Analysis of the Stressed State of Cylindrical Tanks with Defects of Geometrical Shape

Anatoly V. Perelmuter, Sergiy Yu. Fialko, Viktor S. Karpilovsky, Edward Z. Kryksunov*

SCAD Group, 13, Chokolovsky blrd., room 508, 03186, Kiev, Ukraine

e-mail: edk@erriu.ukrtel.net

Abstract

The subject of consideration is a cylindrical oil tank in which a part of its wall has been repaired/replaced and thus its geometrical shape distorted due to mismatched thickness of the wall and its patches. There is presented a comparison between results of a linear analysis and one that allows for geometrical non-linearity. The loading parameter for the step-by-step geometrically nonlinear analysis is assumed to be the level of fluid in the tank.

Keywords: finite element method, optimisation, evolutionary algorithms

1. Introduction

Known theoretical and experimental investigations of cylindrical shells having imperfect geometrical shapes regard, most frequently, constructions with the relative thickness $t/R = 0.001 \div 0.005$ (where t is the thickness; R is the radius) and relative geometrical deviations from the perfect shape Δ/t of some $0.05 \div 0.50$ (Δ is the deviation from the perfect shape). Those investigations were intended chiefly to estimate the effect of the geometrical imperfections on the shells' stability/buckling. For big cylindrical tanks there are also common values $t/R = 0{,}001 \div 0{,}0005$ and $\Delta/t = 5 \div \tilde{8}$. Deviations like these affect the general stressed state of an entire structure, so they must be taken into account even though the buckling may not be a critical issue. Particularly, there can be great local bending stresses that arise in areas of local shape deficiencies and initiate a low-cycle endurance fracture after 1000 to 5000 emptying/filling cycles of the tank (20 to 30 years).

E.O. Paton Institute for Electric Welding (Ukraine) has developed a new technology for repairing damaged tanks with their walls deviated from the designed shapes. This technology includes an insertion of sheets, and this causes distortions of geometrical shapes. Immediately there arises the problem of estimating the effect of these geometrical distortions on the stress/strain distribution in the entire structure [1]. Estimations of this kind have been performed for some real tanks that are capable of storing 20000 m^3 of oil or oil products.

The calculations were performed by the SCAD software system [2] in a geometrically nonlinear formulation. The results of the analysis are discussed below.

2. Finite-element model

As the structure is symmetrical, the analysis involved only a quarter of it. The area of consideration comprises about one half of the tank's height (levels I through VI). Along the lower boundary the stiff clamp conditions are assumed which give some reliability reserve. Along the top edge all linear displacements are allowed, while the slope of the guide is limited. These conditions are in approximate accordance with reality in the middle of the wall where an annular reinforcing rib is installed. At the vertical edges of the fragment, boundary conditions defined by the symmetry of the shape are stated.

The analysis uses a finite-element model consisting of rectangular and triangular shell elements. The sizes of the elements are 626.7 mm horizontally and 745 mm vertically in areas of the wall remote from the reinforced place. These sizes get quartered towards the reinforced area. Fig. 1 shows a view of the wall (here and further, actually an involute) together with its finite element division.

The width of the reinforced area in the annular direction is assumed to be 2500 mm. The thickness of the reinforcing sheets and elements of the wall are based on data presented in Table 1.

Table 1. Thickness of sheets

No. of the	Thickness, mm	
level	Wall	Reinforcement
I	16	16
II	15	16
III	14	16
IV	12	14
V	11	12
VI	10	12

