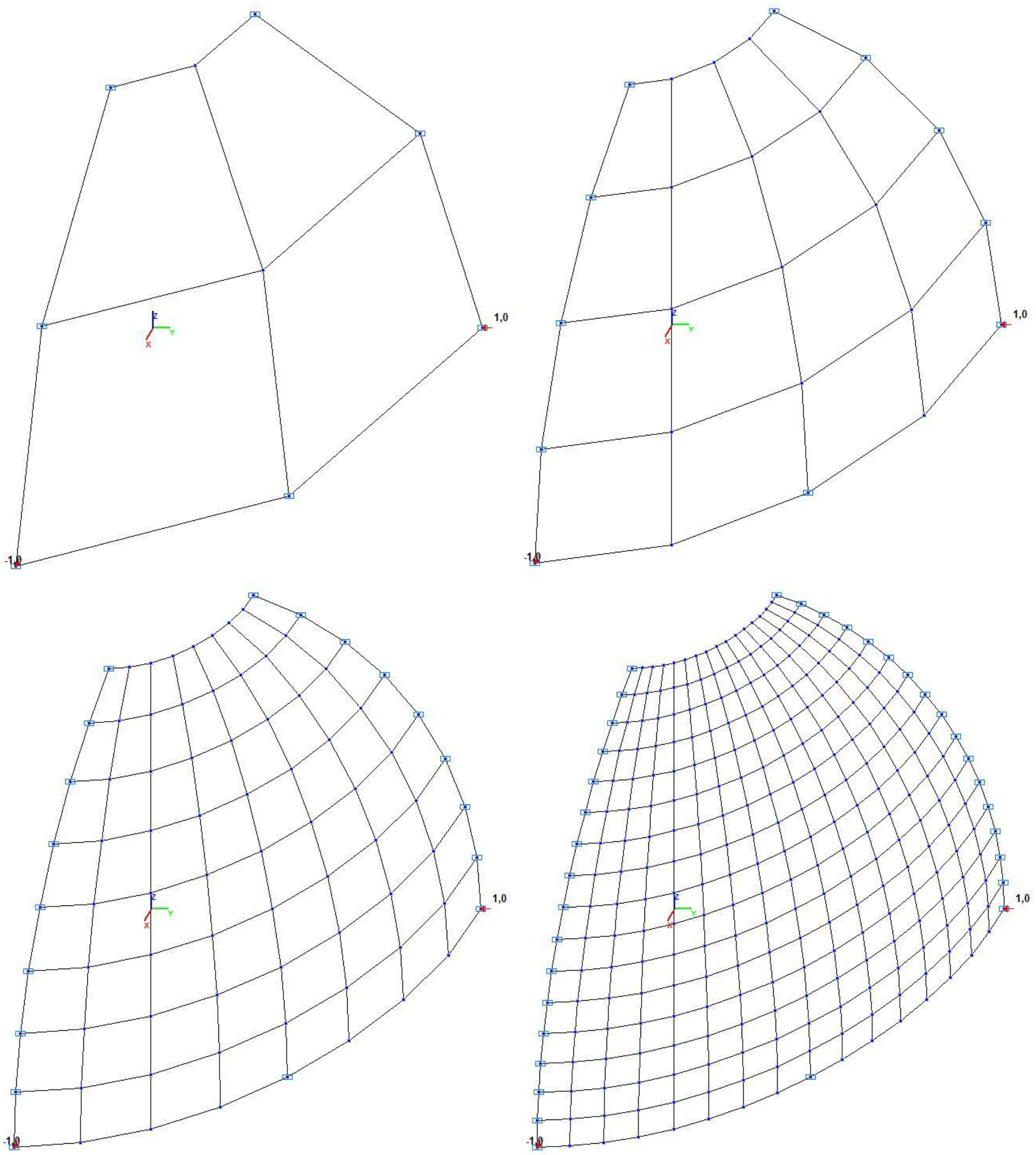

0,092
0.000000 0,0066
0,0066 0,0131
0,0131 0,0197
0,0197 0,0263
0,0263 0,0329
0,0329 0,0394
0,0394 0,046
0,046 0,0526
0,0526 0,0592
0,0592 0,0657
0,0657 0,0723
0,0723 0,0789
0,0789 0,0855
0,0855 0,092

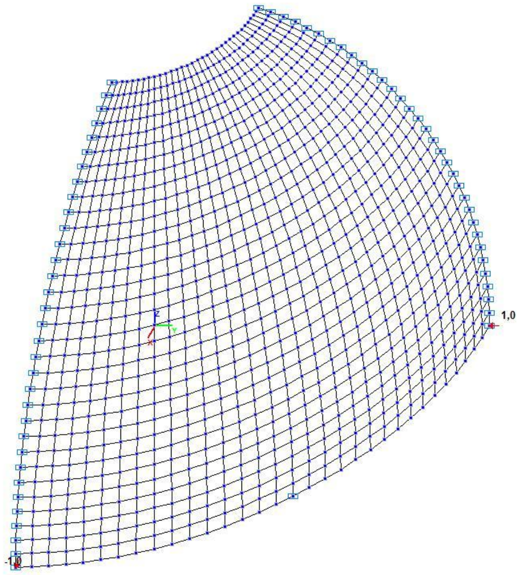

-0,0922
0
-0,0922 -0,0856
-0,0856 -0,079
-0,079 -0,0724
-0,0724 -0,0658
-0,0658 -0,0592
-0,0592 -0,0527
-0,0527 -0,0461
-0,0461 -0,0395
-0,0395 -0,0329
-0,0329 -0,0263
-0,0263 -0,0197
-0,0197 -0,0132
-0,0132 -0,0066
-0,0066 0.000000


0,0927
0 0,0066
0,0066 0,0132
0,0132 0,0199
0,0199 0,0265
0,0265 0,0331
0,0331 0,0397
0,0397 0,0464
0,0464 0,053
0,053 0,0596
0,0596 0,0662
0,0662 0,0728
0,0728 0,0795
0,0795 0,0861
0,0861 0,0927

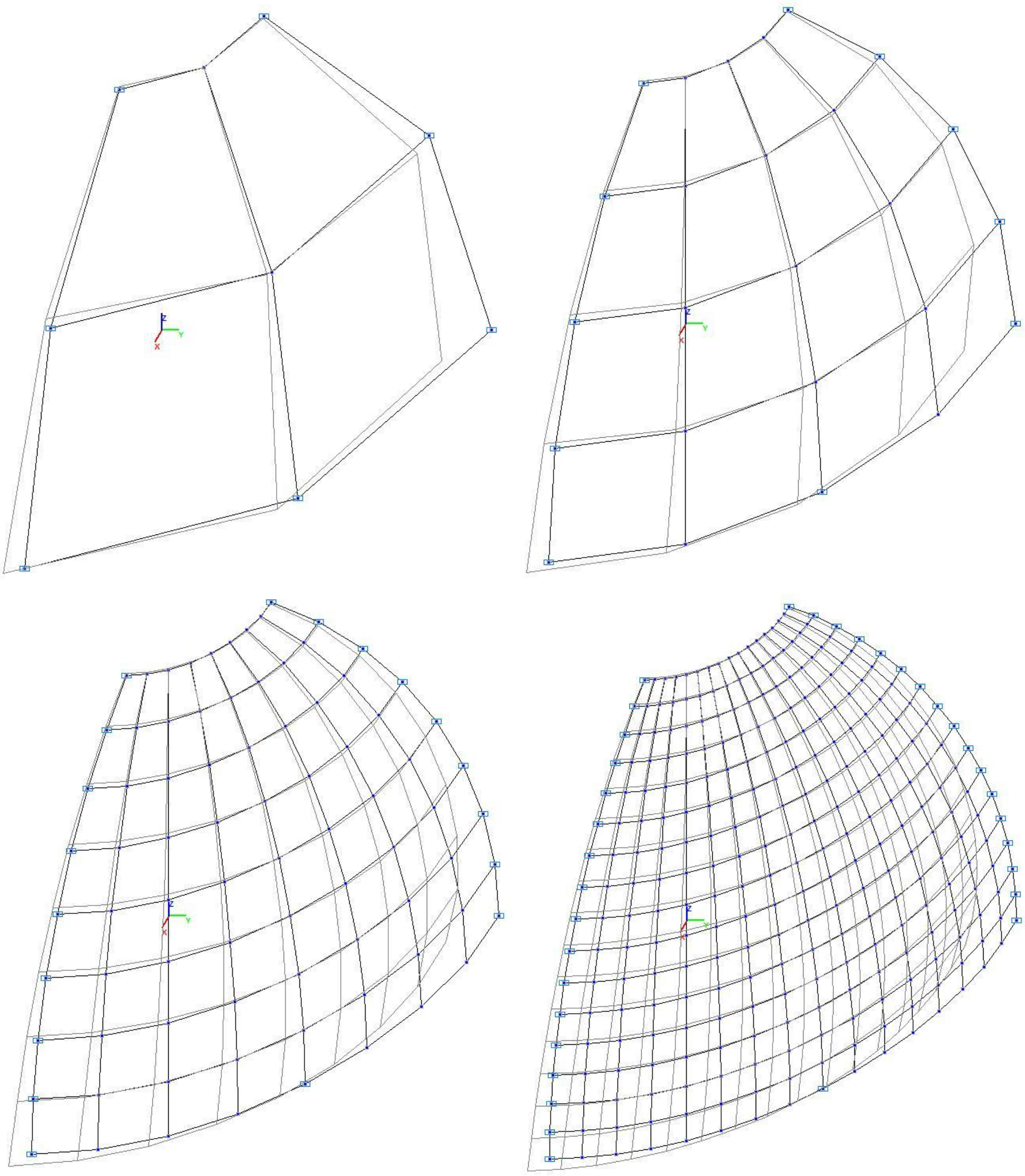


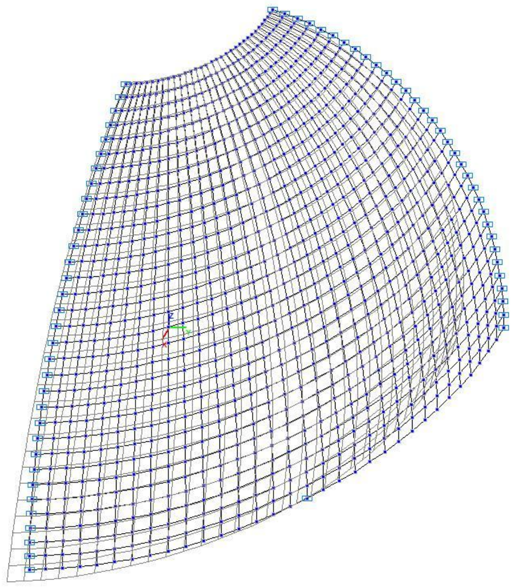
Model 1. Values of the displacements in the direction of the pairs of tensile forces and the pairs of compressive forces along the X and Y axes of the global coordinate system respectively w_{FX} and w_{FY} (m, m)



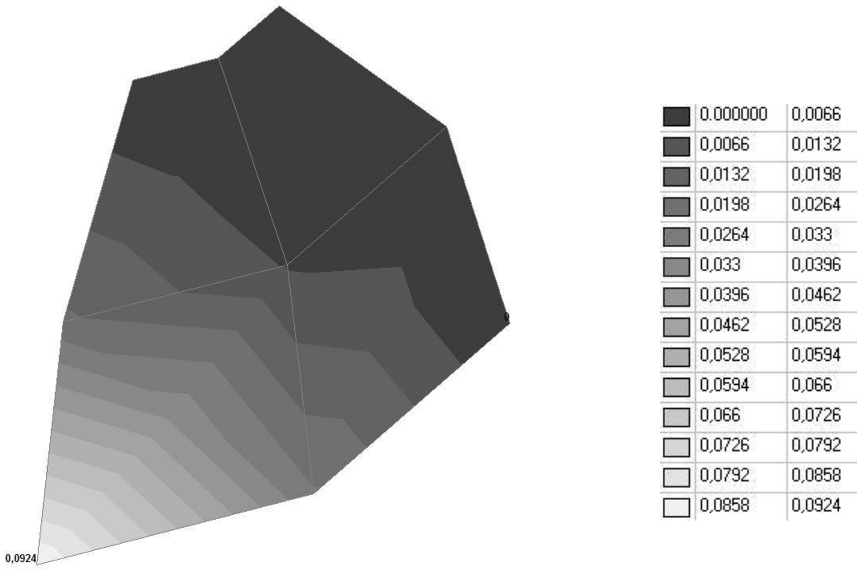


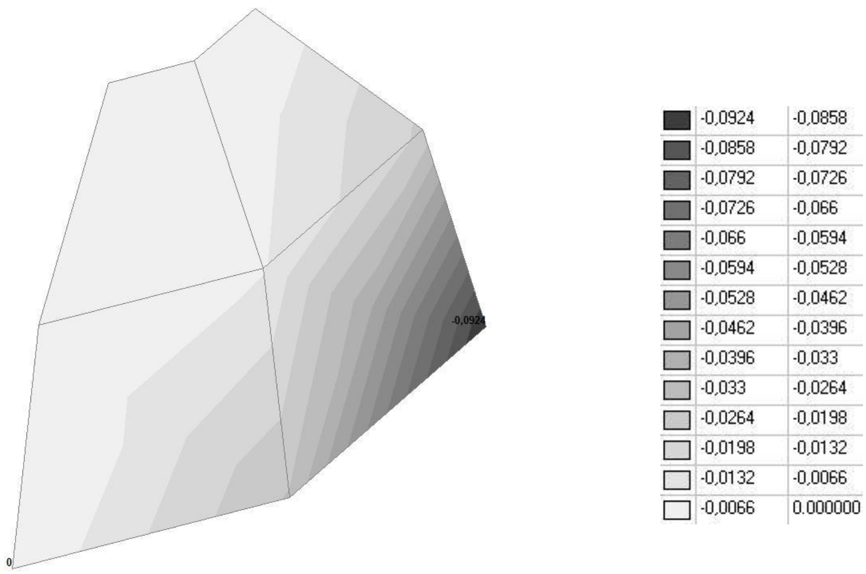
Model 2. Design model

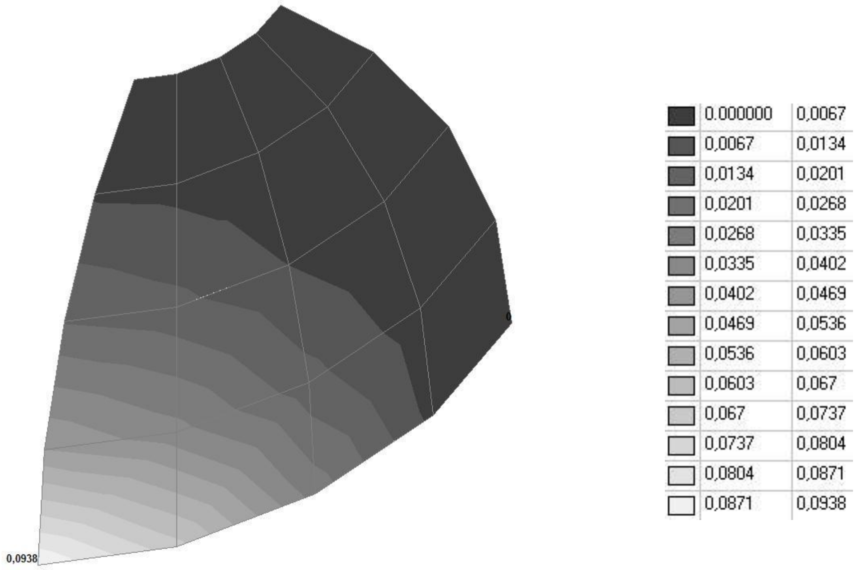


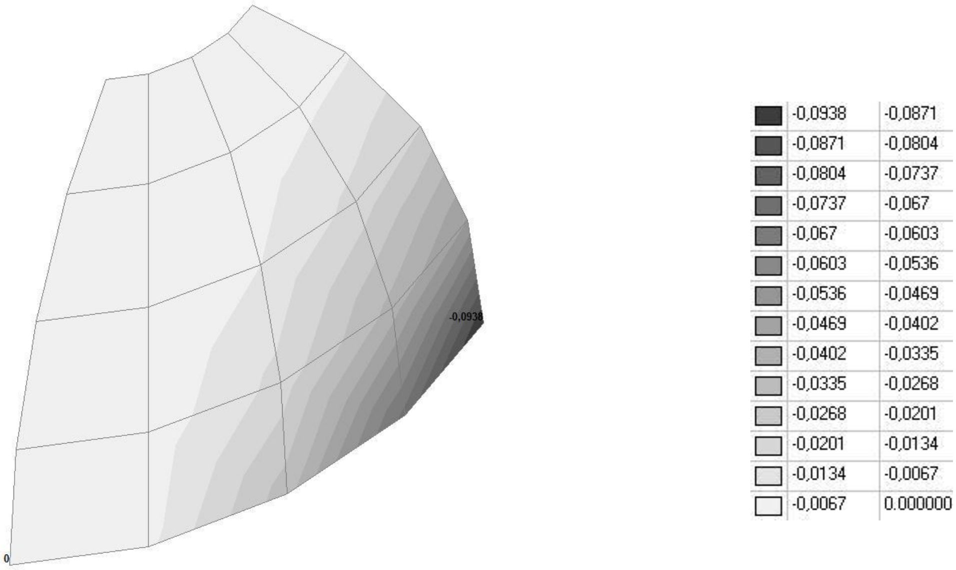


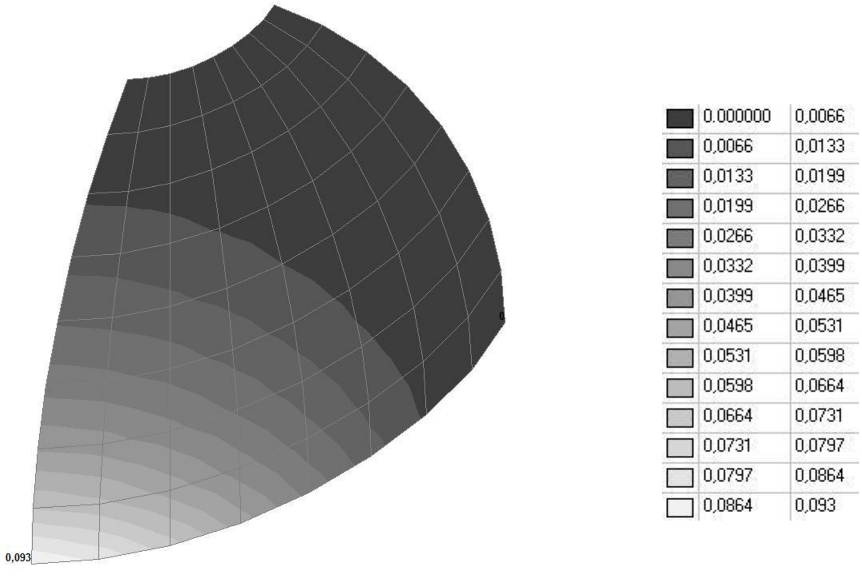
Model 2. Deformed model

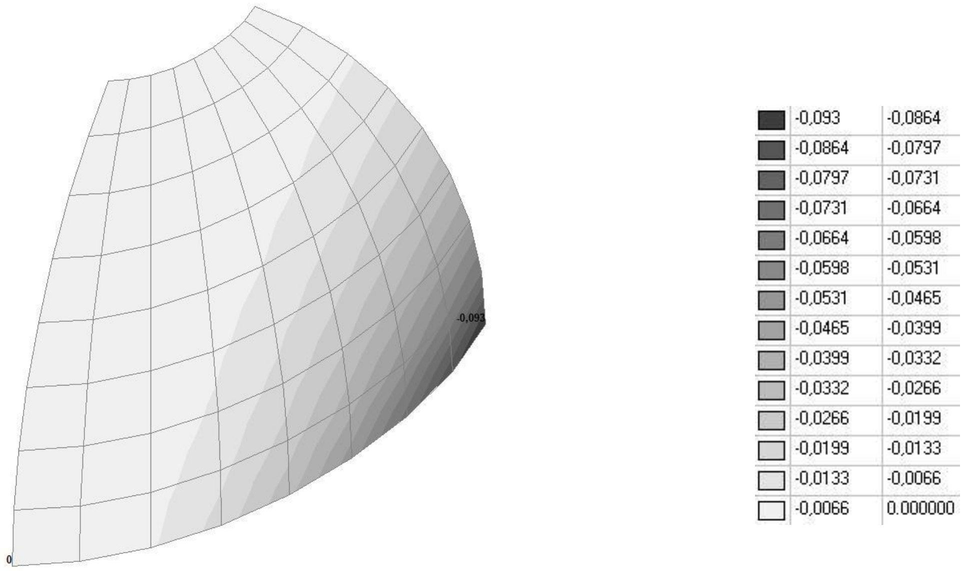


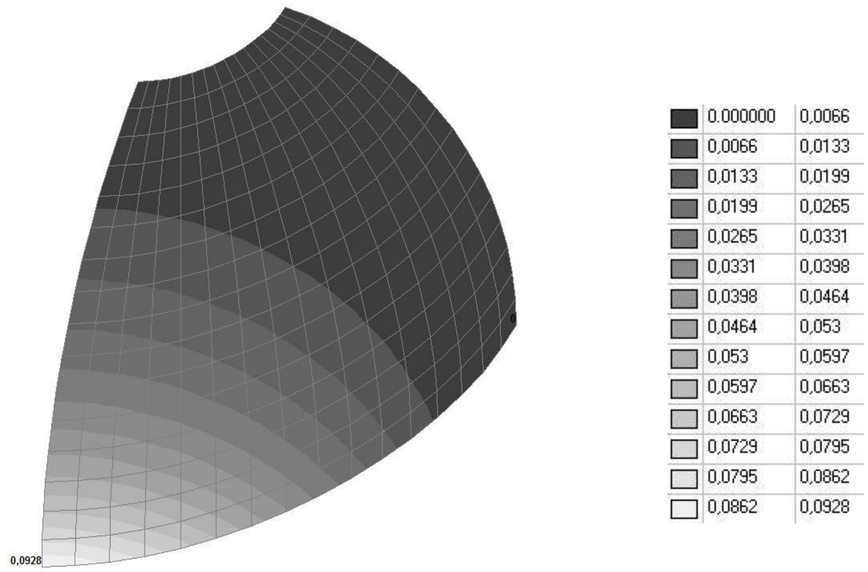


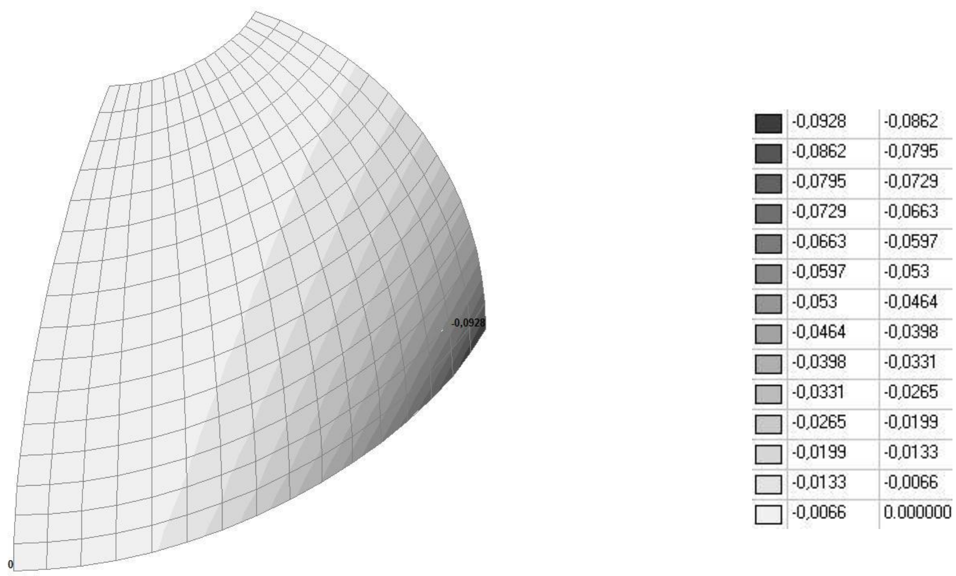

0.000000 0,0067
0,0067 0,0134
0,0134 0,0201
0,0201 0,0268
0,0268 0,0335
0,0335 0,0402
0,0402 0,0469
0,0469 0,0536
0,0536 0,0603
0,0603 0,067
0,067 0,0737
0,0737 0,0804
0,0804 0,0871
0,0871 0,0938
0,0938

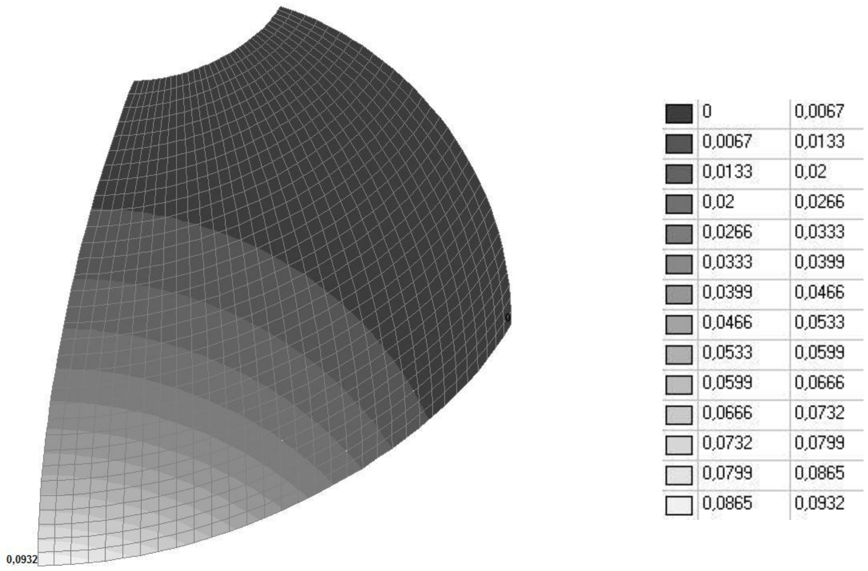

-0,0938 -0,0871
-0,0871 -0,0804
-0,0804 -0,0737
-0,0737 -0,067
-0,067 -0,0603
-0,0603 -0,0536
-0,0536 -0,0469
-0,0469 -0,0402
-0,0402 -0,0335
-0,0335 -0,0268
-0,0268 -0,0201
-0,0201 -0,0134
-0,0134 -0,0067
-0,0067 0.000000
-0,0938
0

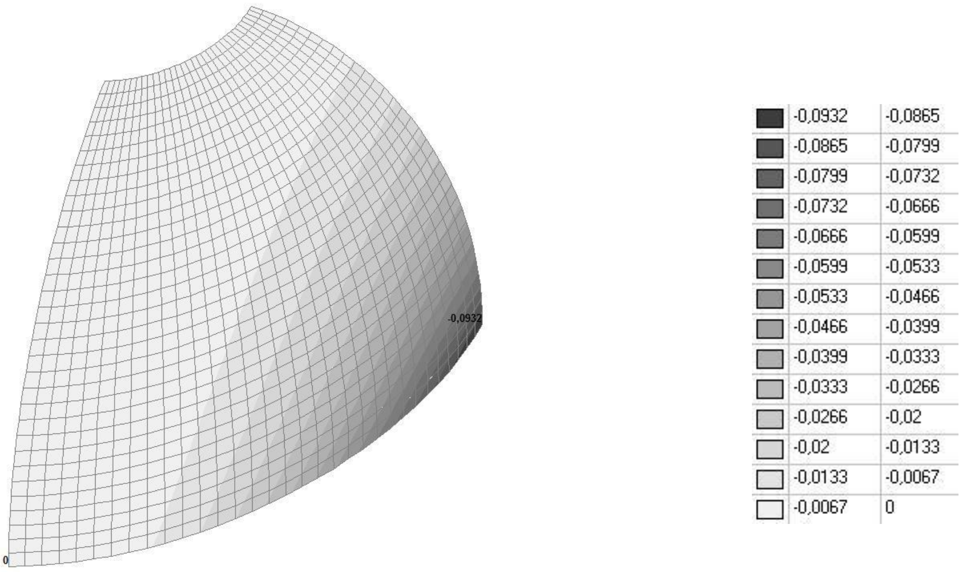

0.000000 0,0066
0,0066 0,0133
0,0133 0,0199
0,0199 0,0266
0,0266 0,0332
0,0332 0,0399
0,0399 0,0465
0,0465 0,0531
0,0531 0,0598
0,0598 0,0664
0,0664 0,0731
0,0731 0,0797
0,0797 0,0864
0,0864 0,093
0,093


-0,093
0
-0,093 -0,0864
-0,0864 -0,0797
-0,0797 -0,0731
-0,0731 -0,0664
-0,0664 -0,0598
-0,0598 -0,0531
-0,0531 -0,0465
-0,0465 -0,0399
-0,0399 -0,0332
-0,0332 -0,0266
-0,0266 -0,0199
-0,0199 -0,0133
-0,0133 -0,0066
-0,0066 0.000000

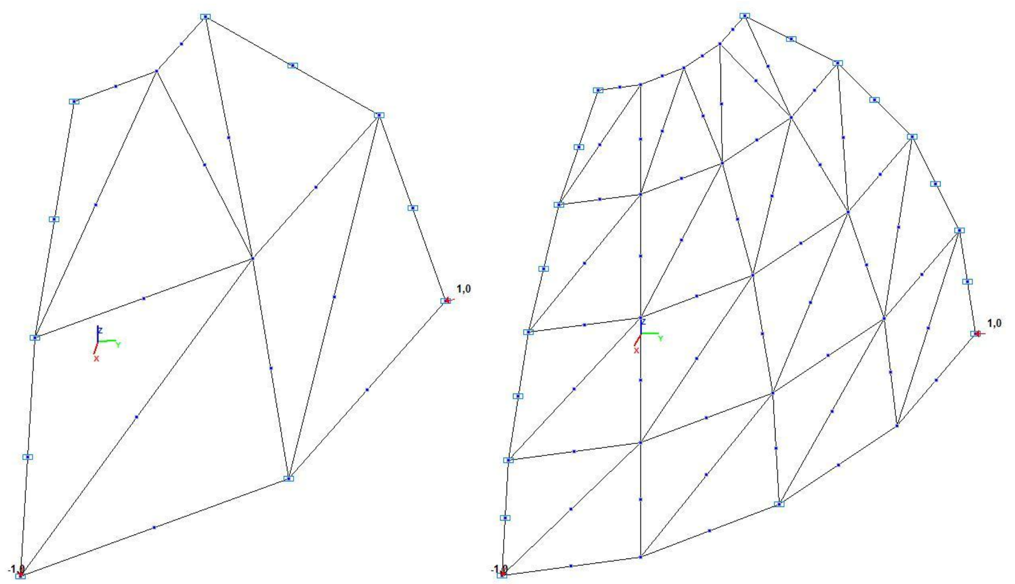

0,0928
0.000000 0,0066
0,0066 0,0133
0,0133 0,0199
0,0199 0,0265
0,0265 0,0331
0,0331 0,0398
0,0398 0,0464
0,0464 0,053
0,053 0,0597
0,0597 0,0663
0,0663 0,0729
0,0729 0,0795
0,0795 0,0862
0,0862 0,0928

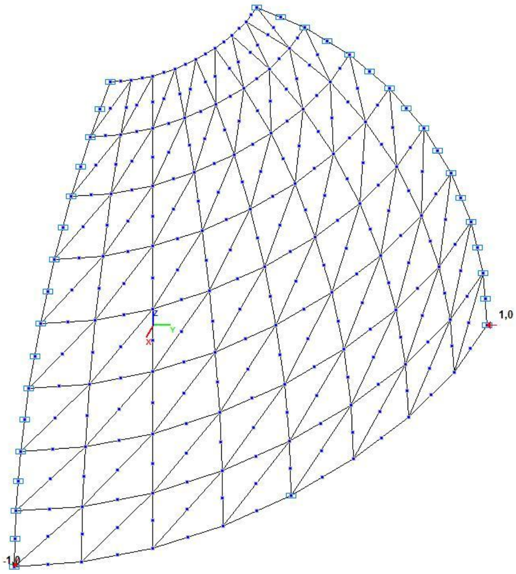

-0,0928
0
-0,0928 -0,0862
-0,0862 -0,0795
-0,0795 -0,0729
-0,0729 -0,0663
-0,0663 -0,0597
-0,0597 -0,053
-0,053 -0,0464
-0,0464 -0,0398
-0,0398 -0,0331
-0,0331 -0,0265
-0,0265 -0,0199
-0,0199 -0,0133
-0,0133 -0,0066
-0,0066 0.000000

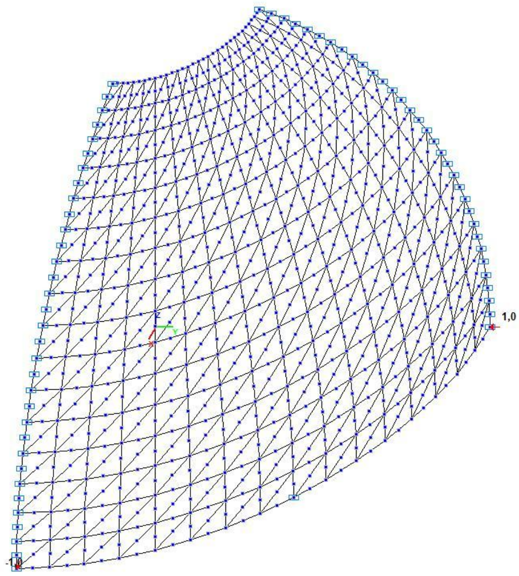


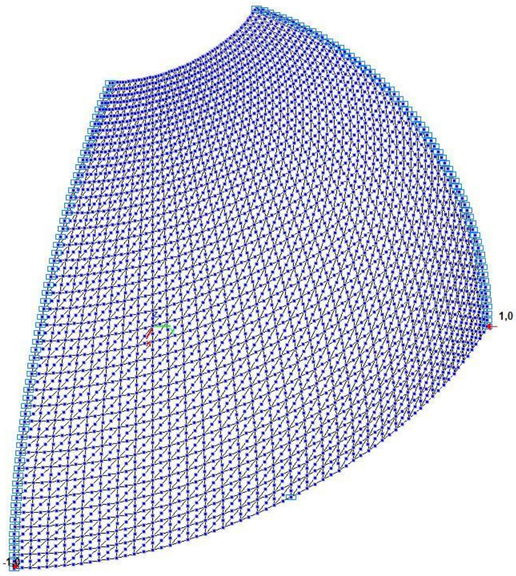


Model 2. Values of the displacements in the direction of the pairs of tensile forces and the pairs of compressive forces along the X and Y axes of the global coordinate system respectively w_{FX} and w_{FY} (m, m)

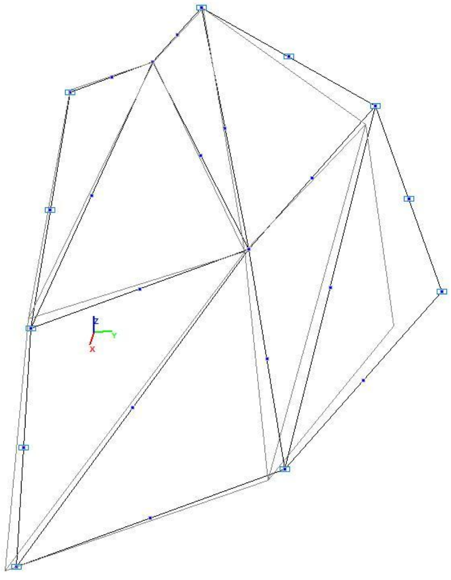


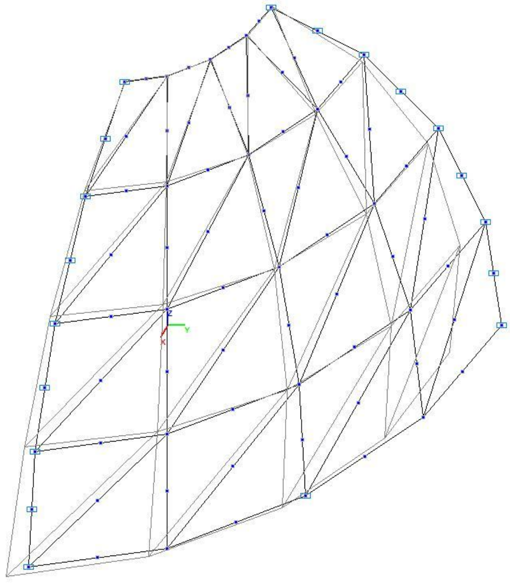


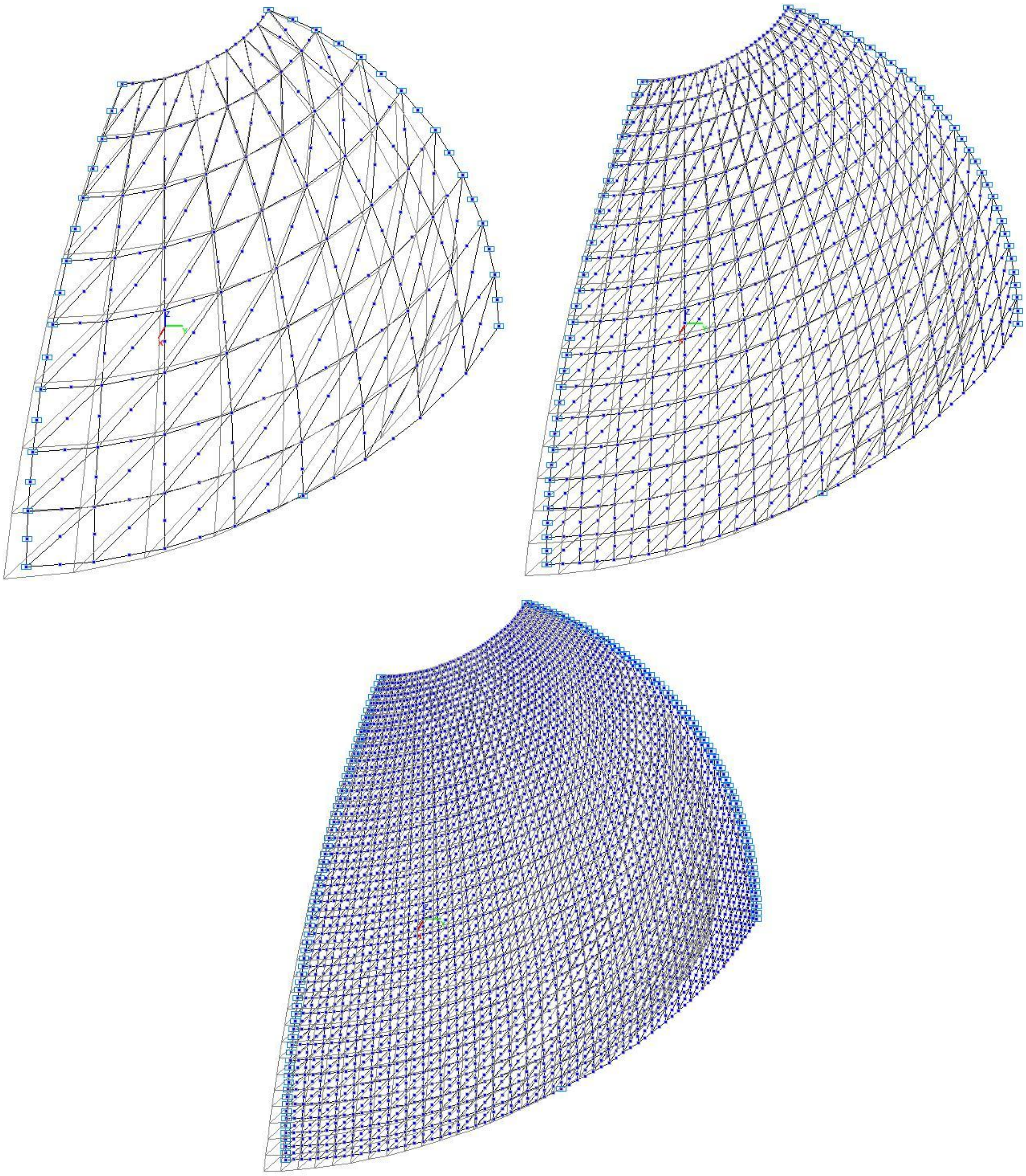




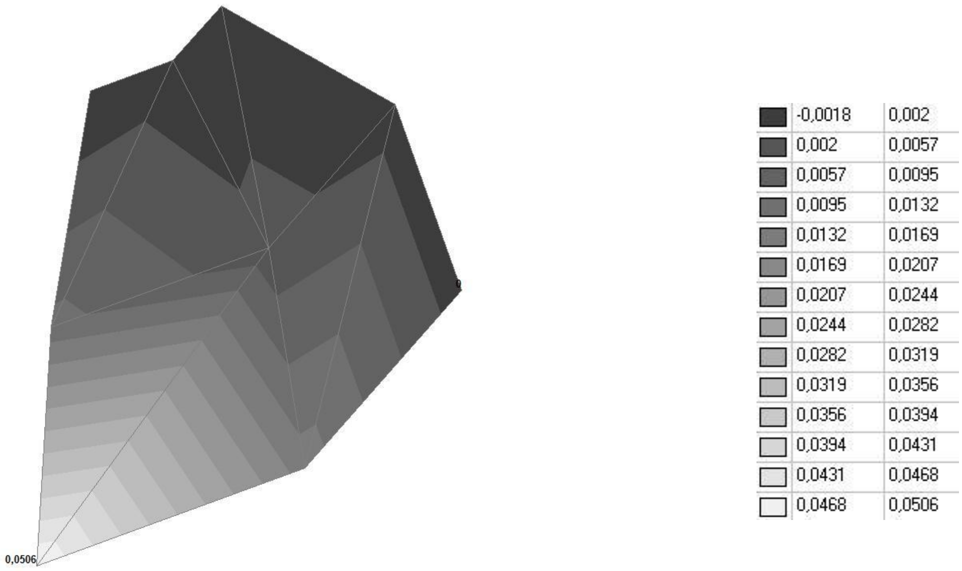
Model 3. Design model

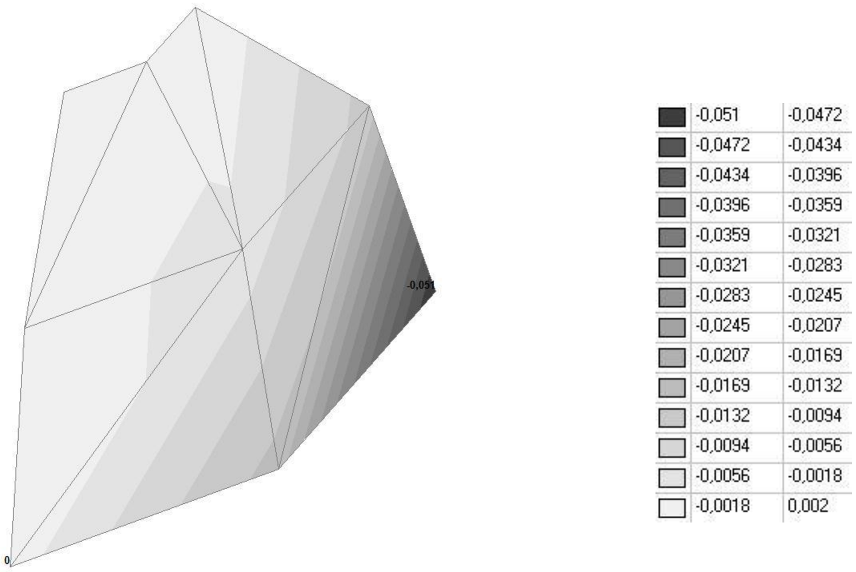


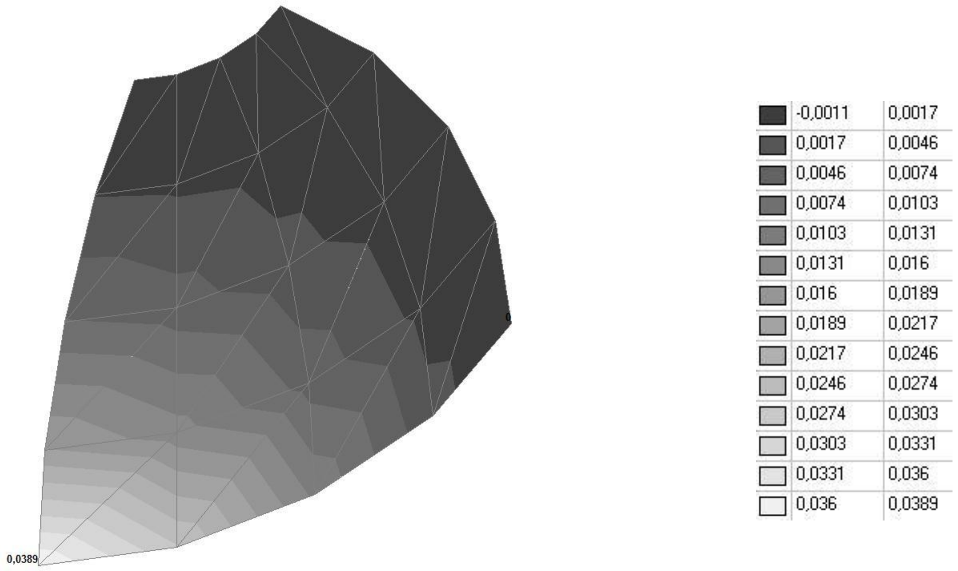


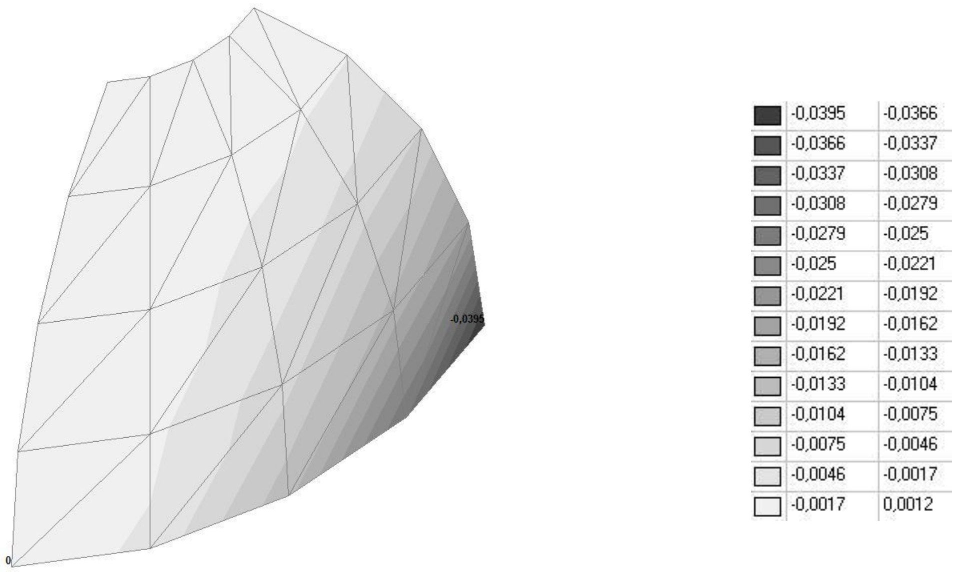


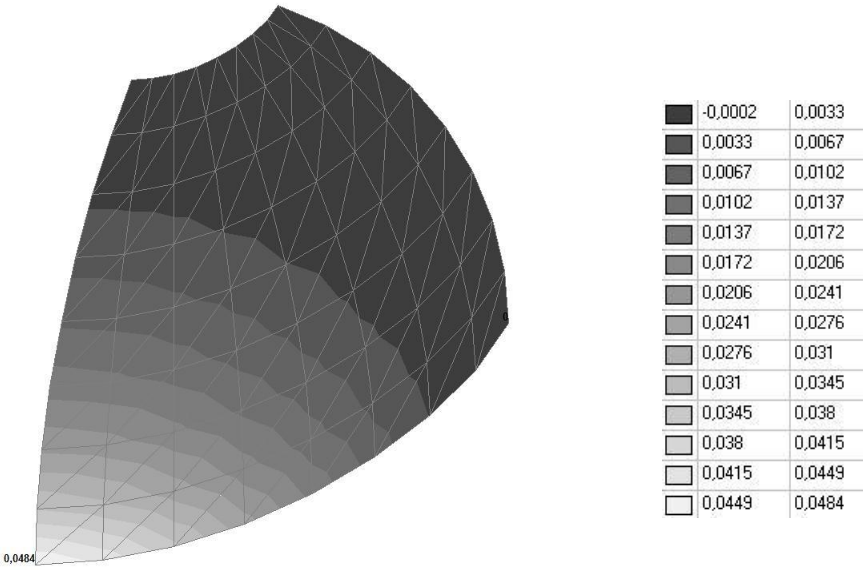
Model 3. Deformed model

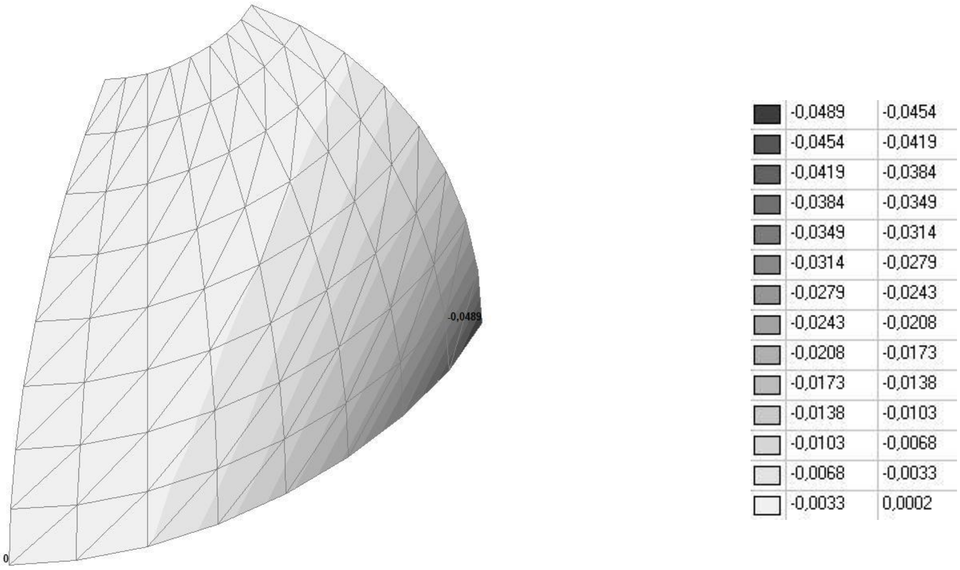


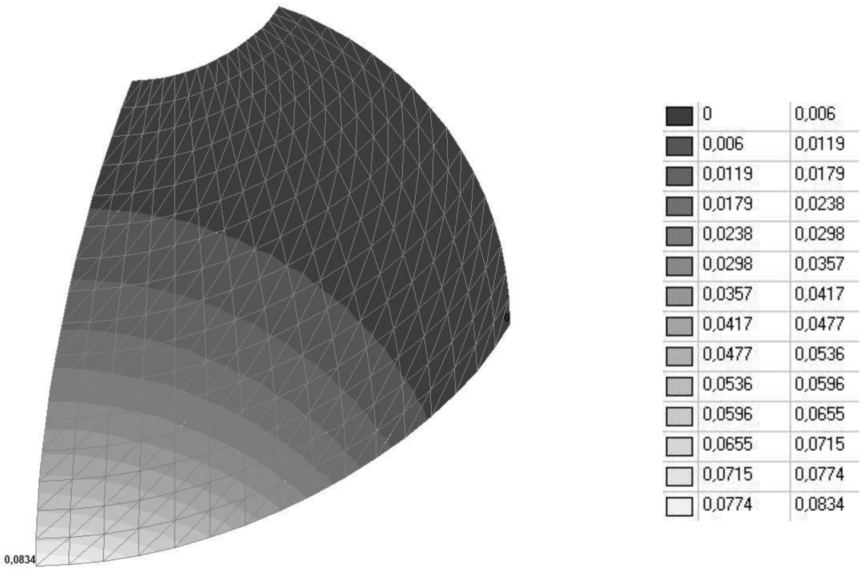

-0,051
0
-0,051 -0,0472
-0,0472 -0,0434
-0,0434 -0,0396
-0,0396 -0,0359
-0,0359 -0,0321
-0,0321 -0,0283
-0,0283 -0,0245
-0,0245 -0,0207
-0,0207 -0,0169
-0,0169 -0,0132
-0,0132 -0,0094
-0,0094 -0,0056
-0,0056 -0,0018
-0,0018 0,002

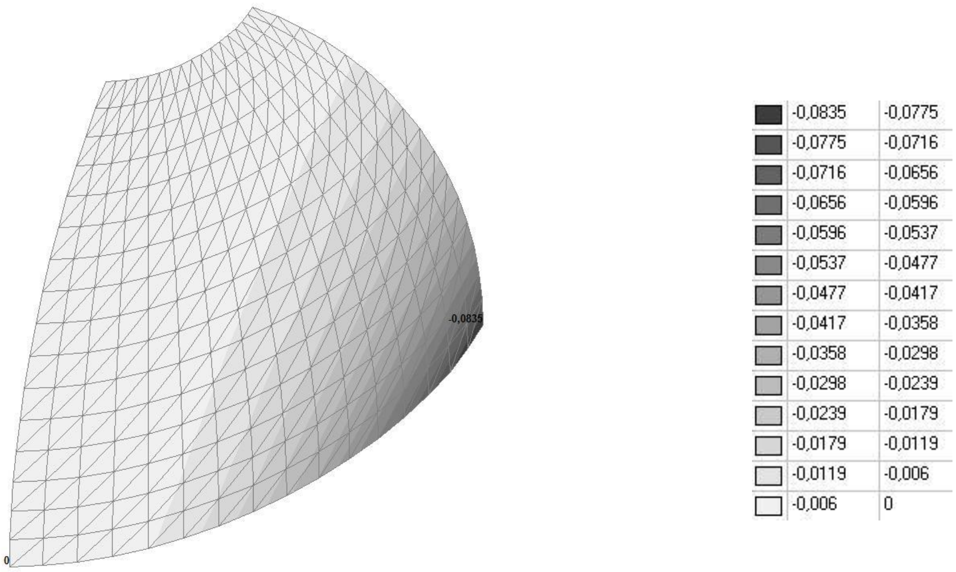

0
0,0389
-0,0011 0,0017
0,0017 0,0046
0,0046 0,0074
0,0074 0,0103
0,0103 0,0131
0,0131 0,016
0,016 0,0189
0,0189 0,0217
0,0217 0,0246
0,0246 0,0274
0,0274 0,0303
0,0303 0,0331
0,0331 0,036
0,036 0,0389

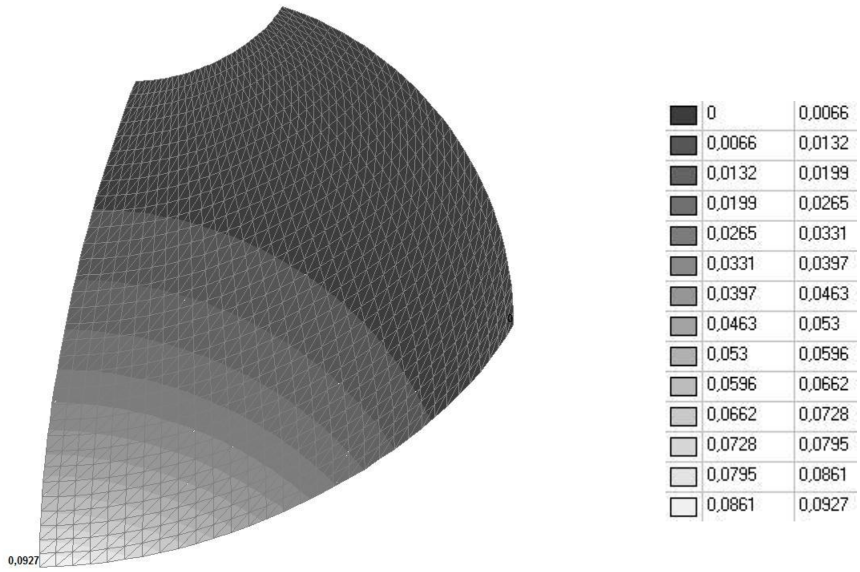

-0,0395
0
-0,0395 -0,0366
-0,0366 -0,0337
-0,0337 -0,0308
-0,0308 -0,0279
-0,0279 -0,025
-0,025 -0,0221
-0,0221 -0,0192
-0,0192 -0,0162
-0,0162 -0,0133
-0,0133 -0,0104
-0,0104 -0,0075
-0,0075 -0,0046
-0,0046 -0,0017
-0,0017 0,0012

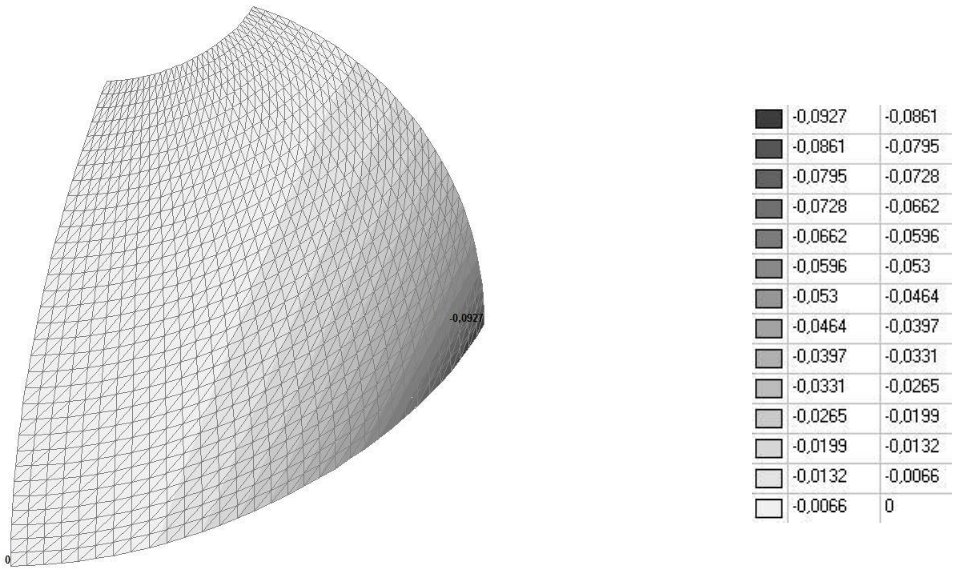

-0,0002 0,0033
0,0033 0,0067
0,0067 0,0102
0,0102 0,0137
0,0137 0,0172
0,0172 0,0206
0,0206 0,0241
0,0241 0,0276
0,0276 0,031
0,031 0,0345
0,0345 0,038
0,038 0,0415
0,0415 0,0449
0,0449 0,0484
0,0484


-0,0489 -0,0454
-0,0454 -0,0419
-0,0419 -0,0384
-0,0384 -0,0349
-0,0349 -0,0314
-0,0314 -0,0279
-0,0279 -0,0243
-0,0243 -0,0208
-0,0208 -0,0173
-0,0173 -0,0138
-0,0138 -0,0103
-0,0103 -0,0068
-0,0068 -0,0033
-0,0033 0,0002
-0,0489
0

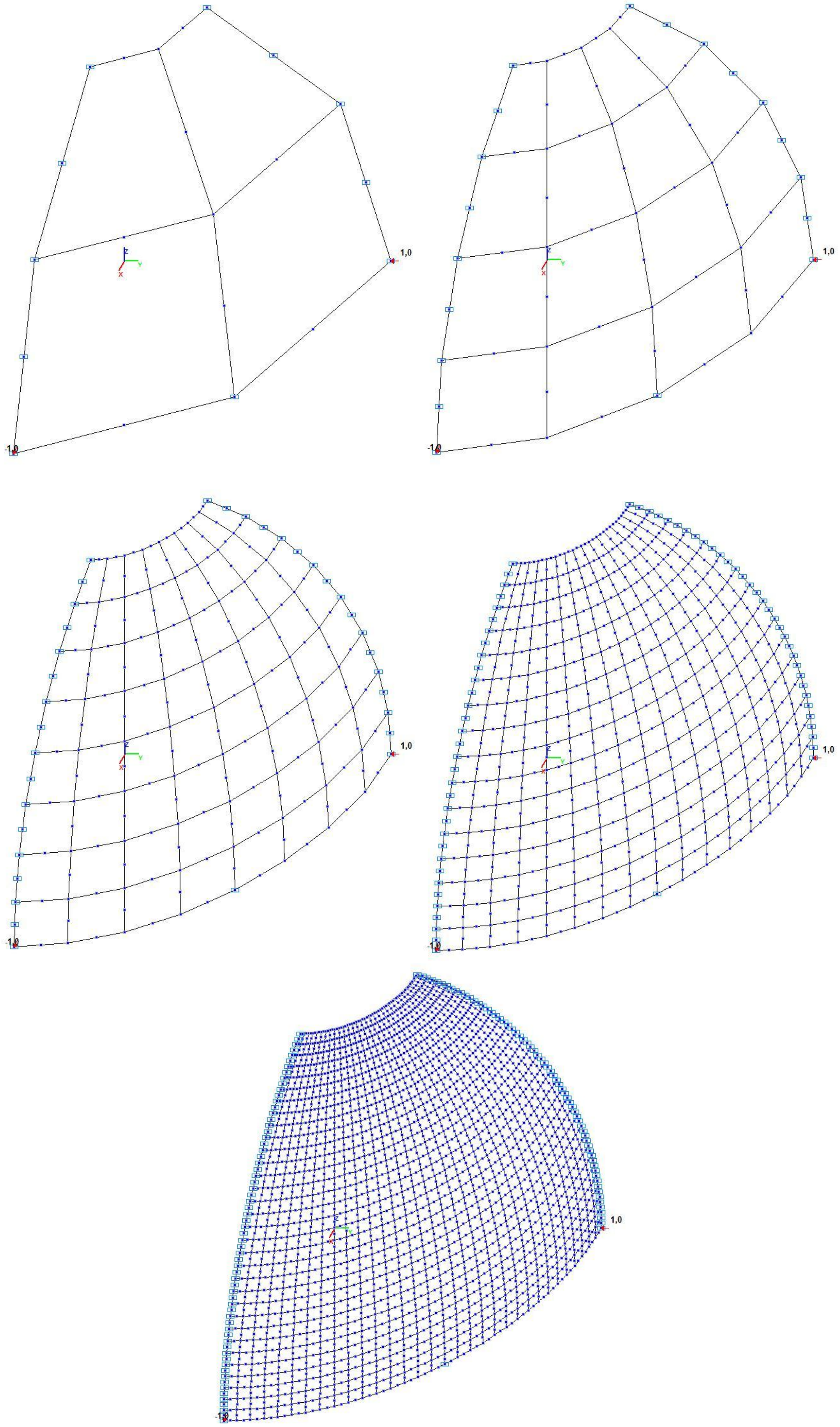

0 0,006
0,006 0,0119
0,0119 0,0179
0,0179 0,0238
0,0238 0,0298
0,0298 0,0357
0,0357 0,0417
0,0417 0,0477
0,0477 0,0536
0,0536 0,0596
0,0596 0,0655
0,0655 0,0715
0,0715 0,0774
0,0774 0,0834
0,0834



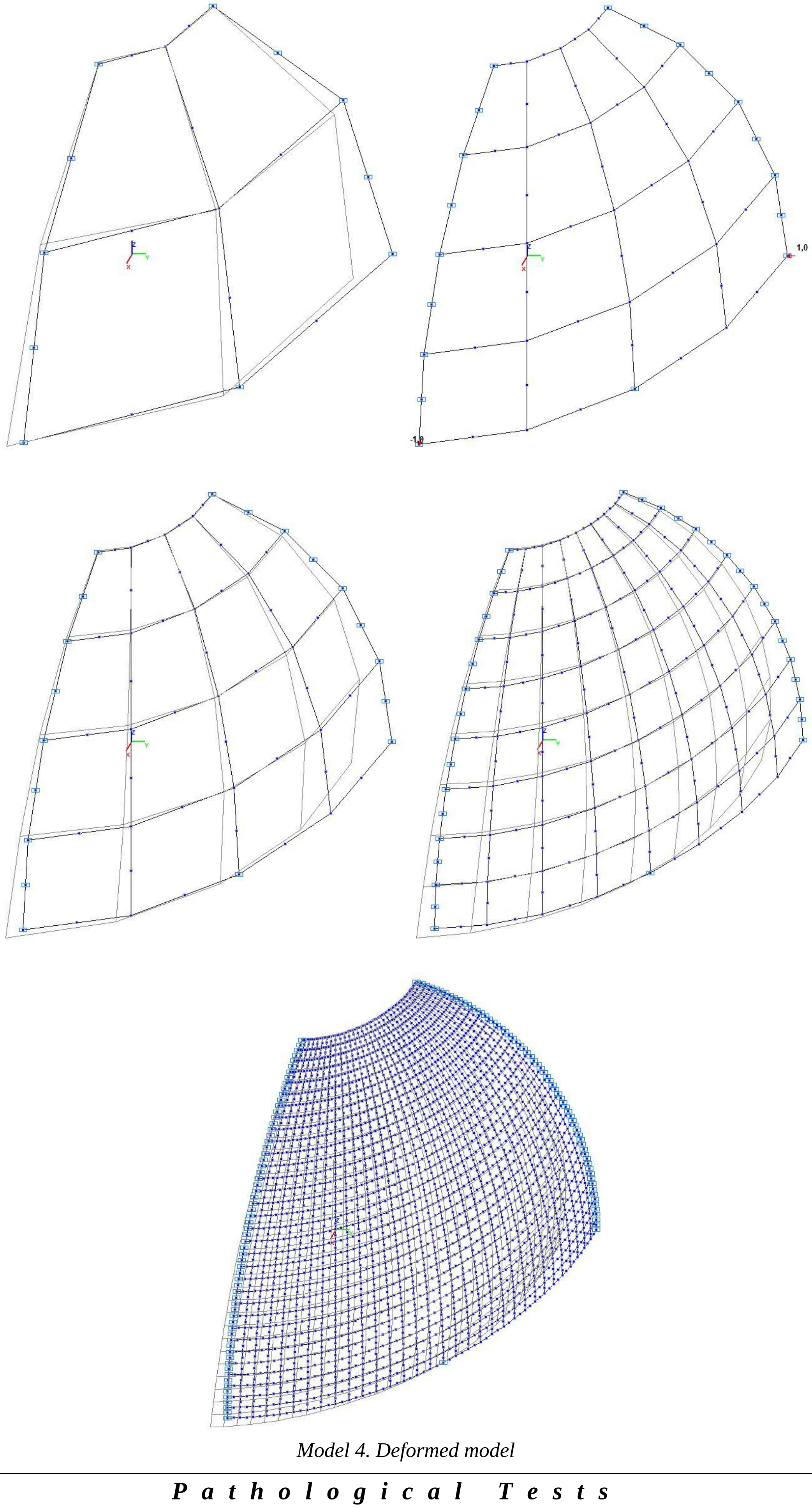




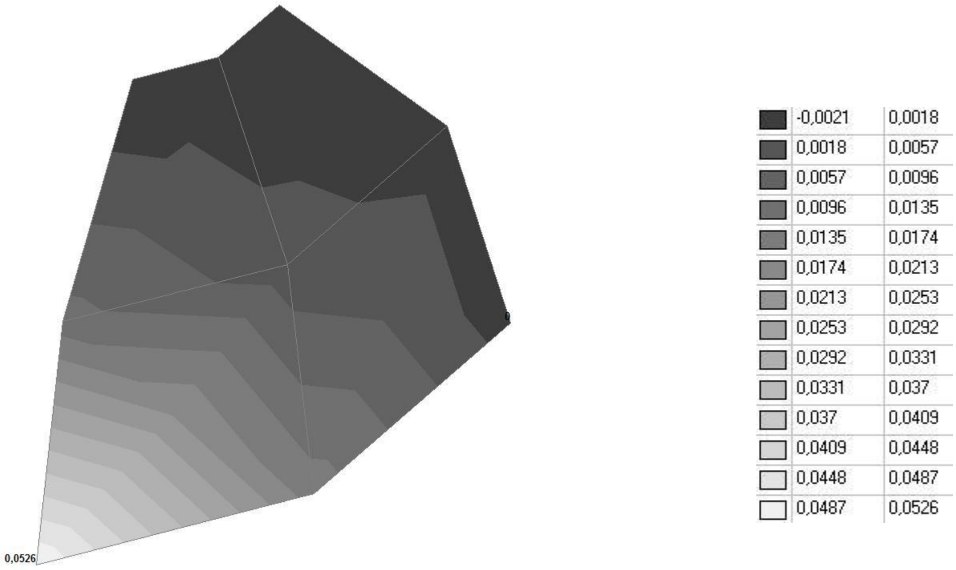
Model 3. Values of the displacements in the direction of the pairs of tensile forces and the pairs of compressive forces along the X and Y axes of the global coordinate system respectively w_{FX} and w_{FY} (m, m)

