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# THE PROBLEM OF INTERPRETATIONS OF THE STABILITY ANALYSIS RESULTS

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**Abstract.** *Analysis of equilibrium stability consists usually of determining the value of a critical parameter of load intensity and a buckling mode. The role of particular elements and subsystems in the system's instability is not commonly revealed which makes the analysis poorer and does not allow making correct conclusions how to reinforce the structure or make it lighter. This report suggests a simple way to do an analysis of this type which can be easily implemented within a structural analysis software.*

## 1 Introduction

When we analyze peculiarities of possible phenomena of instability (buckling), we use the concepts of restrained and forced state of particular parts of the system during the system's loss of stability [1]. It is assumed that a particular part of the system while buckling is in the restrained state if it experiences buckling under a less intensive load transmitted to this part should the part be separated from the rest of the system. If this "isolated" buckling analysis of the separated part requires a larger critical load intensity, or the subsystem does not lose its stability at all, then we talk about a forced buckling of this part of the system. It is natural that these concepts affect the strategy of the system's reinforcement, as soon as the general stability coefficient is found insufficient. Usually, in this case one starts reinforcement from that part of the system which is in restrained state, though one should remember that this measure is efficient only to a certain degree characterized by the reinforced subsystem acquiring the same "power" as the rest of the structure. To achieve further reinforcement efficiency, all compressed elements should be reinforced simultaneously, maintaining the equally stable state of the system's elements that has been achieved.

The concepts of restrained and forced states concerning separate fragments of a structure, which were introduced into engineering practice about half a century ago, enable us to build a purposeful strategy of reinforcement of a structure whose load-carrying ability is determined by its stability conditions. Meanwhile, no formal computation tool especially needed in computer-aided design has been developed until now, as far as the authors are aware.

## 2 Estimates of the influence of particular subsystems

In this connection, we think it useful to classify particular parts of the system (down to its particular elements) into one of two following classes: a class of restraining and a class of pushing elements (or parts) of the system. Restraining elements help maintain the system's equilibrium stability while the role of pushing elements is negative because they force (push) the mechanical system to lose its stability. The algorithm for formal classifying an element into one of the two classes is surprisingly simple and obvious.

Actually, the role of particular subsystems can be verified by calculating the value of energy accumulated in different parts of the system during its deformation into a buckling mode. This energy for the whole system is zero [1, p. 36]:

$$\mathsf{E} = 0{,}5\mathbf{u}^{\mathsf{T}}[\mathbf{K}_0 + \mathbf{K}_1(k)]\mathbf{u} = 0, \tag{1}$$

where $\mathbf{K}_0$ is a usual stiffness matrix and $\mathbf{K}_1$ is a geometrical stiffness matrix.

If the system is conventionally divided into two parts of any kind, then stiffness matrices can be composed for each of them $K_{o,i} + K_{1,i}(k)$ (i = 1,2) which will be assumed filled with zeros up to the full size of the system's stiffness matrix. When calculating the values of energy accumulated by each of the two separate parts of the system:

$$\mathsf{E}_1 = 0{,}5\mathbf{u}^T[\mathbf{K}_{0,1} + \mathbf{K}_{1,1}(k)]\mathbf{u}, \tag{2}$$

$$\mathsf{E}_2 = 0{,}5\mathbf{u}^T[\mathbf{K}_{0,2} + \mathbf{K}_{1,2}(k)]\mathbf{u}\ , \tag{3}$$

we will always obtain values of opposite signs because their sum should be equal to zero. This will determine which subsystem (corresponding to $\mathsf{E}_i < 0$) should be made stiffer or additionally constrained. Comparing these criteria with the concepts introduced above, we can easily guess that the i-th subsystem with $\mathsf{E}_i < 0$ is a pushing one while a subsystem with $\mathsf{E}_i > 0$ is a restraining one.

It is also important that the pushing elements of the system can be ordered by the degree of their "being guilty" in the buckling of the system. If the system is conventionally divided into several, say m, separate parts and a value of energy $\mathsf{E}_i$ (i = 1,…,m) is calculated for each part, then the less (algebraically) the $\mathsf{E}_i$ value the higher the responsibility of the corresponding i-th part of the system in its buckling. In other words, contribution of each element of the system into its common energy balance can serve as a convenient measurement of this responsibility.