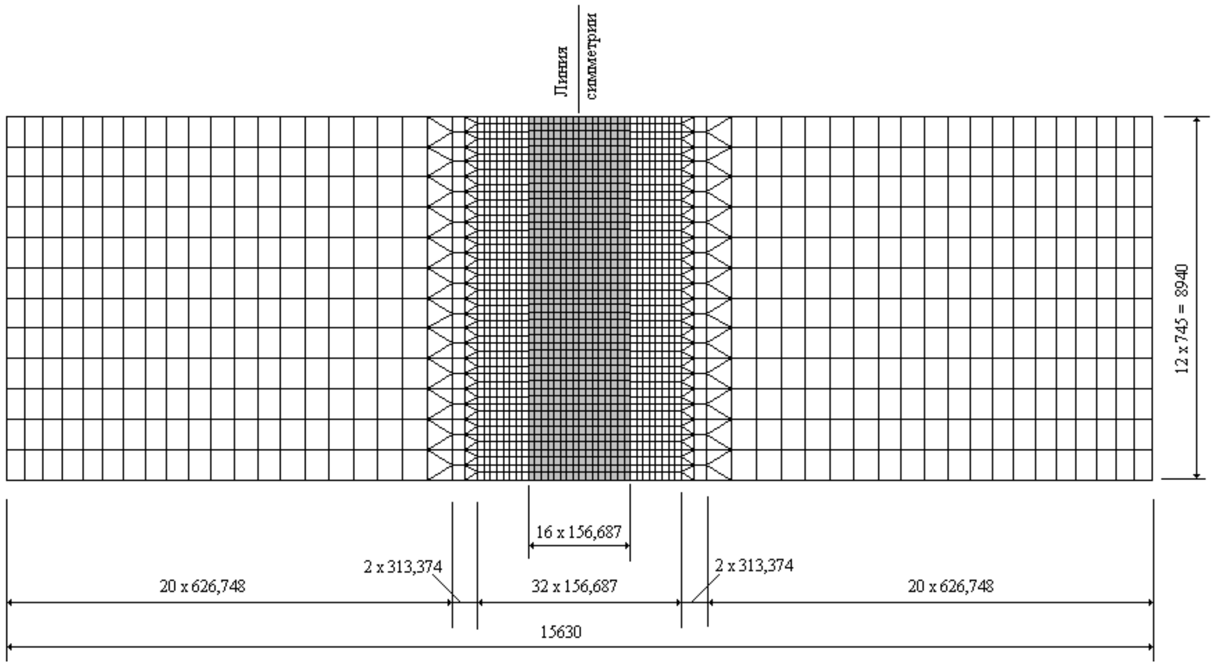

Figure 1: The finite element mesh evolved (the reinforced area is marked)

The analysis uses the assumption of an elastic behavior of the wall's material, and these usual physical constants are adopted:

elasticity modulus E = 2100000 кг/cm^2;

Poisson ratio μ = 0.3.

3. Geometrical shape

The object of consideration is a tank with distortions of its geometrical shape that conform to a real construction of the wall of a tank having the capacity of 20000 m^3 which was actually measured.

The distorted shape of the tank is defined by maximum deviations measured at points in the middle of the reinforcing sheets at the level of horizontal joints. Results of the measurements are presented in Table 2.

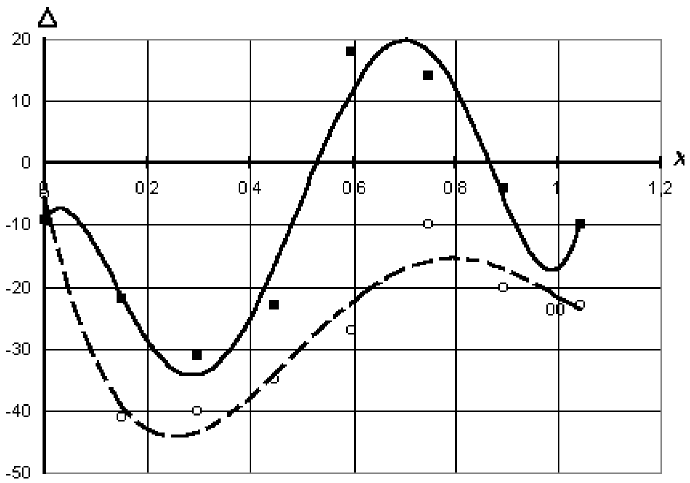

Figure 2: Deviations measured and smoothed

Table 2. Geometrical distortions of the shape

Level, mm	Deviations, mm	
	Area 1	Area 2
0	-5	-9
1490	-41	-22
2980	-40	-31
4470	-35	-23
5960	-27	18
7450	-10	14
8940	-20	-4
10430	-23	-10
11920	-31	22
13410	-33	-5
14900	-18	-9
16390	12	-15
17880	2	-4

Seeing that the measurement caused certain errors to appear, it was decided to smooth them by solving the best root-mean-square approximation problem with the data of Table 2. There were obtained the following relationships between the deviation and the relative height of the checkpoint above the tank's bottom.

For area 1: $\Delta(x) = 4185{,}36x^5 - 10507{,}9x4 + 8657{,}32x3 - -2471{,}16x^2 + 128{,}509x - 9{,}11538$.

For area 2: $\Delta(x) = 91{,}6502x^5 + 124{,}144x^4 - 895{,}411x^3 + +1019{,}66x^2 - 356{,}115x - 5{,}465035$.

Here x is the level mark of the point in mm divided by 10000.

Fig. 2 presents plots of these equations together with the measurement results. Here circles and the dash line correspond to the area 1 while the squares and the solid line to the area 2.

