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# TAKING INTO ACCOUNT THE INDETERMINACY OF PROPERTIES OF A DESIGN ANALYSIS MODEL

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**Abstract.** The necessity of taking into account the indeterminacy of properties of a design model to assess the serviceability of existent structures is pointed out. Most typical examples are given when this type of analysis is a must. The possibility of solving the problem about the most disadvantageous combination of values of undetermined parameters using the SCAD software is illustrated.

#### 1 Introduction

Since our student years, the accuracy of analysis is defined to be one of most important factors. Any deviations and inaccuracies are deemed undesirable. Therefore, our mind tries to reject them as if they did not exist at all, though everybody understands and acknowledges that there cannot be fault-free technologies, absolutely accurate measurements etc. Due to this fact, operating indeterminate parameters requires some changes in one's mind, and also the use of appropriate software tools.

Parameters of a design model that the analyst operates are almost always inaccurate values. Often, these parameters are, in essence, random values, and we actually work with some realization of this randomness when assuming a certain value of the parameter. Sometimes the accurate value of the determinate parameter is just unknown due to limitations of our knowledge (imagine how the analysis of the lunar excursion module was done in the "Apollo" program and what were the assumed properties of its environment).

Loads are among most variable elements of a design model. Even the dead weight of constructions, according to effectual standards, can be assumed to vary by about  $\pm 10 \div 15\%$ , and this is one of most stable loads. Other loads can vary within a much wider range both by intensity or the direction of application and by their position upon the structure. This very peculiarity of loading initiated the development of various techniques of searching for so-called "disadvantageous load", i.e. searching for such a combination of various components of external loads at which the extreme value of some internal force, stress in a section, or a nodal displacement etc. is achieved. From classical works on foundation of methods of search for most disadvantageous position of a group of mobile weights upon the influence line [1] to works based on the application of modern methods of multi-criteria optimization solutions [2], this problem has been attracting the close attention of professionals. The more surprising is the fact that the problem of taking into account other parameters of design models proves to be not really popular though its practical importance is very considerable.

In this paper, we investigate possibilities available in a software system based on ideas of the finite element analysis to assess the role of indeterminacies of design models. All issues are illustrated by examples obtained via the SCAD software [3].

# 2 Special mode of SCAD

The SCAD software suggests a special mode that enables one to process results of the analysis of several close modifications of a design model. The affinity of modifications is to be understood in this sense: they must be topologically similar, contain the same number of nodes and elements, and allow only strictly defined differences between the design models being compared:

• the usage of different types of the corresponding elements is allowed, including the use of the "empty" type elements that simulate the absence of elements without changing the total number thereof:

- the variation of stiffness properties of finite elements is allowed, including the use of zero values of some rigidities;
- the differences are allowed in the system of imposed constraints and/or specified conditions of junctions between elements and nodes (hinges embedded, infinitely stiff inserts installed).

The illustrative example of close (in the said sense) design models is given on Fig. 1 where numbers denote differences between the design (a) and the design (b).

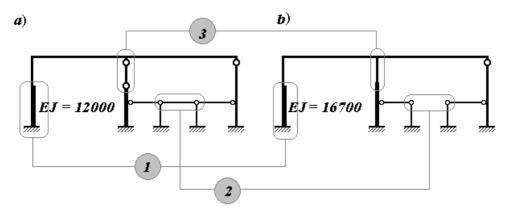


Figure 1: Close design models

The operation mode that is described accepts as source data only the list of files that contain results of solution of particular problems (Fig.2). The system does the logical analysis to make sure the defined affinity of the specified series of problems takes place, and if this validation yields the positive result, the results of analysis of all modifications of the design model are merged into a single array, as if a single design were analyzed under all options of loading that had been specified before in all previous analyses.

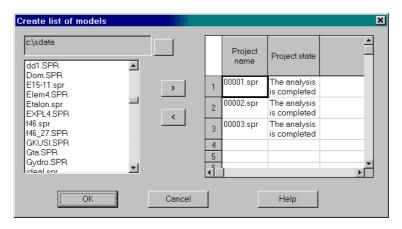


Figure 2: The dialog box of the mode

For the super-problem obtained in this way, one can run the modes of determination of design stress combinations (DSC), create design load combinations (DLC), choose the reinforcement

of ferroconcrete structures, verify or select steelwork sections. Also, one needs to specify the logical relationship between different choices of loading analyzed within each sub-task. For example, loads caused by the dead weight included in every sub-task should, most often, be marked as mutually exclusive to avoid their summation.