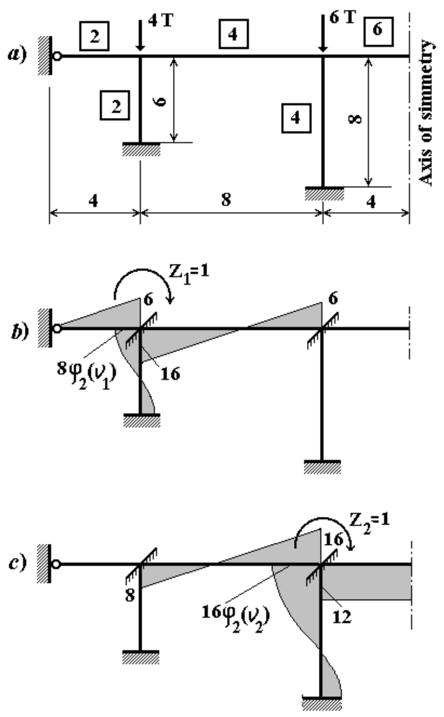


*Fig. 1. Scheme for the analysis example:*
*a — mechanical model; b, c — diagrams of moments in unit states*

## 3 Numerical example

Let us illustrate the above with a numerical example from [1, p.250]. The design of a structure of interest is shown in Fig.1 (to the axis of symmetry) where boxes enclose values of linear rigidities $i = EI / l$, while the function

$$\varphi_2(\nu)=\frac{\nu(tg-\nu)}{8tg\nu\left(tg\frac{\nu}{2}-\frac{\nu}{2}\right)} \tag{4}$$

depends on the argument ν determined by the compression force in the member, and in our case

$$\nu_1=\sqrt{\frac{N_1 l_1}{i_1}}=\sqrt{\frac{4k\times 6}{2}}=\sqrt{12k}\ ,\quad \nu_2=\sqrt{\frac{6k\times 8}{4}}=\sqrt{12k} \tag{5}$$

Critical value of load intensity is determined by the value of $k = 2.51$, and the buckling mode $\mathbf{u}= \|[Z_1,\ Z_2]\|^{T}$ is determined by values of slopes $Z_1 = 1$ and $Z_2 = -1{,}366$. Values of dimensionless parameters $\nu_1$ and $\nu_2$ that correspond to the critical state of the system appear to be equal and are determined as

$$\nu_1 = \nu_2 = \sqrt{12k} = 5{,}488\ .$$

Let us divide the system into three parts, the first one being the left column, the second one the right column, and the third part the frame's crossbar. The full stiffness matrix **K** of the system looks like

$$\left\|\begin{bmatrix} 22 + 8\varphi_2(\nu_1) & 8 \\ 8 & 28 + 16\varphi_2(\nu_2) \end{bmatrix}\right\|$$

and it can be represented by a sum of three matrices:

$$\left\|\begin{bmatrix} 8\varphi_2(\nu_1) & 0 \\ 0 & 0 \end{bmatrix}\right\| + \left\|\begin{bmatrix} 0 & 0 \\ 0 & 16\varphi_2(\nu_2) \end{bmatrix}\right\| + \left\|\begin{bmatrix} 22 & 8 \\ 8 & 28 \end{bmatrix}\right\|$$

which correspond to each of the three selected parts of the system. Now it is easy to calculate $\varphi_2(\nu_1) = \varphi_2(\nu_2) = -1.384$, and therefore

$$E_1 = 0{,}5\times Z_1^2\times 8\varphi_2(\nu_1) = -5{,}536;$$

$$E_2 = 0{,}5\times Z_2^2\times 16\varphi_2(\nu_2) = -20{,}660;$$

$$E_3 = 0{,}5\times(22\times Z_1^2 + 28\times Z_2^2 + 2\times 8\times Z_1\times Z_2) = 26{,}196\ .$$

The balance of energy obtained via these matrices holds: $E_1 + E_2 + E_3 = 0$, exactly as it should be, and comparison between energy contributions of each selected part into the total energy shows that the "bottleneck" of the system is the right column ($E_2 < E_1 < E_3$). This result is completely the same as that from [1], but the algorithm of this analysis is substantially simpler.