It was assumed that the actual surface deviated from the perfect cylinder with the radius 19950 mm could be described by this equation in the cylindrical coordinate system:

$$\rho = 19950 + \Delta(x)\sin^2[\pi s/L],$$

where s is the arc coordinate, L = 2500 mm is the arc length of the distortion area. In other words, the deviations are represented as a product of a function of the height by a function of the arc length.

Fig. 3 presents a scheme of the same surface in the area 2 distorted along the radial coordinate pretty much.

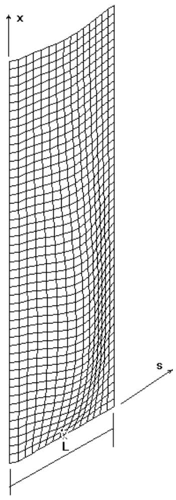

Figure 3: A schematic view of the surface in the distorted areas:
(*a*) photo; (*b*) an evolvent of the distorted surface

4. Effect of mismatched thickness

Because the thickness of the wall and that of the reinforcing sheets are different (see Table 1), the axisymmetric shape of deformation has to be distorted. In order to have an idea of the respective distortion of the stress/strain distribution in the wall, we performed an auxiliary analysis of the tank with its wall of the assumed (designed) cylindrical shape but its thickness being the same as in the reinforced construction. The loading pattern was the same as in the cases of interest: 16 vertical meters of water filling the tank. The analysis of the results showed that the mismatched thickness introduced a distortion of about 20% into the annular stress, and in places where sheets of different thickness contacted there were a narrow strip with bending moments caused by the edge effect.

5. Effect of geometrical distortions

In order to estimate the geometrical distortion, first we performed a geometrically linear analysis. It appeared that the displacements were too great (Fig.4a). The annular stress in the wall (Fig.4b) was distributed pretty smoothly over the surface of the wall, while noticeable deviations could be seen only along a narrow strip near the thickness mismatch between 1st and 2nd levels and in the vicinity of the geometrical distortions.

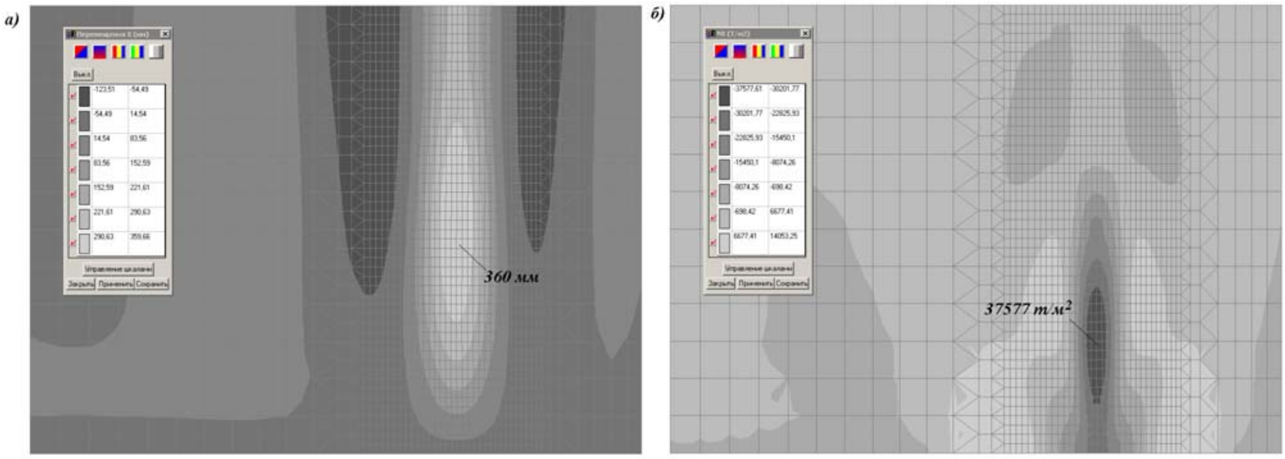

Figure 4:. Results of the linear analysis in the area 1: (a) displacements; (b) annular stresses

Big displacements evidence that the analysis must be performed in the geometrically nonlinear formulation. Such analysis was performed, and Fig. 5 presents its results.

Taking into account the geometrical non-linearity decreased the displacements by an order of magnitude comparing to those yielded by the linear analysis, and these values correspond to those observed on the object. This effect of a "nonlinear reinforcement" is based on the action of the annular tension stress, therefore it appears more clearly at the bottom of the wall where the said stress is big. Also, one should be aware that the stress distribution changes less than the displacements do as the analyst changes from a linear formulation of his problem to a nonlinear one.