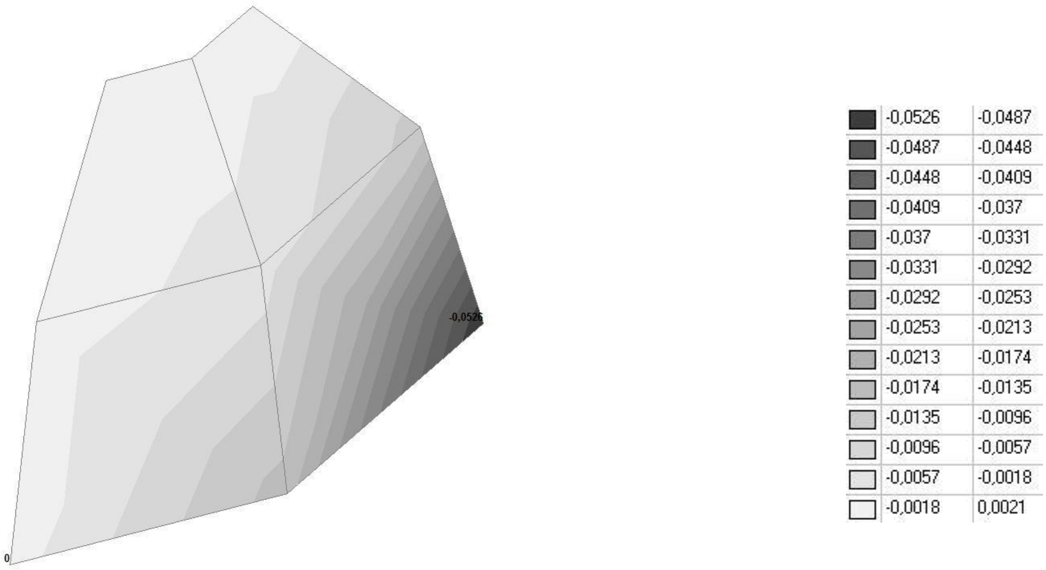


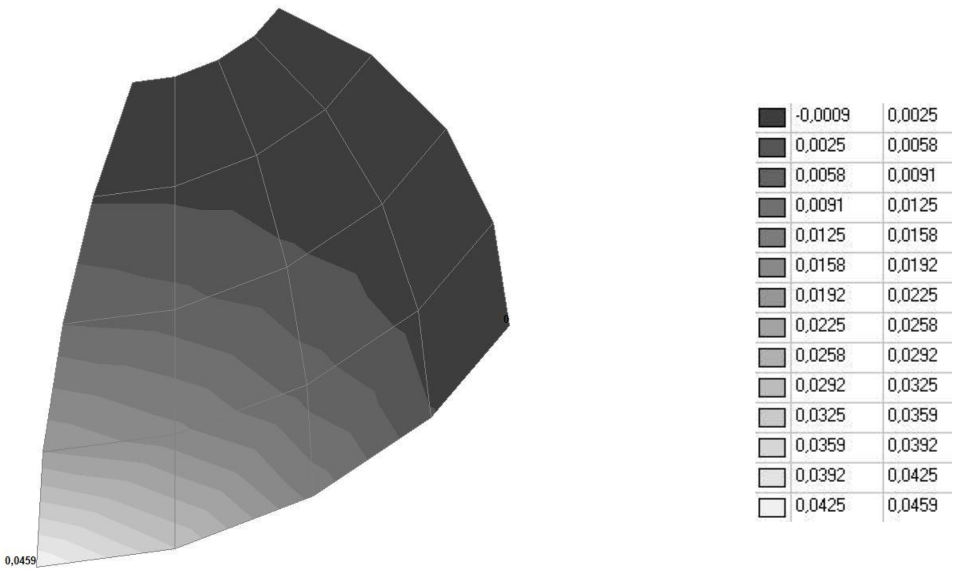
Model 4. Design model

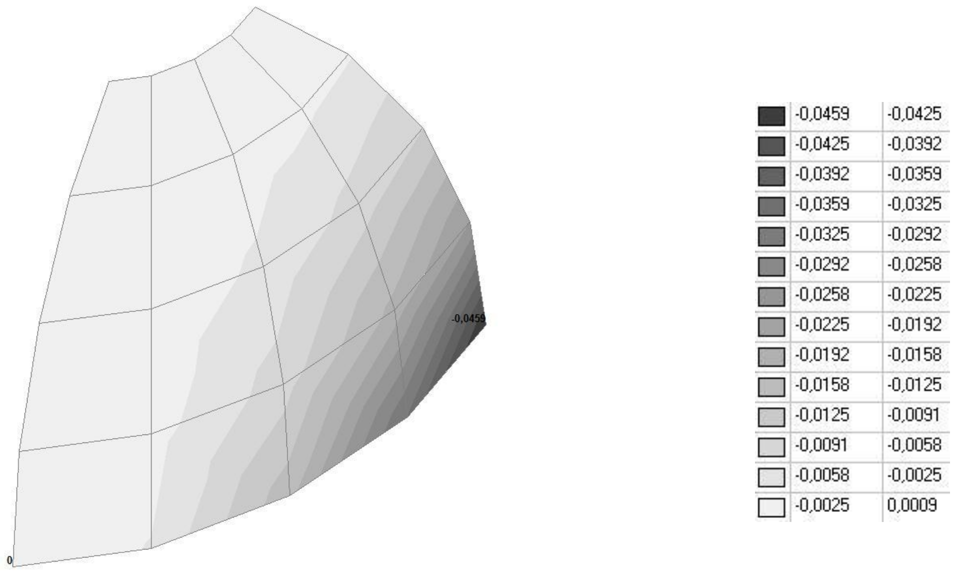


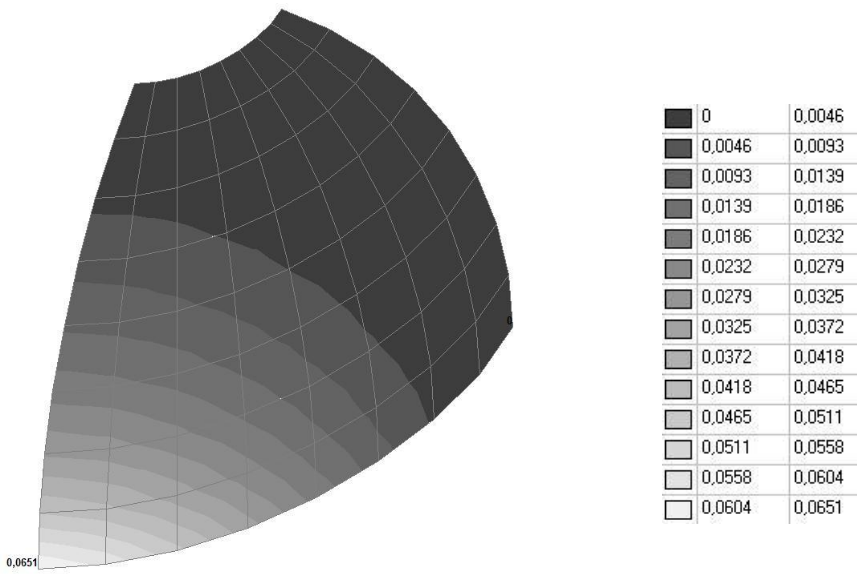
Model 4. Deformed model

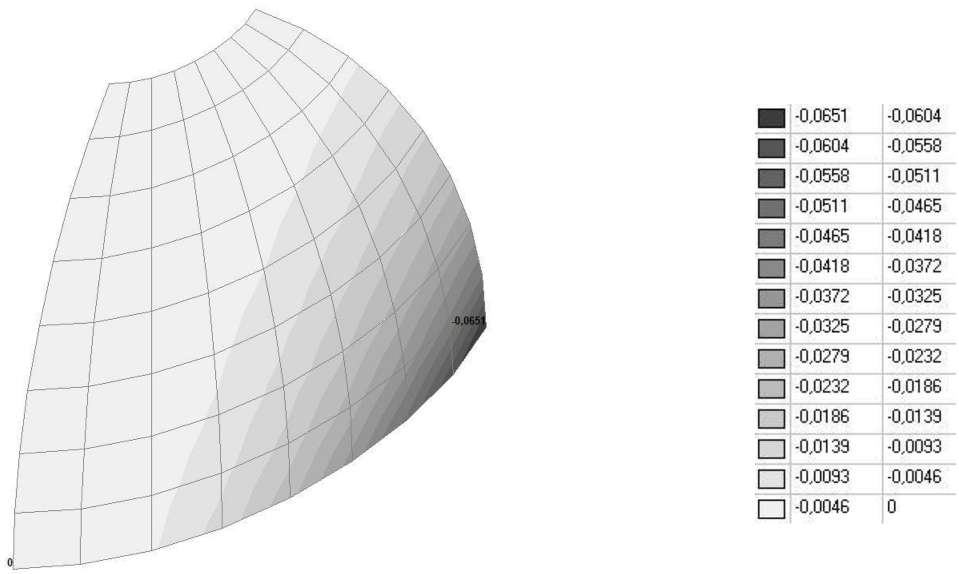

0
0,0526
-0,0021 0,0018
0,0018 0,0057
0,0057 0,0096
0,0096 0,0135
0,0135 0,0174
0,0174 0,0213
0,0213 0,0253
0,0253 0,0292
0,0292 0,0331
0,0331 0,037
0,037 0,0409
0,0409 0,0448
0,0448 0,0487
0,0487 0,0526

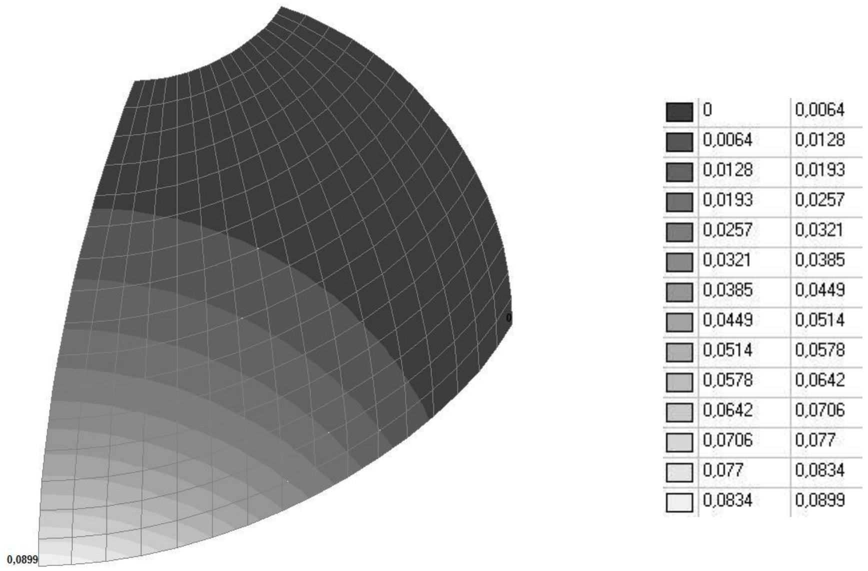

-0,0526
0
-0,0526 -0,0487
-0,0487 -0,0448
-0,0448 -0,0409
-0,0409 -0,037
-0,037 -0,0331
-0,0331 -0,0292
-0,0292 -0,0253
-0,0253 -0,0213
-0,0213 -0,0174
-0,0174 -0,0135
-0,0135 -0,0096
-0,0096 -0,0057
-0,0057 -0,0018
-0,0018 0,0021

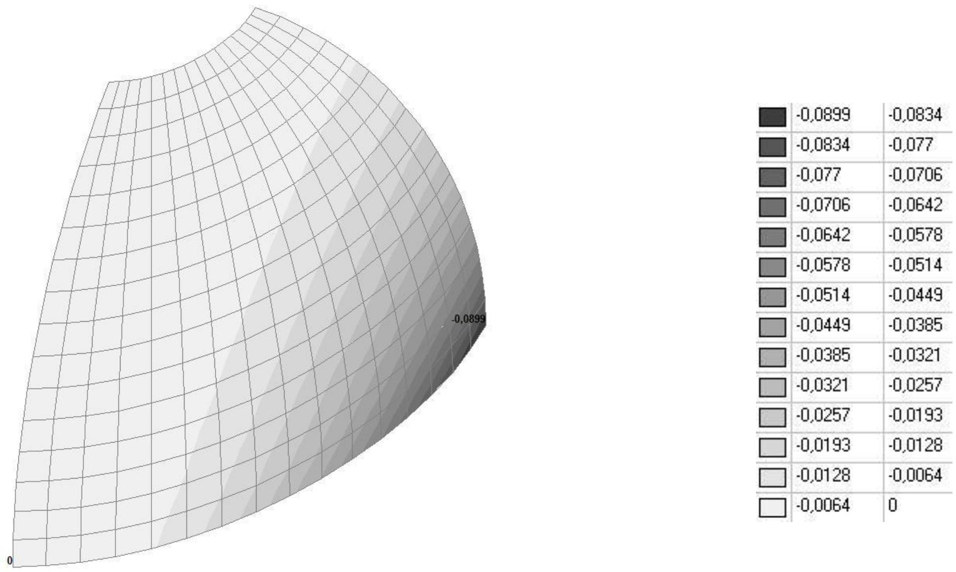

0
0,0459
-0,0009 0,0025
0,0025 0,0058
0,0058 0,0091
0,0091 0,0125
0,0125 0,0158
0,0158 0,0192
0,0192 0,0225
0,0225 0,0258
0,0258 0,0292
0,0292 0,0325
0,0325 0,0359
0,0359 0,0392
0,0392 0,0425
0,0425 0,0459

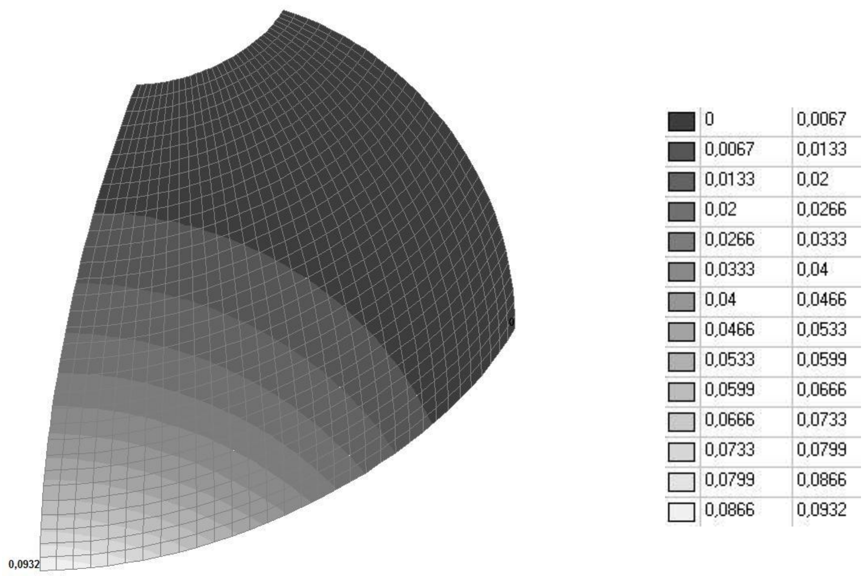

-0,0459
0
-0,0459 -0,0425
-0,0425 -0,0392
-0,0392 -0,0359
-0,0359 -0,0325
-0,0325 -0,0292
-0,0292 -0,0258
-0,0258 -0,0225
-0,0225 -0,0192
-0,0192 -0,0158
-0,0158 -0,0125
-0,0125 -0,0091
-0,0091 -0,0058
-0,0058 -0,0025
-0,0025 0,0009

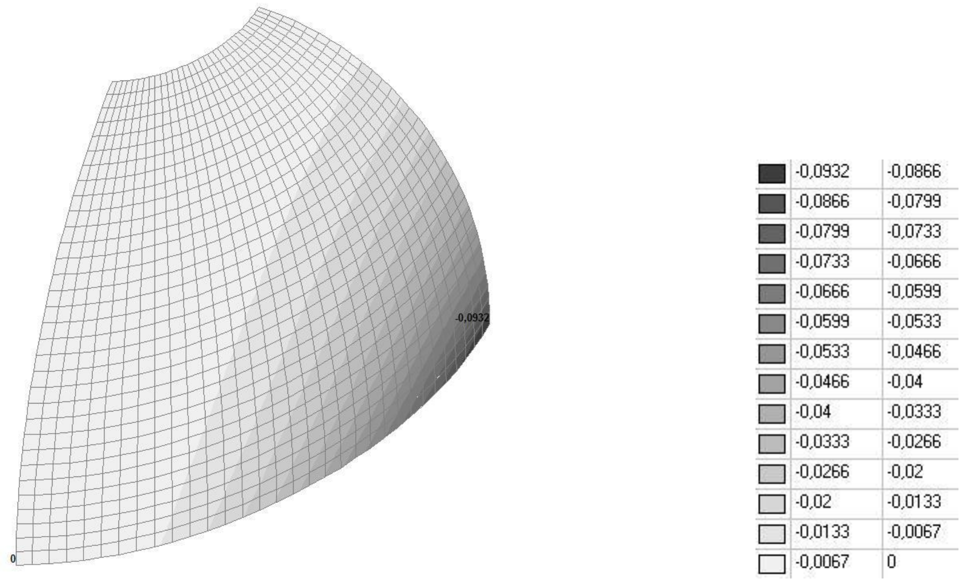

0,0651
0 0,0046
0,0046 0,0093
0,0093 0,0139
0,0139 0,0186
0,0186 0,0232
0,0232 0,0279
0,0279 0,0325
0,0325 0,0372
0,0372 0,0418
0,0418 0,0465
0,0465 0,0511
0,0511 0,0558
0,0558 0,0604
0,0604 0,0651


-0,0651
0
-0,0651 -0,0604
-0,0604 -0,0558
-0,0558 -0,0511
-0,0511 -0,0465
-0,0465 -0,0418
-0,0418 -0,0372
-0,0372 -0,0325
-0,0325 -0,0279
-0,0279 -0,0232
-0,0232 -0,0186
-0,0186 -0,0139
-0,0139 -0,0093
-0,0093 -0,0046
-0,0046 0

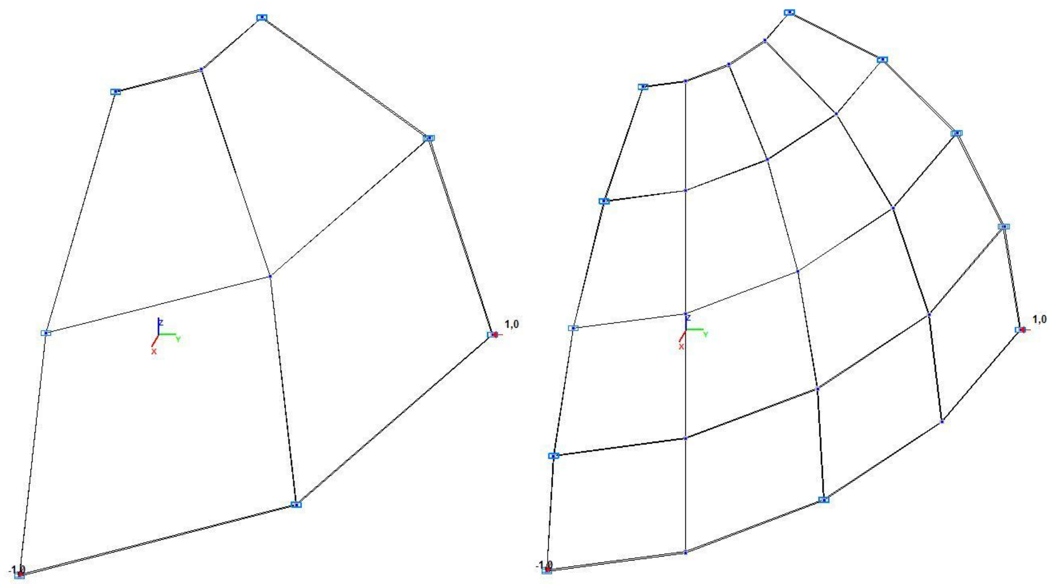

0 0,0064
0,0064 0,0128
0,0128 0,0193
0,0193 0,0257
0,0257 0,0321
0,0321 0,0385
0,0385 0,0449
0,0449 0,0514
0,0514 0,0578
0,0578 0,0642
0,0642 0,0706
0,0706 0,077
0,077 0,0834
0,0834 0,0899
0,0899

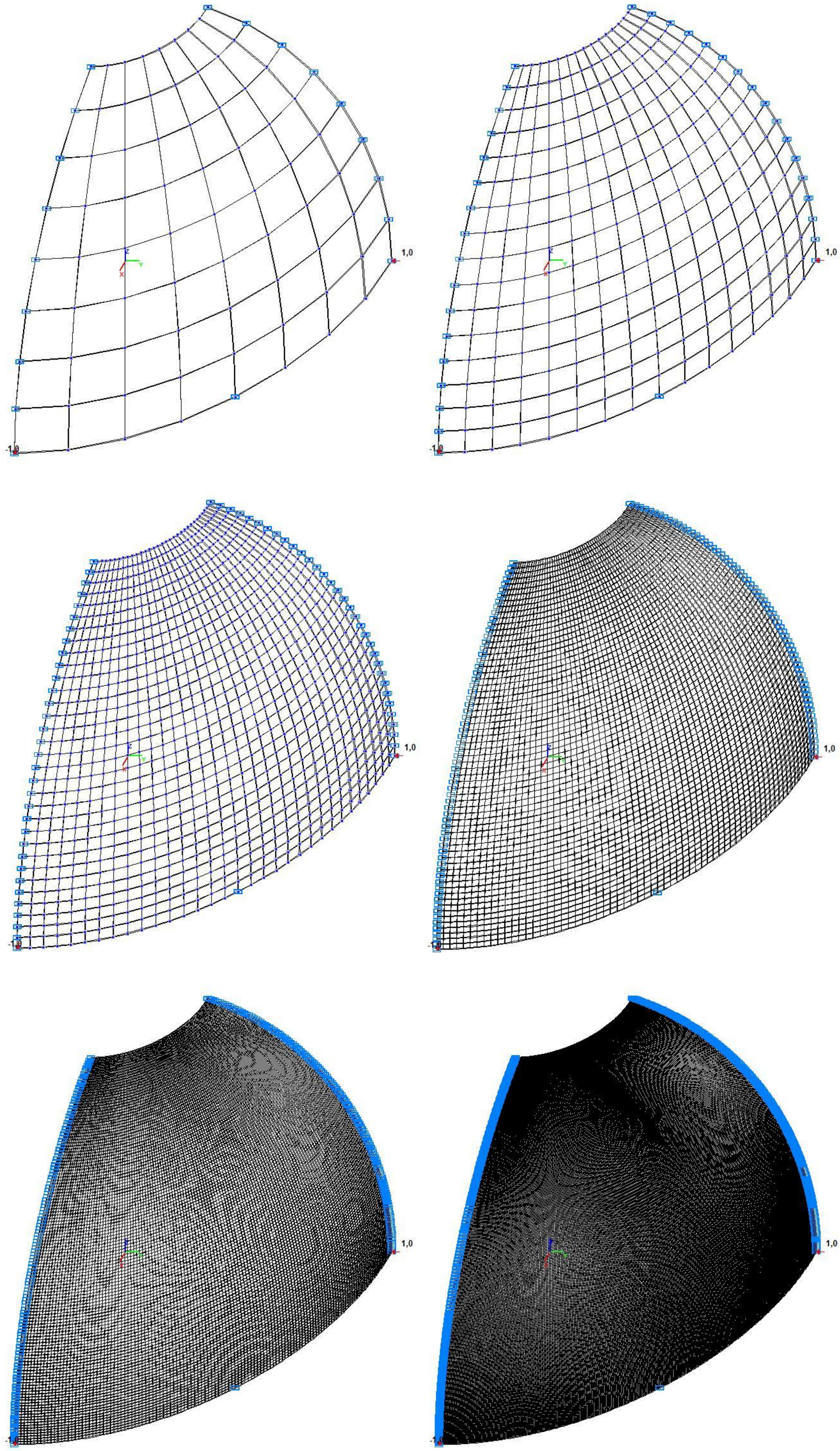

-0,0899 -0,0834
-0,0834 -0,077
-0,077 -0,0706
-0,0706 -0,0642
-0,0642 -0,0578
-0,0578 -0,0514
-0,0514 -0,0449
-0,0449 -0,0385
-0,0385 -0,0321
-0,0321 -0,0257
-0,0257 -0,0193
-0,0193 -0,0128
-0,0128 -0,0064
-0,0064 0
-0,0899
0

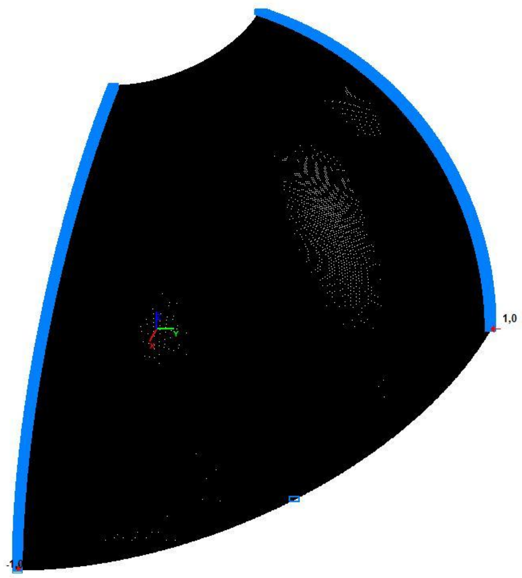

0 0,0067
0,0067 0,0133
0,0133 0,02
0,02 0,0266
0,0266 0,0333
0,0333 0,04
0,04 0,0466
0,0466 0,0533
0,0533 0,0599
0,0599 0,0666
0,0666 0,0733
0,0733 0,0799
0,0799 0,0866
0,0866 0,0932
0,0932



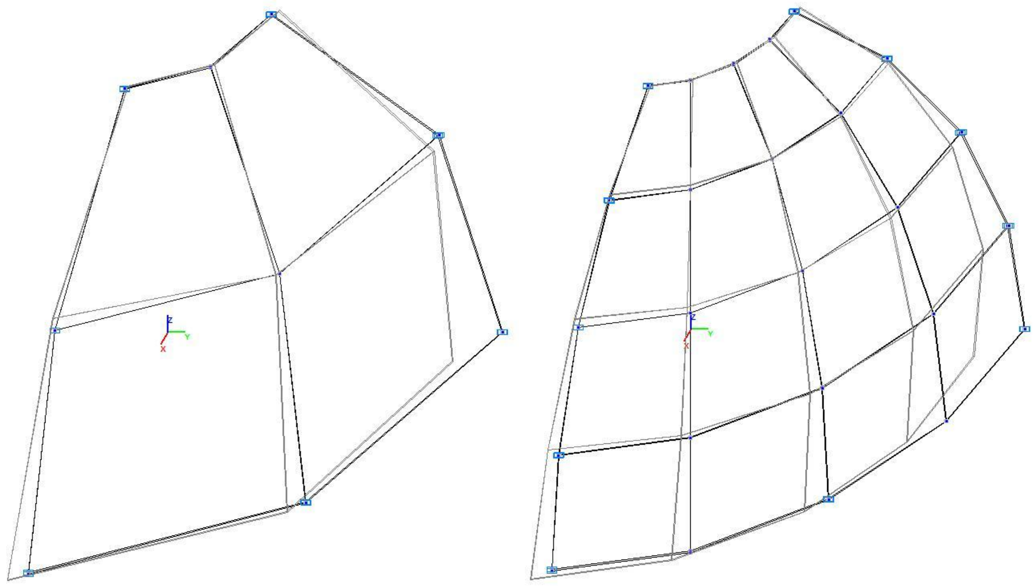
Model 4. Values of the displacements in the direction of the pairs of tensile forces and the pairs of compressive forces along the X and Y axes of the global coordinate system respectively w_{FX} and w_{FY} (m, m)

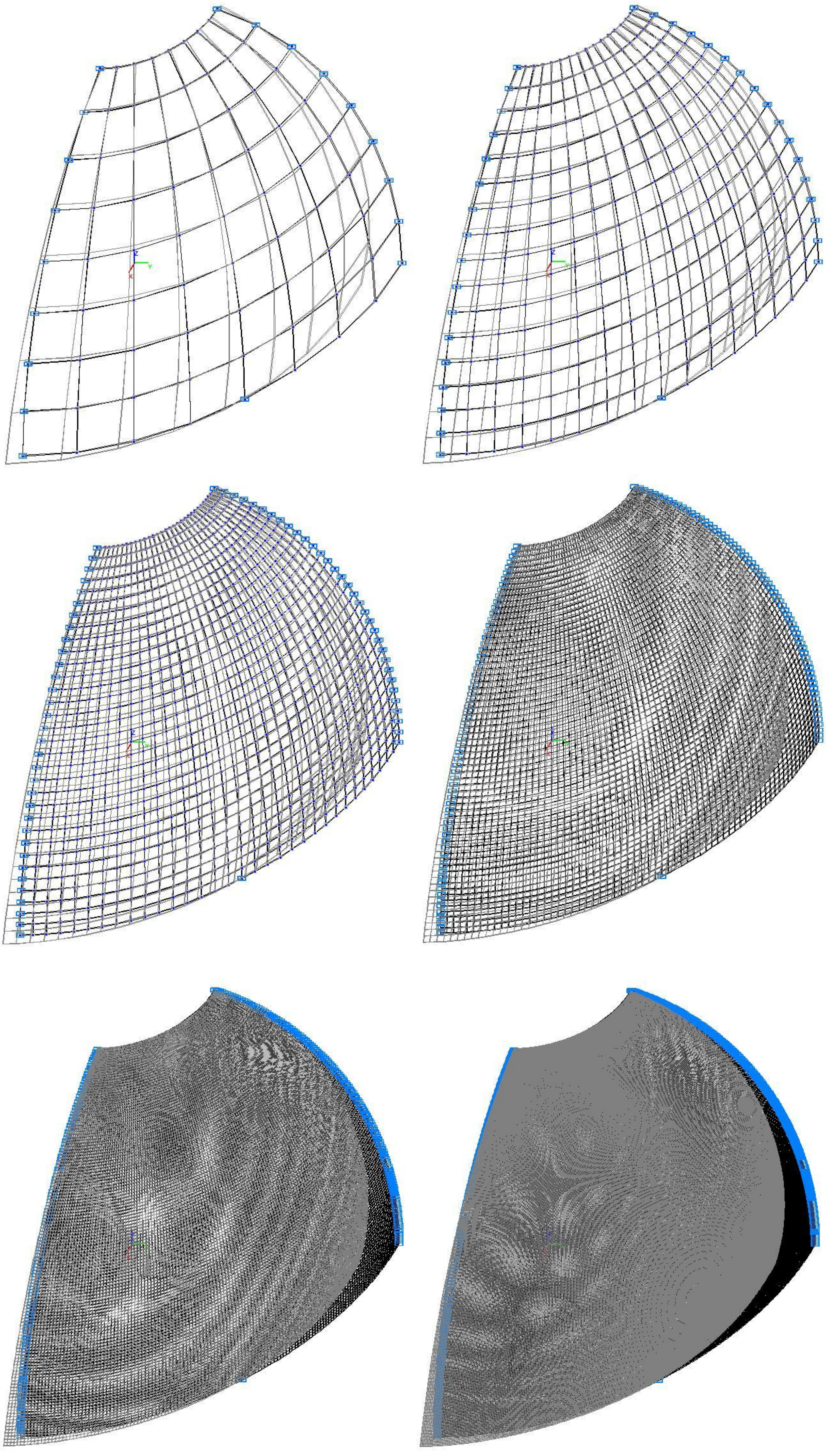


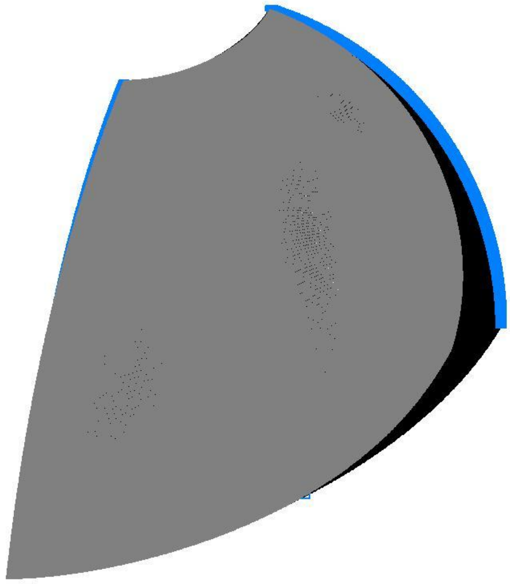

1,0
-1,0



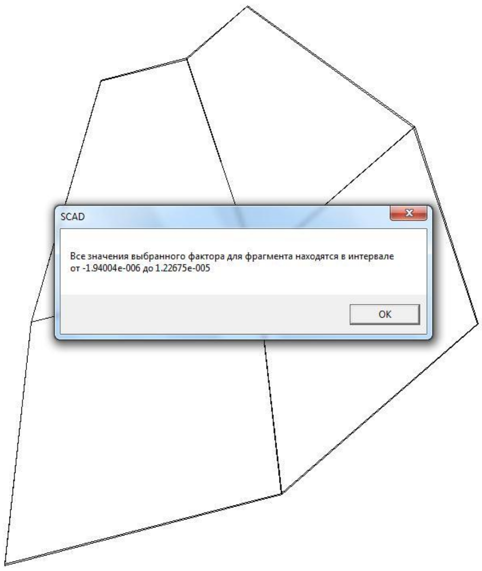
Model 5. Design model

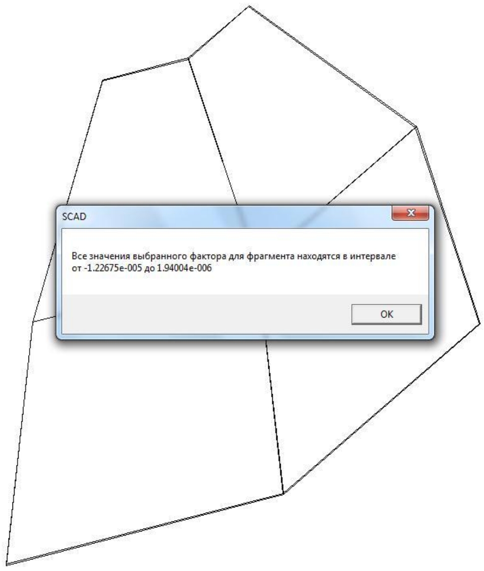


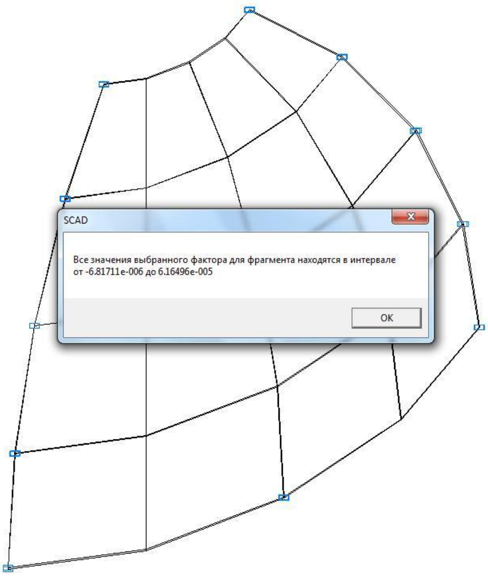


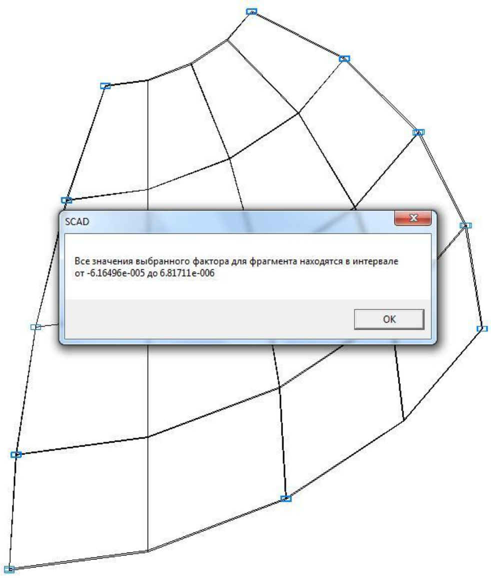


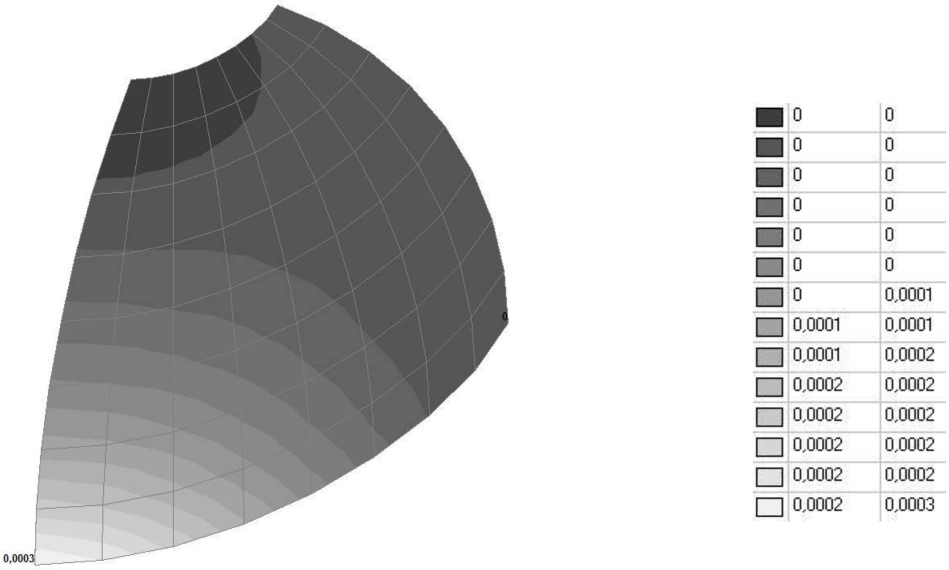
Model 5. Deformed model

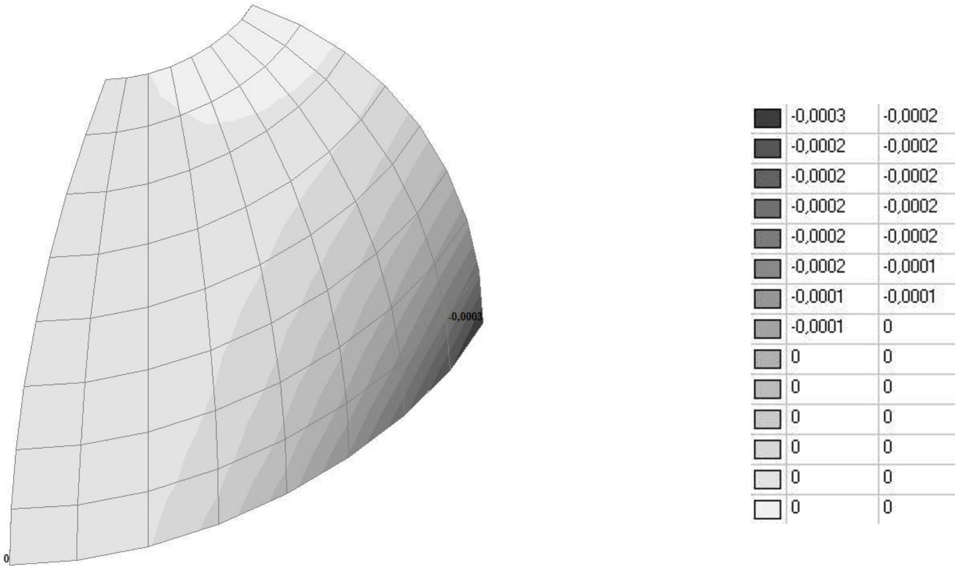


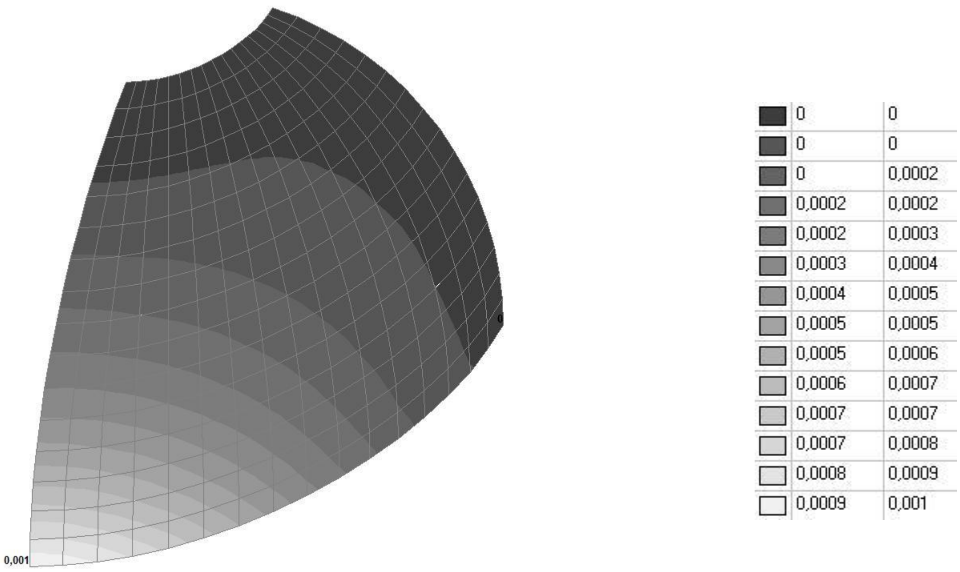


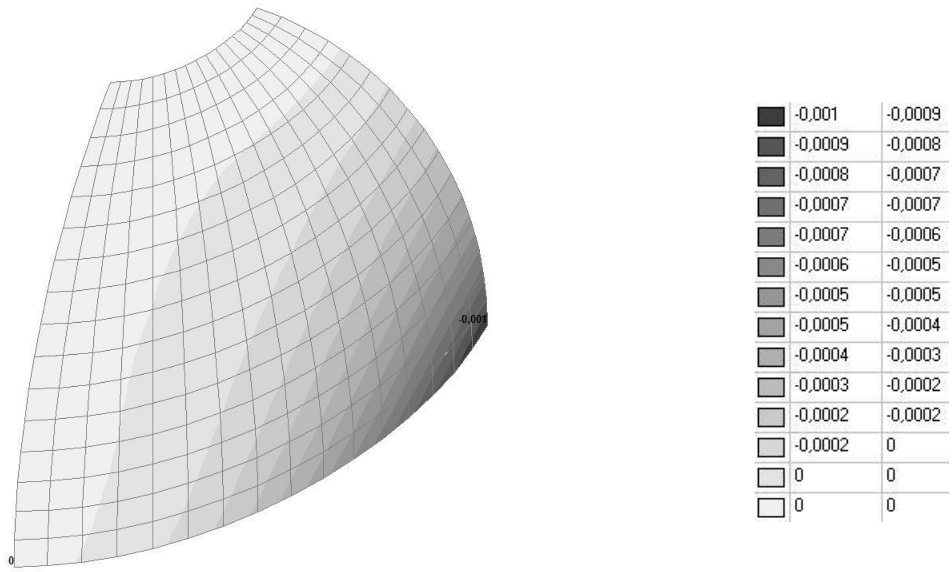


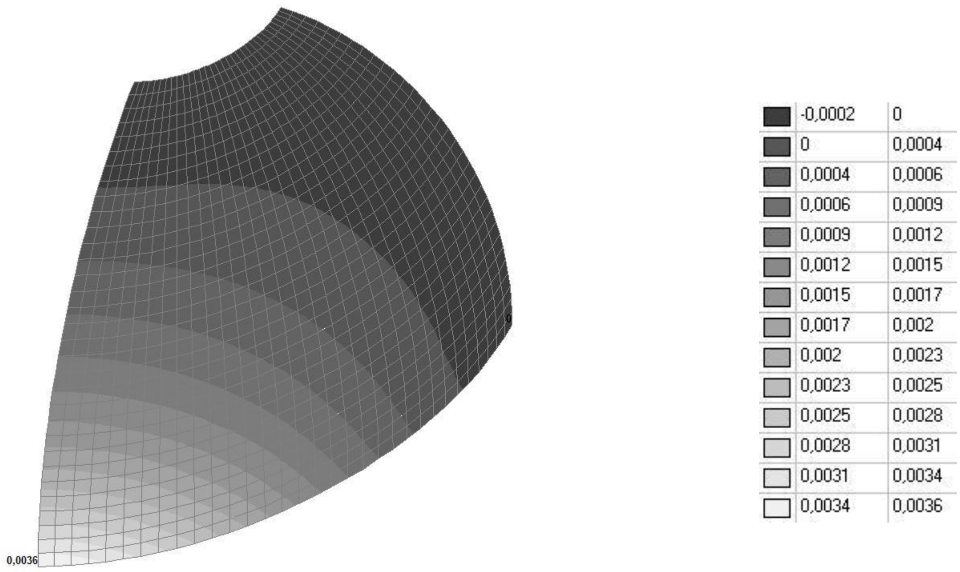


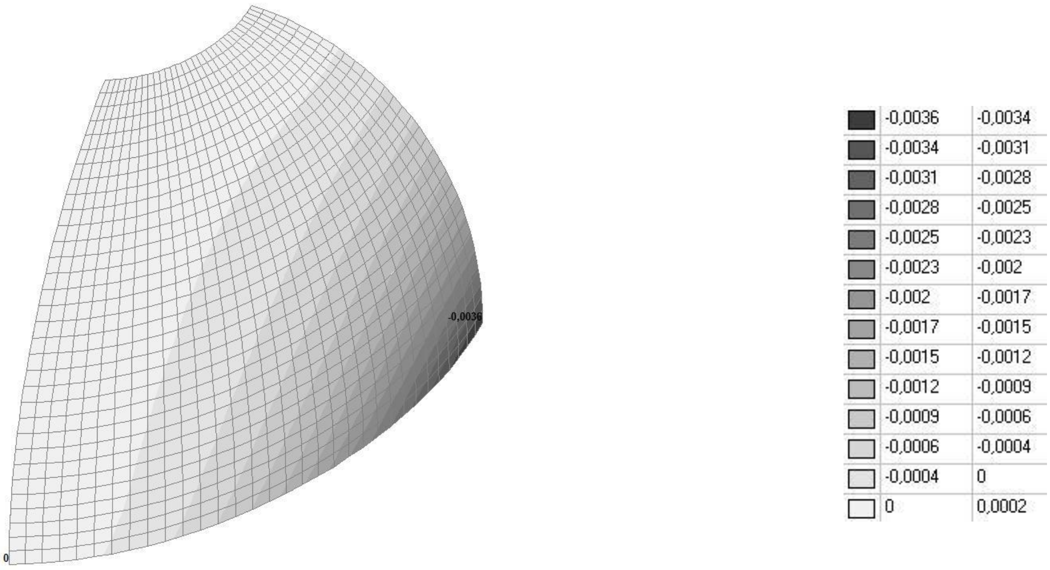

0,0003
0
0 0
0 0
0 0
0 0
0 0
0 0
0 0,0001
0,0001 0,0001
0,0001 0,0002
0,0002 0,0002
0,0002 0,0002
0,0002 0,0002
0,0002 0,0002
0,0002 0,0003

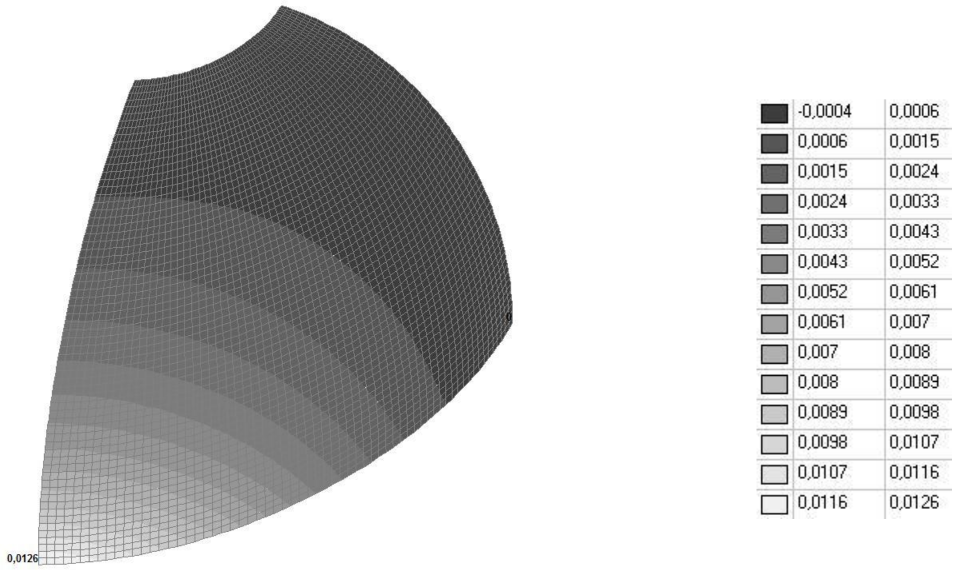

-0,0003
0
-0,0003 -0,0002
-0,0002 -0,0002
-0,0002 -0,0002
-0,0002 -0,0002
-0,0002 -0,0002
-0,0002 -0,0001
-0,0001 -0,0001
-0,0001 0
0 0
0 0
0 0
0 0
0 0
0 0

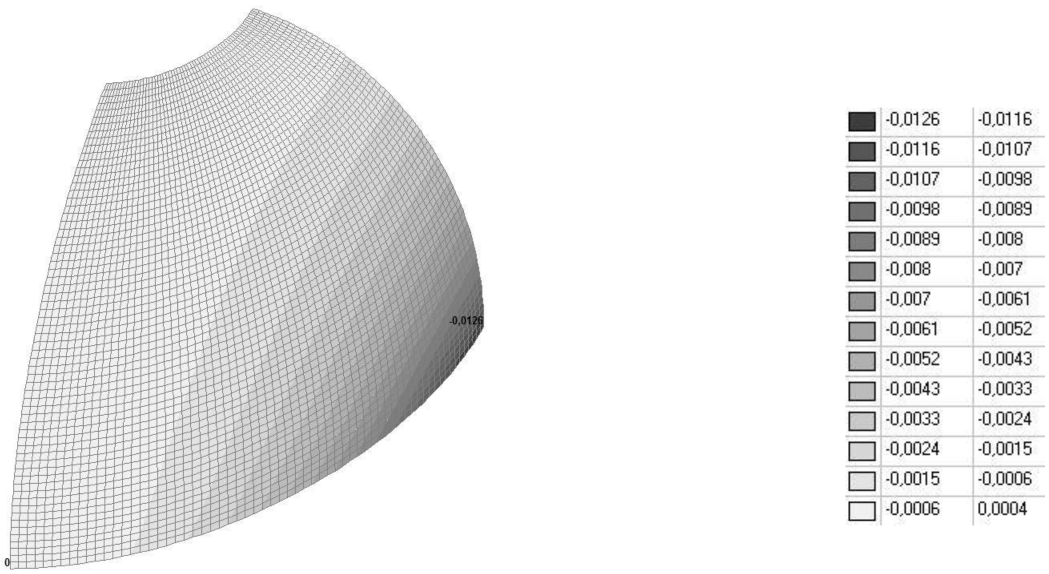

0,001
0
0 0
0 0
0 0,0002
0,0002 0,0002
0,0002 0,0003
0,0003 0,0004
0,0004 0,0005
0,0005 0,0005
0,0005 0,0006
0,0006 0,0007
0,0007 0,0007
0,0007 0,0008
0,0008 0,0009
0,0009 0,001

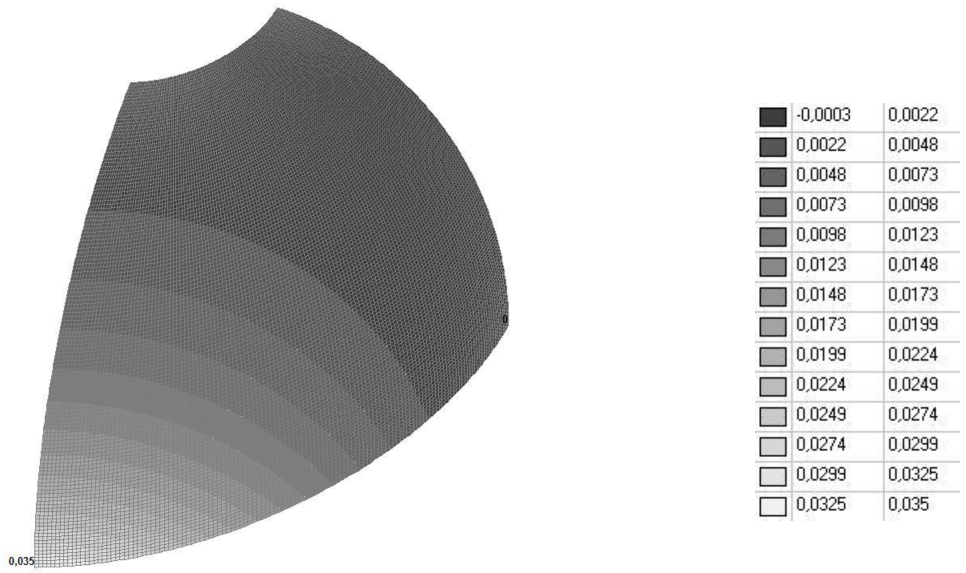

-0,001
0
-0,001 -0,0009
-0,0009 -0,0008
-0,0008 -0,0007
-0,0007 -0,0007
-0,0007 -0,0006
-0,0006 -0,0005
-0,0005 -0,0005
-0,0005 -0,0004
-0,0004 -0,0003
-0,0003 -0,0002
-0,0002 -0,0002
-0,0002 0
0 0
0 0

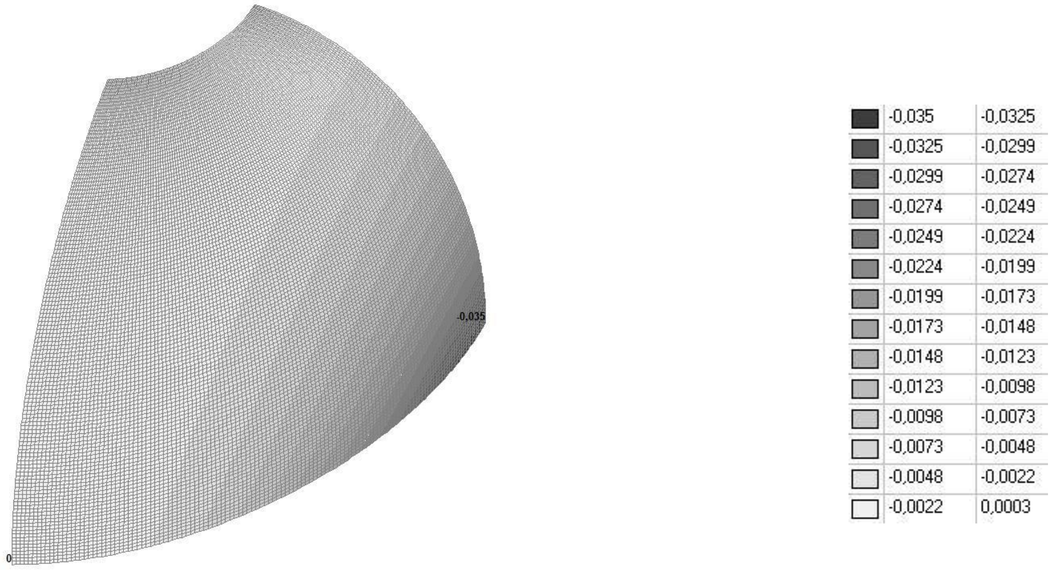

0,0036
-0,0002 0
0 0,0004
0,0004 0,0006
0,0006 0,0009
0,0009 0,0012
0,0012 0,0015
0,0015 0,0017
0,0017 0,002
0,002 0,0023
0,0023 0,0025
0,0025 0,0028
0,0028 0,0031
0,0031 0,0034
0,0034 0,0036

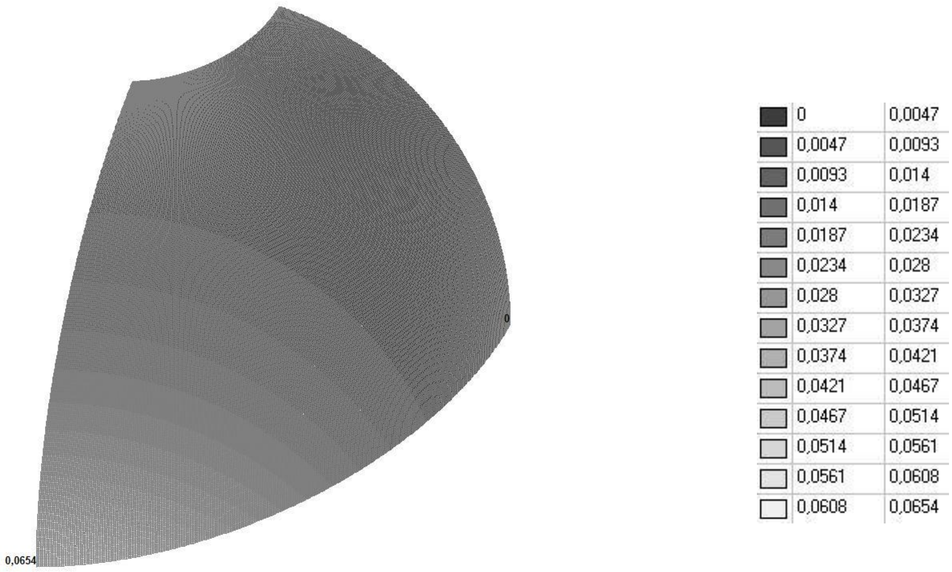

-0,0036
0
-0,0036 -0,0034
-0,0034 -0,0031
-0,0031 -0,0028
-0,0028 -0,0025
-0,0025 -0,0023
-0,0023 -0,002
-0,002 -0,0017
-0,0017 -0,0015
-0,0015 -0,0012
-0,0012 -0,0009
-0,0009 -0,0006
-0,0006 -0,0004
-0,0004 0
0 0,0002

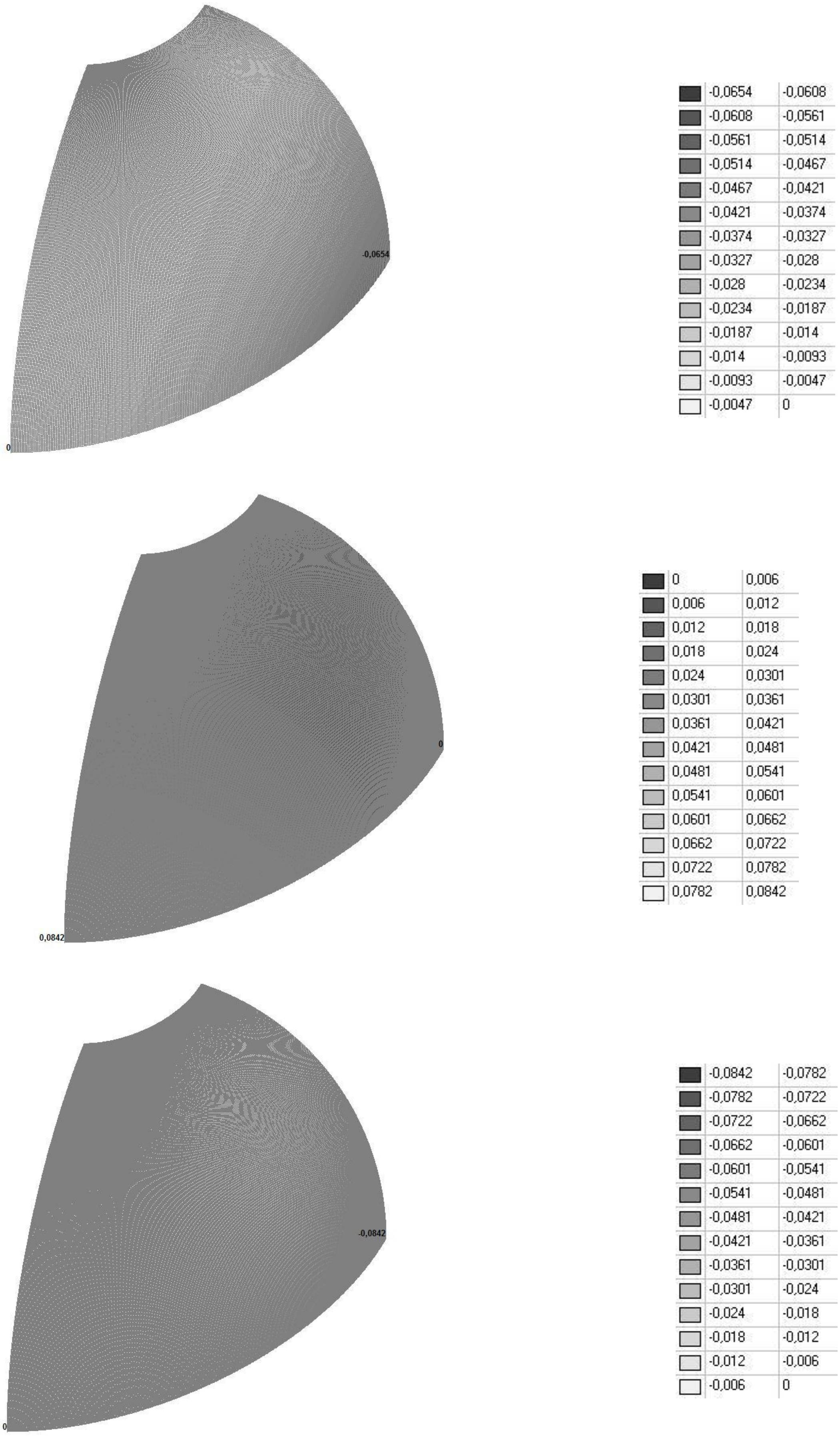

0,0126
-0,0004 0,0006
0,0006 0,0015
0,0015 0,0024
0,0024 0,0033
0,0033 0,0043
0,0043 0,0052
0,0052 0,0061
0,0061 0,007
0,007 0,008
0,008 0,0089
0,0089 0,0098
0,0098 0,0107
0,0107 0,0116
0,0116 0,0126


-0,0126
0
-0,0126 -0,0116
-0,0116 -0,0107
-0,0107 -0,0098
-0,0098 -0,0089
-0,0089 -0,008
-0,008 -0,007
-0,007 -0,0061
-0,0061 -0,0052
-0,0052 -0,0043
-0,0043 -0,0033
-0,0033 -0,0024
-0,0024 -0,0015
-0,0015 -0,0006
-0,0006 0,0004

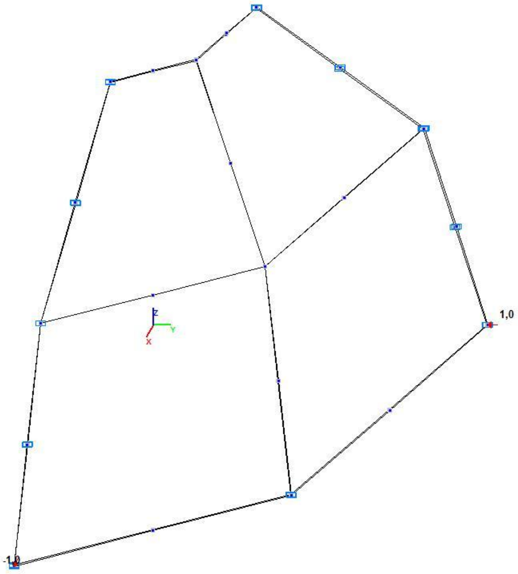

0,035
0
-0,0003 0,0022
0,0022 0,0048
0,0048 0,0073
0,0073 0,0098
0,0098 0,0123
0,0123 0,0148
0,0148 0,0173
0,0173 0,0199
0,0199 0,0224
0,0224 0,0249
0,0249 0,0274
0,0274 0,0299
0,0299 0,0325
0,0325 0,035

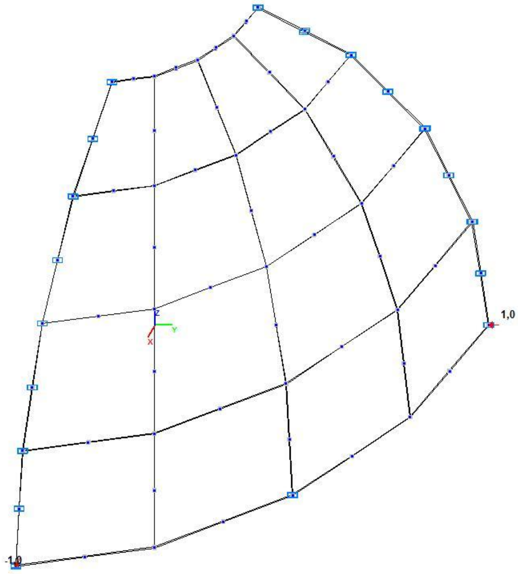

-0,035
0
-0,035 -0,0325
-0,0325 -0,0299
-0,0299 -0,0274
-0,0274 -0,0249
-0,0249 -0,0224
-0,0224 -0,0199
-0,0199 -0,0173
-0,0173 -0,0148
-0,0148 -0,0123
-0,0123 -0,0098
-0,0098 -0,0073
-0,0073 -0,0048
-0,0048 -0,0022
-0,0022 0,0003

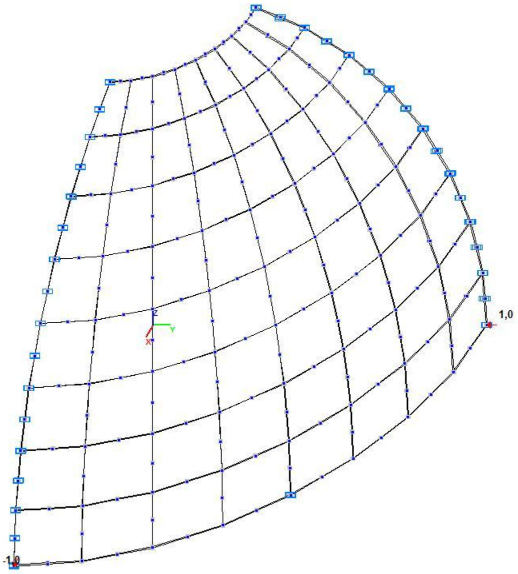

0
0,0654
0 0,0047
0,0047 0,0093
0,0093 0,014
0,014 0,0187
0,0187 0,0234
0,0234 0,028
0,028 0,0327
0,0327 0,0374
0,0374 0,0421
0,0421 0,0467
0,0467 0,0514
0,0514 0,0561
0,0561 0,0608
0,0608 0,0654