# 3 Typical situations of indeterminacy

Below, a list of examples illustrates typical situations of indeterminacy that have been taken from the actual analysis practice. Using these examples, we would like to demonstrate the result of taking into account the assumed deviation of parameters of a design model.

#### 3.1 Rigidity properties

Rigidity properties of elements of a design model can be indeterminate to a fairly large extent. This indeterminacy can occur due to a natural deviation of values of, for example, such a parameter as the elasticity modulus of concrete whose rated value is normally adopted in compliance with design codes, and the possible changeability of realizations, including variations within a single object and with time, is also normally not taken into account. This can be due to a pretty narrow band of variation of elasticity modulus, but a lot of cases give one weighty reasons to allow for changeability of rigidity properties.



Figure 3: The Chernobyl Nuclear Plant UKRITIYE (shelter).

These cases include the consideration of damages that are accumulated in a structure in the course of its operation. These damages must be allowed for in analyses that concern the assessment of the technical condition of existent structures and adopted after appropriate instrumental inspections. Though, many of elements of an existent structure can be inaccessible for inspection, therefore the rigidity properties of those elements are judged by the condition of other parts of the structure that are accessible. The range of indeterminacy can be fairly large in this case.

The typical example can be the destroyed structure of the 4th power unit of Chernobyl Nuclear Power Plant covered with "Sarcophagus" (Fig.3) and inaccessible for examination in most parts of it. At the same time, there is a sore need for assessing the residual load-carrying capacity of these mentioned parts.

Thus, the extent of reduction of the flexural rigidity of damaged elements was experimentally found for the structure of the de-aeration stack (Fig.4, a) whose framework has strong local damages at the level mark 23,900 and is tilted by 300...1040 mm [4]. This fact encourages us to use two design models shown on Fig.4, b and 4, c. The first one conforms to virtually undamaged cross-sections, while the second one describes the most sorely damaged case.

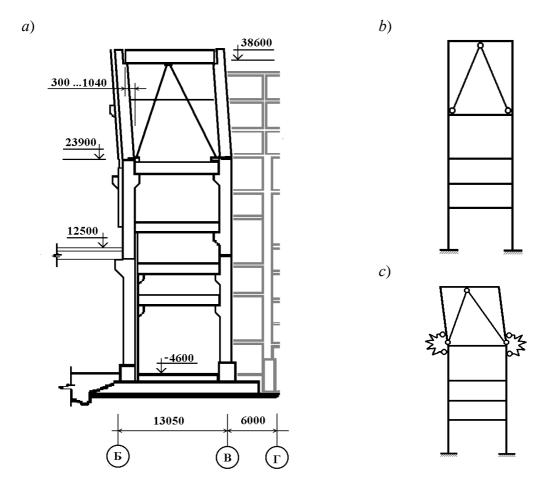


Figure 4: The de-aeration stack framework after the accident.

#### 3.2 Pliability of foundations

The big variability, hence the indeterminacy of values, is characteristic for elasticity properties of natural foundations. This is due to the incompleteness of existent engineering geological data, the approximate nature of the design model of a foundation (Winclaire, elastic layer, half-space etc.). Apparently, the best case is when these properties deviate by  $\pm 15$  ... 30 %, and some situations yield substantially greater deviations. Thus, the U.S. standards for design of nuclear power plants [5] recommend introducing an artificial variation of the shear modulus of soil G, assuming that it can vary from  $G_0(1 + G_v)$  to  $G_0/(1 + G_v)$ , where  $G_0$  is its most probable estimate and  $G_v$  is its variation factor that must not be less than 0.5.

The very instructive example is the analysis of a box-shaped structure located upon an elastic foundation (Fig.5, a) for which the obtained estimate of the compliance factor is within the range from  $C = 500 \text{ t/m}^3$  to  $C = 700 \text{ t/m}^3$ . As the exact distribution of values of C is unknown, several trial choices of this distribution should be built (Fig.5, b, c, d). This procedure is similar to the solution of the problem of a disadvantageous position of a ship on the sea-wave.

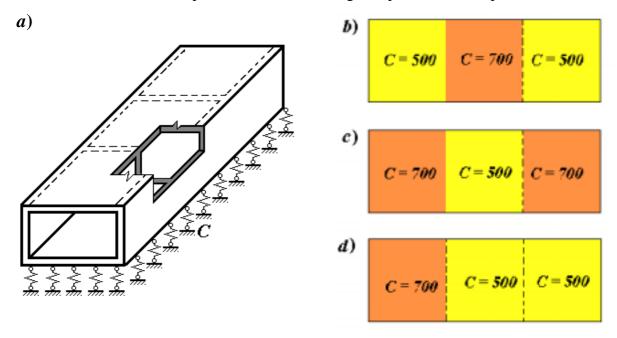


Figure 5. A structure on an elastic foundation

The results of the analysis that has been done are presented on Fig.6 that shows the distributions of stresses in the bottom plate of the box structure for different choices of the elastic foundation's properties, and their most disadvantageous combinations.