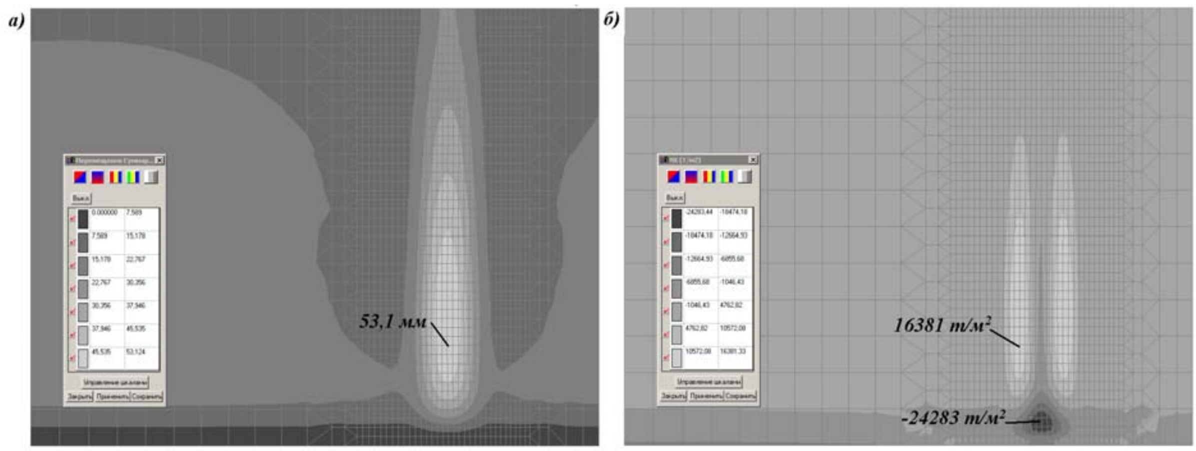

Figure 5: Results of a nonlinear analysis in the area 1: (a) displacements; (b) annular stresses

6. On the nonlinear computation

The nonlinear problem was solved stepwise with iteration refinements by Newton – Rufson method at each step. The accuracy check used the displacement vector's norm, the accuracy requirement was 10^{-4}, and the maximal allowed number of iterations per one step was 20. The linearized problem was solved by a multi-front solver [3] which reduced the computation effort five times or so comparing to Gaussian solvers.

The computation was performed with variable mesh density parameters and at different numbers of steps in order to evaluate the accuracy of the finite-element model and the behavior of the step-by-step process.

One of effective techniques for general estimation of the nonlinear solution involves a loading/unloading repeated cycle which was actually used in this analysis. When using this technique, first we increase the load intensity parameter λ from zero to 1, and then decrease it back from one to zero. We can estimate the accumulation of errors in a computational process by comparing the initial and final states of the system seeing that our mechanical model is conservative.

Notice that in the course of unloading the estimate of the nonlinear algebraic system solution accuracy

$$F(Z) = \lambda P, \tag{1}$$

usually constructed as a ratio of the divergence vector's norm to the load vector's norm

$$\varepsilon = \| F(Z) - F(Z^*)\| / \|\lambda P\| \tag{2}$$

becomes incorrect as $\lambda \to 0$. One should use the estimate (2) at $\lambda = 1$, i.e. divide the error by the norm of the full load rather than the load of a particular step.

For this problem, the step-by-step process was organized in two ways:

(a) as a process of increasing the external load, the parameter being the fluid's density ($\lambda_1 = \gamma$);

(b) as a process of increasing the fluid's level and the level being the load parameter ($\lambda_2 = h$).

Fig. 6 presents plots of displacements as they change in two characteristic points (with a noticeably nonlinear or almost linear behavior). It is natural that the final results are the same, but the behavior of the system in the course of its loading is essentially different in these two cases. Notice that the second way of using the load parameter sped up the process and enabled us to achieve the proper result in half of the number of steps.