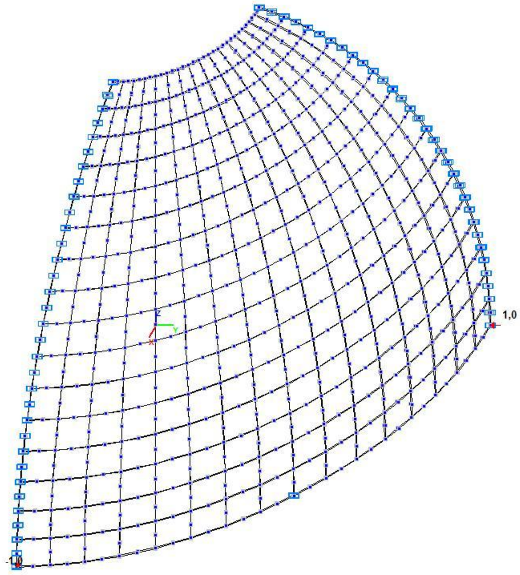


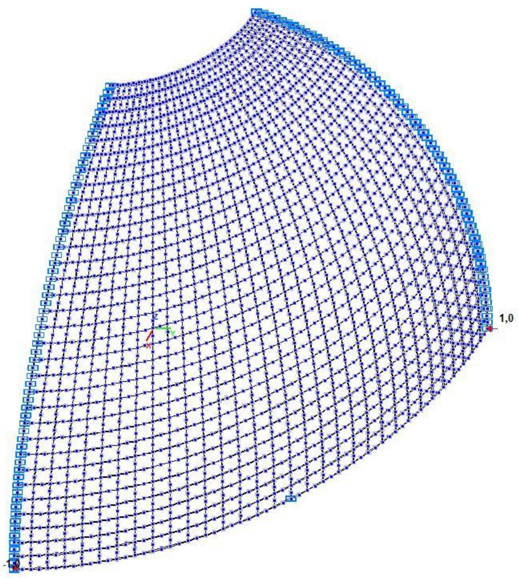
Model 5. Values of the displacements in the direction of the pairs of tensile forces and the pairs of compressive forces along the X and Y axes of the global coordinate system respectively w_{FX} and w_{FY} (m, m)

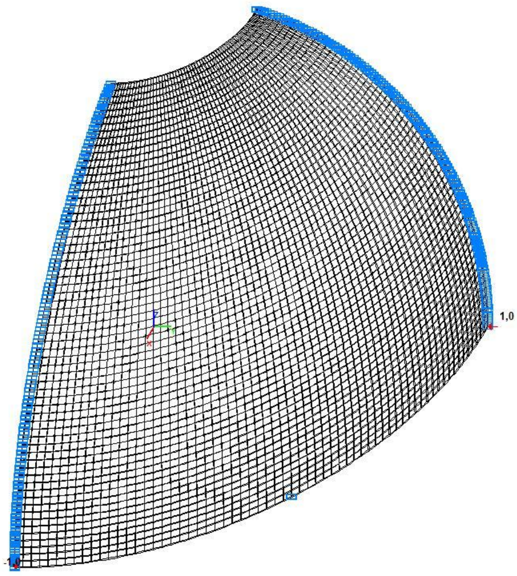

1,0

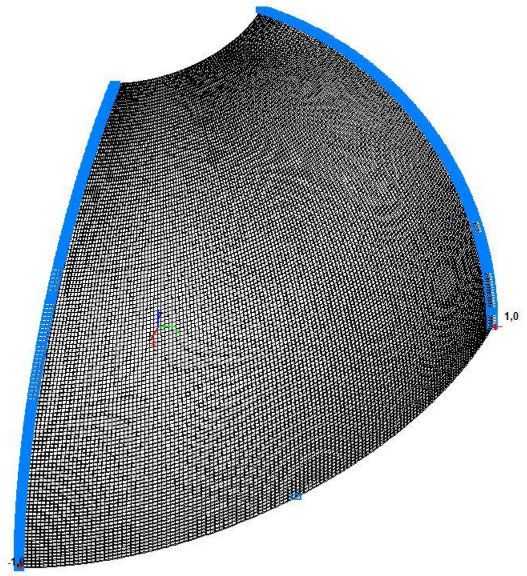

1,0


1,0

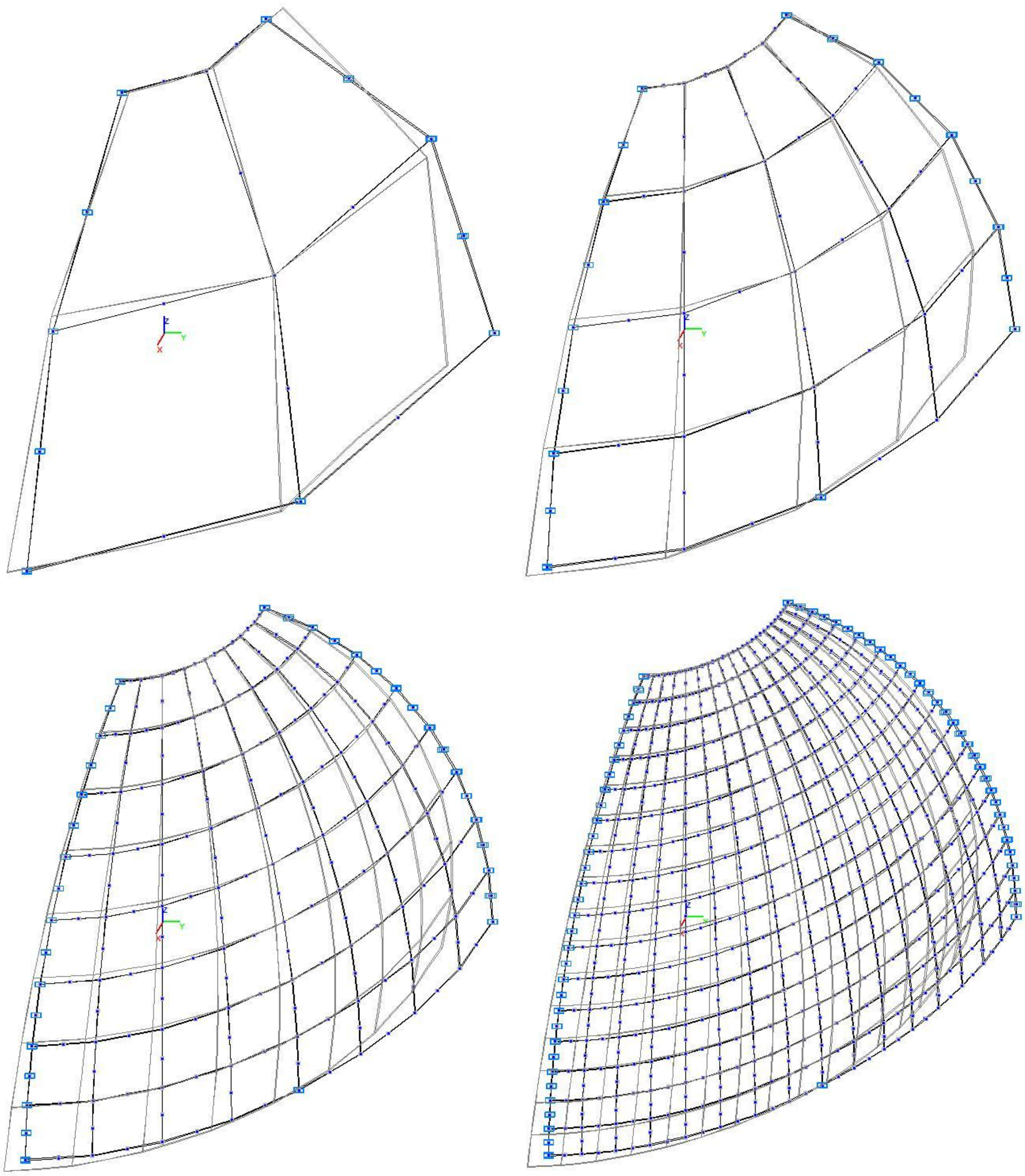

1,0

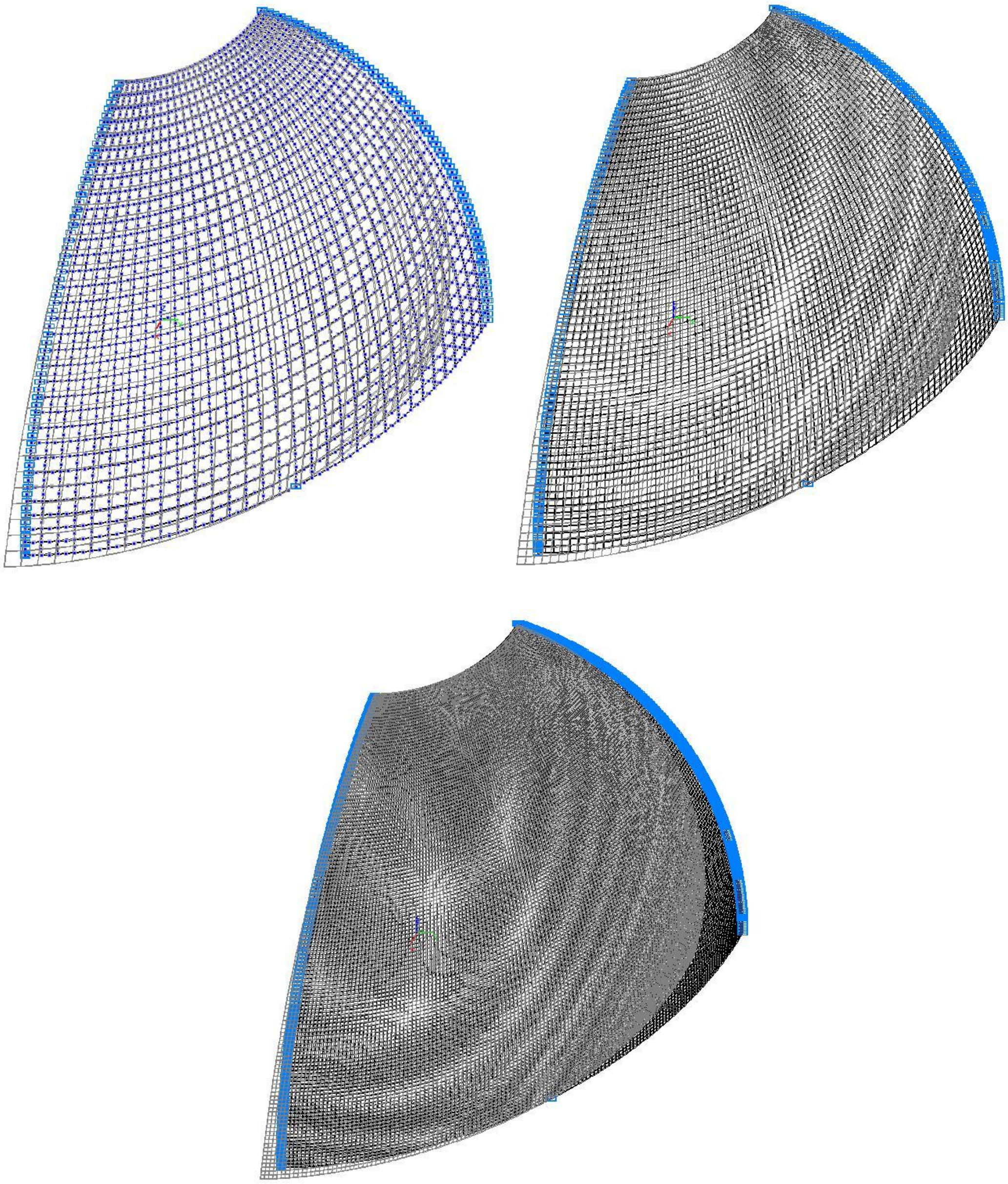

1,0


1,0

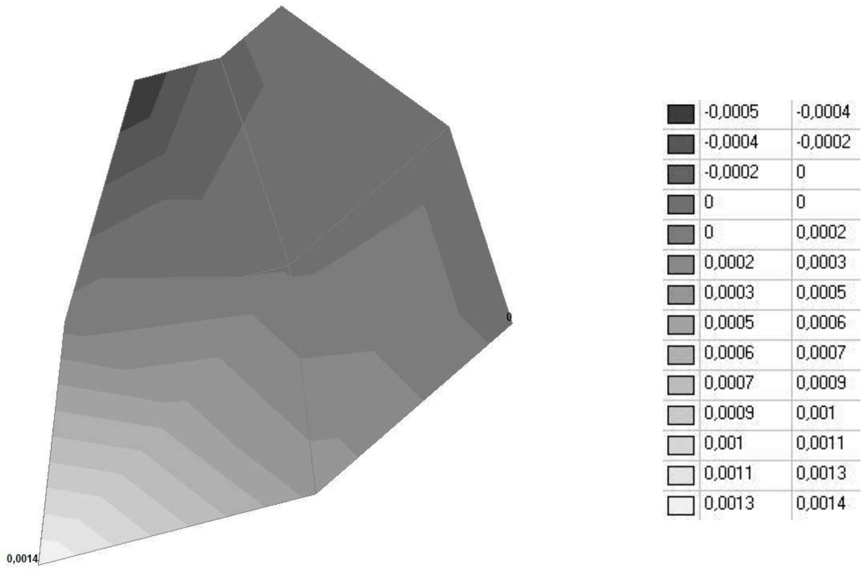


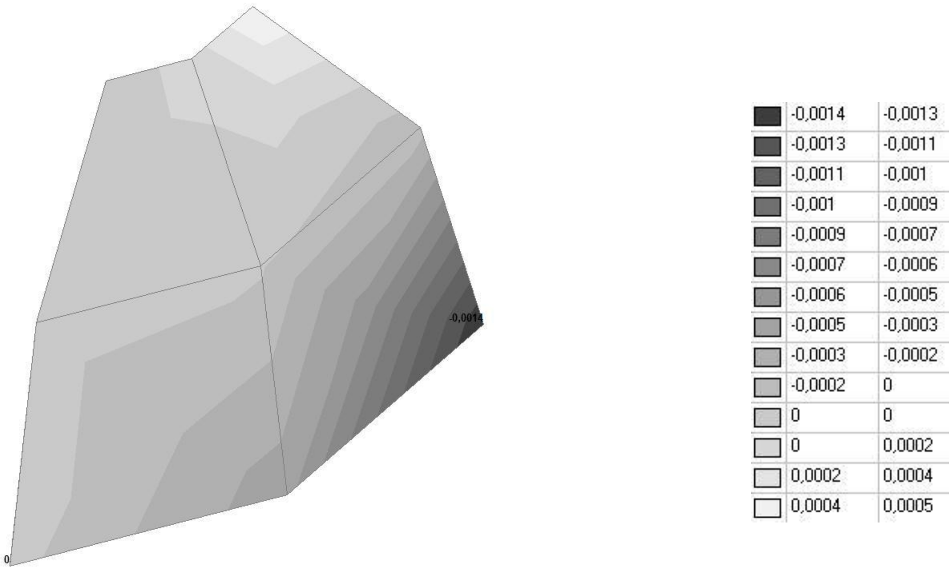
Model 6. Design model

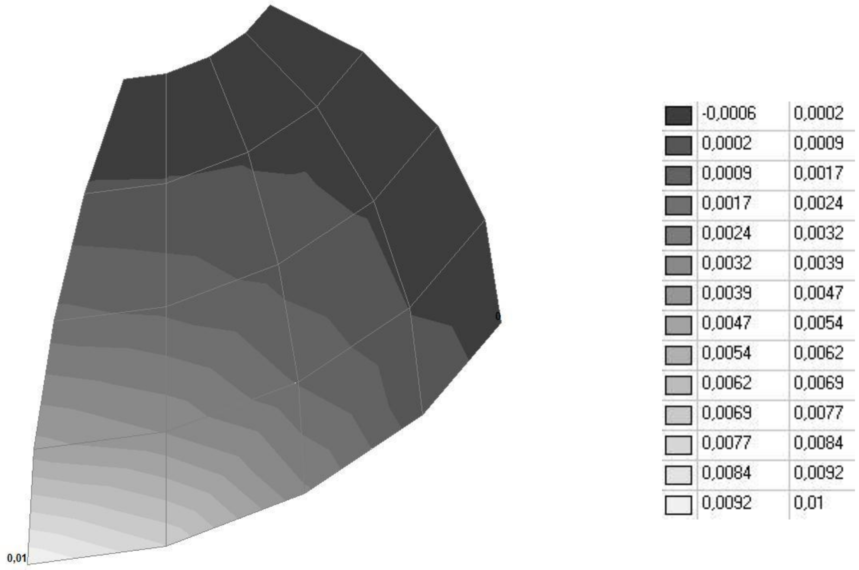


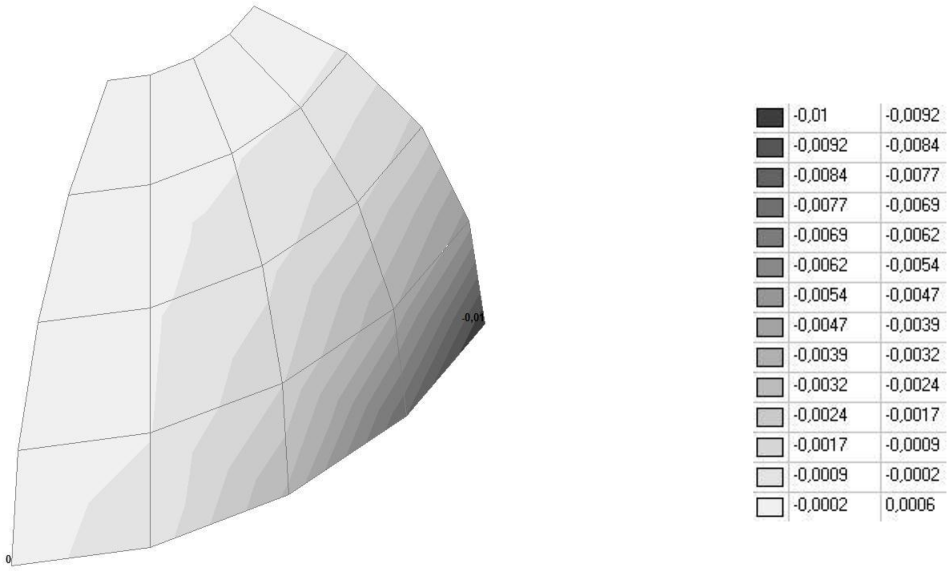


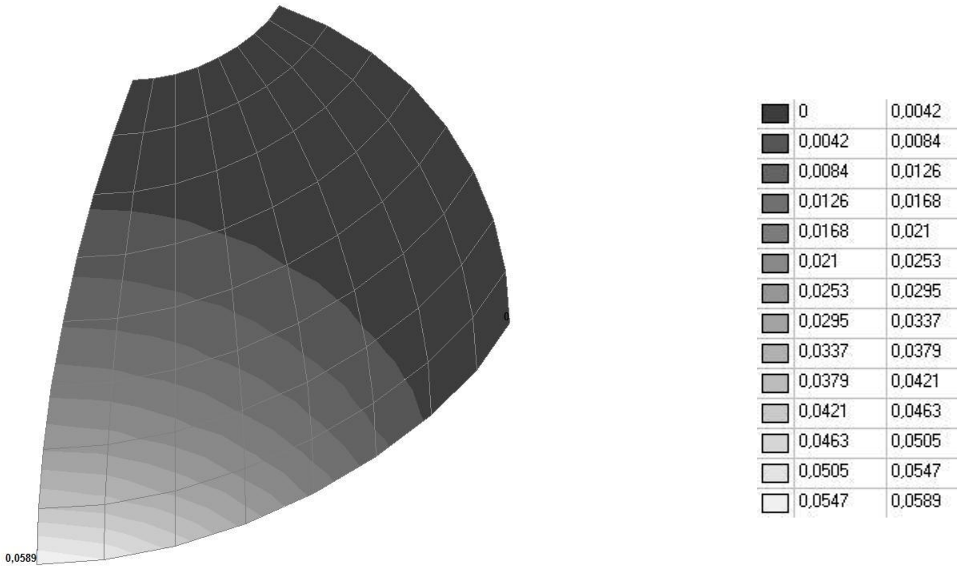
Model 6. Deformed model

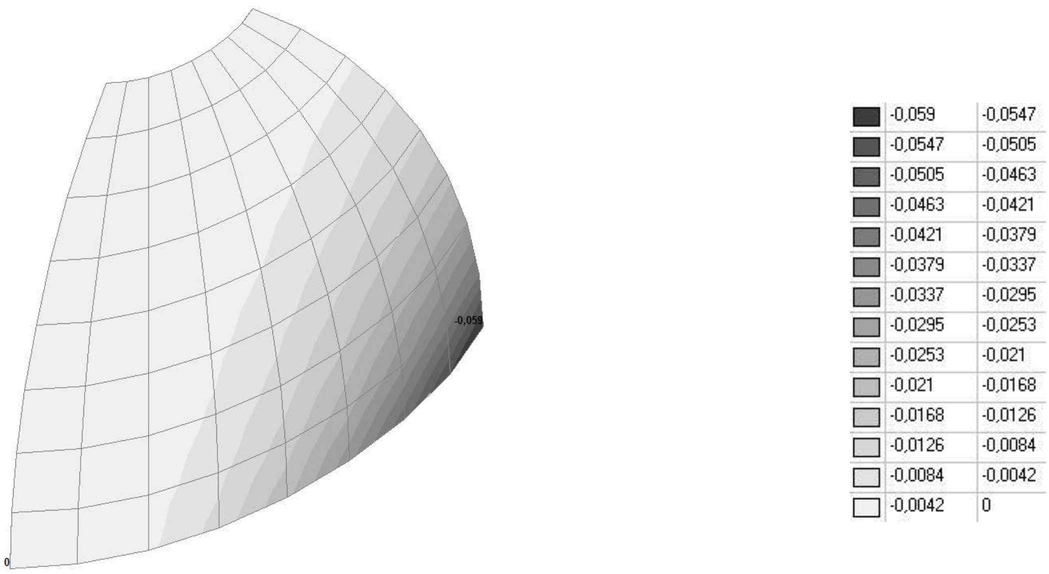


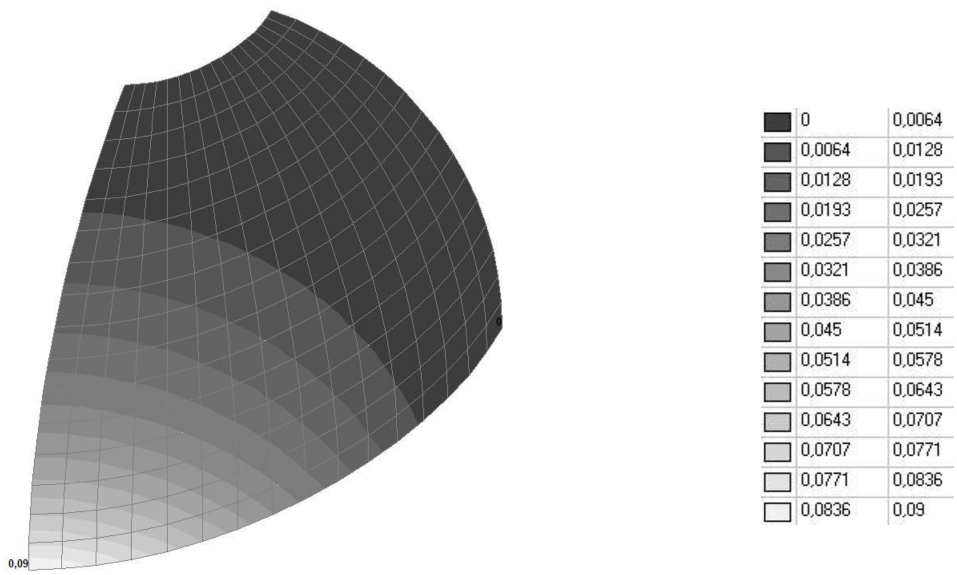

-0,0014 -0,0013
-0,0013 -0,0011
-0,0011 -0,001
-0,001 -0,0009
-0,0009 -0,0007
-0,0007 -0,0006
-0,0006 -0,0005
-0,0005 -0,0003
-0,0003 -0,0002
-0,0002 0
0 0
0 0,0002
0,0002 0,0004
0,0004 0,0005

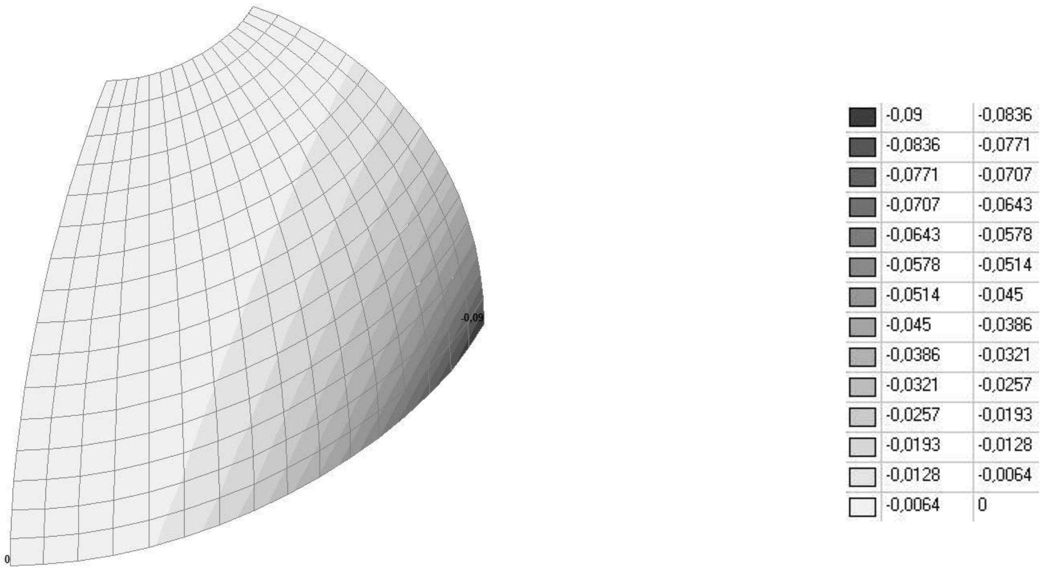

-0,0006 0,0002
0,0002 0,0009
0,0009 0,0017
0,0017 0,0024
0,0024 0,0032
0,0032 0,0039
0,0039 0,0047
0,0047 0,0054
0,0054 0,0062
0,0062 0,0069
0,0069 0,0077
0,0077 0,0084
0,0084 0,0092
0,0092 0,01

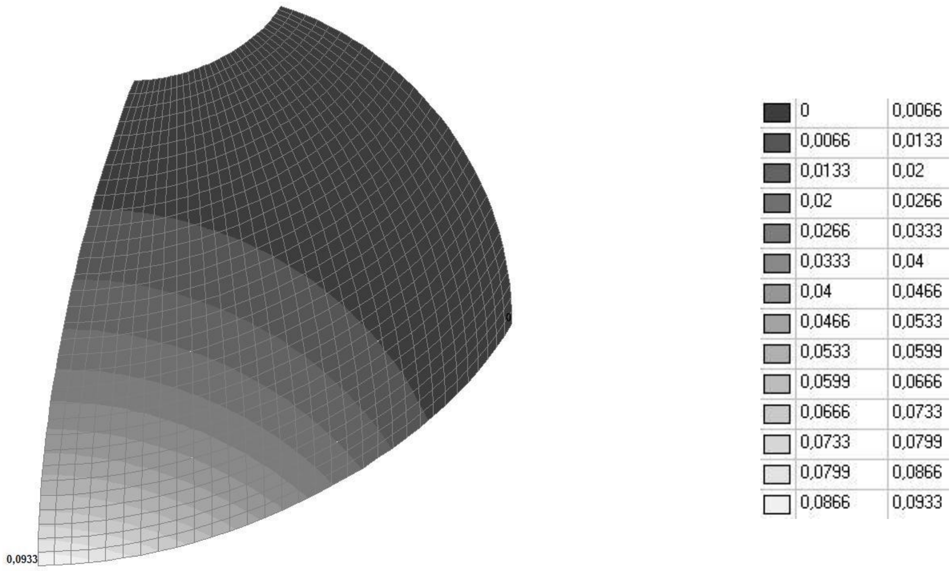

-0,01 -0,0092
-0,0092 -0,0084
-0,0084 -0,0077
-0,0077 -0,0069
-0,0069 -0,0062
-0,0062 -0,0054
-0,0054 -0,0047
-0,0047 -0,0039
-0,0039 -0,0032
-0,0032 -0,0024
-0,0024 -0,0017
-0,0017 -0,0009
-0,0009 -0,0002
-0,0002 0,0006

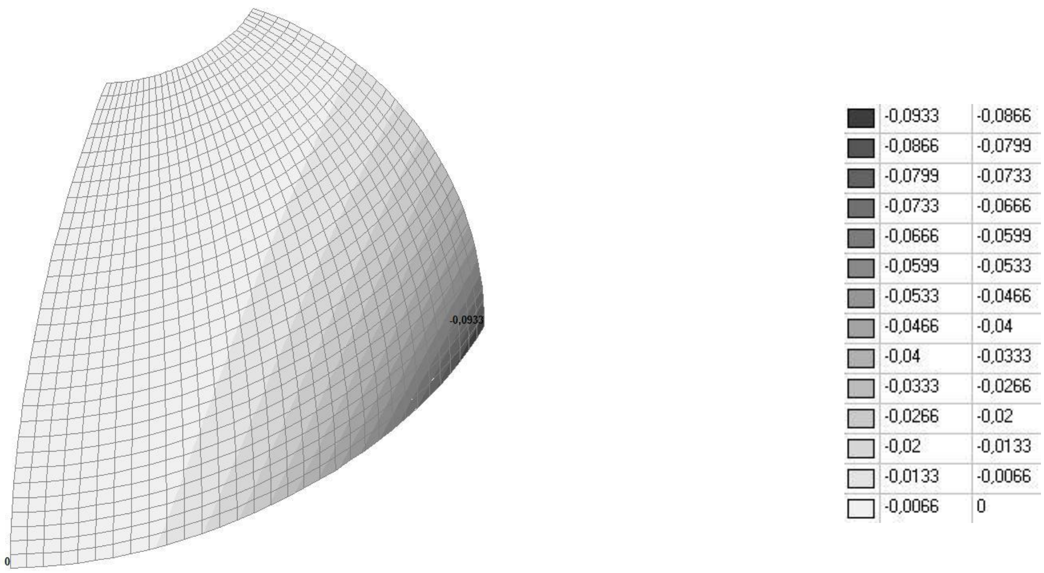

0 0,0042
0,0042 0,0084
0,0084 0,0126
0,0126 0,0168
0,0168 0,021
0,021 0,0253
0,0253 0,0295
0,0295 0,0337
0,0337 0,0379
0,0379 0,0421
0,0421 0,0463
0,0463 0,0505
0,0505 0,0547
0,0547 0,0589
0,0589
0

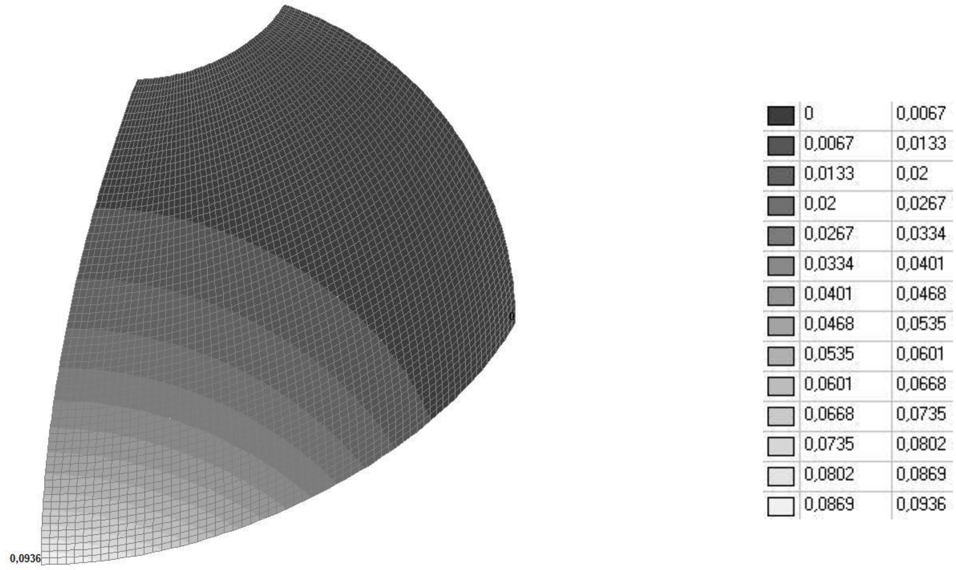

-0,059 -0,0547
-0,0547 -0,0505
-0,0505 -0,0463
-0,0463 -0,0421
-0,0421 -0,0379
-0,0379 -0,0337
-0,0337 -0,0295
-0,0295 -0,0253
-0,0253 -0,021
-0,021 -0,0168
-0,0168 -0,0126
-0,0126 -0,0084
-0,0084 -0,0042
-0,0042 0
-0,059
0

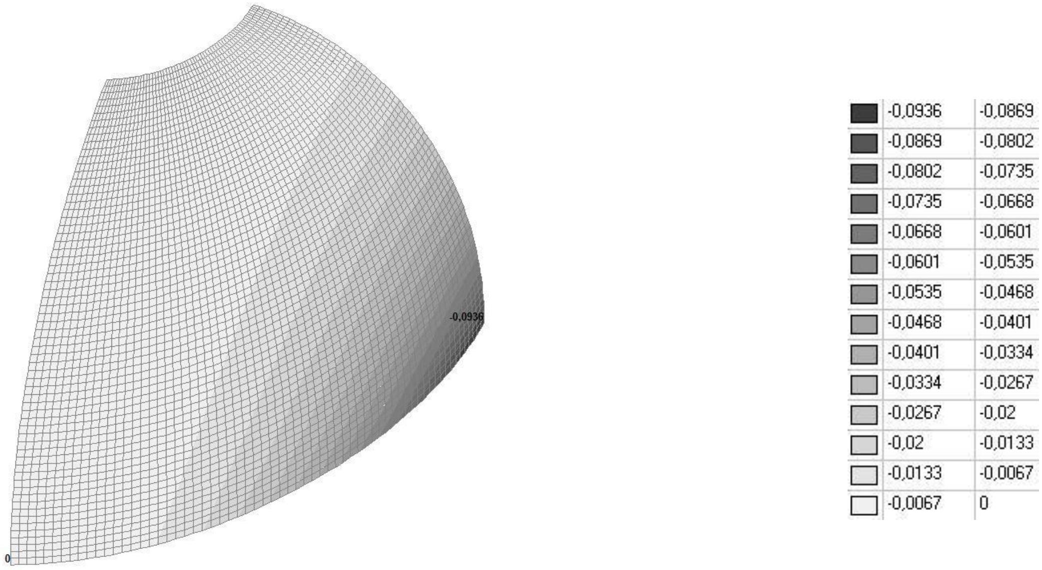

0 0,0064
0,0064 0,0128
0,0128 0,0193
0,0193 0,0257
0,0257 0,0321
0,0321 0,0386
0,0386 0,045
0,045 0,0514
0,0514 0,0578
0,0578 0,0643
0,0643 0,0707
0,0707 0,0771
0,0771 0,0836
0,0836 0,09
0,09
0

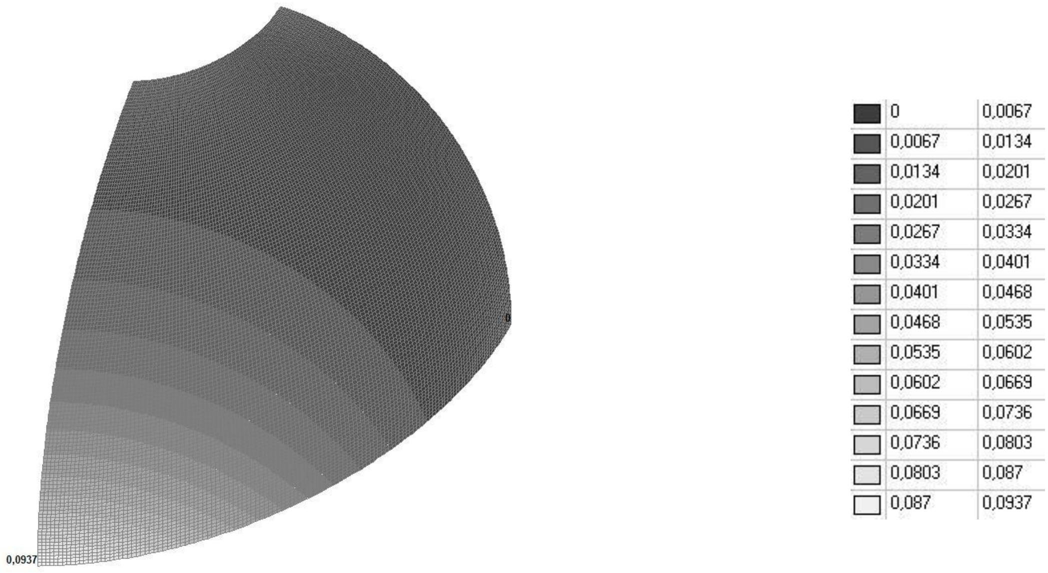

-0,09
0
-0,09 -0,0836
-0,0836 -0,0771
-0,0771 -0,0707
-0,0707 -0,0643
-0,0643 -0,0578
-0,0578 -0,0514
-0,0514 -0,045
-0,045 -0,0386
-0,0386 -0,0321
-0,0321 -0,0257
-0,0257 -0,0193
-0,0193 -0,0128
-0,0128 -0,0064
-0,0064 0

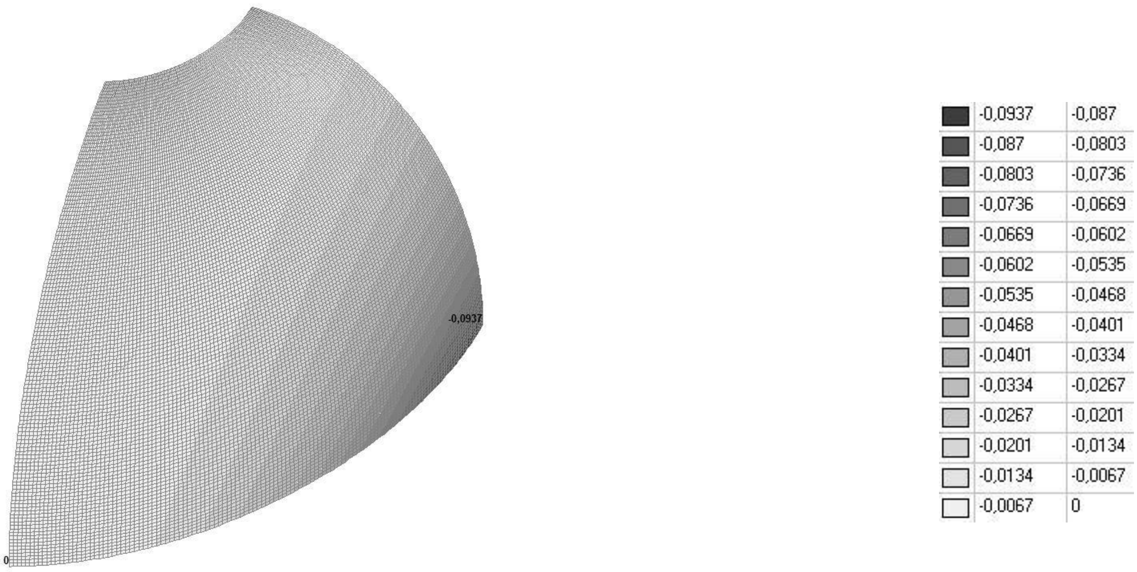

0
0,0933
0 0,0066
0,0066 0,0133
0,0133 0,02
0,02 0,0266
0,0266 0,0333
0,0333 0,04
0,04 0,0466
0,0466 0,0533
0,0533 0,0599
0,0599 0,0666
0,0666 0,0733
0,0733 0,0799
0,0799 0,0866
0,0866 0,0933


-0,0933
0
-0,0933 -0,0866
-0,0866 -0,0799
-0,0799 -0,0733
-0,0733 -0,0666
-0,0666 -0,0599
-0,0599 -0,0533
-0,0533 -0,0466
-0,0466 -0,04
-0,04 -0,0333
-0,0333 -0,0266
-0,0266 -0,02
-0,02 -0,0133
-0,0133 -0,0066
-0,0066 0


0,0936
0
0 0,0067
0,0067 0,0133
0,0133 0,02
0,02 0,0267
0,0267 0,0334
0,0334 0,0401
0,0401 0,0468
0,0468 0,0535
0,0535 0,0601
0,0601 0,0668
0,0668 0,0735
0,0735 0,0802
0,0802 0,0869
0,0869 0,0936


-0,0936
0
-0,0936 -0,0869
-0,0869 -0,0802
-0,0802 -0,0735
-0,0735 -0,0668
-0,0668 -0,0601
-0,0601 -0,0535
-0,0535 -0,0468
-0,0468 -0,0401
-0,0401 -0,0334
-0,0334 -0,0267
-0,0267 -0,02
-0,02 -0,0133
-0,0133 -0,0067
-0,0067 0


0,0937
0
0 0,0067
0,0067 0,0134
0,0134 0,0201
0,0201 0,0267
0,0267 0,0334
0,0334 0,0401
0,0401 0,0468
0,0468 0,0535
0,0535 0,0602
0,0602 0,0669
0,0669 0,0736
0,0736 0,0803
0,0803 0,087
0,087 0,0937



Model 6. Values of the displacements in the direction of the pairs of tensile forces and the pairs of compressive forces along the X and Y axes of the global coordinate system respectively w_{FX} and w_{FY} (m, m)

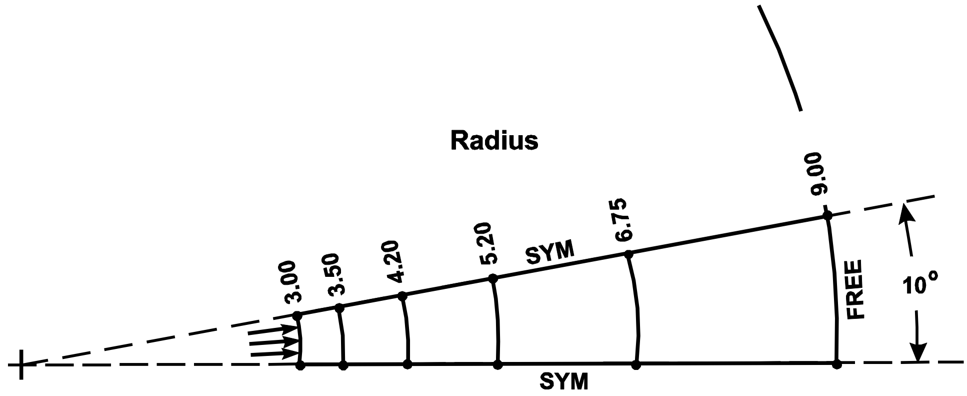
Comparison of solutions:

Displacements in the direction of the pairs of radial tensile forces and the pairs of radial compressive forces F_X and F_Y along the X and Y axes of the global coordinate system respectively w_{FX} and w_{FY} (m, m)

Model	Finite element mesh	Theory	SCAD	Deviation, %
1 (Member type 42)	2x2	+0.0940 –0.0940	+0.0828 –0.0862	11.91 8.30
	4x4		+0.0902 –0.0919	4.04 2.23
	8x8		+0.0917 –0.0922	2.45 1.91
	16x16		+0.0920 –0.0922	2.13 1.91
	32x32		+0.0927 –0.0928	1.38 1.28
2 (Member type 44)	2x2	+0.0940 –0.0940	+0.0924 –0.0924	1.70 1.70
	4x4		+0.0938 –0.0938	0.21 0.21
	8x8		+0.0930 –0.0930	1.06 1.06
	16x16		+0.0928 –0.0928	1.28 1.28
	32x32		+0.0932 –0.0932	0.85 0.85
3 (Member type 45)	2x2	+0.0940 –0.0940	+0.0506 –0.0510	46.17 45.74
	4x4		+0.0389 –0.0395	58.62 57.98
	8x8		+0.0484 –0.0489	48.51 47.98
	16x16		+0.0834 –0.0835	11.28 11.17
	32x32		+0.0927 –0.0927	1.38 1.38
4 (Member type 50)	2x2	+0.0940 –0.0940	+0.0526 –0.0526	44.04 44.04
	4x4		+0.0459	51.17

Model	Finite element mesh	Theory	SCAD	Deviation, %
			–0.0459	51.17
	8x8		+0.0651 –0.0651	30.74 30.74
	16x16		+0.0899 –0.0899	4.36 4.36
	32x32		+0.0932 –0.0932	0.85 0.85
5 (Member type 36)	2x2	+0.0940 –0.0940	+0.0000 –0.0000	100.00 100.00
	4x4		+0.0001 –0.0001	99.89 99.89
	8x8		+0.0003 –0.0003	99.68 99.68
	16x16		+0.0010 –0.0010	98.94 98.94
	32x32		+0.0036 –0.0036	96.17 96.17
	64x64		+0.0126 –0.0126	86.60 86.60
	128x128		+0.0350 –0.0350	62.77 62.77
	256x256		+0.0654 –0.0654	30.43 30.43
	512x512		+0.0842 –0.0842	10.43 10.43
6 (Member type 37)	2x2	+0.0940 –0.0940	+0.0014 –0.0014	98.51 98.51
	4x4		+0.0100 –0.0100	89.36 89.36
	8x8		+0.0589 –0.0590	37.34 37.23
	16x16		+0.0900 –0.0900	4.26 4.26
	32x32		+0.0933 –0.0933	0.74 0.74
	64x64		+0.0936 –0.0936	0.43 0.43
	128x128		+0.0937 –0.0937	0.32 0.32

Nearly Incompressible Thick-Walled Cylinder under Plane Deformation Subjected to Uniformly Distributed Internal Pressure



Objective: Check of the obtained values of radial displacements of the internal surface of a nearly incompressible thick-walled cylinder under plane deformation subjected to uniformly distributed internal pressure.

Initial data files:

File name	Description
Nearly_incompressible_thick_cylinder_Shell_42_ Poisson_ratio_049_.SPR Nearly_incompressible_thick_cylinder_Shell_42_ Poisson_ratio_0499_.SPR Nearly_incompressible_thick_cylinder_Shell_42_ Poisson_ratio_04999_.SPR	Design model with the elements of type 42 for a material with Poisson's ratio 0.49, 0.499, 0.4999
Nearly_incompressible_thick_cylinder_Shell_44_ Poisson_ratio_049_.SPR Nearly_incompressible_thick_cylinder_Shell_44_ Poisson_ratio_0499_.SPR Nearly_incompressible_thick_cylinder_Shell_44_ Poisson_ratio_04999_.SPR	Design model with the elements of type 44 for a material with Poisson's ratio 0.49, 0.499, 0.4999
Nearly_incompressible_thick_cylinder_Shell_45_ Poisson_ratio_049_.SPR Nearly_incompressible_thick_cylinder_Shell_45_ Poisson_ratio_0499_.SPR Nearly_incompressible_thick_cylinder_Shell_45_ Poisson_ratio_04999_.SPR	Design model with the elements of type 45 for a material with Poisson's ratio 0.49, 0.499, 0.4999
Nearly_incompressible_thick_cylinder_Shell_50_ Poisson_ratio_049_.SPR Nearly_incompressible_thick_cylinder_Shell_50_ Poisson_ratio_0499_.SPR Nearly_incompressible_thick_cylinder_Shell_50_ Poisson_ratio_04999_.SPR	Design model with the elements of type 50 for a material with Poisson's ratio 0.49, 0.499, 0.4999
Nearly_incompressible_thick_cylinder_ Solid _36_ Poisson_ratio_049_.SPR Nearly_incompressible_thick_cylinder_ Solid _36_ Poisson_ratio_0499_.SPR Nearly_incompressible_thick_cylinder_ Solid _36_ Poisson_ratio_04999_.SPR	Design model with the elements of type 36 for a material with Poisson's ratio 0.49, 0.499, 0.4999
Nearly_incompressible_thick_cylinder_ Solid _37_ Poisson_ratio_049_.SPR Nearly_incompressible_thick_cylinder_ Solid _37_ Poisson_ratio_0499_.SPR Nearly_incompressible_thick_cylinder_ Solid _37_ Poisson_ratio_04999_.SPR	Design model with the elements of type 37 for a material with Poisson's ratio 0.49, 0.499, 0.4999

***Problem formulation*:** The nearly incompressible thick-walled cylinder is under plane deformation and is subjected to the uniformly distributed internal pressure p. Check the obtained values of the radial displacements of the internal surface u.

References: R. H. Macneal, R. L. Harder, A proposed standard set of problems to test finite element accuracy, North-Holland, Finite elements in analysis and design, 1, 1985, p. 3-20.

Initial data:

E = 1000 kPa	- elastic modulus of the material of the thick-walled cylinder;
ν = 0.49; 0.499; 0.4999	- Poisson's ratio;
R_i = 3.00 m	- radius of the internal surface of the thick-walled cylinder;
R_e = 9.00 m	- radius of the external surface of the thick-walled cylinder;
p = 1.0 kPa	- values of the uniformly distributed internal pressure.

***Finite element model*:** Design model – general type system. Six design models of a sector of the thick-walled cylinder with the thickness of 1.00 m and a central angle θ = 10° according to the symmetry conditions are considered:

Model 1 – 10 three-node shell elements of type 42 of unequal sizes with the spacing of the mesh in the radial direction 3.00 m, 3.50 m, 4.20 m, 5.20 m, 6.75 m, 9.00 m . Boundary conditions are provided by introducing 12 space truss bar elements of type 4 of high axial stiffness (EF = 10^6 kN) in the tangential direction (orthogonal to the lateral surfaces of the sector). Constraints in the directions of the degrees of freedom X, Y, Z are imposed on the support nodes of the bar elements. The dimensional stability is provided by imposing constraints on the lateral surfaces of the sector in the directions of the degrees of freedom Z, UZ. The load uniformly distributed along the line p = 1.0 kN/m is applied to the element on the internal surface of the cylinder. Number of nodes in the model – 24.

Model 2 – 5 four-node shell elements of type 44 of unequal sizes with the spacing of the mesh in the radial direction 3.00 m, 3.50 m, 4.20 m, 5.20 m, 6.75 m, 9.00 m . Boundary conditions are provided by introducing 12 space truss bar elements of type 4 of high axial stiffness (EF = 10^6 kN) in the tangential direction (orthogonal to the lateral surfaces of the sector). Constraints in the directions of the degrees of freedom X, Y, Z are imposed on the support nodes of the bar elements. The dimensional stability is provided by imposing constraints on the lateral surfaces of the sector in the directions of the degrees of freedom Z, UZ. The load uniformly distributed along the line p = 1.0 kN/m is applied to the element on the internal surface of the cylinder. Number of nodes in the model – 24.

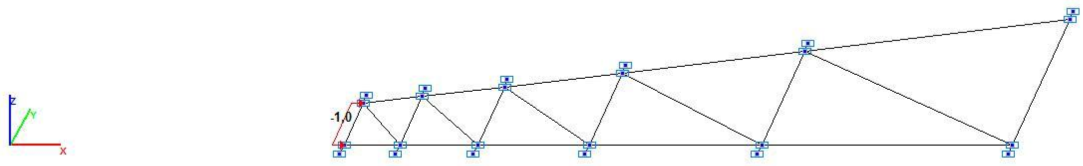
Model 3 – 10 six-node shell elements of type 45 of unequal sizes with the spacing of the mesh in the radial direction 3.00 m, 3.50 m, 4.20 m, 5.20 m, 6.75 m, 9.00 m. Boundary conditions are provided by introducing 22 space truss bar elements of type 4 of high axial stiffness (EF = 10^6 kN) in the tangential direction (orthogonal to the lateral surfaces of the sector). Constraints in the directions of the degrees of freedom X, Y, Z are imposed on the support nodes of the bar elements. The dimensional stability is provided by imposing constraints on the lateral surfaces of the sector in the directions of the degrees of freedom Z, UZ. The load uniformly distributed along the line p = 1.0 kN/m is applied to the element on the internal surface of the cylinder. Number of nodes in the model – 55.

Model 4 – 5 eight-node shell elements of type 50 of unequal sizes with the spacing of the mesh in the radial direction 3.00 m, 3.50 m, 4.20 m, 5.20 m, 6.75 m, 9.00 m. Boundary conditions are provided by introducing 22 space truss bar elements of type 4 of high axial stiffness (EF = 10^6 kN) in the tangential direction (orthogonal to the lateral surfaces of the sector). Constraints in the directions of the degrees of freedom X, Y, Z are imposed on the support nodes of the bar elements. The dimensional stability is provided by imposing constraints on the lateral surfaces of the sector in the directions of the degrees of freedom Z, UZ. The load uniformly distributed along the line p = 1.0 kN/m is applied to the element on the internal surface of the cylinder. Number of nodes in the model – 50.

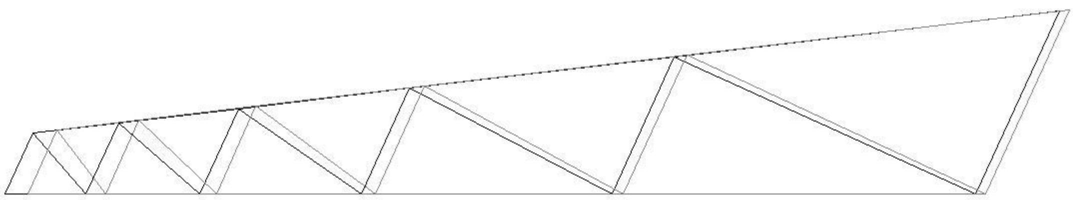
Model 5 – 5 eight-node isoparametric solid elements of type 36 of unequal sizes with the spacing of the mesh in the radial direction 3.00 m, 3.50 m, 4.20 m, 5.20 m, 6.75 m, 9.00 m. Boundary conditions are provided by introducing 24 space truss bar elements of type 4 of high axial stiffness (EF = 10^6 kN) in the tangential direction (orthogonal to the lateral surfaces of the sector). Constraints in the directions of the degrees of freedom X, Y, Z are imposed on the support nodes of the bar elements. The dimensional stability is provided by imposing constraints on the lateral surfaces of the sector in the direction of the degree of freedom Z. The load uniformly distributed over the face p = 1.0 kN/m^2 is applied to the element on the internal surface of the cylinder. Number of nodes in the model – 50.

Model 6 – 5 twenty-node isoparametric solid elements of type 37 of unequal sizes with the spacing of the mesh in the radial direction 3.00 m, 3.50 m, 4.20 m, 5.20 m, 6.75 m, 9.00 m. Boundary conditions are provided by introducing 56 space truss bar elements of type 4 of high axial stiffness (EF = 10^6 kN) in the tangential direction (orthogonal to the lateral surfaces of the sector). Constraints in the directions of the degrees of freedom X, Y, Z are imposed on the support nodes of the bar elements. The dimensional stability is provided by imposing constraints on the lateral surfaces of the sector in the direction of the degree of freedom Z. The load uniformly distributed over the face p = 1.0 kN/m^2 is applied to the element on the internal surface of the cylinder. Number of nodes in the model – 124.

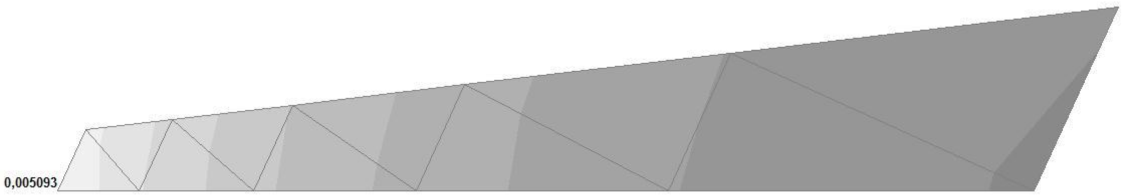
Results in SCAD

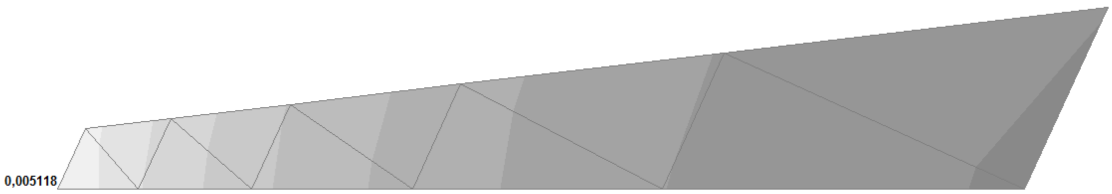


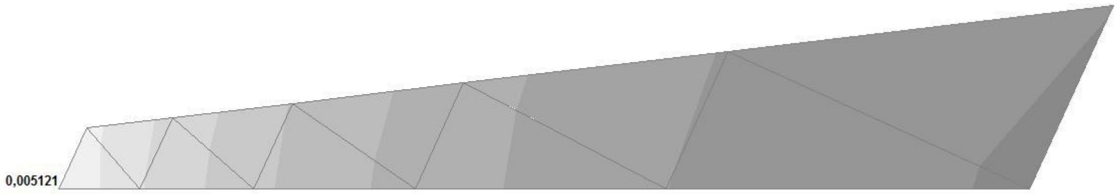
Model 1. Design model



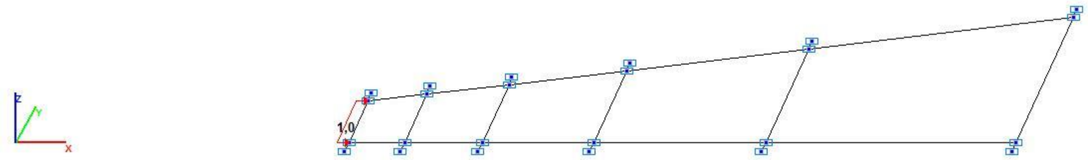
Model 1. Deformed model



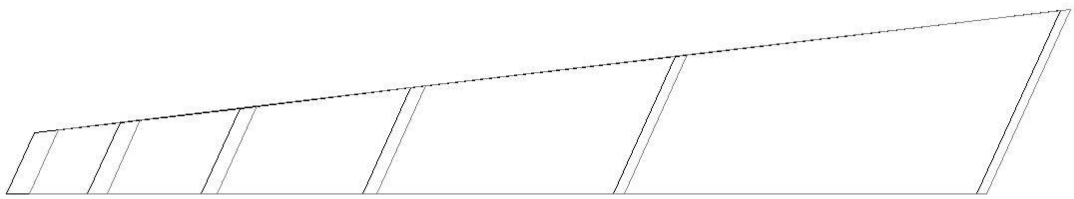




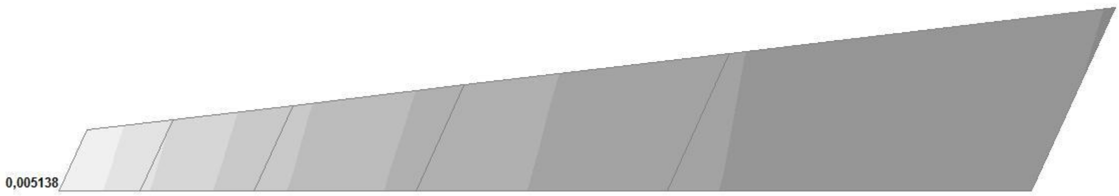
Model 1. Values of the displacements in the direction of the X axis of the global coordinate system (m) for the materials of the thick-walled cylinder with Poisson's ratio 0.49; 0.499; 0.4999

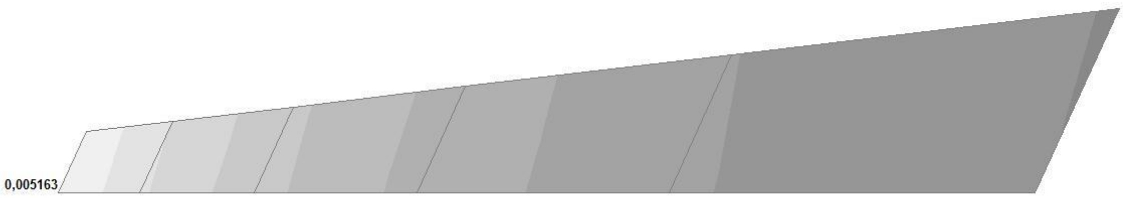


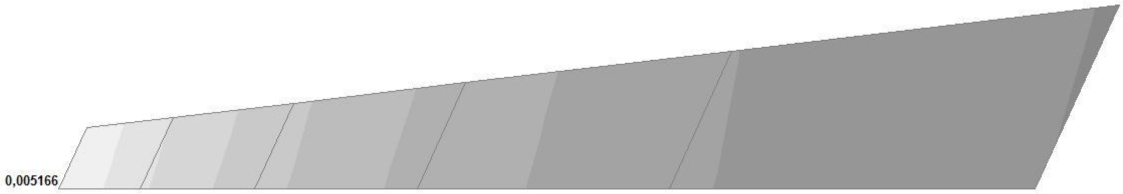
Model 2. Design model



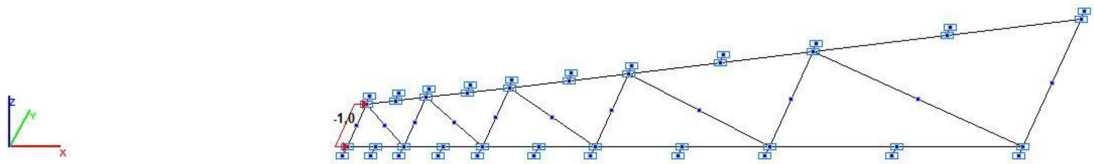
Model 2. Deformed model



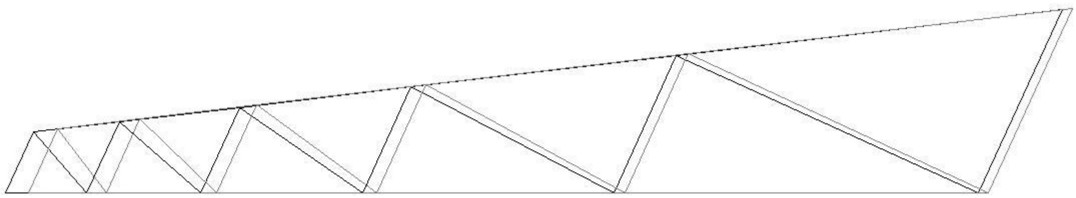




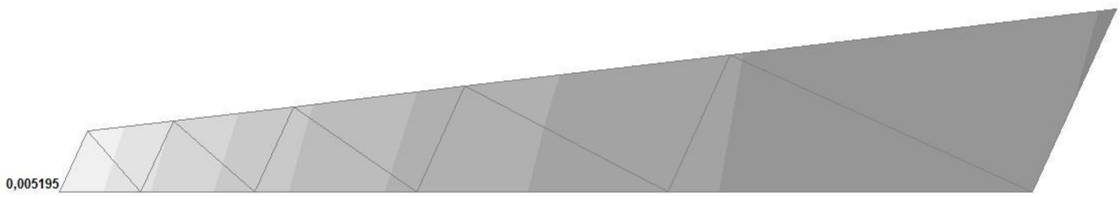
Model 2. Values of the displacements in the direction of the X axis of the global coordinate system (m) for the materials of the thick-walled cylinder with Poisson's ratio 0.49; 0.499; 0.4999

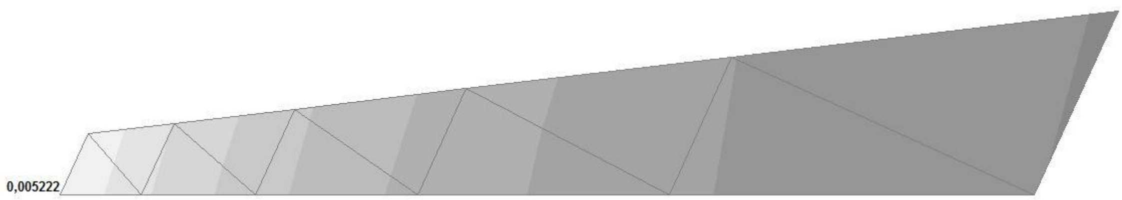


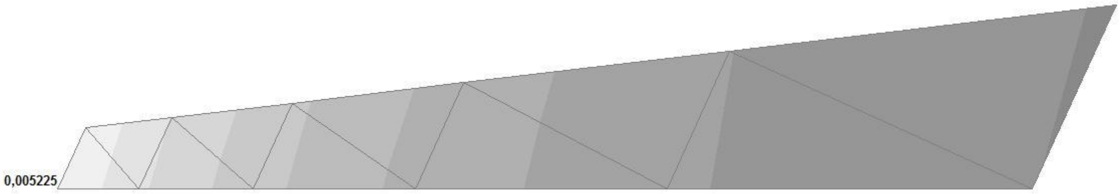
Model 3. Design model



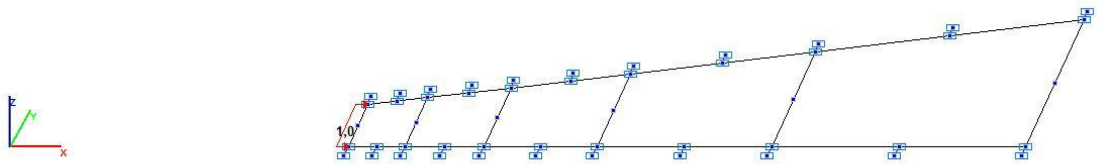
Model 3. Deformed model



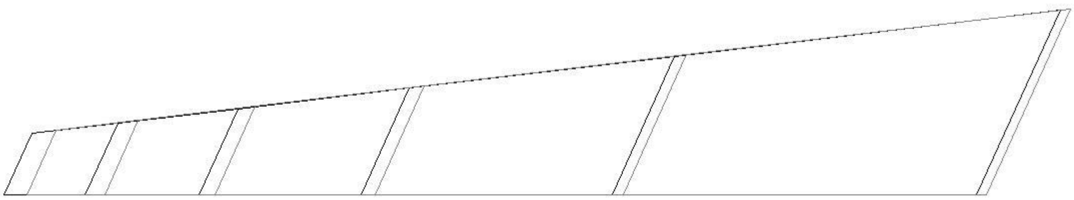




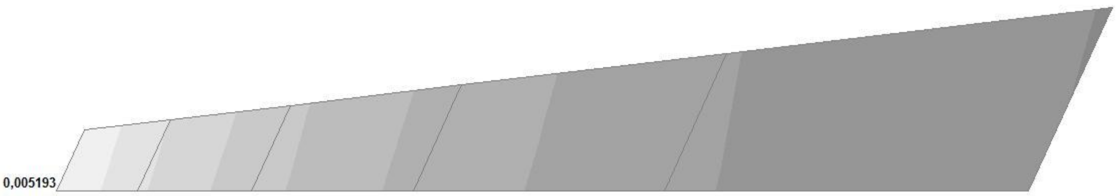
Model 3. Values of the displacements in the direction of the X axis of the global coordinate system (m) for the materials of the thick-walled cylinder with Poisson's ratio 0.49; 0.499; 0.4999