The similar analysis procedure arises when one has to take into account the possibility for a sinkhole the location of which under the foundation plate is unknown. Here, specifying the possible size of the sinkhole, one has to consider different choices of its location simulating the sinkhole by specifying the zero stiffness of the elastic foundation.

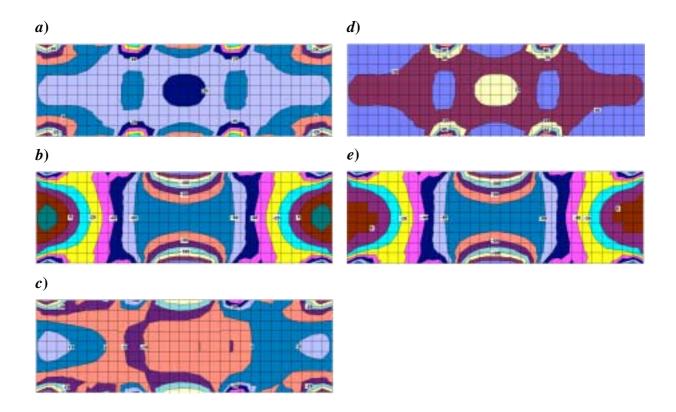


Figure 6: Stresses:

a, b, c — in systems with different choices of the foundation compliance; d,e — the envelop curve

Finally, one has to take into account the different behavior of the foundation soils under long-lasting loads when it is necessary to allow for a certain reduced modulus of the common (elastic-plastic) deformation  $E_0$ , and under short-term (for example, seismic) loads when the elasticity modulus  $E_e$  is taken into account. As a rule,  $E_0/E_e=0.1 \div 0.2$ .

#### 3.3 Nodal joints

When one chooses the design model, idealized conditions of elements' junctions in nodes are usually assumed (absolutely stiff, hinged) though these joints may have certain elastic compliance properties. One can seldom be so successful as to evaluate this compliance, because the experience shows that this evaluation can be practically done only in experiments. Presently, there are few reliable analytical techniques to obtain such evaluations. That's why engineers have to specify the compliance values of nodal joints almost arbitrarily, orienting themselves at the results of experimental studies of "similar" structures. There's a tradition to assume certain structural designs to be absolutely stiff or hinged, and this tradition may well be warrantable in the course of design when the engineer has means to implement his design decisions. But this tradition may prove useless in the conditions of verification of an existent structure when the structural design cannot be adjusted to fit the analytic model.

The expert estimate of the joint type in qualitative terms ("stiff", "compliant" etc.) is fairly typical for the solution of this problem. Table 1 taken from [6] gives the results of expert interrogations with which one can state the relationship between the description of a joint type in words and the numerical property m = R/r where R is the reaction that arises in an isolated joint from a unit deformation, and r is the same reaction under a unit displacement of this node within the structure. As a fuzzy variable is under consideration, the expert estimates are assigned the credibility measure  $\mu$  (the function of belonging to a fuzzy set [7]) whose values are given in Table 1.

Joint type	Values of $m$ , at the $\mu$ factor:					
	1.0	0.8	0.6	0.4	0.2	0.1
Absolutely stiff	1000	100	50	20	10	5
Very stiff	100	50	30	15	10	4
Fairly stiff	30	25	10	5	4	3
Not very stiff	10	5	4	3	2	1
Compliant	3	1	0.5	0.3	0.2	0
Fairly compliant	0.6	0.5	0.3	0.2	0.1	0
Very compliant	0.3	0.25	0.2	0.1	0.0	0
No joint at all	50	0.1	0.1	0.1	0.2	0.3

Table 1: Credibility levels for estimates of stiffness of nodal joints

#### 3.1 History of the structure's erection

The possibility of tracing the erection process and allowing for stresses "closed" in the structure was long implemented in some software products. The typical example of a problem of this type [7] is shown on Fig.7. Using the described mode of SCAD, we have an additional option to trace the changes of stiffness properties of elements through the chain of design models that arise during the erection of the structure. This is particularly relevant for concrete parts of the structure where the elasticity modulus of concrete changes as the strength is acquired.