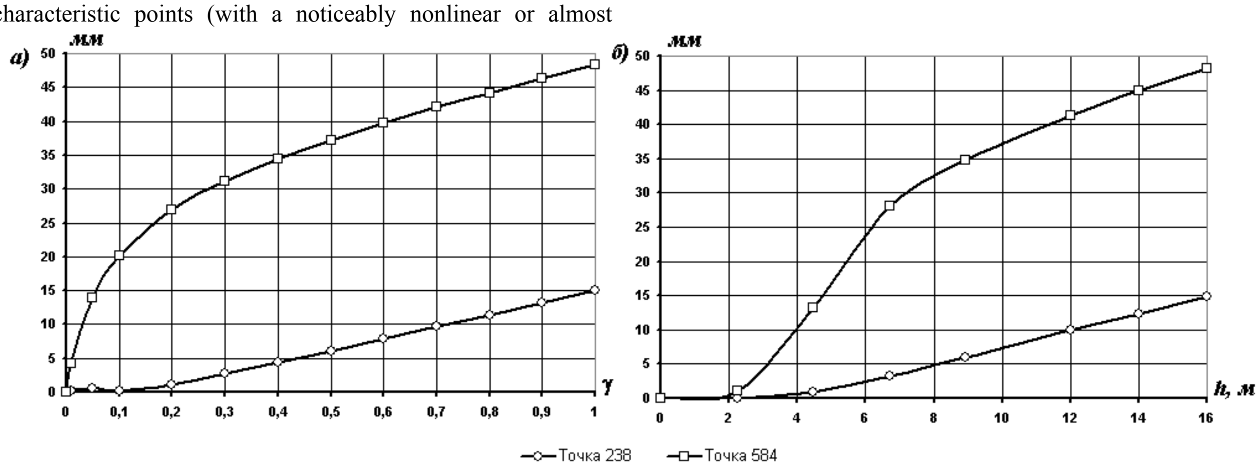

Figure 6: Plots of displacements: (a) parameter $\lambda_1 = \gamma$; (b) parameter $\lambda_2 = h$

Also, it turned out that some geometrical deviations related to deflections inward the tank behave differently in these two cases of the load variation. Some of them remain as the load's intensity is gradually increasing at a constant fluid level, but they disappear as the level starts increasing. Actually, it is the second method of the load variation that happens in reality, therefore we decided to use this one.

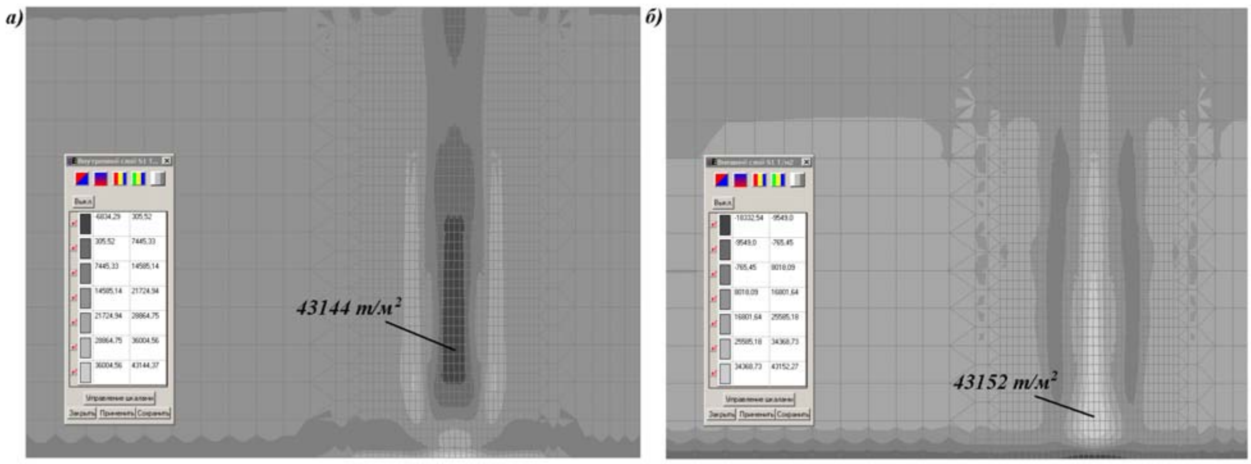

Figure 7: Equivalent stresses in the area 1: (a) in the interior layer; (b) in the exterior layer

Final results of the analysis include values of the equivalent stresses (we used the strain energy theory of failure) at the exterior and interior surfaces of the tank (Fig. 7). The analysis of these has shown that it is acceptable not to make extra efforts of repairing the geometrical shape faults and allow to fill the tank up to the designed level.

References

[1] Barvinko, Yu.P., Golinko, V.M., Barvinko, A.Yu., Perelmuter, A.V., Kuleba, G.V. Improvement of serviceability of vertical erection welded joints of cylindrical tank walls constructed from coiled sheets, *The Paton Welding Journal*, 2001, №7(580), pp. 27–32.

[2] Karpilovsky, V.S., Kryksunov, E.Z., Perelmuter, A.V., Perelmuter, M.A., Trofimchuk, A.N. *SCAD for users*.— Kiev: Compass Publ., 2000.— 332 p. (in Russian)

[3] Fialko, S.Yu., Karpilovsky, V.S., Kryksunov, E.Z. *Sparse direct multifrontal solver in SCAD software* (in these Proceedings)