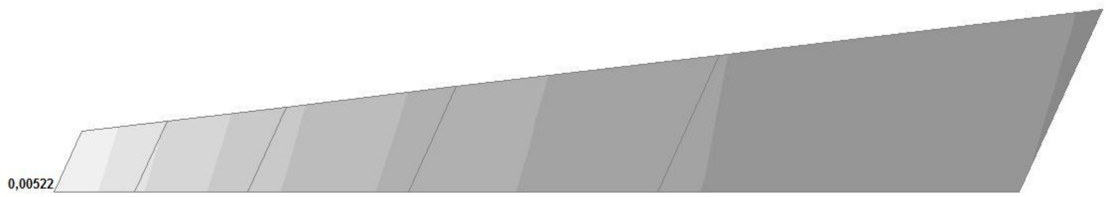


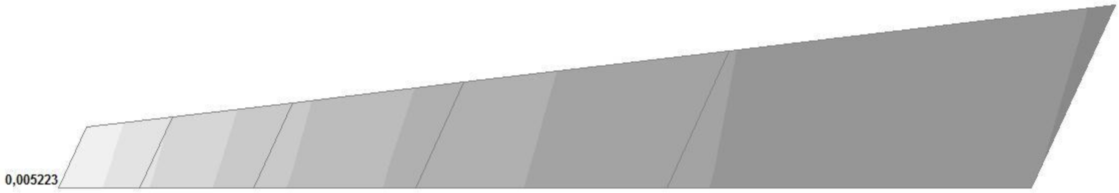
Model 4. Design model



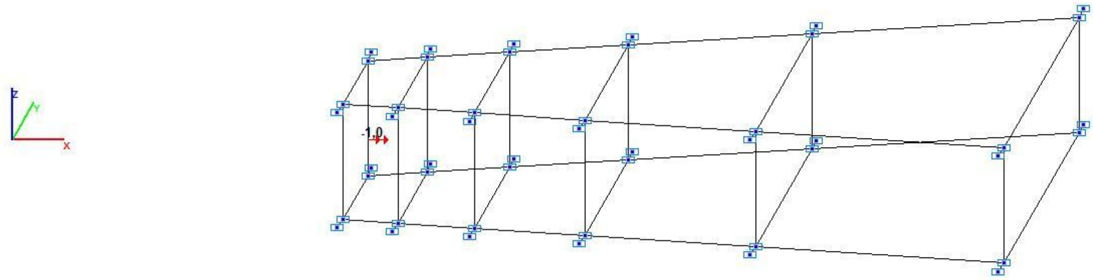
Model 4. Deformed model



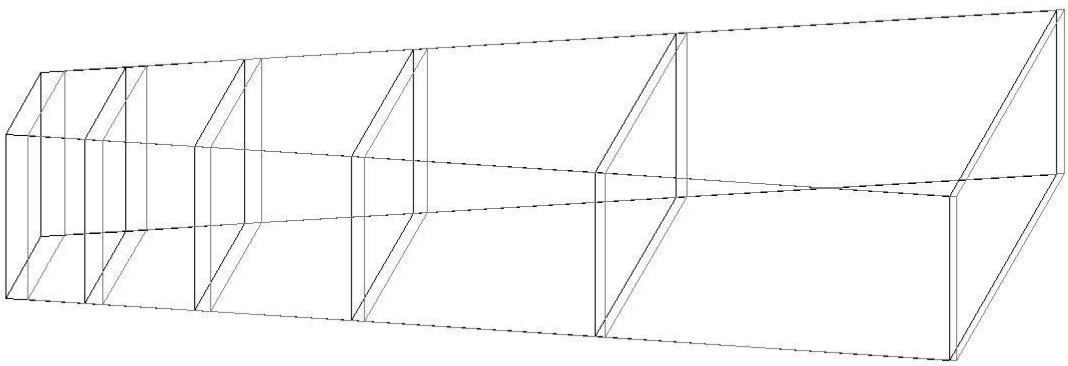




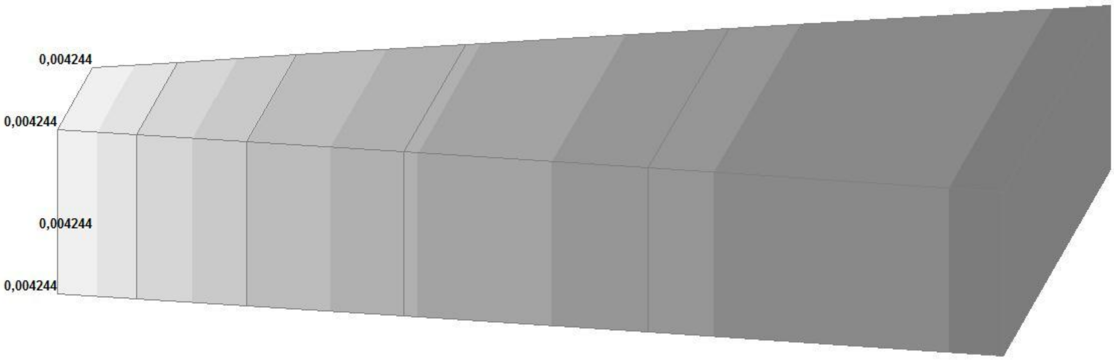
Model 4. Values of the displacements in the direction of the X axis of the global coordinate system (m) for the materials of the thick-walled cylinder with Poisson's ratio 0.49; 0.499; 0.4999

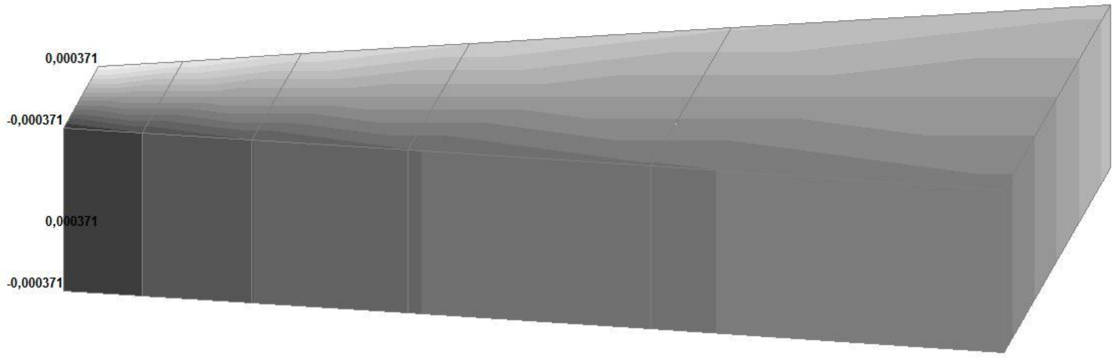


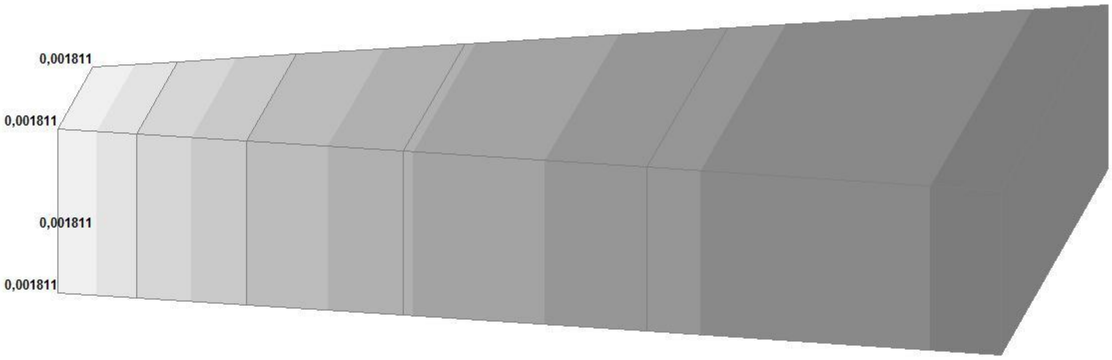
Model 5. Design model

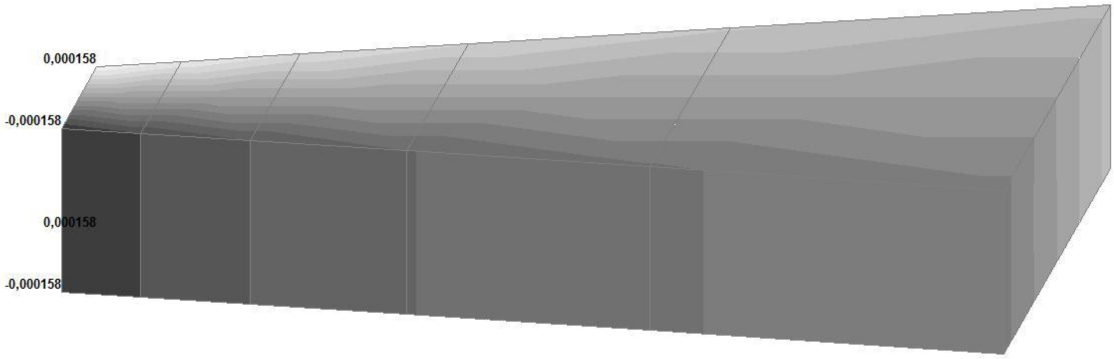


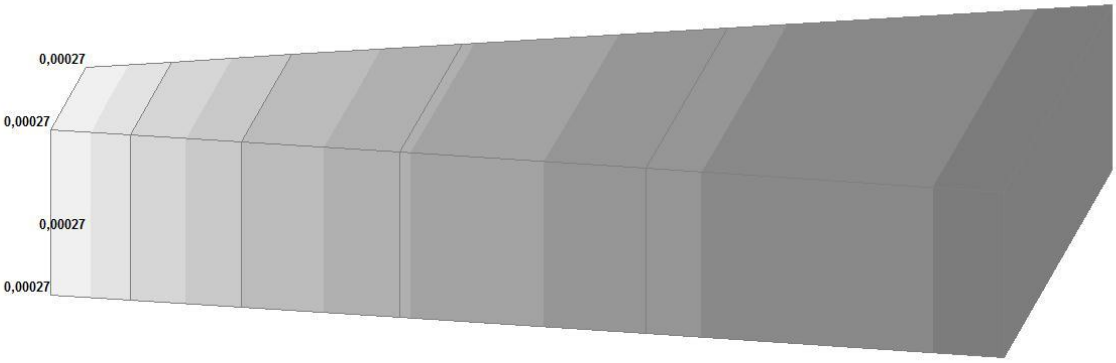
Model 5. Deformed model

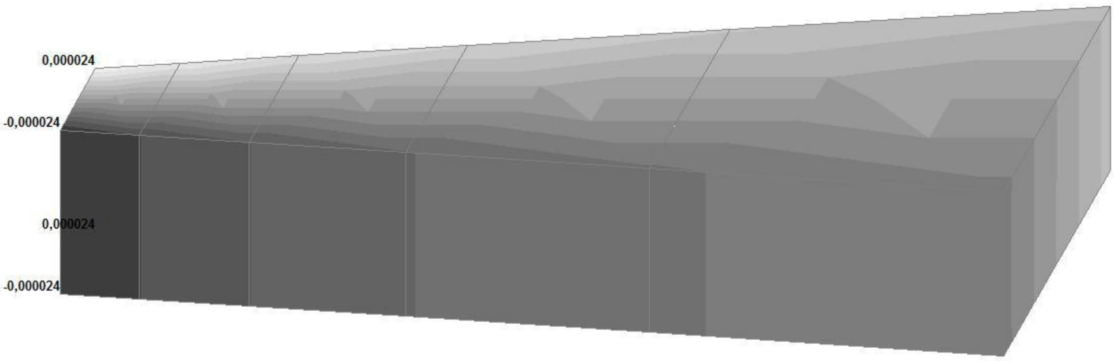



0,000371
-0,000371
0,000371
-0,000371

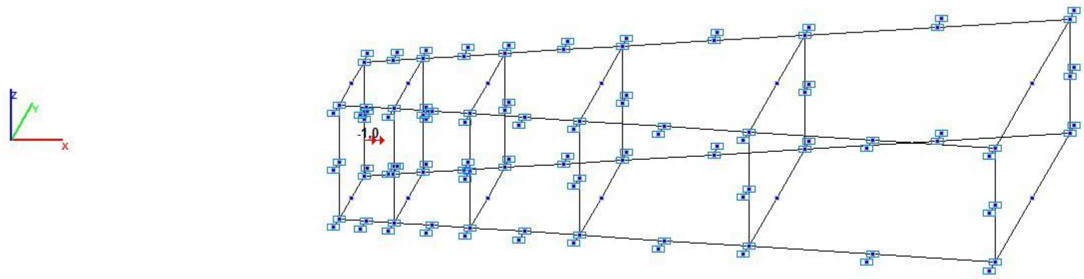

0,001811
0,001811
0,001811
0,001811


0,000158
-0,000158
0,000158
-0,000158

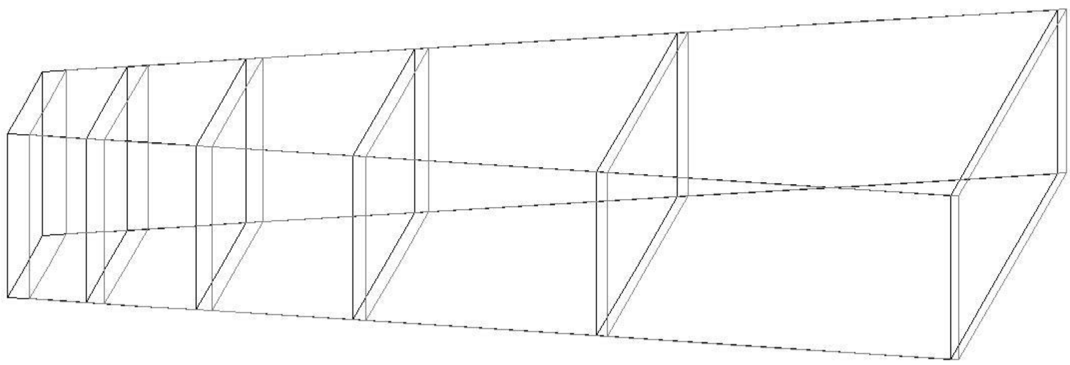

0,00027
0,00027
0,00027
0,00027



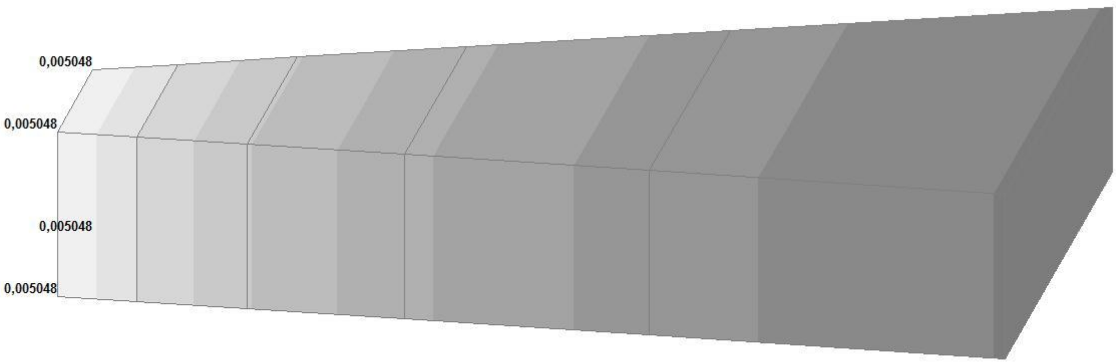
Model 5. Values of the displacements in the directions of the X and Y axes of the global coordinate system (m, m) for the materials of the thick-walled cylinder with Poisson's ratio 0.49; 0.499; 0.4999

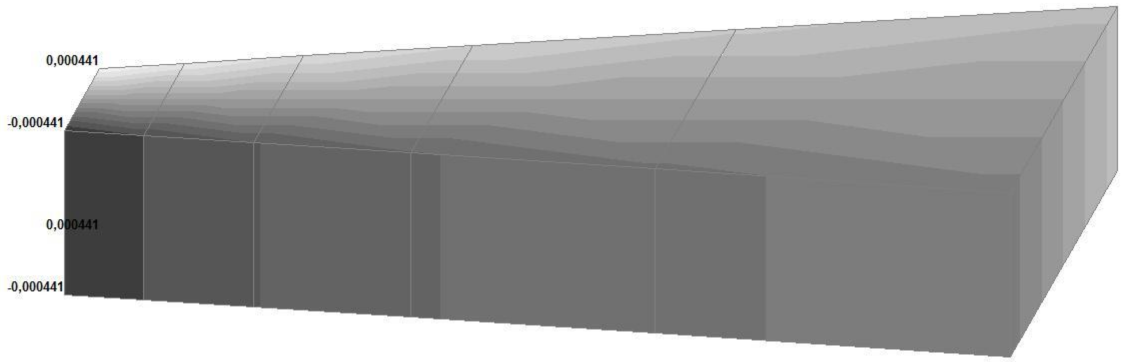


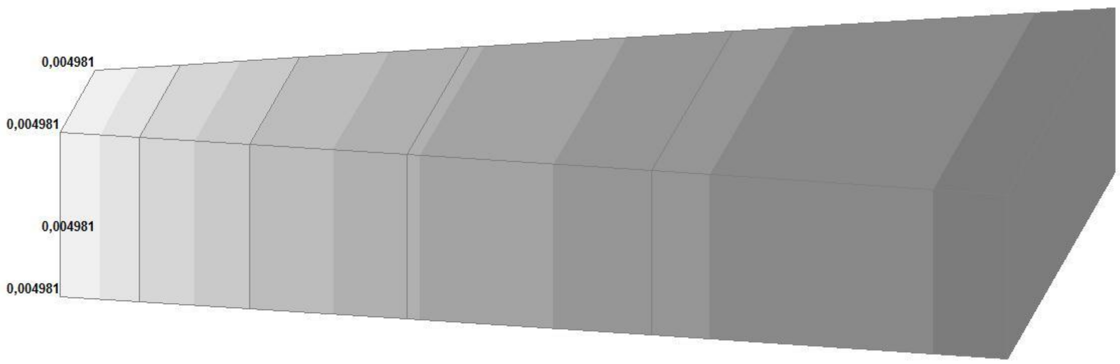
Model 6.Design model

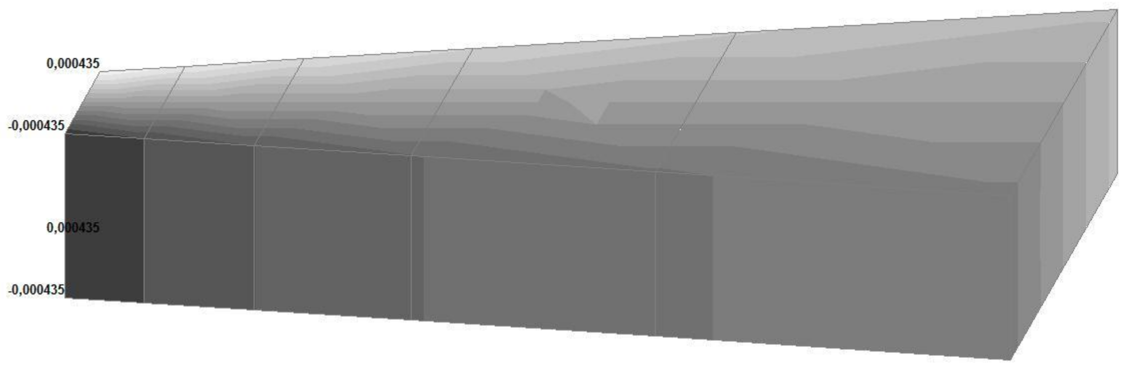


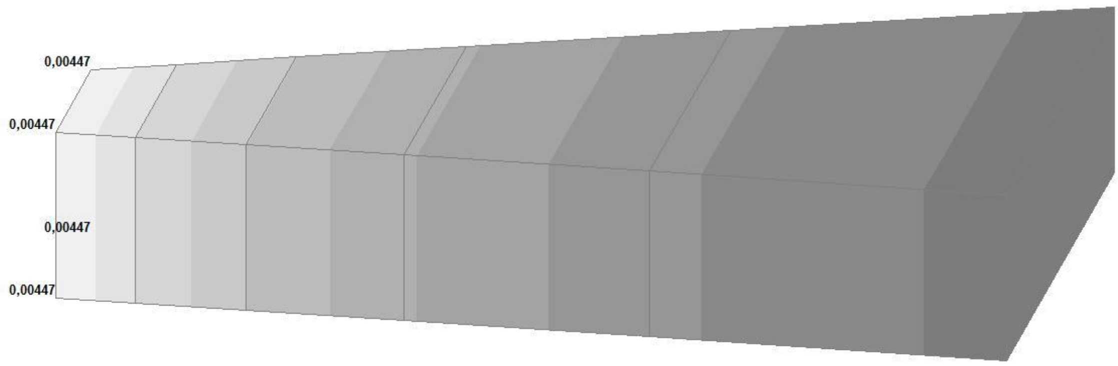
Model 6. Deformed model

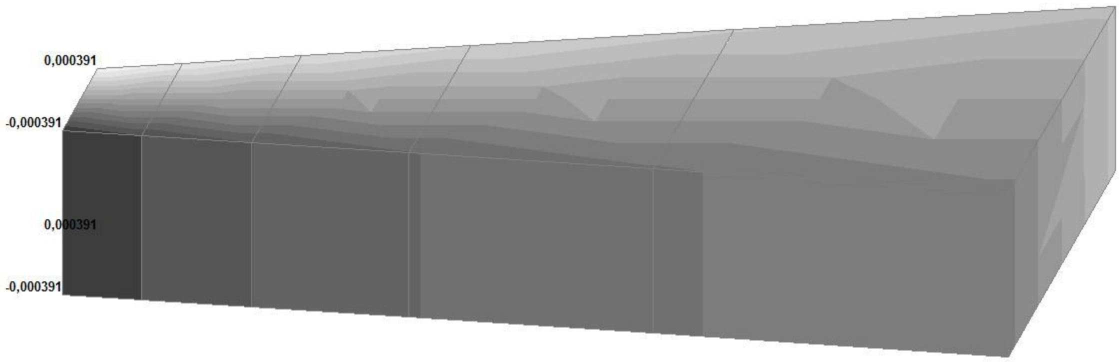



0,000441
-0,000441
0,000441
-0,000441


0,004981
0,004981
0,004981
0,004981


0,000435
-0,000435
0,000435
-0,000435


0,00447
0,00447
0,00447
0,00447



Model 6. Values of the displacements in the directions of the X and Y axes of the global coordinate system (m, m) for the materials of the thick-walled cylinder with Poisson's ratio 0.49; 0.499; 0.4999

Comparison of solutions:

Radial displacements of the internal surface of the thick-walled cylinder u (m) for the materials with Poisson's ratios 0.49; 0.499; 0.4999

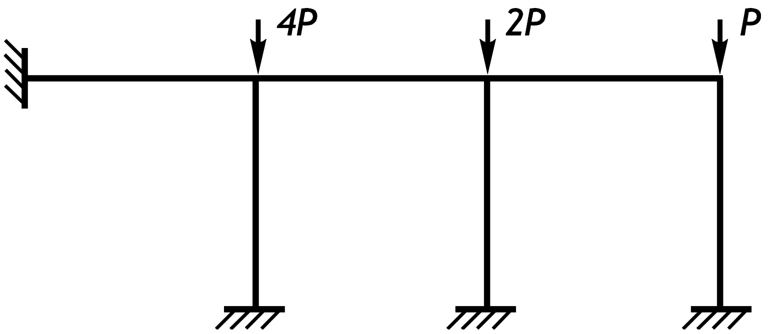
Model	Poisson's ratio	Theory	SCAD	Deviation, %
1 (Member type 42)	0.49	0.005040	0.005093	1.05
	0.499	0.005060	0.005118	1.15
	0.4999	0.005062	0.005121	1.17
2 (Member type 44)	0.49	0.005040	0.005138	1.94
	0.499	0.005060	0.005163	2.04
	0.4999	0.005062	0.005166	2.05
3 (Member type 45)	0.49	0.005040	0.005195	3.08
	0.499	0.005060	0.005222	3.20
	0.4999	0.005062	0.005225	3.22
4 (Member type 50)	0.49	0.005040	0.005193	3.04
	0.499	0.005060	0.005222	3.20
	0.4999	0.005062	0.005223	3.18
5 (Member type 36)	0.49	0.005040	$\sqrt{(0.004244^2 + 0.000371^2)} = 0.004260$	15.48
	0.499	0.005060	$\sqrt{(0.001811^2 + 0.000158^2)} = 0.001818$	64.07
	0.4999	0.005062	$\sqrt{(0.000270^2 + 0.000024^2)} = 0.000271$	94.65
6 (Member type 37)	0.49	0.005040	$\sqrt{(0.005048^2 + 0.000441^2)} = 0.005067$	0.54
	0.499	0.005060	$\sqrt{(0.004981^2 + 0.000435^2)} = 0.005000$	1.19
	0.4999	0.005062	$\sqrt{(0.004470^2 + 0.000391^2)} = 0.004487$	11.36

Notes: In the analytical solution the radial displacements of the internal surface of the nearly incompressible thick-walled cylinder under the plane deformation u from the uniformly distributed internal pressure are determined according to the following formulas:

$$u = \frac{(1+\nu)\cdot p\cdot R_i^{\,2}}{E\cdot\left(R_e^{\,2} - R_i^{\,2}\right)}\cdot\left[\frac{R_e^{\,2}}{R_i} + (1-2\cdot\nu)\cdot R_i\right].$$

Energy Analysis

Frame Subjected to Various Vertical Forces



Objective: Verification of the determination of elements with forced or constricted deformation at buckling.

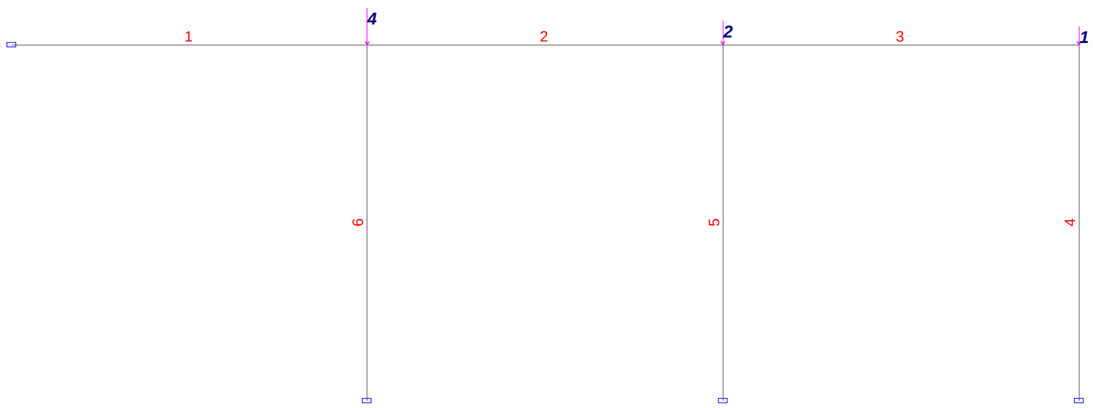
Initial data file: **Energy94A.SPR**

Problem formulation: The plane frame is subjected to different vertical nodal forces. Find elements with positive and negative energy for the first buckling mode.

References: Perelmuter A.V., Slivker V.I., *Design Models of Structures and Possibilities of Their Analysis*. — M, DMK-Press, 2007, § 9.4.

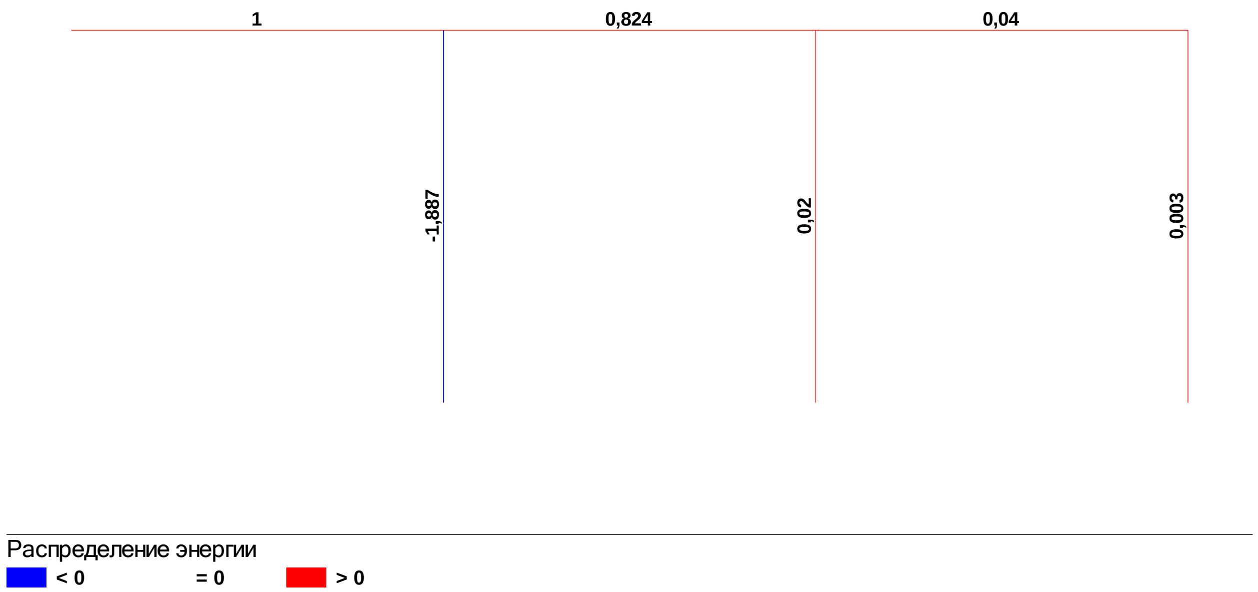
Initial data:
$E = 2.1{\cdot}10^7$ t/m^2 - elastic modulus,
$P = 1$ t - value of the concentrated force.
Bar cross-sections - I-beams No. 40 (bending in the plane of the frame occurs with respect to the axis with the minimum moment I = 667 cm^4, A = 72,6 cm^2, i_y = 3,03 cm, W_y = 86,1 cm^3).

Finite element model: Design model – general type system, 6 bar elements of type 2, 7 nodes.



Design model (with the numbers of elements and loads)

Results in SCAD:

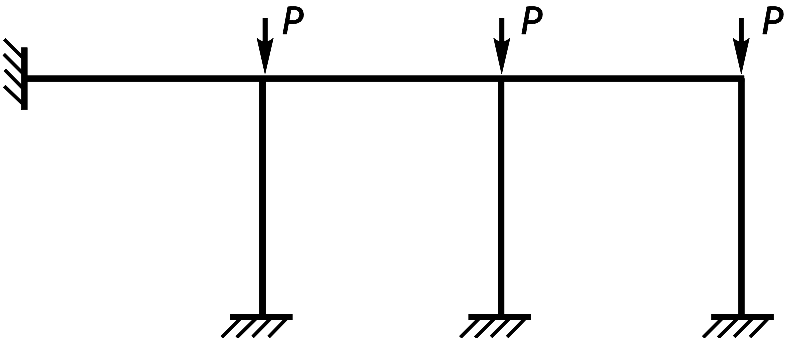


Values of the energy

Comparison of solutions:

Parameter	Theory	SCAD
Numbers of finite elements with the positive energy	1÷5	1÷5
Numbers of finite elements with the negative energy	6	6

Frame Subjected to Vertical Forces



Objective: Verification of the determination of elements with forced or constricted deformation at buckling.

Initial data file: **Energy94B.SPR**

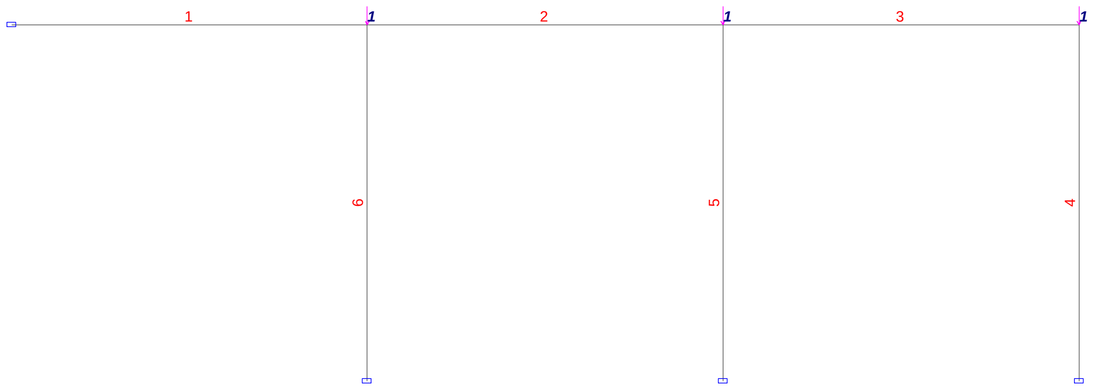
Problem formulation: The plane frame is subjected to vertical nodal forces. Find elements with positive and negative energy for the first buckling mode.

References: Perelmuter A.V., Slivker V.I., Design Models of Structures and Possibilities of Their Analysis. — M, DMK-Press, 2007, § 9.4.

Initial data:

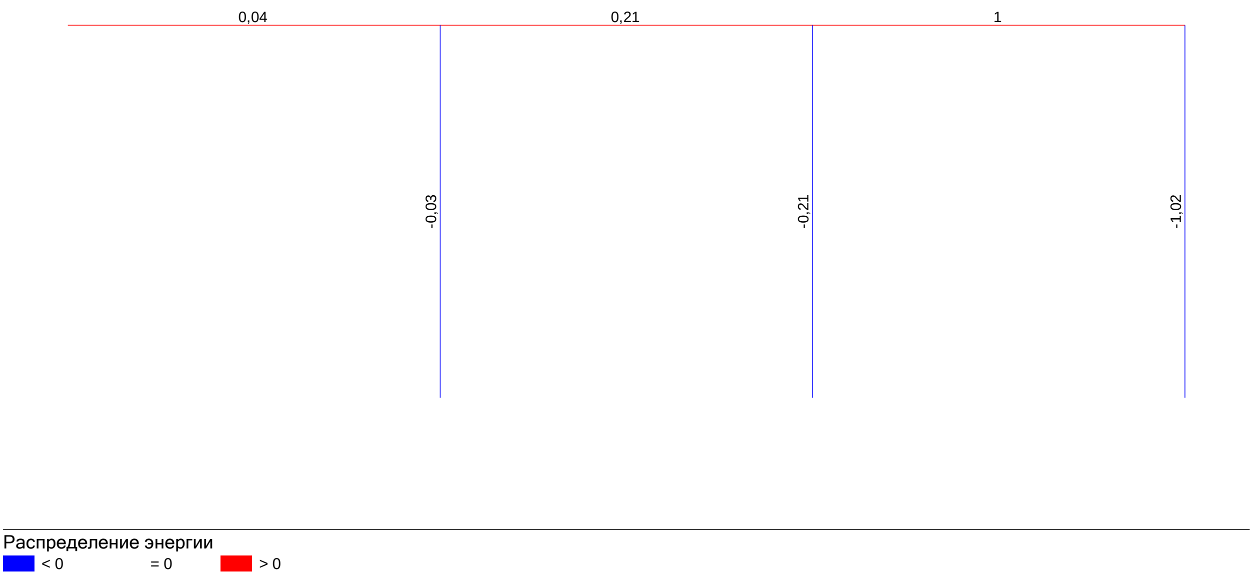
$E = 2.1 \cdot 10^7$ t/m^2 - elastic modulus,
$P = 1$ t - value of the concentrated force.
Bar cross-sections - I-beams No. 40 (bending in the plane of the frame occurs with respect to the axis with the minimum moment I = 667 cm^4, A = 72,6 cm^2, i_y = 3,03 cm, W_y = 86,1 cm^3).

Finite element model: Design model – general type system, 6 bar elements of type 2, 7 nodes.



Design model (with the numbers of elements and loads)

Results in SCAD:

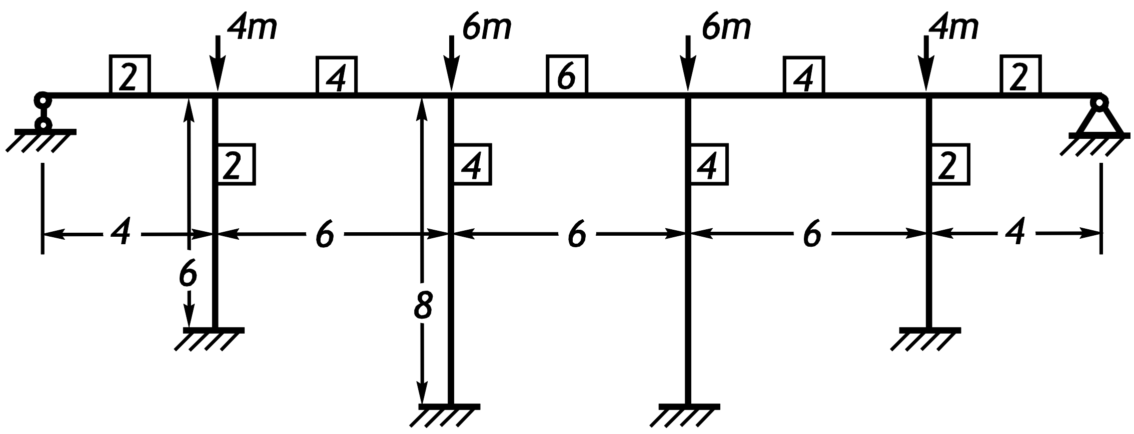


Values of the energy

Comparison of solutions:

Parameter	Theory	SCAD
Numbers of finite elements with the positive energy	1÷3	1÷3
Numbers of finite elements with the negative energy	4÷6	4÷6

Symmetric Frame Subjected to Vertical Forces — Detection of “Weak” Elements



Objective: Verification of the determination of elements with forced or constricted deformation at buckling.

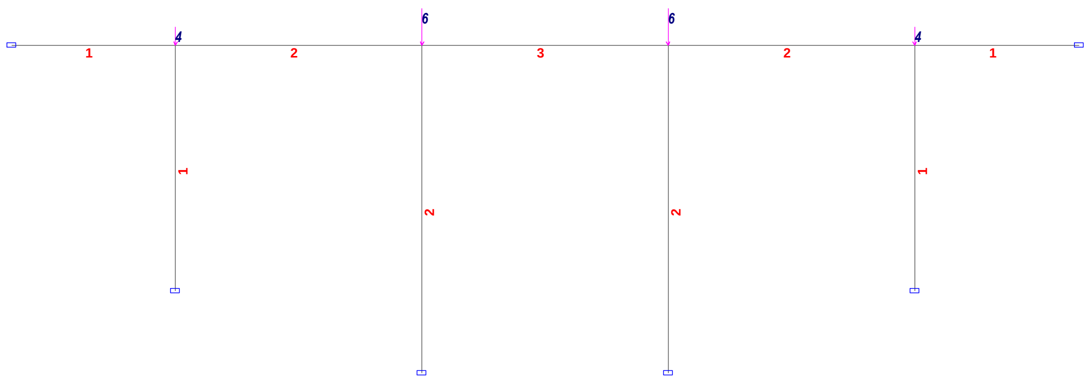
Initial data file: **Energy.SPR**

Problem formulation: The plane frame is subjected to different vertical nodal forces. Find the “weakest” elements in terms of the loss of stability of the system as a whole for the first buckling mode.

References: Perelmuter A.V., Slivker V.I., *Design Models of Structures and Possibilities of Their Analysis*. — M, DMK-Press, 2007, § 9.4.

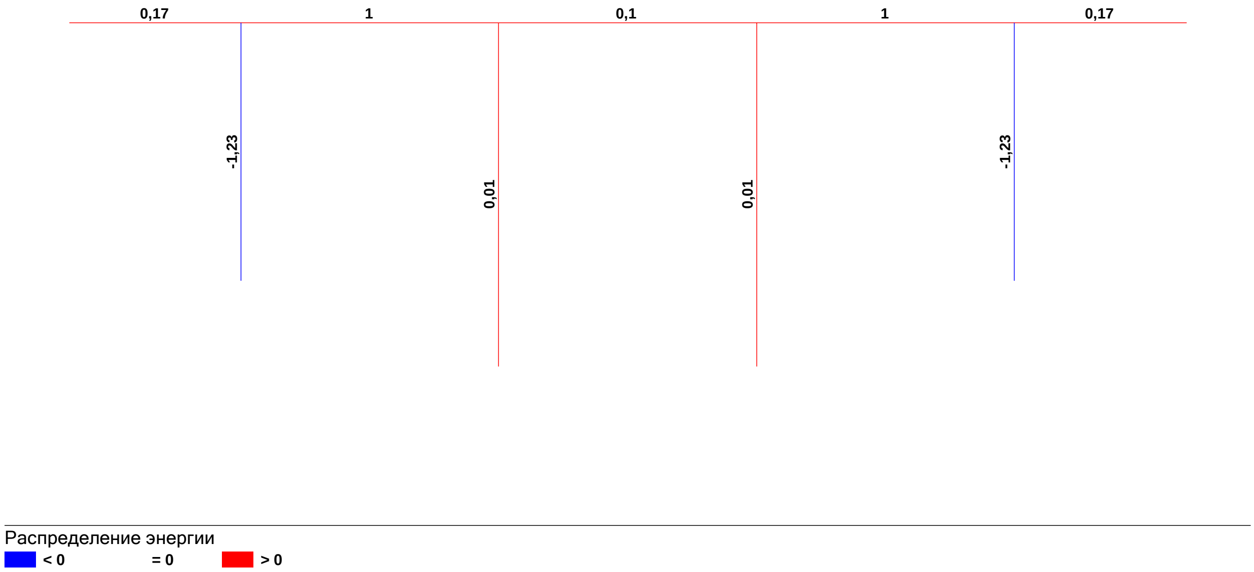
Initial data:
P_1 = 4 t, P_2 = 6 t - values of the concentrated forces.
Bar sections have the following ratios of the stiffnesses per running meter 2:4:6.

Finite element model: Design model – general type system, 9 bar elements of type 2, 10 nodes.



Design model (with the numbers of the rigidity type and loads)

Results in SCAD:



Values of the energy

Comparison of solutions:

Parameter	Theory	SCAD
Elements with the negative energy	edge columns	edge columns

Erection

Static Analysis of Stress-Strain State of a Building Taking into Account Genetic Nonlinearity

Objective: Comparison of the results of the calculations of the stress-strain state of a multi-storey building taking into account genetic nonlinearity performed by SCAD and ANSYS.

Initial data file: **Test-01.MPR**

Problem formulation: Design model – 11-storey building fragment rectangular in plan — spatial model consisting of columns, walls, piers, floor slabs on the rigid subgrade (all linear and angular nodal degrees of freedom are constrained). The model is subjected to the uniformly distributed load (1,5 t/m^2) applied to all floor slabs.

References: O.V. Kabantsev, *Verification of calculation technology* "*Mounting*" *from software complex SCAD*, International Journal for Computational Civil and Structural Engineering , 2011, **7** (3), 103-109.

Initial data:

Physical properties – material of elements of the design model: concrete of the compressive strength class B25; elastic modulus E = 3 10^6 t/m^2 ; Poisson's ratio ν = 0,2.

Geometric properties:

Storey height – 3 m,

Column spacing – 7 m along X and 6 m along Y,

Column section – 50×50 cm.

Floor slab thickness – 20 cm

Wall and pier thickness – 40 cm, pier width – 100 cm.

Boundary conditions: columns, piers and walls are clamped in the plane z = 0 m

Loads:

1) Vertical pressure on the floor slabs q_1 = 1,5 t/m^2 is applied to the newly erected fragments-storeys;

2) Vertical pressure on the floor slabs q_2 = 1 t/m^2 is applied after the erection of the entire building.

Finite element model:

ANSYS

Slabs, walls and piers are modeled by shell finite elements of the SHELL63 type, columns are modeled by beam finite elements of the BEAM44 type.

Stage 1. Resetting the stiffness of all FE to zero (the "element death" function), except for the 1-st floor, and constraining all nodes not belonging to the elements of the 1-st floor in the directions of all degrees of freedom with the application of the load q_1 to the slab of the 1-st floor and the subsequent SSS analysis;

Stage 2. Restoring the previous stiffness of the FE (the "element birth" function) of the 2-nd floor and removing the constraints of the nodes belonging to the elements of the 2-nd floor in the directions of all degrees of freedom with the application of the load q_1 to the slab of the 2-nd floor and the subsequent SSS analysis;

…….

Stage 11. Restoring the previous stiffness of the FE (the "element birth" function) of the 11-th floor and removing the constraints of the nodes belonging to the elements of the 11-th floor in the directions of all degrees of freedom with the application of the load q_1 to the slab of the 11-th floor (roof) and the subsequent SSS analysis;

Stage 12. Application of the load q_2 to all floor slabs of the building with the subsequent SSS analysis.

Nodes not belonging to the "born" elements are constrained in order to fix the structural elements of the building at the design elevations to take into account the actual building erection process.

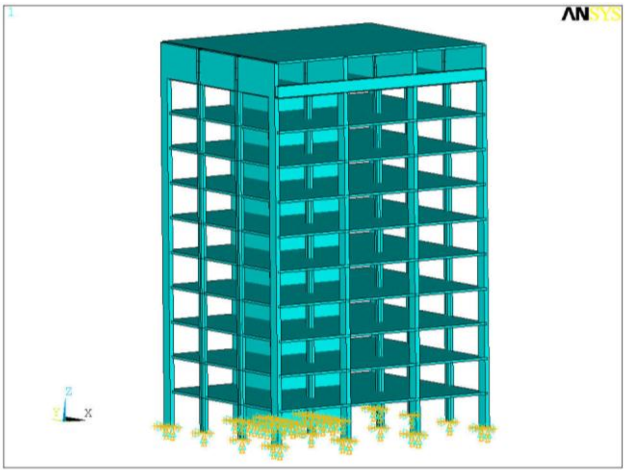
SCAD

Slabs, walls and piers are modeled by shell finite elements of type 44, columns are modeled by bar elements of general type 5.

The dimension of the complete model is 5608 nodes and 5456 finite elements.

Modeling of the building erection process consists of the following stages:

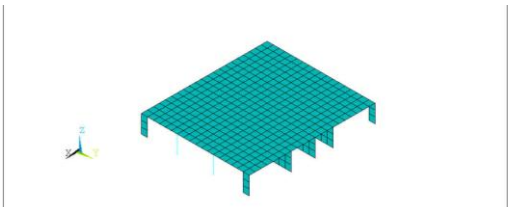
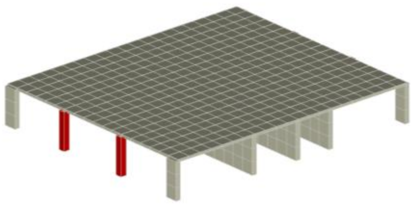
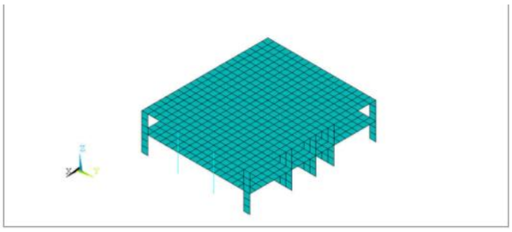
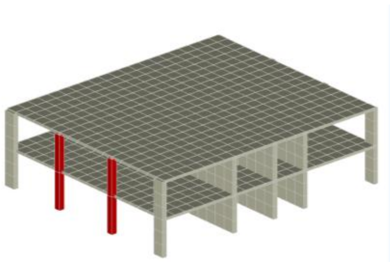
Stage 1. Selection of the set of elements at the level of the 1-st floor which are considered at the stage No.1 with the application of the load q_1 to the slab of the 1-st floor and the subsequent SSS analysis;
Stage 2. Selection of the set of elements at the level of the 1-2-nd floors which are considered at the stage No.2 with the application of the load q_1 to the slabs included in the 2-nd stage and the subsequent SSS analysis;
…………
Stage 11. Selection of the set of elements at the level of the 1-11-th floors which are considered at the stage No.11 with the application of the load q_1 to the slabs included in the 11-th stage and the subsequent SSS analysis;
Stage 12. Application of the load q_2 to all floor slabs of the building with the subsequent SSS analysis.



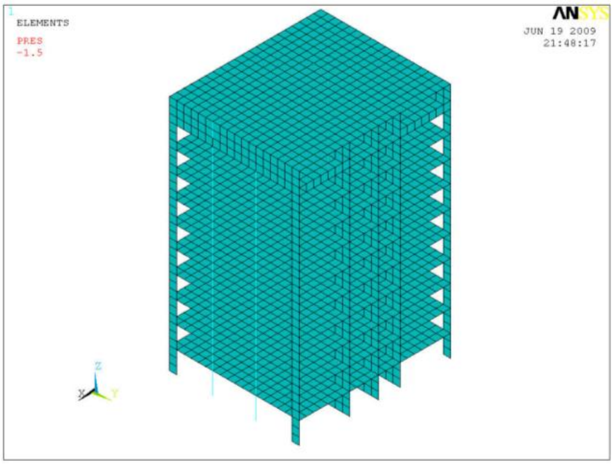
General view of the ANSYS design model.

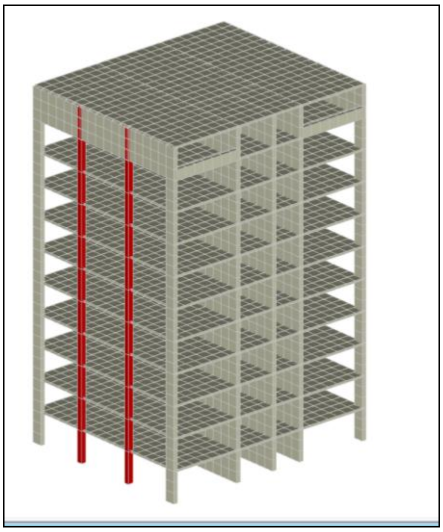
General view of the SCAD design model.

ANSYS and SCAD design models for different calculation stages

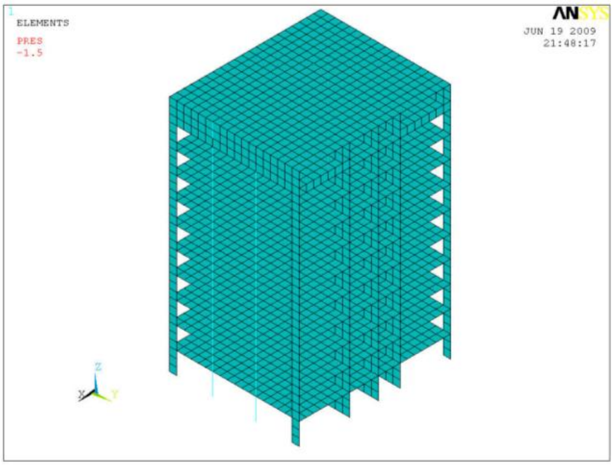
Stage No.	**ANSYS**	**SCAD**
1		
2		

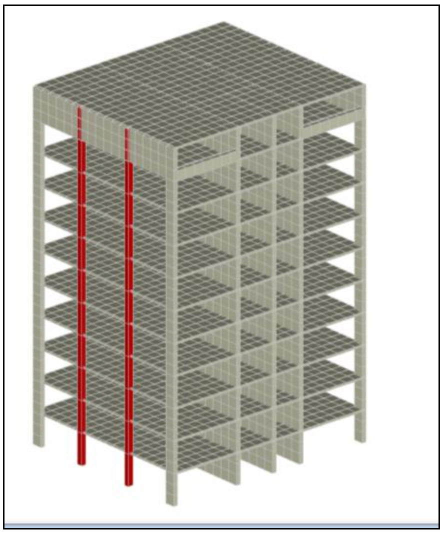
11





12





Loads q_2 have been added

Loads q_2 have been added

Comparison of solutions:

Parameter	Accounting for the 12 erection stages		Deviations, %
	ANSYS	**SCAD**	
Maximum vertical displacement, mm	-24,8	-24,19	2,46
Longitudinal force in a column (1st floor), t	-870,6	-865,1 (FE 386)	0,63
Longitudinal force in a column (10th floor), t	-3,2	-2,99 (FE 4679)	6,56

Determination of Stress-Strain State Taking into Account Genetic Nonlinearity ("Erection" Mode)

Objective: Comparison of the results of the calculations of the stress-strain state of a bar structure taking into account genetic nonlinearity performed by SCAD and the analytical solution.

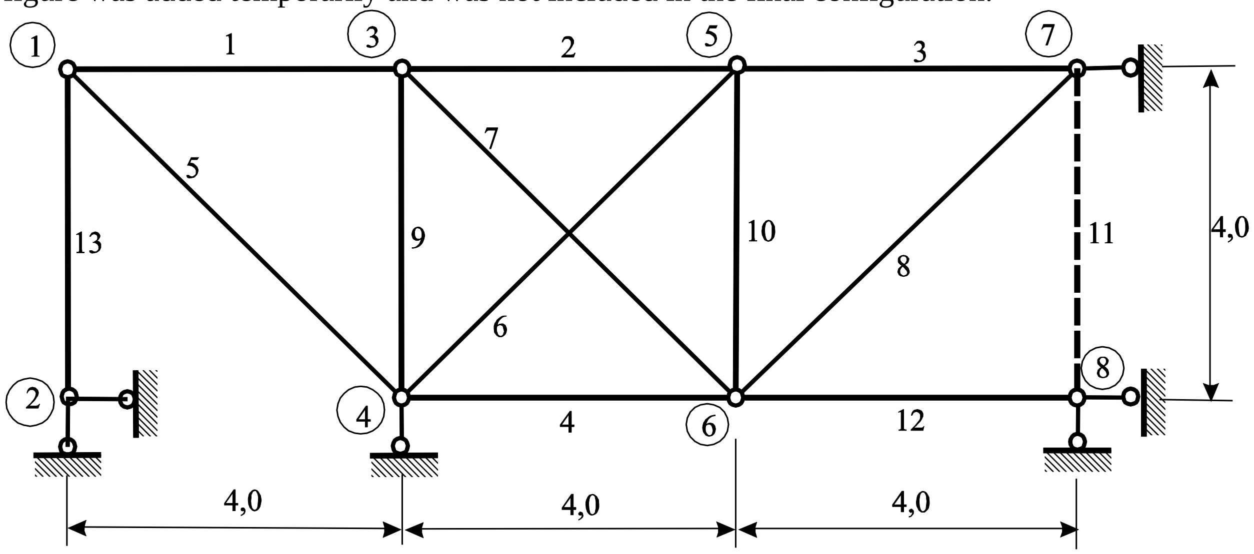
Initial data file: **Truss.MPR**

Problem formulation:

References: A.V.Perelmuter, *Control of the Behavior of Load-Bearing Structures* (2-nd edition revised and supplemented) , Moscow: ASV, 2011, § 5.2.

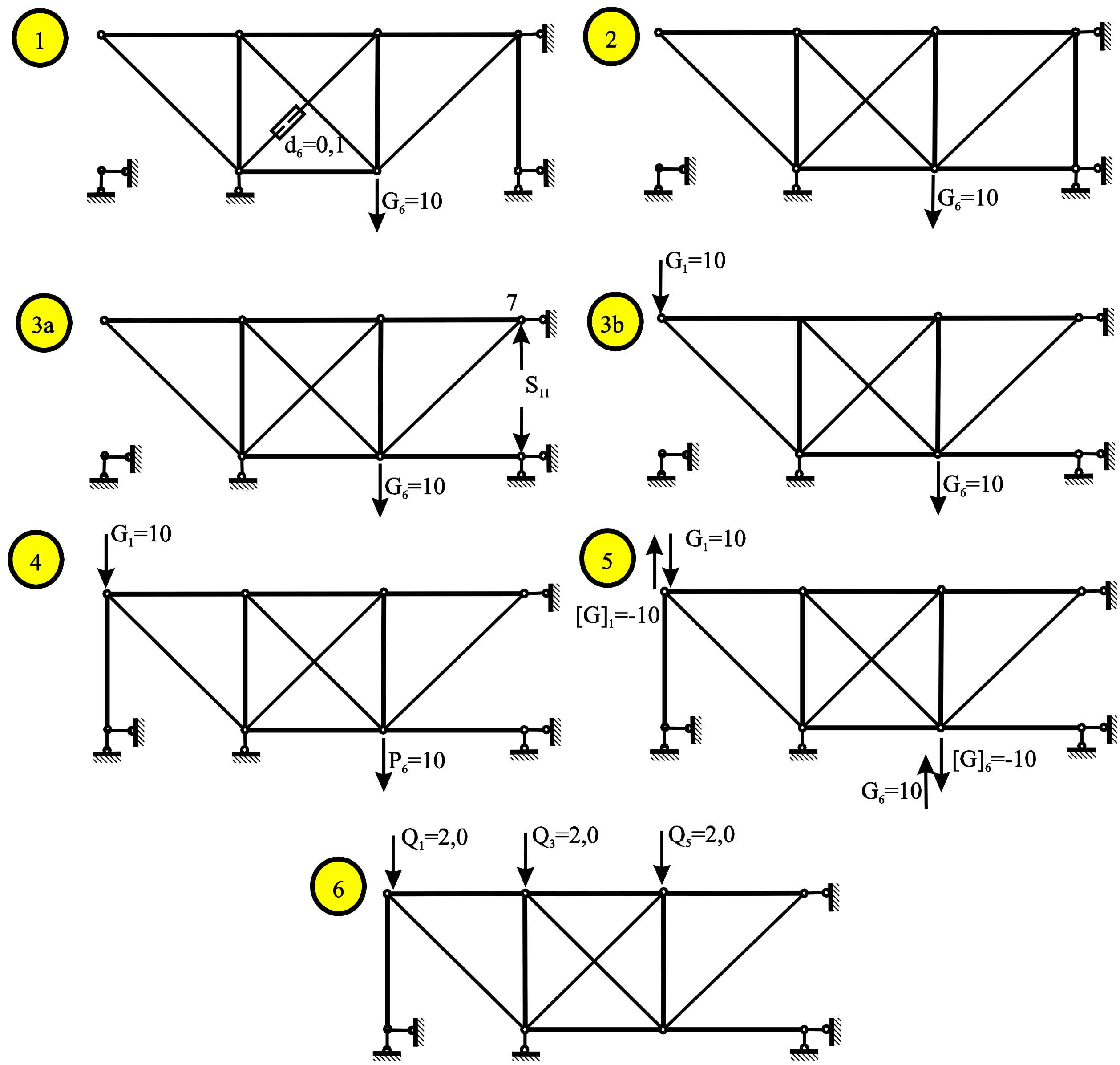
Initial data:

The final model of the analyzed structure is given in the figure (linear dimensions in meters), and some additional information is given in the table. Element 11 shown with a dotted line in this figure was added temporarily and was not included in the final configuration.



Bar numbers	Stiffness EA, t
1	10
2	10
3	10
4	10
5	10
6	2
7	2
8	4
9	5
10	5
11	25
12	10
13	10

The sequence of operations for achieving the prestressing is shown in the figure below.



Finite element model:

The structure is modeled by bar elements of general type 1.

The dimension of the complete model is 8 nodes and 13 finite elements.

Modeling of the building erection process consists of the following stages:

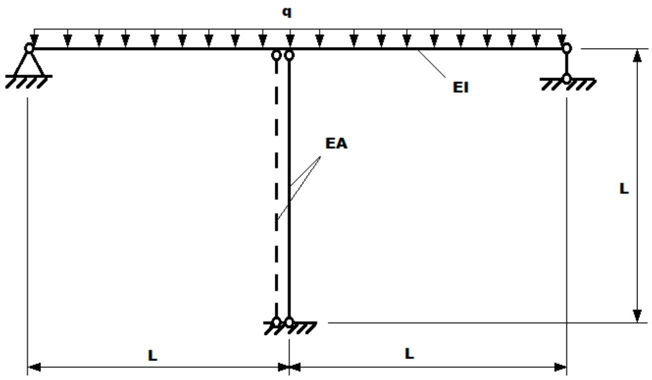
Stage	Description of operations
1	Forced shortening of the bar 6 (dislocation d_6= 0,001 m) and suspension of the ballast weight G_6=10 t in the node 6.
2	Attachment of the bar 12 to the system
3	Removal of the bar 11 performed by the program in two stages: replacement of the effect of the bar by forces S_{11}, which it transfers to the rest of the system (see. 3a), and application of the “compensating” load to the system − S_{11} (see. 3b). Installation of the ballast weight G_1=10 t in the node 1.
4	Attachment of the bar 13 to the system
Working	Removal of the ballast weights G_1 and G_6 and loading the system by the live load Q_1=Q_3=Q_5=2.

Comparison of solutions:

Parameter	Results		Deviations, %
	Theory	**SCAD**	
Stage 1: Vertical displacement of the node 6, cm	-17,078	-17,042	0,21
Force in the element 2, t	-2,510	-2,500	0,40
Stage 2: Vertical displacement of the node 6, cm	-17,078	-17,042	0,21
Force in the element 2, t	-2,510	-2,500	0,40
Stage 3: Vertical displacement of the node 6, cm	-28,220	-28,185	0,12
Force in the element 2, t	4,990	5,000	-0,20
Stage 4: Vertical displacement of the node 6, cm	-28,220	-28,185	0,12
Force in the element 2, t	4,990	5,000	-0,20
Working stage Vertical displacement of the node 6, cm	1,257	1,293	0,21
Force in the element 2, t	5,559	5,61	0,91

Note: There are arithmetic errors in the source. The comparison of solutions was made on the basis of the corrected calculations reported by the author.

Replacement of a Column of a Two-Span Single-Storey Frame Subjected to a Constant Load



Objective: Determination of the internal forces in the elements of a two-span single-storey frame before and after the replacement of a column subjected to a constant load.

Initial data file: **Rearrange_Frame.MPR**

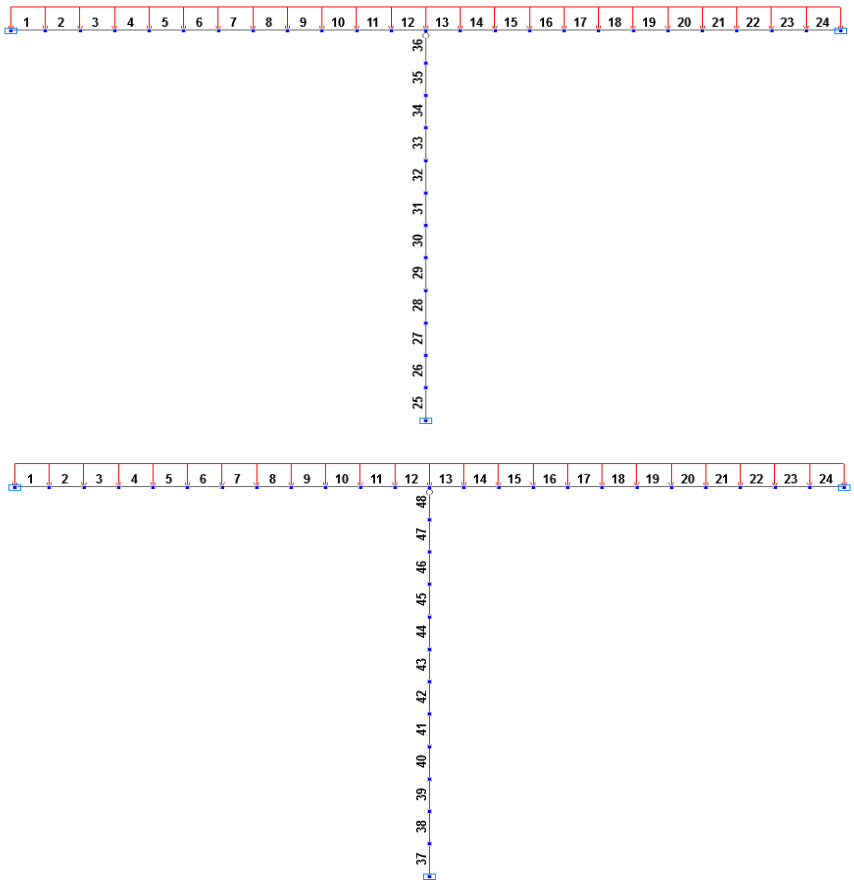
Problem formulation: The two-span girder of the frame simply supported at the ends with a middle support in the form of a hinged column is subjected to the constant uniformly distributed load. During the reconstruction the column is replaced by a column of the same rigidity in the following order: the replacing column is installed and then the original one is dismantled. Determine the maximum bending moments in the girder of the frame M_I, M_{II} and the longitudinal forces in the columns N_I, N_{II} before and after the replacement.

Initial data:

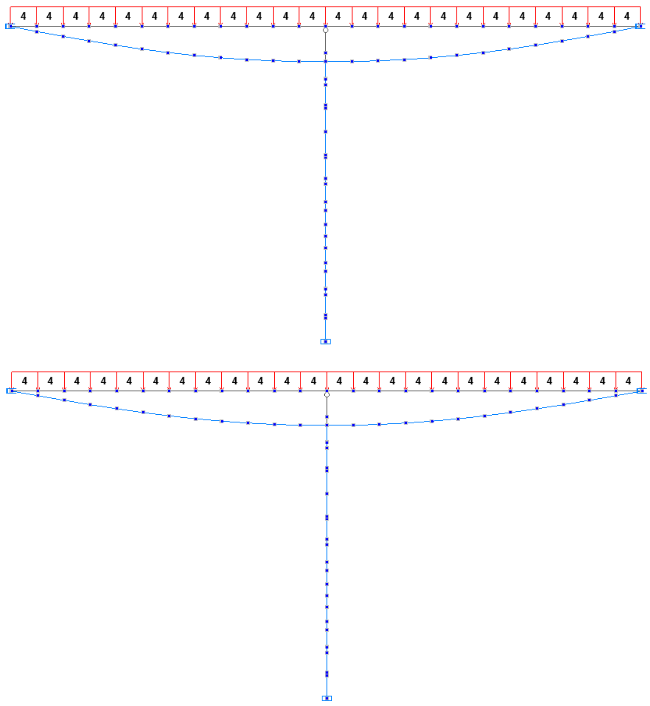
$EF = 2.0 \cdot 10^7$ t	- axial stiffness of the girder and column cross-section;
$EI = 1.2 \cdot 10^8$ t·m^2	- bending stiffness of the girder and column cross-section;
L = 6.0 m	- girder span length and column height;
q = 4.0 t/m	- uniformly distributed constant vertical load applied to the girder spans.

Finite element model: Design model – plane frame, elements of the girder – 24 bar elements of type 2, elements of the columns – 24 bar elements of type 2. The spacing of the finite element mesh along the longitudinal axes of the structural members is 0.5 m. The node of the left end of the girder is constrained in the directions of the degrees of freedom X, Z. The node of the right end of the girder is constrained in the direction of the degree of freedom Z. The nodes of the lower ends of the columns are constrained in the directions of the degrees of freedom X, Z. The elements of the upper ends of the columns have a hinge in the direction of the degree of freedom UY. Number of nodes in the design model – 37. Elements of the girder 1 – 24 and of the original column 25 – 36 are included in the first erection stage. Elements of the girder 1 – 24 and of the replacing column 37 – 48 are included in the second erection stage. The accumulated loading q is acting in both stages.