## 4 On the mathematical theory

Doing the interval assessment of parameters included in the design finite-element model of the system that would yield an interval of values for the coefficients of the stiffness matrix and the right part vector, we encounter the problem of searching for the interval solution of the governing equation system. According to the analysis presented in [8], one should distinguish between three different sets of solutions: combined, boundary and parametric ones. In this paper, we use the boundary solution set, though we believe we should analyze in more detail what results could be obtained via the other approaches.

The search for a solution set is a fairly laborious procedure, and it is usually based on the technique of repetitive solution of the system with different values of a parameter specified on an interval. Considering the high performance and a handy interface of the modern software intended for structural

analysis, one can easily obtain assessments of this type, though varying multiple parameters rather than a single one makes the volume of choices to be exhausted dramatically greater.

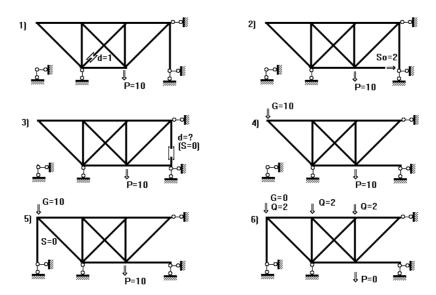


Figure 6: The sequence of assembling

Due to the latter, it is useful to involve methods of planning experiments, in particular, the factor-based planning [9]. The full factor-based experiment should realize all possible combinations of levels of all variable parameters which results in the necessity to run  $2^n$  analyses for a two-level scheme (only the top and the bottom levels are under consideration).

Even for moderate values of n this number can be too great. Due to this, fractional factor plans of the 2n-l type are involved, when one chooses the full factor plan for (n-1) principal parameters that assumes the investigation of all choices of values, and only groups of some indeterminate parameters are to be investigated for each of the other l parameters. In other words, the corresponding group interactions (double, triple etc.) are considered to be essential. The selection of principal parameters and search for their closely related combinations is assumed to base on some qualitative considerations of the a priori type.

# 5 Conclusions

Above, we demonstrated the necessity and possibility for taking into account parameters of design analysis models to properly assess the serviceability of load-carrying structures. Numerical methods used for this type of analysis are not uncommon. The novel technique is rather the very logical scheme of the analysis within which we generalize a well-known problem of choosing a disadvantageous combination of loads applied to the system.

## References

- [1] W. Fränkel. Über die Ungünstigste Einstellung eines Systems von Einzellasten auf Fachwerkträgern mit Hilfe von Influenzkurven. *Ziviling*, 441-459 (1876).
- [2] V.N. Gordeev, V.V. Artemenko, E.I. Minkovich. Choosing the most disadvantageous combination of loads as a solution of the problem of multi-criterial optimization.— In the book: *Numerical methods for analysis and optimization of structural constructions. Proceedings of Kucherenko Central Research Institute of Structural Constructions* (CRISC). Moscow: CRISC, 26-32 (1989). In Russian.
- [3] V.S. Karpilovsky, E.Z. Kryksunov, A.V. Perelmuter, M.A. Perelmuter, O.M. Trophimchuk. *SCAD for users*, Compass Publishing House, Kiev (2000). In Russian.
- [4] Description of the UKRITIYE encasement and requirements for its conversion, "Naukova Dumka" Publishers, Kiev (1992).
- [5] Seismic Analysis of Safety-Related Nuclear Structures and Commentary on Standard for Analysis of Safety-Related Nuclear Structures, *ASCE Standard* (1986).
- [6] V.Y. Malov, Y.M. Pochtman. A fuzzy assessment of properties of nodal joints in model of optimum design, *Izvestiya vuzov. Stroyitelstvo i arkhitektura = Proceedings of universities. Civil engineering and architecture*, No.10, 13–16, (1987) in Russian.
- [7] A.V.Perelmuter, M.A.Mikitarenko, S.G.Burygin. Structural Analysis with Lay-outs Changed During the Action of Load, *Engineering Mechanics*, **3**, No 4, 245-251 (1996).
- [8] Z.Kulpa, K. Roslaniec. Solution sets for systems of linear interval equations, *Proc. of the XIV Polish Conference on Computer Methods in Mechanics*, 26–28 May 1999, Rzeszów, *Poland*, pp. 195–196, Rzeszów, Rzeszów University of Technology, (1999).
- [9] N.I. Johnson and F.C. Leone. *Statistics and Experimental Design in Engineering and the Physical Sciences*, John Wiley & Sons, New York, Second edition, (1977).