## 4 Software implementation

As the calculation of expressions like (2) after obtaining the critical value of parameter k and its appropriate buckling mode is a pretty simple operation, it seems very useful and promising to add a function for determining $E_i$ of any user-specified subset of a structure's elements to a structural analysis software.

The software should be able to divide a structure into any number of parts, down to separate elements, then analyze the contribution of each element into the negative and positive parts of the total energy. This gives the basis for recommendations concerning reinforcement/weakening of particular elements and for determining the role of particular elements in the general process of buckling.

The value of *energy coefficient of stiffness reduction* can be additional useful information calculated for each element of the system as

$$\eta_i = \frac{E_i(N)}{E_i(0)}, \tag{6}$$

where $E_i(N)$ denotes energy accumulated in i–th element and calculated using the stiffness matrix $\mathbf{K}(k)$, while $E_i(0)$ denotes energy of the same element under a zero longitudinal force when its full stiffness matrix is $\mathbf{K}_o$.

It can be easily seen that

$$\eta_i = 1 + \frac{\mathbf{u}_i^T \mathbf{K}_{1,i} \mathbf{u}_i}{\mathbf{u}_i^T \mathbf{K}_{0,i} \mathbf{u}_i}, \tag{7}$$

where $\mathbf{u}_i$ is a subset of components of the vector $\mathbf{u}$ belonging to nodes of the i-th element being analyzed, $\mathbf{K}_{0,i}$ and $\mathbf{K}_{1,i}$ are matrices of usual and geometric stiffness of the same element, accordingly. Apparently, the coefficient $\eta_i$ characterizes the reduction of stiffness of the element in integral sense, caused by presence of compression stresses in the element. The smaller (algebraically) the coefficient $\eta_i$, the more substantial the effect of compression stresses upon the stiffness reduction in the i-th element. Besides, the sign of coefficient $\eta_i$ allows us to classify the appropriate element of the system into one of the two classes: the class of pushing elements (when $\eta_i < 0$) or the class of restraining elements (when $\eta_i > 0$) because the denominator in the formula (6) is always positive.

Certainly, a more accurate analysis of how to reasonably reinforce a structure against buckling should be based on calculation and comparison of sensitivity coefficients of critical value of the load parameter to changes of particular design properties of the system. This analysis can be done as it is recommended in, say, a well-known monograph [2]. Though, a highly laborious computational process needed for determining sensitivity coefficients in buckling analysis problems persuades us to use the simpler algorithm described above, that is based on energy ordering of system elements. From computational viewpoint, it can be reduced to simple calculations by a formula like (7) and sorting of the members.

## 5 Conclusion remarks

It is worth noticing that one should be careful when using the results of the discussed analysis for making decisions on modification of elements or subsystems. The matter is that the calculation of energy is made using buckling mode **u*** which has been determined for the system with its original distribution of stiffnesses and stresses, and changing that distribution will necessarily change the mode. Therefore, the result of modification must be subject to verification. Besides, what is more important, the operation of reinforcing particular elements has a limit above which the operation does not affect the result anymore.

The above-discussed problem confirms this statement. Note that even an infinite increase of rigidity of one of the columns will not cause the same rise of the stability coefficient because the system will experience a “local buckling” without deformation of the over-stiffened column. Apparently, this feature of elastic stability problems was first noticed by I.G.Bubnov. He pointed out that there is some critical stiffness of elastic supports in a problem of stability of a beam those supports. Further increase of the supports’ stiffness does not lead to the increase of the critical force, while the supports with the critical and greater stiffness inherit the features of absolutely rigid supports [3].

## References

[1] A.F.Smirnov. *Stability and oscillations of structures*. Transzheldorizdat, Moscow: (1958) (*In Russian*).

[2] E.J.Haug, K.K.Choi and V.Komkov. *Design sensitivity analysis of structural system*. Academic Press, New York, (1986).

[3] S.P.Timoshenko. *Theory of elastic stability*. McGraw-Hill Book Company, New York, (1951).