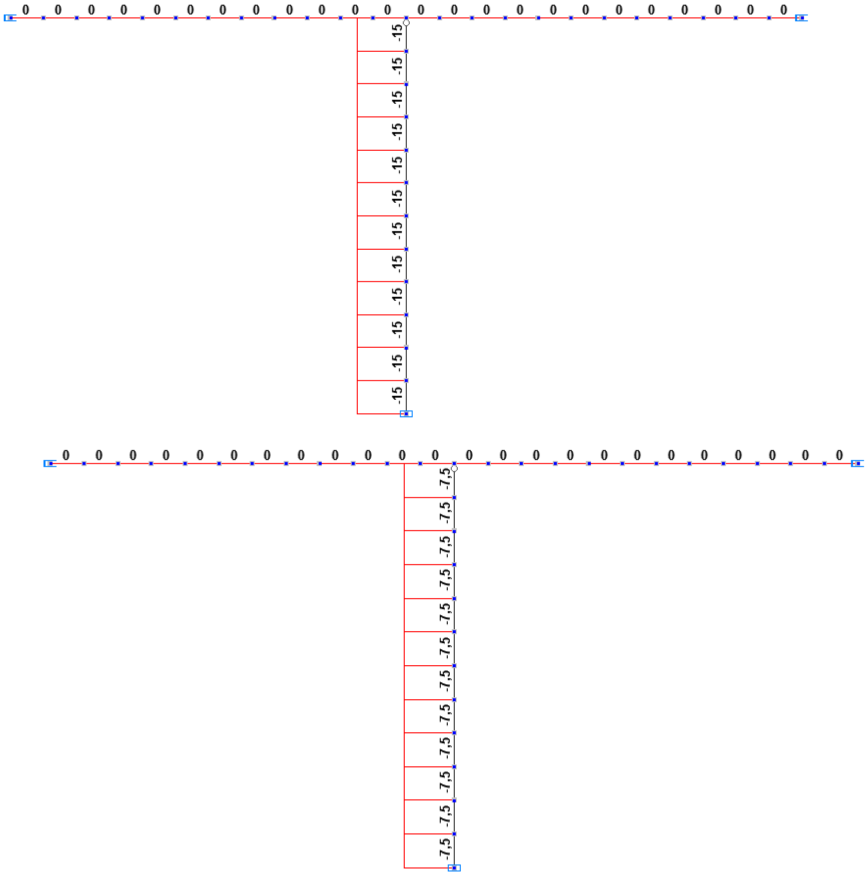
Results in SCAD



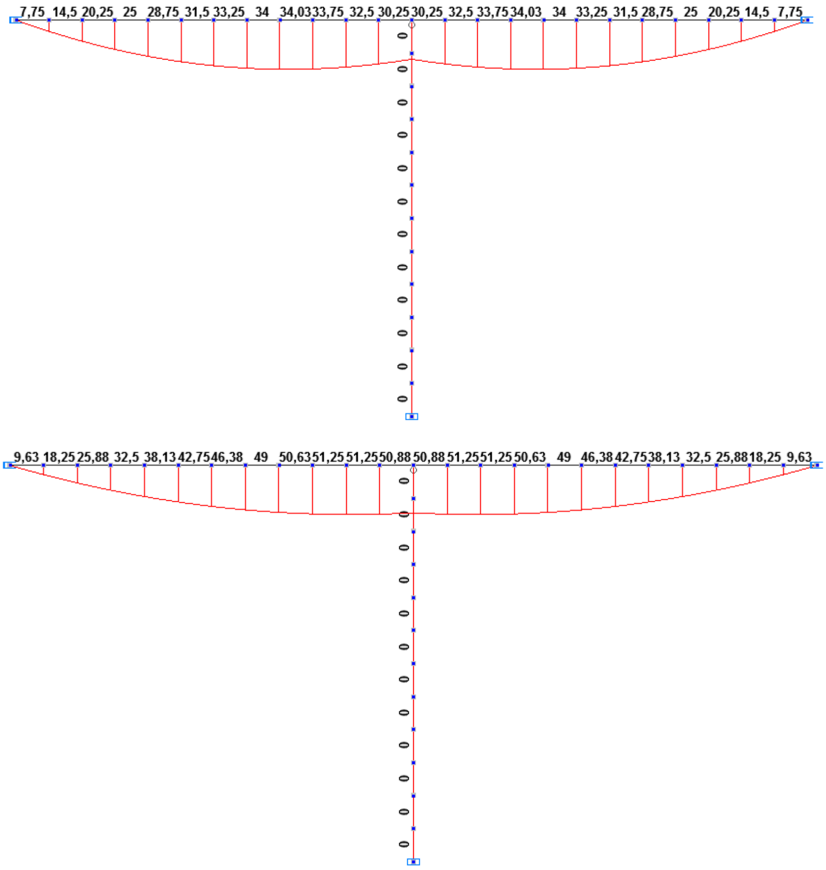
Design models in the first and second erection stages



Deformed models in the first and second erection stages



Longitudinal force diagrams N_I, N_{II} in the first and second erection stages (t)



Bending moment diagrams M_I, M_{II} in the first and second erection stages (t·m)

Comparison of solutions:

Parameter	Theory	SCAD	Deviations, %
Maximum bending moment in the girder of the frame in the first erection stage M_I, t·m	34.03	34.03	0.00
Longitudinal force in the column of the frame in the first erection stage N_I, t	-15.0	-15.0	0.00
Maximum bending moment in the girder of the frame in the second erection stage M_{II}, t·m	51.26	51.25	0.02
Longitudinal force in the column of the frame in the second erection stage N_{II}, t	-7.5	-7.5	0.00

Notes: In the analytical solution the maximum bending moments in the girder of the frame M_I, M_{II} and the longitudinal forces in the columns N_I, N_{II} before and after the replacement are determined according to the following formulas:

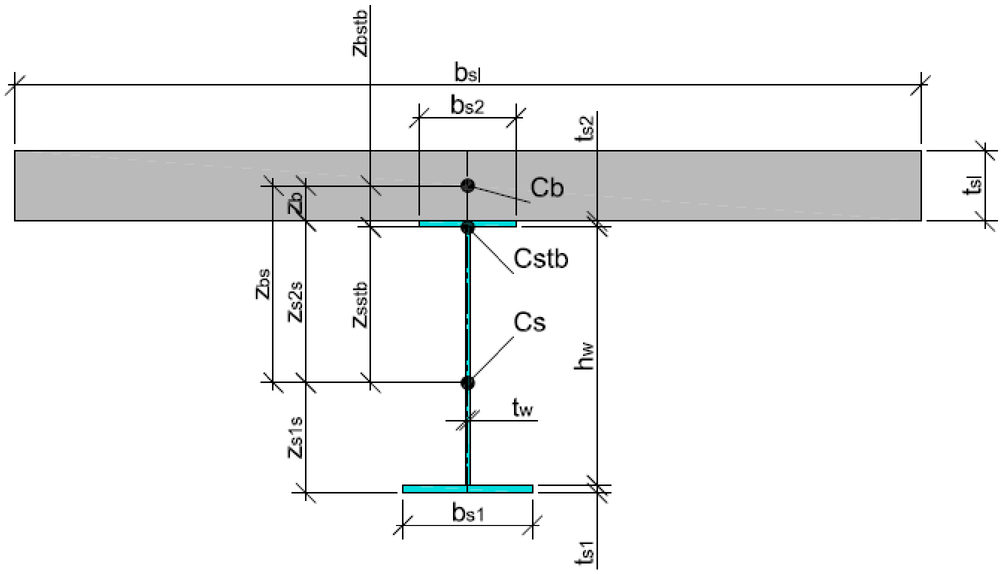
$$M_I = \frac{1}{2} \cdot q \cdot L^2 - \frac{1}{2} \cdot q \cdot L \cdot \frac{\frac{5}{24} \cdot \frac{L^4}{EI}}{\frac{L^3}{6 \cdot EI} + \frac{L}{EA}} + \frac{1}{8} \cdot q \cdot \left(\frac{\frac{5}{24} \cdot \frac{L^4}{EI}}{\frac{L^3}{6 \cdot EI} + \frac{L}{EA}} \right)^2$$

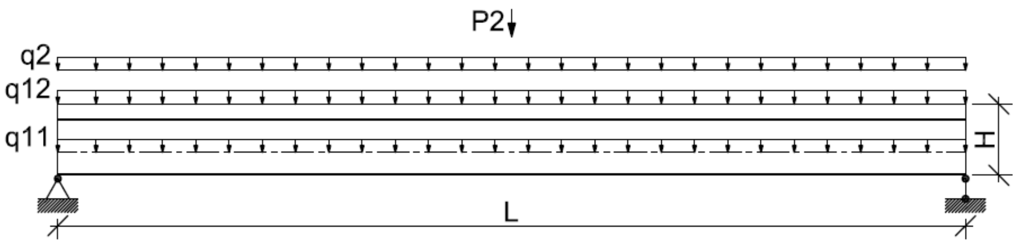
$$N_I = -q \cdot \frac{\frac{5}{24} \cdot \frac{L^4}{EI}}{\frac{L^3}{6 \cdot EI} + \frac{L}{EA}}$$

$$M_{II} = \frac{441}{512} \cdot q \cdot L^2 - \frac{21}{16} \cdot q \cdot L \cdot \frac{\frac{15}{96} \cdot \frac{L^4}{EI}}{\frac{L^2}{6 \cdot EI} + \frac{L}{EA}} + \frac{1}{2} \cdot q \cdot \left(\frac{\frac{15}{96} \cdot \frac{L^4}{EI}}{\frac{L^2}{6 \cdot EI} + \frac{L}{EA}} \right)^2$$

$$N_{II} = -q \cdot \frac{\frac{5}{48} \cdot \frac{L^4}{EI}}{\frac{L^2}{6 \cdot EI} + \frac{L}{EA}}$$

Sequential Erection of a Steel Reinforced Concrete Single-Span Beam





Objective: Determination of the deflections of the steel reinforced concrete single-span beam for the erection stages.

Problem formulation: The erection of the steel reinforced concrete single-span beam is performed in the following order:

- A steel I-beam is installed on the supports, the formwork for the reinforced concrete slab is arranged on the props from the bottom chord of the beam, the reinforcement cage is installed on the formwork, and the monolithic concrete is laid in the first erection stage. The steel beam is subjected to the load from the self-weight q_{11} and from the weight of the fresh concrete q_{12} at this stage.
- The formwork is dismantled, and the reinforced concrete slab starts to bend across the steel beam in the second erection stage.
- The serviceability loads from the weight of the roof structure q_2 and the transport load P_2 are applied to the steel reinforced concrete beam in the third erection stage.

Determine the maximum deflections of the steel reinforced concrete beam in the first w_1 and third w_2 erection stages.

Initial data file: **Wiring_Girder.MPR**

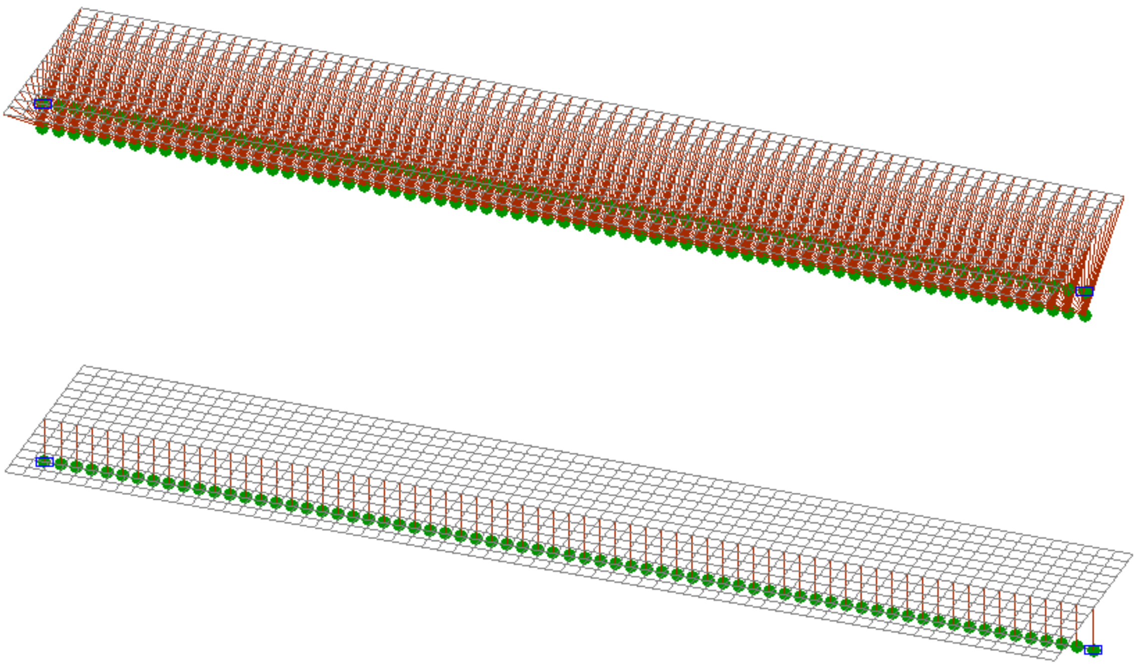
Initial data:

$E_{st} = 2.1 \cdot 10^6$ kgf/cm^2	- elastic modulus of the material of the steel beam;
$E_b = 3.06 \cdot 10^5$ kgf/cm^2	- elastic modulus of the material of the reinforced concrete slab;
$\upsilon_{st} = 0.3$	- Poisson's ratio of steel;
$\upsilon_b = 0.2$	- Poisson's ratio of reinforced concrete;
L = 1365.0 cm	- steel reinforced concrete beam span length;
b_{s1} = 40.0 cm	- width of the bottom chord of the steel beam;

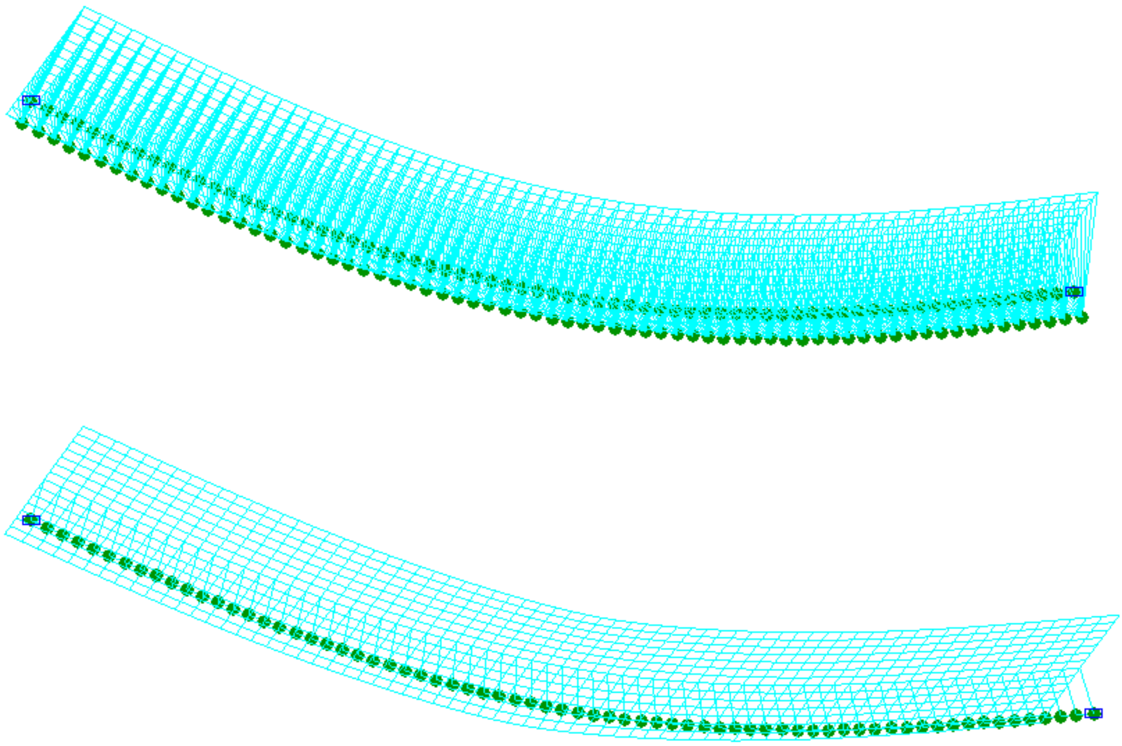
t_{s1} = 2.4 cm - thickness of the bottom chord of the steel beam;
b_{s2} = 30.0 cm - width of the top chord of the steel beam;
t_{s2} = 1.6 cm - thickness of the top chord of the steel beam;
h_w = 80.0 cm - height of the web of the steel beam;
t_w = 1.2 cm - thickness of the web of the steel beam;
b_{sl} = 280.0 cm - width of the reinforced concrete slab;
t_{sl} = 22.0 cm - thickness of the reinforced concrete slab;
q_{11} = 0.2072 t/m - vertical load from the self-weight of the steel beam uniformly distributed along a line;
q_{12} = 0.6050 t/m^2 - vertical load from the self-weight of the reinforced concrete slab uniformly distributed over an area;
q_2 = 0.3770 t/m^2 - vertical load from the self-weight of the roof structure uniformly distributed over an area and applied to the reinforced concrete slab;
P_2 = 39.60 t/m - vertical transport load uniformly distributed along a line.

Finite element model: Design model – general type system, elements of the steel beam – 68 bar elements of type 5, elements of the reinforced concrete slab – 952 shell elements of type 44, elements of the joint between the steel beam and the reinforced concrete slab – 69 elements of type 100, elements of the formwork props – 1035 elements of type 100. The spacing of the finite element mesh of the steel beam along the longitudinal axis is 0.2 m. The spacing of the finite element mesh of the reinforced concrete slab in the longitudinal and transverse directions is 0.2 m. The node of the left end of the beam is constrained in the directions of the degrees of freedom X,Y, Z, UX. The node of the right end of the beam is constrained in the directions of the degrees of freedom Y, Z, UX. Number of nodes in the design model – 1173. Elements of the steel beam, elements of the reinforced concrete slab with the reduced elastic modulus $E_b \cdot 10^{-3}$, elements of the joint and elements of the props are included in the first erection stage. Elements of the steel beam, elements of the reinforced concrete slab with the normal elastic modulus E_b and elements of the joint are included in the second and third erection stages. The loads q_{11} and q_{12} of the accumulated loading q_1 are acting in all stages. The loads q_2 and P_2 of the independent loading q_2 are acting in the third erection stage.

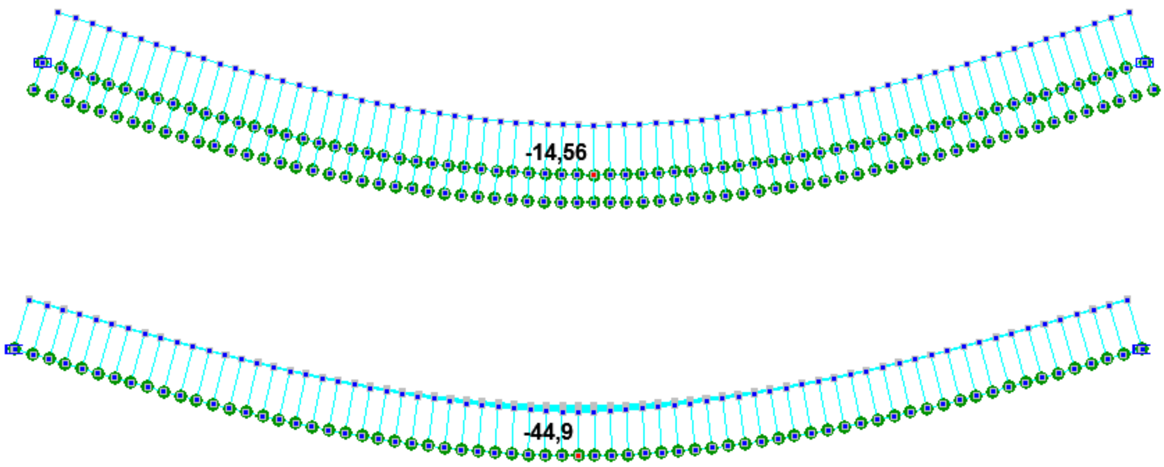
Results in SCAD:



Design models in the first, second and third erection stages



Deformed models in the first and third erection stages



Deflections in the first w_1 and third w_2 erection stages (mm)

Comparison of solutions:

Parameter	Theory	SCAD	Deviations, %
Maximum deflection of the steel reinforced concrete beam in the first erection stage w_1, mm	-14.75	-14,56	1.29
Maximum deflection of the steel reinforced concrete beam in the third erection stage w_2, mm	-44.51	-44,90	0.88

Notes: In the analytical solution the maximum deflections of the steel reinforced concrete beam in the first w_1 and third w_2 erection stages are determined according to the following formulas:

$$w_1 = \frac{5}{384} \cdot \frac{\left(q_{11} + q_{12} \cdot b_{sl}\right) \cdot L^4}{E_{st} \cdot I_s};$$

$$w_1 = \frac{5}{384} \cdot \frac{\left(q_{11} + q_{12} \cdot b_{sl}\right) \cdot L^4}{E_{st} \cdot I_s} + \frac{5}{384} \cdot \frac{q_2 \cdot b_{sl} \cdot L^4}{E_{st} \cdot I_{stb}} + \frac{1}{48} \cdot \frac{P_2 \cdot b_{sl} \cdot L^3}{E_{st} \cdot I_{stb}};$$

$$I_{stb} = I_s + A_s \cdot z_{sstb}^2 + \frac{I_b}{n_b} + \frac{A_b}{n_b} \cdot z_{bstb}^2; \quad z_{bstb} = z_{bs} - z_{sstb}; \quad z_{sstb} = \frac{S_{stb}}{A_{stb}};$$

$$S_{stb} = \frac{A_b}{n_b} \cdot z_{bs}; \quad A_{stb} = A_s + \frac{A_b}{n_b}; \quad z_{bs} = z_{s2s} + z_b; \quad I_b = \frac{b_{sl} \cdot t_{sl}}{12};$$

$$z_b = \frac{S_b}{A_b}; \quad S_b = \frac{b_{sl} \cdot t_{sl}^2}{2}; \quad A_b = b_{sl} \cdot t_{sl}; \quad n_b = \frac{E_{st}}{E_b};$$

$$I_s = I_{s1s} + I_{ws} + I_{s2s}; \quad I_{s2s} = I_{s2} + A_{s2} \cdot \left(t_{s1} + h_w + \frac{t_{s2}}{2} - z_{s1s}\right)^2;$$

$$I_{ws} = I_w + A_w \cdot \left(t_{s1} + \frac{h_w}{2} - z_{s1s}\right)^2; \quad I_{s1s} = I_{s1} + A_{s1} \cdot \left(z_{s1s} - \frac{t_{s1}}{2}\right)^2;$$

$$I_w = \frac{t_w \cdot h_w^3}{12}; \quad I_{s2} = \frac{b_{s2} \cdot t_{s2}^3}{12}; \quad I_{s1} = \frac{b_{s1} \cdot t_{s1}^3}{12};$$

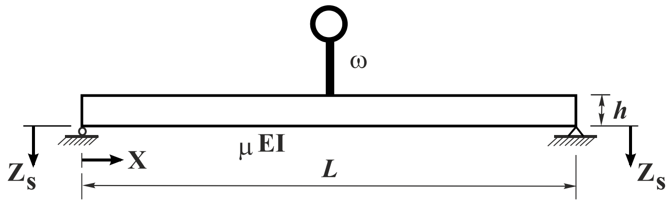
$$z_{s2s} = H_s - z_{s1s}; \quad z_{s1s} = \frac{S_s}{A_s}; \quad S_s = A_{s1} \cdot \frac{t_{s1}}{2} + A_w \cdot \left(t_{s1} + \frac{h_w}{2}\right) + A_{s2} \cdot \left(t_{s1} + h_w + \frac{t_{s2}}{2}\right);$$

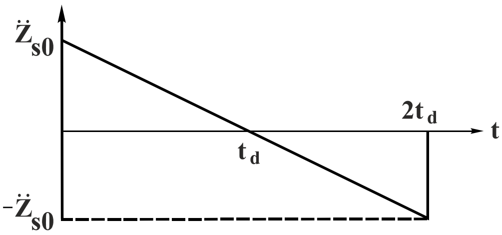
$$A_s = A_{s1} + A_w + A_{s2}; \quad H = H_s + t_{sl}; \quad H_s = t_{s1} + h_w + t_{s2};$$

$$A_w = t_w \cdot h_w; \quad A_{s2} = b_{s2} \cdot t_{s2}; \quad A_{s1} = b_{s1} \cdot t_{s1};$$

Response Spectra

Response Spectrum of Absolute Response Accelerations of a Linear Oscillator Installed in the Middle of the Span of a Simply Supported Beam with a Distributed Mass Subjected to a Kinematic Excitation of Supports (Seismic Action)





Objective: Determination of the response spectrum of response accelerations of a linear oscillator installed in the middle of the span of a simply supported beam with a distributed mass subjected to a kinematic excitation of supports.

Initial data files: **DIN_B_RS.SPR** – design model
DIN_B_RS.SPC – accelerogram

Problem formulation: The simply supported beam of constant cross-section with the uniformly distributed mass μ is subjected to the kinematic excitation of the supports according to the specified accelerogram:

$$\ddot{z}(t) = \ddot{z}_{s0} \cdot \left(1 - \frac{t}{t_d}\right).$$

Determine the response spectrum of the absolute response accelerations of the linear oscillator installed in the middle of the span.

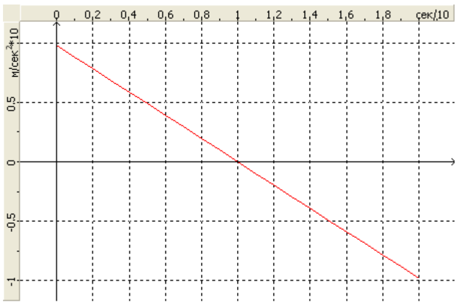
References: John M. Biggs, Introduction to Structural Dynamics, McGraw-Hill Book Companies, New York, 1964, p.256-263;
Kiselev V.A., Structural Mechanics. Special Course. Dynamics and Stability of Structures. Moscow, Stroyizdat, 1980, p. 65-67.

Initial data:
$E = 3.0 \cdot 10^7$ psi $= 2.1092 \cdot 10^7$ tf/m^2 - elastic modulus;
$I = 333.333$ in^4 $= 138.7448 \cdot 10^{-6}$ m^4 - cross-sectional moment of inertia of the beam.
h = 14 in = 0.3556 m - height of the cross-section of the beam;
L = 240 in = 6.0960 m - beam span length;
$\mu = 0.2$ lb·sec^2/in^2 $= 0.1406$ tf·s^2/m^2 - value of the uniformly distributed mass of the beam;
$\ddot{z}_{s0} = \pm 386.2200$ in/sec^2 $= \pm 9.81$ m/s^2 - amplitude values of the acceleration of the supports according to the accelerogram;
$t_d = 0.10$ sec = 0.10 s - half-interval of the kinematic excitation of supports;
$g = 386.2200$ in/sec^2 $= 9.81$ m/s^2 - gravitational acceleration;

Finite element model: Design model – grade beam / plate, 32 bar elements of type 3. Boundary conditions of the simply supported ends of the beam are provided by imposing constraints in the direction of the degree of freedom Z. The dimensional stability of the design model is provided by imposing a constraint in the node of the cross-section along the symmetry axis of the beam in the direction of the degree of freedom UX. The distributed mass is specified by transforming the static load from the self-weight of the beam μ·g.
The kinematic excitation of supports is described by the graph of the acceleration variation with time (accelerogram) and is given in the form of the action along the Z axis of the global coordinate system (direction cosines to the X, Y, Z axes: 0.00, 0.00, 1.00) with the scale factor to the values of the accelerogram equal to 1.00. The height of the beam structure in the model is directed along the Z axis of the global coordinate system. The dissipation factor (energy absorption factor) is taken with the minimum value ξ = 0.000001 for the oscillator and for the structure. The intervals between the time points of the graph of the acceleration variation with time are equal to Δt = 0.01 s. When plotting the graph the

acceleration is taken with the values $\ddot{z}(t) = \ddot{z}_{s0} \cdot (1 - n \cdot \Delta t/t_d)$ at the time points $n{\cdot}\Delta t$. The conversion factor for the added static loading is equal to k = 1.000 (mass generation). Number of nodes in the design model – 33.

Results in SCAD



Design model and the given accelerogram

Comparison of solutions:

The comparison was performed with the solution of the problem obtained in Abaqus (the solution was provided by A.I. Popov — Atomproekt).

Frequency	**Acceleration**	
Hz	**g**	
	Abaqus	**SCAD**
0	0,0000	0,0000
0,05	0,0000	0,0007
0,1	0,0000	0,0029
0,15	0,0000	0,0064
0,2	0,0000	0,0114
0,25	0,0038	0,0178
0,3	0,0027	0,0256
0,35	0,0216	0,0347
0,4	0,0200	0,0452
0,45	0,0490	0,0569
0,5	0,0503	0,0700
0,55	0,0832	0,0842
0,6	0,0881	0,0997
0,65	0,1218	0,1163
0,7	0,1312	0,1340
0,75	0,1642	0,1528
0,8	0,1799	0,1726
0,85	0,2096	0,1934
0,9	0,2310	0,2152
0,95	0,2565	0,2378
1	0,2824	0,2613
1,05	0,3045	0,2855
1,1	0,3338	0,3105
1,15	0,3625	0,3362
1,2	0,3876	0,3626

Frequency	**Acceleration**	
Hz	**g**	
	Abaqus	**SCAD**
1,25	0,4182	0,3895
1,3	0,4481	0,4171
1,35	0,4758	0,4453
1,4	0,5043	0,4739
1,45	0,5395	0,5030
1,5	0,5690	0,5325
1,55	0,5964	0,5625
1,6	0,6324	0,5928
1,65	0,6656	0,6235
1,7	0,6953	0,6545
1,75	0,7270	0,6857
1,8	0,7628	0,7171
1,85	0,7932	0,7487
1,9	0,8267	0,7804
1,95	0,8572	0,8121
2	0,8939	0,8441
2,05	0,9265	0,8760
2,1	0,9559	0,9079
2,15	0,9913	0,9398
2,2	1,0234	0,9717
2,25	1,0561	1,0035
2,3	1,0887	1,0353
2,35	1,1193	1,0669
2,4	1,1498	1,0984
2,45	1,1855	1,1298

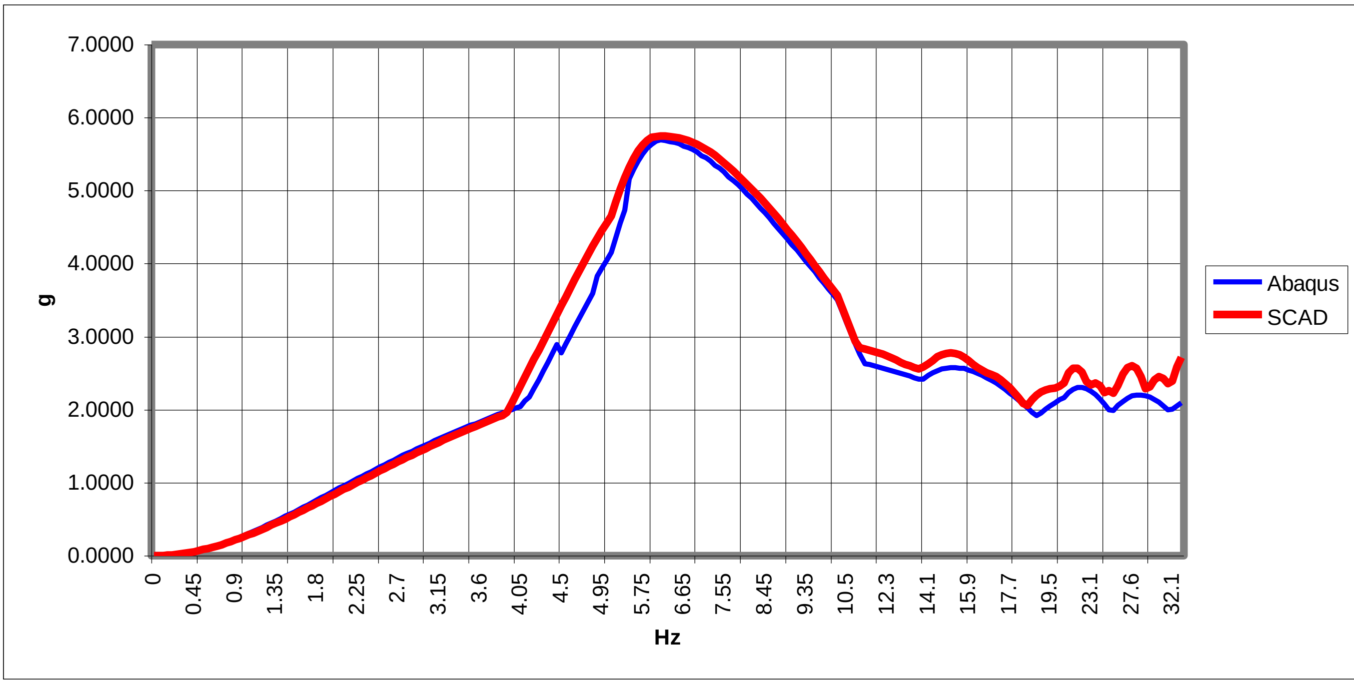
Frequency	**Acceleration**	
Hz	**g**	
	Abaqus	**SCAD**
2,5	1,2171	1,1611
2,55	1,2467	1,1923
2,6	1,2762	1,2234
2,65	1,3048	1,2544
2,7	1,3405	1,2853
2,75	1,3721	1,3160
2,8	1,4027	1,3465
2,85	1,4312	1,3769
2,9	1,4597	1,4071
2,95	1,4862	1,4370
3	1,5158	1,4667
3,05	1,5454	1,4963
3,1	1,5749	1,5255
3,15	1,6045	1,5546
3,2	1,6320	1,5832
3,25	1,6595	1,6115
3,3	1,6860	1,6395
3,35	1,7115	1,6671
3,4	1,7370	1,6943
3,45	1,7604	1,7211
3,5	1,7829	1,7476
3,55	1,8084	1,7736
3,6	1,8318	1,7994
3,65	1,8583	1,8247
3,7	1,8838	1,8499

Frequency	Acceleration	
Hz	g	
	Abaqus	SCAD
3,75	1,9093	1,8744
3,8	1,9337	1,8989
3,85	1,9541	1,9226
3,9	1,9776	1,9629
3,95	2,0000	2,0807
4	2,0224	2,1999
4,05	2,0438	2,3202
4,1	2,1244	2,4415
4,15	2,1713	2,5635
4,2	2,2895	2,6862
4,25	2,4088	2,8092
4,3	2,5291	2,9324
4,35	2,6493	3,0555
4,4	2,7696	3,1784
4,45	2,8899	3,3009
4,5	2,7768	3,4226
4,55	2,8960	3,5434
4,6	3,0143	3,6631
4,65	3,1325	3,7815
4,7	3,2497	3,8982
4,75	3,3660	4,0132
4,8	3,4811	4,1262
4,85	3,5953	4,2370
4,9	3,8267	4,3453
4,95	3,9368	4,4509
5	4,0449	4,5537
5,05	4,1519	4,6535
5,15	4,3568	4,8429
5,25	4,5515	5,0178
5,35	4,7339	5,1765
5,45	5,1580	5,3179
5,55	5,2915	5,4406
5,65	5,4057	5,5436
5,75	5,5025	5,6259
5,85	5,5800	5,6867
5,95	5,6371	5,7255
6,05	5,6799	5,7418
6,15	5,6922	5,7467
6,25	5,6840	5,7459
6,35	5,6667	5,7410
6,45	5,6616	5,7305
6,55	5,6381	5,7172
6,65	5,6106	5,7002
6,75	5,5933	5,6823

Frequency	Acceleration	
Hz	g	
	Abaqus	SCAD
6,85	5,5596	5,6593
6,95	5,5260	5,6326
7,05	5,4760	5,6019
7,15	5,4475	5,5663
7,25	5,4027	5,5263
7,35	5,3435	5,4817
7,45	5,3058	5,4330
7,55	5,2548	5,3803
7,65	5,1906	5,3244
7,75	5,1366	5,2659
7,85	5,0856	5,2063
7,95	5,0214	5,1456
8,05	4,9541	5,0832
8,15	4,8970	5,0199
8,25	4,8298	4,9568
8,35	4,7533	4,8934
8,45	4,6942	4,8276
8,55	4,6259	4,7590
8,65	4,5484	4,6880
8,75	4,4791	4,6150
8,85	4,4108	4,5400
8,95	4,3354	4,4637
9,05	4,2538	4,3856
9,15	4,1876	4,3056
9,25	4,1121	4,2230
9,35	4,0306	4,1386
9,45	3,9602	4,0539
9,55	3,8858	3,9697
9,65	3,8063	3,8864
9,75	3,7278	3,8045
9,85	3,6565	3,7235
9,95	3,5800	3,6425
10,05	3,5005	3,5625
10,25	3,3517	3,4065
10,45	3,1978	3,2517
10,65	3,0510	3,0950
10,85	2,9021	2,9342
11,05	2,7554	2,8493
11,25	2,6320	2,8338
11,45	2,6188	2,8165
11,65	2,6045	2,7987
11,85	2,5851	2,7780
12,05	2,5668	2,7555
12,25	2,5525	2,7311

Frequency	Acceleration	
Hz	g	
	Abaqus	SCAD
12,45	2,5352	2,7038
12,65	2,5138	2,6768
12,85	2,4954	2,6496
13,05	2,4791	2,6235
13,25	2,4608	2,6008
13,45	2,4393	2,5794
13,65	2,4190	2,5580
13,85	2,4200	2,5829
14,05	2,4669	2,6253
14,25	2,5025	2,6754
14,45	2,5321	2,7204
14,65	2,5545	2,7508
14,85	2,5668	2,7680
15,05	2,5770	2,7730
15,25	2,5780	2,7656
15,45	2,5708	2,7468
15,65	2,5627	2,7165
15,85	2,5433	2,6732
16,05	2,5270	2,6209
16,25	2,4995	2,5750
16,45	2,4730	2,5384
16,65	2,4383	2,5071
16,85	2,4037	2,4806
17,05	2,3629	2,4496
17,25	2,3221	2,4087
17,45	2,2742	2,3604
17,65	2,2294	2,3022
17,85	2,1774	2,2363
18,05	2,1284	2,1646
18,25	2,0724	2,0879
18,45	2,0204	2,0594
18,65	1,9602	2,1352
18,85	1,9215	2,1977
19,05	1,9541	2,2423
19,25	2,0071	2,2704
19,45	2,0530	2,2876
19,65	2,0968	2,2987
19,85	2,1356	2,3222
20,05	2,1672	2,3682
20,55	2,2365	2,5018
21,05	2,2783	2,5673
21,55	2,3028	2,5700
22,05	2,3007	2,5106
22,55	2,2854	2,3830

Frequency	Acceleration	
Hz	g	
	Abaqus	SCAD
23,05	2,2528	2,3421
23,55	2,2039	2,3615
24,05	2,1427	2,3313
24,55	2,0734	2,2381
25,05	1,9949	2,2560
25,55	1,9888	2,2237
26,05	2,0601	2,3334
26,55	2,1142	2,4863
27,05	2,1580	2,5792
27,55	2,1865	2,6055
28,05	2,1988	2,5678
28,55	2,1978	2,4566
29,05	2,1876	2,2866
29,55	2,1702	2,3166
30,05	2,1386	2,4101
30,55	2,0989	2,4521
31,05	2,0520	2,4243
31,55	1,9980	2,3600
32,05	2,0071	2,3904
32,55	2,0520	2,5854
33,05	2,0907	2,7152

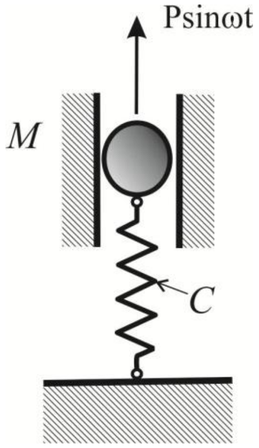


Response Spectra

	Abaqus	**SCAD**	**Deviation**
Frequency at which the maximum acceleration occurs (Hz)	6.15	6.15	0 %
Maximum acceleration (g)	5,6921	5.7467	0.95 %
Spectra correlation coefficient	0.995		

Amplitude-Frequency Characteristics

Amplitude-Frequency Characteristic of a System with One Degree of Freedom



Objective: Plotting the amplitude-frequency characteristic of a single-mass elastic system under harmonic excitation.

Initial data files: **TestAЧX.SPR**

Problem formulation: The behavior of a single-mass elastic system subjected to an excitation by a harmonic time-varying force with different excitation frequencies.

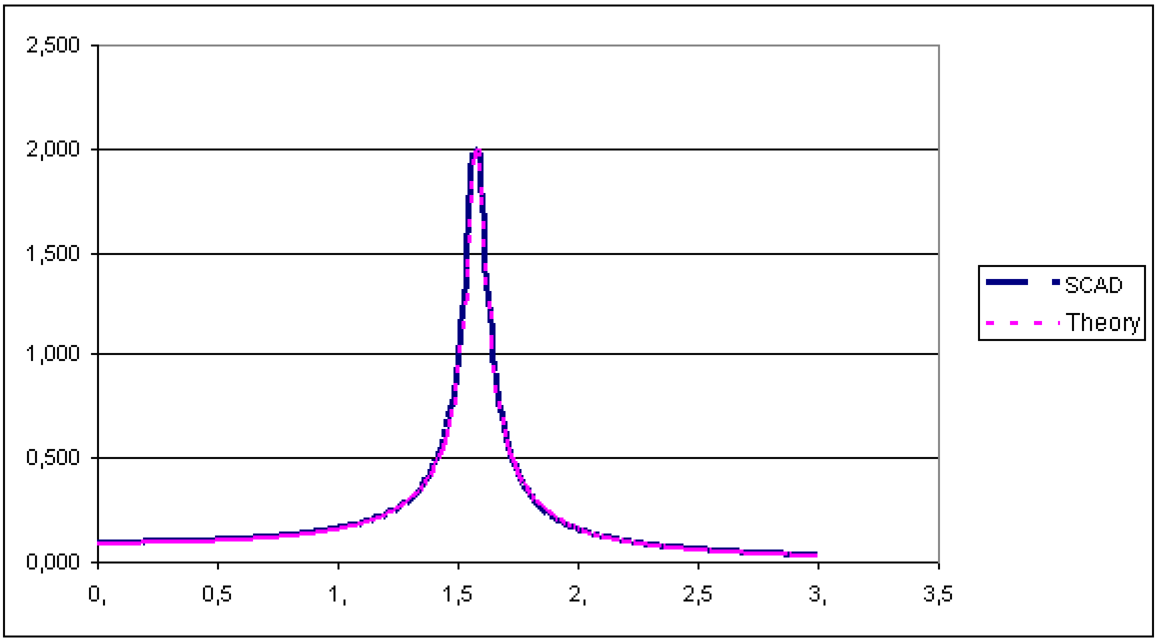
References: Panovko Ya.G. Introduction to the Theory of Mechanical Oscillations — M.: Nauka, 1991, § II.6.

Initial data:

M = 10 kN	- weight of the mass;
C = 100 kN	- stiffness;
P = 10 kN	- amplitude value of the force;
ξ = 0.025	- damping parameter (in fractions of the critical value).

Finite element model: One node where a point mass is specified is supported by a single-node elastic constraint (finite element of type 51).

Results in SCAD



Amplitude-frequency characteristics

Comparison of solutions:

Frequency	Displacement of the node	
	SCAD	Theory
Hz	m	
0,	0,1000	0,1000
0,01	0,1000	0,1000
0,02	0,1000	0,1000
0,03	0,1000	0,1000
0,04	0,1001	0,1001
0,05	0,1001	0,1001
0,06	0,1001	0,1001
0,07	0,1002	0,1002
0,08	0,1003	0,1003
0,09	0,1003	0,1003
0,1	0,1004	0,1004
0,11	0,1005	0,1005
0,12	0,1006	0,1006
0,13	0,1007	0,1007
0,14	0,1008	0,1008
0,15	0,1009	0,1009
0,16	0,1010	0,1010
0,17	0,1012	0,1012
0,18	0,1013	0,1013
0,19	0,1015	0,1015
0,2	0,1016	0,1016
0,21	0,1018	0,1018
0,22	0,1020	0,1020
0,23	0,1022	0,1022
0,24	0,1024	0,1024
0,25	0,1026	0,1026
0,26	0,1028	0,1028
0,27	0,1030	0,1030
0,28	0,1033	0,1032
0,29	0,1035	0,1035
0,3	0,1038	0,1037
0,31	0,1040	0,1040
0,32	0,1043	0,1043
0,33	0,1046	0,1046
0,34	0,1049	0,1048
0,35	0,1052	0,1052
0,36	0,1055	0,1055
0,37	0,1058	0,1058
0,38	0,1062	0,1061
0,39	0,1065	0,1065
0,4	0,1069	0,1068
0,41	0,1072	0,1072
0,42	0,1076	0,1076
0,43	0,1080	0,1080
0,44	0,1084	0,1084
0,45	0,1089	0,1088
0,46	0,1093	0,1092
0,47	0,1097	0,1097
0,48	0,1102	0,1102
0,49	0,1107	0,1106
0,5	0,1112	0,1111
0,51	0,1117	0,1116

Frequency	Displacement of the node	
	SCAD	Theory
Hz	m	
0,52	0,1122	0,1121
0,53	0,1127	0,1127
0,54	0,1133	0,1132
0,55	0,1138	0,1138
0,56	0,1144	0,1143
0,57	0,1150	0,1149
0,58	0,1156	0,1155
0,59	0,1163	0,1162
0,6	0,1169	0,1168
0,61	0,1176	0,1175
0,62	0,1183	0,1182
0,63	0,1190	0,1189
0,64	0,1197	0,1196
0,65	0,1204	0,1203
0,66	0,1212	0,1211
0,67	0,1220	0,1219
0,68	0,1228	0,1227
0,69	0,1237	0,1235
0,7	0,1245	0,1244
0,71	0,1254	0,1253
0,72	0,1263	0,1262
0,73	0,1272	0,1271
0,74	0,1282	0,1280
0,75	0,1292	0,1290
0,76	0,1302	0,1300
0,77	0,1313	0,1311
0,78	0,1324	0,1322
0,79	0,1335	0,1333
0,8	0,1346	0,1344
0,81	0,1358	0,1356
0,82	0,1370	0,1368
0,83	0,1383	0,1380
0,84	0,1396	0,1393
0,85	0,1409	0,1406
0,86	0,1423	0,1420
0,87	0,1437	0,1434
0,88	0,1452	0,1449
0,89	0,1467	0,1464
0,9	0,1482	0,1479
0,91	0,1498	0,1495
0,92	0,1515	0,1512
0,93	0,1532	0,1529
0,94	0,1550	0,1546
0,95	0,1569	0,1564
0,96	0,1588	0,1583
0,97	0,1607	0,1603
0,98	0,1628	0,1623
0,99	0,1649	0,1644
1,	0,1671	0,1666
1,01	0,1694	0,1689
1,02	0,1718	0,1712
1,03	0,1742	0,1736

Frequency	Displacement of the node	
	SCAD	Theory
Hz	m	
1,04	0,1768	0,1762
1,05	0,1794	0,1788
1,06	0,1822	0,1815
1,07	0,1851	0,1844
1,08	0,1881	0,1873
1,09	0,1912	0,1904
1,1	0,1945	0,1936
1,11	0,1979	0,1970
1,12	0,2014	0,2005
1,13	0,2051	0,2042
1,14	0,2090	0,2080
1,15	0,2131	0,2120
1,16	0,2174	0,2162
1,17	0,2219	0,2207
1,18	0,2266	0,2253
1,19	0,2316	0,2302
1,2	0,2368	0,2354
1,21	0,2424	0,2408
1,22	0,2482	0,2465
1,23	0,2544	0,2526
1,24	0,2609	0,2590
1,25	0,2679	0,2658
1,26	0,2752	0,2731
1,27	0,2831	0,2808
1,28	0,2915	0,2890
1,29	0,3004	0,2977
1,3	0,3100	0,3071
1,31	0,3203	0,3172
1,32	0,3314	0,3280
1,33	0,3434	0,3396
1,34	0,3563	0,3522
1,35	0,3704	0,3659
1,36	0,3857	0,3807
1,37	0,4024	0,3970
1,38	0,4207	0,4147
1,39	0,4409	0,4342
1,4	0,4633	0,4558
1,41	0,4881	0,4797
1,42	0,5159	0,5064
1,43	0,5471	0,5364
1,44	0,5824	0,5702
1,45	0,6226	0,6086
1,46	0,6688	0,6525
1,47	0,7222	0,7032
1,48	0,7845	0,7621
1,49	0,8578	0,8312
1,5	0,9449	0,9130
1,51	1,0490	1,0106
1,52	1,1740	1,1276
1,53	1,3230	1,2675
1,54	1,4965	1,4323
1,55	1,6863	1,6178

Frequency	Displacement of the node	
	SCAD	Theory
Hz	m	
1,56	1,8651	1,8048
1,57	1,9824	1,9508
1,58	1,9869	2,0000
1,59	1,8746	1,9270
1,6	1,6930	1,7643
1,61	1,4959	1,5684
1,62	1,3140	1,3792
1,63	1,1575	1,2132
1,64	1,0264	1,0732
1,65	0,9174	0,9565
1,66	0,8265	0,8594
1,67	0,7501	0,7780
1,68	0,6854	0,7092
1,69	0,6301	0,6506
1,7	0,5824	0,6003
1,71	0,5409	0,5566
1,72	0,5045	0,5184
1,73	0,4724	0,4848
1,74	0,4439	0,4550
1,75	0,4184	0,4284
1,76	0,3956	0,4046
1,77	0,3749	0,3831
1,78	0,3561	0,3636
1,79	0,3390	0,3459
1,8	0,3234	0,3297
1,81	0,3091	0,3149
1,82	0,2959	0,3013
1,83	0,2837	0,2887
1,84	0,2724	0,2771
1,85	0,2619	0,2663
1,86	0,2521	0,2562
1,87	0,2430	0,2468
1,88	0,2344	0,2381
1,89	0,2264	0,2299
1,9	0,2189	0,2222
1,91	0,2119	0,2149
1,92	0,2052	0,2081
1,93	0,1989	0,2017
1,94	0,1930	0,1956
1,95	0,1873	0,1898
1,96	0,1820	0,1844
1,97	0,1769	0,1792
1,98	0,1721	0,1743
1,99	0,1675	0,1696
2,	0,1631	0,1651
2,01	0,1590	0,1609
2,02	0,1550	0,1568
2,03	0,1512	0,1529
2,04	0,1475	0,1492
2,05	0,1440	0,1457
2,06	0,1407	0,1423
2,07	0,1375	0,1390
2,08	0,1344	0,1359
2,09	0,1314	0,1329

Frequency	Displacement of the node	
	SCAD	Theory
Hz	m	
2,1	0,1286	0,1300
2,11	0,1259	0,1272
2,12	0,1232	0,1245
2,13	0,1207	0,1219
2,14	0,1182	0,1194
2,15	0,1159	0,1170
2,16	0,1136	0,1147
2,17	0,1114	0,1125
2,18	0,1093	0,1103
2,19	0,1072	0,1082
2,2	0,1052	0,1062
2,21	0,1033	0,1043
2,22	0,1014	0,1024
2,23	0,0996	0,1005
2,24	0,0979	0,0988
2,25	0,0962	0,0971
2,26	0,0945	0,0954
2,27	0,0929	0,0938
2,28	0,0914	0,0922
2,29	0,0899	0,0907
2,3	0,0884	0,0892
2,31	0,0870	0,0877
2,32	0,0856	0,0863
2,33	0,0842	0,0850
2,34	0,0829	0,0836
2,35	0,0817	0,0823
2,36	0,0804	0,0811
2,37	0,0792	0,0799
2,38	0,0780	0,0787
2,39	0,0769	0,0775
2,4	0,0757	0,0764
2,41	0,0747	0,0753
2,42	0,0736	0,0742
2,43	0,0725	0,0731
2,44	0,0715	0,0721
2,45	0,0705	0,0711
2,46	0,0696	0,0701
2,47	0,0686	0,0692
2,48	0,0677	0,0682
2,49	0,0668	0,0673
2,5	0,0659	0,0664
2,51	0,0650	0,0655
2,52	0,0642	0,0647
2,53	0,0634	0,0639
2,54	0,0626	0,0630
2,55	0,0618	0,0622
2,56	0,0610	0,0615
2,57	0,0602	0,0607
2,58	0,0595	0,0599
2,59	0,0588	0,0592
2,6	0,0581	0,0585
2,61	0,0574	0,0578
2,62	0,0567	0,0571
2,63	0,0560	0,0564

Frequency	Displacement of the node	
	SCAD	Theory
Hz	m	
2,64	0,0553	0,0557
2,65	0,0547	0,0551
2,66	0,0541	0,0545
2,67	0,0535	0,0538
2,68	0,0528	0,0532
2,69	0,0522	0,0526
2,7	0,0517	0,0520
2,71	0,0511	0,0514
2,72	0,0505	0,0509
2,73	0,0500	0,0503
2,74	0,0494	0,0498
2,75	0,0489	0,0492
2,76	0,0484	0,0487
2,77	0,0479	0,0482
2,78	0,0473	0,0477
2,79	0,0469	0,0472
2,8	0,0464	0,0467
2,81	0,0459	0,0462
2,82	0,0454	0,0457
2,83	0,0449	0,0452
2,84	0,0445	0,0448
2,85	0,0440	0,0443
2,86	0,0436	0,0439
2,87	0,0432	0,0435
2,88	0,0427	0,0430
2,89	0,0423	0,0426
2,9	0,0419	0,0422
2,91	0,0415	0,0418
2,92	0,0411	0,0414
2,93	0,0407	0,0410
2,94	0,0403	0,0406
2,95	0,0399	0,0402
2,96	0,0396	0,0398
2,97	0,0392	0,0394
2,98	0,0388	0,0391
2,99	0,0385	0,0387
3,	0,0381	0,0384

	Theory	SCAD	Deviation
Frequency at which the maximum displacement occurs (Hz)	1.58	1.58	0 %
Maximum displacement (m)	2,0000	1,9869	0.65 %

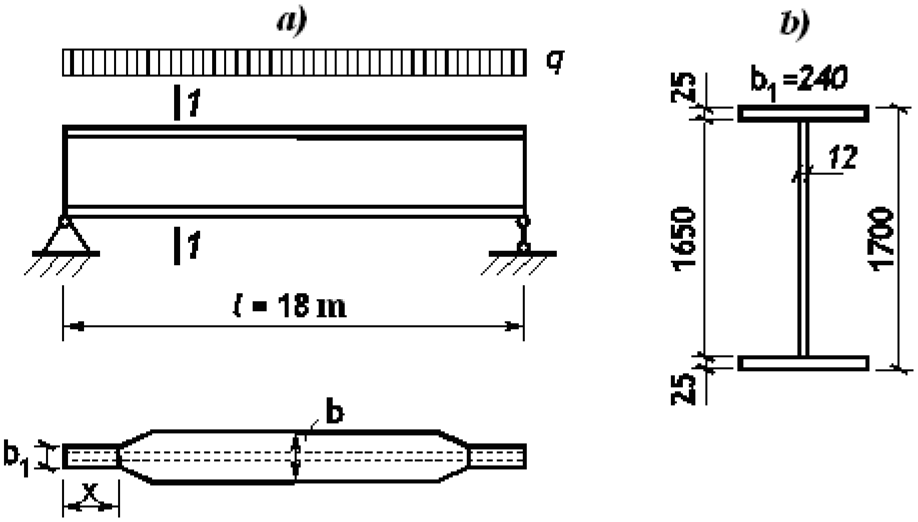
Notes: In the analytical solution the vertical displacement is described by the following transfer function:

$$\frac{1}{\sqrt{\left(1-\frac{\theta^2}{\omega^2}\right)^2+\left(2\xi\theta/\omega\right)^2}},$$

where ω — natural frequency of the undamped system.

Steel Structural Members

Strength and Stiffness Analysis of a Welded I-beam



a – cross-section variation along the beam length; b – beam section and stress diagrams.

Objective: Check of the **Resistance of Sections** mode in the “Steel” postprocessor of SCAD

Task: Check the design section of a welded I-beam for the main beams with a span of 18 m in a normal stub girder system. The top chord of the main beams is restrained by the stringers arranged with a spacing of 1,125 m.

Source: Steel Structures: Student Handbook / [Kudishin U.I., Belenya E.I., Ignatieva V.S and others] - 13-th ed. rev. - M.: Publishing Center "Academy", 2011. p 195.

Compliance with the codes: SNiP II-23-81*, SP 16.13330.2011, DBN B.2.6-163:2010.

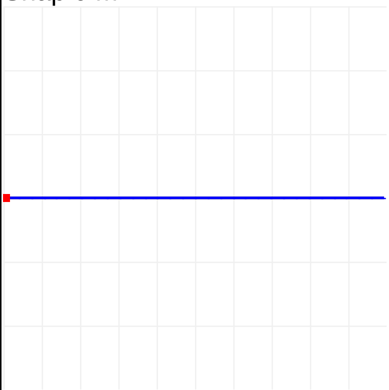
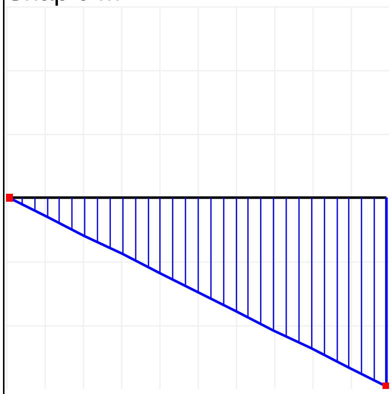
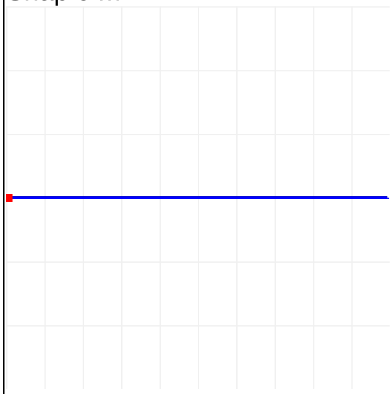
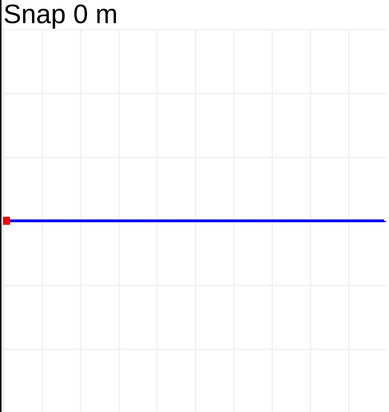
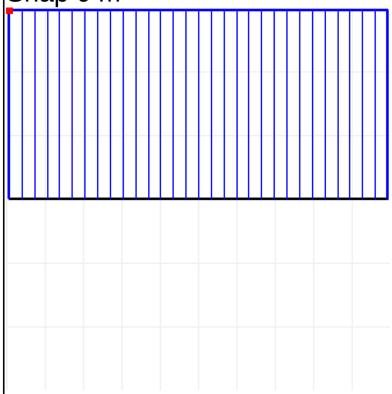
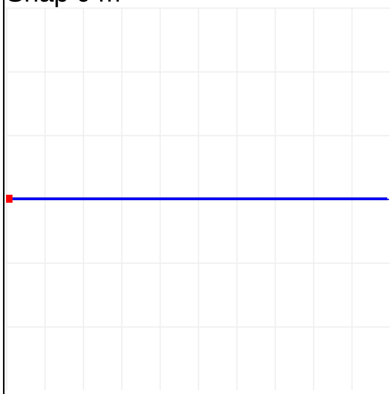
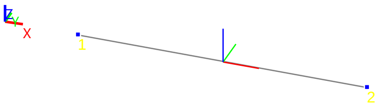
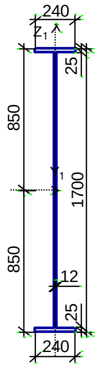
Initial data file:
4.1 SectionResistance_Example_4.1.spr;
report – 4.1 SectionResistance _Example_4.1.doc

Initial data:

M_1 = 3469,28 kNm = 353,6473 Tm	Design bending moment;
Q_1 = 925 kN = 94,29 T	Design shear force;
R_y = 23 kN/cm^2, R_s = 0,58*23=13,3 kN/cm^2	Steel grade C255 with thickness t>20 mm;
l = 18 m	Beam span;
$W_y = 15187,794$ cm^3	Geometric properties for a welded
$I_y = 1290962,5$ cm^4	I-section with flanges 240×25 mm and a web 1650×12 mm.
$S_y = 9108,75$ cm^3	
$i_y = 63,715$ cm, $i_z = 4,265$ cm	

SCAD Parameters. STEEL Postprocessor:

[Element No. 1] Forces

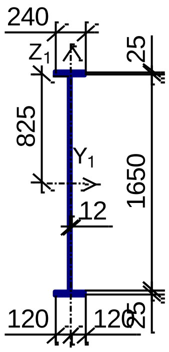
N Max. 0 T Snap 0 m  Max. 0 T Snap 0 m	**M_y** Max. 0 T*m Snap 0 m  Max. 353,65 T*m Snap 3,75 m	**M_z** Max. 0 T*m Snap 0 m  Max. 0 T*m Snap 0 m
M_k Max. 0 T*m Snap 0 m  Max. 0 T*m Snap 0 m	**Q_z** Max. 94,31 T Snap 0 m 	**Q_y** Max. 0 T Snap 0 m  Max. 0 T Snap 0 m
 	Length of the bar 3,75 m Length of the flexible part 3,75 m Loading L1	

Analysis complies with SNiP II-23-81*
Structural member Section

Steel: C255
Member length 3,75 m
Limit slenderness for members in compression: 220
Limit slenderness for members in tension: 300
Service factor 1
Importance factor 1
Effective length factor in the XoY plane 1,125 m
Effective length factor in the XoZ plane 18 m

Length between out-of-plane restraints 1,125 m

Section



Results	Check	Utilization factor
Sec.5.12	Strength under action of bending moment My	0,99
Sec.5.12,5.18	Strength under action of lateral force Qz	0,41
Sec.5.24,5.25	Strength under combined action of longitudinal force and bending moments, no plasticity	0,99
Sec. 5.14*	Strength for reduced stresses at the simultaneous action of the bending moment and the lateral force	0,86
Sec.5.15	Stability of in-plane bending	0,99
Sec.6.15,6.16	Limit slenderness in XoY plane	0,09
Sec.6.15,6.16	Limit slenderness in XoZ plane	0,09

Utilization factor 0,99 - Strength under action of bending moment My

Manual calculation (SNiP II-23-81*):

1. Necessary beam section modulus:

$$W_{nes} = \frac{M_{\max}}{R_y \gamma_c} = \frac{3469,28 \cdot 100}{23} = 15083,826 \text{ cm}^3.$$

2. Maximum tangential stresses in support sections of the beam:

$$\tau_{\max} = \frac{Q_{\max} S_y}{I_y t_w} = \frac{925 \cdot 9108,75}{1290962,5 \cdot 1,2} = 5,4388 \text{ kN/cm}^2.$$

3. Reduced stresses in the considered beam section:

$$\sigma_y = \frac{M_y}{I_y}\frac{h_w}{2} = \frac{3469,28 \cdot 100 \cdot 165}{1290962,5 \cdot 2} = 22,1707 \text{ kN/cm}^2$$

$$\tau_{yz} = \frac{Q_z S_{yf}}{I_y t_w} = \frac{925 \cdot \left(24 \cdot 2,5 \cdot \left(0,5 \cdot 165 + 0,5 \cdot 2,5\right)\right)}{1290962,5 \cdot 1,2} = 3,00 \text{ kN/cm}^2$$

$$\sigma_{red} = \sqrt{\sigma_y^2 + 3\tau_{yz}^2} = \sqrt{22,1707^2 + 3 \cdot 3,00^2} = 22,7715 \text{ kN/cm}^2$$

4. Slenderness of the member in the moment plane:

$$\lambda_y = \frac{\mu l}{i_y} = \frac{18 \cdot 100}{63,715} = 28,2508.$$

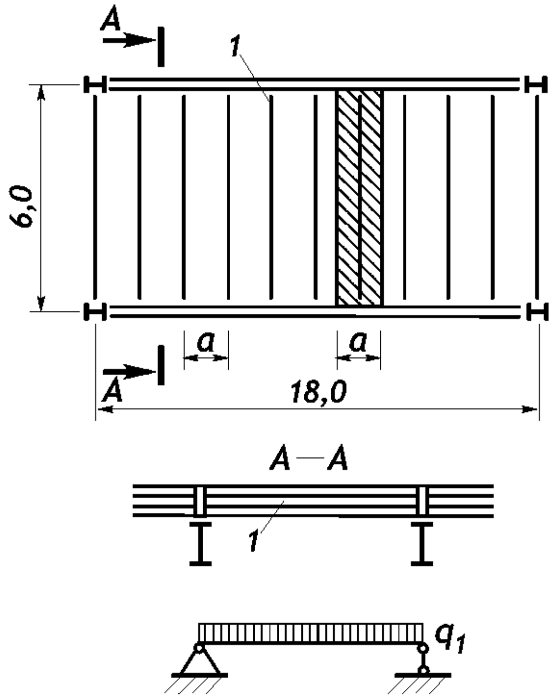
5. Slenderness of the member out of the moment plane:

$$\lambda_y = \frac{\mu l}{i_y} = \frac{1,125 \cdot 100}{4,265} = 26,3775.$$

Comparison of solutions:

Factor	**Manual calculation**	**SCAD**	**Deviation, %**
Strength under action of bending moment *M*y	15083,826/15187,794 = 0,993	0,993	0,0
Strength under action of lateral force *Q*z	5,4388/13,3 = 0,4089	0,408	0,0
Strength for reduced stresses	22,7715/1,15/23 = 0,861	0,86	0,0
Strength under combined action of longitudinal force and bending moments, no plasticity	–	0,993	0,0
Stability of in-plane bending	–	0,993	0,0
Limit slenderness in XoY plane	26,3775/300 = 0,088	0,088	0,0
Limit slenderness in XoZ plane	28,2508/300 = 0,094	0,094	0,0

Strength and Stiffness Analysis of a Rolled I-beam



1 – stringer

Objective: Check of the **Resistance of Sections** mode in the “Steel” postprocessor of SCAD.

Task: Check the design section of a rolled I-beam for the stringers with a span of 6 m in a normal stub girder system. The top chord of the stringers is continuously restrained by the floor plate.

Source: Steel Structures: Student Handbook / [Kudishin U.I., Belenya E.I., Ignatieva V.S and others] - 13-th ed. rev. - M.: Publishing Center "Academy", 2011. p. 183.

Compliance with the codes: SNiP II-23-81*, SP 16.13330.2011, DBN B.2.6-163:2010.

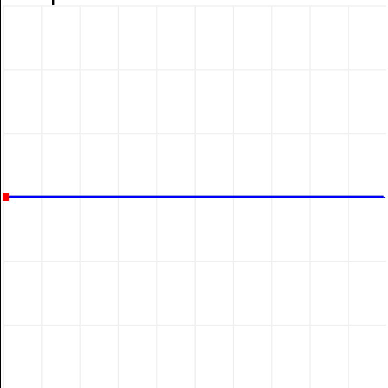
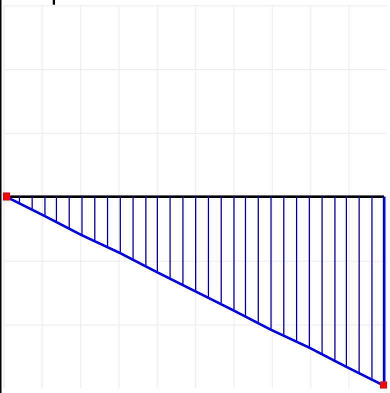
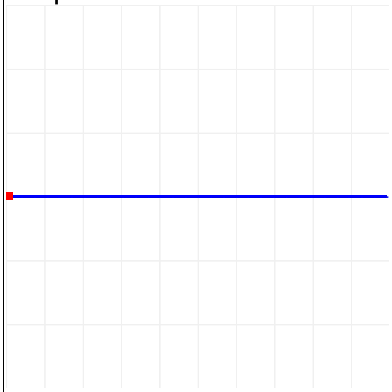
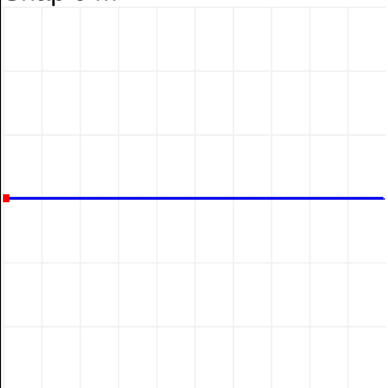
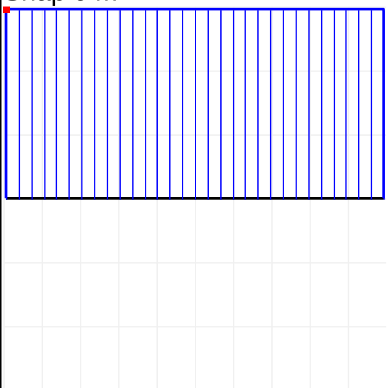
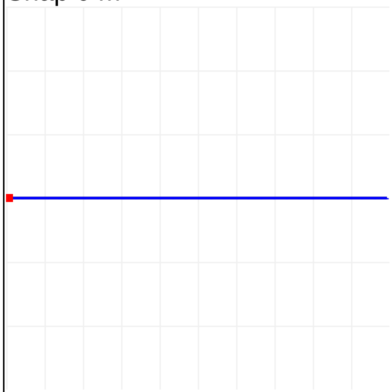
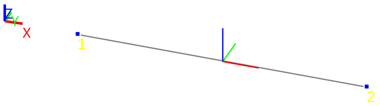
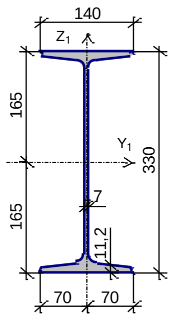
Initial data file:
4.2 SectionResistance_Example_4.2.spr;
report – 4.2 SectionResistance _Example_4.2.doc

Initial data:

$a = 1{,}125$ m	Spacing of stringers;
$R_y = 23$ kN/cm^2,	Steel grade C235;
$M = 125{,}55$ kNm = 12,798 Tm	Design bending moment;
$\gamma_c = 1$	Service factor;
$l = 6$ m	Beam span;
$c_x = 1{,}1$	Coefficient allowing for plastic deformations;
$W_x = 597$ cm^3	Selected I-beam No.33 GOST 8239-89.
$i_y = 13{,}524$ cm, $i_z = 2{,}791$ cm	

SCAD Parameters. STEEL Postprocessor:

[Element No. 1] Forces

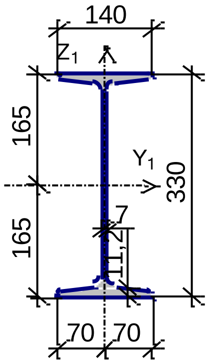
N Max. 0 T Snap 0 m  Max. 0 T Snap 0 m	**M_y** Max. 0 T*m Snap 0 m  Max. 12,8 T*m Snap 6 m	**M_z** Max. 0 T*m Snap 0 m  Max. 0 T*m Snap 0 m
M_k Max. 0 T*m Snap 0 m  Max. 0 T*m Snap 0 m	**Q_z** Max. 2,13 T Snap 0 m 	**Q_y** Max. 0 T Snap 0 m  Max. 0 T Snap 0 m
	Length of the bar 6 m Length of the flexible part 6 m Loading L1	

Analysis complies with SNiP II-23-81*
Structural member section

Steel: C235
Member length 6 m
Limit slenderness for members in compression: 250
Limit slenderness for members in tension: 250
Service factor 1
Importance factor 1
Effective length factor XoZ -- 1
Effective length factor XoY -- 1

Length between out-of-plane restraints 1,125 m

Section



Profile: I-beam with sloped inner flange surfaces GOST 8239-89 33

Results	Check	Utilization factor
Sec.5.12	Strength under action of bending moment My	0,92
Sec.5.12,5.18	Strength under action of lateral force Qz	0,08
Sec.5.24,5.25	Strength under combined action of longitudinal force and bending moments, no plasticity	0,92
Sec.5.15	Stability of in-plane bending	0,92
Sec.6.15,6.16	Limit slenderness in XoY plane	0,86
Sec.6.15,6.16	Limit slenderness in XoZ plane	0,18

Utilization factor 0,92 - Strength under action of bending moment My

Manual calculation (SNiP II-23-81*):

1. Necessary beam section modulus:

$$W_{nes} = \frac{M_{\max}}{R_y \gamma_c} = \frac{125,55 \cdot 100}{23} = 545,8696 \text{ cm}^3.$$

2. Slenderness of the member in the moment plane:

$$\lambda_y = \frac{\mu l}{i_y} = \frac{6 \cdot 100}{13,524} = 44,3656\,.$$

3. Slenderness of the member out of the moment plane:

$$\lambda_z = \frac{l_{ef,z}}{i_z} = \frac{6 \cdot 100}{2,791} = 214,9767\,.$$

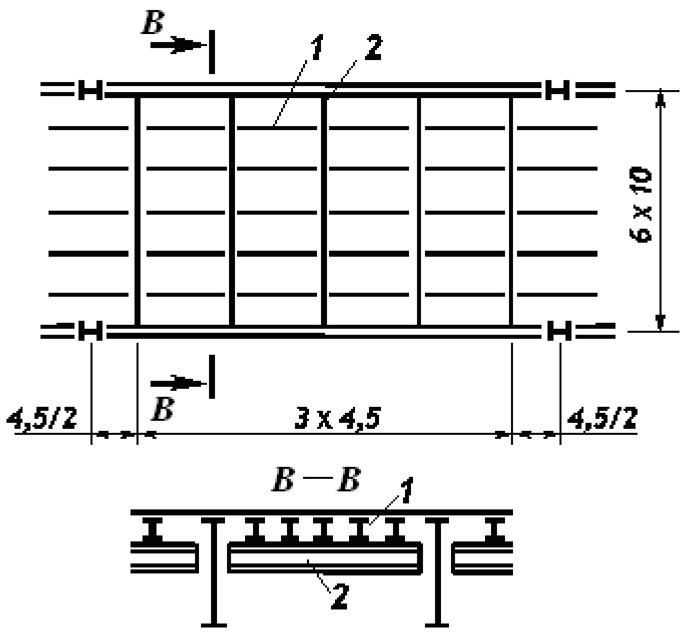
Comparison of solutions:

Factor	Manual calculation	SCAD	Deviation, %
Strength under action of bending moment *M*y	545,8696/597 = 0,914	0,915	–
Strength under combined action of longitudinal force and bending moments, no plasticity	–	0,915	–
Stability of in-plane bending	–	0,915	–
Limit slenderness in XoY plane	214,9767/250 = 0,86	0,86	–
Limit slenderness in XoZ plane	44,3656/250 = 0,177	0,177	–

Comments:

The check of the beam strength taking into account the development of the limited plastic deformations was not performed in the manual calculation, because according to the codes this calculation is possible only when the beam web has stiffeners. In the initial data of the example the stringer was specified without any intermediate stiffeners.

Strength and Stiffness Analysis of a Rolled I-beam



1 – stringer; 2 – secondary beam

Objective: Check of the **Resistance of Sections** mode in the "Steel" postprocessor of SCAD.

Task: Check the design section of a rolled I-beam for the stringers with a span of 4,5 m in a normal stub girder system. The top chord of the stringers is continuously restrained by the floor plate.

Source: Steel Structures: Student Handbook / [Kudishin U.I., Belenya E.I., Ignatieva V.S and others] - 13-th ed. rev. - M.: Publishing Center "Academy", 2011. p. 183.

Compliance with the codes: SNiP II-23-81*, SP 16.13330.2011, DBN B.2.6-163:2010.

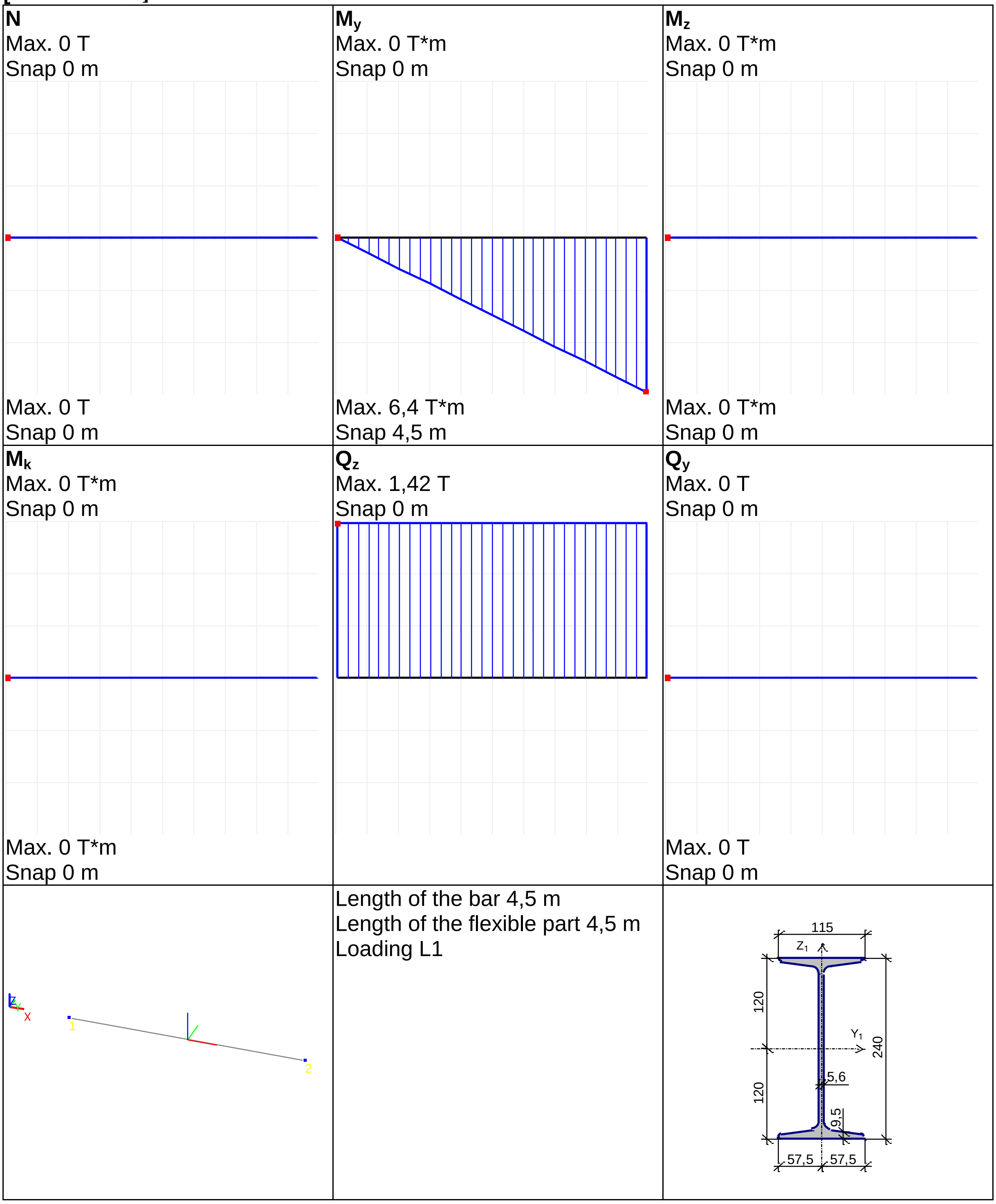
Initial data file:
4.3 SectionResistance_Example_4.3.spr;
report – 4.3 SectionResistance _Example_4.3.doc

Initial data:

R_y = 23 kN/cm^2	Steel grade C235;
M = 62,78 kNm= 6,4 Tm	Design bending moment;
γ_c = 1	Service factor;
l = 4,5 m	Beam span;
c_x = 1,1	Coefficient allowing for plastic deformations;
W_x = 288,33 cm^3	Selected I-beam No.24 GOST 8239-89.
$i_y = 9,971\,\text{cm},\ i_z = 2,385\,\text{cm}$	

SCAD Parameters. STEEL Postprocessor:

[Element No. 1] Forces

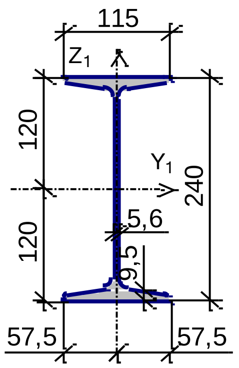


Analysis complies with SNiP II-23-81*
Structural member section

Steel: C235
Member length 4,5 m
Limit slenderness for members in compression: 250
Limit slenderness for members in tension: 250
Service factor 1
Importance factor 1
Effective length factor XoZ -- 1
Effective length factor XoY -- 1

Length between out-of-plane restraints 1,125 m

Section



Profile: I-beam with sloped inner flange surfaces GOST 8239-89 24

Results	Check	Utilization factor
Sec.5.12	Strength under action of bending moment My	0,95
Sec.5.12,5.18	Strength under action of lateral force Qz	0,09
Sec.5.24,5.25	Strength under combined action of longitudinal force and bending moments, no plasticity	0,95
Sec.5.15	Stability of in-plane bending	0,95
Sec.6.15,6.16	Limit slenderness in XoY plane	0,75
Sec.6.15,6.16	Limit slenderness in XoZ plane	0,18

Utilization factor 0,95 - Strength under action of bending moment My

Manual calculation (SNiP II-23-81*):

1. Necessary beam section modulus:

$$W_{nes} = \frac{M_{\max}}{R_y \gamma_c} = \frac{62,78 \cdot 100}{23} = 272,9565 \text{ cm}^3.$$

2. Slenderness of the member in the moment plane:

$$\lambda_y = \frac{\mu l}{i_y} = \frac{4,5 \cdot 100}{9,971} = 45,131.$$

3. Slenderness of the member out of the moment plane:

$$\lambda_z = \frac{\mu l}{i_z} = \frac{4,5 \cdot 100}{2,385} = 188,679.$$

Comparison of solutions:

Factor	Manual calculation	SCAD	Deviation, %
Strength under action of bending moment *My*	272,9565/288,33 = 0,9467	0,947	0.0
Strength under combined action of longitudinal force and bending moments, no plasticity	–	0,947	0.0
Stability of in-plane bending	–	0,947	0.0
Limit slenderness in XoY plane	188,679/250 = 0,755	0,755	0.0
Limit slenderness in XoZ plane	45,131/250 = 0,1805	0,181	0.0

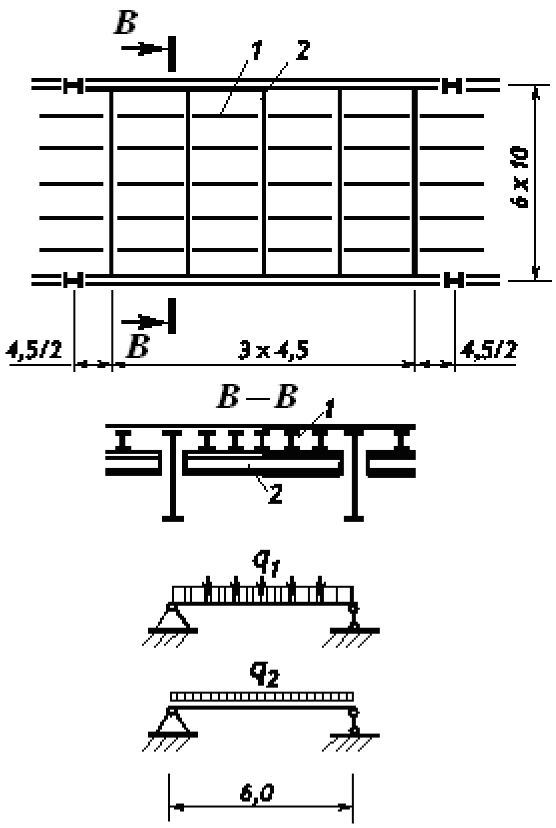
Comments:

1. The check of the beam strength taking into account the development of the limited plastic deformations was not performed in the manual calculation, because according to the codes this calculation is possible

only when the beam web has stiffeners. In the initial data of the example the stringer was specified without any intermediate stiffeners.

2. The check for the stability of in-plane bending was performed in the computer-aided calculation according to the codes at $\varphi_b = 1,0$.

Strength and Stiffness Analysis of a Rolled I-beam



1 – stringer;
2 – secondary beam

Objective: Check of the **Resistance of Sections** mode in the "Steel" postprocessor of SCAD.

Task: Check the design section of a rolled I-beam for the secondary beams with a span of 6 m in a complex stub girder system. The top chord of the secondary beams is restrained by the stringers arranged with a spacing of 1 m.

Source: Steel Structures: Student Handbook / [Kudishin U.I., Belenya E.I., Ignatieva V.S and others] - 13-th ed. rev. - M.: Publishing Center "Academy", 2011. p. 183.

Compliance with the codes: SNiP II-23-81*, SP 16.13330.2011, DBN B.2.6-163:2010.

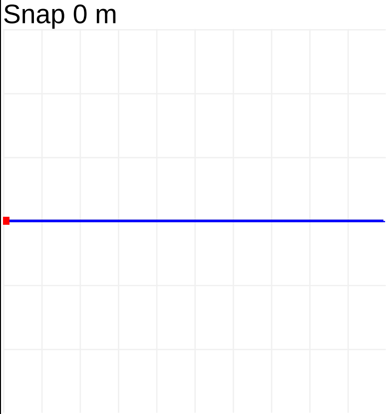
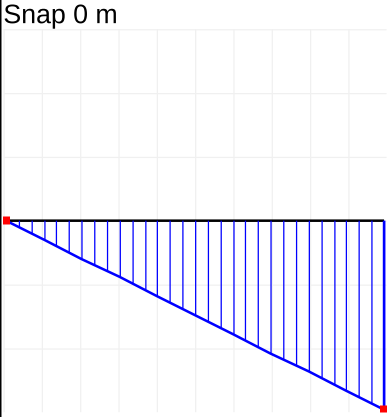
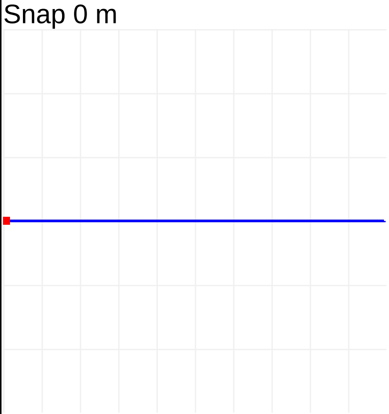
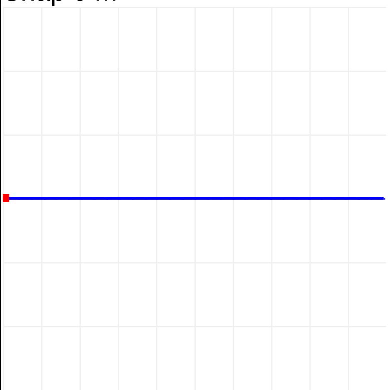
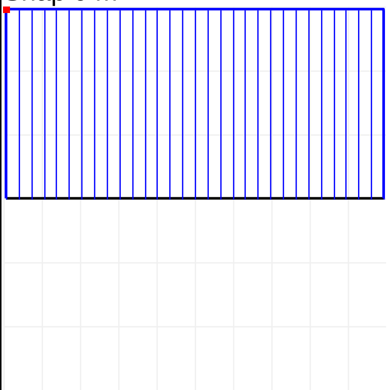
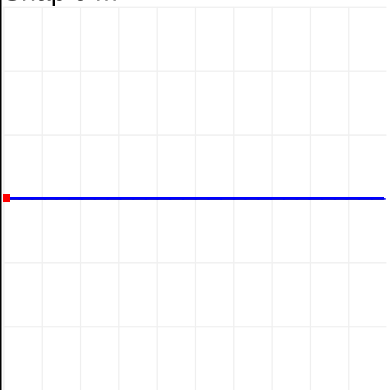
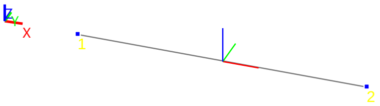
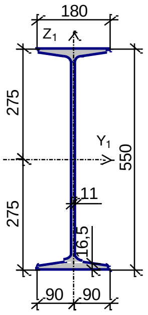
Initial data file:
4.4 SectionResistance_Example_4.4.spr;
report – 4.4 SectionResistance _Example_4.4.doc

Initial data:

R_y = 23 kN/cm^2,	Steel grade C235;
M = 508,5 kNm = 51,83486 Tm	Design bending moment;
γ_c = 1	Service factor;
l = 6 m	Beam span;
c_x = 1,1	Coefficient allowing for plastic deformations;
W_x = 2034,982 cm^3	Selected I-beam No.55 GOST 8239-89
$i_y = 21,777$ cm, $i_z = 3,39$ cm.	

SCAD Results. STEEL Postprocessor:

[Element No. 1] Forces

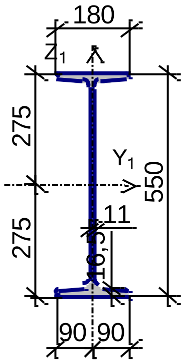
N Max. 0 T Snap 0 m  Max. 0 T Snap 0 m	**M_y** Max. 0 T*m Snap 0 m  Max. 51,83 T*m Snap 6 m	**M_z** Max. 0 T*m Snap 0 m  Max. 0 T*m Snap 0 m
M_k Max. 0 T*m Snap 0 m  Max. 0 T*m Snap 0 m	**Q_z** Max. 8,64 T Snap 0 m 	**Q_y** Max. 0 T Snap 0 m  Max. 0 T Snap 0 m
	Length of the bar 6 m Length of the flexible part 6 m Loading L1	

Analysis complies with SNiP II-23-81*
Structural member section

Steel: C235
Member length 6 m
Limit slenderness for members in compression: 250
Limit slenderness for members in tension: 250
Service factor 1
Importance factor 1
Effective length factor XoZ -- 1
Effective length factor XoY -- 1

Length between out-of-plane restraints 1,125 m

Section



Profile: I-beam with sloped inner flange surfaces GOST 8239-89 55

Results	Check	Utilization factor
Sec.5.12	Strength under action of bending moment My	**1,09**
Sec.5.12,5.18	Strength under action of lateral force Qz	0,12
Sec.5.24,5.25	Strength under combined action of longitudinal force and bending moments, no plasticity	**1,09**
Sec.5.15	Stability of in-plane bending	**1,09**
Sec.6.15,6.16	Limit slenderness in XoY plane	0,71
Sec.6.15,6.16	Limit slenderness in XoZ plane	0,11

Utilization factor 1,09 - Strength under action of bending moment My

Manual calculation (SNiP II-23-81*):

1. Necessary beam section modulus:

$$W_{nes} = \frac{M_{\max}}{R_y \gamma_c} = \frac{508,5 \cdot 100}{23} = 2210,8696 \text{ cm}^3.$$

2. Slenderness of the member in the moment plane and out of the moment plane:

$$\lambda_y = \frac{\mu l}{i_y} = \frac{6,0 \cdot 100}{21,777} = 27,552;$$

$$\lambda_z = \frac{\mu l}{i_z} = \frac{6,0 \cdot 100}{3,39} = 176,99.$$

Comparison of solutions:

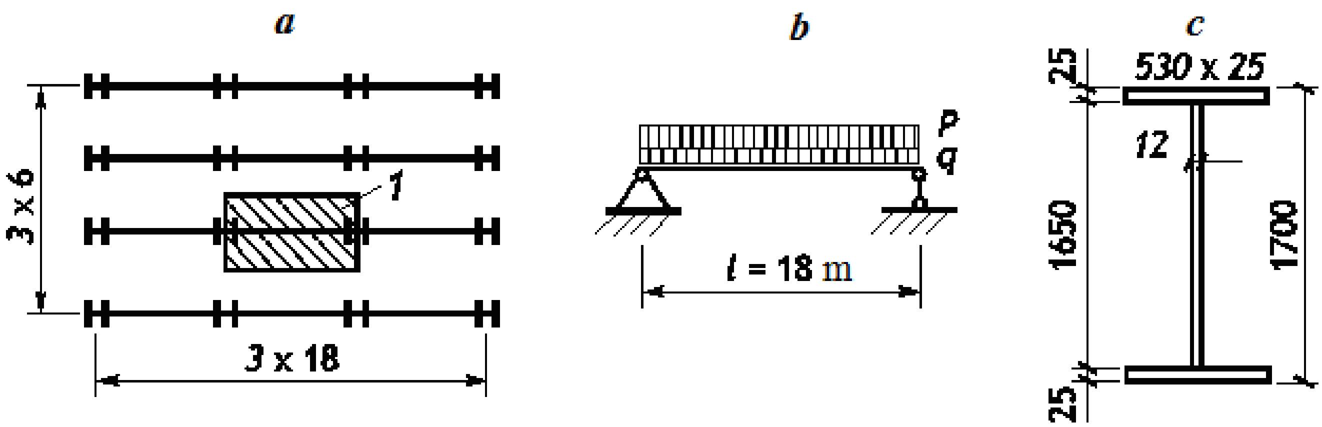
Factor	Manual calculation	SCAD	Deviation, %
Strength under action of bending moment *M*y	2210,8696/2034,982 = 1,086	1,086	0,0
Strength under combined action of longitudinal force and bending moments, no plasticity	–	1,086	0,0
Stability of in-plane bending	–	1,086	0,0
Limit slenderness in XoY plane	176,99/250 = 0,708	0,708	0,0
Limit slenderness in XoZ plane	27,552/250 = 0,110	0,11	0,0

Comments:

1. The check of the beam strength taking into account the development of the limited plastic deformations was not performed in the manual calculation, because according to the codes this calculation is possible only when the beam web has stiffeners. In the initial data of the example the stringer was specified without any intermediate stiffeners.

2. The check for the stability of in-plane bending was performed in the computer-aided calculation according to the codes at $\varphi_b = 1,0$ for the effective length $l_{ef} = 1$ m.

Strength and Stiffness Analysis of a Welded I-beam



a – floor plan; b – design model of the main beam; c – beam section; 1 – load area

Objective: Check of the **Resistance of Sections** mode in the "Steel" postprocessor of SCAD.

Task: Check the design section of a welded I-beam for the main beams with a span of 18 m in a normal stub girder system. The top chord of the main beams is restrained by secondary beams arranged with a spacing of 1,0 m.

Source: Steel Structures: Student Handbook / [Kudishin U.I., Belenya E.I., Ignatieva V.S and others] - 13-th ed. rev. - M.: Publishing Center "Academy", 2011. p. 192.

Compliance with the codes: SNiP II-23-81*, SP 16.13330.2011, DBN B.2.6-163:2010.

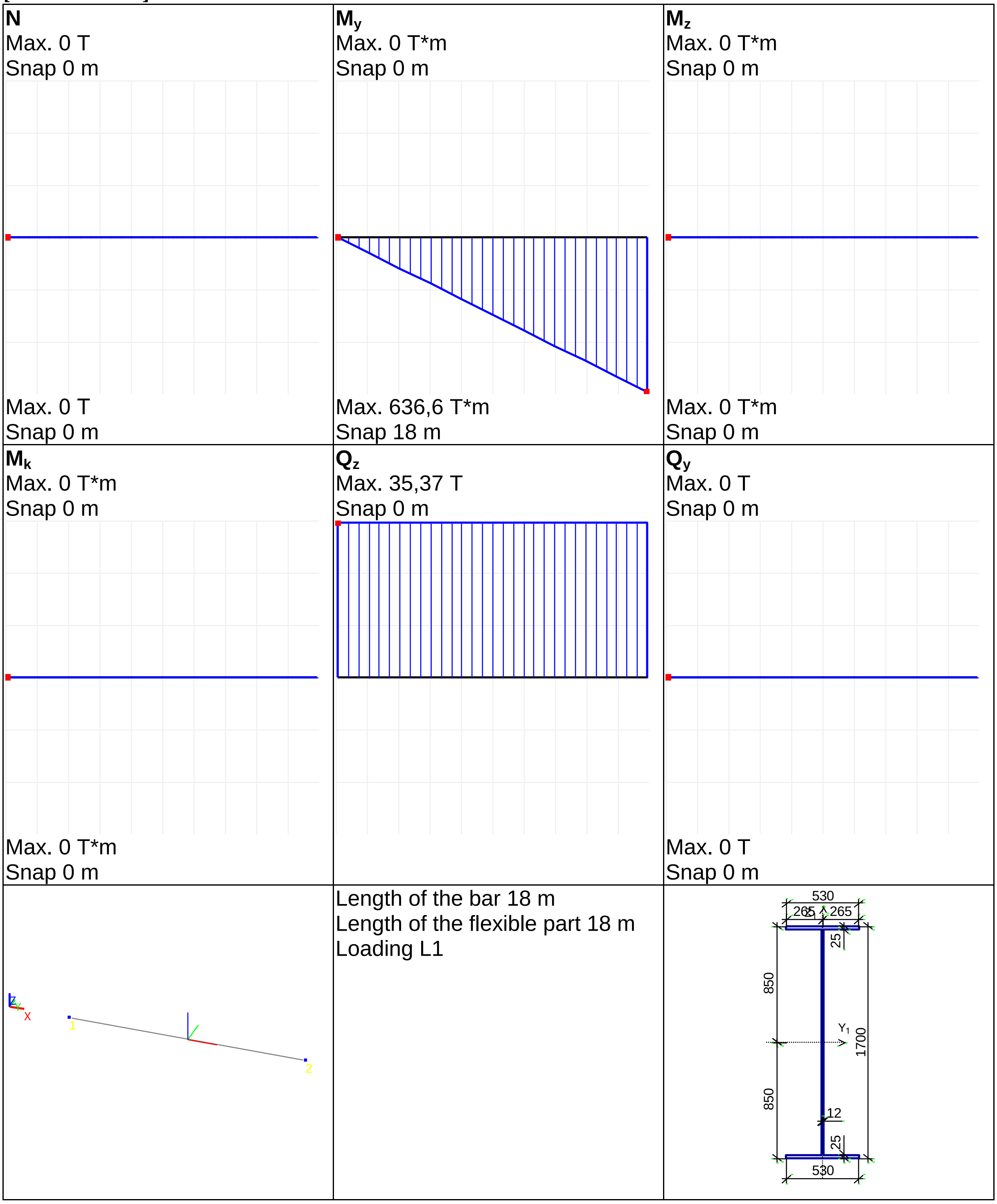
Initial data file:
4.5 SectionResistance_Example_4.5.spr;
report – 4.5 SectionResistance _Example_4.5.doc

Initial data:

R_y = 23 kN/cm^2, R_s = 0,58*23=13,3 kN/cm^2	Steel grade C255 with thickness t>20 mm;
M = 6245 kNm = 636,595 Tm	Design bending moment;
$\gamma_c = 1$	Service factor;
l = 18 m	Beam span;
$I_y = 2308077{,}083\text{cm}^4$	Geometric properties for a welded
$W_y = 27153{,}848\text{cm}^3$	I-section with flanges 1650×12 mm and a web 530×25 mm.
$i_y = 70{,}605\text{cm},\ i_z = 11{,}577$ cm	

SCAD Results. STEEL Postprocessor:

[Element No. 1] Forces

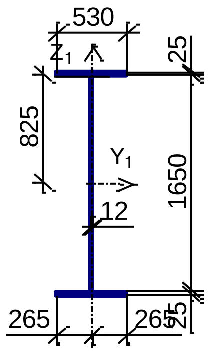


Analysis complies with SNiP II-23-81*
Structural member section

Steel: C255
Member length 18 m
Limit slenderness for members in compression: 250
Limit slenderness for members in tension: 250
Service factor 1
Importance factor 1
Effective length factor XoZ -- 1
Effective length factor XoY -- 1

Length between out-of-plane restraints 1,125 m

Section



Results	Check	Utilization factor
Sec.5.12	Strength under action of bending moment My	1
Sec.5.12,5.18	Strength under action of lateral force Qz	0,14
Sec.5.24,5.25	Strength under combined action of longitudinal force and bending moments, no plasticity	1
Sec.5.15	Stability of in-plane bending	1
Sec.6.15,6.16	Limit slenderness in XoY plane	0,62
Sec.6.15,6.16	Limit slenderness in XoZ plane	0,1

Utilization factor 1 - Strength under action of bending moment My

Manual calculation (SNiP II-23-81*):

1. Necessary beam section modulus:

$$W_{nes} = \frac{M_{\max}}{R_y \gamma_c} = \frac{6245 \cdot 100}{23} = 27152{,}174\ \text{cm}^3.$$

2. Slenderness of the member in the moment plane and out of the moment plane:

$$\lambda_y = \frac{\mu l}{i_y} = \frac{18{,}0 \cdot 100}{70{,}605} = 25{,}4939\ ;$$

$$\lambda_z = \frac{\mu l}{i_z} = \frac{18{,}0 \cdot 100}{11{,}577} = 155{,}481\,.$$

Comparison of solutions:

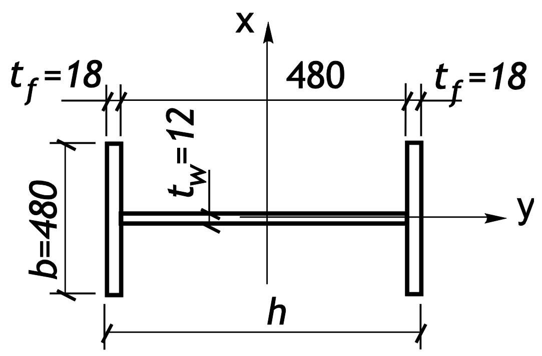
Factor	Manual calculation	SCAD	Deviation, %
Strength under action of bending moment *M*y	27152,174/27153,848 = 1,0	1,0	0,0
Strength under combined action of longitudinal force and bending moments, no plasticity	–	1,0	0,0
Stability of in-plane bending	–	1,0	0,0
Limit slenderness in XoZ plane	25,4939/250 = 0,102	0,102	0,0
Limit slenderness in XoY plane	155,481/250 = 0,622	0,622	0,0

Comments:

1. The check of the beam strength taking into account the development of the limited plastic deformations was not performed, because according to the codes this calculation is possible only when the beam web has stiffeners. In the initial data of the example the stringer was specified without any intermediate stiffeners.

2. The check for the stability of in-plane bending was performed in the computer-aided calculation according to the codes at $\varphi_b = 1,0$ for the effective length $l_{ef} = 1$ m.

Analysis of an Axially Compressed Welded I-beam Column



Objective: Check of the **Resistance of Sections** mode in the “Steel” postprocessor of SCAD

Task: Check the design section of a welded I-beam for the axially compressed column with a height of 6,5 m.

Source: Steel Structures: Student Handbook / [Kudishin U.I., Belenya E.I., Ignatieva V.S and others] - 13-th ed. rev. - M.: Publishing Center "Academy", 2011. p. 256.

Compliance with the codes: SNiP II-23-81*, SP 16.13330.2011, DBN B.2.6-163:2010.

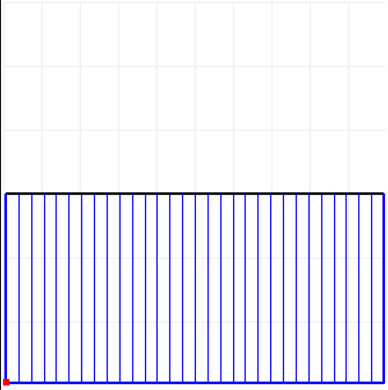
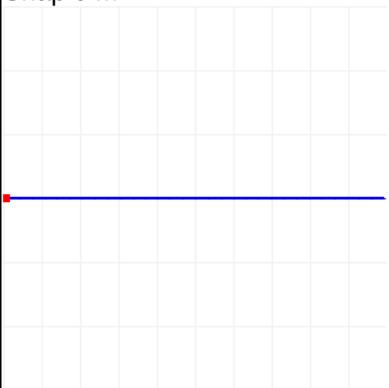
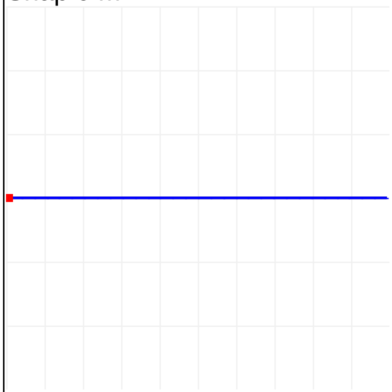
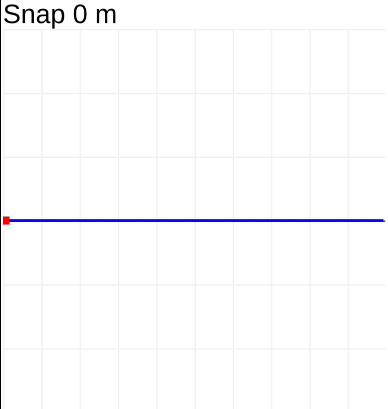
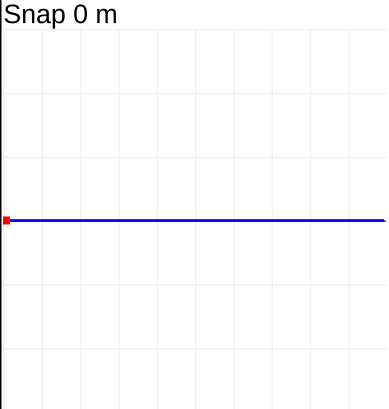
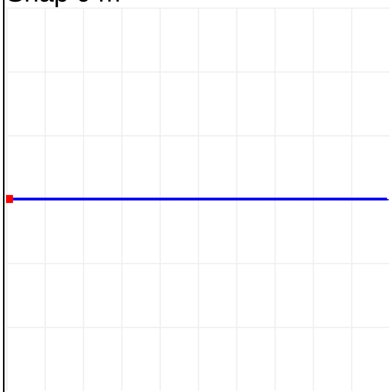
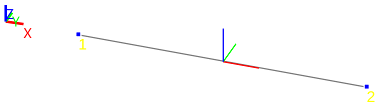
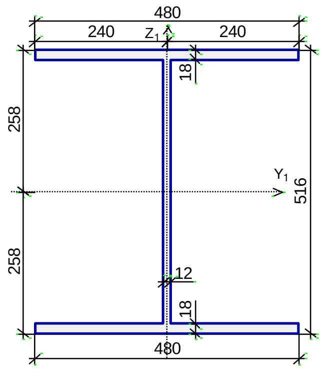
Initial data file:
4.6 SectionResistance_Example_4.6.spr;
report – 4.6 SectionResistance _Example_4.6.doc

Initial data:

R_y = 24 kN/cm^2	Steel grade C245;
l = 6,5 m	Column height;
N = 5000 kN = 509,684 T	Design longitudinal compressive force;
μ = 0,7	The lower restraint is rigid and the upper one is pinned for both principal planes of inertia;
γ_c = 1	Service factor;
$A = 230,4\,\text{cm}^2$, $I_y = 118243,584\,\text{cm}^4$, $I_z = 33184,512\,\text{cm}^4$	Geometric properties for a welded I-section with a web 480×12 mm and flanges 480×18 mm;
$W_y = 4583,085\,\text{cm}^3$, $W_z = 1382,688\,\text{cm}^3$	
$i_y = 22,654\,\text{cm}$, $i_z = 12,001\,\text{cm}$	

SCAD Results. STEEL Postprocessor:

[Element No. 1] Forces

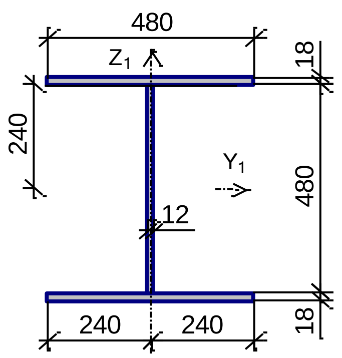
N  Max. -509,68 T Snap 0 m	**M_y** Max. 0 T*m Snap 0 m  Max. 0 T*m Snap 0 m	**M_z** Max. 0 T*m Snap 0 m  Max. 0 T*m Snap 0 m
M_k Max. 0 T*m Snap 0 m  Max. 0 T*m Snap 0 m	**Q_z** Max. 0 T Snap 0 m  Max. 0 T Snap 0 m	**Q_y** Max. 0 T Snap 0 m  Max. 0 T Snap 0 m
	Length of the bar 6,5 м Length of the flexible part 6,5 м Loading L1	

Analysis complies with SNiP II-23-81*
Structural member section

Steel: C245
Member length 6,5 m
Limit slenderness for members in compression: 180 - 60□
Limit slenderness for members in tension: 250
Service factor 1
Importance factor 1
Effective length factor XoZ -- 0,7
Effective length factor XoY -- 0,7

Length between out-of-plane restraints 6,5 m

Section



Results	Check	Utilization factor
Sec.5.24,5.25	Strength under combined action of longitudinal force and bending moments, no plasticity	0,9
Sec.5.3	Stability under compression in XoY (XoU) plane	**1**
Sec.5.3	Stability under compression in XoZ (XoV) plane	0,94
Sec.5.1	Strength under axial compression/tension	0,9
Sec.6.15,6.16	Limit slenderness in XoY plane	0,316
Sec.6.15,6.16	Limit slenderness in XoZ plane	0,162

Utilization factor 1 - Stability under compression in XoY (XoU) plane

Manual calculation (SNiP II-23-81*):

1. Load-bearing capacity of the element under axial compression/tension:

$$N = AR_y\gamma_c = 230,4 \cdot 24 \cdot 1 = 5529,6 \text{ kN.}$$

2. Slenderness of the element for both principal planes of inertia:

$$\lambda_y = \frac{l_{ef,y}}{i_y} = \frac{\mu l}{i_y} = \frac{0,7 \cdot 6,5 \cdot 100}{22,654} = 20,08475\,;$$

$$\bar{\lambda}_z = \frac{l_{ef,z}}{i_z} = \frac{\mu l}{i_z} = \frac{0,7 \cdot 6,5 \cdot 100}{12,001} = 37,9135\,.$$

3. Conditional slenderness of the element for both principal planes of inertia:

$$\bar{\lambda}_y = \frac{l_{ef,y}}{i_y}\sqrt{\frac{R_y}{E}} = \frac{\mu l}{i_y}\sqrt{\frac{R_y}{E}} = \frac{0,7 \cdot 6,5 \cdot 100}{22,654}\sqrt{\frac{240}{2,06 \cdot 10^5}} = 0,68555\,;$$

$$\bar{\lambda}_z = \frac{l_{ef,z}}{i_z}\sqrt{\frac{R_y}{E}} = \frac{\mu l}{i_z}\sqrt{\frac{R_y}{E}} = \frac{0,7 \cdot 6,5 \cdot 100}{12,001}\sqrt{\frac{240}{2,06 \cdot 10^5}} = 1,2941\,.$$

4. Buckling coefficients under axial compression:

$$\varphi_y = 1 - \left(0,073 - 5,53\frac{R_y}{E}\right)\bar{\lambda}_y\sqrt{\bar{\lambda}_y} = 1 - \left(0,073 - 5,53 \cdot \frac{240}{2,06 \cdot 10^5}\right) \cdot 0,68555\sqrt{0,68555} = 0,9622\,;$$

$$\varphi_z = 1 - \left(0,073 - 5,53\frac{R_y}{E}\right)\bar{\lambda}_z\sqrt{\bar{\lambda}_z} = 1 - \left(0,073 - 5,53 \cdot \frac{240}{2,06 \cdot 10^5}\right) \cdot 1,2941\sqrt{1,2941} = 0,902\,;$$

5. Load-bearing capacity of the element at its buckling:

$$N_{b,y} = \varphi_y AR_y\gamma_c = 0,9622 \cdot 230,4 \cdot 24 \cdot 1 = 5320,58 \text{ kN;}$$

$$N_{b,z} = \varphi_z AR_y\gamma_c = 0,902 \cdot 230,4 \cdot 24 \cdot 1 = 4987,7 \text{ kN.}$$

6. Limit slenderness:

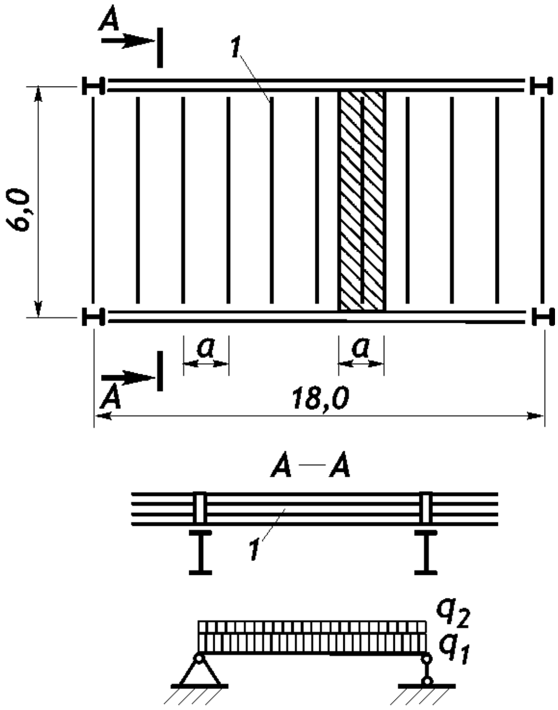
$$\lambda_{uy} = 180 - 60 \cdot \frac{N}{\varphi_y A R_y \gamma_c} = 180 - 60 \cdot \frac{5000}{0,9622 \cdot 230,4 \cdot 24 \cdot 1} = 123,615;$$

$$\lambda_{uz} = 180 - 60 \cdot \frac{N}{\varphi_z A R_y \gamma_c} = 180 - 60 \cdot \frac{5000}{0,902 \cdot 230,4 \cdot 24 \cdot 1} = 119,852.$$

Comparison of solutions:

Factor	**Source**	**Manual calculation**	**SCAD**	**Deviation, %**
Strength under combined action of longitudinal force and bending moments, no plasticity	–	5000/5529,6 = 0,904	0,904	0,0
Stability under compression in XoY (XoU) plane	23,69/24 = 0,987	5000/4987,7 = 1,002	1,002	0,0
Stability under compression in XoZ (XoV) plane	–	5000/5320,58 = 0,94	0,94	0,0
Strength under axial compression/tension	0,904	5000/5529,6 = 0,904	0,904	0,0
Limit slenderness in XoY plane	–	37,9135/119,852 = 0,316	0,316	0,0
Limit slenderness in XoZ plane	–	20,085/123,615 = 0,162	0,162	0,0

Strength and Stiffness Analysis of Stringers for a Normal Stub Girder System



1 – stringer

Objective: Check the mode for the beam analysis in the "Steel" postprocessor of SCAD.

Task: Select a rolled I-beam for the stringers with a span of 6 m in a normal stub girder system. The top chord of the stringers is continuously restrained by the floor plate.

Source: Steel Structures: Student Handbook / [Kudishin U.I., Belenya E.I., Ignatieva V.S and others] - 13-th ed. rev. - M.: Publishing Center "Academy", 2011. p. 183.

Compliance with the codes: SNiP II-23-81*, SP 16.13330.2011, DBN B.2.6-163:2010.

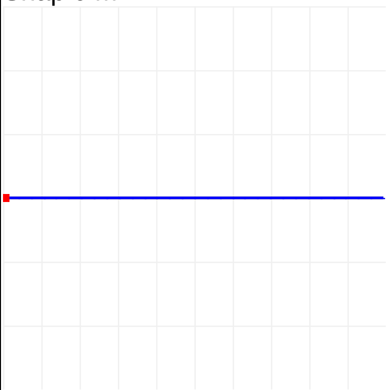
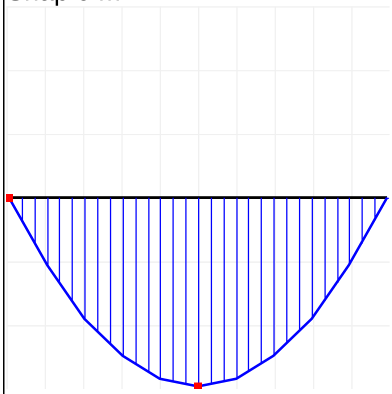
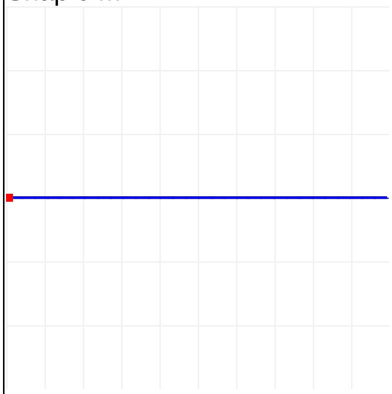
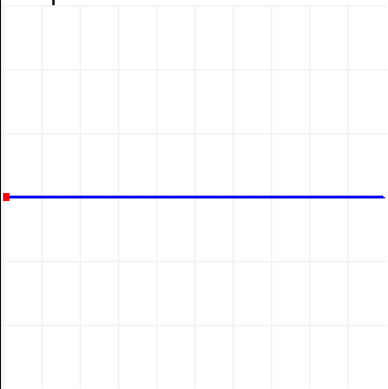
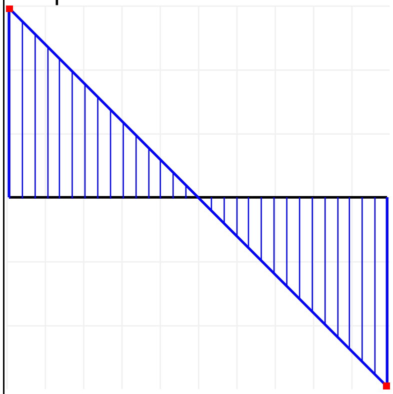
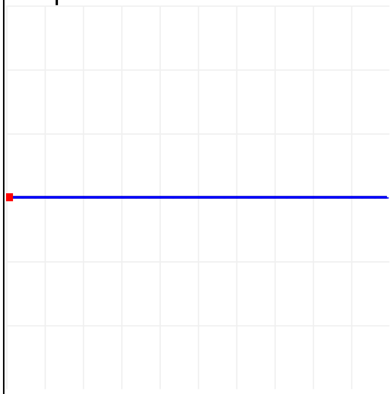
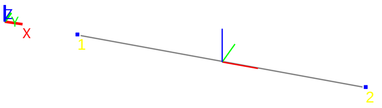
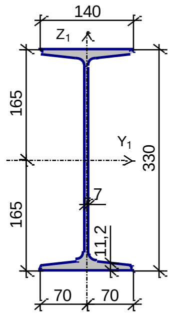
Initial data file:
3.1 Beam_Example_3.1.spr;
report – 3.1 Beam_Example_3.1.doc

Initial data:

$a = 1{,}125$ m	Spacing of stringers
q_{ch} = (0,77 + 20) kN/m^2 ×1,125 m = 23,37 kN/m	Total characteristic load
q_1 = 1,05×0,77 kN/m^2 ×1,125 m = 0,91 kN/m	Design permanent load
q_2 = 1,2×20 kN/m^2 ×1,125 m = 27 kN/m	Design temporary load
R_y = 23 kN/cm^2	Steel grade C235
l = 6 m	Beam span
$[f]$ = 1/250×6,0 m = 24 mm	Limit deflection
γ_c = 1	Service factor
W_x = 596,364 cm^3	Selected I-beam No.33 GOST 8239-89
I_x = 9840 cm^4, $S_x = 339$ cm^3, $t_w = 7$ mm.	

SCAD Results. STEEL Postprocessor:

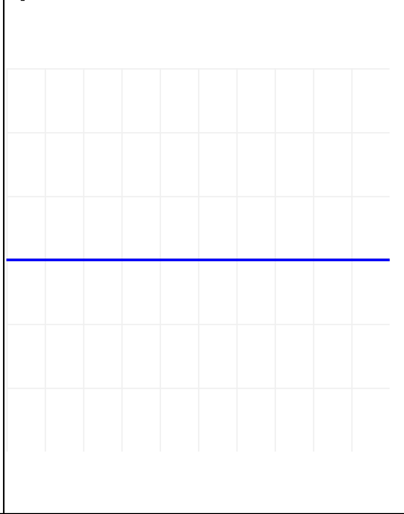
[Element No. 1] Forces

<table>
<tr>
<td>N
Max. 0 T
Snap 0 m

Max. 0 T
Snap 0 m</td>
<td>M_y
Max. 0 T*m
Snap 0 m

Max. 12,8 T*m
Snap 3 m</td>
<td>M_z
Max. 0 T*m
Snap 0 m

Max. 0 T*m
Snap 0 m</td>
</tr>
<tr>
<td>M_k
Max. 0 T*m
Snap 0 m

Max. 0 T*m
Snap 0 m</td>
<td>Q_z
Max. 8,54 T
Snap 0 m

Max. -8,54 T
Snap 6 m</td>
<td>Q_y
Max. 0 T
Snap 0 m

Max. 0 T
Snap 0 m</td>
</tr>
<tr>
<td>
</td>
<td>Length of the bar 6 m
Length of the flexible part 6 m
Loading L1 - "1"</td>
<td>
</td>
</tr>
</table>

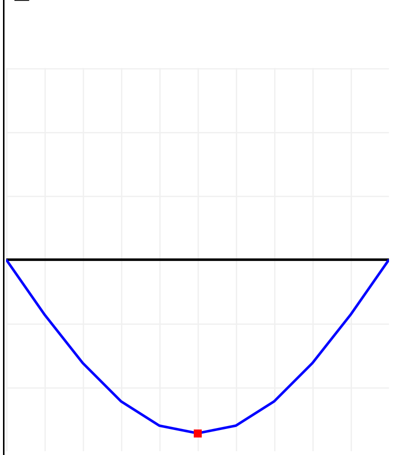
[Element No. 1] Deflections

X

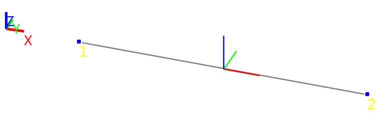
Y



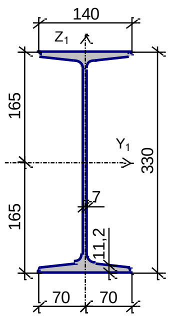
Z



Max. -23,23 mm
Snap 3 m



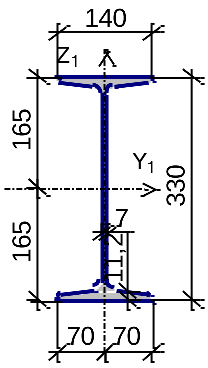
Length of the bar 6 m
Length of the flexible part 6 m
Loading L1 - "1"



Analysis complies with SNiP II-23-81*
Structural member beam

Steel: C235
Member length 6 m
Limit slenderness for members in compression: 180
Limit slenderness for members in tension: 300
Service factor 1
Importance factor 1
Effective length factor XoZ -- 1
Effective length factor XoY -- 1
Length between out-of-plane restraints 0,01 m

Section



Profile: I-beam with sloped inner flange surfaces GOST 8239-89 33

Results	Check	Utilization factor
Sec.5.12	Strength under action of bending moment My	0,92
Sec.5.12,5.18	Strength under action of lateral force Qz	0,31
Sec.5.24,5.25	Strength under combined action of longitudinal force and bending moments, no plasticity	0,92
Sec.5.15	Stability of in-plane bending	0,92
Sec.6.15,6.16	Limit slenderness in XoY plane	0,72
Sec.6.15,6.16	Limit slenderness in XoZ plane	0,15

Utilization factor 0,92 - Strength under action of bending moment My

Manual calculation:

1. Design bending moment and shear force:

$$M_{\max} = \frac{q_{\Sigma} l^2}{8} = \frac{(0.91 + 27) \cdot 6.0^2}{8} = 125.593 \text{ kNm;}$$

$$Q_{\max} = \frac{q_{\Sigma} l}{2} = \frac{(0.91 + 27) \cdot 6.0}{2} = 83,73 \text{ kN.}$$

2. Necessary beam section modulus assuming that the deformations of steel are elastic:

$$W = \frac{M_{\max}}{R_y} = \frac{125.593 \cdot 100}{23} = 546.057 \text{ cm}^3.$$

3. Maximum deflection occurring in the middle of the beam span:

$$f_{\max} = \frac{5}{384} \cdot \frac{q_{н} l^4}{EI_x} = \frac{5}{384} \cdot \frac{23,37 \cdot 6^4}{2,06 \cdot 10^5 \cdot 10^3 \cdot 9840 \cdot 10^{-8}} = 19,46 \text{ mm.}$$

4. Check of the maximum shear stresses:

$$\tau_{\max} = \frac{Q_{\max} S_x}{I_x t_w} = \frac{83,73 \cdot 339}{9840 \cdot 0,7} = 4,12577 \text{ kN/cm}^2 < R_s \gamma_c = 0,58 \cdot 23 = 13,34 \text{ kN/cm}^2.$$

Comparison of solutions:

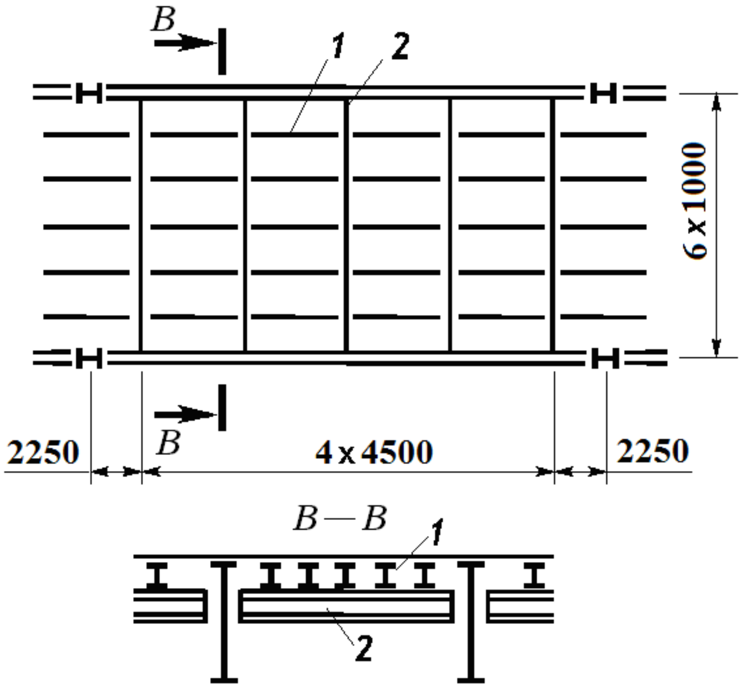
Factor	Strength under action of lateral force	Strength under action of bending moment	Stability of in-plane bending under moment	Maximum deflection
Manual calculation	4,126/13,34 = 0,309	546,06/596,36 = 0,916	–	19,46/24 = 0,81
SCAD	0,309	0,916	0,916	23,23/1,1945/24 = 0,81
Deviation, %	0,0	0,0	0,0	0,0

Comments:

1. The check of the general stability of the beam was not performed in the manual calculation, because the compressed beam chord is restrained against lateral displacements out of the bending plane by a welded floor plate.

2. The check of the beam strength taking into account the development of the limited plastic deformations was not performed, because according to the codes this calculation is possible only when the beam web has stiffeners. In the initial data of the example the stringer was specified without any intermediate stiffeners.

Strength and Stiffness Analysis of Stringers for a Complex Stub Girder System



1 – stringer
2 – secondary beam

Objective: Check the mode for the beam analysis in the “Steel” postprocessor of SCAD.

Task: Select a rolled I-beam for the stringers with a span of 4,5 m in a complex stub girder system. The top chord of the stringers is continuously restrained by the floor plate.

Source: Steel Structures: Student Handbook / [Kudishin U.I., Belenya E.I., Ignatieva V.S and others] - 13-th ed. rev. - M.: Publishing Center "Academy", 2011. p. 183.

Compliance with the codes: SNiP II-23-81*, SP 16.13330.2011, DBN B.2.6-163:2010.

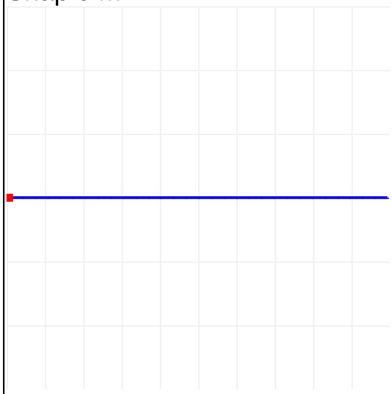
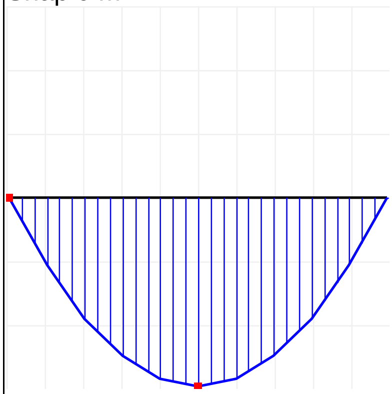
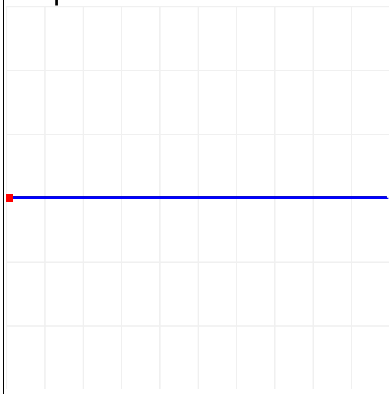
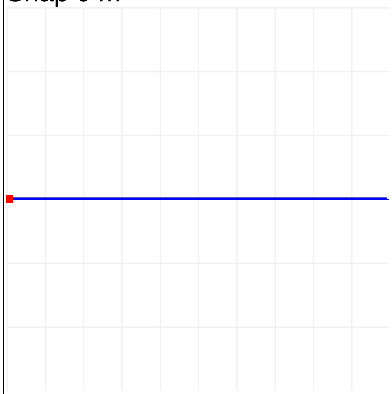
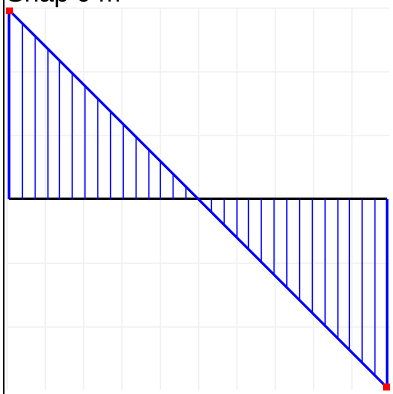
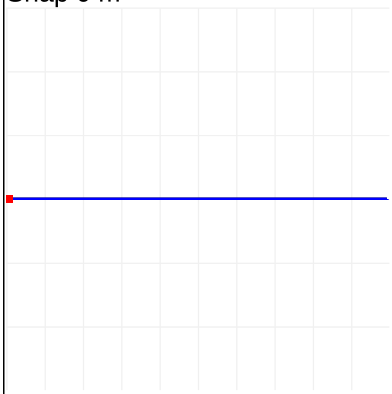
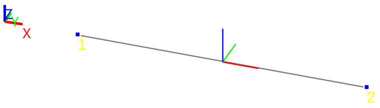
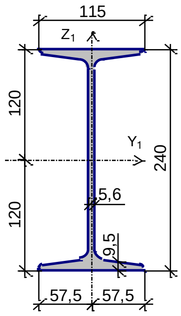
Initial data file:
3.2 Beam_Example_3.2.spr;
report – 3.2 Beam_Example_3.2.doc

Initial data:

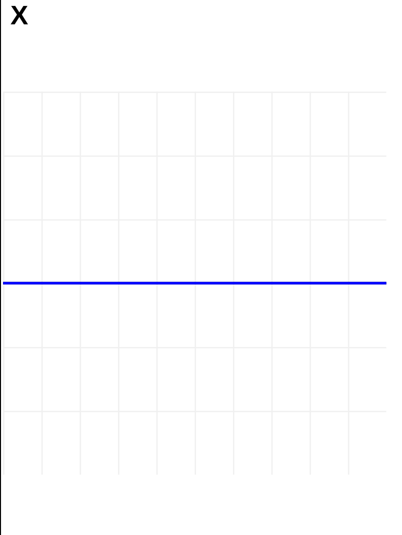
$a = 1{,}0$ m	Spacing of stringers;
$q_{ch} = (0{,}77 + 20)$ kN/m^2 ×1 m = 20,77 kN/m	Total characteristic load;
$q_1 = 1{,}05 \times 0{,}77$ kN/m^2 ×1 m = 0,8085 kN/m	Design permanent load;
$q_2 = 1{,}2 \times 20$ kN/m^2 ×1 m = 24 kN/m	Design temporary load;
$R_y = 23$ kN/cm^2,	Steel grade C235;
$l = 4{,}5$ m	Beam span;
$[f] = 1/250 \times 4{,}5$ m = 18 mm	Limit deflection;
$\gamma_c = 1$	Service factor;
$W_x = 288{,}33$ cm^3	Selected I-beam No.24 GOST 8239-89.
$I_x = 3460$ cm^4	

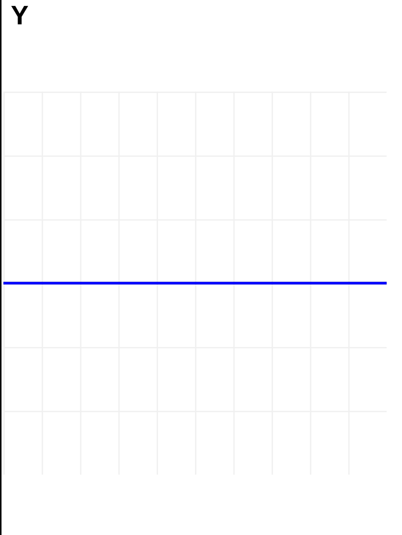
SCAD Results. STEEL Postprocessor:

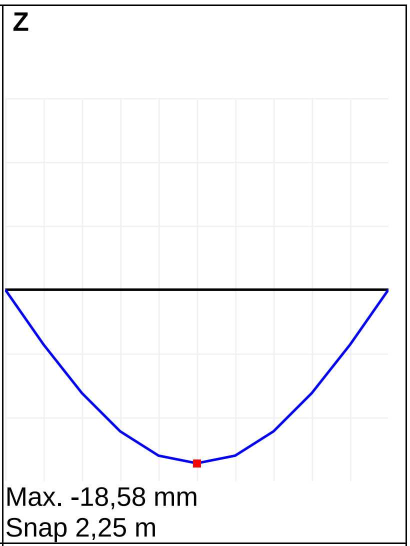
[Element No. 1] Forces

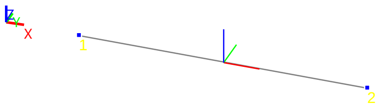
N Max. 0 T Snap 0 m  Max. 0 T Snap 0 m	**M_y** Max. 0 T*m Snap 0 m  Max. 6,4 T*m Snap 2,25 m	**M_z** Max. 0 T*m Snap 0 m  Max. 0 T*m Snap 0 m
M_k Max. 0 T*m Snap 0 m  Max. 0 T*m Snap 0 m	**Q_z** Max. 5,69 T Snap 0 m  Max. -5,69 T Snap 4,5 m	**Q_y** Max. 0 T Snap 0 m  Max. 0 T Snap 0 m
 	Length of the bar 4,5 m Length of the flexible part 4,5 m Loading L1 - "1"	

[Element No. 1] Deflections

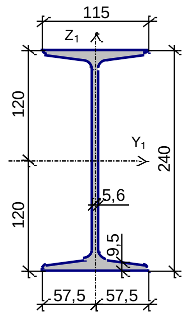








Length of the bar 4,5 m
Length of the flexible part 4,5 m
Loading L1 - "1"

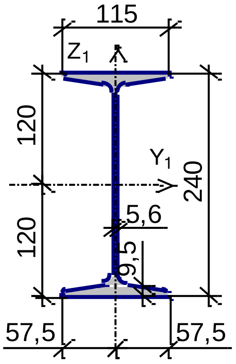


Analysis complies with SNiP II-23-81*
Structural member *beam*

Steel: C235

Importance factor 1
Service factor 1
Limit slenderness for members in compression: 180
Limit slenderness for members in tension: 300
Member length 4,5 m
Effective length factor XoZ -- 1
Effective length factor XoY -- 1
Length between out-of-plane restraints 0,01 m

Section



Profile: I-beam with sloped inner flange surfaces GOST 8239-89 24

Results	Check	Utilization factor
Sec.5.12	Strength under action of bending moment My	0,95
Sec.5.12,5.18	Strength under action of lateral force Qz	0,35
Sec.5.24,5.25	Strength under combined action of longitudinal force and bending moments, no plasticity	0,95
Sec.5.15	Stability of in-plane bending	0,95
Sec.6.15,6.16	Limit slenderness in XoY plane	0,63
Sec.6.15,6.16	Limit slenderness in XoZ plane	0,15

Utilization factor 0,95 - Strength under action of bending moment My

Manual calculation:

1. Design bending moment acting in the beam span:

$$M_{\max} = \frac{q_{\Sigma} l^2}{8} = \frac{(0,8085 + 24) \cdot 4,5^2}{8} = 62,7965 \text{ kNm.}$$

2. Necessary beam section modulus assuming that the deformations of steel are elastic:

$$W = \frac{M_{\max}}{R_y} = \frac{62,7965 \cdot 100}{23} = 273,028 \text{ cm}^3.$$

3. Maximum deflection occurring in the middle of the beam span:

$$f_{\max} = \frac{5}{384} \cdot \frac{q_{н} l^4}{EI_x} = \frac{5}{384} \cdot \frac{20,77 \cdot 4,5^4}{2,06 \cdot 10^5 \cdot 10^3 \cdot 3460 \cdot 10^{-8}} = 15,56 \text{ mm.}$$

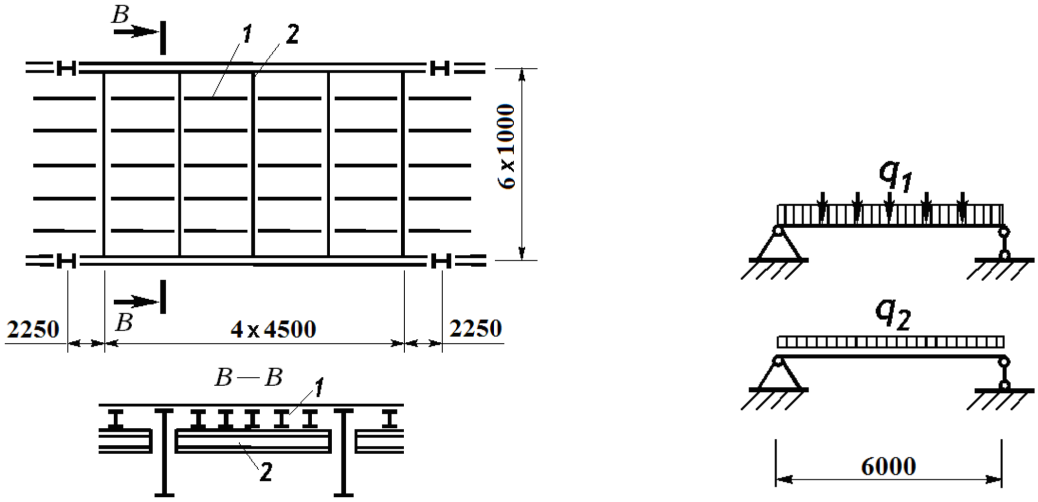
Comparison of solutions:

Factor	Strength under action of lateral force	Strength under action of bending moment	Stability of in-plane bending under moment	Maximum deflection
Manual calculation	not defined	273,028/288,33 = 0,947	not defined	15,56/18 = 0,864
SCAD	0,352	0,947	0,947	18,58/1,1944/18 = 0,864
Deviation, %	0,0	0,0	0,0	0,0

Comments:

1. The check of tangential stresses was not performed in the manual calculation due to the absence of weakenings and a relatively large thickness of the beam webs.
2. The check of the general stability of the beam was not performed in the manual calculation, because the compressed beam chord is restrained against lateral displacements out of the bending plane by a welded floor plate.
3. The check of the beam strength taking into account the development of the limited plastic deformations was not performed, because according to the codes this calculation is possible only when the beam web has stiffeners. In the initial data of the example the stringer was specified without any intermediate stiffeners.

Strength and Stiffness Analysis of Secondary Beams for a Complex Stub Girder System



1 – stringer
2 – secondary beam

Objective: Check the mode for the beam analysis in the “Steel” postprocessor of SCAD.

Task: Select a rolled I-beam for the secondary beams with a span of 6 m in a complex stub girder system. The top chord of the secondary beams is restrained by the stringers arranged with a spacing of 1 m.

Source: Steel Structures: Student Handbook / [Kudishin U.I., Belenya E.I., Ignatieva V.S and others] - 13-th ed. rev. - M.: Publishing Center "Academy", 2011. p. 183.

Compliance with the codes: SNiP II-23-81*, SP 16.13330.2011, DBN B.2.6-163:2010.

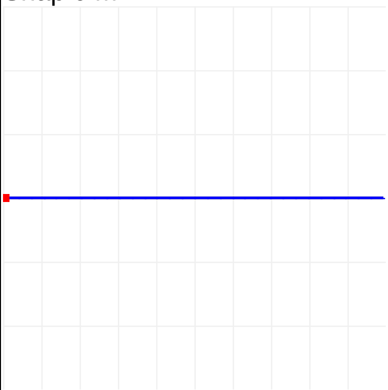
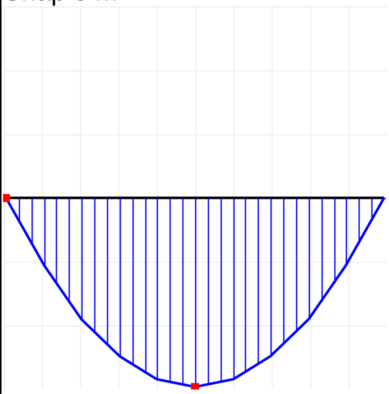
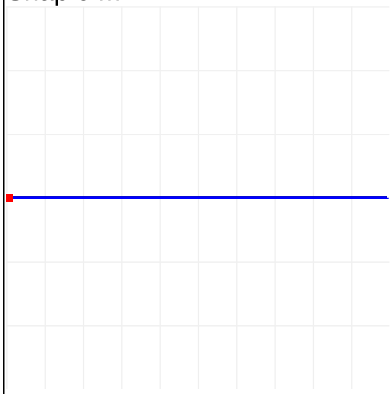
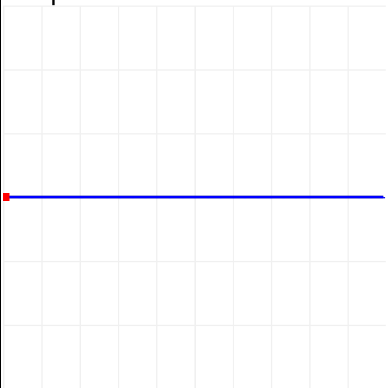
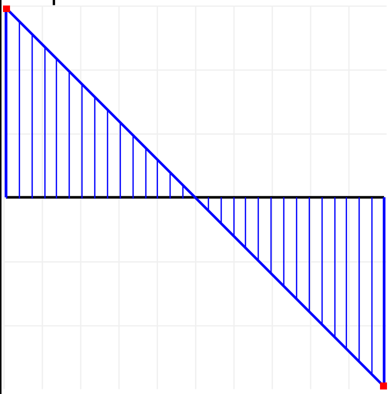
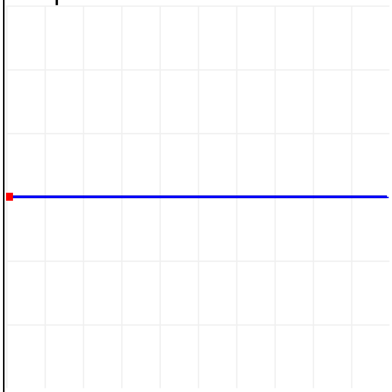
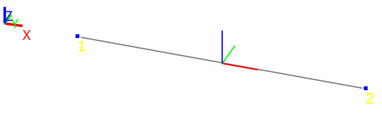
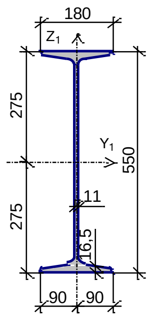
Initial data file:
3.3 Beam_Example_3.3.spr;
report – 3.3 Beam_Example_3.3.doc

Initial data:

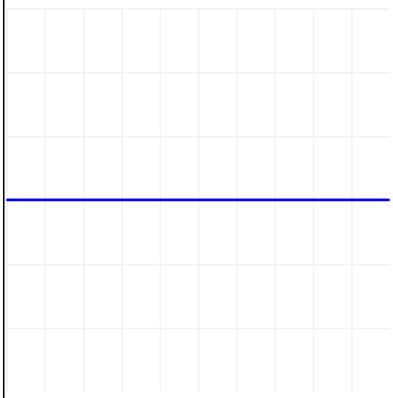
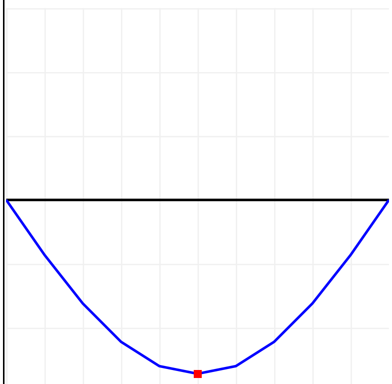
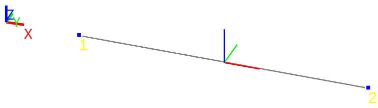
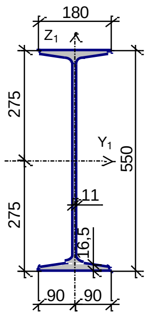
a = 4,5 m	Spacing of secondary beams;
q_{ch} = (0,77 + 27,3/102 + 20) kN/m^2 × 4,5 m = 94,67 kN/m	Total characteristic load;
q_1 = 1,05×(0,77 + 27,3/102) kN/m^2 × 4,5 m = 4,9 kN/m	Design permanent load;
q_2 = 1,2×20 kN/m^2 × 4,5 м = 108 kN/m	Design temporary load;
R_y = 23 kN/cm^2,	Steel grade C235;
l = 6,0 m	Beam span;
$[f]$ = 1/250×6,0 m = 24 mm	Limit deflection;
γ_c = 1	Service factor;
W_y = 2034,98 cm^3	Selected I-beam No.55 GOST 8239-89.
I_y = 55962 cm^4	

SCAD Results. STEEL Postprocessor:

[Element No. 1] Forces

N Max. 0 T Snap 0 m  Max. 0 T Snap 0 m	**M_y** Max. 0 T*m Snap 0 m  Max. 51,79 T*m Snap 3 m	**M_z** Max. 0 T*m Snap 0 m  Max. 0 T*m Snap 0 m
M_k Max. 0 T*m Snap 0 m  Max. 0 T*m Snap 0 m	**Q_z** Max. 34,53 T Snap 0 m  Max. -34,53 T Snap 6 m	**Q_y** Max. 0 T Snap 0 m  Max. 0 T Snap 0 m
	Length of the bar 6 m Length of the flexible part 6 m Loading L1 - "1"	

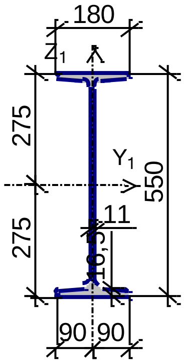
[Element No. 1] Deflections

X	Y	Z
		 Max. -16,53 mm Snap 3 m
	Length of the bar 6 m Length of the flexible part 6 m Loading L1 - "1"	

Analysis complies with SNiP II-23-81*
Structural member beam

Steel: C235
Member length 6 m
Limit slenderness for members in compression: 180
Limit slenderness for members in tension: 300
Service factor 1
Importance factor 1
Effective length factor XoZ -- 1
Effective length factor XoY -- 1
Length between out-of-plane restraints 0,01 m

Section



Profile: I-beam with sloped inner flange surfaces GOST 8239-89 55

Results	Check	Utilization factor
Sec.5.12	Strength under action of bending moment My	**1,09**
Sec.5.12,5.18	Strength under action of lateral force Qz	0,49
Sec.5.24,5.25	Strength under combined action of longitudinal force and bending moments, no plasticity	**1,09**
Sec.5.15	Stability of in-plane bending	**1,09**
Sec.6.15,6.16	Limit slenderness in XoY plane	0,59
Sec.6.15,6.16	Limit slenderness in XoZ plane	0,09

Utilization factor 1,09 - Strength under action of bending moment My

Manual calculation:

1. Design bending moment acting in the beam span:

$$M_{\max} = \frac{q_{\Sigma} l^2}{8} = \frac{(4,9+108)\cdot 6,0^2}{8} = 508,05 \text{ kNm.}$$

2. Necessary beam section modulus assuming that the deformations of steel are elastic:

$$W_{nes} = \frac{M_{\max}}{R_y} = \frac{508,05\cdot 100}{23} = 2208,913 \text{ cm}^3.$$

3. Maximum deflection occurring in the middle of the beam span:

$$f_{\max} = \frac{5}{384}\cdot\frac{q_н l^4}{EI_y} = \frac{5}{384}\cdot\frac{94,67\cdot 6,0^4}{2,06\cdot 10^5\cdot 10^3\cdot 55962\cdot 10^{-8}} = 13,858 \text{ mm.}$$

4. Conditional limit slenderness of the compressed beam chord:

$$\bar{\lambda}_{ub} = 0,35 + 0,0032\frac{b_f}{t_f} + \left(0,76 - 0,02\frac{b_f}{t_f}\right)\frac{b_f}{h_f} = 0,35 + 0,0032\frac{180}{16,5} +$$

$$\left(0,76 - 0,02\frac{180}{16,5}\right)\frac{180}{533,5} = 0,5677.$$

5. Conditional actual slenderness of the compressed beam chord:

$$\bar{\lambda}_b = \frac{l_{ef}}{b_f}\sqrt{\frac{R_y}{E}} = \frac{1000}{180}\sqrt{\frac{230}{2,06\cdot 10^5}} = 0,1856 < \bar{\lambda}_{ub} = 0,5677$$ – the stability check is not required.

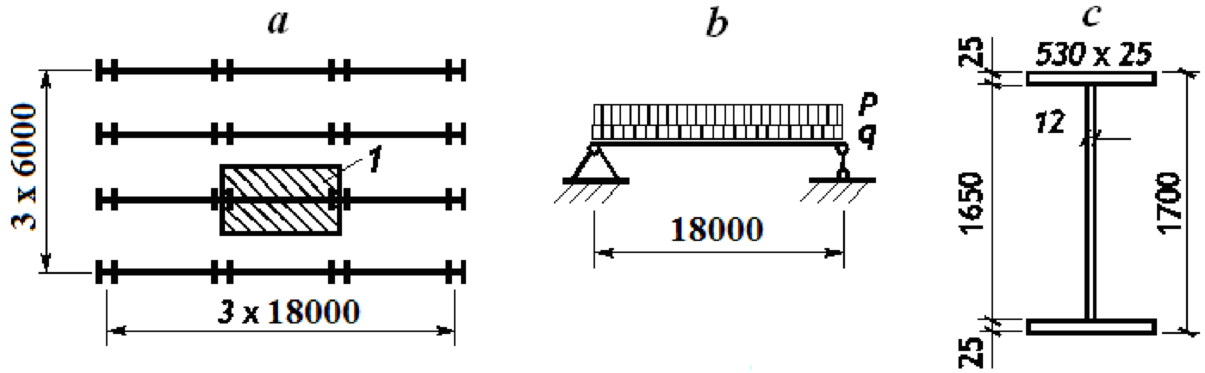
Comparison of solutions:

Factor	Strength under action of lateral force	Strength under action of bending moment	Stability of in-plane bending under moment	Maximum deflection
Manual calculation	not defined	2208,913/2034,98 =1,085	check is not required	13,858/24 = 0,577
SCAD	0,488	1,085	1,085	16,53/1,1925/24 = 0,577
Deviation, %	0,0	0,0	0,0	0,0

Comments:

1. The check of tangential stresses was not performed in the manual calculation due to the absence of weakenings and a relatively large thickness of the beam webs.
2. The check for the stability of in-plane bending of the beam was performed in the computer-aided calculation according to the codes at $\varphi_b = 1,0$.
3. The check of the beam strength taking into account the development of the limited plastic deformations was not performed, because according to the codes this calculation is possible only when the beam web has stiffeners. In the initial data of the example a rolled beam without intermediate stiffeners was selected for the secondary beam.

Strength and Stiffness Analysis of Main Beams of Complex Stub Girder Systems



a – floor plan; b – design model of the main beam; c – beam section;
1 – load area

Objective: Check the mode for the beam analysis in the "Steel" postprocessor of SCAD

Task: Select a welded I-beam for the main beams with a span of 18 m in a normal stub girder system. The top chord of the main beams is restrained by the stringers arranged with a spacing of 1 m.

Source: Steel Structures: Student Handbook / [Kudishin U.I., Belenya E.I., Ignatieva V.S and others] - 13-th ed. rev. - M.: Publishing Center "Academy", 2011. p. 192.

Compliance with the codes: SNiP II-23-81*, SP 16.13330.2011, DBN B.2.6-163:2010.

Initial data file:
3.4 Beam_Example_3.4.spr;
report – 3.4 Beam_Example_3.4.doc

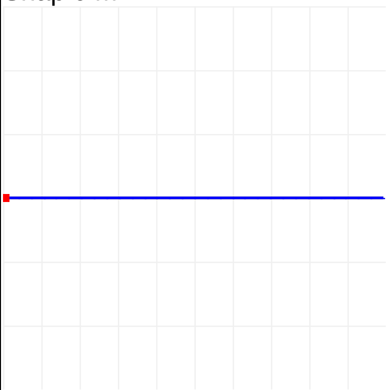
Initial data:

a = 6 m	Spacing of main beams
g_1 = 1,16 kN/m^2	Weight of the floor plate and stringers
p = 20 kN/m^2	Temporary (live) load
q_{ch} = 127,099 kN/m	Total characteristic load on the beam
q_1 = 1,05*1,16 kN/m^2 * 6 m*1,02 = 7,454 kN/m	Design permanent load (coefficient 1,02 allows for the self-weight of the main beam)
q_2 = 1,2*20 kN/m^2 * 6 m = 144,0 kN/m	Design live load
l = 18 m	Main beam span
R_y = 23 kN/cm^2	Steel grade C255 with thickness t>20 mm
R_s = 0,58*23=13,34 kN/cm^2	
$[f] = l/400$ = 45 mm	Limit deflection
$b_p \times t_p = 530 \times 20$ mm	Section of the bearing stiffener
$k_p = 6$ mm	Fillet weld leg in a welded connection between a bearing stiffener and a beam
γ_c = 1	Service factor
$W_y = 27153,85$ cm^3	Geometric properties for a welded I-section with flanges 530×25 mm
$I_y = 2308077,083$ cm^4	and a web 1650×12 mm
$S_y = 15180,625$ cm^3	

SCAD Results. STEEL Postprocessor:

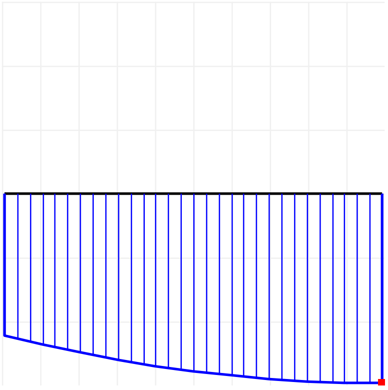
[Element No 3] Forces

N
Max. 0 T
Snap 0 m



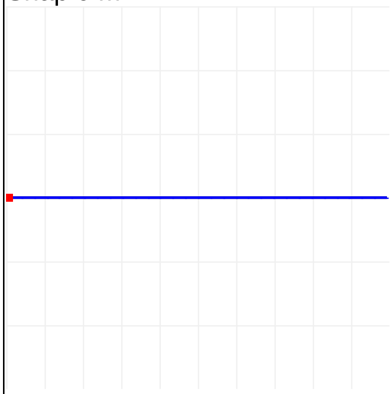
Max. 0 T
Snap 0 m

M_y



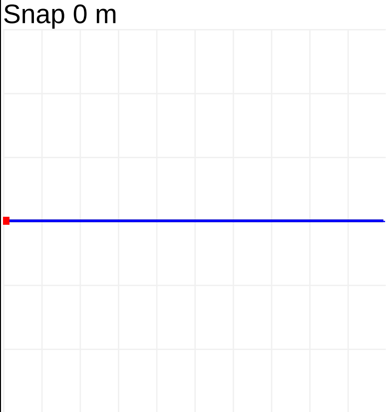
Max. 625,27 T*m
Snap 4,5 m

M_z
Max. 0 T*m
Snap 0 m



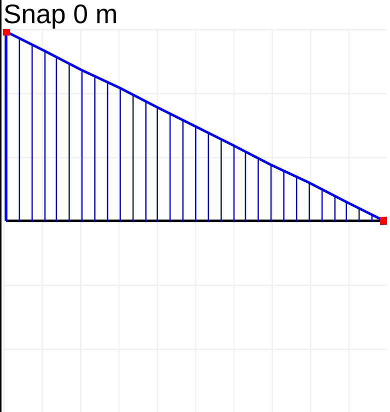
Max. 0 T*m
Snap 0 m

M_k
Max. 0 T*m
Snap 0 m



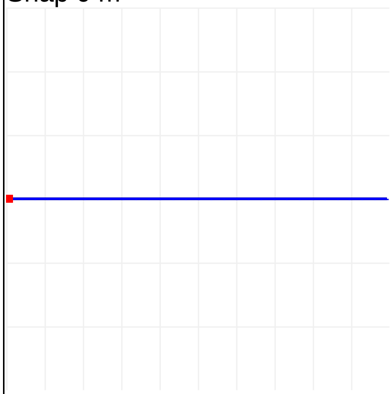
Max. 0 T*m
Snap 0 m

Q_z
Max. 69,47 T
Snap 0 m

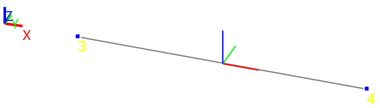


Max. 0 T
Snap 4,5 m

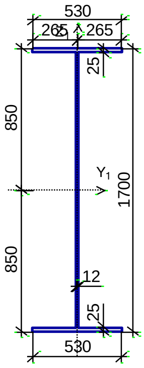
Q_y
Max. 0 T
Snap 0 m



Max. 0 T
Snap 0 m

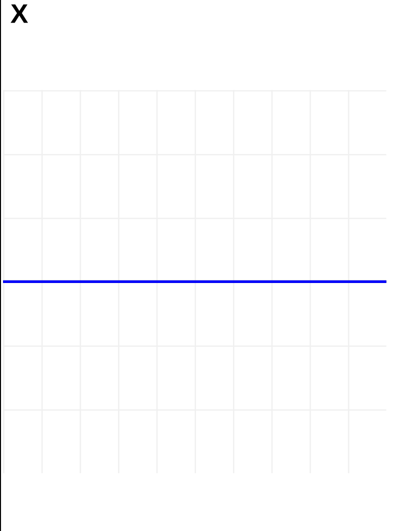


Length of the bar 4,5 m
Length of the flexible part 4,5 m
Loading L1 - "1"

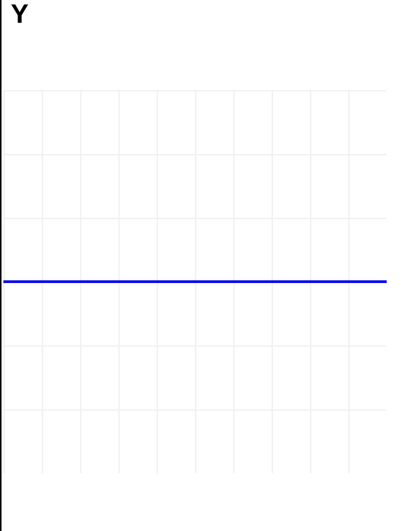


[Element No. 3] Deflections

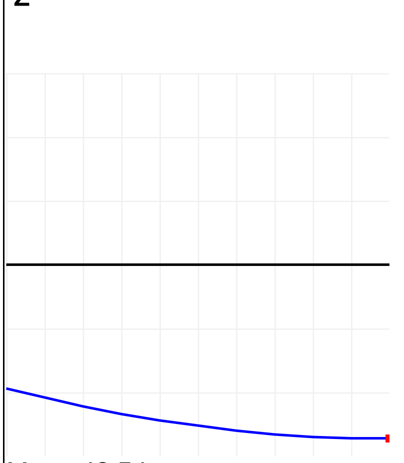
X



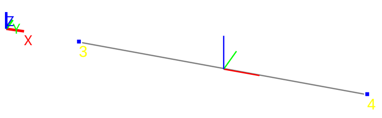
Y



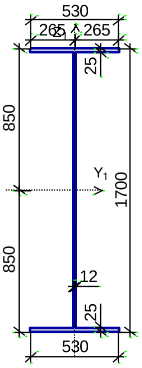
Z



Max. -43,54 mm
Snap 4,5 m



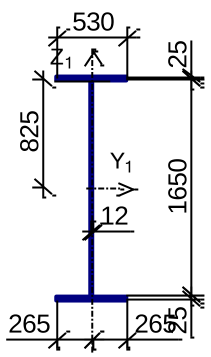
Length of the bar 4,5 m
Length of the flexible part 4,5 m
Loading L1 - "1"



Analysis complies with SNiP II-23-81*
Structural member main beam

Steel: C255
Member length 18 m
Limit slenderness for members in compression: 180
Limit slenderness for members in tension: 300
Service factor 1
Importance factor 1
Effective length factor XoZ -- 1
Effective length factor XoY -- 1
Length between out-of-plane restraints 1 m

Section



Results	Check	Utilization factor
Sec.5.12	Strength under action of bending moment My	0,98
Sec.5.12,5.18	Strength under action of lateral force Qz	0,56
Sec.5.24,5.25	Strength under combined action of longitudinal force and bending moments, no plasticity	0,98
Sec.5.15	Stability of in-plane bending	0,98
Sec.6.15,6.16	Limit slenderness in XoY plane	0,52
Sec.6.15,6.16	Limit slenderness in XoZ plane	0,08

Utilization factor 0,98 - Strength under action of bending moment My

Manual calculation (SNiP II-23-81)*

1. Maximum bending moment and shear force acting in the design sections of the beam:

$$M_{\max} = \frac{q_{\Sigma} l^2}{8} = \frac{(7,454+144)\cdot 18,0^2}{8} = 6133,887 \text{ kNm.}$$

$$Q_{\max} = \frac{q_{\Sigma} l}{2} = \frac{(7,454+144)\cdot 18,0}{2} = 1363,086 \text{ kN.}$$

2. Necessary beam section modulus:

$$W_{nes} = \frac{M_{\max}}{R_y \gamma_c} = \frac{6133,887\cdot 100}{23} = 26669,074 \text{ cm}^3.$$

3. Maximum tangential stresses in the support section of the beam:

$$\tau_{\max} = \frac{Q_{\max} S_y}{I_y t_w} = \frac{1363,086\cdot 15180,625}{2308077,083\cdot 1,2} = 7,471 \text{ kN/cm}^2.$$

4. Maximum deflection occurring in the middle of the beam span:

$$f_{\max} = \frac{5}{384}\cdot\frac{q_{и} l^4}{EI_y} = \frac{5}{384}\cdot\frac{127,099\cdot 18,0^4}{2,06\cdot 10^5\cdot 10^3\cdot 2308077,083\cdot 10^{-8}} = 36,539 \text{ mm.}$$

5. Conditional limit slenderness of the compressed beam chord:

$$\bar{\lambda}_{ub} = 0,35 + 0,0032\frac{b_f}{t_f} + \left(0,76 - 0,02\frac{b_f}{t_f}\right)\frac{b_f}{h_f} = 0,35 + 0,0032\frac{530}{25} + \left(0,76 - 0,02\frac{530}{25}\right)\frac{530}{1675} = 0,524.$$

6. Conditional actual slenderness of the compressed beam chord:

$$\bar{\lambda}_b = \frac{l_{ef}}{b_f}\sqrt{\frac{R_y}{E}} = \frac{1000}{530}\sqrt{\frac{230}{2,06\cdot 10^5}} = 0,063 < \bar{\lambda}_{ub} = 0,524$$ – the stability check is not required.

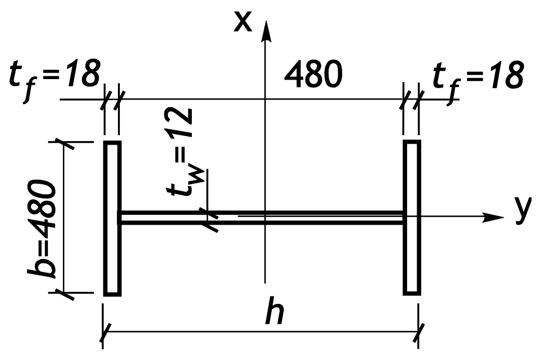
Comparison of solutions:

Factor	Manual calculation	SCAD	Deviation, %
Strength under action of lateral force	7,471/13,34 = 0,56	0,56	0,0
Strength under action of bending moment	26669,074/ 27153,85 =0,982	0,982	0,0
Stability of in-plane bending under moment	–	0,982	0,0
Maximum deflection	36,539/45 = 0,812	43,54/1,1916/45= 0,812	0,0

Comments:

The check for the stability of in-plane bending of the beam was performed in the computer-aided calculation according to the codes at $\varphi_b = 1{,}0$.

Analysis of an Axially Compressed Welded I-beam Column



Objective: Check the mode for calculating columns of solid cross-section in the “Steel” postprocessor of SCAD.

Task: Check the design section of a welded I-beam for the axially compressed column with a height of 6,5 m.

Source: Steel Structures: Student Handbook / [Kudishin U.I., Belenya E.I., Ignatieva V.S and others] - 13-th ed. rev. - M.: Publishing Center "Academy", 2011. p. 256.

Compliance with the codes: SNiP II-23-81*, SP 16.13330.2011, DBN B.2.6-163:2010.

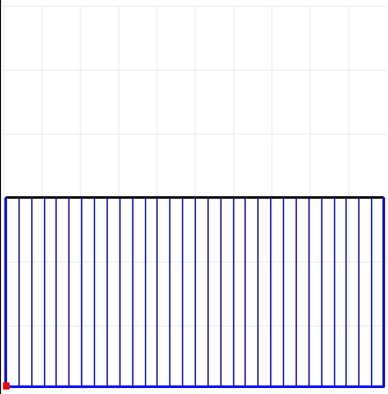
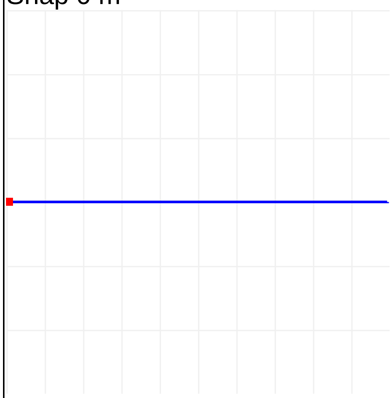
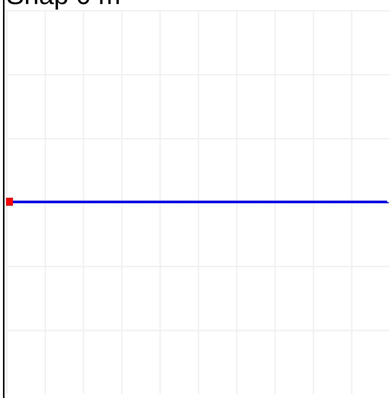
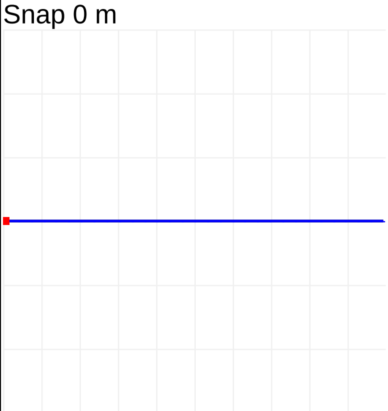
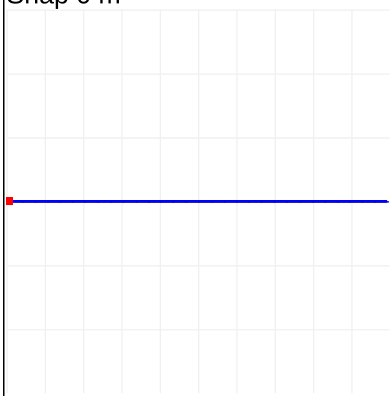
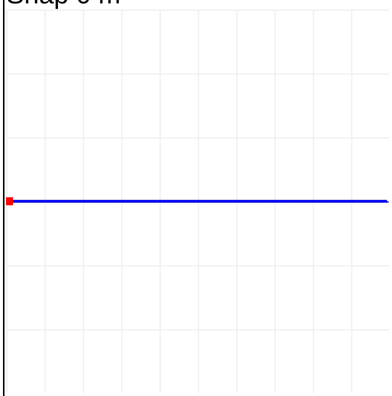
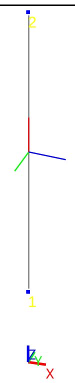
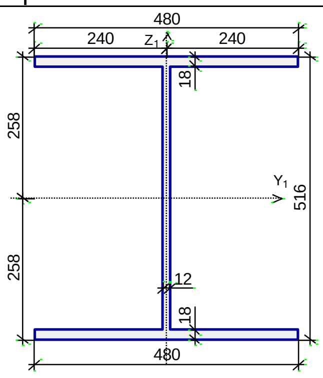
Initial data file:
5.1 Column_Example_5.1.spr;
report – 5.1 Column_Example_5.1.doc

Initial data:

l = 6,5 m	Column height
μ = 0,7	The lower restraint is rigid and the upper one is pinned
N = 5000 kN	Design compressive force
γ_c = 1	Service factor
R_y = 24 kN/cm^2	Steel grade C245
$A = 230,4\,\text{cm}^2$	Geometric properties of
$I_x = 118243,584\,\text{cm}^4,\ I_y = 33184,512\,\text{cm}^4$	the selected section
$i_x = 22,654\,\text{cm},\ i_y = 12,001\,\text{cm}$	

SCAD Results. STEEL Postprocessor:

[Element No 1] Forces

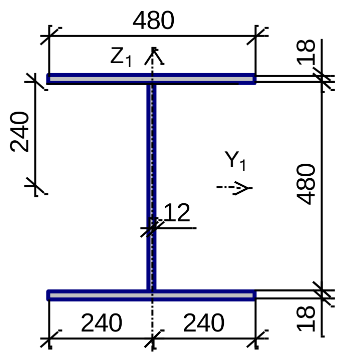
N  Max. -509,68 T Snap 0 m	**M_y** Max. 0 T*m Snap 0 m  Max. 0 T*m Snap 0 m	**M_z** Max. 0 T*m Snap 0 m  Max. 0 T*m Snap 0 m
M_k Max. 0 T*m Snap 0 m  Max. 0 T*m Snap 0 m	**Q_z** Max. 0 T Snap 0 m  Max. 0 T Snap 0 m	**Q_y** Max. 0 T Snap 0 m  Max. 0 T Snap 0 m
	Length of the bar 6,5 m Length of the flexible part 6,5 m Loading L1	

Analysis complies with SNiP II-23-81*
Structural member column

Steel: C245
Member length 6,5 m
Limit slenderness for members in compression: 180 - 60□
Limit slenderness for members in tension: 300
Service factor 1
Importance factor 1

Effective length factor XoZ -- 0,7
Effective length factor XoY -- 0,7
Length between out-of-plane restraints 0 m

Section



Results	Check	Utilization factor
Sec.5.24,5.25	Strength under combined action of longitudinal force and bending moments, no plasticity	0,9
Sec.5.3	Stability under compression in XoY (XoU) plane	1
Sec.5.3	Stability under compression in XoZ (XoV) plane	0,94
Sec.5.1	Strength under axial compression/tension	0,9
Sec.6.15,6.16	Limit slenderness in XoY plane	0,316
Sec.6.15,6.16	Limit slenderness in XoZ plane	0,162

Utilization factor 1 - Stability under compression in XoY (XoU) plane

Manual calculation (SNiP II-23-81*):

1. Strength check of the selected column section:

$$\frac{N}{AR_y\gamma_c} = \frac{5000}{230,4\cdot 24\cdot 1} = 0,904\,.$$

2. Slenderness of the column:

$$\lambda_x = \frac{l_{ef,x}}{i_x} = \frac{0,7\cdot 6,5\cdot 100}{22,654} = 20,08475\,;$$

$$\lambda_y = \frac{l_{ef,y}}{i_y} = \frac{0,7\cdot 6,5\cdot 100}{12,001} = 37,9135\,.$$

3. Conditional slenderness of the column:

$$\bar{\lambda}_x = \frac{l_{ef,x}}{i_x}\sqrt{\frac{R_y}{E}} = \frac{0,7\cdot 6,5\cdot 100}{22,654}\sqrt{\frac{240}{2,06\cdot 10^5}} = 0,68555\,;$$

$$\bar{\lambda}_y = \frac{l_{ef,y}}{i_y}\sqrt{\frac{R_y}{E}} = \frac{0,7\cdot 6,5\cdot 100}{12,001}\sqrt{\frac{240}{2,06\cdot 10^5}} = 1,2941\,.$$

4. Buckling coefficients:

$$\varphi_y = 1-\left(0,073-5,53\frac{R_y}{E}\right)\bar{\lambda}_y\sqrt{\bar{\lambda}_y} = 1-\left(0,073-\frac{5,53\cdot 240}{2,06\cdot 10^5}\right)\cdot 0,68555\sqrt{0,68555} = 0,9622\,;$$

$$\varphi_y = 1 - \left(0,073 - 5,53\frac{R_y}{E}\right)\bar{\lambda}_y\sqrt{\bar{\lambda}_y} = 1 - \left(0,073 - \frac{5,53\cdot 240}{2,06\cdot 10^5}\right)\cdot 1,2941\sqrt{1,2941} = 0,902\,.$$

5. Strength of the column from the condition of providing the general stability under axial compression:

$$N_{b,x} = \varphi_x A R_y \gamma_c = 0,9622\cdot 230,4\cdot 24\cdot 1 = 5320,58 \text{ kN};$$

$$N_{b,y} = \varphi_y A R_y \gamma_c = 0,902\cdot 230,4\cdot 24\cdot 1 = 4987,7 \text{ kN}.$$

6. Limit slenderness of the column:

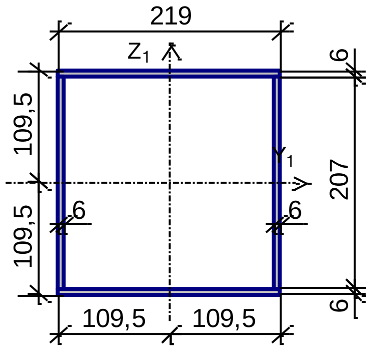
$$[\lambda]_x = 180 - 60\alpha_x = 180 - 60\cdot\frac{N}{\varphi_x A R_y \gamma_c} = 180 - 60\cdot\frac{5000}{5320,58} = 123,615\,;$$

$$[\lambda]_y = 180 - 60\alpha_y = 180 - 60\cdot\frac{N}{\varphi_y A R_y \gamma_c} = 180 - 60\cdot 1 = 120\,.$$

Comparison of solutions:

Factor	Source	Manual calculation	SCAD	Deviation, %
Strength under combined action of longitudinal force and bending moments, no plasticity	–	0,904	0,904	0,0
Stability under compression in XoY (XoU) plane	23,69/24=0,987	5000/4987,7 = 1,002	1,002	0,0
Stability under compression in XoZ (XoV) plane	–	5000/5320,58 = 0,940	0,94	0,0
Strength under axial compression/tension	5000/230,4/24= 0,904	0,904	0,904	0,0
Limit slenderness in XoY plane	–	37,9135/120 = 0,316	0,316	0,0
Limit slenderness in XoZ plane	–	20,08475/123,615 = 0,162	0,162	0,0

Analysis of an Axially Compressed Electric Welded Circular Hollow Section Column



Objective: Check the mode for calculating columns of solid cross-section in the “Steel” postprocessor of SCAD

Task: Check the design section of an axially compressed electric welded circular hollow section column with a height of 7,7 m.

Source: Kuznetsov A.F., Kozmin N.B., Amelkovich S.V. Examples of the analysis of steel structures of civil and industrial buildings. Textbook for students of construction specialties. - Chelyabinsk, 2009. – p. 11, 12.

Compliance with the codes: SNiP II-23-81*, SP 16.13330.2011, DBN B.2.6-163:2010, DBN B.2.6-198:2014.

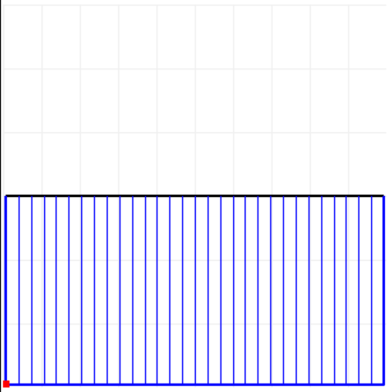
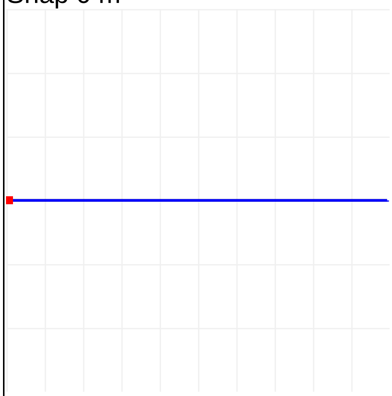
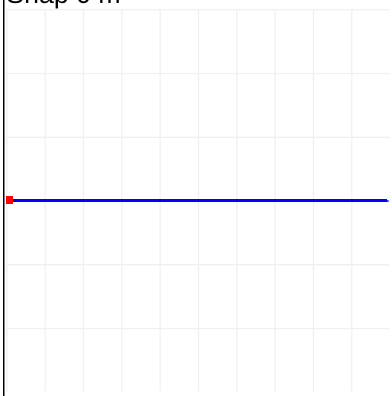
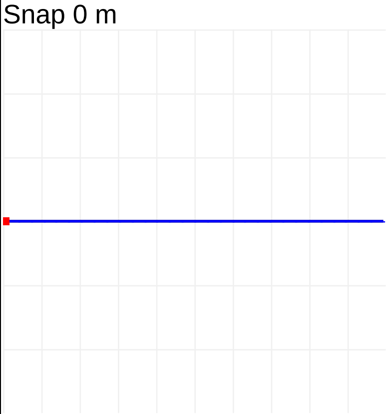
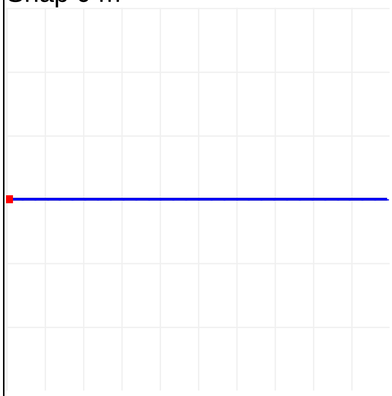
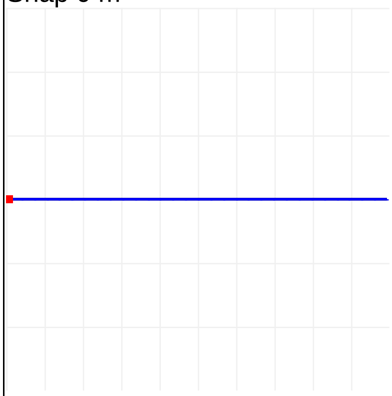
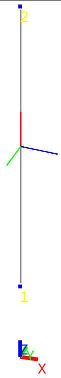
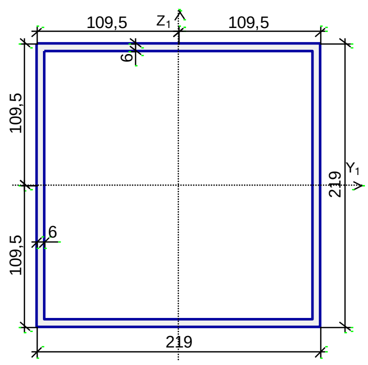
Initial data file:
5.3 Column_Example_5.3.spr;
report – 5.3 Column_Example_5.3.doc

Initial data:

l = 7,7 m	Column height
μ = 1,0	The lower and upper restraints are pinned
N = 472,5 kN	Design compressive force
γ_c = 1	Service factor
R_y = 23 kN/cm^2	Steel grade C235
$A = 51{,}12\ \text{cm}^2$	Geometric properties of
$I_y = I_z = 3868{,}506\ \text{cm}^4$	the selected section
$i_y = i_z = 8{,}699\ \text{cm}$	

SCAD Results. STEEL Postprocessor:

[Element No. 1] Forces

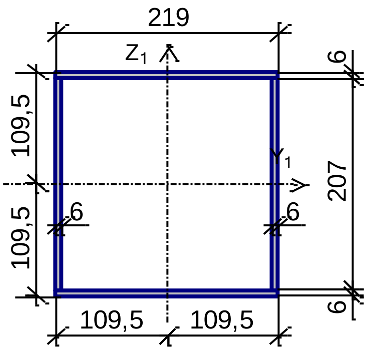
N  Max. -48,17 T Snap 0 m	**M_y** Max. 0 T*m Snap 0 m  Max. 0 T*m Snap 0 m	**M_z** Max. 0 T*m Snap 0 m  Max. 0 T*m Snap 0 m
M_k Max. 0 T*m Snap 0 m  Max. 0 T*m Snap 0 m	**Q_z** Max. 0 T Snap 0 m  Max. 0 T Snap 0 m	**Q_y** Max. 0 T Snap 0 m  Max. 0 T Snap 0 m
	Length of the bar 7,7 m Length of the flexible part 7,7 m Loading L1	

Analysis complies with SNiP II-23-81*
Structural member column1

Steel: C235
Member length 7,7 m
Limit slenderness for members in compression: 180 - 60□
Limit slenderness for members in tension: 300
Service factor 1
Importance factor 1
Effective length factor XoZ -- 1,0
Effective length factor XoY -- 1,0

Length between out-of-plane restraints 7,7 m

Section:



Results	Check	Utilization factor
Sec. 5.24,5.25	Strength under combined action of longitudinal force and bending moments, no plasticity	0,4
Sec. 5.3	Stability under compression in XoY (XoU) plane	0,63
Sec. 5.3	Stability under compression in XoZ (XoV) plane	0,63
Sec. 5.34	Stability under compression and bending in two planes	0,63
Sec. 5.1	Strength under axial compression/tension	0,4
Sec. 6.15,6.16	Limit slenderness in XoY plane	0,62
Sec. 6.15,6.16	Limit slenderness in XoZ plane	0,62

Utilization factor 0,63 - Stability under compression in XoY (XoU) plane

Manual calculation (SNiP II-23-81):*

1. Strength check of the selected column section:

$$\frac{N}{AR_y\gamma_c} = \frac{472,5}{51,12\cdot 23\cdot 1} = 0,402.$$

2. Slenderness of the column:

$$\lambda_y = \frac{l_{ef,y}}{i_y} = \frac{1,0\cdot 7,7\cdot 100}{8,699} = 88,516;$$

$$\lambda_z = \frac{l_{ef,z}}{i_z} = \frac{1,0\cdot 7,7\cdot 100}{8,699} = 88,516.$$

3. Conditional slenderness of the column:

$$\bar{\lambda}_y = \frac{l_{ef,y}}{i_y}\sqrt{\frac{R_y}{E}} = \frac{1,0\cdot 7,7\cdot 100}{8,699}\sqrt{\frac{230}{2,06\cdot 10^5}} = 2,9577;$$

$$\bar{\lambda}_z = \frac{l_{ef,z}}{i_z}\sqrt{\frac{R_y}{E}} = \frac{1,0\cdot 7,7\cdot 100}{8,699}\sqrt{\frac{230}{2,06\cdot 10^5}} = 2,9577.$$

4. Buckling coefficients at $2,5 < \bar{\lambda} \le 4,5$:

$$\varphi_y = \varphi_z = 1,47 - 13,0\frac{R_y}{E} - \left(0,371 - 27,3\frac{R_y}{E}\right)\bar{\lambda}_y + \left(0,0275 - 5,53\frac{R_y}{E}\right)\bar{\lambda}_y^2 =$$

$$= 1,47 - \frac{13,0\cdot 230}{2,06\cdot 10^5} - \left(0,371 - \frac{27,3\cdot 230}{2,06\cdot 10^5}\right)\cdot 2,9577 + \left(0,0275 - \frac{5,53\cdot 230}{2,06\cdot 10^5}\right)\cdot 2,9577^2 = 0,6349.$$

5. Strength of the column from the condition of providing the general stability under axial compression:

$$N_{b,y} = \varphi_y A R_y \gamma_c = 0,6349\cdot 23\cdot 51,12\cdot 1 = 746,476 \text{ kN};$$

$$N_{b,z} = \varphi_z A R_y \gamma_c = 0,6349 \cdot 23 \cdot 51,12 \cdot 1 = 746,476 \text{ kN.}$$

6. Limit slenderness of the column:

$$[\lambda]_y = 180 - 60\alpha_y = 180 - 60 \cdot \frac{N}{\varphi_y A R_y \gamma_c} = 180 - 60 \cdot \frac{472,5}{746,476} = 142,022;$$

$$[\lambda]_z = 180 - 60\alpha_z = 180 - 60 \cdot \frac{N}{\varphi_z A R_y \gamma_c} = 180 - 60 \cdot \frac{472,5}{746,476} = 142,022.$$

Comparison of solutions:

Factor	**Source**	**Manual calculation**	**SCAD**	**Deviation, %**
Strength under combined action of longitudinal force and bending moments, no plasticity	–	0,402	0,4	0,0
Stability under compression in XoY (XoU) plane	0,966	472,5/746,476 = 0,633	0,63	0,0
Stability under compression in XoZ (XoV) plane	0,966	472,5/746,476 = 0,633	0,63	0,0
Strength under axial compression/tension	0,511	0,402	0,4	0,0
Limit slenderness in XoY plane	–	88,516/142,022 = 0,62	0,62	0,0
Limit slenderness in XoZ plane	–	88,516/142,022 = 0,62	0,62	0,0

Analysis of a Top Truss Chord from Unequal Angles

Objective: Check the mode for calculating truss members in the “Steel” postprocessor of SCAD

Task: Check the top truss chord section from two unequal angles L160x100x9 mm. The truss panel length is 2,58 m. The truss is restrained out of the bending plane through the panel.

Source: Steel Structures: Student Handbook / [Kudishin U.I., Belenya E.I., Ignatieva V.S and others] - 13-th ed. rev. - M.: Publishing Center "Academy", 2011. p. 280.

Compliance with the codes: SNiP II-23-81*, SP 16.13330.2011, DBN B.2.6-163:2010, DBN B.2.6-198:2014.

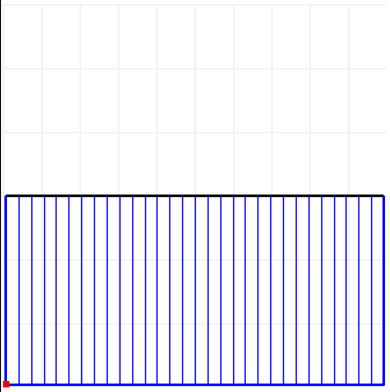
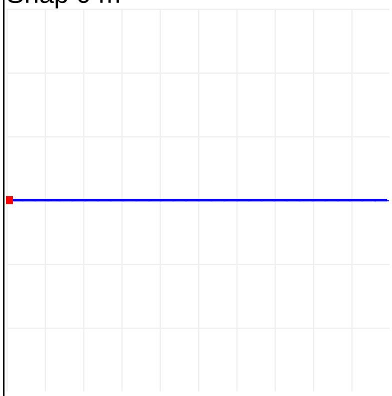
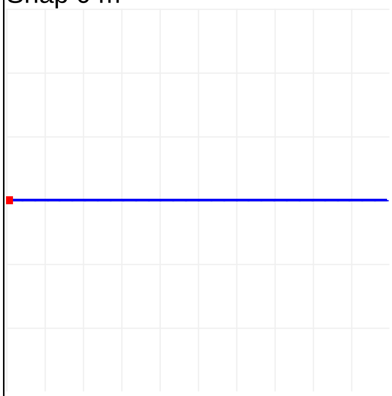
Initial data file:
7.1 Truss_Element_Example_7.1.spr;
report – 7.1 Truss_Element_Example_7.1.doc

Initial data:

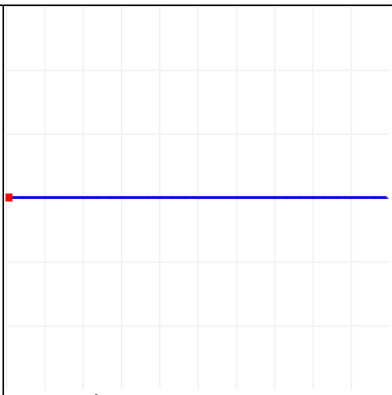
N = 535 kN	Design compressive force
R_y = 24 kN/cm^2	Steel grade C245
γ_c = 0,95	Service factor
g = 12 mm	Thickness of the gusset plate
l_y = 2,58, l_z = 5,16	Effective lengths of the bar
i_y = 2,851 cm, A = 45,74 cm^2	Geometric properties of
i_z = 7,745 cm	the top chord section from two angles 160x100x9

SCAD Results. STEEL Postprocessor:

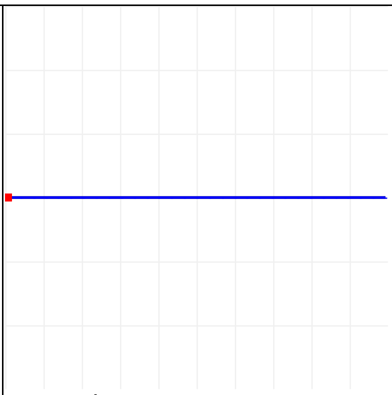
[Element No 1] Forces

N  Max. -535 kN Snap 0 m	**M_y** Max. 0 kN*m Snap 0 m  Max. 0 kN*m Snap 0 m	**M_z** Max. 0 kN*m Snap 0 m  Max. 0 kN*m Snap 0 m
M_k Max. 0 kN*m Snap 0 m	**Q_z** Max. 0 kN Snap 0 m	**Q_y** Max. 0 kN Snap 0 m

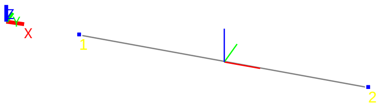
Max. 0 kN*m
Snap 0 m



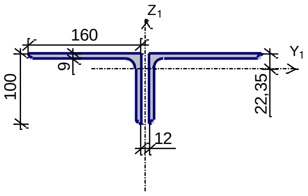
Max. 0 kN
Snap 0 m



Max. 0 kN
Snap 0 m



Length of the bar 2,58 m
Length of the flexible part 2,58 m
Loading L1 - "ff"

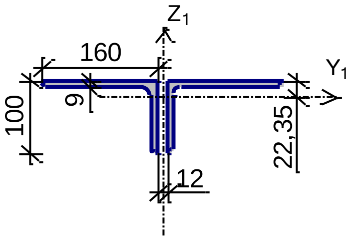


Analysis complies with SNiP II-23-81*
Structural member Truss chord

Steel: C245
Member length 2,58 m
Limit slenderness for members in compression: 180 - 60□
Limit slenderness for members in tension: 300
Service factor 0,95
Importance factor 1
Inelasticity is forbidden

Effective length factor in the X_1OZ_1 plane 1
Effective length factor in the X_1OY_1 plane 2
Length between the restraints out of the bending plane 2,58 m

Section:



Profile: Unequal angle GOST 8510-86* L160x100x9

Results	Check	Utilization factor
Sec.5.24,5.25	Strength under combined action of longitudinal force and bending moments, no plasticity	0,51
Sec.5.3	Stability under compression in XoY (XoU) plane	0,66
Sec.5.3	Stability under compression in XoZ (XoV) plane	0,84
Sec.5.1	Strength under axial compression/tension	0,51
Sec.6.15,6.16	Limit slenderness in XoY plane	0,48
Sec.6.15,6.16	Limit slenderness in XoZ plane	0,7

Utilization factor 0,84 - Stability under compression in XoZ (XoV) plane

Manual calculation (SNiP II-23-81):*

1. Strength check

$$\frac{N}{A}=\frac{535}{45,74}=11,69655 \text{ kN/cm}^2 < R_y\gamma_c = 24\cdot 0,95 = 22,8 \text{ kN/cm}^2.$$

2. Slenderness of the truss member:

$$\lambda_y=\frac{l_{ef,y}}{i_y}=\frac{2,58\cdot 100}{2,851}=90,49456;$$

$$\lambda_z=\frac{l_{ef,z}}{i_z}=\frac{5,16\cdot 100}{7,745}=66,6236.$$

3. Conditional slenderness of the truss member:

$$\bar{\lambda}_y=\frac{l_{ef,y}}{i_y}\sqrt{\frac{R_y}{E}}=\frac{2,58\cdot 100}{2,851}\sqrt{\frac{240}{2,06\cdot 10^5}}=3,0888;$$

$$\bar{\lambda}_z=\frac{l_{ef,z}}{i_z}\sqrt{\frac{R_y}{E}}=\frac{5,16\cdot 100}{7,745}\sqrt{\frac{240}{2,06\cdot 10^5}}=2,274.$$

4. Buckling coefficients:

$$\varphi_y=1,47-13,0\frac{R_y}{E}-\left(0,371-27,3\frac{R_y}{E}\right)\bar{\lambda}_y+\left(0,0275-5,53\frac{R_y}{E}\right)\bar{\lambda}_y^2=$$

$$=1,47-\frac{13,0\cdot 240}{2,06\cdot 10^5}-\left(0,371-\frac{27,3\cdot 240}{2,06\cdot 10^5}\right)\cdot 3,0888+\left(0,0275-\frac{5,53\cdot 240}{2,06\cdot 10^5}\right)\cdot 3,0888^2=0,60805$$

$$\varphi_z=1-\left(0,073-5,53\frac{R_y}{E}\right)\bar{\lambda}_z\sqrt{\bar{\lambda}_z}=1-\left(0,073-\frac{5,53\cdot 240}{2,06\cdot 10^5}\right)\cdot 2,274\sqrt{2,274}=0,77176.$$

5. Strength of the truss member from the condition of providing the general stability under axial compression:

$$N_{b,y}=\varphi_y A R_y \gamma_c = 0,60805\cdot 45,74\cdot 24\cdot 0,95 = 634,118 \text{ kN};$$

$$N_{b,z}=\varphi_z A R_y \gamma_c = 0,77176\cdot 45,74\cdot 24\cdot 0,95 = 804,847 \text{ kN}.$$

6. Limit slenderness of the truss member:

$$[\lambda]_y=180-60\alpha_y=180-60\cdot\frac{N}{\varphi_y A R_y\gamma_c}=180-60\cdot\frac{535}{634,118}=129,3785;$$

$$[\lambda]_z=180-60\alpha_z=180-60\cdot\frac{N}{\varphi_z A R_y\gamma_c}=180-60\cdot\frac{535}{804,847}=140,1166.$$

Comparison of solutions:

Factor	Source	Manual calculation	SCAD	Deviation, %
Strength of member	535/45,8/22,8=0,512	11,6966/22,8 = 0,513	0,51	0,0
Stability of member in the truss plane	21,4/22,8=0,938	535/634,118 = 0,844	0,84	0,0
Stability of member out of the truss plane	not defined	535/804,847 = 0,665	0,66	0,0
Slenderness of the member in the truss plane	not defined	90,4946/129,3785 = 0,7	0,7	0,0
Slenderness of the member out of the truss plane	not defined	66,6236/140,1166 = 0,4755	0,48	0,0

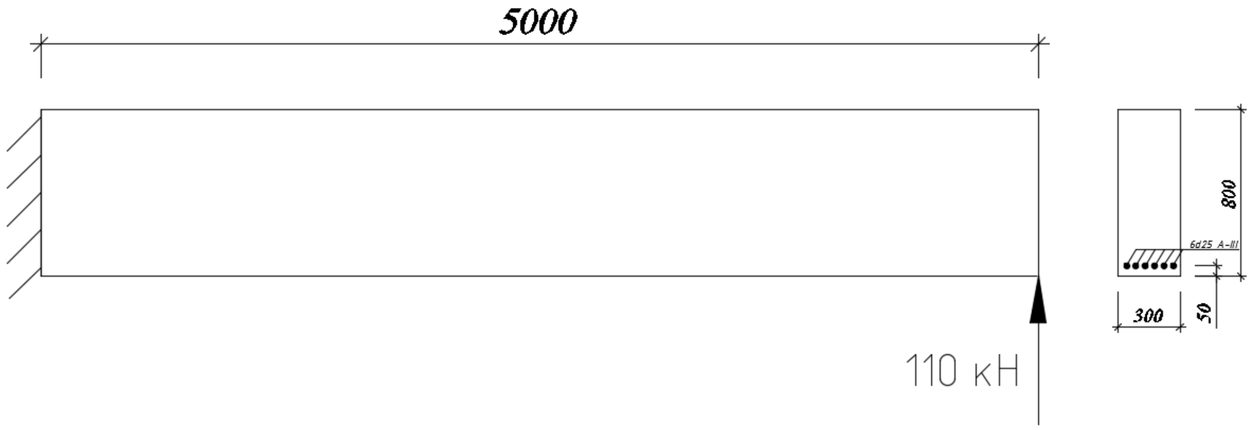
Comments:

In the source the buckling coefficient for the conditional slenderness of the bar of 3.09 was mistakenly taken as 0.546 instead of 0.6081, which caused the differences in the results of the stability analysis.

Reinforced Concrete Structural Members

Calculations according to SNiP 2.03.01-84*

Strength Analysis of a Rectangular Beam



Objective: Check the mode for calculating reinforced concrete structures in the “Reinforced Concrete” postprocessor of SCAD

Task: Check the strength of the cantilever beam section for the specified reinforcement

References: Guide on designing of concrete and reinforced concrete structures made of heavy-weight or lightweight concrete (no prestressing) (to SNiP 2.03.01-84), 1989, p. 26.

Initial data file:
SCAD 3 SNiP.spr
report – SCAD 3 SNiP.doc

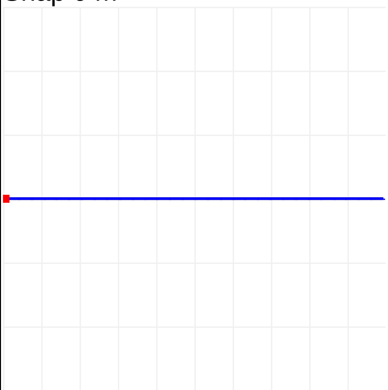
Compliance with the codes: SNiP 2.03.01-84.

Initial data:

b = 200 mm	Beam section sizes
h = 800 mm	
a = 50 mm	Distance from the center of gravity of the reinforcement to the compressed edge of the section
A_s = 2945 mm^2 (6Ø25)	Cross-sectional area of reinforcement
Concrete class	B25
Class of reinforcement	A-III
l = 4,8 m	Beam span
q = 191 kN/m	Load on the beam
M = 550 kNm	Bending moment in the section under the load

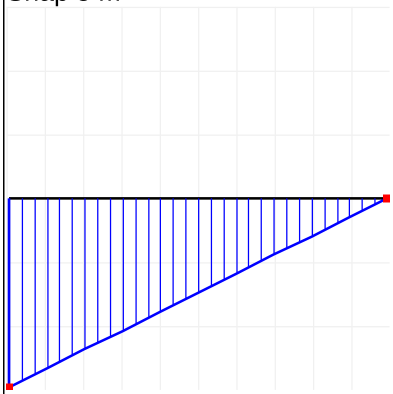
Results of the SCAD analysis:

N
Max. 0 T
Snap 0 m



Max. 0 T
Snap 0 m

M_y
Max. 0 kN*m
Snap 5 m

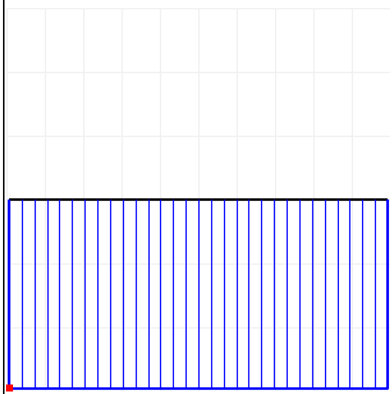


Max. 550,55 kN*m
Snap 0 m

M_z

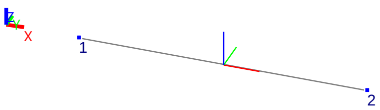
M_k

Q_z

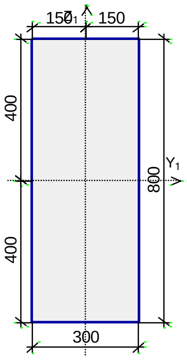


Max. -11,21 T
Snap 0 m

Q_y



Length of the bar 5 m
Length of the flexible part 5 m
Loading L1 - "110 kN"



Structural group Beam

Distance between the rebars in the first row S1 is less than the allowable value (see Sec. 5.12 of SNiP 2.03.01-84*) .
Elements: 1

Importance factor γ_n = 1

Importance factor (serviceability limit state) = 1

Member type – Flexural

Stress state - Uniaxial bending

Coefficients allowing for seismic action	
Normal sections	0
Oblique sections	0

Distance to the c.o.g. of reinforcement	
a_1	a_2
mm	mm
50	50

Reinforcement	Class	Service factor
Longitudinal	A-III	1
Transverse	A-I	1

Concrete
Concrete type: Heavy-weight
Concrete class: B25
Hardening conditions: Natural
Hardening factor 1

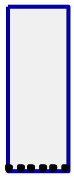
Service factor for concrete		
γ_{b2}	allowance for the sustained loads	0,9
	resulting factor without γ_{b2}	1

Humidity of environmental air - 40-75%

Crack resistance
Category of crack resistance - 3
Conditions of operation: Indoors
Mode of concrete humidity - Natural humidity
Allowable crack opening width:
Short-term opening 0,4 mm
Long-term opening 0,3 mm

Structural group Beam. Element No. 1
Member length 5,0 m

Specified reinforcement

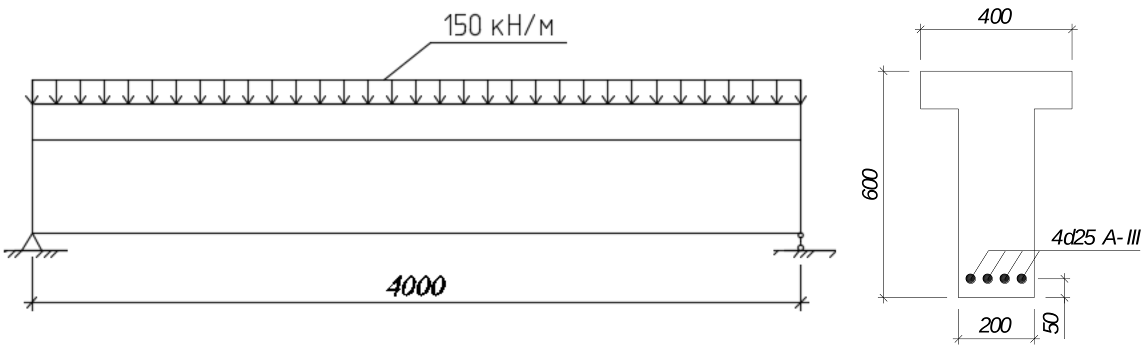
Segment	Reinforcement	Section
1	S_1 - 6∅25	

Results			
Segment	**Utilization factor**	**Check**	**Checked according to SNiP**
1	0,83	Ultimate moment strength of the section	Sec. 3.15-3.20, 3.27-3.28

Comparison of solutions

Check	strength of the section
Guide	550/636,4 = 0,864
SCAD	0,83
Deviation, %	4,1 %

Strength Analysis of a T-section



Objective: Check of the strength analysis of the section

Task: Check the strength of a simply supported T-beam with the length of 4,0 m and the specified reinforcement

References: Guide on designing of concrete and reinforced concrete structures made of heavy-weight or lightweight concrete (no prestressing) (to SNiP 2.03.01-84), 1989, p. 27-28.

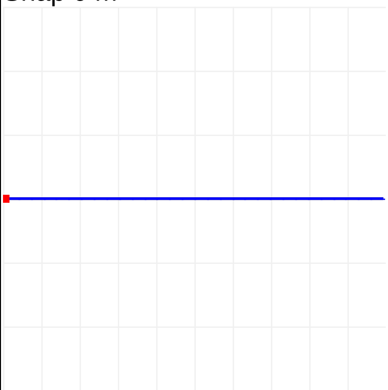
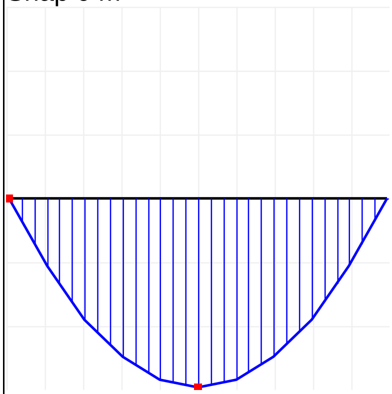
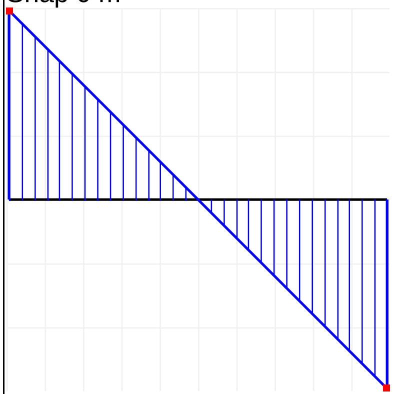
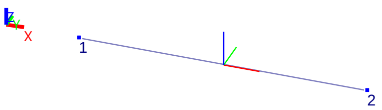
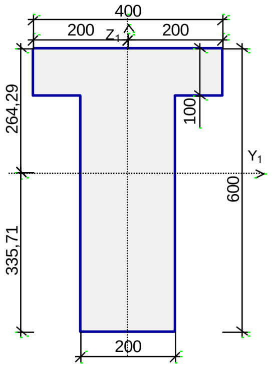
Initial data file:
SCAD 7 SNiP.spr
report – SCAD 7 SNiP.doc

Compliance with the codes: SNiP 2.03.01-84.

Initial data:

b = 200 mm	Beam section sizes
h = 600 mm	
b'_f = 400 mm	
h'_f = 100 mm	
a = 50 mm	Distance from the center of gravity of the reinforcement to the compressed edge of the section
A_s = 1964 mm^2 (4Ø25)	Cross-sectional area of reinforcement
Concrete class	B25
Class of reinforcement	A-III
q = 191 kN/m	Load on the beam
M = 300 kNm	Bending moment in the section

Results of the SCAD analysis:

N Max. 0 T Snap 0 m  Max. 0 T Snap 0 m	**M_y** Max. 0 kN*m Snap 0 m  Max. 300 kN*m Snap 2 m	**M_z**
M_k	**Q_z** Max. 30,58 T Snap 0 m  Max. -30,58 T Snap 4 m	**Q_y**
	Length of the bar 4 m Length of the flexible part 4 m Loading L1 - "150 kN/m"	

Structural group Beam

Distance between the rebars in the first row S1 is less than the allowable value (see Sec. 5.12 of SNiP 2.03.01-84*) .
Elements: 1

Importance factor γ_n = 1

Importance factor (serviceability limit state) = 1

Member type – Flexural

Stress state - Uniaxial bending

Coefficients allowing for seismic action	
Normal sections	0
Oblique sections	0

Distance to the c.o.g. of reinforcement	
a_1	**a_2**
mm	**mm**
58,5	20

Reinforcement	Class	Service factor
Longitudinal	A-III	1
Transverse	A-I	1

Concrete
Concrete type: Heavy-weight
Concrete class: B25
Hardening conditions: Natural
Hardening factor 1

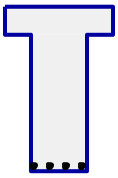
Service factor for concrete		
γ_{b2}	allowance for the sustained loads	0,9
	resulting factor without γ_{b2}	1

Humidity of environmental air - 40-75%

Crack resistance
Category of crack resistance - 3
Conditions of operation: Indoors
Mode of concrete humidity - Natural humidity
Allowable crack opening width:
Short-term opening 0,4 mm
Long-term opening 0,3 mm

Structural group Beam. Element No. 1
Member length 4 m

Specified reinforcement

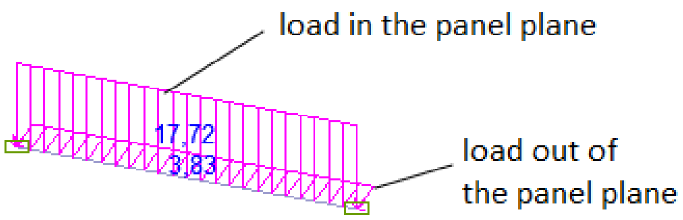
Segment	Reinforcement	Section
1	S_1 - 4∅25	

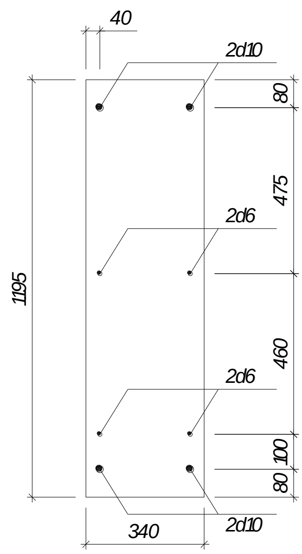
Results			
Segment	**Utilization factor**	**Check**	**Checked according to SNiP**
1	0,89	Ultimate moment strength of the section	Sec. 3.15-3.20, 3.27-3.28

Comparison of solutions

Check	strength of the section
Guide	300/327,1 = 0,917
SCAD	0,89
Deviation, %	3,0 %

Strength Analysis of a Wall Panel





Objective: Check of the strength of the wall panel

Task: Check the strength of the section

References: Guide on designing of concrete and reinforced concrete structures made of heavy-weight or lightweight concrete (no prestressing) (to SNiP 2.03.01-84), 1989, p. 32-34.

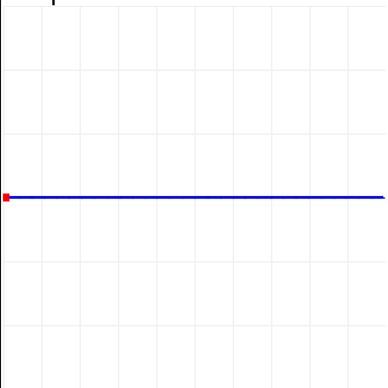
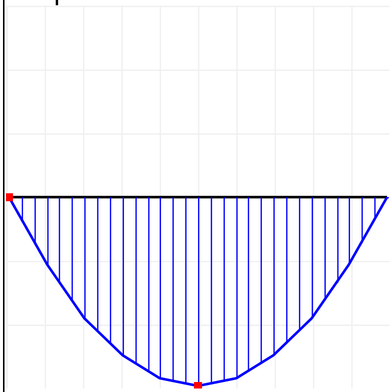
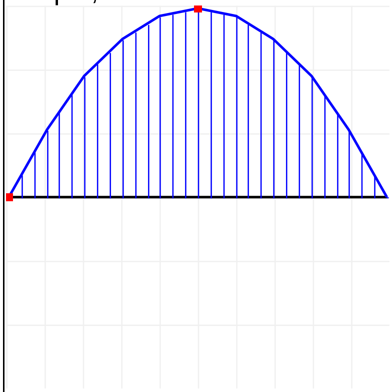
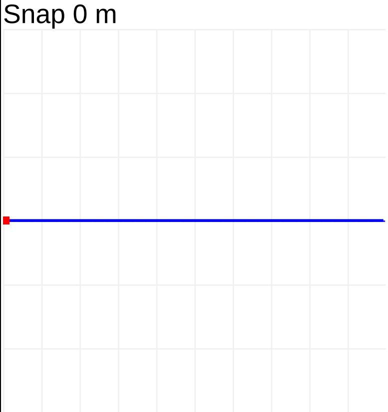
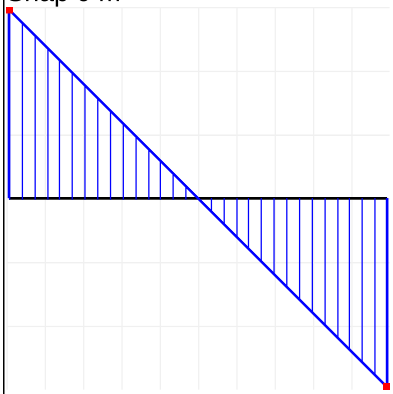
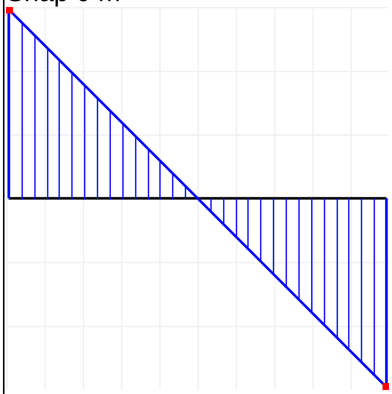
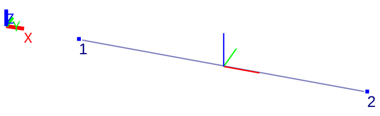
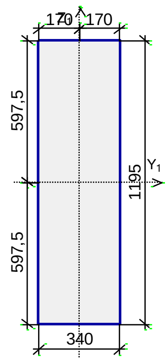
Initial data file:
SCAD 12 SNiP.spr
report – SCAD 12 SNiP.doc

Compliance with the codes: SNiP 2.03.01-84.

Initial data:

l = 5,8 m	Wall panel span
$b \times h$ = 340×1195 mm	Wall panel section sizes
q_{tot} = 3,93 kN/m^2	Total vertical uniformly distributed load
(q_x = 17,72 kN/m)	Reduced load in the panel plane
q_w = 0,912 kN/m^2	Wind load
(q_y = 3,83 kN/m)	Reduced load out of the panel plane
Concrete class	B3,5; D1100
Class of reinforcement	A-III

Results of the SCAD analysis:

N Max. 0 T Snap 0 m  Max. 0 T Snap 0 m	**M_y** Max. 0 kN*m Snap 0 m  Max. 74,51 kN*m Snap 2,9 m	**M_z** Max. -16,11 kN*m Snap 2,9 m  Max. 0 kN*m Snap 0 m
M_k Max. 0 kN*m Snap 0 m  Max. 0 kN*m Snap 0 m	**Q_z** Max. 5,24 T Snap 0 m  Max. -5,24 T Snap 5,8 m	**Q_y** Max. 1,13 T Snap 0 m  Max. -1,13 T Snap 5,8 m
	Length of the bar 5,8 m Length of the flexible part 5,8 m Loading L1 - "17,72 kN/m + 3,83 kN/m"	

Structural group Wall panel

Importance factor γ_n = 1

Member type - Member under compression and bending (in tension)

Stress state - Biaxial bending

Maximum percentage of reinforcement 10
Random eccentricity along Z_1 0 mm
Random eccentricity along Y_1 0 mm

Structure is statically indeterminate
Effective length factor in the X_1OZ_1 plane 1
Effective length factor in the X_1OY_1 plane 1

Coefficients allowing for seismic action	
Normal sections	0
Oblique sections	0

Distance to the c.o.g. of reinforcement	
a_1	**a_2**
mm	**mm**
75	75

Reinforcement	Class	Service factor
Longitudinal	A-III	1
Transverse	A-I	1

Concrete
Concrete type: Lightweight
Concrete class: B3,5
Grade by average density: D1100
Aggregate: Artificial dense
Hardening conditions: Natural
Hardening factor 1

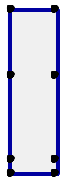
Service factor for concrete		
γ_{b2}	allowance for the sustained loads	1
	resulting factor without γ_{b2}	1,1

Humidity of environmental air - 40-75%

Structural group Wall panel. Element No. 1

Member length 5,8 m

Specified reinforcement

Segment	Reinforcement	Section
1	S_1 - 2∅10, second row 2∅6 Clear distance between rows 92 mm S_2 - 2∅10, second row 2∅6 Clear distance between rows 467 mm	

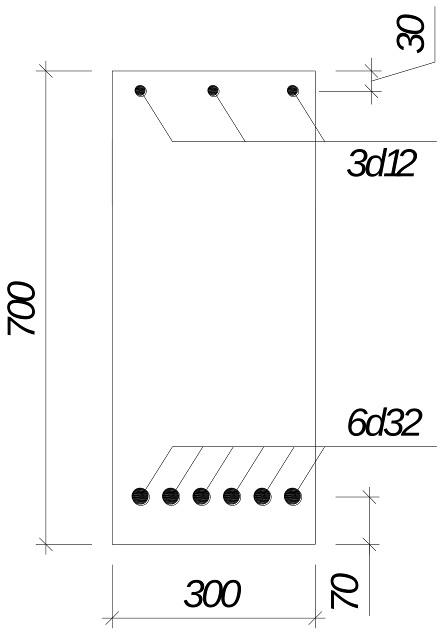
Results			
Segment	**Utilization factor**	**Check**	**Checked according to SNiP**
1	0,99	Ultimate moment strength of the section	Sec. 3.15-3.20, 3.27-3.28

Comparison of solutions:

Check	Strength of the section
Guide	74,5/78,4 = 0,95
SCAD	0,99
Deviation, %	4,2 %

Calculations according to SNiP 52-01-2003

Strength Analysis of a Rectangular Beam



Objective: Check the mode for calculating reinforced concrete structures in the "Reinforced Concrete" postprocessor of SCAD

Task: Check the strength of the beam section for the specified reinforcement

References: Guide on designing of concrete and reinforced concrete structures made of heavy-weight concrete (no prestressing) (to SP 52-101-2003), 2005, p. 28.

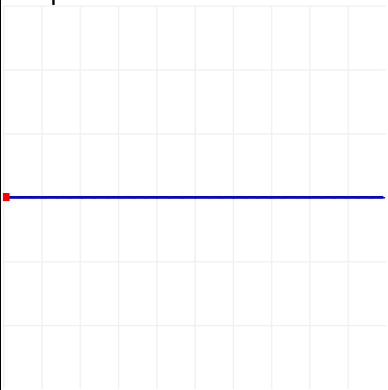
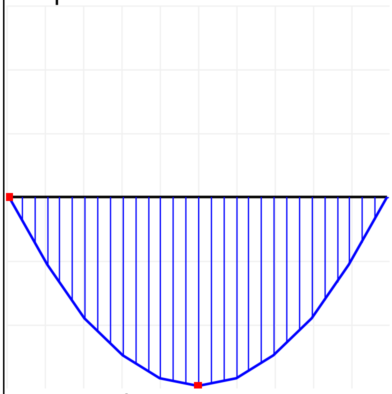
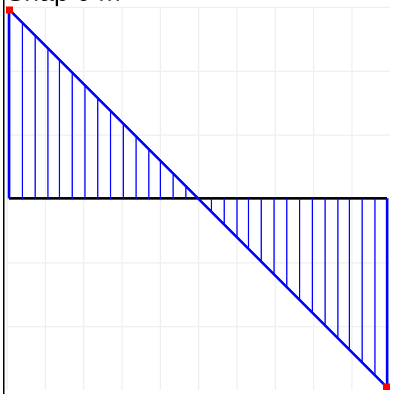
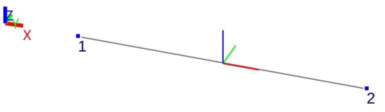
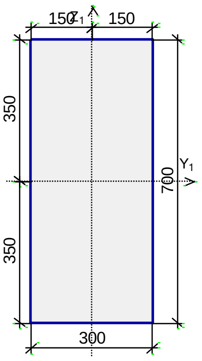
Initial data file:
SCAD 6 SP.spr
report – SCAD 6 SP.doc

Compliance with the codes: SP 52-101-2003.

Initial data:

$b \times h$ = 300×700 mm	Section sizes
a = 70 mm	Distance to the c.o.g. of tensile reinforcement
a'= 30 mm	Distance to the c.o.g. of compressed reinforcement
A_s = 4826 mm^2 (6∅32)	Cross-sectional area of tensile reinforcement
A'_s = 339 mm^2 (3∅12)	Cross-sectional area of compressed reinforcement
l = 6,0 m	Beam span
q = 140 kN/m	Load on the beam
M = 630 kNm	Bending moment
Concrete class	B20
Class of reinforcement	A400

Results of the SCAD analysis:

<table>
<tr>
<td>**N**
Max. 0 T
Snap 0 m

Max. 0 T
Snap 0 m</td>
<td>**M_y**
Max. 0 kN*m
Snap 0 m

Max. 630 kN*m
Snap 3 m</td>
<td>**M_z**</td>
</tr>
<tr>
<td>**M_k**</td>
<td>**Q_z**
Макс. 42,81 T
Snap 0 m

Max. -42,81 T
Snap 6 m</td>
<td>**Q_y**</td>
</tr>
<tr>
<td>
</td>
<td>Length of the bar 6 m
Length of the flexible part 6 m
Loading L1 - "140 kN/m"</td>
<td>
</td>
</tr>
</table>

Structural group Beam

Distance between the rebars in the first row S1 is less than the allowable value (see Sec. 8.3.3 of SP 52-101-2003)   .
Elements: 1

Importance factor $\gamma_n = 1$

Member type - Flexural

Stress state - Uniaxial bending

Coefficients allowing for seismic action	
Normal sections	0
Oblique sections	0

Distance to the c.o.g. of reinforcement	
a_1	**a_2**
mm	**mm**
70	30

Reinforcement	Class	Service factor
Longitudinal	A400	1
Transverse	A240	1

Concrete
Concrete type: Heavy-weight
Concrete class: B20

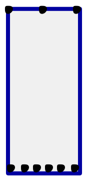
Service factor for concrete		
γ_{b1}	allowance for the sustained loads	1
γ_{b2}	allowance for the failure behavior	1
γ_{b3}	allowance for the vertical position during concreting	1
γ_{b4}	allowance for the freezing/thawing and negative temperatures	1

Humidity of environmental air - 40-75%

Crack resistance
Limited crack opening width
Requirements to crack opening width are based on the preservation of reinforcement
Allowable crack opening width:
Short-term opening 0,4 mm
Long-term opening 0,3 mm

Structural group Beam. Element No. 1
Member length 6 m

Specified reinforcement

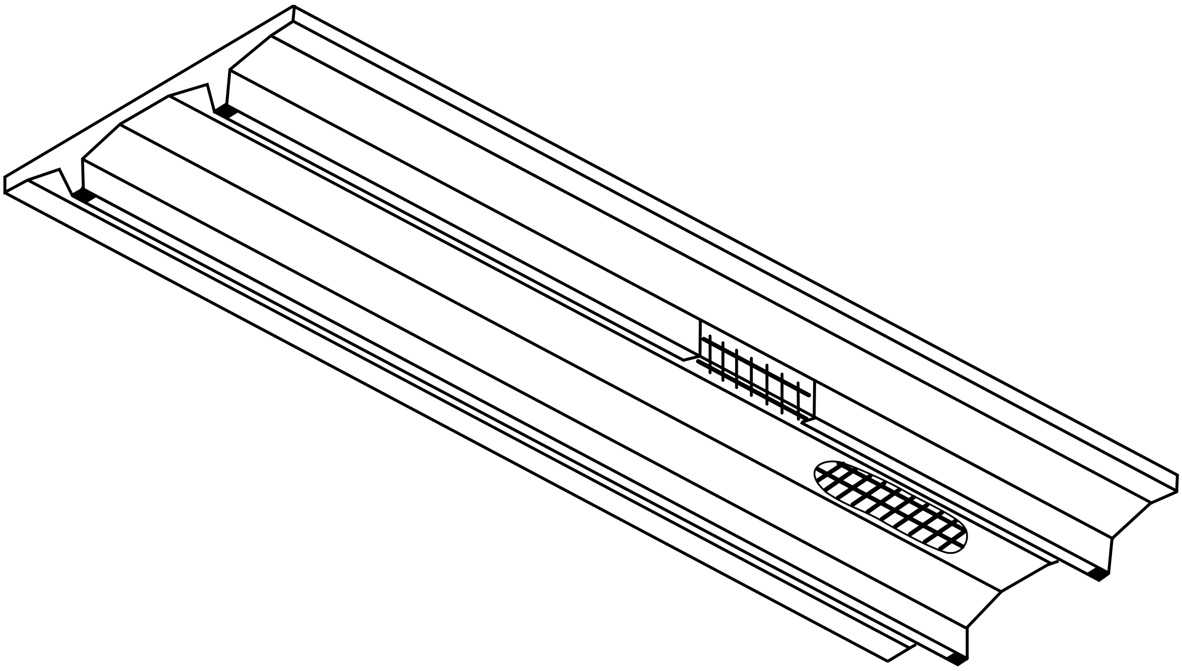
Segment	Reinforcement	Section
1	S_1 - 6∅32 S_2 - 3∅12	

Results			
Segment	**Utilization factor**	**Check**	**Checked according to SNiP**
1	**1,02**	Ultimate moment strength of the section	

Comparison of solutions:

Check	Strength of the section
Guide	630/606,2 = 1,039
SCAD	1,02
Deviation, %	1,9 %

Calculation of a Rib of a TT-shaped Floor Slab for Load-bearing Capacity under Lateral Forces



Objective: Check the mode for calculating reinforced concrete structures in the “Reinforced Concrete” postprocessor of SCAD

Task: Verify the correctness of the strength analysis of oblique sections and a concrete strip between the oblique sections.

References: Guide on designing of concrete and reinforced concrete structures made of heavy-weight concrete (no prestressing) (to SP 52-101-2003), 2005, p. 56-57.

Initial data file:

when the lateral force is Q = 62 kN – SCAD 12.1.SP. spr
report – SCAD 12.1.SP.doc.
when the lateral force is Q = 58,4 kN – SCAD 12.2.SP. spr
report – SCAD 12.2.SP.doc.

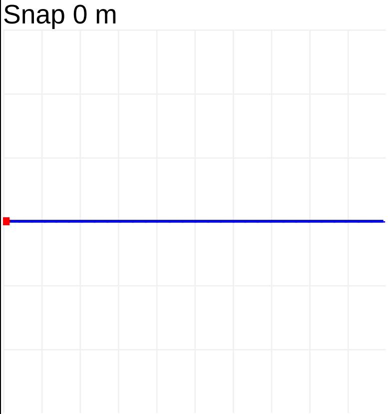
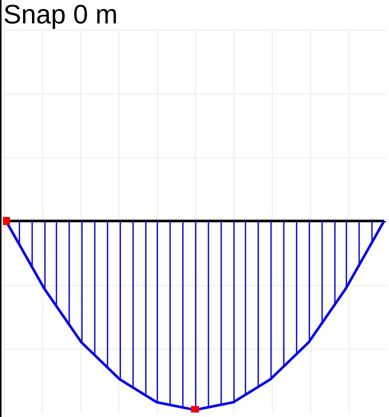
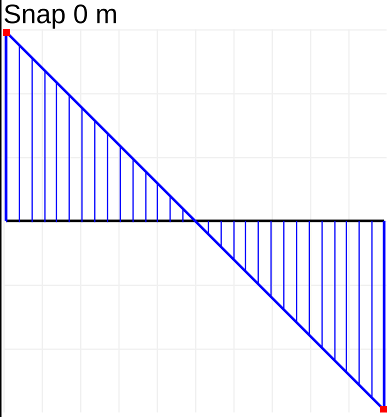
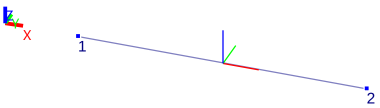
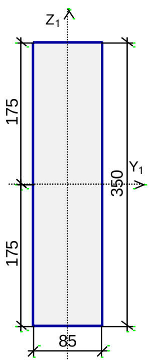
Compliance with the codes: SP 52-101-2003, SP 63.13330.2012.

Initial data:

$b \times h$ = 85×350 mm	Section sizes
a = 35 mm	Distance to the c.o.g. of tensile reinforcement
d = 8 mm	Diameter of transverse reinforcement
s = 100 mm	Spacing of transverse reinforcement
q = 21,9 kN/m	Load on the rib
q = 18 kN/m	Temporary equivalent load
Q = 62 kN	Lateral force on the support

Concrete class B15
Class of transverse reinforcement A400

Results of the SCAD analysis (when the lateral force is Q = 62 kN):

N Max. 0 kN Snap 0 m  Max. 0 kN Snap 0 m	**M_y** Max. 0 kN*m Snap 0 m  Max. 48,18 kN*m Snap 1,55 m	**M_z**
M_k	**Q_z** Max. 62 kN Snap 0 m  Max. -62 kN Snap 3,11 m	**Q_y**
	Length of the bar 3,11 m Length of the flexible part 3,11 m Loading L1 - "39,9 kN/m"	

Structural group Beam

Number of rebars in a row must be not less than two (see Sec. 8.3.7 of SP 52-101-2003)
Distance between the rebars in the first row S1 is less than the allowable value (see Sec. 8.3.3 of SP 52-101-2003) .
Elements: 1

Importance factor γ_n = 1

Member type - Flexural

Stress state - Uniaxial bending

Coefficients allowing for seismic action	
Normal sections	0
Oblique sections	0

Distance to the c.o.g. of reinforcement	
a_1	**a_2**
mm	**mm**
32	32

Reinforcement	Class	Service factor
Longitudinal	A400	1
Transverse	A400	1

Concrete
Concrete type: Heavy-weight
Concrete class: B15

Service factor for concrete		
γ_{b1}	allowance for the sustained loads	1
γ_{b2}	allowance for the failure behavior	1
γ_{b3}	allowance for the vertical position during concreting	1
γ_{b4}	allowance for the freezing/thawing and negative temperatures	1

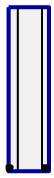
Humidity of environmental air - 40-75%

Crack resistance
No cracks

Structural group Beam. Element No. 1

Member length 3,11 m

Specified reinforcement

Segment	Reinforcement	Section
1	S_1 - 2∅6 Transverse reinforcement along the Z axis 1∅8, spacing of transverse reinforcement 100 mm	

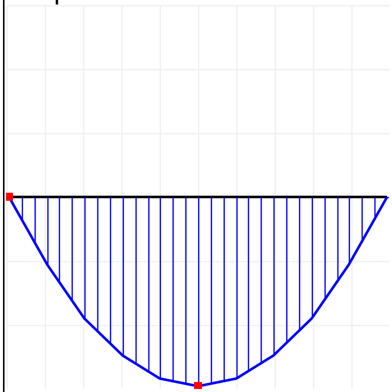
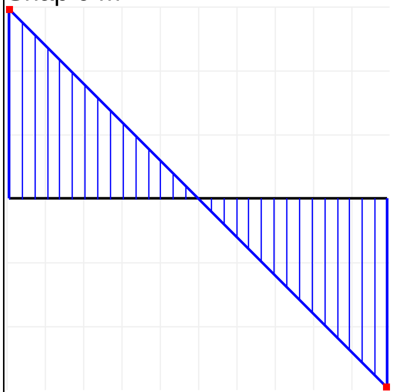
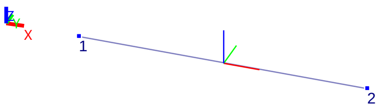
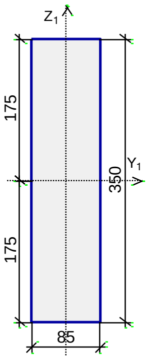
Results			
Segment	**Utilization factor**	**Check**	**Checked according to SNiP**
1	0,9	Strength in a concrete strip between oblique sections	Sec. 6.2.33, Sec. 3.52 of the Guide

Comparison of solutions

Check	Strength in a concrete strip between oblique sections
Guide	62/68,276 = 0,908
SCAD	0,9
Deviation, %	0,9 %

Results of the SCAD analysis (when the lateral force is Q = 58,4 kN):

<table>
<tr>
<td>N
Max. 0 kN
Snap 0 m

Max. 0 kN
Snap 0 m</td>
<td>M_y
Max. 0 kN*m
Snap 0 m

Max. 42,74 kN*m
Snap 1,46 m</td>
<td>M_z</td>
</tr>
<tr>
<td>M_k</td>
<td>Q_z
Max. 58,4 kN
Snap 0 m

Max. -58,4 kN
Snap 2,93 m</td>
<td>Q_y</td>
</tr>
<tr>
<td>
</td>
<td>Length of the bar 2,93 m
Length of the flexible part 2,93 m
Loading L1 - "39,9 kN/m"</td>
<td>
</td>
</tr>
</table>

Structural group Beam

Number of rebars in a row must be not less than two (see Sec. 8.3.7 of SP 52-101-2003)
Distance between the rebars in the first row S1 is less than the allowable value (see Sec. 8.3.3 of SP 52-101-2003) .
Elements: 1

Importance factor γ_n = 1

Member type - Flexural

Stress state - Uniaxial bending

Coefficients allowing for seismic action	
Normal sections	0
Oblique sections	0

Distance to the c.o.g. of reinforcement	
a_1	**a_2**
mm	**mm**
32	32

Reinforcement	Class	Service factor
Longitudinal	A400	1
Transverse	A400	1

Concrete
Concrete type: Heavy-weight
Concrete class: B15

Service factor for concrete		
γ_{b1}	allowance for the sustained loads	1
γ_{b2}	allowance for the failure behavior	1
γ_{b3}	allowance for the vertical position during concreting	1
γ_{b4}	allowance for the freezing/thawing and negative temperatures	1

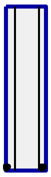
Humidity of environmental air - 40-75%

Crack resistance
No cracks

Structural group Beam. Element No. 1

Member length 2,93 m

Specified reinforcement

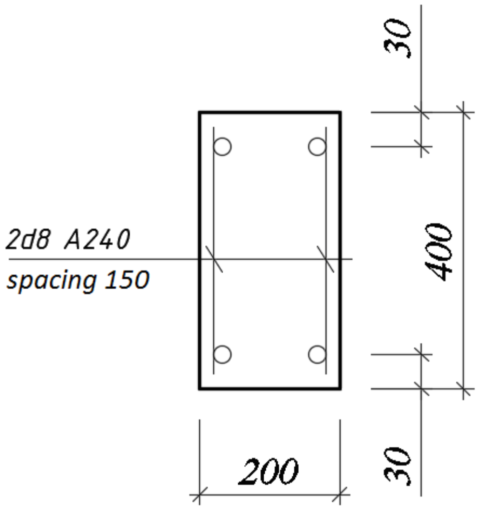
Segment	Reinforcement	Section
1	S_1 - 2∅6 Transverse reinforcement along the Z axis 1∅8, spacing of transverse reinforcement 100 mm	

Results			
Segment	**Utilization factor**	**Check**	**Checked according to SNiP**
1	0,9	Strength for an oblique section	Sec. 6.2.34, Sec. 3.52,3.71 of the Guide

Comparison of solutions:

Check	Strength for an oblique section
Guide	58,4/63,97 = 0,913
SCAD	0,9
Deviation, %	1,4 %

Calculation of a Simply Supported Rectangular Beam under Lateral Forces



Objective: Check the mode for calculating reinforced concrete structures in the "Reinforced Concrete" postprocessor of SCAD

Task: Check the strength of the oblique section of the beam for the specified reinforcement

References: Guide on designing of concrete and reinforced concrete structures made of heavy-weight concrete (no prestressing) (to SP 52-101-2003), 2005, p. 57-58.

Initial data file:
SCAD 13 SP.spr
report – SCAD 13 SP.doc

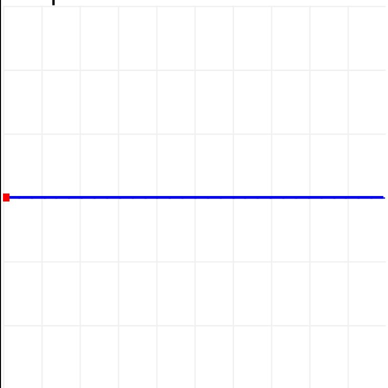
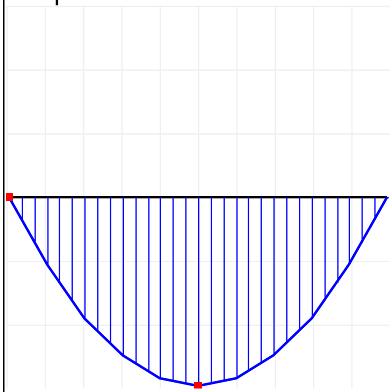
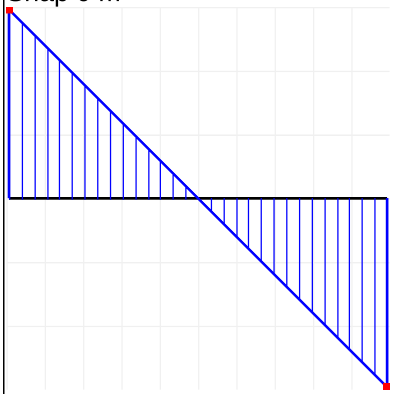
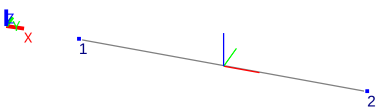
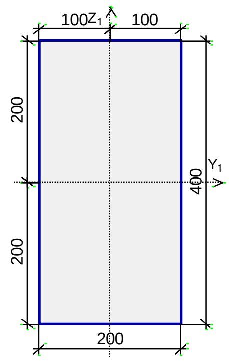
Compliance with the codes: SP 52-101-2003.

Initial data:

$b \times h$ = 200×400 mm	Section sizes
a = 30 mm	Distance to the c.o.g. of tensile reinforcement
a' = 30 mm	Distance to the c.o.g. of compressed reinforcement
A_{sw} = 101mm^2 (2∅8)	Cross-sectional area of transverse reinforcement
s_w = 150 mm	Spacing of transverse reinforcement
q_v = 36 kN/m	Temporary load on the beam
q_g = 14 kN/m	Permanent load on the beam
Q = 100,35 kNm	Lateral force on the support

Concrete class B25
Class of reinforcement 240

Results of the SCAD analysis:

N Max. 0 kN Snap 0 m  Max. 0 kN Snap 0 m	**M_y** Max. 0 kN*m Snap 0 m  Max. 100,7 kN*m Snap 2,01 m	**M_z**
M_k	**Q_z** Max. 100,35 kN Snap 0 m  Max. -100,35 kN Snap 4,01 m	**Q_y**
	Length of the bar 4,01 m Length of the flexible part 4,01 m Loading L1 - "50 kN/m"	

Structural group Beam

Structural group Beam. Element No. 1

Importance factor γ_n = 1

Member type – Flexural

Stress state - Uniaxial bending

Coefficients allowing for seismic action	
Normal sections	0
Oblique sections	0

Distance to the c.o.g. of reinforcement	
a_1	**a_2**
mm	**mm**
30	30

Reinforcement	Class	Service factor
Longitudinal	A240	1
Transverse	A240	1

Concrete
Concrete type: Heavy-weight
Concrete class: B25

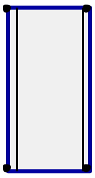
Service factor for concrete		
γ_{b1}	allowance for the sustained loads	1
γ_{b2}	allowance for the failure behavior	1
γ_{b3}	allowance for the vertical position during concreting	1
γ_{b4}	allowance for the freezing/thawing and negative temperatures	1

Humidity of environmental air - 40-75%

Crack resistance
No cracks

Structural group Beam. Element No. 1
Member length 4,01 m

Specified reinforcement

Segment	Reinforcement	Section
1	S_1 - 2∅6 S_2 - 2∅6 Transverse reinforcement along the Z axis 2∅8, spacing of transverse reinforcement 150 mm Transverse reinforcement along the Y axis 2∅8, spacing of transverse reinforcement 150 mm	

Results			
Segment	**Utilization factor**	**Check**	**Checked according to SNiP**
1	0,98	Strength for an oblique section	Sec. 6.2.34, Sec. 3.52,3.71 of the Guide

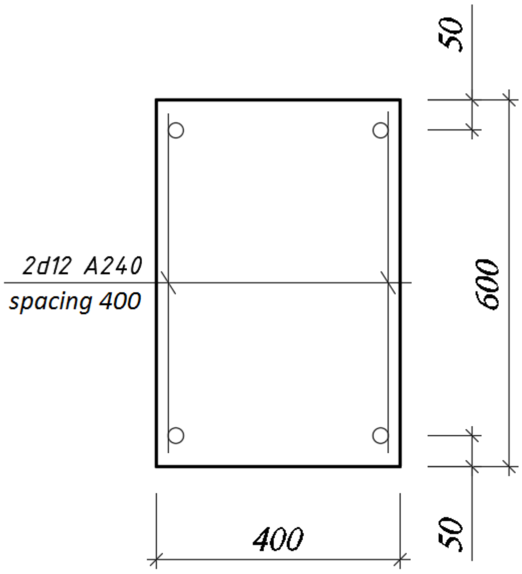
Comparison of solutions

Check	Strength of the section
Guide	100,35/100,69 = 0,997
SCAD	0,98
Deviation, %	1,7 %

Comments:

1. The strength check of oblique sections is performed by comparing a sum of lateral forces resisted by concrete and stirrups in the oblique section (Q_b + Q_{sw}), with a lateral force Q in the oblique section which is determined as a projection on the normal to the longitudinal axis of the element of the resultant of all external forces acting on the element on one side of the considered oblique section ($Q = Q_{max} - q_1 c$). The lateral force in the normal section is taken as Q = 100,35 kN according to the Guide.
2. The data on the longitudinal reinforcement has to be specified in SCAD. Since it is not defined in the problem, the following reinforcement is used: class A240, rebars 2Ø6.

Calculation of a Column of a Multi-storey Frame for Load-bearing Capacity under a Lateral Force



Objective: Check the mode for calculating reinforced concrete structures in the "Reinforced Concrete" postprocessor of SCAD

Task: Check the strength of the column section for the specified reinforcement

References: Guide on designing of concrete and reinforced concrete structures made of heavy-weight concrete (no prestressing) (to SP 52-101-2003), 2005, p. 104-105.

Initial data file:
SCAD 34 SP.spr
report – SCAD 34 SP-2003.doc
report – SCAD 34 SP-2012.doc

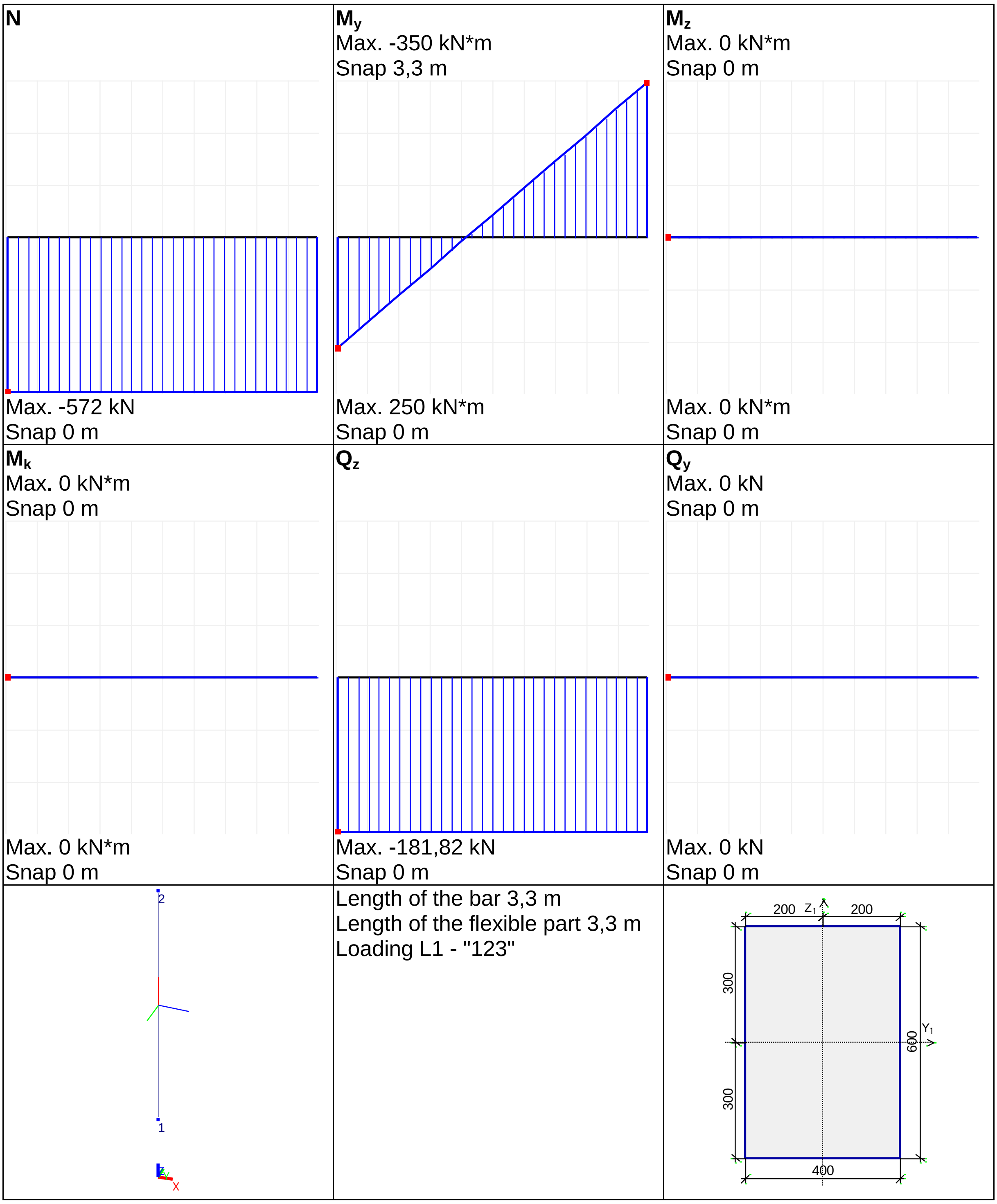
Compliance with the codes: SP 52-101-2003, SP 63.13330.2012.

Initial data:

$b \times h$ = 400×600 mm	Section sizes
a = 50 mm	Distance to the c.o.g. of tensile reinforcement
a' = 50 mm	Distance to the c.o.g. of compressed reinforcement
A_{sw} = 226 mm^2 (2∅12)	Cross-sectional area of tensile reinforcement
s_w = 400 mm	Cross-sectional area of compressed reinforcement
l = 3,3 m	Column length
M_{inf} = 250 kNm	Bending moment in the lower support section
M_{sup} = 350 kNm	Bending moment in the upper support section
N = 572 kN	Longitudinal compressive force

Concrete class B25
Class of transverse reinforcement A240

Results of the SCAD analysis:



Structural group Column

Spacing of transverse reinforcement is greater than the allowable value (see Sec. 8.3.11 of SP 52-101-2003) .
Elements: 1

Importance factor $\gamma_n = 1$

Member type - Member under compression and bending (in tension)

Stress state - Uniaxial bending

Maximum percentage of reinforcement 10
Random eccentricity along Z_1 0 mm
Random eccentricity along Y_1 0 mm

Structure is statically indeterminate
Effective length factor in the X_1OZ_1 plane 1
Effective length factor in the X_1OY_1 plane 1

Coefficients allowing for seismic action	
Normal sections	0
Oblique sections	0

Distance to the c.o.g. of reinforcement	
a_1	**a_2**
mm	**mm**
50	50

Reinforcement	**Class**	**Service factor**
Longitudinal	A240	1
Transverse	A240	1

Concrete
Concrete type: Heavy-weight
Concrete class: B25

Service factor for concrete		
γ_{b1}	allowance for the sustained loads	1
γ_{b2}	allowance for the failure behavior	1
γ_{b3}	allowance for the vertical position during concreting	1
γ_{b4}	allowance for the freezing/thawing and negative temperatures	1

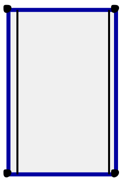
Humidity of environmental air - 40-75%

Crack resistance
No cracks

Structural group Column. Element No. 1

Member length 3,3 m

Specified reinforcement

Segment	Reinforcement	Section
1	S_1 - 2∅6 S_2 - 2∅6 Transverse reinforcement along the Z axis 2∅12, spacing of transverse reinforcement 400 mm Transverse reinforcement along the Y axis 2∅12, spacing of transverse reinforcement 400 mm	

Results			
Segment	**Utilization factor**	**Check**	**Checked according to SNiP**
1	0,98	Strength for an oblique section	Sec. 6.2.34, Sec. 3.52,3.71 of the Guide

Comparison of solutions (according to SNiP 52-101-2003):

Check	Strength for an oblique section
Guide	181,8/184,8 = 0,984
SCAD	0,98
Deviation, %	0,4 %

Structural group Column

Spacing of transverse reinforcement is greater than the allowable value (see Sec. 10.3.13 of SP 63.13330.2012)   .
Elements: 1

Importance factor γ_n = 1

Member type - Member under compression and bending (in tension)

Stress state - Uniaxial bending

Maximum percentage of reinforcement 10
Random eccentricity along Z_1 0 mm
Random eccentricity along Y_1 0 mm

Structure is statically indeterminate
Effective length factor in the X_1OZ_1 plane 1
Effective length factor in the X_1OY_1 plane 1

Coefficients allowing for seismic action	
Normal sections	0
Oblique sections	0

Distance to the c.o.g. of reinforcement	
a_1	**a_2**
mm	**mm**
50	50

Reinforcement	Class	Service factor
Longitudinal	A240	1
Transverse	A240	1

Concrete
Concrete type: Heavy-weight
Concrete class: B25

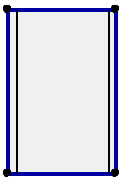
Service factor for concrete		
γ_{b1}	allowance for the sustained loads	1
γ_{b2}	allowance for the failure behavior	1
γ_{b3}	allowance for the vertical position during concreting	1
γ_{b5}	allowance for the freezing/thawing and negative temperatures	1

Humidity of environmental air - 40-75%

Crack resistance
No cracks

Structural group Column. Element No. 1
Member length 3,3 m

Specified reinforcement

Segment	Reinforcement	Section
1	S_1 - 2∅6 S_2 - 2∅6 Transverse reinforcement along the Z axis 2∅12, spacing of transverse reinforcement 400 mm Transverse reinforcement along the Y axis 2∅12, spacing of transverse reinforcement 400 mm	

Results			
Segment	**Utilization factor**	**Check**	**Checked according to SNiP**
1	0,84	Strength for an oblique section	Sec. 8.1.33, 8.1.34

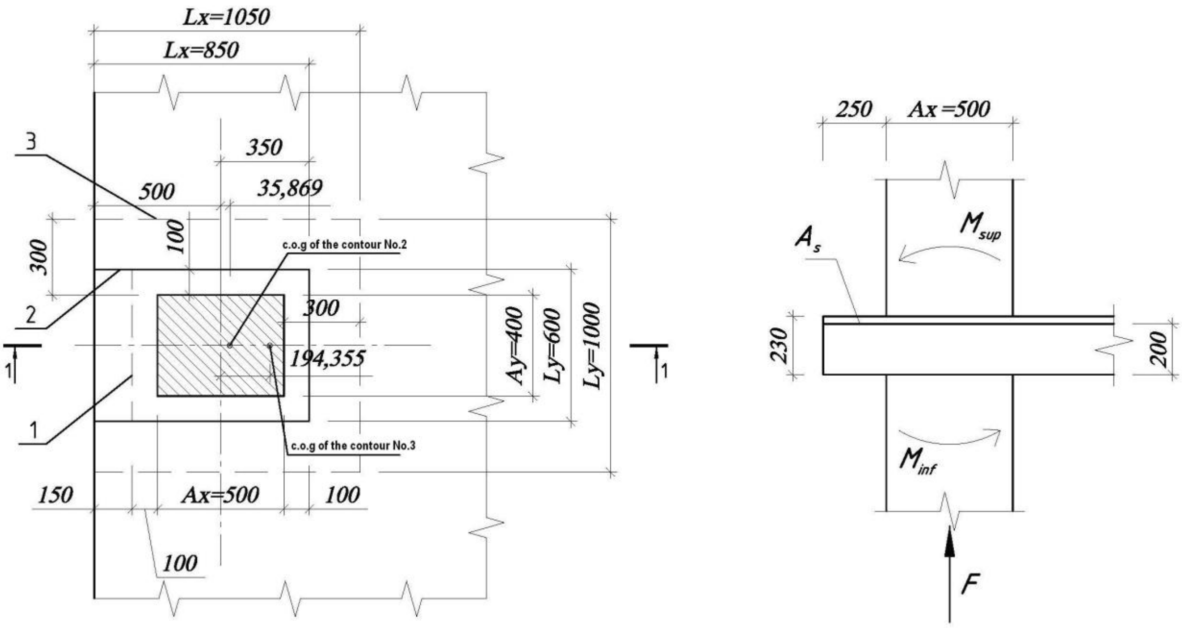
Comparison of solutions (according to SP 63.13330.2012)

Check	Strength for an oblique section
Guide	181,8/184,8 = 0,984
SCAD	0,84
Deviation, %	14,6 %

Comment:

The difference between the utilization factors of 14,6% in the results of the solution in the Guide and in SCAD according to SP 63.13330.2012 is due to the fact that compressive stresses are taken into account in different ways according to the given codes (Sec. 8.1.34) and according to SNiP 52-101-2003.

Example of Punching Near the Edge of the Slab



1 – closed design contour No.1, 2 – open design contour No.2, 3 – open design contour No.3.

Punching Analysis of a Flat Monolithic Floor Slab

Objective: Check the **Punching** mode in the “Reinforced Concrete” postprocessor of SCAD
Task: Verify the correctness of the punching strength analysis of a concrete element under a concentrated force and a bending moment when the load application area is near the edge of the slab.
Compliance with the codes: SNiP 52-101-2003, SP 63.13330.2012.
Initial data file:
SCAD 41 SP-2003.spr, SCAD 41 SP-2012.spr
report – SCAD 41 SP-2003.doc
report – SCAD 41 SP-2012.doc

Initial data:

h = 230 mm	Slab thickness
h_0 = 200 mm	Average effective height of the slab
$a \times b$ = 500×400 mm	Column section sizes
F = 150 kN	Load transferred from the floor slab to the column
M_{sup} = 80 kN·m	Moment in the column section on the upper face of the slab
M_{inf} = 90 kN·m	Moment in the column section on the lower face of the slab
x_0 = 500 mm	Distance from the center of the column section to the free edge of the slab
Concrete class	B25

Analytical solution:

In this case it is necessary to check the strength of three contours of the design cross-section:

contour No.1 – closed contour around the column section at a distance of $0,5h_0$ from the column contour;

contour No.2 – open contour around the column section at a distance of $0,5h_0$ from the column contour with the extension of the contour to the free edge of the slab;

contour No.3 – open contour around the column section at a distance of $1,5h_0$ from the column contour (contour of the verification analysis without the consideration of the reinforcement).

1. Closed contour No.1:

$L_x = A_x + h_0$ = 500 + 200 = 700 mm = 0,7 m,

$L_y = A_y + h_0$ = 400 + 200 = 600 mm = 0,6 m,

Perimeter of the design contour of the cross-section:

$u = 2(L_x + L_y) = 2$ (0,7 + 0,6) = 2,6 m.

Area of the design contour of the cross-section:

$A_b = uh_0 =$ 2,6 x 0,2 = 0,52 m^2.

Ultimate force resisted by concrete:

$F_{b,ult} = R_{bt}A_b =$ 1,05 x10^3 x 0,52 = 546 kN.

Moment of inertia of the design contour with respect to the X axis passing through its center of gravity:

$$I_{bx} = 2\frac{L_y^3}{12} + 2L_x\left(\frac{L_y}{2}\right)^2 = 2\frac{0,6^3}{12} + 2\cdot 0,7\left(\frac{0,6}{2}\right)^2 = 0{,}162 \text{ m}^3.$$

Section modulus of the design contour of concrete

$$W_{bx} = \frac{I_{bx}}{y_{\max}} == \frac{0,162}{0,3} = 0{,}54 \text{ m}^2.$$

Moment of inertia of the design contour with respect to the Y axis passing through its center of gravity:

$$I_{by} = 2\frac{L_x^3}{12} + 2\cdot L_y\left(\frac{L_x}{2}\right)^2 = 2\frac{0,7^3}{12} + 2\cdot 0,6\left(\frac{0,7}{2}\right)^2 = 0{,}204 \text{ m}^3.$$

Section modulus of the design contour of concrete

$$W_{by} = \frac{I_{by}}{x_{\max}} == \frac{0,204}{0,35} = 0{,}583 \text{ m}^2.$$

Bending moment which can be resisted by concrete in the design cross-section:

$M_{bx,ult} = R_{bt}W_{bx}h_0 =$ 1,05 x10^3 x 0,54 x 0,2 = 113,4 kNm.

$M_{by,ult} = R_{bt}W_{by}h_0 =$ 1,05 x10^3 x 0,583 x 0,2 = 122,4 kNm.

For SNiP 52-101-2003:

$$\frac{M_x}{M_{bx,ult}} \le \frac{F}{F_{b,ult}}; \qquad \frac{M_y}{M_{by,ult}} \le \frac{F}{F_{b,ult}}$$

$\frac{M_y}{M_{by,ult}} = \frac{85}{122,4} = 0,694 \le \frac{F}{F_{b,ult}} = \frac{150}{546} = 0,275$ – condition is not met.

Assume

$\frac{M_y}{M_{by,ult}} = \frac{F}{F_{b,ult}} = 0,275$

Punching strength of the slab:

$$K1 = \left[\frac{F}{F_{b,ult}} + \frac{M_x}{M_{bx,ult}} + \frac{M_y}{M_{by,ult}}\right] \le 1,0$$

K1 = 0,275+0+0,275 = 0,55

For SP 63.13330.2012:

$$\frac{M_x}{M_{bx,ult}} + \frac{M_y}{M_{by,ult}} \le 0,5\frac{F}{F_{b,ult}}$$

$\frac{M_y}{M_{by,ult}} = \frac{85}{122,4} = 0,694 \le 0,5\frac{F}{F_{b,ult}} = \frac{150}{546} = 0,5 \cdot 0,275 = 0,1375$ – condition is not met.

Assume

$\frac{M_y}{M_{by,ult}} = \frac{F}{F_{b,ult}} = 0,1375$

Punching strength of the slab:

$$K1 = \left[\frac{F}{F_{b,ult}} + \frac{M_x}{M_{bx,ult}} + \frac{M_y}{M_{by,ult}}\right] \le 1,0$$

K1 = 0,275+0+0,1375 = 0,413

Open contour No.2:

$L_x = A_x + h_0 + 150$ = 500 + 200 + 150 = 850 mm = 0,85 m,

$L_y = A_y + h_0$ = 400 + 200 = 600 mm = 0,6 m,

Perimeter of the design contour of the cross-section:

$u = 2L_x + L_y$ = 2x0,85 + 0,6 = 2,3 m.

Area of the design contour of the cross-section:

$A_b = uh_0 =$ 2,3 x 0,2 = 0,46 m^2.

X coordinate of the center of gravity of the open contour with respect to the left edge of the slab:

$$X = \frac{425 \cdot 850 \cdot 2 + 850 \cdot 600}{850 \cdot 2 + 600} = 535{,}869 \text{ mm}$$

Ultimate force resisted by concrete:

$F_{b,ult} = R_{bt} A_b =$ 1,05 x10^3 x 0,46 = 483 kN.

Moment of inertia of the design contour with respect to the X axis passing through its center of gravity:

$$I_{bx} = \frac{L_y^3}{12} + 2L_x\left(\frac{L_y}{2}\right)^2 = \frac{0,6^3}{12} + 2 \cdot 0,85\left(\frac{0,6}{2}\right)^2 = 0{,}171 \text{ m}^3.$$

Section modulus of the design contour of concrete

$$W_{bx} = \frac{I_{bx}}{y_{max}} = \frac{0{,}171}{0,3} = 0{,}57 \text{ m}^2.$$

Moment of inertia of the design contour with respect to the Y axis passing through its center of gravity:

$$I_{by} = 2\frac{L_x^3}{12} + 2L_x(0,075 + 0,035869)^2 + L_y\left(0,35 - 0,035869\right)^2 = 2\frac{0,85^3}{12} + 2 \cdot 0,85(0,075 + 0,035869)^2 + 0,6\left(0,35 - 0,035869\right)^2$$

=0,183 m^3.

Section modulus of the design contour of concrete

$$W_{by} = \frac{I_{by}}{x_{max}} = \frac{0{,}183}{0,535869} = 0{,}341 \text{m}^2.$$

Bending moment which can be resisted by concrete in the design cross-section:

$M_{bx,ult} = R_{bt} W_{bx} h_0 =$ 1,05 x10^3 x 0,57 x 0,2 = 119,7 kNm.

$M_{by,ult} = R_{bt} W_{by} h_0 =$ 1,05 x10^3 x 0,341 x 0,2 = 71,6 kNm.

$M_y = M_y - Fe_0 =$ 85 – 150x0,035869 = 85 – 5,38 = 79,62 kNm.

For SNiP 52-101-2003:

$$\frac{M_x}{M_{bx,ult}} \le \frac{F}{F_{b,ult}}; \qquad \frac{M_y}{M_{by,ult}} \le \frac{F}{F_{b,ult}}$$

$\frac{M_y}{M_{by,ult}} = \frac{79,62}{71,6} = 1,112 \le \frac{F}{F_{b,ult}} = \frac{150}{483} = 0,311$ – condition is not met.

Assume

$\frac{M_y}{M_{by,ult}} = \frac{F}{F_{b,ult}} = 0,311$

Punching strength of the slab:

$$K1 = \left[\frac{F}{F_{b,ult}} + \frac{M_x}{M_{bx,ult}} + \frac{M_y}{M_{by,ult}}\right] \le 1,0$$

К1 = 0,311+0+0,311 = 0,622

For SP 63.13330.2012:

$$\frac{M_x}{M_{bx,ult}} + \frac{M_y}{M_{by,ult}} \le 0,5\frac{F}{F_{b,ult}}$$

$\frac{M_y}{M_{by,ult}} = \frac{79,62}{71,6} = 1,112 \le 0,5\frac{F}{F_{b,ult}} = \frac{150}{483} = 0,5 \cdot 0,311 = 0,155$ – condition is not met.

Assume

$\frac{M_y}{M_{by,ult}} = \frac{F}{F_{b,ult}} = 0,155$

Punching strength of the slab:

$$K1 = \left[\frac{F}{F_{b,ult}} + \frac{M_x}{M_{bx,ult}} + \frac{M_y}{M_{by,ult}}\right] \le 1,0$$

К1 = 0,311+0+0,155 = 0,466

Open contour No.3:

$L_x = A_x + 1,5h_0 + 250$ = 500 +1,5x200 + 250 = 1050 mm = 1,05 m,

$L_y = A_y + 2 \cdot 1,5h_0$ = 400 + 2x1,5x200 = 1000 mm = 1,0 m,

Perimeter of the design contour of the cross-section:

$u = 2L_x + L_y$ = 2x1,05 + 1,0 = 3,1 m.

Area of the design contour of the cross-section:

$A_b = uh_0 =$ 3,1 x 0,2 = 0,62 m^2.

X coordinate of the center of gravity of the open contour with respect to the left edge of the slab:

$$X = \frac{525 \cdot 1050 \cdot 2 + 1050 \cdot 1000}{1050 \cdot 2 + 1000} = 694,355 \text{ mm}$$

Ultimate force resisted by concrete:

$F_{b,ult} = R_{bt} A_b =$ 1,05 x10^3 x 0,62 = 651 kN.

Moment of inertia of the design contour with respect to the X axis passing through its center of gravity:

$$I_{bx} = \frac{L_y^3}{12} + 2L_x \left(\frac{L_y}{2}\right)^2 = \frac{1,05^3}{12} + 2 \cdot 1,05 \left(\frac{1,0}{2}\right)^2 = 0,608 \text{ m}^3.$$

Section modulus of the design contour of concrete

$$W_{bx} = \frac{I_{bx}}{y_{\max}} = \frac{0,608}{0,5} = 1,217 \text{ m}^2.$$

Moment of inertia of the design contour with respect to the Y axis passing through its center of gravity:

$$I_{by} = 2\frac{L_x^3}{12} + 2L_x(0,194355 - 0,025)^2 + L_y\left(1,05 - 0,694355\right)^2 = 2\frac{1,05^3}{12} + 2 \cdot 1,05(0,194355 - 0,025)^2 + 1,0\left(1,05 - 0,694355\right)^2 = 0,38 \text{ m}^3.$$

Section modulus of the design contour of concrete

$$W_{by} = \frac{I_{by}}{x_{\max}} = \frac{0,38}{0,694355} = 0,547 \text{ m}^2.$$

Bending moment which can be resisted by concrete in the design cross-section:

$M_{bx,ult} = R_{bt} W_{bx} h_0 =$ 1,05 x10^3 x 1,217 x 0,2 = 255,57 kNm.

$M_{by,ult} = R_{bt} W_{by} h_0 =$ 1,05 x10^3 x 0,547 x 0,2 = 114,87 kNm.

$M_y = M_y - Fe_0 =$ 85 – 150x0,194355 = 85 – 29,15 = 55,85 kNm.

For SNiP 52-101-2003:

$$\frac{M_x}{M_{bx,ult}} \le \frac{F}{F_{b,ult}}; \qquad \frac{M_y}{M_{by,ult}} \le \frac{F}{F_{b,ult}}$$

$\frac{M_y}{M_{by,ult}} = \frac{55,85}{114,87} = 0,486 \le \frac{F}{F_{b,ult}} = \frac{150}{651} = 0,23$ – condition is not met.

Assume

$\frac{M_y}{M_{by,ult}} = \frac{F}{F_{b,ult}} = 0,23$

Punching strength of the slab:

$$K1 = \left[\frac{F}{F_{b,ult}} + \frac{M_x}{M_{bx,ult}} + \frac{M_y}{M_{by,ult}}\right] \le 1,0$$

K1 = 0,23+0+0,23 = 0,46

For SP 63.13330.2012:

$$\frac{M_x}{M_{bx,ult}} + \frac{M_y}{M_{by,ult}} \le 0,5\frac{F}{F_{b,ult}}$$

$\frac{M_y}{M_{by,ult}} = \frac{55,85}{114,87} = 0,486 \le 0,5\frac{F}{F_{b,ult}} = \frac{150}{651} = 0,5 \cdot 0,23 = 0,115$ – condition is not met.

Assume

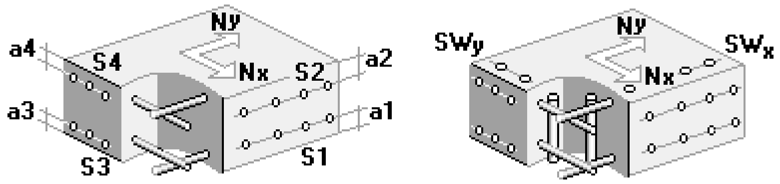
$\frac{M_y}{M_{by,ult}} = \frac{F}{F_{b,ult}} = 0,155$

Punching strength of the slab:

$$K1 = \left[\frac{F}{F_{b,ult}} + \frac{M_x}{M_{bx,ult}} + \frac{M_y}{M_{by,ult}}\right] \le 1,0$$

K1 = 0,23+0+0,115 = 0,345

Results of the SCAD analysis:



Node No. 5

Importance factor γ_n = 1

Concrete
Concrete type: Heavy-weight
Concrete class: B25

Service factor for concrete		
γ_{b1}	allowance for the sustained loads	1
γ_{b2}	allowance for the failure behavior	1
γ_{b3}	allowance for the vertical position during concreting	1
γ_{b4}	allowance for the freezing/thawing and negative temperatures	1

Distance to the c.o.g. of reinforcement			
a_1	a_2	a_3	a_4
mm	**mm**	**mm**	**mm**
30	30	0	0

Results

Design case – edge column

Length of the upper base of the bearing pyramid - 1800 mm
Length of the lower base of the bearing pyramid - 2300 mm

Comparison of solutions (according to SNiP 52-101-2003)

Checked according to SNiP	Check	Utilization factor
Sec.6.2.49	Strength without the consideration of the reinforcement	0,62

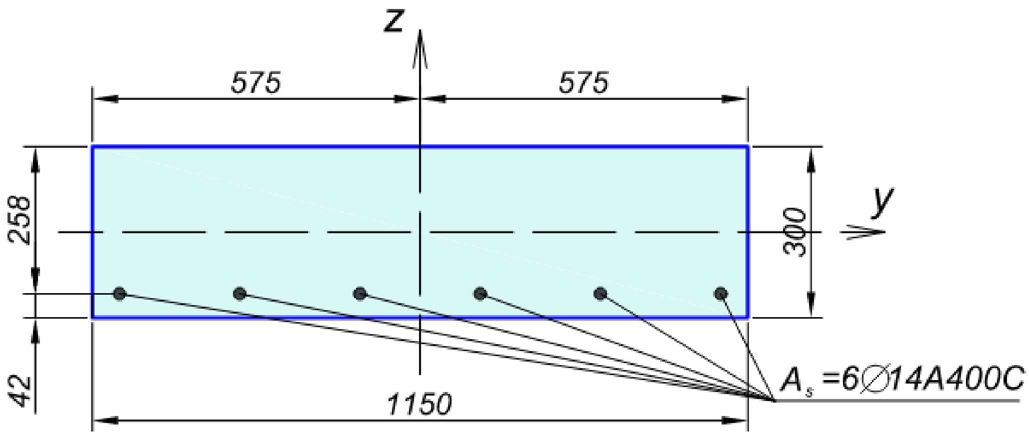
Check	punching strength of an unclosed concrete element under a concentrated force and bending moments (including additional ones caused by the eccentric application of a force with respect to the punched contour) with their vectors along X,Y-axes (load application area is near the edge of the slab)
Analytical solution	0,622
SCAD	0,62
Deviation, %	0,1 %

Comparison of solutions (according to SP 63.13330.2012)

Checked according to SP	Check	Utilization factor
Sec.8.1.49	Strength without the consideration of the reinforcement	0,47

Check	punching strength of an unclosed concrete element under a concentrated force and bending moments (including additional ones caused by the eccentric application of a force with respect to the punched contour) with their vectors along X,Y-axes (load application area is near the edge of the slab)
Analytical solution	0,466
SCAD	0,47
Deviation, %	0,1 %

Analysis of a Reinforced Concrete Foundation Slab for Normal Crack Opening



Objective: Check the calculation of the crack opening width in the “Reinforced Concrete” postprocessor of SCAD

Task: Verify the correctness of the analysis of normal crack opening.

References:

1. Guide on designing of concrete and reinforced concrete structures made of heavy-weight concrete (no prestressing) (to SP 52-101-2003), 2005, p. 155-157.
2. M.A. Perelmuter, K.V. Popok, L.N. Skoruk, *Calculation of the Normal Crack Opening Width for SP 63.13330.2012*, Concrete and. Reinforced Concrete, 2014, №1, p.21-22.

Initial data file:

SCAD 43 SP.spr

report – SCAD 43 SP-2003.doc

Compliance with the codes: SP 52-101-2003.

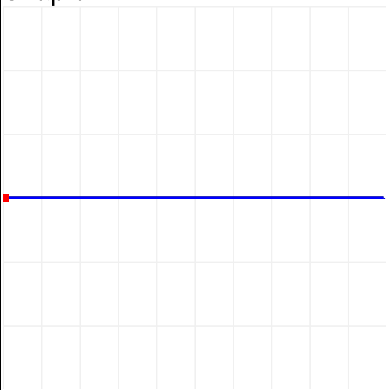
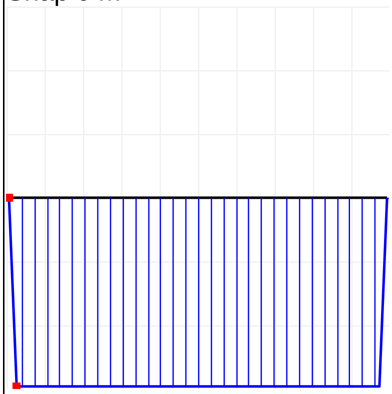
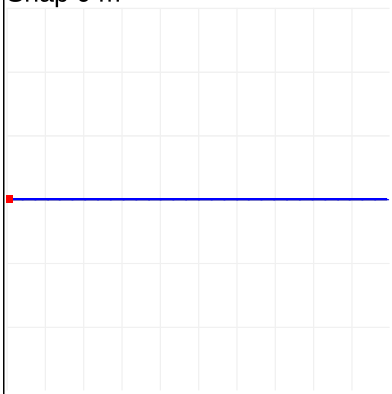
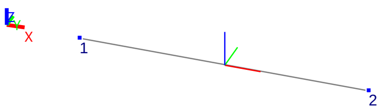
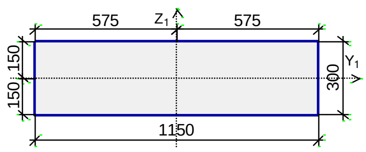
Initial data:

$b \times h$ = 1150×300 mm	Slab section sizes
a = 42 mm	Distance to the c.o.g. of tensile reinforcement
A_{sw} = 923 mm^2 (6∅14)	Cross-sectional area of tensile reinforcement
M_l = 50 kNm	Moment in the design section from permanent and long-term loads
M_{sh} = 10 kNm	Moment from short-term loads

Concrete class B15

Class of reinforcement A400

Results of the SCAD analysis:

N Max. 0 T Snap 0 m  Max. 0 T Snap 0 m	**M_y** Max. 0 kN*m Snap 0 m  Max. 60 kN*m Snap 0,02 m	**M_z**
M_k	**Q_z** Max. 0 T Snap 0 m  Max. 0 T Snap 0 m	**Q_y**
	Length of the bar 1 m Length of the flexible part 1 m Loading L1 - "Moment"	

Structural group Beam

Importance factor γ_n = 1

Member type - Flexural

Stress state - Uniaxial bending

Maximum percentage of reinforcement 10

Coefficients allowing for seismic action	
Normal sections	0
Oblique sections	0

Distance to the c.o.g. of reinforcement	
a_1	a_2
mm	mm
42	42

Reinforcement	Class	Service factor
Longitudinal	A400	1
Transverse	A240	1

Concrete
Concrete type: Heavy-weight
Concrete class: B15

Service factor for concrete		
γ_{b1}	allowance for the sustained loads	1
γ_{b2}	allowance for the failure behavior	1
γ_{b3}	allowance for the vertical position during concreting	1
γ_{b4}	allowance for the freezing/thawing and negative temperatures	1

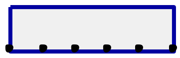
Humidity of environmental air - 40-75%

Crack resistance
Limited crack opening width
Requirements to crack opening width are based on the preservation of reinforcement
Allowable crack opening width:
  Short-term opening 0,4 mm
  Long-term opening 0,3 mm

Structural group Beam. Element No. 1

Member length 1 m

Specified reinforcement

Segment	Reinforcement	Section
1	S_1 - 6∅14	

Results			
Segment	**Utilization factor**	**Check**	**Checked according to SNiP**
1	0,97	crack opening width (long-term)	Sec. 7.2.3, 7.2.4, 7.2.12

Comparison of solutions

Check	crack opening width (long-term)
Guide	0,306/0,3 = 1,02
SCAD	0,97
Deviation, %	4,9 %

Comments:

1. The value of the total moment acting in the section, $M = M_l + M_{sh} = 50 + 10 = 60$ kN·m, factor for sustained load is equal to $Ml\ /M = 50/60 = 0{,}833$.
2. The deviation of the results of SCAD from the theoretical solution is due to the fact that in order to provide computational stability, diagrams in which the horizontal part of the graph σ(ε) has a small slope are used in SCAD instead of the perfect diagrams of the material behavior.